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An Asymmetric Measure of Comparison of Neutrosophic Sets

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Abstract: The single-valued neutrosophic (SVNS) set is a beneficial and significant tool to deal with uncertainty with the neutrality of truth. This article introduces a non-conventional asymmetric measure of comparison in the single-valued neutrosophic framework. Such a measure is applied to the problems where the conventional symmetric measures of comparison do not produce valid computational results and require a directed closeness or discrimination between two abstractions represented by neutrosophic sets. We prove some properties of the proposed neutrosophic comparison measure and empirically justify its utilization in a problem of strategic decision-making, pattern recognition and medical diagnosis. The assessment of the performance of the proposed measure using "Degree of Confidence" shows the advantage of the proposed measure.

Keywords: Single-valued neutrosophic set (SVNS); asymmetric measure; inaccuracy measure; pattern recognition.

1. Introduction

Many problems concerning decision-making, identification of patterns, machine learning, computer vision, data analytics, etc., predominantly utilize some measure of comparisons. Several studies are available regarding comparison measures in various uncertain and vague settings. The prevalently investigated comparison measures in uncertain environments are divergence, distance, dissimilarity, and similarity measures. One common characteristic of divergence measures, distance measures, dissimilarity measures, and similarity measures in fuzzy and non-standard fuzzy settings is that these are symmetric. But, in specific comparisons, the symmetric comparison is not suitable. For instance, "P is like Q" may be preferred over "Q is like P" or vice-versa. Such situations need an asymmetric or directed comparison measure. This study proposes an asymmetric measure of comparison for single-valued neutrosophic sets.

However, in an uncertain environment due to randomness, an asymmetric measure of comparison of two probability distributions was proposed by Kullback-Leibler [1-2]. These measures found vital application in communication theory and economics. Kerridge inaccuracy is a non-parametric generalization of Shannon's entropy [3]. Kerridge [4] termed it an 'inaccuracy measure' for measuring the inaccuracy between two probability distributions. Moreover, numerous probabilistic information measures were put forward during the second half of the twentieth century.

In 1965, Zadeh [5] coined a new form of uncertainty due to vagueness or linguistic imprecision and developed fuzzy theory. As Shannon's entropy [3] quantifies the uncertainty due to randomness, De-Luca and Termini [6] introduced fuzzy entropy to quantify the uncertainty due to vagueness. This pioneered fuzzy entropy is structurally similar to that of Shannon's entropy [3] but practically different. Various extensions of fuzzy theory and information measures in these frameworks have been developed in the last three decades.

Smarandache [7] developed a more advanced notion, "Neutrosophy," to more comprehensively model the vagary of information. The neutrosophic theory reconciles certain pitfalls of the fuzzy approach and its extensions. Wang *et al*. [8] proposed a single-valued neutrosophic set as a subclass of a neutrosophic set. In SVNs, the data information is indicated with 3-tuple, i.e., degree of membership, degree of indeterminacy, and degree of non-membership. Hence, the information evaluation in terms of neutrosophic sets seems more suitable for studies concerning decision-making, pattern recognition, clustering analysis, etc. A single-valued neutrosophic set allows us to choose truth membership, false membership, and indeterminacy in an unrestrictive manner in contrast to other fuzzy extensions.

Several neutrosophic information measures have been proposed, such as entropy, similarity, distance, and divergence measures over the years. Some prominent researches are due to Chai *et al.* [9], Wang [10], Biswas, *et al.* [11], Wu *et al.* [12], Aydogdu [13], Bourmi and Smarandache [14], Bourmi and Smarandache [15], Khan *et al.*, [16], Majumdar and Samanta [17], Ye [18-19], Ye and Fu [20], Chakraborty *et al.* [21], Chakraborty *et al.* [22], Haque *et al.* [23], Haque *et al.* [24], and Chakraborty *et al.* [25], Bonissone [26], Eshragh and Mamdani [27], etc. References [28-30] also report the work on developing new similarity/distance measures for fuzzy and SVNSs.

Motivation and contribution

Recent trends notice that all the existing measures of comparison (distance/similarity/divergence) in the neutrosophic framework are symmetric. But there are practical circumstances where the asymmetric comparison is more suitable. For example, we consider the following two sentences:

I. Saddam Hussain was like Hitler.
II. Hitler was like Saddam Hussain.

Off course, sentence-I would be the apparent preference for the comparison, probably due to the genocide instinct of the latter. In such a situation, one concept is the target, another is the base, and the main focus is the target. In sentence-I and -II, Saddam Hussain is the target, and Hitler is the base.

Further, a problem of medical diagnosis, where the symptoms of the patient are compared with symptoms of certain diseases (as established by medical experts), also seems to be better dealt with using asymmetric comparison measures. In such a problem, the patient's symptoms in single-valued neutrosophic representation (P) must be treated as a target, and the pre-assigned symptoms of the disease (Q) may be treated as a base. The direction of comparison in this problem must be Q→P instead of P→Q. Such asymmetric comparisons are unavoidable in any discipline and can essentially need to be investigated using some asymmetric measures of comparison. In view of these facts, natural question arise what is the concept of asymmetric measure of comparison? How to construct an asymmetric measure of comparison in neutrosophic environment? Does such a comparison measure practically valid and effective? Moreover, to best of our knowledge there is no asymmetric measure of information in the literature concerning neutrosophic information theory. Here-mentioned facts and research gap in neutrosophic information theory motivated us to consider this study.

The novel contribution of this article is as follows.

- We introduce a novel concept of asymmetric comparison measure for SVNSs and term it a "Single-Valued Neutrosophic Inaccuracy Measure."
- We prove some algebraic properties of the proposed comparison measure for SVNSs.
- We also deduce a performance index "Degree of Confidence" to examine the performance of various neutrosophic comparison measures in the classification problems
- We also discuss applications of the proposed measure in pattern recognition and medical diagnosis problem.

The remaining paper is structured as follows. Section 2 presents preliminaries. Section 3 introduces inaccuracy measures/asymmetric measure of comparison between SVNSs. In section 4, we discuss some properties of the proposed measure. Section 5 presents an application of the proposed inaccuracy measure. Section 6 includes the comparative study. Finally, Section 7 concludes the article.

2. Preliminaries

This section considers some notions related to single-valued neutrosophic sets and inaccuracy measures.

Definition 2.1[3]. Let $Y = (y_1, y_2, y_{3,\ldots,} y_n)$ be a random variable associated with an experiment. Let $P = (p_1, p_2, p_{3,\ldots} p_n)$ be the probability distribution of random variable Y. Shannon's entropy measure is given by

$$H(P) = -\sum_{i=1}^{n} p_i log_2 p_i.$$

Definition 2.2[1][2]. Let $Y = (y_1, y_2, y_{3,\ldots,} y_n)$ be a random variable associated with an experiment. Let $P = (p_1, p_2, p_{3,\ldots} p_n)$ and $Q = (q_1, q_2, q_{3,\ldots} q_n)$ be two probability distributions. Then the divergence measure between P and Q is given by

$$D\,(P,Q) = \sum_{i=1}^{n} p_i log_2 \frac{p_i}{q_i}.$$

Definition 2.3[4]. Let $P = (p_1, p_2, p_{3,\ldots} p_n)$ and $Q = (q_1, q_2, q_{3,\ldots} q_n)$ be two probability distributions. Then inaccuracy of distribution Q with respect to distribution P is given by

$$I(P,Q) = -\sum_{i=1}^{n} p_i \, log_2 \, q_i.$$

A particular case of a neutrosophic set is a single-valued neutrosophic set which was proposed by Wang *et al*. [8]

Definition 2.5[8]. Let y_i be a generic element of the universal set Y. A truth-membership function characterizes a single-valued neutrosophic set $\rho_A(y_i)$, indeterminacy-membership function $\theta_A(y_i)$ and falsity-membership function$\delta_A(y_i)$. Also, for each $y_i \in Y$, $\rho_A(y_i)$, $\theta_A(y_i)$, $\delta_A(y_i) \in [0,1]$ with condition $\rho_A(y_i) + \theta_A(y_i) + \delta_A(y_i) \in [0,3]$.

