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Augmented Latin Square Designs for Imprecise Data

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Abstract: This paper addresses a novel approach for analyzing augmented Latin square design with uncertain observations, the so-called neutrosophic augmented Latin square design (NALSD). The contribution of our work lies in estimating the effects of rows, columns, control and new treatments, as well as formulating their sums of squares. Moreover, by determining the neutrosophic hypotheses and decision rule, the F_N-statistic in NANOVA table is given. The performance of the proposed design is evaluated using a numerical example and simulation study. In light of the results observed, it can find that the NALSD performs better than the classic augmented Latin square design (ALSD) in the presence of uncertainty.
Keywords: Augmented Latin square design, neutrosophic statistics, imprecise data, neutrosophic ANOVA.

1. Introduction

In the field of experimental design, the Latin square is one of the most common designs to control systematic error by two-way blocking. In this design, each treatment occurs once, and only once, in each row and column. Thus, the number of treatments, rows, and columns are all equal. In this context, Fisher [1] was the first to apply Latin Square designs. Many studies have been published on this design; however, several problems arose when using large samples, such as many genotypes in the early stages of plant breeding. Researchers have devised an appropriate solution to this problem using augmented designs. The augmented design is appropriate since it incorporates many additional entries for various treatments. This design aims to compare new genotypes against standard treatments, known as checks. The first research on augmented design as a blocking design was conducted by Federer [2]. There have been several classes of augmented designs, including the augmented randomized complete block and augmented Latin squares [3, 4], augmented Lattice squares [5], and augmented row-column designs with a small number of checks [6]. A review of augmented designs has been given by Federer and Crossa [7]. In a newer study, an augmented design without replicating all treatments was discussed by Burgueño, et al. [8]. More about the augmented designs can be viewed in [9-15]. None of the above-mentioned researches is applicable if there is uncertainty in data set regarding to collected unreliable observation.

Recently, neutrosophic logic has been extensively studied by Smarandache [16]. Smarandache [17] developed the idea of basic neutrosophic statistics (NS) as an extension of classical basic statistics and suggested that these statistics can be used effectively in uncertain situations. The difference between fuzzy statistics, neutrosophic statistics, and classical statistics were explained by Aslam [18]. The concept of neutrosophic ANOVA was introduced by Aslam [19]. Neutrosophic analysis of covariance has been applied to completely randomized designs as well as randomized complete block designs and split-plot designs by AlAita and Aslam [20]. AlAita, et al. [21] provided a discussion on the application of neutrosophic statistical analysis in split-plot designs. AlAita and Talebi [22] furnished exact neutrosophic analysis of missing value in augmented randomized complete block design. Aslam and Albassam [23] proposed post hoc multiple comparison tests under NS. Salama, et al. [24] suggested neutrosophic correlation and simple linear regression. Nagarajan, et al. [25] discussed the analysis of neutrosophic multiple regression. Numerous neutrosophic statistical studies have been discussed in [26-33].

Based on our knowledge no research on augmented Latin square designs is in indeterminate environments. This study aims to solve problems associated with studies and experiments that use imprecise and uncertain data in augmented Latin square designs. Also, we developed our proposed design under NS to provide additional information on the indeterminacy measure that classic statistics cannot provide.

2. Neutrosophic Basic Definitions

This section provides some basic concepts about neutrosophic statistics that will be useful throughout of this paper. Throughout this paper, suppose that $X_N \in [X_L, X_U]$ is a neutrosophic random variable (NRV) that follows the neutrosophic normal distribution (NND).

Definition 1: Consider the neutrosophic random variable (NRV) $X_N = X_L + X_U I_N$, the neutrosophic population mean and variance can be found as follows:

$$\mu_N \in \left[\frac{\sum_{i=1}^{N} x_{Li}}{N}, \frac{\sum_{i=1}^{N} x_{Ui}}{N}\right];\ \mu_N \in [\mu_L, \mu_U] \text{ and } \sigma_N^2 \in \left[\frac{\sum_{i=1}^{N}(x_{Li}-\mu_L)^2}{N}, \frac{\sum_{i=1}^{N}(x_{Ui}-\mu_U)^2}{N}\right];\ \sigma_N^2 \in [\sigma_L^2, \sigma_U^2],$$

where X_L and $X_U I_N$ are determinate and indeterminate parts, respectively, and $I_N \in [I_L, I_U]$ is the measure of uncertainty.

Definition 2: Suppose n be a neutrosophic random sample selected from a population of size N having indeterminate observations. The estimated neutrosophic sample mean $\bar{x}_N$ and the variance s_N^2, are expressed by

$$\bar{x}_N \in \left[\frac{\sum_{i=1}^{n} x_{Li}}{n}, \frac{\sum_{i=1}^{n} x_{Ui}}{n}\right];\ \bar{x}_N \in [\bar{x}_L, \bar{x}_U] \text{ and } s_N^2 \in \left[\frac{\sum_{i=1}^{n}(x_{Li}-\bar{x}_L)^2}{n-1}, \frac{\sum_{i=1}^{n}(x_{Ui}-\bar{x}_U)^2}{n-1}\right];\ s_N^2 \in [s_L^2, s_U^2].$$

