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A Hybrid Approach to Micro Vague Topological Space via Neutrosophic Topological Space

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Abstract. Plenty of topologists are exploring and discovering multiple forms of topological spaces. The chief objective of the current enquiry is to establish and evaluate an original hybrid topological space named Neutrosophic Micro Vague Topological Space. When contrasted with distinctive fuzzy sets, Neutrosophic Micro Vague sets give more adaptive framework for dealing with uncertainties and obscurity because they allow more nuanced portrayal of the multitude of the elements of inconsistencies. Some of the basic definitions and operations on Neutrosophic Micro Vague Sets are defined and examined with numerical examples. Furthermore,some of the basic algebraic properties of Neutrosophic Micro Vague Sets are described and investigated with appropriate examples.

Keywords: Neutrosophic Micro Vague set; Neutrosophic Micro Vague Topology; Neutrosophic Micro Vague Topological Space; Absolute Neutrosophic Micro Vague set; Null Neutrosophic Micro Vague Set.

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1. **Introduction**

Fuzzy sets (FS) which were first coined by L.A. Zadeh [1] in 1965 are the most required perspectives in modern mathematics. C.L. Chang [2] pioneered a version of fuzzy topology in 1967. Atanassov [3] recommended the Intuitionistic Fuzzy Set (IFS) in 1986, which has been widely utilised in various fields of mathematics. Around 1993, Gau and Buehrer [4] identified Vague sets (VS) as a further development of FS research and they are considered as a unique instance of context-aware FS. Bustince. H along with Burillo. P [15] demonstrated that VSs are IFSs in 1996. Florentin Smarandache [5], [6] an eminent mathematician and analyst, premiered neutrosophy as a broadening of fuzzy set theory. The referrals "neutrosophy" speaks to the scrutiny of not just truth and falsehood as essential components but also indeterminacy which deals with uncertainty or unpredictability in precisely the same way as fuzzy systems operate. Mathematically a Neutrosophic set (NS) has been assigned over the universal set X and contains three subsets. 1. Truth component: this subset reflects the components that are unquestionably part of the set. 2. Indeterminacy component: this subset represents elements that are unsure whether or not they belong to the set. 3. Falsity component: this subset indicates the elements which are unambiguously not in the set. The Neutrosophic Two Fold Algebra was first presented by F. Smarandache [17] in 2024.

Implementing the concept of neutrosophic frequency and neutrosophic relative frequency distribution, Adebisi S. A. and Broumi S. [16] have examined the educational progress of a group of students in their primary disciplines. Shawkat Alkhazaleh [7] formed the Neutrosophic Vague set (NVS) idea during 2015 as an amalgamation of NS and VS. He defined the basic operations for NVSs such as Union, Intersection, complement and inclusion. These operations are intended to manage the triple membership degrees while retaining the traits of unpredictability and indeterminate nature. NVSs have specific features that set them apart from other set enhancements. In 2024, Smarandache et.al [18] conducted an evaluation of Blockchain Cybersecurity Based on Tree Soft and Opinion Weight Criteria Method under Uncertainty Climate.

M. Lellis Thivagar [9], [13] coined the Nano topology (NT) and Neutrosophic Nano Topology (NNT) in the year 2013 and 2018 respectively. NT is centred on the concepts of lower approximation, higher approximation and boundary region which was brought by Z. Pawlak [8]. S. Chandrasekar [10] constructed Micro Topology (MT) later in 2019 by employing the basic extension idea on NT. MT affords a lens that enables mathematicians and scientists to study and comprehend the multifaceted intricacies of spaces, structures and networks particularly where smaller factors are important. Emergent features at tiny sizes can be shown by MT. MT entails scale relying analysis in which the features of a space are explored at different degrees of detail or resolution. MT is also attributed to more technical topics like sheaf theory and homotopy theory which deal with local patterns and continual deviations respectively. In an environment of metric spaces, MT may entail investigating neighbourhood qualities convergence at a point and other local aspects. In 2023, Vargees Vahini T and Trinita Pricilla M [11] established the novel topological space named Micro Vague Topological Space. Mary Margaret A et.al [12] in 2021 pioneered Neutrosophic Vague Nano Topological Space and have studies some of the basic characteristics.

In this paper, we suggested an innovative topology called the Neutrosophic Micro Vague Topology. Some novel sets in Neutrosophic Micro Vague Topological Space are introduced and discussed. Additionally, using numerical examples certain fundamental definitions, operations and some of the basic algebraic characteristics on Neutrosophic Micro Vague Sets are addressed and explained. Learning about and employing these qualities enables academics and practitioners to capitalise on Neutrosophic Micro Vague sets in several types of domains contributing to more robust and flexible modelling of highly complex and volatile data. It can be useful in an assortment of domains including pure mathematics, mathematical physics and some areas of computer science where fine-grained spatial relationships are noteworthy. The particular application and methodologies used may differ depending on the situation in which Neutrosophic Micro Vague topology is used.

2. Preliminaries

Definition 2.1. [4] A VS $\mathfrak{F}$ in the universe of discourse Λ is characterized by a truth membership function $\vartheta_{\mathfrak{F}}$ and a falsity membership function $\lambda_{\mathfrak{F}}$ as follows: $\vartheta_{\mathfrak{F}} : \Lambda \to [0,1]\,;\ \lambda_{\mathfrak{F}} : \Lambda \to [0,1]$ and $\vartheta_{\mathfrak{F}} + \lambda_{\mathfrak{F}} \leq 1$ where $\vartheta_{\mathfrak{F}}(\grave{m})$ is a lower bound on the grade of membership of $\grave{m}$ derived from the evidence for $\grave{m}$ and $\lambda_{\mathfrak{F}}(\grave{m})$ is a lower bound on the grade of membership of the negation of $\grave{m}$ derived from the evidence against $\grave{m}$. The Vague set $\mathfrak{F}$ is written as $A = \{\langle \grave{m}, \vartheta_{\mathfrak{F}}(\grave{m}), 1-\lambda_{\mathfrak{F}}(\grave{m})\rangle \,|\grave{m} \in \Lambda\}$.

Definition 2.2. [6] Let Λ be a non-empty set and $\mathfrak{I}$ be the unit interval $[0,1]$. A $Neut.Set$ is an object of the form $\mathfrak{D} = \{\langle \grave{m}, \pi_{\mathfrak{D}}(\grave{m}), \phi_{\mathfrak{D}}(\grave{m}), \varphi_{\mathfrak{D}}(\grave{m})\,; \grave{m} \in \Lambda\rangle\}$ where $\pi_{\mathfrak{D}}(\grave{m}), \phi_{\mathfrak{D}}(\grave{m}), \varphi_{\mathfrak{D}}(\grave{m}) \in [0,1]$ with $0 \leq \pi_{\mathfrak{D}}(\grave{m}) + \phi_{\mathfrak{D}}(\grave{m}) + \varphi_{\mathfrak{D}}(\grave{m}) \leq 3\ \ \forall\ \grave{m} \in \Lambda$. Here, $\pi_{\mathfrak{D}}(\grave{m}), \phi_{\mathfrak{D}}(\grave{m})\ and\ \varphi_{\mathfrak{D}}(\grave{m})$ are respectively denote the Degree of truth membership, Degree of indeterminacy membership and Degree of falsity membership.

Definition 2.3. [7] A Neut. Vag. Set $\mathfrak{Z}$ on the universe of discourse Λ is written as $\mathfrak{Z} = \left\{\left\langle \grave{m}; \hat{\pi}_{\mathfrak{Z}}(\grave{m}), \hat{\phi}_{\mathfrak{Z}}(\grave{m}), \hat{\varphi}_{\mathfrak{Z}}(\grave{m})\right\rangle |\grave{m} \in \Lambda\right\}$ whose truth membership, indeterminacy membership and false membership function are defined as follows:

$$\hat{\pi}_{\mathfrak{Z}}(\grave{m}) = \left[\pi^-, \pi^+\right],\ \hat{\phi}_{\mathfrak{Z}}(\grave{m}) = \left[\phi^-, \phi^+\right],\ \ \hat{\varphi}_{\mathfrak{Z}}(\grave{m}) = \left[\varphi^-, \varphi^+\right]$$

Where,

(1) $\pi^+ = 1 - \varphi^-$
(2) $\varphi^+ = 1 - \pi^-$ and
(3) ${}^-0 \leq (\pi^-)^2 + (\phi^-)^2 + (\varphi^-)^2 \leq 2^+$.

Definition 2.4. [11] Assume $(S, \sigma_Y(\mathfrak{Z}))$ a Nano.Vag. topological space. Let $\theta_Y(\mathfrak{Z}) = \{\mathfrak{H} \cup (\mathfrak{H}' \cap \theta)$: $\mathfrak{H}, \mathfrak{H}' \in \sigma_Y(\mathfrak{Z})\}$. Then $\eta_Y(\mathfrak{Z})$ is termed as Mic.Vag. topology (Shortly $\mathcal{MV}$ Topology) of $\sigma_Y(\mathfrak{Z})$ by θ where $\theta \notin \sigma_Y(X)$; Then, $\theta_Y(X)$ fulfills the criteria listed here:

(1) $0_{\mathcal{MV}}, 1_{\mathcal{MV}} \in \theta_Y(\mathfrak{Z})$
(2) Arbitrary union of any sub collection of $\theta_Y(\mathfrak{Z})$ is in $\theta_Y(\mathfrak{Z})$
(3) Finite intersection of sub collection of $\theta_Y(\mathfrak{Z})$ is in $\theta_Y(\mathfrak{Z})$.

The triplet $(S, \sigma_Y(\mathfrak{Z}), \theta_Y(\mathfrak{Z}))$ is called the Micro Vague Topological Space. The elements of $\theta_Y(\mathfrak{Z})$ are called Micro Vague open sets and the complement of a Micro Vague Open set is called a Micro Vague Closed set.

Definition 2.5. [12] Let $\acute{\mathfrak{L}}$ be a non-empty set and $\mathfrak{R}$ be an equivalence relation on $\acute{\mathfrak{L}}$. Let $\mathfrak{S}$ be a Neut.Vag. set in $\acute{\mathfrak{L}}$. If the collection $\vartheta_{\mathfrak{R}}(\mathfrak{S}) = \{0_{NV}, 1_{NV}, \underline{\text{NV}}(\mathfrak{S}), \overline{\text{NV}}(\mathfrak{S}), B_{NV}(\mathfrak{S})\}$ satisfies the following axioms:

(1) $0_{nv}, 1_{nv} \in \vartheta_{\mathfrak{R}}(\mathfrak{S})$.
(2) Arbitrary union of any sub collection of $\vartheta_{\mathfrak{R}}(\mathfrak{S})$ is in $\vartheta_{\mathfrak{R}}(\mathfrak{S})$.
(3) Finite intersection of sub collection of $\vartheta_{\mathfrak{R}}(\mathfrak{S})$ is in $\vartheta_{\mathfrak{R}}(\mathfrak{S})$.

then, $\vartheta_{\mathfrak{R}}(\mathfrak{S})$is called the NVNT and $(\acute{\mathfrak{L}}, \vartheta_{\mathfrak{R}}(\mathfrak{S}))$ is called the NVNTS. The elements of $\vartheta_{\mathfrak{R}}(\mathfrak{S})$ are called NVNOS and the complement of it is called NVNCS.

3. Proposed Neutrosophic Micro Vague Topological Space

Definition 3.1. Let $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}))$ be a $\mathcal{NVNTS}$. Let $\Omega_{\mathcal{Y}}(\mathfrak{S}) = \{\Phi \cup (\Phi' \cap \Omega) : \Omega \notin \Psi_{\mathcal{Y}}(\mathfrak{S})\}$. Then $\Omega_{\mathcal{Y}}(\mathfrak{S})$ is called the Neutrosophic Micro Vague Topology (*shortly* $\mathcal{NMVT}$) of $\vartheta_{\mathfrak{R}}(\mathfrak{S})$ by η if it satisfies the following axioms:

(1) $0_{\mathcal{NMV}}, 1_{\mathcal{NMV}} \in \Omega_{\mathcal{Y}}(\mathfrak{S})$.
(2) The union of the elements of any sub collection of $\Omega_{\mathcal{Y}}(\mathfrak{S})$ is in $\Omega_{\mathcal{Y}}(\mathfrak{S})$.
(3) The intersection of the elements of any finite sub collection of $\Omega_{\mathcal{Y}}(\mathfrak{S})$ is in $\Omega_{\mathcal{Y}}(\mathfrak{S})$.

The triplet $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ is called the *Neutrosophic Micro Vague Topological Space* (*denoted by* $\mathcal{NMVTS}$). The elements of $\Omega_{\mathcal{Y}}(\mathfrak{S})$ are called $\mathcal{NMVOS}$ and the complement is called as $\mathcal{NMVCS}$.

