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On Derived Superhyper BE-Algebras

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Abstract. Florentin Smarandache, introduced a creative idea named superhyper algebras as a popularization of algebras, via the nested power sets, superhyper operation, and superhyper axiom. This paper used the significance of superhyper BEsubalgebras as a popularization of BE subalgebras and investigated attributes of this pioneer significance in logic algebra. We prove that every BE algebras can be extended to a superhyper BE subalgebra and show how a superhyper BE algebra is a popularization of a hyper BE algebra and a BE algebra. The identity element in superhyper BE algebras acts a main designation in the form, attribute, analysis, and communication of other elements in these types of superhyper logic algebras.

Keywords: BE algebra, hyper BE algebra, superhyper BE algebra, generalized operation.

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1. Introduction

Logic algebra is one of the superlative important algebraic interdisciplinary structures that is used in various engineering sciences, especially computer science and all related branches. In this theory, on each finite and arbitrary collection, axiom principles are defined according to the ruling logic and its application in the actual religion, which gives this collection a special rule and rule. Based on these principles, a logical algebra is formed that all elements under this algebra must follow a certain rule. The importance of logical algebras is so great that many researchers in different fields are investigating its attribute and importance. During the research, they also found some deficiencies in the field of these algebras, which they solved with new definitions and popularizations of its principles. One of the ways to fix the defects in these algebras is to generalize them to logical algebraic significances and superstructures, which leads to the popularization and correction of the subject principles in logical algebras. Logical algebraic hyperstructures by covering the shortcomings of logical algebras can have more applications in the actual universe, especially when the proportion amongst a set of objects is discussed. One of the important logic algebras is the class of BE algebras, which is applied in the computer sciences. An mentioned, the BE algebras as a special of logic algebras, has and still has shortcomings, so the of hyper BE algebras is started to solve the defects in these logic algebras. The hyper BE algebras extended in recent years and has many applications. In this theory, for every given two elements, is created a set of elements that are related to axiom principles and must follow their principles. In this theory, just two elements can be combined and related to a set. But if we want to have more than two elements, we can't follow the axiom principles in hyper BE algebras. Therefore, this causes a defect in the application of these logical hyperalgebras and it is necessary to eliminate this defect. Based on these defects, Smarandache presented a new significance titled superhyper algebras as a popularization of hyperalgebras which have disparate attribute and are relevant with the actual universe [12–14]. In this theory, we can for every given more than two elements consider a set of sets of elements that are related under axiom principles and so we cover the defects in the of hyperalgebras. After then and based on these unprecedented significances, some researchers investigated some varius of superhyperalgebras. Hamidi, et al. in the realm of indeterminate logic (hyper) algebra, bring forward the significance of neutro BCK subalgebras. Beside, Rahmati et al. introduced the significance of superhyper BCK algebras as a popularization of BCK algebras and investigated some attribute of this unprecedented significance [6]. They published the significance of eextension of G algebras to superhyper G Algebras, wherethrough has nice outcomes in superhyper logic algebras. Some kinds of literature in this scops wherethrough we use for our work is such as on hyper K algebras [1], systems of propositional calculi [8], on hyper BE algebras [9], on fuzzy subalgebras of BE algebras [10], extension of G algebras to superhyper G algebras [11], popularizations and alternatives of classical algebraic structures to neutroalgebraic structures and antialgebraic structures [15], Implicative ideals of BCK algebras based on MBJ neutrosophic sets, on commutative BE algebras [17], extended fuzzy BCK Subalgebras [18], compactness and neutrosophic topological space via grills, neutrosophic systems with applications [2], separation axioms in neutrosophic topological spaces, neutrosophic systems with applications [3], separation axioms in neutrosophic topological spaces [4] and new types of topologies and neutrosophic topologies [16]. In this research, we try to apply the axioms in superhyper algebras and present the notation of superhyper BE algebras and analyze the connection between of BE algebras and superhyper BE algebras.