3. Neutrosophic Augmented Latin Square Design (NALSD)

3.1. Neutrosophic Model and Notations

Consider a $b \times b$ Latin square, the neutrosophic statistical model for a NALSD can be formulated as follows:

$$y_{Nhijkg} = \mu_N + \alpha_{Ni} + \beta_{Nj} + \tau_{Nqk} + \tau_{Nlijg} + \varepsilon_{Nhijkg}, \begin{cases} i = 1,2,\dots,b \\ j = 1,2,\dots,b \\ k = 1,2,\dots,b \\ g = 1,2,\dots,n_{(lij)} \end{cases} \quad (1)$$

The neutrosophic form of y_{Nhijkg} can be expressed as

$$y_{Nhijkg} = y_{Lhijkg} + y_{Uhijkg} I_N;\ I_N \in [I_L, I_U],$$

where $h = l \text{ or } q$ stands for the neutrosophic effects associated with new treatments or checks, respectively, μ_N is a neutrosophic overall mean, α_{Ni} is the neutrosophic effect of the ith row, β_{Nj} is the neutrosophic effect of the jth column, τ_{Nqk} is the neutrosophic effect of the kth check, τ_{Nlijg} is the neutrosophic effect of the gth new treatment in ith row and jth column, and ε_{Nhijkg} is the neutrosophic random error assumed to have mean zero and variance σ_N^2. We denote $\mathrm{v} = \sum_{i=1}^{b}\sum_{j=1}^{b} n_{(lij)}$ for the number of new treatments, c for the number of check treatments, a for the number of rows, and b for the number of columns; therefore, $e = \mathrm{v} + b$ is the total number of new and check treatments and the total number of all plots in the blocks (rows and columns) is n; i.e., $n = \mathrm{v} + b^2$. Throughout the paper in the context of neutrosophic ANOVA, the SS_{NT}, SS_{NR}, SS_{NC}, SS_{NTr}, and SS_{NE} stand for the neutrosophic sum of squares (NSS) total, row, column, treatment, and error, respectively and the subscript N denotes the neutrosophic context.

3.2. Estimation of Neutrosophic Parameters

To estimate the neutrosophic model parameters in a NALSD, first, the least squares normal equations (NE) are obtained and given below.

μ_N: $(\mathrm{v}+b^2)\hat{\mu}_N + (b-1)\sum_{k=1}^{b}\hat{\tau}_{Nqk} + \sum_{i=1}^{b}\sum_{j=1}^{b} n_{(lij)}\,\hat{\alpha}_{Ni} + \sum_{j=1}^{b}\sum_{i=1}^{b} n_{(lij)}\,\hat{\beta}_{Nj} = y_{N....}$,

α_{Ni}: $(b+\sum_{j=1}^{b} n_{(lij)})(\hat{\mu}_N + \hat{\alpha}_{Ni}) + \sum_{k=1}^{b}\hat{\tau}_{Nqk} + \sum_{j=1}^{b}\sum_{g=1}^{n(lij)}\hat{\tau}_{Nlijg} + \sum_{j=1}^{b} n_{(lij)}\hat{\beta}_{Nj} = y_{N.i..}$,

β_{Nj}: $(b+\sum_{i=1}^{b} n_{(lij)})(\hat{\mu}_N + \hat{\beta}_{Nj}) + \sum_{k=1}^{b}\hat{\tau}_{Nqk} + \sum_{i=1}^{b}\sum_{g=1}^{n(lij)}\hat{\tau}_{Nlijg} + \sum_{i=1}^{b} n_{(lij)}\hat{\alpha}_{Ni} = y_{N..j.}$,

τ_{Nqk}: $b(\hat{\mu}_N + \hat{\tau}_{Nqk}) = y_{Nq..k}$,

τ_{Nlijg}: $\hat{\mu}_N + \hat{\alpha}_{Ni} + \hat{\beta}_{Nj} + \hat{\tau}_{Nlijg} = y_{Nlijg}$.

By solving the above NE using the constraints $\sum_{i=1}^{b}\hat{\alpha}_{Ni} = 0$, $\sum_{j=1}^{b}\hat{\beta}_{Nj} = 0$, and $\sum_{k=1}^{b}\hat{\tau}_{Nqk} + \sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{g=1}^{n(lij)}\hat{\tau}_{Nlijg} = 0$, the estimates of the neutrosophic parameters of the model (1) are

$\hat{\mu}_N = \frac{1}{(\mathrm{v}+b)}\left(y_{N....} - (b-1)\sum_{k=1}^{b}\bar{y}_{Nq..k}\right) = \frac{1}{(\mathrm{v}+b)}(y_{N....} - (b-1)m_N)$; $\hat{\mu}_N \in [\hat{\mu}_L, \hat{\mu}_U]$,

$\hat{\alpha}_{Ni} = \frac{1}{b}\left(y_{Nqi..} - \sum_{k=1}^{b}\bar{y}_{Nq..k}\right) = \frac{1}{b}(y_{Nqi..} - m_N)$; $\hat{\alpha}_{Ni} \in [\hat{\alpha}_{Li}, \hat{\alpha}_{Ui}]$,

$\hat{\beta}_{Nj} = \frac{1}{b}\left(y_{Nq.j.} - \sum_{k=1}^{b}\bar{y}_{Nq..k}\right) = \frac{1}{b}(y_{Nq.j.} - m_N)$; $\hat{\beta}_{Nj} \in [\hat{\beta}_{Lj}, \hat{\beta}_{Uj}]$,

$\hat{\tau}_{Nqk} = \frac{y_{Nq..k}}{b} - \hat{\mu}_N$; $\hat{\tau}_{Nqk} \in [\hat{\tau}_{Lqk}, \hat{\tau}_{Uqk}]$,

$\hat{\tau}_{Nlijg} = y_{Nlijg} - \hat{\alpha}_{Ni} - \hat{\beta}_{Nj} - \hat{\mu}_N$; $\hat{\tau}_{Nlijg} \in [\hat{\tau}_{Llijg}, \hat{\tau}_{Ulijg}]$,

where $i, j, k = 1,2,\ldots,b$, $g = 1,2,\ldots,n_{(lij)}$ and $m_N = \sum_{i=1}^{b}\bar{y}_{Nqi..} = \sum_{j=1}^{b}\bar{y}_{Nq.j.} = \sum_{k=1}^{b}\bar{y}_{Nq..k}$.