Example 3.2. Let $\acute{\mathfrak{L}} = \{\alpha, \beta, \gamma, \delta\}$ be the Universe. Let $\acute{\mathfrak{L}}/\mathcal{Y} = \{\{\alpha, \delta\}, \{\beta, \gamma\}\}$ be an equivalence relation on $\acute{\mathfrak{L}}$. Let $\mathfrak{S} = \{<\alpha, [0.3, 0.5], [0.2, 0.6], [0.8, 0.9]>, <\beta, [0.6, 0.7], [0.5, 0.7], [0.2, 0.5]>, <\gamma, [0.2, 0.5], [0.9, 0.9], [0.3, 0.4]>, <\delta, [0.6, 0.8], [0.5, 0.9], [0.3, 0.8]>\}$ be a subset of $\acute{\mathfrak{L}}$. Then, $\Psi_{\mathcal{Y}}(\mathfrak{S}) = \{0_{NV}, 1_{NV}, \{<\alpha, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9]>, <\beta, [0.2, 0.5], [0.9, 0.9], [0.3, 0.5]>, <\gamma, [0.2, 0.5], [0.9, 0.9], [0.3, 0.5]>, <\delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9]>\}, \{<\alpha, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>, <\beta, [0.6, 0.7], [0.5, 0.7], [0.2, 0.4]>, <\gamma, [0.6, 0.7], [0.5, 0.7], [0.2, 0.4]>, <\delta, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>\}, \{<\alpha, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>, <\beta, [0.3, 0.5], [0.5, 0.7][0.2, 0.5]>, <\gamma, [0.3, 0.5], [0.5, 0.7], [0.2, 0.5]>, <\delta, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>\}\}$ is a $\mathcal{NVNT}$ on $\acute{\mathfrak{L}}$. Let $\Omega = \{<\alpha, [0.2, 0.7], [0.1, 0.6], [0.8, 0.9]>, <\beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5]>, <\gamma, [0.1, 0.4], [0.9, 0.9], [0.2, 0.5]>, <\delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9]>\}$. Then, $\Omega_{\mathcal{Y}}(\mathfrak{S}) = \{0_{NMV}, 1_{NMV}, \{<\alpha, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9]>, <\beta, [0.2, 0.5], [0.9, 0.9], [0.3, 0.5]>, <\gamma, [0.2, 0.5], [0.9, 0.9], [0.3, 0.5]>, <\delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9]>\}, \{<\alpha, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>,$

$<\beta, [0.6, 0.7], [0.5, 0.7], [0.2, 0.4]>$,$<\gamma, [0.6, 0.7], [0.5, 0.7], [0.2, 0.4]>$,$< \delta, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>\}$,$\{<\alpha, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>$, $<\beta, [0.3, 0.5], [0.5, 0.7], [0.2, 0.5]>$, $< \gamma, [0.3, 0.5], [0.5, 0.7], [0.2, 0.5]>$, $< \delta, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>\}$,$\{< \alpha, [0.2, 0.7], [0.1, 0.6], [0.8, 0.9]>$,$<\beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5]>$,$<\gamma, [0.1, 0.4], [0.9, 0.9], [0.2, 0.5]>$,$<\delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9]>\}$,$\{<\alpha, [0.6, 0.8], [0.1, 0.6], [0.3, 0.8]>$, $<\beta, [0.3, 0.5], [0.5, 0.7], [0.2, 0.5]>$,$<\gamma, [0.3, 0.5], [0.5, 0.7], [0.2, 0.5]>$, $<\delta, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>\}$,$\{< \alpha, [0.2, 0.5], [0.5, 0.9], [0.8, 0.9]>$,$<\beta, [0.2, 0.5], [0.9, 0.9], [0.3, 0.5]>$, $<\gamma, [0.1, 0.4], [0.9, 0.9], [0.3, 0.5]>$, $< \delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9]>\}$,$\{< \alpha, [0.2, 0.7], [0.2, 0.6], [0.8, 0.9]>$, $< \beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5]>$, $< \gamma, [0.1, 0.4], [0.9, 0.9], [0.2, 0.5]>$, $< \delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9]>\}$, $\{< \alpha, [0.3, 0.7], [0.1, 0.6], [0.8, 0.9]>$, $< \beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5]>$, $<\gamma, [0.2, 0.5], [0.9, 0.9], [0.2, 0.5]>$, $<\delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9]>\}$, $\{< \alpha, [0.3, 0.7], [0.2, 0.6], [0.8, 0.9]>$, $< \beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5]>$, $< \gamma, [0.2, 0.5], [0.9, 0.9], [0.2, 0.5]>$,$<\delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9]>\}$,$\{< \alpha, [0.6, 0.8], [0.1, 0.6], [0.3, 0.8]>$,$< \beta, [0.6, 0.7], [0.5, 0.7], [0.2, 0.4]>$, $<\gamma, [0.6, 0.7], [0.5, 0.7], [0.2, 0.4]>$, $<\delta, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>\}\}$ is called the $\mathcal{NMVT}$ on $\mathfrak{S}$. The triplet $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ is called the $\mathcal{NMVTS}$.

Definition 3.3. Let $P_{\mathcal{NMV}}$ and $Q_{\mathcal{NMV}}$ be two $\mathcal{NMV}$ sets in $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$. If $\forall \nu_s \in \acute{\mathfrak{L}}$, $\widehat{\Gamma}_P(\nu_s) \leq \widehat{\Gamma}_Q(\nu_s)$, $\widehat{\Delta}_P(\nu_s) \geq \widehat{\Delta}_Q(\nu_s)$, $\widehat{\Upsilon}_P(\nu_s) \geq \widehat{\Upsilon}_Q(\nu_s)$ then the $\mathcal{NMV}$ set $P_{\mathcal{NMV}}$ is included or contained in the $\mathcal{NMV}$ set $Q_{\mathcal{NMV}}$, denoted by $P_{\mathcal{NMV}} \subseteq Q_{\mathcal{NMV}}$ where $1 \leq s \leq n$.

Remark 3.4. Here, the set $P_{\mathcal{NMV}} = \{ < \nu_s, \widehat{\Gamma}_P(\nu_s), \widehat{\Delta}_P(\nu_s), \widehat{\Upsilon}_P(\nu_s) > \}$ denotes the $\mathcal{NMV}$ set in $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ where $\widehat{\Delta}_P(\nu_s) = \left[\Gamma_P^-(\nu_s), \Gamma_P^+(\nu_s)\right]$, $\widehat{\Delta}_P(\nu_s) = \left[\Delta_P^-(\nu_s), \Delta_P^+(\nu_s)\right]$ *and* $\widehat{\Upsilon}_P(\nu_s) = \left[\Upsilon_P^-(\nu_s), \Upsilon_P^+(\nu_s)\right]$.

Example 3.5. Let us consider the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ defined in ex.3.2. Let $P_{\mathcal{NMV}} = \{ < \alpha, [0.2, 0.5], [0.5, 0.9], [0.8, 0.9] >, < \beta, [0.2, 0.5], [0.9, 0.9], [0.3, 0.5] >, < \gamma, [0.1, 0.4], [0.9, 0.9], [0.3, 0.5] >, < \delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9] >\}$ and $Q_{\mathcal{NMV}} = \{ < \alpha, [0.6, 0.8], [0.1, 0.6], [0.3, 0.8] >, < \beta, [0.6, 0.7], [0.5, 0.7], [0.2, 0.4] >, < \gamma, [0.6, 0.7], [0.5, 0.7], [0.2, 0.4] >, < \delta, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8] >\}$ be two $\mathcal{NMV}$ sets. Here, $P_{\mathcal{NMV}} \subseteq Q_{\mathcal{NMV}}$.

Definition 3.6. Let $P_{\mathcal{NMV}}$ and $Q_{\mathcal{NMV}}$ be two $\mathcal{NMV}$ in $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$. If $\forall \nu_s \in \acute{\mathfrak{L}}$, $\widehat{\Gamma}_P(\nu_s) = \widehat{\Gamma}_Q(\nu_s)$, $\widehat{\Delta}_P(\nu_s) = \widehat{\Delta}_Q(\nu_s)$, $\widehat{\Upsilon}_P(\nu_s) = \widehat{\Upsilon}_Q(\nu_s)$, then the $\mathcal{NMV}$ set $P_{\mathcal{NMV}}$ is equal to the $\mathcal{NMV}$ set $Q_{\mathcal{NMV}}$, denoted by $P_{\mathcal{NMV}} = Q_{\mathcal{NMV}}$ where $1 \leq s \leq n$.

Definition 3.7. The complement of a $\mathcal{NMV}$ set $P_{\mathcal{NMV}}$ in $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ denoted by $P^C_{\mathcal{NMV}}$ is defined as $P^C_{\mathcal{NMV}} = \{< \nu_s, [1 - \Gamma_P^+(\nu_s),\ 1 - \Gamma_P^-(\nu_s)], [1 - \Delta_P^+(\nu_s),\ 1 - \Delta_P^-(\nu_s)], [1 - \Upsilon_P^+(\nu_s),\ 1 - \Upsilon_P^-(\nu_s)] >\}$.

Example 3.8. Let us consider the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ defined in ex.3.2. Let $P_{\mathcal{NMV}} = \{ < \alpha, [0.3, 0.7], [0.1, 0.6], [0.8, 0.9] >, < \beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5] >,$

$< \gamma, [0.2, 0.5]\,, \quad [0.9, 0.9]\,, \quad [0.2, 0.5] >, < \delta, \; [0.3, 0.5]\,, [0.5, 0.9], [0.8, 0.9] >\}$ be a $\mathcal{NMV}$ set. Then the complement of P is as follows:

$$P^{C}_{\mathcal{NMV}} = \{ < \alpha, [0.3, 0.7], \quad [0.4, 0.9], \quad [0.1, 0.2] >, < \beta, \quad [0.5, 0.8]\,, \quad [0.1, 0.3]\,, \quad [0.5, 0.7] >, \\ < \gamma, [0.5, 0.8]\,, \quad [0.1, 0.1]\,, \quad [0.5, 0.8] >, < \delta, \; [0.5, 0.7]\,, [0.1, 0.5], [0.1, 0.2] >\}.$$

Definition 3.9. Let $P_{\mathcal{NMV}}$ be a $\mathcal{NMV}$ set in $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$. If $\forall \nu_s \in \acute{\mathfrak{L}}$, $\widehat{\Gamma}_P(\nu_s) = [1, 1]$, $\widehat{\Delta}_P(\nu_s) = [0, 0]$ and $\widehat{\Upsilon}_P(\nu_s) = [0, 0]$, then $P_{\mathcal{NMV}}$ is called Absolute $\mathcal{NMV}$ set where $1 \leq s \leq n$.

Definition 3.10. Let $P_{\mathcal{NMV}}$ be a $\mathcal{NMV}$ set in $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$. If $\forall \nu_s \in \acute{\mathfrak{L}}$, $\widehat{\Gamma}_P(\nu_s) = [0, 0]$, $\widehat{\Delta}_P(\nu_s) = [1, 1]$ and $\widehat{\Upsilon}_P(\nu_s) = [1, 1]$, then $P_{\mathcal{NMV}}$ is called Null $\mathcal{NMV}$ set where $1 \leq s \leq n$.