Motivation: Considering the ideas and creativity presented in new mathematics, especially Smarandache mathematics, we are looking to increase the communication of mathematics with applied and interdisciplinary topics. Development and expansion of logical algebra of superhypr BE algebras, can be the basis for discussions related to the communication of engineering sciences, especially computer sciences. Our whole motive for presenting this research is that superhypr BE algebras considering the nested super actions, it can study and research the sets of elements of whole-to-whole and part-to-whole communication elements in the first, middle and outer layers.

2. Preliminaries

In what lies ahead, we recollect some significances that require at follows.

Definition 2.1. [7] A system $(X;\sigma,\iota)$ known as a BE algebra provided,

$(BE1)$ $x\sigma x=\iota$,

$(BE2)$ $x\sigma\iota=\iota$,

$(BE3)$ $\iota\sigma x=x$,

$(BE4)$ $x\sigma(y\sigma z)=y\sigma(x\sigma z)$,

wherethrough $x,y,z\in X$ are arbitrary. An arbitrary BE algebra (X,σ,ι) is commutative, provided for each $x,y\in X,(x\sigma y)\sigma y=(y\sigma x)\sigma x$.

Proposition 2.2. [7] *Assume X is an arbitrary BE algebra. Afterwards for all $x,y\in X$*

(i) $x\sigma(y\sigma x)=\iota$,

(ii) $y\sigma((y\sigma x)\sigma x)=\iota$.

Definition 2.3. [10] Assume H is an arbitrary nondevoid set, $\epsilon\in X$ and $\varrho:H^2\to\mathcal{P}^*(H)$ be a map. Afterwards (H,ϱ,ι) known as a hyper BE–algebra, provided

(HBE_1) $\iota\in x\varrho\iota$ and $\iota\in x\varrho x$,

(HBE_2) $x\varrho(y\varrho z)=y\varrho(x\varrho z)$,

(HBE_3) $x\in\iota\varrho x$,

(HBE_4) provided $\iota\in\iota\varrho x$, so $x=\iota$,

wherethrough $x,y,z\in H$.

Theorem 2.4. [10] *In every hyper BE algebra H,*

(i) $A\varrho(B\varrho C)=B\varrho(A\varrho C)$,

(ii) $\iota\in A\varrho A$,

(iii) *provided* $\iota\in\iota\varrho A$, *afterwards* $\iota\in A$,

(iv) $\iota\in x\varrho(y\varrho x)$,

(v) *provided* $\iota\in x\varrho(y\varrho z)$, *afterwards* $\iota\in y\varrho(x\varrho z)$,

(vi) $\iota\in(x\varrho y)\varrho y$,

(vii) *provided* $z\in x\varrho y$, *afterwards* $\iota\in x\varrho(z\varrho y)$,

$(viii)$ *provided* $y\in\iota\varrho x$, *afterwards* $\iota\in y\varrho x$,

wherethrough $A,B,C\subseteq H$.

Definition 2.5. [12,13] Let X be an arbitrary nondevoid set and $\iota \in X$. For a map $\vartheta^*_{\{m\to n\}} : X^m \to \mathcal{P}^n_*(X)$, $(X, \vartheta^*_{\{m\to n\}})$ known as an $\{m \to n\}$-super hyperalgebra, that $\mathcal{P}^n_*(X)$ is the n^{th} nested powerset of X and $\emptyset \notin \mathcal{P}^n_*(X)$.

3. On development of BE subalgebra

In what follows, present the significance of superhyper BE subalgebras based on the popularization of axioms of BE algebras and make a connection between of these logic superhyper algebras and BE algebras.

In the following, we prove a proposition that is fundamental in our work.

Lemma 3.1. *Assume (X, σ, ι) is a BE algebra. Afterwards $x\sigma(y\sigma z) = \iota\sigma(x\sigma(y\sigma z))$, whence $x, y, z \in X$.*

Proof. Whereof for every $x \in X$,, we have $x\varrho\iota = x$, we get $x\sigma(y\sigma z) = \iota\sigma(x\sigma(y\sigma z))$, whence $x, y, z \in X$. □

According to Lemma 3.1, we describe the significance of $\{m \to n\}$ superhyper BE subalgebras.