In other words, a single-valued neutrosophic set A can be denoted by a triplet, *i.e.*,

$$A = \{\langle \rho_A(y_i), \theta_A(y_i), \delta_A(y_i)\rangle |\ y_i \in Y\}$$

Notation: SVNS (Y) denotes the set of all neutrosophic elements in Y.

Some of the basic and useful operations on SVNS are defined as follows:

Definition 2.6[8]. Let $A = \{\langle \rho_A(y_i), \theta_A(y_i), \delta_A(y_i)\rangle |\ y_i \in Y\}$ and

$$B = \{\langle \rho_B(y_i), \theta_B(y_i), \delta_B(y_i)\rangle |\ y_i \in Y\}.$$

be two SVNSs, then the union of A and B is defined as

$$A \cup B = \{< max.(\rho_A(y_i), \rho_B(y_i)), min.(\theta_A(y_i), \theta_B(y_i)), min.(\delta_A(y_i), \delta_B(y_i))> \ |\ y_i \in Y\ \}.$$

Definition 2.7 [8] For two SVNSs, A and B, the intersection of A and B is

$$A \cap B = \{< min.(\rho_A(y_i), \rho_B(y_i)), max.(\theta_A(y_i), \theta_B(y_i)), max.(\delta_A(y_i),\ \delta_B(y_i)) > |\ y_i \in Y\}.$$

Definition 2.8[8]. Let $A = \{\langle \rho_A(y_i), \theta_A(y_i), \delta_A(y_i)\rangle |\ y_i \in Y\}$ be an SVNs. Then the complement of A is defined as

$$A^c = \{< 1 - \rho_A(y_i), 1 - \theta_A(y_i), 1 - \delta_A(y_i) > |\ y_i \in Y\}.$$

Definition 2.9[8]. Let $A = \{\langle \rho_A(y_i), \theta_A(y_i), \delta_A(y_i)\rangle |\ y_i \in Y\}$ and $B = \{\langle \rho_B(y_i), \theta_B(y_i), \delta_B(y_i)\rangle |\ y_i \in Y\}$ be two SVNSs, then $A \subseteq B$ if

$$\rho_A(y_i) \leq \rho_B(y_i),\ \theta_A(y_i) \geq \theta_B(y_i),\ \delta_A(y_i) \geq \delta_B(y_i),\ \ \forall\ y_i \in Y.$$

Hatzimichailidis [31] introduced the notion of Degree of confidence (DoC) in intuitionistic fuzzy environment. The definition of DoC in neutrosophic settings is as follows.

Definition 2.10. Let P_i be an unknown pattern classified to some pattern from the class P_j. Degree of confidence of neutrosophic comparison measure *M* estimates the confidence level that comparison measure in classifying a pattern P_i to the pattern P_k (belongs to a class of patterns) and it can be computed as

$$DOC = \sum_{j=1\ j\neq k}^{n} |M(P_i, P_k) - M(P_i, P_j)|.$$

The greater the degree of confidence (*DOC*) for a comparison measure, the more confident the classification result of the measure is.

3. Inaccuracy Measure of a Single-Valued Neutrosophic Set

In this section, we propose an inaccuracy measure of single-valued neutrosophic sets and discuss their properties. Verma and Sharma [32] presented an inaccuracy measure of fuzzy sets as follows:

$$I\,(A, B) = -\frac{1}{n}\sum_{i=1}^{n}[\rho_A(y_i)\log\,(\rho_B(y_i)) + (1 - \rho_A(y_i))\log\ (1 - \rho_B(y_i))]\,. \quad (1)$$

where ρ_A and ρ_B are the membership functions associated with fuzzy sets A and B.

We can write

$$I\,(A,B) = -\frac{1}{n}\sum_{i=1}^{n} S(x,y). \tag{2}$$

where, $S\,(x,y) = -xlogy - (1-x)\log\,(1-y)$ is called Karridge's inaccuracy function for two events.

Since in a single-valued neutrosophic set, the non-membership and indeterminacy are independent of the membership function; therefore, the inaccuracy function utilized in equation (2) can be modified as

$$S\,(x,y) = -x_1 logy_1 - x_2 logy_2 - x_3 logy_3, \tag{3}$$

where $x_i\,, y_i \in [0,1]\,, i = 1,2,3.$

Consequently, the inaccuracy measure for two single-valued neutrosophic sets $A = \{\langle \rho_A(y_i), \theta_A(y_i), \delta_A(y_i)\rangle |\, y_i \in Y\,\}$ and $B = \{\langle \rho_B(y_i), \theta_B(y_i), \delta_B(y_i)\rangle |\, y_i \in Y\,\}$, is defined as follows:

$$I_{SVNS}(A,B) = -\frac{1}{n}\sum_{i=1}^{n}[\rho_A(y_i)\log\,(\rho_B(y_i)) + \theta_A(y_i)\log\,\theta_B(y_i) + \delta_A(y_i)\log\,\delta_B(y_i)]. \tag{4}$$

with convention $0.\log 0 = 0$.

Next, we prove some properties of the proposed inaccuracy measure of SVNSs.

Theorem 3.1. Let $A, B, C \in SVNS(Y)$, the proposed inaccuracy measure satisfies the following properties:

a) $I_{SVNS}(A,B) = 0$ if and only if either $\rho_A(y_i) = \rho_B(y_i) = 0, \theta_A(y_i) = \theta_B(y_i) = 0, \delta_A(y_i) = \delta_B(y_i) = 0$ or $\rho_A(y_i) = \rho_B(y_i) = 1, \theta_A(y_i) = \theta_B(y_i) = 1, \delta_A(y_i) = \delta_B(y_i) = 1\; where\; i = 1,2,3,\dots,n\,;\; \forall A,B,C \in SVNS(Y).$

b) $I_{SVNS}(A, B \cup C) + I_{SVNS}(A, B \cap C) = I_{SVNS}(A,B) + I_{SVNS}(A,C)\;\; \forall A,B,C \in SVNS(Y).$

c) $I_{SVNS}(A \cup B, C) + I_{SVNS}(A \cap B, C) = (\,I_{SVNS}(A,C) + I_{SVNS}(B,C);\;\; \forall A,B,C \in SVNS(Y).$

d) $I_{SVNS}(A \cup B, A \cap B) + I_{SVNS}(A \cap B, A \cup B) = I_{SVNS}(A,B) + I_{SVNS}(B,A);\;\; \forall A,B \in SVNS(Y).$

Proof. a)

Let $I_{SVNS}(A,B) = 0$, then, from Eq. (4), we have

$$-\frac{1}{n}\sum_{i=1}^{n}[\rho_A(y_i)\log\,(\rho_B(y_i)) + \theta_A(y_i)\log\,\theta_B(y_i) + \delta_A(y_i)\log\,\delta_B(y_i)] \;\; = 0,$$

$[\rho_A(y_i)\log\,(\rho_B(y_i)) + \theta_A(y_i)\log\,\theta_B(y_i) + \delta_A(y_i)\log\,\delta_B(y_i)] \;\; = 0\;\; \forall\, i = 1,2,3,\dots,n.$

The above relation holds, if and only if

$\rho_A(y_i) = \rho_B(y_i) = 0, \theta_A(y_i) = \theta_B(y_i) = 0, \delta_A(y_i) = \delta_B(y_i) = 0$

or $\rho_A(y_i) = \rho_B(y_i) = 1, \theta_A(y_i) = \theta_B(y_i) = 1, \delta_A(y_i) = \delta_B(y_i) = 1\; \forall\, i = 1,2,3,\dots,n$.