Definition 3.11. The Union of two $\mathcal{NMV}$ sets $P_{\mathcal{NMV}}$ and $Q_{\mathcal{NMV}}$ is a $\mathcal{NMV}$ set R which is written as $R = P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}$ whose $\widehat{\Gamma}_R(\nu_s), \widehat{\Delta}_R(\nu_s), \widehat{\Upsilon}_R(\nu_s)$ are defined $\forall\ \nu_s \in \acute{\mathfrak{L}}$ where $1 \leq s \leq n$ as follows:

$$\widehat{\Gamma}_R(\nu_s) = \left[\bigvee\left(\Gamma^-_P(\nu_s),\ \Gamma^-_Q(\nu_s)\right),\ \bigvee\left(\Gamma^+_P(\nu_s),\ \Gamma^+_Q(\nu_s)\right)\right]$$
$$\widehat{\Delta}_R(\nu_s) = \left[\bigwedge\left(\Delta^-_P(\nu_s),\ \Delta^-_Q(\nu_s)\right),\ \bigwedge\left(\Delta^+_P(\nu_s),\ \Delta^+_Q(\nu_s)\right)\right]$$
$$\widehat{\Upsilon}_R(\nu_s) = \left[\bigwedge\left(\Upsilon^-_P(\nu_s),\ \Upsilon^-_Q(\nu_s)\right),\ \bigwedge\left(\Upsilon^+_P(\nu_s),\ \Upsilon^+_Q(\nu_s)\right)\right]$$

Example 3.12. Let us consider the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ defined in ex.3.2. Let $P_{\mathcal{NMV}} = \{ < \alpha, [0.6, 0.8], \quad [0.2, 0.6], \quad [0.3, 0.8] >, < \beta, \quad [0.6, 0.7]\,, \quad [0.5, 0.7]\,, \quad [0.2, 0.4] >, < \gamma, [0.6, 0.7]\,, \quad [0.5, 0.7]\,, \quad [0.2, 0.4] >, < \delta, \; [0.6, 0.8]\,, [0.2, 0.6], [0.3, 0.8] >\}$ and $Q_{\mathcal{NMV}} = \{ < \alpha, [0.6, 0.8], \quad [0.1, 0.6], \quad [0.3, 0.8] >, < \beta, \quad [0.3, 0.5]\,, \quad [0.5, 0.7]\,, \quad [0.2, 0.5] >, < \gamma, [0.3, 0.5]\,, \quad [0.5, 0.7]\,, \quad [0.2, 0.5] >, < \delta, \; [0.6, 0.8]\,, [0.2, 0.6], [0.3, 0.8] >\}$ be two $\mathcal{NMV}$ sets. Then the union $R_{\mathcal{NMV}} = P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}$ is given as follows:

$$R_{\mathcal{NMV}} = \{ < \alpha, [0.6, 0.8], \quad [0.1, 0.6], \quad [0.3, 0.8] >, < \beta, \quad [0.6, 0.7]\,, \quad [0.5, 0.7]\,, \quad [0.2, 0.4] >, \\ < \gamma, [0.6, 0.7]\,, \quad [0.5, 0.7]\,, \quad [0.2, 0.4] >, < \delta, \; [0.6, 0.8]\,, [0.2, 0.6], [0.3, 0.8] >\}.$$

Definition 3.13. The Intersection of two $\mathcal{NMV}$ sets $P_{\mathcal{NMV}}$ and $Q_{\mathcal{NMV}}$ is a $\mathcal{NMV}$ set $S_{\mathcal{NMV}}$ which is written as $S_{\mathcal{NMV}} = P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}$ whose $\widehat{\Gamma}_S(\nu_s), \widehat{\Delta}_S(\nu_s), \widehat{\Upsilon}_S(\nu_s)$ are defined $\forall\ \nu_s \in \acute{\mathfrak{L}}$ where $1 \leq s \leq n$ as follows:

$$\widehat{\Gamma}_S(\nu_s) = \left[\bigwedge\left(\Gamma^-_P(\nu_s),\ \Gamma^-_Q(\nu_s)\right),\ \bigwedge\left(\Gamma^+_P(\nu_s),\ \Gamma^+_Q(\nu_s)\right)\right]$$
$$\widehat{\Delta}_S(\nu_s) = \left[\bigvee\left(\Delta^-_P(\nu_s),\ \Delta^-_Q(\nu_s)\right),\ \bigvee\left(\Delta^+_P(\nu_s),\ \Delta^+_Q(\nu_s)\right)\right]$$
$$\widehat{\Upsilon}_S(\nu_s) = \left[\bigvee\left(\Upsilon^-_P(\nu_s),\ \Upsilon^-_Q(\nu_s)\right),\ \bigvee\left(\Upsilon^+_P(\nu_s),\ \Upsilon^+_Q(\nu_s)\right)\right]$$

Example 3.14. Let us consider the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ defined in ex.3.2. Let $P_{\mathcal{NMV}} = \{<\alpha, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>, <\beta, [0.6, 0.7], [0.5, 0.7], [0.2, 0.4]>, <\gamma, [0.6, 0.7], [0.5, 0.7], [0.2, 0.4]>, <\delta, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>\}$ and $Q_{\mathcal{NMV}} = \{<\alpha, [0.6, 0.8], [0.1, 0.6], [0.3, 0.8]>, <\beta, [0.3, 0.5], [0.5, 0.7], [0.2, 0.5]>, <\gamma, [0.3, 0.5], [0.5, 0.7], [0.2, 0.5]>, <\delta, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>\}$ be two $\mathcal{NMV}$ sets. Then the intersection $S_{\mathcal{NMV}} = P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}$ is given as follows:

$$S_{\mathcal{NMV}} = \{<\alpha, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>, <\beta, [0.3, 0.5], [0.5, 0.7], [0.2, 0.5]>, <\gamma, [0.3, 0.5], [0.5, 0.7], [0.2, 0.5]>, <\delta, [0.6, 0.8], [0.2, 0.6], [0.3, 0.8]>\}.$$

Definition 3.15. Let $\{P_{s_{\mathcal{NMV}}} : s \in D\}$ where $D = \{1, 2, \ldots, n\}$ be an arbitrary family of $\mathcal{NMV}$ sets. Then

(1) $\cup P_{s_{\mathcal{NMV}}} =$

$$\left\{\left\langle \begin{array}{c} \grave{m}; \left[\bigvee_{s\in D}\left(\Gamma^{-}_{P_{s_{\mathcal{NMV}}}}\right), \bigvee_{s\in D}\left(\Gamma^{+}_{P_{s_{\mathcal{NMV}}}}\right)\right], \\ \left[\bigwedge_{s\in D}\left(\Delta^{-}_{P_{s_{\mathcal{NMV}}}}\right), \bigwedge_{s\in D}\left(\Delta^{+}_{P_{s_{\mathcal{NMV}}}}\right)\right], \left[\bigwedge_{s\in D}\left(\Upsilon^{-}_{P_{s_{\mathcal{NMV}}}}\right), \bigwedge_{s\in D}\left(\Upsilon^{+}_{P_{s_{\mathcal{NMV}}}}\right)\right] \end{array} \right\rangle\right\}$$

(2) $\cap P_{s_{\mathcal{NMV}}} =$

$$\left\{\left\langle \grave{m}; \begin{array}{c} \left[\bigwedge_{s\in D}\left(\Gamma^{-}_{P_{s_{\mathcal{NMV}}}}\right), \bigwedge_{s\in D}\left(\Gamma^{+}_{P_{s_{\mathcal{NMV}}}}\right)\right], \\ \left[\bigvee_{s\in D}\left(\Delta^{-}_{P_{s_{\mathcal{NMV}}}}\right), \bigvee_{s\in D}\left(\Delta^{+}_{P_{s_{\mathcal{NMV}}}}\right)\right], \left[\bigvee_{s\in D}\left(\Upsilon^{-}_{P_{s_{\mathcal{NMV}}}}\right), \bigvee_{s\in D}\left(\Upsilon^{+}_{P_{s_{\mathcal{NMV}}}}\right)\right] \end{array} \right\rangle\right\}$$

Proposition 3.16. *Let $P_{\mathcal{NMV}}$, $Q_{\mathcal{NMV}}$, $R_{\mathcal{NMV}}$ and $S_{\mathcal{NMV}}$ be $\mathcal{NMV}$ sets in $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$.*

(1) *If $P_{\mathcal{NMV}} \subseteq Q_{\mathcal{NMV}}$ and $R_{\mathcal{NMV}} \subseteq S_{\mathcal{NMV}}$, then*
 (a) $(P_{\mathcal{NMV}} \cup R_{\mathcal{NMV}}) \subseteq (Q_{\mathcal{NMV}} \cup S_{\mathcal{NMV}})$
 (b) $(P_{\mathcal{NMV}} \cap R_{\mathcal{NMV}}) \subseteq (Q_{\mathcal{NMV}} \cap S_{\mathcal{NMV}})$

(2) *If $P_{\mathcal{NMV}} \subseteq Q_{\mathcal{NMV}}$ and $P_{\mathcal{NMV}} \subseteq R_{\mathcal{NMV}}$, then $P_{\mathcal{NMV}} \subseteq (Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}})$*

(3) *If $P_{\mathcal{NMV}} \subseteq R_{\mathcal{NMV}}$ and $Q_{\mathcal{NMV}} \subseteq R_{\mathcal{NMV}}$, then $(P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}) \subseteq R_{\mathcal{NMV}}$*

(4) *If $P_{\mathcal{NMV}} \subseteq Q_{\mathcal{NMV}}$ and $Q_{\mathcal{NMV}} \subseteq R_{\mathcal{NMV}}$, then $P_{\mathcal{NMV}} \subseteq R_{\mathcal{NMV}}$*

(5) *If $P_{\mathcal{NMV}} \subseteq Q_{\mathcal{NMV}}$, then $\overline{Q_{\mathcal{NMV}}} \subseteq \overline{P_{\mathcal{NMV}}}$*

(6) $\overline{1}_{\mathcal{NMV}} = 0_{\mathcal{NMV}}$

(7) $\overline{0}_{\mathcal{NMV}} = 1_{\mathcal{NMV}}$

Proof. Proof is obvious. □

Corollary 3.17. *Let $P_{s\mathcal{NMV}}(s \in D)$ and $Q_{\mathcal{NMV}}$ be $\mathcal{NMV}$ sets in $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$. Then*

(1) $P_{s\mathcal{NMV}} \subseteq Q_{\mathcal{NMV}}$ *for each* $s \in D$ *implies that* $\cup(P_{s\mathcal{NMV}}) \subseteq Q_{\mathcal{NMV}}$

(2) $Q_{\mathcal{NMV}} \subseteq P_{s\mathcal{NMV}}$ *for each* $s \in D$ *implies that* $Q_{\mathcal{NMV}} \subseteq \cap(P_{s\mathcal{NMV}})$

Proof. Proof is obvious. □

Remark 3.18. (1) In $\mathcal{NMVTS}$, the boundary region cannot be empty.

(2) Let $\{ \eta_i | i \in \mathrm{l}\}$ be the family of $\mathcal{NMVT}s$ on X_i, then $\bigcap_{i\in \mathrm{l}} \eta_i$ is a $\mathcal{NMVT}$ in X.

(3) Let $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ and $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), {\eta'}_{\mathfrak{R}}(\mathfrak{S}))$ be two $\mathcal{NMVTS}$s over X. Then $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S})) \cup {\eta'}_{\mathfrak{R}}(\mathfrak{S}))$ need not to be a $\mathcal{NMVTS}$.

4. Properties of Neutrosophic Micro Vague Sets

Theorem 4.1. ***(Idempotent law)*** *For any non-empty* $\mathcal{NMV}$ *set* $P_{\mathcal{NMV}}$ *in* $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$,

(1) $P_{\mathcal{NMV}} \cup P_{\mathcal{NMV}} = P_{\mathcal{NMV}}$.

(2) $P_{\mathcal{NMV}} \cap P_{\mathcal{NMV}} = P_{\mathcal{NMV}}$.

Proof. The proof is obvious. □

Example 4.2. (1). Let us consider the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ defined in ex.3.2. Let $P_{\mathcal{NMV}} = \{ < \alpha, [0.2, 0.7], [0.2, 0.6], [0.8, 0.9] >, < \beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5] >, < \gamma, [0.1, 0.4], [0.9, 0.9], [0.2, 0.5] >, < \delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9] >\}$ be a $\mathcal{NMV}$ set. Then, $P_{\mathcal{NMV}} \cup P_{\mathcal{NMV}} = \{ < \alpha, [0.2, 0.7], [0.2, 0.6], [0.8, 0.9] >, < \beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5] >, < \gamma, [0.1, 0.4], [0.9, 0.9], [0.2, 0.5] >, < \delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9] >\}$

(2). Similar to (1).

Theorem 4.3. ***(Identity law)*** *For any non-empty* $\mathcal{NMV}$ *sets* $P_{\mathcal{NMV}}$ *and* $Q_{\mathcal{NMV}}$ *in* $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$*, identity law holds:*

(1) $P_{\mathcal{NMV}} \cup 0_{\mathcal{NMV}} = P_{\mathcal{NMV}}$.

(2) $Q_{\mathcal{NMV}} \cap 1_{\mathcal{NMV}} = Q_{\mathcal{NMV}}$.

Proof. **(1).** Let $P_{\mathcal{NMV}}$ be a $\mathcal{NMV}$ set in the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ of the form $P_{\mathcal{NMV}} = \{ < \nu_s, \left[\Gamma^-_P(\nu_s), \Gamma^+_P(\nu_s)\right], \left[\Delta^-_P(\nu_s), \Delta^+_P(\nu_s)\right], \left[\Upsilon^-_P(\nu_s), \Upsilon^+_P(\nu_s)\right] > \}$. Let the null $\mathcal{NMV}$ set be of the form $0_{\mathcal{NMV}} = \{< \nu_s, [0, 0], [1, 1], [1, 1] >\}$. Then,

$$P_{\mathcal{NMV}} \cup 0_{\mathcal{NMV}} = \left\{ \left\langle \nu_s,\ \begin{matrix} \left[\bigvee\left(\Gamma_P^-(\nu_s),\ 0\right),\ \bigvee\left(\Gamma_P^+(\nu_s),\ 0\right)\right], \\ \left[\bigwedge\left(\Delta_P^-(\nu_s),\ 1\right),\ \bigwedge\left(\Delta_P^+(\nu_s),\ 1\right)\right], \\ \left[\bigwedge\left(\Upsilon_P^-(\nu_s),1\right),\ \bigwedge\left(\Upsilon_P^+(\nu_s),1\right)\right] \end{matrix} \right\rangle \right\}$$

$= P_{\mathcal{NMV}}$.