In the same manner, the estimation of the parameters in corresponding neutrosophic treatment-reduced, row-reduced, and column-reduced models can be obtained.

3.3. Neutrosophic Testing of Parameters

Under the normality assumption of the data, it can use the ANAVO method to test neutrosophic parameters in NALSD. Therefore, we need to formulate the SS_{NT} and neutrosophic adjusted (adj) and unadjusted (unadj) sums of squares for rows, columns, treatments (new and check), and the NSS for error. Following, the calculated sums of squares are given.

$SS_{NT} = \sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{k=1}^{b} y_{Nqijk}^2 + \sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{g=1}^{n(lij)} y_{Nlijg}^2 - \frac{y_{N....}^2}{n}$; $SS_{NT} \in [SS_{LT}, SS_{UT}]$,

$SS_{NR(\text{unadj})} = \sum_{i=1}^{b}\frac{y_{N.i..}^2}{b+n_{(lij)}} - \frac{y_{N....}^2}{n}$; $SS_{NR(\text{unadj})} \in [SS_{LR(\text{unadj})}, SS_{UR(\text{unadj})}]$,

$SS_{NC(\text{unadj})} = \sum_{j=1}^{b}\frac{y_{N..j.}^2}{b+n_{(lij)}} - \frac{y_{N....}^2}{n}$; $SS_{NC(\text{unadj})} \in [SS_{LC(\text{unadj})}, SS_{UC(\text{unadj})}]$,

$SS_{NTr(\text{unadj})} = \frac{1}{b}\sum_{k=1}^{b} y_{Nq..k}^2 + \sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{g=1}^{n(lij)} y_{Nlijg}^2 - \frac{y_{N....}^2}{n}$; $SS_{NTr(\text{unadj})} \in [SS_{LTr(\text{unadj})}, SS_{UTr(\text{unadj})}]$,

$SS_{NR(\text{adj.})} = \frac{1}{b}\left[\sum_{i=1}^{b}(y_{Nqi..} - m_N)\,y_{N.i..} - \sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{g=1}^{n(lij)}(y_{Nqi..} - m_N)y_{Nlijg}\right]$; $SS_{NR(\text{adj})}[SS_{LR(\text{adj})}, SS_{UR(\text{adj})}]$,

$SS_{NC(\text{adj.})} = \frac{1}{b}\left[\sum_{j=1}^{b}(y_{Nq.j.} - m_N)\,y_{N..j.} - \sum_{j=1}^{b}\sum_{i=1}^{b}\sum_{g=1}^{n(lij)}(y_{Nq.j.} - m_N)y_{Nlijg}\right]$; $SS_{NC(\text{adj})}[SS_{LC(\text{adj})}, SS_{UC(\text{adj})}]$,

$SS_{NTr(\text{adj})} = \frac{1}{b}\left(\sum_{i=1}^{b} y_{Nqi..}^2 + \sum_{j=1}^{b} y_{Nq.j.}^2 + \sum_{k=1}^{b} y_{Nq..k}^2\right) - \frac{\left(\sum_{i=1}^{b} y_{N.i..}^2 + \sum_{j=1}^{b} y_{N..j.}^2\right)}{(b+\mathrm{v})} + \sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{g=1}^{n(lij)} y_{Nlijg}^2 - 2m_N^2 + \frac{y_{N....}^2}{n}$; $SS_{NTr(\text{adj})} \in [SS_{LTr(\text{adj})}, SS_{UTr(\text{adj})}]$,

$SS_{NCheck} = \frac{1}{b}\sum_{k=1}^{b} y_{Nq..k}^2 - \frac{y_{Nq...}^2}{b^2}$; $SS_{NCheck} \in [SS_{LCheck}, SS_{UCheck}]$,

$SS_{Nnew} = \sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{g=1}^{n(lij)} y_{Nlijg}^2 - \frac{y_{NL...}^2}{\mathrm{v}}$; $SS_{Nnew} \in [SS_{Lnew}, SS_{Unew}]$,

$SS_{N\text{new and new}\times\text{ch}} = SS_{NTr(\text{adj})} - SS_{NCheck}$; $SS_{N\text{new and new}\times\text{ch}} \in [SS_{L\text{new and new}\times\text{ch}}, SS_{U\text{new and new}\times\text{ch}}]$,

$SS_{N\text{new}\times\text{check}} = SS_{NTr(\text{unadj})} - SS_{NCheck} - SS_{Nnew}$; $SS_{N\text{new}\times\text{check}} \in [SS_{L\text{new}\times\text{check}}, SS_{U\text{new}\times\text{check}}]$, and

$SS_{NE} = SS_{NT} - SS_{NTr(\text{adj})} - SS_{NR(\text{unadj})} - SS_{NC(\text{unadj})}$.