Conversely,

Suppose, $\rho_A(y_i) = \rho_B(y_i) = 0, \theta_A(y_i) = \theta_B(y_i) = 0, \delta_A(y_i) = \delta_B(y_i) = 0$

or $\rho_A(y_i) = \rho_B(y_i) = 1, \theta_A(y_i) = \theta_B(y_i) = 1, \delta_A(y_i) = \delta_B(y_i) = 1\; \forall\, i = 1,2,3,\dots,n.$

i.e., $[\rho_A(y_i)\log\,(\rho_B(y_i)) + \theta_A(y_i)\log\,\theta_B(y_i) + \delta_A(y_i)\log\,\delta_B(y_i)] \;\; = 0$

or,

$$-\frac{1}{n}\sum_{i=1}^{n}[\rho_A(y_i)\log(\rho_B(y_i)) + \theta_A(y_i)\log\ \theta_B(y_i) +\ \delta_A(y_i)\log\ \delta_B(y_i)]\ \ = 0$$

Which implies that $I_{SVNS}(A, B) = 0.$ □

Proof. b)

For this, we divide the universal set Y into two disjoint subsets, *i.e.*,

$$Y_1 = \{\rho_A(y_i) \geq \rho_B(y_i) \geq \rho_C(y_i);\ \theta_A(y_i) \leq \theta_B(y_i) \leq \theta_C(y_i);\ \delta_A(y_i) \leq \delta_B(y_i) \leq \delta_C(y_i) |\ y_i \in Y\} \qquad (5)$$

$$Y_2 = \{\rho_A(y_i) \leq \rho_B(y_i) \leq \rho_C(y_i);\ \theta_A(y_i) \geq \theta_B(y_i) \geq \theta_C(y_i);\ \delta_A(y_i) \geq \delta_B(y_i) \geq \delta_C(y_i) |\ y_i \in Y\} \qquad (6)$$

Then by taking L.H.S, we have

$$I_{SVNS}(A, B \cup C) = -\tfrac{1}{n}\textstyle\sum_{Y_1}[\rho_A(y_i)\log(\rho_{B\cup C}(y_i)) + \theta_A(y_i)\log\ \theta_{B\cup C}(y_i) +\ \delta_A(y_i)\log\ \delta_{B\cup C}(y_i)] +$$

$$\left(-\frac{1}{n}\sum_{Y_2}[\rho_A(y_i)\log(\rho_{B\cup C}(y_i)) + \theta_A(y_i)\log\ \theta_{B\cup C}(y_i) +\ \delta_A(y_i)\log\ \delta_{B\cup C}(y_i)]\right)$$

Now using (5) and (6), we get

$$I_{SVNS}(A, B \cup C) = -\tfrac{1}{n}\textstyle\sum_{Y_1}[\rho_A(y_i)\log(\rho_B(y_i)) + \theta_A(y_i)\log\ \theta_C(y_i) +\ \delta_A(y_i)\log\ \delta_C(y_i)] -$$

$$\tfrac{1}{n}\textstyle\sum_{Y_2}[\rho_A(y_i)\log(\rho_C(y_i)) + \theta_A(y_i)\log\ \theta_B(y_i) +\ \delta_A(y_i)\log\ \delta_B(y_i)].$$

Which implies that

$$I_{SVNS}(A, B \cup C) =\ I_{SVNS}(A, B) + I_{SVNS}(A, C). \qquad (7)$$

Now by taking,

$$I_{SVNS}(A, B \cap C) = -\frac{1}{n}\sum_{Y_1}[\rho_A(y_i)\log(\rho_{B\cap C}(y_i)) + \theta_A(y_i)\log\ \theta_{B\cap C}(y_i) +\ \delta_A(y_i)\log\ \delta_{B\cap C}(y_i)]$$

$$-\frac{1}{n}\sum_{Y_2}[\rho_A(y_i)\log(\rho_{B\cap C}(y_i)) + \theta_A(y_i)\log\ \theta_{B\cap C}(y_i) +\ \delta_A(y_i)\log\ \delta_{B\cap C}(y_i)]$$

Now using (5) and (6), we get

$$I_{SVNS}(A, B \cap C) = -\frac{1}{n}\sum_{Y_1}[\rho_A(y_i)\log(\rho_C(y_i)) + \theta_A(y_i)\log\ \theta_B(y_i) +\ \delta_A(y_i)\log\ \delta_B(y_i)]$$

$$-\frac{1}{n}\sum_{Y_2}[\rho_A(y_i)\log(\rho_B(y_i)) + \theta_A(y_i)\log\ \theta_C(y_i) +\ \delta_A(y_i)\log\ \delta_C(y_i)]$$

which implies that

$$I_{SVNS}(A, B \cap C) =\ I_{SVNS}(A, B) + I_{SVNS}(A, C) \qquad (8)$$

Adding (7) and (8), we get

$$I_{SVNS}(A, B \cup C) + I_{SVNS}(A, B \cap C) \leq \left(I_{SVNS}(A, B) + I_{SVNS}(A, C)\right). \square$$

Proof. c)

By taking L.H.S, we have

$I_{SVNS}(A \cup B, C) = -\frac{1}{n}\sum_{Y_1}[\rho_{A\cup B}(y_i)\log(\rho_C(y_i)) + \theta_{A\cup B}(y_i)\log\ \theta_C(y_i) + \ \delta_{A\cup B}(y_i)\log\ \delta_C(y_i)]+$

$\left(-\frac{1}{n}\sum_{Y_2}[\rho_{A\cup B}(y_i)\log(\rho_C(y_i)) + \theta_{A\cup B}(y_i)\log\ \theta_C(y_i) + \ \delta_{A\cup B}(y_i)\log\ \delta_C(y_i)]\right).$

Now using (5) and (6), we get

$$I_{SVNS}(A \cup B, C) = -\frac{1}{n}\sum_{Y_1}[\rho_A(y_i)\log(\rho_C(y_i)) + \theta_B(y_i)\log\ \theta_C(y_i) + \ \delta_B(y_i)\log\ \delta_C(y_i)]$$