Therefore, $P_{\mathcal{NMV}} \cup 0_{\mathcal{NMV}} = P_{\mathcal{NMV}}$.

(2). Let $Q_{\mathcal{NMV}}$ be a $\mathcal{NMV}$ set in the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ of the form $Q_{\mathcal{NMV}} = \{ < \nu_s, [\Gamma_Q^-(\nu_s), \Gamma_Q^+(\nu_s)], [\Delta_Q^-(\nu_s), \Delta_Q^+(\nu_s)], [\Upsilon_Q^-(\nu_s), \Upsilon_Q^+(\nu_s)]> \}$. Let the absolute $\mathcal{NMV}$ set be of the form $1_{\mathcal{NMV}} = \{< \nu_s, [1,1], [0,0], [0,0] >\}$. Then,

$$Q_{\mathcal{NMV}} \cap 1_{\mathcal{NMV}} = \left\{ \left\langle \nu_s,\ \begin{matrix} \left[\bigwedge\left(\Gamma_Q^-(\nu_s),1\right),\ \bigwedge\left(\Gamma_Q^+(\nu_s),\ 1\right)\right], \\ \left[\bigvee\left(\Delta_Q^-(\nu_s),0\right),\ \bigvee\left(\Delta_Q^+(\nu_s),0\right)\right], \\ \left[\bigvee\left(\Upsilon_Q^-(\nu_s),0\right),\ \bigvee\left(\Upsilon_Q^+(\nu_s),0\right)\right] \end{matrix} \right\rangle \right\}$$

$= Q_{\mathcal{NMV}}$.

Therefore, $Q_{\mathcal{NMV}} \cap 1_{\mathcal{NMV}} = Q_{\mathcal{NMV}}$. □

Example 4.4. (1). Let us consider the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ defined in ex.3.2. Let $P_{\mathcal{NMV}} = \{ < \alpha, [0.2, 0.7], [0.2, 0.6], [0.8, 0.9] >, < \beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5] >, < \gamma, [0.1, 0.4], [0.9, 0.9], [0.2, 0.5] >, < \delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9] >\}$ be a $\mathcal{NMV}$ set. Then, $P_{\mathcal{NMV}} \cup 0_{\mathcal{NMV}} = \{ < \alpha, [0.2, 0.7], [0.2, 0.6], [0.8, 0.9] >, < \beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5] >, < \gamma, [0.1, 0.4], [0.9, 0.9], [0.2, 0.5] >, < \delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9] >\}$

(2). Similar to (1).

Theorem 4.5. ***(Dominance law)*** *Let $P_{\mathcal{NMV}}$ and $Q_{\mathcal{NMV}}$ be $\mathcal{NMV}$ subsets of the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$. Then for the null set and the absolute set the following conditions holds:*

(1) $P_{\mathcal{NMV}} \cap 0_{\mathcal{NMV}} = 0_{\mathcal{NMV}}$.
(2) $Q_{\mathcal{NMV}} \cup 1_{\mathcal{NMV}} = 1_{\mathcal{NMV}}$.

Proof. **(1).** Let $P_{\mathcal{NMV}}$ be a $\mathcal{NMV}$ set in the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ of the form $P_{\mathcal{NMV}} = \{ < \nu_s, \left[\Gamma_P^-(\nu_s), \Gamma_P^+(\nu_s)\right], \left[\Delta_P^-(\nu_s), \Delta_P^+(\nu_s)\right], \left[\Upsilon_P^-(\nu_s), \Upsilon_P^+(\nu_s)\right] > \}$. Let the empty $\mathcal{NMV}$ set be of the form $0_{\mathcal{NMV}} = \{< \nu_s, [0,0], [1,1], [1,1] >\}$. Then,

$$P_{\mathcal{NMV}} \cap 0_{\mathcal{NMV}} = \left\{ \left\langle \nu_s, \begin{matrix} \left[\bigwedge \left(\Gamma_P^-(\nu_s), 0\right), \ \bigwedge \left(\Gamma_P^+(\nu_s), \ 0\right)\right], \\ \left[\bigvee \left(\ \Delta_P^-(\nu_s), 1\right), \ \bigvee \left(\Delta_P^+(\nu_s), 1\right)\right], \\ \left[\bigvee \left(\ \Upsilon_P^-(\nu_s), 1\right), \ \bigvee \left(\Upsilon_P^+(\nu_s), 1\right)\right] \end{matrix} \right\rangle \right\}$$

$= 0_{\mathcal{NMV}}$.

Therefore, $P_{\mathcal{NMV}} \cap 0_{\mathcal{NMV}} = 0_{\mathcal{NMV}}$.

(2). Let $Q_{\mathcal{NMV}}$ be a $\mathcal{NMV}$ set in the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ of the form $Q_{\mathcal{NMV}} = \{ < \nu_s, [\Gamma_Q^-(\nu_s), \Gamma_Q^+(\nu_s)], [\Delta_Q^-(\nu_s), \Delta_Q^+(\nu_s)], \ [\Upsilon_Q^-(\nu_s), \Upsilon_Q^+(\nu_s)] > \}$. Let the absolute $\mathcal{NMV}$ set be of the form $1_{\mathcal{NMV}} = \{< \nu_s, [1, 1], [0, 0], [0, 0] >\}$. Then,

$$Q_{\mathcal{NMV}} \cup 1_{\mathcal{NMV}} = \left\{ \left\langle \nu_s, \begin{matrix} \left[\bigvee \left(\Gamma_Q^-(\nu_s), \ 1\right), \ \bigvee \left(\Gamma_Q^+(\nu_s), \ 1\right)\right], \\ \left[\bigwedge \left(\Delta_Q^-(\nu_s), \ 0\right), \ \bigwedge \left(\Delta_Q^+(\nu_s), \ 0\right)\right], \\ \left[\bigwedge \left(\Upsilon_Q^-(\nu_s), 0\right), \ \bigwedge \left(\Upsilon_Q^+(\nu_s), 0\right)\right] \end{matrix} \right\rangle \right\}$$

$= 1_{\mathcal{NMV}}$.

Hence $Q_{\mathcal{NMV}} \cup 1_{\mathcal{NMV}} = 1_{\mathcal{NMV}}$. □

Example 4.6. (1). Let us consider the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ defined in ex.3.2. Let $P_{\mathcal{NMV}} = \{ < \alpha, [0.2, 0.7], [0.2, 0.6], [0.8, 0.9] >, < \beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5] >, < \gamma, [0.1, 0.4], [0.9, 0.9], [0.2, 0.5] >, < \delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9] >\}$ be a $\mathcal{NMV}$ set and $0_{\mathcal{NMV}} = \{< \alpha, [0, 0], [1, 1], [1, 1] >, < \beta, [0, 0], [1, 1], [1, 1] >, < \gamma, [0, 0], [1, 1], [1, 1] >, < \delta, [0, 0], [1, 1], [1, 1] >\}$. Then, $P_{\mathcal{NMV}} \cap 0_{\mathcal{NMV}} = 0_{\mathcal{NMV}}$

(2). Similar to (1).

Theorem 4.7. ***(Double Complement law)*** *For any $\mathcal{NMV}$ subset $P_{\mathcal{NMV}}$ in the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$, $\left(\mathcal{P}^C_{\mathcal{NMV}}\right)^C = P_{\mathcal{NMV}}$.*

Proof. Let $P_{\mathcal{NMV}} = \left\{\left\langle \nu_s, \left[\Gamma_P^-(\nu_s), \Gamma_P^+(\nu_s)\right], \left[\Delta_P^-(\nu_s), \Delta_P^+(\nu_s)\right], \left[\Upsilon_P^-(\nu_s), \Upsilon_P^+(\nu_s)\right]\right\rangle\right\}$ be a $\mathcal{NMV}$ subset in a $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$. Then,

$$\mathcal{P}^C_{\mathcal{NMV}} = \left\{ \left\langle \nu_s, \begin{matrix} \left[1 - \Gamma_P^+(\nu_s), 1 - \Gamma_P^-(\nu_s)\right], \left[1 - \Delta_P^+(\nu_s), 1 - \Delta_P^-(\nu_s)\right], \\ \left[1 - \Upsilon_P^+(\nu_s), 1 - \Upsilon_P^-(\nu_s)\right] \end{matrix} \right\rangle \right\}$$

Now,

$$\left(\mathcal{P}^{C}_{\mathcal{NMV}}\right)^{C} = \left\{ \left\langle \nu_s, \begin{array}{c} \left[1-\left(1-\Gamma^{+}_{P}(\nu_s)\right), 1-\left(1-\Gamma^{-}_{P}(\nu_s)\right)\right], \\ \left[1-\left(1-\Delta^{+}_{P}(\nu_s)\right), 1-\left(1-\Delta^{-}_{P}(\nu_s)\right)\right], \\ \left[1-\left(1-\Upsilon^{+}_{P}(\nu_s)\right), 1-\left(1-\Upsilon^{-}_{P}(\nu_s)\right)\right] \end{array} \right\rangle \right\}$$

$$= \left\{\left\langle \nu_s, \left[\Gamma^{-}_{P}(\nu_s), \Gamma^{+}_{P}(\nu_s)\right], \left[\Delta^{-}_{P}(\nu_s), \Delta^{+}_{P}(\nu_s)\right], \left[\Upsilon^{-}_{P}(\nu_s), \Upsilon^{+}_{P}(\nu_s)\right]\right\rangle\right\}$$

Therefore, $\left(\mathcal{P}^{C}_{\mathcal{NMV}}\right)^{C} = P_{\mathcal{NMV}}$. □

Example 4.8. (1). Let us consider the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ defined in ex.3.2. Let $P_{\mathcal{NMV}} = \{ <\alpha, [0.2, 0.7], [0.2, 0.6], [0.8, 0.9]>, <\beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5]>, <\gamma, [0.1, 0.4], [0.9, 0.9], [0.2, 0.5]>, <\delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9]>\}$ be a $\mathcal{NMV}$ set. Then,

$$\mathcal{P}^{C}_{\mathcal{NMV}} = \{ <\alpha, [0.3, 0.8], [0.4, 0.8], [0.1, 0.2]>, <\beta, [0.5, 0.8], [0.1, 0.3], [0.5, 0.7]>, < \gamma, [0.6, 0.9], [0.1, 0.1], [0.5, 0.8]>, <\delta, [0.5, 0.7], [0.1, 0.5], [0.1, 0.2]>\}$$

$$\left(\mathcal{P}^{C}_{\mathcal{NMV}}\right)^{C} = \{ <\alpha, [0.2, 0.7], [0.2, 0.6], [0.8, 0.9]>, <\beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5]>, <\gamma, [0.1, 0.4], [0.9, 0.9], [0.2, 0.5]>, <\delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9]>\} = \mathcal{P}_{\mathcal{NMV}}$$

Theorem 4.9. ***(Absorption law)*** *For any two $\mathcal{NMV}$ subsets $P_{\mathcal{NMV}}$ and $R_{\mathcal{NMV}}$ in the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$,*

(1) $Q_{\mathcal{NMV}} \cup (Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}}) = Q_{\mathcal{NMV}}$
(2) $Q_{\mathcal{NMV}} \cap (Q_{\mathcal{NMV}} \cup R_{\mathcal{NMV}}) = Q_{\mathcal{NMV}}$

Proof. **(1).** Let $Q_{\mathcal{NMV}} = \{ <\nu_s, [\Gamma^{-}_{Q}(\nu_s), \Gamma^{+}_{Q}(\nu_s)], [\Delta^{-}_{Q}(\nu_s), \Delta^{+}_{Q}(\nu_s)], [\Upsilon^{-}_{Q}(\nu_s), \Upsilon^{+}_{Q}(\nu_s)]>\}$ and $R_{\mathcal{NMV}} = \{ <\nu_s, [\Gamma^{-}_{R}(\nu_s), \Gamma^{+}_{R}(\nu_s)], [\Delta^{-}_{R}(\nu_s), \Delta^{+}_{R}(\nu_s)], [\Upsilon^{-}_{R}(\nu_s), \Upsilon^{+}_{R}(\nu_s)]>\}$ be the subsets of the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$. Then,

$$Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}} = \left\{ \left\langle \nu_s, \begin{array}{c} \left[\left(\Gamma^{-}_{Q}(\nu_s) \wedge \Gamma^{-}_{R}(\nu_s)\right), \left(\Gamma^{+}_{Q}(\nu_s) \wedge \Gamma^{+}_{R}(\nu_s)\right)\right], \\ \left[\left(\Delta^{-}_{Q}(\nu_s) \wedge \Delta^{-}_{R}(\nu_s)\right), \left(\Delta^{+}_{Q}(\nu_s) \wedge \Delta^{+}_{R}(\nu_s)\right)\right], \\ \left[\left(\Upsilon^{-}_{Q}(\nu_s) \wedge \Upsilon^{-}_{R}(\nu_s)\right), \left(\Upsilon^{+}_{Q}(\nu_s) \wedge \Upsilon^{+}_{R}(\nu_s)\right)\right] \end{array} \right\rangle \right\}$$