Neutrosophic mean squares for all source of variations are obtained in the ranges of the form $[MS_{L(.)}, MS_{U(.)}]$. Based on the calculated MSEs, the neutrosophic test statistics F_N are:

$F_{NTr(\text{adj})} = \frac{MS_{NTr(\text{adj})}}{MS_{NE}}$; $F_{NTr(\text{adj})} \in [F_{LTr(\text{adj})}, F_{UTr(\text{adj})}]$,
$F_{NR(\text{adj})} = \frac{MS_{NR(\text{adj})}}{MS_{NE}}$; $F_{NR(\text{adj})} \in [F_{LR(\text{adj})}, F_{UR(\text{adj})}]$,
$F_{NC(\text{adj})} = \frac{MS_{NC(\text{adj})}}{MS_{NE}}$; $F_{NC(\text{adj})} \in [F_{LC(\text{adj})}, F_{UC(\text{adj})}]$,
$F_{NCheck} = \frac{MS_{NCheck}}{MS_{NE}}$; $F_{NCheck} \in [F_{LCheck}, F_{UCheck}]$,
$F_{Nnew} = \frac{MS_{Nnew}}{MS_{NE}}$; $F_{Nnew} \in [F_{Lnew}, F_{Unew}]$,
$F_{N\text{new and new} \times \text{ch}} = \frac{MS_{N\text{new and new} \times \text{ch}}}{MS_{NE}}$; $F_{N\text{new and new} \times \text{ch}} \in [F_{L\text{new and new} \times \text{ch}}, F_{U\text{new and new} \times \text{ch}}]$, and
$F_{N\text{new} \times \text{check}} = \frac{MS_{N\text{new} \times \text{check}}}{MS_{NE}}$; $F_{N\text{new} \times \text{check}} \in [F_{L\text{new} \times \text{check}}, F_{U\text{new} \times \text{check}}]$.

The neutrosophic form of F_N is $F_N = F_L + F_U I_{F_N}$; $I_{F_N} \in [I_{F_L}, I_{F_U}]$, where F_L and $F_U I_{F_N}$ are determinate and indeterminate parts of each proposed test. This test reduces to a test under classic statistics if $I_{F_N} = 0$.

3.4. Neutrosophic Hypotheses and Decision Rules

In order to test the rows, columns, checks, and new treatments, the null and alternative hypotheses are as follows, respectively:

$H_{N0}: \alpha_{Ni} = 0$ vs $H_{N1}: at\ least\ one\ \alpha_{Ni} \neq 0,\ i = 1.2.,,,.b,$
$H_{N0}: \beta_{Nj} = 0$ vs $H_{N1}: at\ least\ one\ \beta_{Nj} \neq 0,\ j = 1.2.,,,.b,$
$H_{N0}: \tau_{Nqk} = 0$ vs $H_{N1}: at\ least\ one\ \tau_{Nqk} \neq 0,\ k = 1.2.,,,.b,$
$H_{N0}: \tau_{Nlijg} = 0$ vs $H_{N1}: at\ least\ one\ \tau_{Nlijg} \neq 0,\ g = 1.2.,,,.n_{(lij)}.$

The null hypothesis does not reject if $min\{p_N - value\} > \alpha$, where α is a level of significance. Meanwhile, we reject the null hypothesis if $max\{p_N - value\} \leq \alpha$.
All the above testing process are summarized in the NANOVA Tables 1 and 2 for the NALSD under NS.

Table 1 NANOVA Table (A) for NALSD

Sources of variation	**Ndf**	**NSS**	**NMS**	**F_N-value**
Rows (unadj)	$b-1$	$SS_{NB(\text{unadj})}$	$\frac{SS_{NR(\text{unadj})}}{b-1}$	
Columns (unadj)	$b-1$	$SS_{NB(\text{unadj})}$	$\frac{SS_{NC(\text{unadj})}}{b-1}$	
Treatments (adj)	$b+\text{v}-1$	$SS_{NTr(\text{adj})}$	$\frac{SS_{NTr(\text{adj})}}{b+\text{v}-1}$	$\frac{MS_{NTr(\text{adj})}}{MS_{NE}}$
Checks	$b-1$	SS_{NCheck}	$\frac{SS_{NCheck}}{b-1}$	$\frac{MS_{NCheck}}{MS_{NE}}$
New and New × Check	v	$SS_{N\text{new and new} \times \text{ch}}$	$\frac{SS_{N\text{new and new} \times \text{ch}}}{\text{v}}$	$\frac{MS_{N\text{new and new} \times \text{ch}}}{MS_{NE}}$
Error	$(b-1)(b-2)$	SS_{NE}	$\frac{SS_{NE}}{(b-1)(b-2)}$	
Total	$n-1$	SS_{NT}		