$-\frac{1}{n}\sum_{Y_2}[\rho_B(y_i)\log(\rho_C(y_i)) + \theta_A(y_i)\log\ \theta_C(y_i) + \ \delta_A(y_i)\log\ \delta_C(y_i)].$

which implies that

$$I_{SVNS}(A \cup B, C) = \ I_{SVNS}(A, C) + I_{SVNS}(B, C). \tag{9}$$

Again, by taking L.H.S, we have

$$\begin{aligned} I_{SVNS}(A \cap \ B, C) = &-\frac{1}{n}\sum_{Y_1}[\rho_{A\cap B}(y_i)\log(\rho_C(y_i)) + \theta_{A\cap B}(y_i)\log\ \theta_C(y_i) + \ \delta_{A\cap B}(y_i)\log\ \delta_C(y_i)] \\ &-\frac{1}{n}\sum_{Y_2}[\rho_{A\cap B}(y_i)\log(\rho_C(y_i)) + \theta_{A\cap B}(y_i)\log\ \theta_C(y_i) + \ \delta_{A\cap B}(y_i)\log\ \delta_C(y_i)]\,. \end{aligned}$$

Now using (5) and (6), we get

$$\begin{aligned} I_{SVNS}(A \cap B, C) = &-\frac{1}{n}\sum_{Y_1}[\rho_B(y_i)\log(\rho_C(y_i)) + \theta_A(y_i)\log\ \theta_C(y_i) + \ \delta_A(y_i)\log\ \delta_C(y_i)] \\ &-\frac{1}{n}\sum_{Y_2}[\rho_A(y_i)\log(\rho_C(y_i)) + \theta_B(y_i)\log\ \theta_C(y_i) + \ \delta_B(y_i)\log\ \delta_C(y_i)]. \end{aligned}$$

Which implies that

$$I_{SVNS}(A \cap \ B, C) = \ I_{SVNS}(A, C) + I_{SVNS}(B, C). \tag{10}$$

Adding (9) and (10), we get

$$I_{SVNS}(A, B \cup C) + I_{SVNS}(A, B \cap C) \leq \left(I_{SVNS}(A, C) + I_{SVNS}(B, C)\right). \ \square$$

Proof. d)

By taking two disjoint subsets of universal set Y, *i.e.,*

$$Y_1 = \{\rho_A(y_i) \geq \rho_B(y_i);\ \theta_A(y_i) \leq \theta_B(y_i);\ \delta_A(y_i) \leq \delta_B(y_i) |\ y_i \in Y\}, \tag{11}$$

$$Y_2 = \{\rho_A(y_i) \leq \rho_B(y_i);\ \theta_A(y_i) \geq \theta_B(y_i);\ \delta_A(y_i) \geq \delta_B(y_i) |\ y_i \in Y\}. \tag{12}$$

By using equations (5), (6) and by taking L.H.S, we have

$$\begin{aligned} I_{SVNS}(A \cup B, A \cap B) \\ = &-\frac{1}{n}\sum_{Y_1}[\rho_{A\cup B}(y_i)\log(\rho_{A\cap B}(y_i)) + \theta_{A\cup B}(y_i)\log\ \theta_{A\cap B}(y_i) + \ \delta_{A\cup B}(y_i)\log\ \delta_{A\cap B}(y_i)] \\ &-\frac{1}{n}\sum_{Y_2}[\rho_{A\cup B}(y_i)\log(\rho_{A\cap B}(y_i)) + \theta_{A\cup B}(y_i)\log\ \theta_{A\cap B}(y_i) + \ \delta_{A\cup B}(y_i)\log\ \delta_{A\cap B}(y_i)]. \end{aligned}$$

Now using (5) and (6), we get

$$I_{SVNS}(A \cup B, A \cap B) = -\frac{1}{n}\sum_{Y_1}[\rho_A(y_i)\log(\rho_B(y_i)) + \theta_B(y_i)\log\ \theta_A(y_i) + \ \delta_B(y_i)\log\ \delta_A(y_i)]$$

$$-\frac{1}{n}\sum_{Y_2}[\rho_B(y_i)\log(\rho_A(y_i)) + \theta_A(y_i)\log\ \theta_B(y_i) + \ \delta_A(y_i)\log\ \delta_B(y_i)].$$

Which implies that

$$I_{SVNS}(A \cup B, A \cap B)$$
$$= -\frac{1}{n}\sum_{Y_1 \cup Y_2}[\rho_A(y_i)\log(\rho_B(y_i)) + \theta_A(y_i)\log\ \theta_B(y_i) + \ \delta_A(y_i)\log\ \delta_B(y_i)]$$
$$-\frac{1}{n}\sum_{Y_1 \cup Y_2}[\rho_B(y_i)\log(\rho_A(y_i)) + \theta_B(y_i)\log\ \theta_A(y_i) + \ \delta_B(y_i)\log\ \delta_A(y_i)].$$

or

$$I_{SVNS}(A \cup B, A \cap B) = \ I_{SVNS}(A, B) + I_{SVNS}(B, A). \tag{13}$$

Now,

$$I_{SVNS}(A \cap \ B, A \cup B)$$
$$= \ -\frac{1}{n}\sum_{Y_1}[\rho_{A\cap B}(y_i)\log(\rho_{A\cup B}(y_i)) + \theta_{A\cap B}(y_i)\log\ \theta_{A\cup B}(y_i) + \ \delta_{A\cap B}(y_i)\log\ \delta_{A\cup B}(y_i)]$$
$$-\frac{1}{n}\sum_{Y_2}[\rho_{A\cap B}(y_i)\log(\rho_{A\cup B}(y_i)) + \theta_{A\cap B}(y_i)\log\ \theta_{A\cup B}(y_i) + \ \delta_{A\cap B}(y_i)\log\ \delta_{A\cup B}(y_i)].$$

Now using (5) and (6), we get

$$I_{SVNS}(A \cap B, A \cup B)$$
$$= -\frac{1}{n}\sum_{Y_1}[\rho_B(y_i)\log(\rho_A(y_i)) + \theta_A(y_i)\log\ \theta_B(y_i) + \ \delta_A(y_i)\log\ \delta_B(y_i)]$$
$$-\frac{1}{n}\sum_{Y_2}[\rho_A(y_i)\log(\rho_B(y_i)) + \theta_B(y_i)\log\ \theta_A(y_i) + \ \delta_B(y_i)\log\ \delta_A(y_i)].$$

Which implies that

$$I_{SVNS}(A \cup B, A \cap B)$$
$$= -\frac{1}{n}\sum_{Y_1 \cup Y_2}[\rho_A(y_i)\log(\rho_B(y_i)) + \theta_A(y_i)\log\ \theta_B(y_i) + \ \delta_A(y_i)\log\ \delta_B(y_i)]$$
$$-\frac{1}{n}\sum_{Y_1 \cup Y_2}[\rho_B(y_i)\log(\rho_A(y_i)) + \theta_B(y_i)\log\ \theta_A(y_i) + \ \delta_B(y_i)\log\ \delta_A(y_i)].$$

or

$$I_{SVNS}(A \cap \ B, A \cup B) \leq \ I_{SVNS}(A, B) + I_{SVNS}(B, A) \tag{14}$$

Adding (13) and (14), we get

$$I_{SVNS}(A \cup B, A \cap B) + I_{SVNS}(A \cap B, A \cup B) \leq \left(I_{SVNS}(A, B) + I_{SVNS}(B, A)\right).\square$$

In the next section, we investigate the application of the proposed inaccuracy measure.

4. Applications

In this section, we empirically illustrate the practical application of our proposed asymmetric measure of comparison in strategic decision-making and medical diagnosis.