Case (i): If $Q_{\mathcal{NMV}} \subseteq R_{\mathcal{NMV}}$, then,

$$Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}} = \left\{\left\langle \nu_s, \left[\Gamma^{-}_{Q}(\nu_s), \Gamma^{+}_{Q}(\nu_s)\right], \left[\Delta^{-}_{Q}(\nu_s), \Delta^{+}_{Q}(\nu_s)\right], \left[\Upsilon^{-}_{Q}(\nu_s), \Upsilon^{+}_{Q}(\nu_s)\right] \right\rangle\right\}$$

$Q_{\mathcal{NMV}} \cup (Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}})$

$$= \left\{ \left\langle \ \nu_s, \left[\Gamma_Q^- (\nu_s), \Gamma_Q^+ (\nu_s) \right], \left[\Delta_Q^- (\nu_s), \Delta_Q^+ (\nu_s) \right], \left[\Upsilon_Q^- (\nu_s), \Upsilon_Q^+ (\nu_s) \right] \ \right\rangle \right\} \cup$$
$$\left\{ \left\langle \ \nu_s, \left[\Gamma_Q^- (\nu_s), \Gamma_Q^+ (\nu_s) \right], \left[\Delta_Q^- (\nu_s), \Delta_Q^+ (\nu_s) \right], \left[\Upsilon_Q^- (\nu_s), \Upsilon_Q^+ (\nu_s) \right] \ \right\rangle \right\}$$
$$= Q_{\mathcal{NMV}}.$$

Case (ii): If $R_{\mathcal{NMV}} \subseteq Q_{\mathcal{NMV}}$, then,
$Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}} = \left\{ \left\langle \nu_s, \left[\Gamma_R^- (\nu_s), \Gamma_R^+ (\nu_s) \right], \left[\Delta_R^- (\nu_s), \Delta_R^+ (\nu_s) \right], \left[\Upsilon_R^- (\nu_s), \Upsilon_R^+ (\nu_s) \right] \right\rangle \right\}$

$$Q_{\mathcal{NMV}} \cup (Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}})$$

$$= \left\{ \left\langle \ \nu_s, \left[\Gamma_Q^- (\nu_s), \Gamma_Q^+ (\nu_s) \right], \left[\Delta_Q^- (\nu_s), \Delta_Q^+ (\nu_s) \right], \left[\Upsilon_Q^- (\nu_s), \Upsilon_Q^+ (\nu_s) \right] \ \right\rangle \right\} \cup$$
$$\left\{ \left\langle \ \nu_s, \left[\Gamma_R^- (\nu_s), \Gamma_R^+ (\nu_s) \right], \left[\Delta_R^- (\nu_s), \Delta_R^+ (\nu_s) \right], \left[\Upsilon_R^- (\nu_s), \Upsilon_R^+ (\nu_s) \right] \ \right\rangle \right\}$$

$$= Q_{\mathcal{NMV}}.$$

Hence $Q_{\mathcal{NMV}} \cup (Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}}) = Q_{\mathcal{NMV}}$.

(2). Proof of (2) is similar to (1). □

Example 4.10. (1). Let us consider the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ defined in ex.3.2. Let $Q_{\mathcal{NMV}} = \{ < \alpha, [0.2, 0.7], [0.2, 0.6], [0.8, 0.9] >, < \beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5] >, < \gamma, [0.1, 0.4], [0.9, 0.9], [0.2, 0.5] >, < \delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9] >\}$ and $R_{\mathcal{NMV}} = \{< \alpha, [0, 0], [1, 1], [1, 1] >, < \beta, [0, 0], [1, 1], [1, 1] >, < \gamma, [0, 0], [1, 1], [1, 1] >, < \delta, [0, 0], [1, 1], [1, 1] > \}$ be $\mathcal{NMV}$ sets. Then,

$Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}}$
$$= \{ < \alpha, [0.2, 0.7], [0.2, 0.6], [0.8, 0.9] >, < \beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5] >,$$
$$< \gamma, [0.1, 0.4], [0.9, 0.9], [0.2, 0.5] >, < \delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9] >\}.$$

$Q_{\mathcal{NMV}} \cup (Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}})$
$$= \{ < \alpha, [0.2, 0.7], [0.2, 0.6], [0.8, 0.9] >, < \beta, [0.2, 0.5], [0.7, 0.9], [0.3, 0.5] >,$$
$$< \gamma, [0.1, 0.4], [0.9, 0.9], [0.2, 0.5] >, < \delta, [0.3, 0.5], [0.5, 0.9], [0.8, 0.9] >\} = Q_{\mathcal{NMV}}$$

(2). Similar to (1).

Theorem 4.11. ***(De-Morgan law)*** *Let $Q_{\mathcal{NMV}}$ and $R_{\mathcal{NMV}}$ be any two subsets of $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$. Then the following statements hold true.*

(1) $(Q_{\mathcal{NMV}} \cup R_{\mathcal{NMV}})^C = (Q_{\mathcal{NMV}})^C \cap (R_{\mathcal{NMV}})^C$.
(2) $(Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}})^C = (Q_{\mathcal{NMV}})^C \cup (R_{\mathcal{NMV}})^C$.

Proof. Let $Q_{\mathcal{NMV}} = \{ < \nu_s, [\Gamma_Q^-(\nu_s), \Gamma_Q^+(\nu_s)], [\Delta_Q^-(\nu_s), \Delta_Q^+(\nu_s)], [\Upsilon_Q^-(\nu_s), \Upsilon_Q^+(\nu_s)]>\}$ and $R_{\mathcal{NMV}} = \{ < \nu_s, [\Gamma_R^-(\nu_s), \Gamma_R^+(\nu_s)], [\Delta_R^-(\nu_s), \Delta_R^+(\nu_s)], [\Upsilon_R^-(\nu_s), \Upsilon_R^+(\nu_s)]>\}$ be the subsets of

$\mathcal{NMVTS}\ (\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$.

1. LHS:

$$Q_{\mathcal{NMV}} \cup R_{\mathcal{NMV}} = \left\{\left\langle \nu_s, \left[\Gamma_Q^-(\nu_s), \Gamma_Q^+(\nu_s)\right], \left[\Delta_Q^-(\nu_s), \Delta_Q^+(\nu_s)\right], \left[\Upsilon_Q^-(\nu_s), \Upsilon_Q^+(\nu_s)\right]\right\rangle\right\} \cup$$

$$\left\{\left\langle \nu_s, \left[\Gamma_R^-(\nu_s), \Gamma_R^+(\nu_s)\right], \left[\Delta_R^-(\nu_s), \Delta_R^+(\nu_s)\right], \left[\Upsilon_R^-(\nu_s), \Upsilon_R^+(\nu_s)\right]\right\rangle\right\}$$

$$Q_{\mathcal{NMV}} \cup R_{\mathcal{NMV}} = \left\{\left\langle \nu_s, \begin{array}{c} \left[\Gamma_Q^-(\nu_s) \vee \Gamma_R^-(\nu_s), \Gamma_Q^+(\nu_s) \vee \Gamma_R^+(\nu_s)\right], \\ \left[\Delta_Q^-(\nu_s) \wedge \Delta_R^-(\nu_s), \Delta_Q^+(\nu_s) \wedge \Delta_R^+(\nu_s)\right], \\ \left[\Upsilon_Q^-(\nu_s) \wedge \Upsilon_R^-(\nu_s), \Upsilon_Q^+(\nu_s) \wedge \Upsilon_R^+(\nu_s)\right] \end{array} \right\rangle\right\}$$

$$(Q_{\mathcal{NMV}} \cup R_{\mathcal{NMV}})^c = \left\{\left\langle \nu_s, \begin{array}{c} \left[1-(\Gamma_Q^+(\nu_s) \vee \Gamma_R^+(\nu_s)), 1-(\Gamma_Q^-(\nu_s) \vee \Gamma_R^-(\nu_s))\right], \\ \left[1-(\Delta_Q^+(\nu_s) \wedge \Delta_R^+(\nu_s)), 1-(\Delta_Q^-(\nu_s) \wedge \Delta_R^-(\nu_s))\right], \\ \left[1-(\Upsilon_Q^+(\nu_s) \wedge \Upsilon_R^+(\nu_s)), 1-\left(\Upsilon_Q^-(\nu_s) \wedge \Upsilon_R^-(\nu_s)\right)\right] \end{array} \right\rangle\right\}$$

2. RHS:

$$Q^C_{\mathcal{NMV}} = \left\{\left\langle \nu_s, \begin{array}{c} \left[1-\Gamma_Q^+(\nu_s), 1-\Gamma_Q^-(\nu_s)\right], \\ \left[1-\Delta_Q^+(\nu_s), 1-\Delta_Q^-(\nu_s)\right], \\ \left[1-\Upsilon_Q^+(\nu_s), 1-\Upsilon_Q^-(\nu_s)\right] \end{array} \right\rangle\right\} \text{ and}$$

$$R^C_{\mathcal{NMV}} = \left\{\left\langle \nu_s, \begin{array}{c} \left[1-\Gamma_R^+(\nu_s), 1-\Gamma_R^-(\nu_s)\right], \\ \left[1-\Delta_R^+(\nu_s), 1-\Delta_R^-(\nu_s)\right], \\ \left[1-\Upsilon_R^+(\nu_s), 1-\Upsilon_R^-(\nu_s)\right] \end{array} \right\rangle\right\}$$

$Q^C_{\mathcal{NMV}} \cap R^C_{\mathcal{NMV}}$

$$= \left\{ \left\langle \nu_s, \begin{array}{c} \left[1-\Gamma_Q^+(\nu_s), 1-\Gamma_Q^-(\nu_s)\right], \\ \left[1-\Delta_Q^+(\nu_s), 1-\Delta_Q^-(\nu_s)\right], \\ \left[1-\Upsilon_Q^+(\nu_s), 1-\Upsilon_Q^-(\nu_s)\right] \end{array} \right\rangle \right\} \cap \left\{ \left\langle \nu_s, \begin{array}{c} \left[1-\Gamma_R^+(\nu_s), 1-\Gamma_R^-(\nu_s)\right], \\ \left[1-\Delta_R^+(\nu_s), 1-\Delta_R^-(\nu_s)\right], \\ \left[1-\Upsilon_R^+(\nu_s), 1-\Upsilon_R^-(\nu_s)\right] \end{array} \right\rangle \right\}$$

$$= \left\{ \left\langle \nu_s, \begin{array}{c} \left[1-\Gamma_Q^+(\nu_s) \wedge 1-\Gamma_R^+(\nu_s), 1-\Gamma_Q^-(\nu_s) \wedge 1-\Gamma_R^-(\nu_s)\right], \\ \left[1-\Delta_Q^+(\nu_s) \vee 1-\Delta_R^+(\nu_s), 1-\Delta_Q^-(\nu_s) \vee 1-\Delta_R^-(\nu_s)\right], \\ \left[1-\Upsilon_Q^+(\nu_s) \vee 1-\Upsilon_R^+(\nu_s), 1-\Upsilon_Q^-(\nu_s) \vee 1-\Upsilon_R^-(\nu_s)\right] \end{array} \right\rangle \right\}$$

$$= \left\{ \left\langle \nu_s, \begin{array}{c} \left[1-(\Gamma_Q^+(\nu_s) \vee \Gamma_R^+(\nu_s)), 1-(\Gamma_Q^-(\nu_s) \vee \Gamma_R^-(\nu_s))\right], \\ \left[1-(\Delta_Q^+(\nu_s) \wedge \Delta_R^+(\nu_s)), 1-(\Delta_Q^-(\nu_s) \wedge \Delta_R^-(\nu_s))\right], \\ \left[1-(\Upsilon_Q^+(\nu_s) \wedge \Upsilon_R^+(\nu_s)), 1-\left(\Upsilon_Q^-(\nu_s) \wedge \Upsilon_R^-(\nu_s)\right)\right] \end{array} \right\rangle \right\}$$

So, LHS = RHS.