Table 2 NANOVA Table (B) for NALSD

Sources of variation	**Ndf**	**NSS**	**NMS**	**F_N-value**

Rows (adj)	$b-1$	$SS_{NB(\text{adj})}$	$\frac{SS_{NR(\text{adj})}}{b-1}$	$\frac{MS_{NR(\text{adj})}}{MS_{NE}}$
Columns (adj)	$b-1$	$SS_{NB(\text{adj})}$	$\frac{SS_{NC(\text{adj})}}{b-1}$	$\frac{MS_{NC(\text{adj})}}{MS_{NE}}$
Treatments (unadj)	$c+\text{v}-1$	$SS_{NTr(\text{unadj})}$	$\frac{SS_{NTr(\text{unadj})}}{c+\text{v}-1}$	
Checks	$c-1$	SS_{NCheck}	$\frac{SS_{NCheck}}{c-1}$	$\frac{MS_{NCheck}}{MS_{NE}}$
New treatments	$\text{v}-1$	SS_{Nnew}	$\frac{SS_{Nnew}}{\text{v}-1}$	$\frac{MS_{Nnew}}{MS_{NE}}$
New × Check	1	$SS_{N\text{new}\times\text{check}}$	$\frac{SS_{N\text{new}\times\text{check}}}{1}$	$\frac{MS_{N\text{new}\times\text{check}}}{MS_{NE}}$
Error	$(b-1)(b-2)$	SS_{NE}	$\frac{SS_{NE}}{(b-1)(b-2)}$	
Total	$n-1$	SS_{NT}		

4. Numerical Examples and Simulation

In this section, the performance of the proposed design is numerically assessed by an example and a simulation study. For assessing the efficiency of the proposed methods, the proposed tests $F_N \in [F_L, F_U]$ of the proposed design under NS are calculated and compared with the existing tests under classic statistics.

4.1. Numerical Example

In this example, we have generated neutrosophic data for NALSD. Five neutrosophic check treatments named A, B, C, D and E, and 50 neutrosophic new treatments, denoted by 1,2,…,50, are arranged in an augmented Latin square with 5 rows and 5 columns. The neutrosophic data are given in Table 5.

Using the computational software R, we can obtain neutrosophic data randomly for this example by running the following code

```
y_L<-rnorm(75,40,10)
z<-length(y_L)
I<-rnorm(75,3,0.5)
y_U<-c()
for(i in 1:z){
 y_U[i]<-y_L[i]+I[i]}
```

We applied the proposed method to calculate F_N-tests, where $F_N \in [F_L, F_U]$. The corresponding NANOVA results for the NALSD are presented in Tables 3 and 4.

4.2. Simulation Study

This section evaluates the quality of the proposed F test for NALSD using simulated data from the Monte Carlo (MC) procedure for the proposed model (1). In this study, MC simulations have been performed 10,000 times. The data have been generated using neutrosophic normal standard distribution. Furthermore, the neutrosophic variances have been assumed to be homogeneous, and the NALSDs are balanced. Also, to simulate type I error, the significance levels of 0.05 and 0.01 have been chosen as the initial values. Moreover, it has been assumed that the treatments all have zero mean under the null hypothesis. It has

Table 3 ANOVA Table (A) for the NALSD

Sources of variation	Ndf	NSS	NMS	F_N	Neutrosophic form F_N	p_N-value
Rows (unadj)	4	[373.906, 409.200]	[93.476, 102.300]			
Columns (unadj)	4	[355.476, 347.226]	[88.869, 86.806]			
Treatments (adj)	54	[6312.786, 6486.414]	[116.903, 120.119]	[1.054,1.062]	$1.054 + 1.062I_{F_N}$; $I_{F_N} \in [0, 0.007]$	[0.493, 0.486]
Checks	4	[163.869, 181.964]	[40.967, 45.491]	[0.369,0.402]	$0.369 + 0.402I_{F_N}$; $I_{F_N} \in [0, 0.082]$	[0.826, 0.804]
New and New × Check	50	[6148.917, 6304.450]	[122.978, 126.089]	[1.109,1.115]	$1.109 + 1.115I_{F_N}$; $I_{F_N} \in [0, 0.005]$	[0.449, 0.444]
Error	12	[1331.077, 1357.583]	[110.923, 113.132]			
Total	74	[8373.244, 8600.422]				

Table 4 ANOVA Table (B) for the NALSD

Sources of variation	Ndf	NSS	NMS	F_N	Neutrosophic form F_N	p_N-value
Rows (adj)	4	[591.937, 613.179]	[147.984, 153.295]	[1.334,1.355]	$1.334 + 1.355I_{F_N}$; $I_{F_N} \in [0, 0.015]$	[0.313, 0.306]
Columns (adj)	4	[193.643, 198.487]	[48.411, 49.622]	[0.436,0.439]	$0.436 + 0.439I_{F_N}$; $I_{F_N} \in [0, 0.007]$	[0.780, 0.778]
Treatments (unadj)	54	[6256.587, 6431.174]	[115.863, 119.096]			
Checks	4	[163.869, 181.964]	[40.967, 45.491]	[0.369,0.402]	$0.369 + 0.402I_{F_N}$; $I_{F_N} \in [0, 0.082]$	[0.826, 0.804]
New treatments	49	[6085.317, 6239.574]	[124.190, 127.338]	[1.120,1.126]	$1.120 + 1.126I_{F_N}$; $I_{F_N} \in [0, 0.005]$	[0.440, 0.436]
New × Check	1	[7.401, 9.637]	[7.401, 9.637]	[0.067,0.085]	$0.067 + 0.085I_{F_N}$; $I_{F_N} \in [0,0.212]$	[0.801, 0.775]
Error	12	[1331.077, 1357.583]	[110.923, 113.132]			
Total	74	[8373.244, 8600.422]				

been compared the significance levels and power of the test for the proposed test with the existing test under classic statistics.

To calculate the neutrosophic empirical Type I error rate and the test power for an MC experiment; the following steps need to be completed:

MC simulation for computing $\boldsymbol{\alpha}_{\textbf{Empirical}}$

Step 1: We generate the random sample $x_{N1}^{(i)}, x_{N2}^{(i)}, \dots, x_{Nn}^{(i)}$ from the neutrosophic normal standard distribution under H_{N0}, $i = 1,2,\dots,10000$.