4.1 Application to strategic decision-making

Let us consider a very pertinent corporate problem in which corporation Y wants to launch one of its five products using five strategies. Let the set of products be $P = \{C_1, C_2, C_3, C_4, C_5\}$, and the set of strategies be $S = \{Z_1, Z_2, Z_3, Z_4, Z_5\}$. Table 1 represents the weights of strategies of corporation Y in terms of memberships $\rho_y(Z_i)$, indeterminacy $\theta_y(Z_i)$ and non-membership value $\delta_y(Z_i)$ where $i = 1, 2, 3, 4, 5$. The weights have been assigned to these strategies because of their feasibility based on certain factors.

Table 1. Weights of strategies of the corporation as Single-Valued neutrosophic number

$(\rho_y(Z_1), \theta_y(Z_1), \delta_y(Z_1))$	$(\rho_y(Z_2), \theta_y(Z_2), \delta_y(Z_2))$	$(\rho_y(Z_3), \theta_y(Z_3), \delta_y(Z_3))$	$(\rho_y(Z_4), \theta_y(Z_4), \delta_y(Z_4))$	$(\rho_y(Z_5), \theta_y(Z_5), \delta_y(Z_5))$
(0.8, 0.2, 0.3)	(0.4, 0.4, 0.2)	(0.5, 0.4, 0.2)	(0.6, 0.2, 0.1)	(0.7, 0.5, 0.3)

It may be noted that $(\rho_y(Z_i), \theta_y(Z_i), \delta_y(Z_i))$ indicates the degree of importance, degree of inconclusiveness, and degree of the unimportance of strategy Z_i to the corporation in its implementation.

Table 2. represent the degree of importance, degree of inconclusiveness, and degree of the unimportance of products $(\rho_{C_j}(Z_i), \theta_{C_j}(Z_i), \delta_{C_j}(Z_i))$ concerning the strategy Z_i , where $j = 1, 2, 3, 4, 5$.

Table 2. Weights of strategies implementation for product launch as Single-Valued neutrosophic number

	$(\rho_{C_1}(Z_1), \theta_{C_1}(Z_1), \delta_{C_1}(Z_1))$	$(\rho_{C_2}(Z_2), \theta_{C_2}(Z_2), \delta_{C_2}(Z_2))$	$(\rho_{C_3}(Z_3), \theta_{C_3}(Z_3), \delta_{C_3}(Z_3))$	$(\rho_{C_4}(Z_4), \theta_{C_4}(Z_4), \delta_{C_4}(Z_4))$	$(\rho_{C_5}(Z_5), \theta_{C_5}(Z_5), \delta_{C_5}(Z_5))$
C_1	(0.5, 0.4, 0.4)	(0.6, 0.3,0.2)	(0.5, 0.2, 0.1)	(0.6, 0.2, 0.8)	(0.9, 0.2, 0.1)
C_2	(0.9, 0.3, 0.2)	(0.8, 0.7, 0.3)	(0.6, 0.3, 0.2)	(0.3, 0.4, 0.5)	(0.4, 0.2, 0.2)
C_3	(0.6, 0.5, 0.2)	(0.7, 0.6, 0.5)	(0.5, 0.3, 0.3)	(0.2, 0.5, 0.6)	(0.5, 0.4, 0.3)
C_4	(0.9, 0.2, 0.1)	(0.8, 0.8, 0.7)	(0.4, 0.3, 0.2)	(0.8, 0.5, 0.4)	(0.3, 0.4, 0.5)
C_5	(0.7, 0.7, 0.6)	(0.1, 0.5, 0.2)	(0.4, 0.3, 0.7)	(0.6, 0.3, 0.2)	(0.5, 0.4, 0.2)

In an objective of the corporation to launch a suitable product because of the suitability of the five strategies, with the minimum risk (inaccuracy in our case), we use Eq. (4) to compute inaccuracy measures $I_{SVNS}(Y, C_i)$, $i = 1, 2, 3, 4, 5$ using the data of Table 1 and Table 2.

Table 3. Inaccuracy measures between products and strategies

$\boldsymbol{I_{SVNS}(Y, C_1)}$	$\boldsymbol{I_{SVNS}(Y, C_2)}$	$\boldsymbol{I_{SVNS}(Y, C_3)}$	$\boldsymbol{I_{SVNS}(Y, C_4)}$	$\boldsymbol{I_{SVNS}(Y, C_5)}$
1.6421	1.5129	1.4554	**1.2930**	1.4933

According to the inaccuracy measures presented in Table 3, the product C_4 will be more suitable for launch because of the available strategies.

4.2 Application to Medical Diagnosis

First, we state the problem of medical diagnosis and present it in the framework of a neutrosophic environment.

Medical diagnosis: The process of identifying an actual disease of a patient based on their symptoms is termed a medical diagnosis.

Substantial uncertainties occur in most diagnostic decisions and can be handled using fuzzy methodologies. In medical science, several diseases have many symptoms in common. Therefore, identifying the appropriate illness from which a patient is suffering is difficult for physicians/experts. Let $D = \{D_1, D_2, D_3 \ldots . D_n\}$ be the set of diseases with several common symptoms and S $= \{r_1, r_2, r_3, \ldots, r_n\}$ be the set of symptoms of the patient under investigation. In this scenario, S is an SVNS and $D_1, D_2, D_3 \ldots . D_n$ are also SVNSs. We compare each of D_i with S. The patient is diagnosed with a disease D_i with which S is maximum directed closeness.

The flowchart of the process is shown in the figure 1.