2. Proof of (2) is similar as proof of (1). □

Corollary 4.12. *Let $P_{\mathcal{NMV}}$, $Q_{\mathcal{NMV}}$, $R_{\mathcal{NMV}}$ and $S_{\mathcal{NMV}}$ be $\mathcal{NMV}$ sets in $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$. Then,*

(1) $\overline{\cup (P_{i\mathcal{NMV}})} = \cap \left(\overline{P_{i\mathcal{NMV}}}\right)$

(2) $\overline{\cap (P_{i\mathcal{NMV}})} = \cup \left(\overline{P_{i\mathcal{NMV}}}\right)$

Proof. The proof is obvious from the above theorem. □

Theorem 4.13. ***(Commutative law)*** *Let $P_{\mathcal{NMV}}$ and $Q_{\mathcal{NMV}}$ be $\mathcal{NMV}$ sets in the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$. Then the following statements hold true.*

(1) $P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}} = Q_{\mathcal{NMV}} \cup P_{\mathcal{NMV}}$.

(2) $P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}} = Q_{\mathcal{NMV}} \cap P_{\mathcal{NMV}}$.

Proof. Let $P_{\mathcal{NMV}}$ and $Q_{\mathcal{NMV}}$ be $\mathcal{NMV}$ sets in the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ of the form $P_{\mathcal{NMV}} = \{ < \nu_s, \left[\Gamma_P^-(\nu_s), \Gamma_P^+(\nu_s)\right], \left[\Delta_P^-(\nu_s), \Delta_P^+(\nu_s)\right], \left[\Upsilon_P^-(\nu_s), \Upsilon_P^+(\nu_s)\right] > \}$ and $Q_{\mathcal{NMV}} = \{ < \nu_s, [\Gamma_Q^-(\nu_s), \Gamma_Q^+(\nu_s)], [\Delta_Q^-(\nu_s), \Delta_Q^+(\nu_s)], [\Upsilon_Q^-(\nu_s), \Upsilon_Q^+(\nu_s)] > \}$.

1.$P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}$

$$= \left\{ \left\langle \nu_s,\ \begin{array}{c} \left[\bigvee\left(\Gamma_P^-(\nu_s),\ \Gamma_Q^-(\nu_s)\right),\ \bigvee\left(\Gamma_P^+(\nu_s),\ \Gamma_Q^+(\nu_s)\right)\right], \\ \left[\bigwedge\left(\Delta_P^-(\nu_s),\ \Delta_Q^-(\nu_s)\right),\ \bigwedge\left(\Delta_P^+(\nu_s),\ \Delta_Q^+(\nu_s)\right)\right], \\ \left[\bigwedge\left(\Upsilon_P^-(\nu_s),\ \Upsilon_Q^-(\nu_s)\right),\ \bigwedge\left(\Upsilon_P^+(\nu_s),\ \Upsilon_Q^+(\nu_s)\right)\right] \end{array} \right\rangle \right\}$$

$$= \left\{ \left\langle \nu_s,\ \begin{array}{c} \left[\bigvee\left(\Gamma_Q^-(\nu_s),\Gamma_P^-(\nu_s)\right),\ \bigvee\left(\Gamma_Q^+(\nu_s),\Gamma_P^+(\nu_s)\right)\right], \\ \left[\bigwedge\left(\Delta_Q^-(\nu_s),\Delta_P^-(\nu_s)\right),\ \bigwedge\left(\Delta_Q^+(\nu_s),\Delta_P^+(\nu_s)\right)\right], \\ \left[\bigwedge\left(\Upsilon_Q^-(\nu_s),\Upsilon_P^-(\nu_s)\right),\ \bigwedge\left(\Upsilon_Q^+(\nu_s),\Upsilon_P^+(\nu_s)\right)\right] \end{array} \right\rangle \right\}$$

$= Q_{\mathcal{NMV}} \cup P_{\mathcal{NMV}}$.

Therefore, $P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}} = Q_{\mathcal{NMV}} \cup P_{\mathcal{NMV}}$.

2.Proof of (2) is similar to (1). □

Theorem 4.14. ***(Associative law)*** *Following conditions are true for the $\mathcal{NMV}$ sets $P_{\mathcal{NMV}}$, $Q_{\mathcal{NMV}}$ and $R_{\mathcal{NMV}}$ of the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$.*

(1) $(P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}) \cup R_{\mathcal{NMV}} = P_{\mathcal{NMV}} \cup (Q_{\mathcal{NMV}} \cup R_{\mathcal{NMV}})$.

(2) $(P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}) \cap R_{\mathcal{NMV}} = P_{\mathcal{NMV}} \cap (Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}})$.

Proof. Let $P_{\mathcal{NMV}}$, $Q_{\mathcal{NMV}}$ and $R_{\mathcal{NMV}}$ be subsets of $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ defined as $P_{\mathcal{NMV}} = \{ < \nu_s, \left[\Gamma_P^-(\nu_s), \Gamma_P^+(\nu_s)\right], \left[\Delta_P^-(\nu_s), \Delta_P^+(\nu_s)\right], \left[\Upsilon_P^-(\nu_s), \Upsilon_P^+(\nu_s)\right] > \}$, $Q_{\mathcal{NMV}} = \{ < \nu_s, [\Gamma_Q^-(\nu_s), \Gamma_Q^+(\nu_s)], [\Delta_Q^-(\nu_s), \Delta_Q^+(\nu_s)], [\Upsilon_Q^-(\nu_s), \Upsilon_Q^+(\nu_s)] > \}$ and $R_{\mathcal{NMV}} = \{ < \nu_s, [\Gamma_R^-(\nu_s), \Gamma_R^+(\nu_s)], [\Delta_R^-(\nu_s), \Delta_R^+(\nu_s)], [\Upsilon_R^-(\nu_s), \Upsilon_R^+(\nu_s)] > \}$.

1.$P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}$

$$= \left\{ \left\langle \nu_s,\ \begin{array}{c} \left[\bigvee\left(\Gamma_P^-(\nu_s),\ \Gamma_Q^-(\nu_s)\right),\ \bigvee\left(\Gamma_P^+(\nu_s),\ \Gamma_Q^+(\nu_s)\right)\right], \\ \left[\bigwedge\left(\Delta_P^-(\nu_s),\ \Delta_Q^-(\nu_s)\right),\ \bigwedge\left(\Delta_P^+(\nu_s),\ \Delta_Q^+(\nu_s)\right)\right], \\ \left[\bigwedge\left(\Upsilon_P^-(\nu_s),\ \Upsilon_Q^-(\nu_s)\right),\ \bigwedge\left(\Upsilon_P^+(\nu_s),\ \Upsilon_Q^+(\nu_s)\right)\right] \end{array} \right\rangle \right\}$$

$$= \left\{ \left\langle \nu_s, \begin{array}{c} \left[\left(\Gamma^-_P(\nu_s) \vee \Gamma^-_Q(\nu_s)\right), \left(\Gamma^+_P(\nu_s) \vee \Gamma^+_Q(\nu_s)\right)\right], \\ \left[\left(\Delta^-_P(\nu_s) \wedge \Delta^-_Q(\nu_s)\right), \left(\Delta^+_P(\nu_s) \wedge \Delta^+_Q(\nu_s)\right)\right], \\ \left[\left(\Upsilon^-_P(\nu_s) \wedge \Upsilon^-_Q(\nu_s)\right), \left(\Upsilon^+_P(\nu_s) \wedge \Upsilon^+_Q(\nu_s)\right)\right] \end{array} \right\rangle \right\}$$

Then,

$(P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}) \cup R_{\mathcal{NMV}}$

$$= \left\{ \left\langle \nu_s, \begin{array}{c} \left[\left(\Gamma^-_P(\nu_s) \vee \Gamma^-_Q(\nu_s)\right) \vee \left(\Gamma^-_R(\nu_s)\right), \left(\Gamma^+_P(\nu_s) \vee \Gamma^+_Q(\nu_s)\right) \vee \left(\Gamma^+_R(\nu_s)\right)\right], \\ \left[\left(\Delta^-_P(\nu_s) \wedge \Delta^-_Q(\nu_s)\right) \wedge \left(\Delta^-_R(\nu_s)\right), \left(\Delta^+_P(\nu_s) \wedge \Delta^+_Q(\nu_s)\right) \wedge \left(\Delta^+_R(\nu_s)\right)\right], \\ \left[\left(\Upsilon^-_P(\nu_s) \wedge \Upsilon^-_Q(\nu_s)\right) \wedge \left(\Upsilon^-_R(\nu_s)\right), \left(\Upsilon^+_P(\nu_s) \wedge \Upsilon^+_Q(\nu_s)\right) \wedge \left(\Upsilon^+_R(\nu_s)\right)\right] \end{array} \right\rangle \right\}$$

$$= \left\{ \left\langle \nu_s, \begin{array}{c} \left[\left(\Gamma^-_P(\nu_s) \vee \Gamma^-_Q(\nu_s) \vee \Gamma^-_R(\nu_s)\right), \left(\Gamma^+_P(\nu_s) \vee \Gamma^+_Q(\nu_s) \vee \Gamma^+_R(\nu_s)\right)\right], \\ \left[\left(\Delta^-_P(\nu_s) \wedge \Delta^-_Q(\nu_s) \wedge \Delta^-_R(\nu_s)\right), \left(\Delta^+_P(\nu_s) \wedge \Delta^+_Q(\nu_s) \wedge \Delta^+_R(\nu_s)\right)\right], \\ \left[\left(\Upsilon^-_P(\nu_s) \wedge \Upsilon^-_Q(\nu_s) \wedge \Upsilon^-_R(\nu_s)\right), \left(\Upsilon^+_P(\nu_s) \wedge \Upsilon^+_Q(\nu_s) \wedge \Upsilon^+_R(\nu_s)\right)\right] \end{array} \right\rangle \right\}$$

$$= \left\{ \left\langle \nu_s, \begin{array}{c} \left[\left(\Gamma^-_P(\nu_s)\right) \vee \left(\Gamma^-_Q(\nu_s) \vee \Gamma^-_R(\nu_s)\right), \left(\Gamma^+_P(\nu_s)\right) \vee \left(\Gamma^+_Q(\nu_s) \vee \Gamma^+_R(\nu_s)\right)\right], \\ \left[\left(\Delta^-_P(\nu_s)\right) \wedge \left(\Delta^-_Q(\nu_s) \wedge \Delta^-_R(\nu_s)\right), \left(\Delta^+_P(\nu_s)\right) \wedge \left(\Delta^+_Q(\nu_s) \wedge \Delta^+_R(\nu_s)\right)\right], \\ \left[\left(\Upsilon^-_P(\nu_s)\right) \wedge \left(\Upsilon^-_Q(\nu_s) \wedge \Upsilon^-_R(\nu_s)\right), \left(\Upsilon^+_P(\nu_s)\right) \wedge \left(\Upsilon^+_Q(\nu_s) \wedge \Upsilon^+_R(\nu_s)\right)\right] \end{array} \right\rangle \right\}$$

$= P_{\mathcal{NMV}} \cup (Q_{\mathcal{NMV}} \cup R_{\mathcal{NMV}})$.

2. Proof of (2) is similar to proof of (1). □

Theorem 4.15. ***(Distributive law)*** *Let $P_{\mathcal{NMV}}$, $Q_{\mathcal{NMV}}$ and $R_{\mathcal{NMV}}$ be $\mathcal{NMV}$ sets in the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$. Then distributive law holds.*

(1) $P_{\mathcal{NMV}} \cup (Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}}) = (P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}) \cap (P_{\mathcal{NMV}} \cup R_{\mathcal{NMV}})$.

(2) $P_{\mathcal{NMV}} \cap (Q_{\mathcal{NMV}} \cup R_{\mathcal{NMV}}) = (P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}) \cup (P_{\mathcal{NMV}} \cap R_{\mathcal{NMV}})$.

Proof.

Let $P_{\mathcal{NMV}}$, $Q_{\mathcal{NMV}}$ and $R_{\mathcal{NMV}}$ be subsets of $\mathcal{NMV}$ topological space $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ defined as $P_{\mathcal{NMV}} = \{ < \nu_s, [\Gamma_P^-(\nu_s), \Gamma_P^+(\nu_s)], [\Delta_P^-(\nu_s), \Delta_P^+(\nu_s)], [\Upsilon_P^-(\nu_s), \Upsilon_P^+(\nu_s)] > \}$, $Q_{\mathcal{NMV}} = \{ < \nu_s, [\Gamma_Q^-(\nu_s), \Gamma_Q^+(\nu_s)], [\Delta_Q^-(\nu_s), \Delta_Q^+(\nu_s)], [\Upsilon_Q^-(\nu_s), \Upsilon_Q^+(\nu_s)] > \}$ and $R_{\mathcal{NMV}} = \{ < \nu_s, [\Gamma_R^-(\nu_s), \Gamma_R^+(\nu_s)], [\Delta_R^-(\nu_s), \Delta_R^+(\nu_s)], [\Upsilon_R^-(\nu_s), \Upsilon_R^+(\nu_s)] > \}$.