Step 2: We compute the F_{Ni}-test under H_{N0}.

Step 3: We record the results by recording $I_{Ni} = 1$ when the H_{N0} is rejected, and $I_{Ni} = 0$ otherwise.

Step 4: We compute the ratio $\frac{1}{10000}\sum_{i=1}^{10000} I_{Ni}$ and take it as $\alpha_{\text{Empirical}}$.

MC simulation for computing $\boldsymbol{Power_{Empirical}}$

Step 1: We generate the random sample $x_{N1}^{(i)}, x_{N2}^{(i)}, \dots, x_{Nn}^{(i)}$ from the neutrosophic normal standard distribution under H_{N1}, $i = 1,2,\dots,10000$. For instance, $(\mu_{N1}, \mu_{N2}, \mu_{N3}, \mu_{N4}) = (1,2,3,4)$.

Step 2: We compute the F_{Ni}-test under H_{N1}.

Step 3: We record the results by recording $I_{Ni} = 1$ when the H_{N1} is rejected, and $I_{Ni} = 0$ otherwise.

Step 4: We compute the ratio $\frac{1}{10000}\sum_{i=1}^{10000} I_{Ni}$ and take it as $Power_{Empirical}$.

Table 5 Data for NALSD

Row	Column 1			Column 2			Column 3			Column 4			Column 5		
1	C [41.82, 44.92]	19 [39.79, 43.76]	37 [36.57, 39.68]	44 [25.12, 28.5]	8 [28.33, 31.23]	A [46.65, 50.38]	25 [37.95, 41.12]	B [35.98, 38.8]	35 [44.6, 47.91]	E [35.84, 38.27]	2 [42.52, 45.4]	26 [33.44, 36.42]	10 [32.99, 36.1]	D [31.88, 34.88]	24 [26.79, 29.21]
2	46 [36.41, 39.49]	12 [47.62, 50.68]	E [32.62, 36.32]	39 [28.09, 31.2]	C [55.38, 58.39]	4 [44.2, 47.73]	A [38.73, 42.47]	16 [60.19, 63.64]	11 [41.38, 44.3]	27 [38.9, 41.03]	D [38, 41.06]	28 [36.38, 39.33]	5 [45.34, 48.56]	45 [33.31, 36.51]	B [31.93, 34.42]
3	34 [39.22, 41.81]	D [36, 38.08]	21 [32.41, 36.03]	15 [39.74, 42.45]	1 [20.84, 23.21]	E [30, 32.73]	C [36.74, 39.25]	42 [29.98, 33.29]	36 [39.55, 42.18]	7 [39.95, 42.29]	B [20.34, 22.76]	32 [38.65, 40.58]	20 [36.98, 40.51]	38 [49.72, 52.26]	A [46.27, 50.09]
4	17 [47.01, 49.78]	B [49.68, 52.08]	6 [11.91, 14.94]	D [34.69, 37.77]	13 [53.81, 57.21]	31 [52.44, 56.24]	E [22.18, 24.44]	30 [40.75, 43.62]	9 [36.94, 39.56]	22 [38.69, 41.36]	A [29.62, 33.42]	41 [51.11, 54.6]	C [17.42, 20.22]	49 [33.59, 36.65]	33 [27.33, 30.05]
5	A [33.52, 36.34]	23 [19.12, 22.91]	43 [36.9, 39.8]	18 [11.39, 14.4]	48 [63.66, 66.76]	B [44.54, 47.56]	47 [43.05, 45.73]	D [48.56, 51.31]	29 [43.98, 47.66]	3 [49.46, 52.71]	50 [56.28, 60.34]	C [52.67, 55.82]	40 [17.73, 19.86]	E [45.29, 48.38]	14 [43.91, 47.75]

Table 6 Simulation results for NALSD with parameters $(b = 4, \mathrm{v} = 32, n = 48)$ *for Check treatment means* $(\mu_{N1}, \mu_{N2}, \mu_{N3}, \mu_{N4})$ *and different values of new treatments* $\left(\mu_{Ni} = 0, \mu_{Nj} = 0, \mu_{Nk} = 1, \mu_{Nl} = 2\right)$, $i = 1,\ldots,10, j = 11, \ldots, 20, k = 21,\ldots,30, l = 31, \ldots, 40$.

Test	α	Mean $\alpha_{\text{Empirical}}$	Mean $Power_{Empirical}$ $\delta_1 = (0,1,1,2)$	$\delta_2 = (1,2,2,3)$	$\delta_3 = (1,1,3,3)$	$\delta_4 = (0,1,2,3)$	$\delta_5 = (0,1,3,4)$	$\delta_6 = (0,3,4,4)$	$\delta_7 = (0,3,4,5)$	$\delta_8 = (0,2,4,6)$
NALSD	0.01	[0.0088, 0.0093]	[0.0412,0.0420]	[0.0688, 0.0734]	[0.0840, 0.0905]	[0.1072, 0.1188]	[0.1314, 0.1502]	[0.2130, 0.2366]	[0.2821, 0.3144]	[0.3514, 0.3940]
	0.05	[0.0476, 0.0477]	[0.1647, 0.1800]	[0.2463, 0.2732]	[0.2977, 0.3203]	[0.3495, 0.3857]	[0.4202, 0.4550]	[0.5702, 0.6135]	[0.6696, 0.7036]	[0.7493, 0.7884]

Table 7 Simulation results for NALSD with parameters $(b = 5, \mathrm{v} = 50, n = 75)$ *for Check treatment means* $(\mu_{N1}, \mu_{N2}, \mu_{N3}, \mu_{N4}, \mu_{N5})$ *and different values of new treatments* $\left(\mu_{Ni} = 0, \mu_{Nj} = 0, \mu_{Nk} = 1, \mu_{Nl} = 1, \mu_{Nu} = 2\right)$, $i = 1,\ldots,10, j = 11, \ldots, 20, k = 21,\ldots,30, l = 31, \ldots, 40, u = 41, \ldots, 50$.