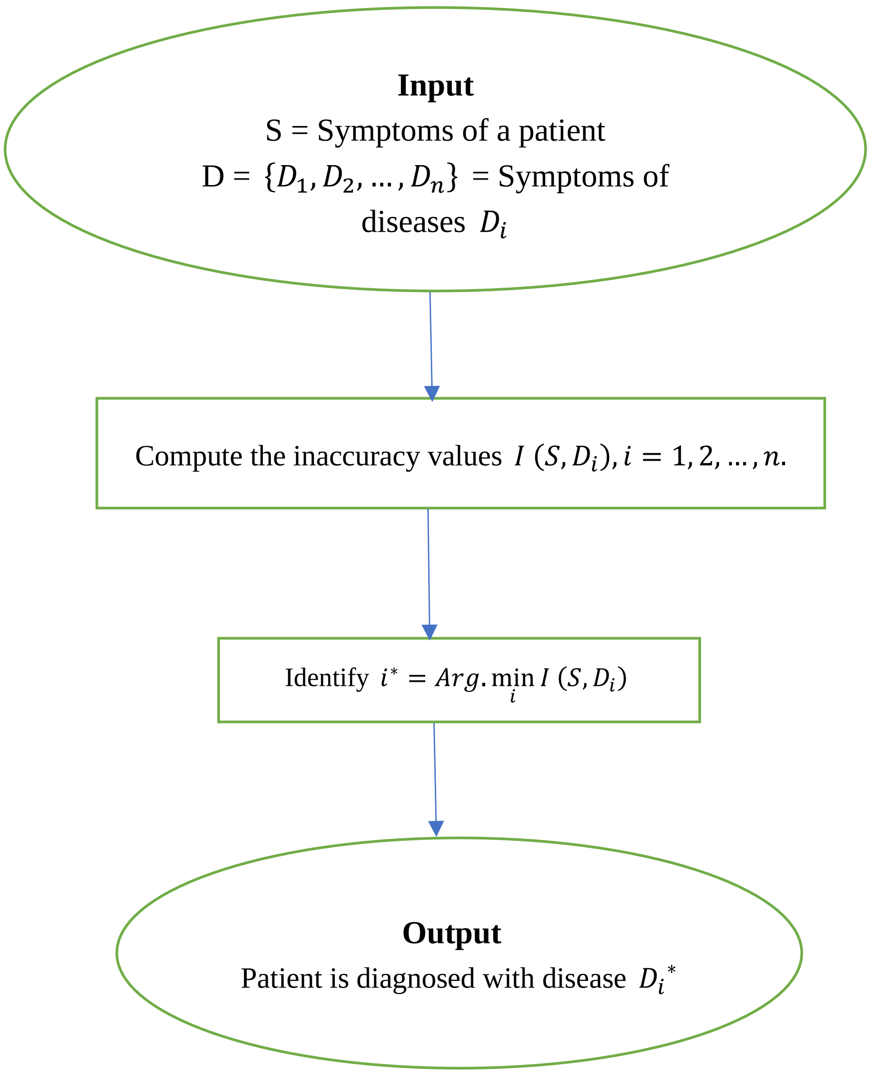

Figure1: Flowchart of Medical Diagnosis using Inaccuracy Measure

We consider the following numerical example to illustrate the procedure.

We present a numerical example to check the impact of the proposed inaccuracy measure.

Numerical Example: We consider a set of three diseases, $D = \{(D_1, viral\ fever)\ (D_2, malaria)\ (D_3, typhoid)\}$, each of which has three common symptoms given in set $R = \{(r_1, fever),\ (r_2, headache)\ \ (r_3, cough)\}$.

An expert team of doctors in the form of SVNSs assesses the characteristic information of the given diseases. The indicative information of symptoms and diagnosis for patients represented in the form of SVNSs as shown in Table 4:

Table 4. Characteristics information of the diseases described in the form of SVNSs

	r_1	r_2	r_3
D_1	(0.3, 0.2, 0.5)	(0.1, 0.3, 0.7)	(0.4, 0.3, 0.3)
D_2	(0.2, 0.2. 0.6)	(0.1, 0.1, 0.8)	(0.2, 0.3, 0.6)
D_3	(0.2, 0.1, 0.7)	(0.6, 0.3, 0.1)	(0.3, 0.4, 0.3)

The set P_1 represents the symptoms of the patient under investigation as an SVNS.

$P_1 = \{(r_1, (0.1, 0.2, 0.7))(r_2, (0.8, 0.2, 0.3))(r_3, (0.2, 0.4, 0.4))\}$.

Our task is to evaluate the closeness of P_1 with D_i using various SVN comparison measures.

To check the effectiveness of the proposed inaccuracy measure, we consider the following similarity/ distance measures for SVNSs.

$$S_1 = 1 - \frac{1}{n}\sum_{i=1}^{n} max.\{|\rho_A(y_i) - \rho_B(y_i)|, |\theta_A(y_i) - \theta_B(y_i)|, |\delta_A(y_i) - \delta_B(y_i)|\}.$$

(Bourmi and Smarandache [33])

$$S_2 = \frac{\sum_{i=1}^{n}\{min.(\rho_A(y_i),\rho_B(y_i)) + min.(\theta_A(y_i),\theta_B(y_i)) + min.(\delta_A(y_i),\delta_B(y_i))\}}{\sum_{i=1}^{n}\{max.(\rho_A(y_i),\rho_B(y_i)) + max.(\theta(y_i),\theta_B(y_i)) + max.(\delta_A(y_i),\delta_B(y_i))\}}.$$ (Majumdar and Samanta [17])

$$S_3 = \frac{1}{3n}\sum_{i=1}^{n}\left[\frac{min.(\rho_A(y_i),\rho_B(y_i))}{max.(\rho_A(y_i),\rho_B(y_i))} + \frac{min.(\theta_A(y_i),\delta_B(y_i))}{max.(\delta_A(y_i),\delta_B(y_i))} + \frac{min.(\delta_A(y_i),\delta_B(y_i))}{max.(\delta_A(y_i),\delta_B(y_i))}\right].$$ (Ye and Zhang [34])

$$S_4 = \frac{1}{n}\sum_{i=1}^{n}\left[1 - \frac{|\rho_A(y_i) - \rho_B(y_i)| + |\theta_A(y_i) - \theta_B(y_i)| + |\delta_A(y_i) - \delta_B(y_i)|}{3}\right].$$ (Ali Aydogdu [13])

$$S_5 = 1 - \frac{1}{3n}\sum_{y\in n}\left|\left(\rho_A{}^2(y_i) - \rho_B{}^2(y_i)\right) - \left(\theta_A{}^2(y_i) - \theta_B{}^2(y_i)\right) - \left(\delta_A{}^2(y_i) - \delta_B{}^2(y_i)\right)\right|.$$ (Chai *et al.* [9])

$$S_6 = \frac{1}{n}\sum_{i=1}^{n}\frac{2(\rho_A(y_i).\rho_B(y_i) + \theta_A(y_i).\theta_A(y_i) + \delta_A(y_i).\delta_A(y_i))}{(\rho_A{}^2(y_i) + \theta_A{}^2(y_i) + \delta_A{}^2(y_i)) + (\rho_B{}^2(y_i) + \theta_B{}^2(y_i) + \delta_B{}^2(y_i))}.$$ (Ye [35])

$$DM_1 = 1 - \frac{1}{n}\sum_{i=1}^{n}\left[1 - \frac{|\rho_A(y_i) - \rho_B(y_i)| + |\theta_A(y_i) - \theta_B(y_i)| + |\delta_A(y_i) - \delta_B(y_i)|}{3}\right].$$

(Ali Aydogdu [13])

$$DM_2 = \frac{1}{3n}\sum_{y\in n}\left|\left(\rho_A{}^2(y_i) - \rho_B{}^2(y_i)\right) - \left(\theta_A{}^2(y_i) - \theta_B{}^2(y_i)\right) - \left(\delta_A{}^2(y_i) - \delta_B{}^2(y_i)\right)\right|.$$ (Chai *et al.* [9])

$$DM_3 = \frac{1}{n}\sum_{y\in n}\left(|\rho_A{}^2(y_i) - \rho_B{}^2(y_i)| \vee |\theta_A{}^2(y_i) - \theta_B{}^2(y_i)| \vee |\delta_A{}^2(y_i) - \delta_B{}^2(y_i)|\right).$$ (Chai *et al.* [9])

Now, compute the similarity/distance measure between patient P_1 and diagnosis D. Similarly, we compute the proposed inaccuracy measure between the patient and the diagnosis. Table 5 shows the result obtained by calculating the different existing and proposed inaccuracy measures.