Definition 3.2. Let $m-2 = \kappa$, X be a nonavoided set and $\iota \in X$. Afterwards $(X, \vartheta^*_{\{m\to n\}}, \iota)$ known as an $\{m \to n\}$ superhyper BE subalgebra, provided

(i) $\iota \in \vartheta^*_{\{m\to n\}}\underbrace{(x, x, \ldots, x, x)}_{m-times}$,

(ii) $\iota \in \vartheta^*_{\{m\to n\}}(x, \underbrace{(\iota, \iota, \ldots, \iota, \iota)}_{(\lambda)-times})$,

(iii) $x \in \vartheta^*_{\{m\to n\}}(\underbrace{(\iota, \iota, \ldots, \iota, \iota)}_{(\lambda)-times}, x)$,

(iv)

$$\vartheta^*_{\{m\to n\}}(\underbrace{(\iota, \iota, \ldots, \iota, \iota)}_{(\kappa)-times}, x, \vartheta^*_{\{m\to n\}}(y, x_1, \ldots, x_{m-1})) = \vartheta^*_{\{m\to n\}}(\underbrace{(\iota, \iota, \ldots, \iota, \iota)}_{(\kappa)-times}, y, \vartheta^*_{\{m\to n\}}(x, x_1, \ldots, x_{m-1})).$$

From now on, we will use $\{m \to n\}$ S.H BE algebra instead of $\{m \to n\}$ superhyper BE subalgebra, for simplify.

Example 3.3. (i) Suppose $(X, \vartheta^*_{\{m\to n\}})$ is an $\{m \to n\}$ S.H BE algebra. Afterwards $(X, \vartheta^*_{(2,0)})$ is a BE subalgebra.

(ii) Assume $(X, \vartheta^*_{\{m\to n\}})$ is an $\{m \to n\}$ S.H BE algebra. Afterwards $(X, \vartheta^*_{\{2\to 1\}})$ is a hyper BE subalgebra.

Example 3.4. Let $X = \{\iota, s\}$.

(i) Afterwards (X, ϑ^*) is a $\{3 \to 3\}$ S.H BE algebra what comes next:

$$\vartheta^*_{(3,3)}(r,r,r) = \mathcal{P}^3_*(\{\iota, r\}), \vartheta^*_{(3,3)}(r,s,\iota) = \mathcal{P}^3_*(\{\iota\}),$$
$$\vartheta^*_{(3,3)}(\iota,s,t) = \mathcal{P}^3_*(\{r\}), \text{and, for each another cases } \vartheta^*_{(3,3)}(r,s,t) = \mathcal{P}^3_*(\{r,s,t\}),$$

where

$$\mathcal{P}_*(\{s\}) = \mathcal{P}^2_*(\{s\}) = \mathcal{P}^3_*(\{s\}) = \{s\},$$
$$\mathcal{P}_*(\{\iota, s\}) = \{\iota, s, \{\iota, s\}\},$$
$$\mathcal{P}^2_*(\{\iota, s\}) = \{\iota, s, \{\iota, s\}, \{\iota, \{\iota, s\}\}, \{s, \{\iota, s\}\}\},$$
$$\mathcal{P}^3_*(\{\iota, s\}) = \big\{\iota, s, \{\iota, s\}, \{\iota, \{\iota, s\}\},$$
$$\{s, \{\iota, s\}\}, \{\iota, \{\iota, \{\iota, s\}\}, \{\iota, \{s, \{\iota, s\}\}, \{s, \{\iota, \{\iota, s\}\},$$
$$\{s, \{s, \{\iota, s\}\}, \{\{\iota, s\}, \{\iota, \{\iota, s\}\}\}, \{\{\iota, s\}, \{s, \{\iota, s\}\}, \{\{\iota, \{\iota, s\}\}, \{s, \{\iota, s\}\}\}\big\}.$$

(i) By definition, $\iota \in \vartheta^*_{\{3\to 3\}}(r,r,r) = \mathcal{P}^3_*(\{\iota, r\})$.

(ii) By definition, $\iota \in \vartheta^*_{\{3\to 3\}}(r,\iota,\iota) = \mathcal{P}^3_*(\{\iota\})$.