Test	$\boldsymbol{\alpha}$	Mean $\alpha_{\text{Empirical}}$	Mean $Power_{Empirical}$ $\delta_1 = (0,0,1,1,1)$	$\delta_2 = (0,1,1,2,2)$	$\delta_3 = (0,1,2,2,2)$	$\delta_4 = (1,1,2,2,3)$	$\delta_5 = (2,2,3,3,3)$	$\delta_6 = (2,3,3,3,4)$	$\delta_7 = (0,1,3,4,4)$	$\delta_8 = (0,2,4,5,6)$
NALSD	0.01	[0.0094,0.0099]	[0.0581,0.0683]	[0.0820,0.0980]	[0.0980, 0.1207]	[0.1159, 0.1442]	[0.2317, 0.2865]	[0.3651, 0.4467]	[0.4485, 0.5319]	[0.8435, 0.9010]
	0.05	[0.0485,0.0494]	[0.2006, 0.2314]	[0.2668, 0.3080]	[0.2975, 0.3495]	[0.3616, 0.4112]	[0.5460, 0.6190]	[0.7164, 0.7826]	[0.7885, 0.8474]	[0.9845, 0.9945]

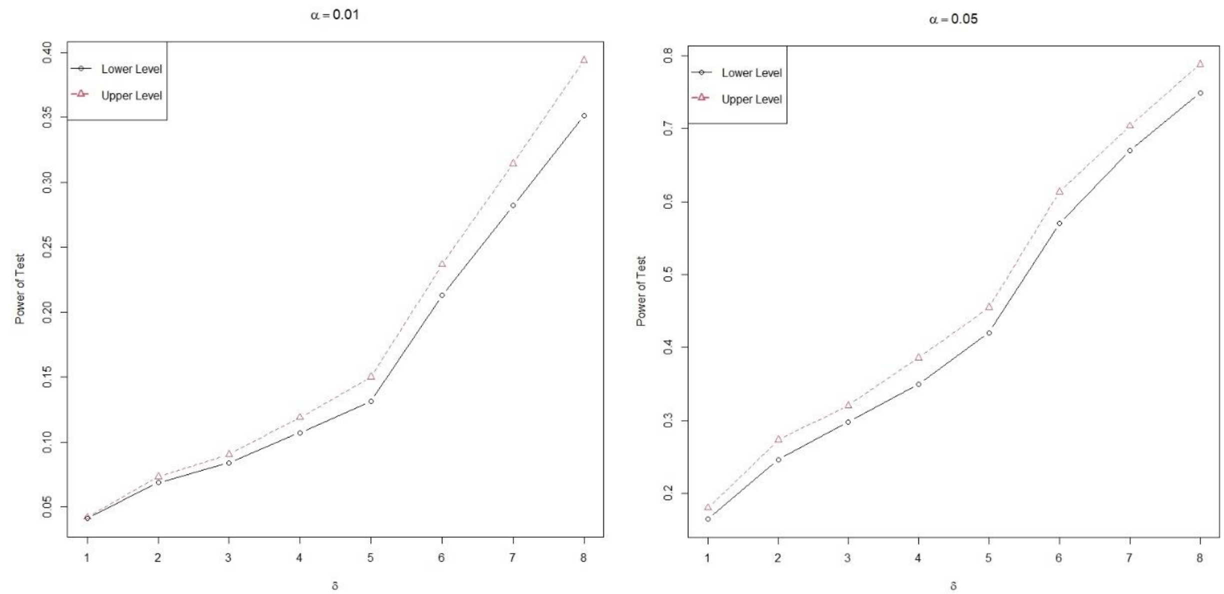

Figure 1 Power curves of the new and existing tests for NALSD with parameters $(b = 4, \mathrm{v} = 32, n = 48)$