Table 5: The similarity /distance measures between the symptoms of a patient $P_1$and diagnosis D_i

	D_1	D_2	D_3	Ranking
$S_1(P_1, D_i)$	0.5867	0.68	0.8734	$D_3 > D_2 > D_1$
$S_2(P_1, D_i)$	0.5238	0.5609	0.75	$D_3 > D_2 > D_1$
$S_3(P_1, D_i)$	0.0582	0.0623	0.25	$D_3 > D_2 > D_1$
$S_4(P_1, D_i)$	0.3108	0.3108	0.33	$D_3 > D_2 > D_1$
$S_5(P_1, D_i)$	0.8556	0.8389	0.9523	$D_3 > D_1 > D_2$
$S_6(P_1, D_i)$	0.7806	0.7987	0.9598	$D_3 > D_2 > D_1$
$DM_1(P_1, D_i)$	0.6892	0.6892	0.67	$D_3 > D_2 > D_1$
$DM_3(P_1, D_i)$	0.1444	0.1611	0.0477	$D_3 > D_1 > D_2$
$DM_4(P_1, D_i)$	0.33	0.32	0.1266	$D_3 > D_2 > D_1$
$I(D_i, P_1)$	1.7949	1.5818	1.314	$D_3 > D_2 > D_1$
$I(P_1, D_i)$	2.0504	2.0283	1.5870	$D_3 > D_2 > D_1$

Analysis: From the Table 5, we observe that all the comparison measures diagnosing the patient P_1 for Typhoid. Our proposed asymmetric comparison measure from both directions $D_i \rightarrow P_1$ and $P_1 \rightarrow D_i$ also resulting the same diagnosis (refer last two rows of the Table 5). Thus, we conclude that our proposed measure is consistent with existing models. The proposed model is more effective from the following observations.

In the figure 2, the directed comparison $I(D_i, P_1)$ shows the greater discriminating capability within the diseases. Thus, the proposed asymmetric measure is sensitive to the direction of comparison from the view point of the discriminating power. In the considered numerical problem, the diagnostic result due to both directed comparisons ($I(D_i, P_1)$ and $I(P_1, D_i)$) remains same but discriminating power is different.

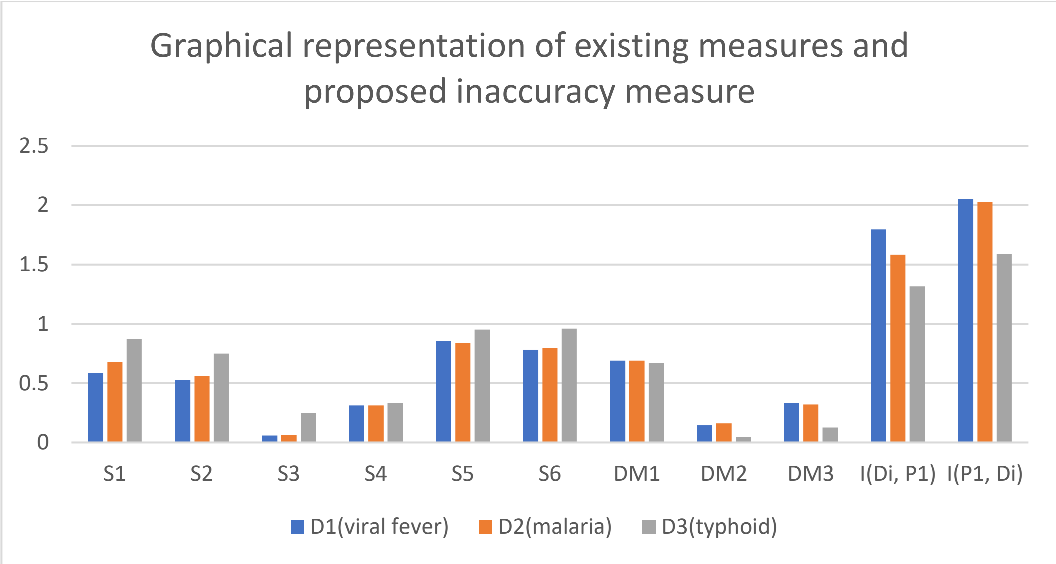

Figure 2: Graphical representation of computed values of closeness due to various measures in Table 5

From the above graph, we see that the diagnosis of patient P_1 is typhoid. It shows that the proposed inaccuracy measure is feasible and effective.

In the next section, we compare some existing divergence measures, similarity measures, and the proposed inaccuracy measure.

5. Comparative Study

To check the superiority of the proposed inaccuracy measure, we consider the numerical example obtained from Thao and Smarandache [36].

Let us suppose, for universal set $U = \{u_1, u_2, u_3, \dots, u_m\}$, there are n patterns in the form of neutrosophic set $\{A_1, A_2, A_3, \dots, A_n\}$. Suppose that we have an unknown sample B. Our goal is to classify sample B into which pattern A_i.

For this, we have to calculate the proposed inaccuracy measure, existing divergence measures, and similarity measures of unknown sample B with each pattern $A_i(n = 1, 2, 3, \dots n)$.

Assume A_1= {(u_1,0.7,0.7,0.2), (u_2,0.7,0.8,0.4), (u_3,0.6,0.8,0.2)}.

A_2= {(u_1,0.5,0.7,0.3), (u_2,0.7,0.7,0.5), (u_3,0.8,0.6,0.1)}.

A_3={(u_1,0.9,0.5,0.1), (u_2,0.7,0.6,0.4), (u_3,0.8,0.5,0.2)}.

Unknown Sample

B = {(u_1,0.7,0.8,0.4), (u_2,0.8,0.5,0.3), (u_3,0.5,0.8,0.5)}.

For the comparative study, we consider all measures listed in section 4 along with the following existing divergence measures and similarity measures:

$$S_7 = \frac{1}{n}\sum_{i=1}^{n}\left\{\frac{\left(\rho_A{}^2(y_i)\wedge\rho_B{}^2(y_i)\right)}{\left(\rho_A{}^2(y_i)\vee\rho_B{}^2(y_i)\right)} + \frac{\left((1-\theta_A{}^2(y_i))\wedge(1-\theta_B{}^2(y_i)\right)}{\left((1-\theta_A{}^2(y_i))\vee(1-\theta_B{}^2(y_i)\right)} + \frac{\left((1-\delta_A{}^2(y_i))\wedge(1-\delta_B{}^2(y_i)\right)}{\left((1-\delta_A{}^2(y_i))\vee(1-\delta_B{}^2(y_i)\right)}\right\}. \qquad \text{(Chai \textit{et al.} [9])}$$

$$DM_4(A,B) = \frac{1}{n}\sum_{i=1}^{n}\left[D_T{}^i(A,B) + D_I{}^i(A,B) + D_F{}^i(A,B)\right]. \qquad \text{(Thao and Smarandache [36])}$$

where, $DM_T{}^i(A,B) = \rho_A(y_i)\ln\frac{2\rho_A(y_i)}{\rho_A(y_i)+\rho_B(y_i)} + \rho_B(y_i)\ln\frac{2\rho_B(y_i)}{\rho_A(y_i)+\rho_B(y_i)}$

$$DM_I{}^i(A,B) = \theta_A(y_i)\ln\frac{2\theta_A(y_i)}{\theta_A(y_i)+\theta_B(y_i)} + \theta_B(y_i)\ln\frac{2\theta_B(y_i)}{\theta_A(y_i)+\theta_B(y_i)}$$

$$D_F{}^i(A,B) = \delta_A(y_i)\ln\frac{2\delta_A(y_i)}{\delta_A(y_i)+\delta_B(y_i)} + \delta_B(y_i)\ln\frac{2\delta_B(y_i)}{\delta_A(y_i)+\delta_B(y_i)}$$