1. LHS: $$Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}} = \left\{ \left\langle \nu_s, \begin{array}{c} \left[\left(\Gamma_Q^-(\nu_s) \wedge \Gamma_R^-(\nu_s)\right), \left(\Gamma_Q^+(\nu_s) \wedge \Gamma_R^+(\nu_s)\right)\right], \\ \left[\left(\Delta_Q^-(\nu_s) \vee \Delta_R^-(\nu_s)\right), \left(\Delta_Q^+(\nu_s) \vee \Delta_R^+(\nu_s)\right)\right], \\ \left[\left(\Upsilon_Q^-(\nu_s) \vee \Upsilon_R^-(\nu_s)\right), \left(\Upsilon_Q^+(\nu_s) \vee \Upsilon_R^+(\nu_s)\right)\right] \end{array} \right\rangle \right\}$$

$P_{\mathcal{NMV}} \cup (Q_{\mathcal{NMV}} \cap R_{\mathcal{NMV}})$

$$= \left\{ \left\langle \nu_s, \begin{array}{c} \left[\Gamma_P^-(\nu_s), \Gamma_P^+(\nu_s)\right], \\ \left[\Delta_P^-(\nu_s), \Delta_P^+(\nu_s)\right], \\ \left[\Upsilon_P^-(\nu_s), \Upsilon_P^+(\nu_s)\right] \end{array} \right\rangle \right\} \cup \left\{ \left\langle \nu_s, \begin{array}{c} \left[\left(\Gamma_Q^-(\nu_s) \wedge \Gamma_R^-(\nu_s)\right), \left(\Gamma_Q^+(\nu_s) \wedge \Gamma_R^+(\nu_s)\right)\right], \\ \left[\left(\Delta_Q^-(\nu_s) \wedge \Delta_R^-(\nu_s)\right), \left(\Delta_Q^+(\nu_s) \wedge \Delta_R^+(\nu_s)\right)\right], \\ \left[\left(\Upsilon_Q^-(\nu_s) \wedge \Upsilon_R^-(\nu_s)\right), \left(\Upsilon_Q^+(\nu_s) \wedge \Upsilon_R^+(\nu_s)\right)\right] \end{array} \right\rangle \right\}$$

$$= \left\{ \left\langle \nu_s, \begin{array}{c} \left[\left(\Gamma_P^-(\nu_s)\right) \vee \left(\Gamma_Q^-(\nu_s) \wedge \Gamma_R^-(\nu_s)\right), \left(\Gamma_P^+(\nu_s)\right) \vee \left(\Gamma_Q^+(\nu_s) \wedge \Gamma_R^+(\nu_s)\right)\right], \\ \left[\left(\Delta_P^-(\nu_s)\right) \vee \left(\Delta_Q^-(\nu_s) \wedge \Delta_R^-(\nu_s)\right), \left(\Delta_P^+(\nu_s)\right) \vee \left(\Delta_Q^+(\nu_s) \wedge \Delta_R^+(\nu_s)\right)\right], \\ \left[\left(\Upsilon_P^-(\nu_s)\right) \vee \left(\Upsilon_Q^-(\nu_s) \wedge \Upsilon_R^-(\nu_s)\right), \left(\Upsilon_P^+(\nu_s)\right) \vee \left(\Upsilon_Q^+(\nu_s) \wedge \Upsilon_R^+(\nu_s)\right)\right] \end{array} \right\rangle \right\}$$

RHS:

$P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}$

$$= \left\{ \left\langle \nu_s, \begin{array}{c} \left[\bigvee\left(\Gamma_P^-(\nu_s), \Gamma_Q^-(\nu_s)\right), \bigvee\left(\Gamma_P^+(\nu_s), \Gamma_Q^+(\nu_s)\right)\right], \\ \left[\bigwedge\left(\Delta_P^-(\nu_s), \Delta_Q^-(\nu_s)\right), \bigwedge\left(\Delta_P^+(\nu_s), \Delta_Q^+(\nu_s)\right)\right], \\ \left[\bigwedge\left(\Upsilon_P^-(\nu_s), \Upsilon_Q^-(\nu_s)\right), \bigwedge\left(\Upsilon_P^+(\nu_s), \Upsilon_Q^+(\nu_s)\right)\right] \end{array} \right\rangle \right\}$$

$$= \left\{ \left\langle \nu_s,\ \begin{matrix} \left[\left(\Gamma^-_P(\nu_s) \vee \Gamma^-_Q(\nu_s)\right), \left(\Gamma^+_P(\nu_s) \vee \Gamma^+_Q(\nu_s)\right)\right], \\ \left[\left(\Delta^-_P(\nu_s) \wedge \Delta^-_Q(\nu_s)\right), \left(\Delta^+_P(\nu_s) \wedge \Delta^+_Q(\nu_s)\right)\right], \\ \left[\left(\Upsilon^-_P(\nu_s) \wedge \Upsilon^-_Q(\nu_s)\right), \left(\Upsilon^+_P(\nu_s) \wedge \Upsilon^+_Q(\nu_s)\right)\right] \end{matrix} \right\rangle \right\}$$

$(P_{\mathcal{NMV}} \cup R_{\mathcal{NMV}})$

$$= \left\{ \left\langle \nu_s,\ \begin{matrix} \left[\bigvee\left(\Gamma^-_P(\nu_s),\ \Gamma^-_R(\nu_s)\right),\ \bigvee\left(\Gamma^+_P(\nu_s),\ \Gamma^+_R(\nu_s)\right)\right], \\ \left[\bigwedge\left(\Delta^-_P(\nu_s),\ \Delta^-_R(\nu_s)\right),\ \bigwedge\left(\Delta^+_P(\nu_s),\ \Delta^+_R(\nu_s)\right)\right], \\ \left[\bigwedge\left(\Upsilon^-_P(\nu_s),\ \Upsilon^-_R(\nu_s)\right),\ \bigwedge\left(\Upsilon^+_P(\nu_s),\ \Upsilon^+_R(\nu_s)\right)\right] \end{matrix} \right\rangle \right\}$$

$$= \left\{ \left\langle \nu_s,\ \begin{matrix} \left[\left(\Gamma^-_P(\nu_s) \vee \Gamma^-_R(\nu_s)\right), \left(\Gamma^+_P(\nu_s) \vee \Gamma^+_R(\nu_s)\right)\right], \\ \left[\left(\Delta^-_P(\nu_s) \wedge \Delta^-_R(\nu_s)\right), \left(\Delta^+_P(\nu_s) \wedge \Delta^+_R(\nu_s)\right)\right], \\ \left[\left(\Upsilon^-_P(\nu_s) \wedge \Upsilon^-_R(\nu_s)\right), \left(\Upsilon^+_P(\nu_s) \wedge \Upsilon^+_R(\nu_s)\right)\right] \end{matrix} \right\rangle \right\}$$

Then, $(P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}) \cap (P_{\mathcal{NMV}} \cup R_{\mathcal{NMV}})$

$$= \left\{ \left\langle \nu_s,\ \begin{matrix} \left[\left(\Gamma^-_P(\nu_s) \vee \Gamma^-_Q(\nu_s)\right), \left(\Gamma^+_P(\nu_s) \vee \Gamma^+_Q(\nu_s)\right)\right], \\ \left[\left(\Delta^-_P(\nu_s) \wedge \Delta^-_Q(\nu_s)\right), \left(\Delta^+_P(\nu_s) \wedge \Delta^+_Q(\nu_s)\right)\right], \\ \left[\left(\Upsilon^-_P(\nu_s) \wedge \Upsilon^-_Q(\nu_s)\right), \left(\Upsilon^+_P(\nu_s) \wedge \Upsilon^+_Q(\nu_s)\right)\right] \end{matrix} \right\rangle \right\} \cap$$

$$\left\{ \left\langle \nu_s,\ \begin{matrix} \left[\left(\Gamma^-_P(\nu_s) \vee \Gamma^-_R(\nu_s)\right), \left(\Gamma^+_P(\nu_s) \vee \Gamma^+_R(\nu_s)\right)\right], \\ \left[\left(\Delta^-_P(\nu_s) \wedge \Delta^-_R(\nu_s)\right), \left(\Delta^+_P(\nu_s) \wedge \Delta^+_R(\nu_s)\right)\right], \\ \left[\left(\Upsilon^-_P(\nu_s) \wedge \Upsilon^-_R(\nu_s)\right), \left(\Upsilon^+_P(\nu_s) \wedge \Upsilon^+_R(\nu_s)\right)\right] \end{matrix} \right\rangle \right\}$$

$$=\left\{\left\langle \nu_s,\ \begin{matrix} \left[\left(\Gamma^-_P(\nu_s)\vee\Gamma^-_Q(\nu_s)\right)\wedge\left(\Gamma^-_P(\nu_s)\vee\Gamma^-_R(\nu_s)\right),\left(\Gamma^+_P(\nu_s)\vee\Gamma^+_Q(\nu_s)\right)\wedge\left(\Gamma^+_P(\nu_s)\vee\Gamma^+_R(\nu_s)\right)\right], \\ \left[\left(\Delta^-_P(\nu_s)\wedge\Delta^-_Q(\nu_s)\right)\vee\left(\Delta^-_P(\nu_s)\wedge\Delta^-_R(\nu_s)\right),\left(\Delta^+_P(\nu_s)\wedge\Delta^+_Q(\nu_s)\right)\vee\left(\Delta^+_P(\nu_s)\wedge\Delta^+_R(\nu_s)\right)\right], \\ \left[\left(\Upsilon^-_P(\nu_s)\wedge\Upsilon^-_Q(\nu_s)\right)\vee\left(\Upsilon^-_P(\nu_s)\wedge\Upsilon^-_R(\nu_s)\right),\left(\Upsilon^+_P(\nu_s)\wedge\Upsilon^+_Q(\nu_s)\right)\vee\left(\Upsilon^+_P(\nu_s)\wedge\Upsilon^+_R(\nu_s)\right)\right] \end{matrix} \right\rangle\right\}$$

$$=\left\{\left\langle \nu_s,\ \begin{matrix} \left[\left(\Gamma^-_P(\nu_s)\right)\vee\left(\Gamma^-_Q(\nu_s)\wedge\Gamma^-_R(\nu_s)\right),\left(\Gamma^+_P(\nu_s)\right)\vee\left(\Gamma^+_Q(\nu_s)\wedge\Gamma^+_R(\nu_s)\right)\right], \\ \left[\left(\Delta^-_P(\nu_s)\right)\vee\left(\Delta^-_Q(\nu_s)\wedge\Delta^-_R(\nu_s)\right),\left(\Delta^+_P(\nu_s)\right)\vee\left(\Delta^+_Q(\nu_s)\wedge\Delta^+_R(\nu_s)\right)\right], \\ \left[\left(\Upsilon^-_P(\nu_s)\right)\vee\left(\Upsilon^-_Q(\nu_s)\wedge\Upsilon^-_R(\nu_s)\right),\left(\Upsilon^+_P(\nu_s)\right)\vee\left(\Upsilon^+_Q(\nu_s)\wedge\Upsilon^+_R(\nu_s)\right)\right] \end{matrix} \right\rangle\right\}$$

Therefore, LHS = RHS. Hence Distributive law holds.

2. Proof of (2) is similar to (1). □

5. Closure and Interior of $\mathcal{NMV}$ set

Definition 5.1. Let $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ be a $\mathcal{NMVTS}$. Let $P_{\mathcal{NMV}}$ be a $\mathcal{NMV}$ set. The $\mathcal{NMV}$ interior of $P_{\mathcal{NMV}}$ is defined as the union of all $\mathcal{NMVOS}s$ contained in $P_{\mathcal{NMV}}$. (i.e) $\mathcal{NMV}int(P_{\mathcal{NMV}}) = \cup\{\text{G} : \text{G is a } \mathcal{NMV} \text{ open set and G} \subseteq P_{\mathcal{NMV}}\}$. Clearly, $\mathcal{NMV}$-int $(P_{\mathcal{NMV}})$ is the largest $\mathcal{NMV}$ open set that is contained in $P_{\mathcal{NMV}}$.

Definition 5.2. Let $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ be a $\mathcal{NMVTS}$. Let $P_{\mathcal{NMV}}$ be a $\mathcal{NMV}$ set. The $\mathcal{NMV}$ closure of $P_{\mathcal{NMV}}$ is defined as the intersection of all $\mathcal{NMVCS}s$ containing $P_{\mathcal{NMV}}$. (i.e) $\mathcal{NMV}cl(P_{\mathcal{NMV}}) = \cap\{\text{K} : \text{K is } \mathcal{NMV} \text{ closed set and } P_{\mathcal{NMV}} \subseteq \text{K}\}$. Clearly, $\mathcal{NMV}$-cl $(P_{\mathcal{NMV}})$ is the smallest $\mathcal{NMV}$ closed set that contains $P_{\mathcal{NMV}}$.