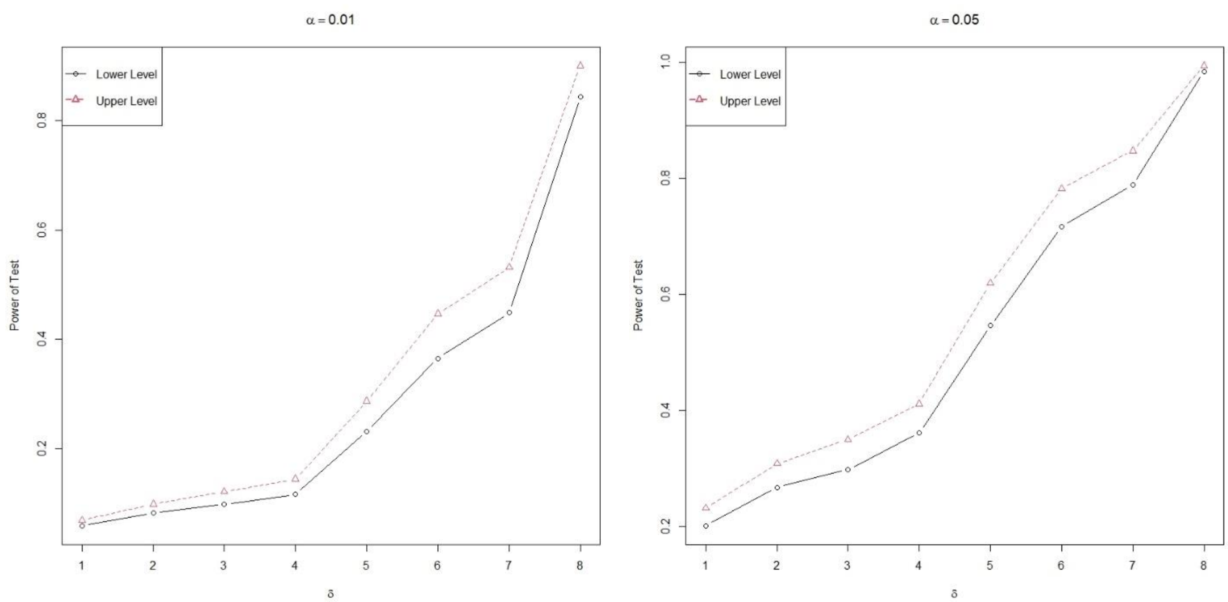

Figure 2 Power curves of the new and existing tests for NALSD with parameters $(b = 5, \mathrm{v} = 50, n = 75)$

The power of the test for the treatment effects through the NAN was calculated for neutrosophic data. The results are given in Tables 6 and 7 for NALSD with $(b = 4, \mathrm{v} = 32, n = 48)$ and $(b = 5, \mathrm{v} = 50, n = 75)$, for different sets of neutrosophic check means, $(\mu_{N1}, \mu_{N2}, \mu_{N3}, \mu_{N4})$ and $(\mu_{N1}, \mu_{N2}, \mu_{N3}, \mu_{N4}, \mu_{N5})$. The power of the test for the proposed and existing approaches' performance in Tables 6 and 7 is displayed in Figures 1 and 2.

Without loss of generality, the powers were plotted in ascending order. Evidently, the power of the test for the indeterminate part is higher than the power for the determinate part; so, the proposed approach performs better than the existing one in testing the treatment effects.

5. Comparative Study

As mentioned earlier, the proposed design is a generalization of the augmented Latin square design under classical statistics. The proposed F_N-test for NALSD reduces to the existing F-test for ALSD when all observations in the data are exact, determined and certain. Throughout this section, the proposed F_N-test is compared to the existing F-test in terms of the measure of indeterminacy, accuracy, flexibility, and information. For the purpose of comparison, the neutrosophic form of the F_N-test for the proposed design of the effects of treatments can be expressed as follows:

$$1.054 + 1.062I_{F_N};\ I_{F_N} \in [0, 0.007]$$

Note that the neutrosophic form can be reduced to a statistic under classical statistics when $I_{F_N} = 0$; So, the first part of the neutrosophic form 1.054 describes the value of the test statistic under classical statistics. The second part $1.062I_{F_N}$ illustrates the indeterminate portion of the neutrosophic form. Additionally, this test has a measure of indeterminacy of 0.007. According to the proposed test, the values of the F_N-test, $F_N \in [F_L, F_U]$ are flexible and lie in the indeterminate interval that is $F_N \in [1.054, 1.062]$. Based on the proposed test, it is expected that $F_N \in [F_L, F_U]$ may range from 1.054 to 1.062 under an uncertain environment. This range distinguishes the proposed test from the existing test under classical statistics, which is based on the determined value, which does not appropriate under uncertain conditions. Additionally, this test provides additional information about the testing approach when indeterminacy is present; namely, it provides additional information about the testing procedure which is the measure of indeterminacy. To illustrate the numerical example, for testing H_{N0} (means of treatment are equal), the probability that it will be accepted is 0.95, the probability that it will be rejected when true is 0.05, and the probability of uncertainty about it is 0.007.

Moreover, Tables 6 and 7 provide a comparative evaluation of the relative effectiveness of the proposed test in terms of the $\alpha_{\text{Empirical}}$ and $Power_{Empirical}$. The results indicate that the $\alpha_{\text{Empirical}}$ of the proposed test is close to 0.05 under NS. In addition, Figures 1 and 2 indicate that the curves of the power of the test for the indeterminate part are higher than those for the determinate part. This emphasizes that the indeterminate part plays an important role in uncertain environments. According to the results of the study, the proposed test for NALSD under NS is more informative, accuracy, and flexible than the test for ALSD under classical statistics.

6. Conclusion

This article introduces neutrosophic augmented Latin square design as a generalization to the existing augmented Latin square design. In this context, the statistical model and a NANOVA approach have been presented for the proposed design to deal with neutrosophic hypotheses and the decision rule about the treatment effects in the design. Besides, the performance of the proposed design has been evaluated using a numerical example and a simulation study. According to the results, the proposed design led to more accuracy in analyzing practical problems in uncertainty. It is conjectured that, based on the proposed design, many new investigations will be carried out in the future. Moreover, in practical experiments using the proposed design with uncertain data will be analyzed more precisely.

Declarations

Conflicts of Interest: The authors declare no conflict of interest.

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