$DM_j(A,B) =$

$$\sum_{i=1}^{n} 2^\alpha\left[\frac{\left(\sqrt{\rho_A(y_i)} - \sqrt{\rho_B(y_i)}\right)^{2(\alpha+1)}}{(\rho_A(y_i)+\rho_B(y_i))^\alpha} + \frac{\left(\sqrt{1-\rho_A(y_i)} - \sqrt{1-\rho_B(y_i)}\right)^{2(\alpha+1)}}{(2-\rho_A(y_i)+\rho_B(y_i))^\alpha}\right] +$$

$$\sum_{i=1}^{n} 2^\alpha\left[\frac{\left(\sqrt{\theta_A(y_i)} - \sqrt{\theta_B(y_i)}\right)^{2(\alpha+1)}}{\left(\theta_A(y_i)+\theta_B(y_i)\right)^\alpha} + \frac{\left(\sqrt{1-\theta_A(y_i)} - \sqrt{1-\theta_B(y_i)}\right)^{2(\alpha+1)}}{\left(2-\theta_A(y_i)+\theta_B(y_i)\right)^\alpha}\right] +$$

$$\sum_{i=1}^{n} 2^\alpha\left[\frac{\left(\sqrt{\delta_A(y_i)} - \sqrt{\delta_B(y_i)}\right)^{2(\alpha+1)}}{\left(\delta_A(y_i)+\delta_B(y_i)\right)^\alpha} + \frac{\left(\sqrt{1-\delta_A(y_i)} - \sqrt{1-\delta_B(y_i)}\right)^{2(\alpha+1)}}{\left(2-\delta_A(y_i)+\delta_B(y_i)\right)^\alpha}\right]; \; j = 5,6. \quad \text{(Guleria et al. [37])}$$

The result obtained by calculating the proposed inaccuracy measure, existing divergence measure, and similarity measure of unknown sample B with each pattern A_i is shown in Table 7.

Table 7. Result of the Existing Similarity Measure and Proposed Divergence Measure, along with the Degree of Confidence

	(A_1, B)	(A_2, B)	(A_3, B)	DOC
S_1	0.7333	0.7333	**0.7666**	**0.0666**
S_2	**0.7762**	0.7036	0.6745	**0.1743**
S_3	**0.7620**	0.6781	0.64	**0.2059**
S_4	**0.8666**	0.7996	0.7776	**0.156**
S_5	**0.9933**	0.9622	0.9555	**0.0689**
S_6	**0.9844**	0.9319	0.9203	**0.1166**
S7	**0.8021**	0.7063	0.6993	**0.1986**
D_1	**0.1333**	0.2004	0.2224	**0.1562**
D_2	**0.0067**	0.0378	0.0445	**0.0689**
D_3	**0.25**	0.29	0.2966	**0.0866**
DM_4	**0.1537**	0.2674	0.2951	**0.25513**
$DM_5(when\ \alpha = 1)$	**0.0352**	0.1090	0.1161	**0.1547**
$DM_6(when\ \alpha = 4)$	**0.0001**	0.0103	0.0032	**0.0133**
$I_{SVNS}(B_i, A)$	**1.4854**	1.7288	1.8444	**0.6024**
$I_{SVNS}(A_i, B))$	1.2092	1.2554	**1.1457**	**0.1732**

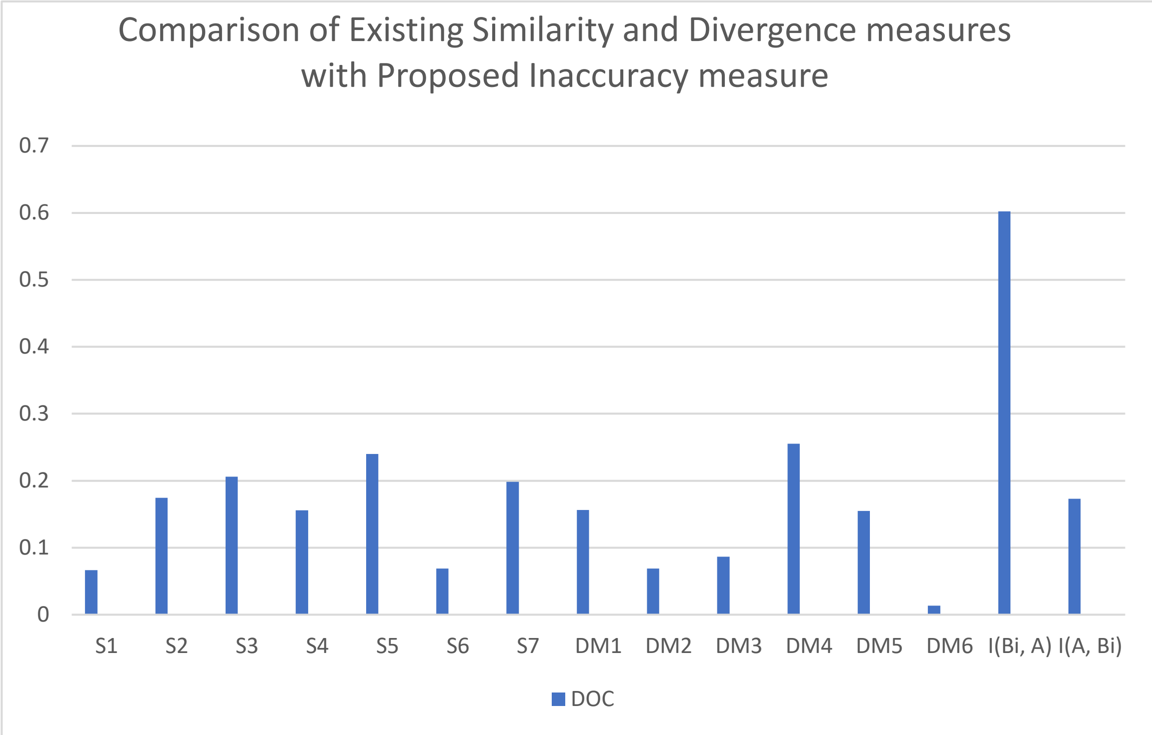

Figure 3: Graphical representation of degree of confidence of various comparison measures

Analysis: The highest value of similarity, the lowest value of divergence/inaccuracy, and the degree of confidence of the existing similarity, divergence measure, and the proposed inaccuracy measure are in bold in Table 7. The computed values of the comparison measures indicate that unknown sample B belongs to the pattern A_1. Only S_1 shows a different result. Our proposed inaccuracy measure's highest value of DOC, when the direction of comparison is $B_i \rightarrow A$. This justifies its effectiveness over other comparison measures, as illustrated graphically in the figure 3.

6. Conclusion

In this work, we have proposed an inaccuracy measure for SVNSs, to find directed discrimination between two SVNSs and studied some of their mathematical properties. The illustrative numerical problem in a corporate crisis of product launch has shown the applicability of the proposed measure. In addition, the advantage of the proposed measure has been justified by using a performance index DOC and in a medical diagnosis problem. The proposed asymmetric comparison measure may be impactful to the various studies in data science, machine learning and computer vision requiring a directed comparative analysis. The limitation of this article is that all the investigations have been done using hypothetical data. In future, we plan to investigate the applications of the suggested asymmetric comparison metrics in cluster analysis, multiple attribute decision-making, and medical diagnosis using real data sets. However, applying the proposed measures to actual data sets needs an efficient method of converting the crisp data to single valued neutrosohic data set without potential loss of information. Thus, formulating a suitable data conversion process because of the given scenario is also a problem for future investigations. Some recent studies [21-25] investigate the applicability of neutrosophic methods in various disciplines like decision-making, pattern recognition, inventory management, pollution in megacities, etc. We also plan to explore the relevance of the proposed approach to these disciplines.

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