Proposition 5.3. *Let $P_{\mathcal{NVM}}$ be any $\mathcal{NMV}$ set in $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$. Then*

(1) $\mathcal{NMV}int(1 - P_{\mathcal{NMV}}) = 1 - (\mathcal{NMV}cl(P_{\mathcal{NMV}}))$
(2) $\mathcal{NMV}cl(1 - P_{\mathcal{NMV}}) = 1 - (\mathcal{NMV}int(P_{\mathcal{NMV}}))$

Proof. **(1)** By definition, $\mathcal{NMV}cl(P_{\mathcal{NMV}}) = \cap\{\text{K} : \text{K is } \mathcal{NMV} \text{ closed set and } P_{\mathcal{NMV}} \subseteq \text{K}\}$. Therefore, $1 - (\mathcal{NMV}cl(P_{\mathcal{NMV}})) = 1 - \cap\{\text{K} : \text{K is } \mathcal{NMV} \text{ closed set and } P_{\mathcal{NMV}} \subseteq \text{K}\}$

$$= \cup\{(1-\text{K}) : \text{K is } \mathcal{NMV} \text{ closed set and } P_{\mathcal{NMV}} \subseteq \text{K}\}$$

$$= \cup\{\text{G} : \text{G is a } \mathcal{NMV} \text{ open set and G} \subseteq (1-P_{\mathcal{NMV}})\}$$

$$= \mathcal{NMV}int(1 - P_{\mathcal{NMV}})$$

(2) The proof is similar to (1). □

Proposition 5.4. *For any two $\mathcal{NMV}$ sets $P_{\mathcal{NMV}}$ and $Q_{\mathcal{NMV}}$ in $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ the following statements hold:*

(1) $P_{\mathcal{NMV}}$ *is* $\mathcal{NMV}$ *Closed set if and only if* $\mathcal{NMV}-cl\ (P_{\mathcal{NMV}}) = P_{\mathcal{NMV}}$

(2) $P_{\mathcal{NMV}}$ *is* $\mathcal{NMV}$ *Open set if and only if* $\mathcal{NMV}-int\ (P_{\mathcal{NMV}}) = P_{\mathcal{NMV}}$

(3) $P_{\mathcal{NMV}} \subseteq Q_{\mathcal{NMV}}$ *implies* $\mathcal{NMV}-int\ (P_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-int\ (Q_{\mathcal{NMV}})$

(4) $P_{\mathcal{NMV}} \subseteq Q_{\mathcal{NMV}}$ *implies* $\mathcal{NMV}-cl\ (P_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-cl\ (Q_{\mathcal{NMV}})$

(5) $\mathcal{NMV}-cl\ (\mathcal{NMV}-cl\ (P_{\mathcal{NMV}})) = \mathcal{NMV}-cl\ (P_{\mathcal{NMV}})$

(6) $\mathcal{NMV}-int\ (\mathcal{NMV}-int\ (P_{\mathcal{NMV}})) = \mathcal{NMV}-int\ (P_{\mathcal{NMV}})$

Proof. (1) If $P_{\mathcal{NMV}}$ is a $\mathcal{NMV}$ closed set, then $P_{\mathcal{NMV}}$ is the smallest $\mathcal{NMV}$ closed set containing itself and hence $\mathcal{NMV}-cl(P_{\mathcal{NMV}}) = P_{\mathcal{NMV}}$. Conversely if $\mathcal{NMV}-cl(P_{\mathcal{NMV}}) = P_{\mathcal{NMV}}$, then $P_{\mathcal{NMV}}$ is the smallest $\mathcal{NMV}$ closed set containing itself and hence $P_{\mathcal{NMV}}$ is $\mathcal{NMV}$ closed set.

(2) Let $P_{\mathcal{NMV}}$ be a Neutrosophic Micro Vague Open set in the $\mathcal{NMVTS}$ $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$. We know that $\mathcal{NMV}-int(P_{\mathcal{NMV}})$ of any set is a subset of the set $P_{\mathcal{NMV}}$. So, $\mathcal{NMV}-int(P_{\mathcal{NMV}}) \subseteq P_{\mathcal{NMV}}$. Since, $P_{\mathcal{NMV}}$ is a Neutrosophic Micro Vague open set, we have $P_{\mathcal{NMV}} \subseteq \mathcal{NMV}-int(P_{\mathcal{NMV}})$. Therefore, $\mathcal{NMV}-int(P_{\mathcal{NMV}}) = P_{\mathcal{NMV}}$. Conversely suppose if $\mathcal{NMV}-int(P_{\mathcal{NMV}}) = P_{\mathcal{NMV}}$, then since $\mathcal{NMV}-int(P_{\mathcal{NMV}})$ is a $\mathcal{NMV}$ open set, clearly $P_{\mathcal{NMV}}$ is also a $\mathcal{NMV}$ open set.

(3) Let $P_{\mathcal{NMV}} \subseteq Q_{\mathcal{NMV}}$, then $1-P_{\mathcal{NMV}} \subseteq 1-Q_{\mathcal{NMV}}$, this implies that $\mathcal{NMV}-cl\ (1-P_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-cl\ (1-Q_{\mathcal{NMV}}) \Longrightarrow \mathcal{NMV}-int(P_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-int(Q_{\mathcal{NMV}})$.

(4) Similarly, it is proved that $\mathcal{NMV}-cl(P_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-cl(Q_{\mathcal{NMV}})$.

(5) Since $\mathcal{NMV}-int(P_{\mathcal{NMV}})$ is a $\mathcal{NMV}$ open set, $\mathcal{NMV}-int\ (\mathcal{NMV}-int(P_{\mathcal{NMV}})) = \mathcal{NMV}-int(P_{\mathcal{NMV}})$.

(6) Similarly, since $\mathcal{NMV}-int(P_{\mathcal{NMV}})$ is a $\mathcal{NMV}$ closed set, then $\mathcal{NMV}-cl\ (\mathcal{NMV}-cl(P_{\mathcal{NMV}})) = \mathcal{NMV}-cl(P_{\mathcal{NMV}})$.

Hence Proved. □

Proposition 5.5. *For any two $\mathcal{NMV}$ sets $P_{\mathcal{NMV}}$ and $Q_{\mathcal{NMV}}$ in $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ the following statements hold:*

(1) $\mathcal{NMV}-cl\ (P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}) = \mathcal{NMV}-cl\ (P_{\mathcal{NMV}}) \cup \mathcal{NMV}-cl\ (Q_{\mathcal{NMV}})$

(2) $\mathcal{NMV}-cl\ (P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-cl\ (P_{\mathcal{NMV}}) \cap \mathcal{NMV}-cl\ (Q_{\mathcal{NMV}})$

Proof. (1) Since $P_{\mathcal{NMV}} \subseteq P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}$ and $Q_{\mathcal{NMV}} \subseteq P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}$, then $\mathcal{NMV}-cl(P_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-cl(P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}})$ and $\mathcal{NMV}-cl(Q_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-cl(P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}})$. Therefore, $\mathcal{NMV}-cl(P_{\mathcal{NMV}}) \cup \mathcal{NMV}-cl(Q_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-cl(P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}})$.

Conversely since $P_{\mathcal{NMV}} \subseteq \mathcal{NMV}-cl(P_{\mathcal{NMV}})$ and $Q_{\mathcal{NMV}} \subseteq \mathcal{NMV}-cl(Q_{\mathcal{NMV}})$, then $P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}} \subseteq \mathcal{NMV}-cl(P_{\mathcal{NMV}}) \cup \mathcal{NMV}-cl(Q_{\mathcal{NMV}})$. Besides $\mathcal{NMV}-cl(P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}})$ is the smallest $\mathcal{NMV}$ closed set that containing $P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}$. Therefore, $\mathcal{NMV}-cl\ (P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-cl(P_{\mathcal{NMV}}) \cup \mathcal{NMV}-cl(Q_{\mathcal{NMV}})$. Thus, $\mathcal{NMV}-cl\ (P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}) = \mathcal{NMV}-cl(P_{\mathcal{NMV}}) \cup \mathcal{NMV}-cl(Q_{\mathcal{NMV}})$.

(2) Since, $P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}} \subseteq \mathcal{NMV}-cl(P_{\mathcal{NMV}}) \cap \mathcal{NMV}-cl(Q_{\mathcal{NMV}})$ and $\mathcal{NMV}-cl(P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}})$ is the smallest $\mathcal{NMV}$ closed set that containing $P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}$, then $\mathcal{NMV}-cl(P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-cl(P_{\mathcal{NMV}}) \cap \mathcal{NMV}-cl(Q_{\mathcal{NMV}})$.

Hence Proved. □

Proposition 5.6. *For any two $\mathcal{NMV}$ sets $P_{\mathcal{NMV}}$ and $Q_{\mathcal{NMV}}$ in $(\acute{\mathfrak{L}}, \Psi_{\mathcal{Y}}(\mathfrak{S}), \Omega_{\mathcal{Y}}(\mathfrak{S}))$ the following statements hold:*

(1) $\mathcal{NMV}-int\ (P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}) \supseteq \mathcal{NMV}-int\ (P_{\mathcal{NMV}}) \cup \mathcal{NMV}-int\ (Q_{\mathcal{NMV}})$

(2) $\mathcal{NMV}-int\ (P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}) = \mathcal{NMV}-int\ (P_{\mathcal{NMV}}) \cap \mathcal{NMV}-int\ (Q_{\mathcal{NMV}})$

Proof. (1) Since $P_{\mathcal{NMV}} \subseteq P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}$ and $Q_{\mathcal{NMV}} \subseteq P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}}$, then $\mathcal{NMV}-int(P_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-int(P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}})$ and $\mathcal{NMV}-int(Q_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-int(P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}})$. Therefore, $\mathcal{NMV}-int(P_{\mathcal{NMV}}) \cup \mathcal{NMV}-int(Q_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-int\ (P_{\mathcal{NMV}} \cup Q_{\mathcal{NMV}})$.

(2) Since $P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}} \subseteq P_{\mathcal{NMV}}$ and $P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}} \subseteq Q_{\mathcal{NMV}}$, then $\mathcal{NMV}-int(P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-int(P_{\mathcal{NMV}})$ and $\mathcal{NMV}-int(P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-int(Q_{\mathcal{NMV}})$. So, $\mathcal{NMV}-int(P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-int(P_{\mathcal{NMV}}) \cap \mathcal{NMV}-int(Q_{\mathcal{NMV}})$.

On the other hand, since $\mathcal{NMV}-int(P_{\mathcal{NMV}}) \subseteq P_{\mathcal{NMV}}$ and $\mathcal{NMV}-int(Q_{\mathcal{NMV}}) \subseteq Q_{\mathcal{NMV}}$, then $\mathcal{NMV}-int(P_{\mathcal{NMV}}) \cap \mathcal{NMV}-int(Q_{\mathcal{NMV}}) \subseteq P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}$. Besides $\mathcal{NMV}-int(P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}) \subseteq P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}$ and it is the biggest $\mathcal{NMV}$ open set that contained in $P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}$. Therefore, $\mathcal{NMV}-int(P_{\mathcal{NMV}}) \cap \mathcal{NMV}-int(Q_{\mathcal{NMV}}) \subseteq \mathcal{NMV}-int(P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}})$. Thus $\mathcal{NMV}-int(P_{\mathcal{NMV}} \cap Q_{\mathcal{NMV}}) = \mathcal{NMV}-int(P_{\mathcal{NMV}}) \cap \mathcal{NMV}-int(Q_{\mathcal{NMV}})$.

Hence the proof. □

Proposition 5.7. (1) $\mathcal{NMV}-cl\ (Q^C_{\mathcal{NMV}}) = [\mathcal{NMV}-int\ (Q_{\mathcal{NMV}})]^C$

(2) $\mathcal{NMV}-int\ (Q^C_{\mathcal{NMV}}) = [\mathcal{NMV}-cl\ (Q_{\mathcal{NMV}})]^C$

Proof. Proof of the above proposition is obvious. □

6. Conclusion

By applying simple extension on Neutrosophic Vague Nano topological spaces, this research presents a brand-new topological space known as Neutrosophic Micro Vague Topological Space. Various operations on Neutrosophic Micro Vague sets such as union, intersection, inclusion and complement are defined with suitable examples. Moreover, some of the fundamental algebraic set properties for Neutrosophic Micro Vague sets have been described and evaluated with appropriate examples. Neutrosophic micro vague set encourages inventiveness in variety of domains. In dynamic contexts where information is continuously shifting or evolving, these sets are advantageous and they offer a strong foundation for managing uncertainty in a variety of applications which enhances decision-making and problem-solving skills. Utilizing this advanced framework researchers may explore applications in fields including image processing, natural language comprehension, robotics and optimization. In future, Neutrosophic Micro Vague sets can be used in association with other mathematical infrastructure to represent various characteristics of unknowns and insufficient accuracy.

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