(iii) By definition, $x \in \vartheta^*_{\{3\to 3\}}(\iota,\iota,r) = \mathcal{P}^3_*(\{r\})$.

(iv) By definition,

$$\begin{aligned} \vartheta^*_{\{3\to 3\}}(\iota, x, \vartheta^*_{\{3\to 3\}}(y,z,w)) &= \vartheta^*_{\{3\to 3\}}(\iota, r, \mathcal{P}^3_*(\{u,v,w\})) \\ &= \mathcal{P}^3_*(\{r,u,v,w\}) \\ &= \vartheta^*_{\{3\to 3\}}(\iota, u, \vartheta^*_{\{3\to 3\}}(r,v,w)). \end{aligned}$$

(ii) Afterwards (X, ϑ^*) is a $\{3 \to 0\}$ S.H BE subalgebra what comes next:

$$\vartheta^*_{(3,\iota)}(r,r,r) = \{\iota\}, \text{and for another cases}, \vartheta^*_{(3,\iota)}(s,r,t) = \{t\}.$$

Theorem 3.5. *Assume $(X, \vartheta^*_{\{m\to n\}})$ is an $\{m \to n\}$ S.H BE algebra. Afterwards for every $k \geq n$, $(X, \vartheta^*_{\{m\to n\}})$ is an $\{m \to k\}$ superhyper BE subalgebra.*

Proof. Suppose $(X, \vartheta^*_{\{m\to n\}})$ is an $\{m \to n\}$ S.H BE algebra and $k \geq n$. Whereof $\mathcal{P}^n_*(X) \subseteq \mathcal{P}^k_*(X)$, for every $y_1, y_2, \ldots, y_m \in X, \vartheta^*_{\{m\to n\}}(y_1, y_2, \ldots, y_m) \subseteq \vartheta^*_{(m,k)}(y_1, y_2, \ldots, y_m)$. Thus $\iota \in \vartheta^*_{\{m\to n\}}(y_1, y_2, \ldots, y_m)$ implies that $\iota \in \vartheta^*_{(m,k)}(y_1, y_2, \ldots, y_m)$ and every principles are accurate. □

Example 3.6. Let $X = \{0, s\}$. Afterwards for each $n \geq 3$, using above Theorem, (X, ϑ^*) is a $\{3 \to n\}$ S.H BE algebra what comes next:

$$\vartheta^*_{(3,3)}(u,u,u) = \mathcal{P}^n_*(\{\iota, u\}), \vartheta^*_{(3,3)}(u,v,\iota) = \mathcal{P}^n_*(\{\iota\}),$$
$$\vartheta^*_{(3,3)}(\iota,v,w) = \mathcal{P}^n_*(\{u\}),$$
$$\text{and for another cases } \vartheta^*_{(3,3)}(u,v,w) = \mathcal{P}^n_*(\{u,v,w\}).$$

Let $k \in \mathbb{N}$, $m = 2k$, $(X, \vartheta^*_{\{m \to n\}})$ be an $\{m \to n\}$ S.H BE algebra. For every given $c_1, c_2, \ldots, c_m \in X$, define $(c_1, c_2, \ldots, c_{\frac{m}{2}}) \leq (c_{\frac{m}{2}+1}, c_{\frac{m}{2}+2}, \ldots, c_m)$ iff $\iota \in \vartheta^*_{\{m \to n\}}(c_1, c_2, \ldots, c_m)$.

Theorem 3.7. *Let $k \in \mathbb{N}$, $m = 2k$, $c_1, c_2, \ldots, c_m \in X$. Afterwards $(X, \vartheta^*_{\{m \to n\}})$ is an $\{m \to n\}$ S.H BE algebra provided,*

(i) $\vartheta^*_{\{m \to n\}}(y, x, \ldots, x) \leq \vartheta^*_{\{m \to n\}}(y, x, \ldots, x)$,
(ii) $\vartheta^*_{\{m \to n\}}(c_1, c_2, \ldots, c_m) \leq \vartheta^*_{\{m \to n\}}(c_1, c_2, \ldots, c_{m-1}, \iota)$,
(iii) $\vartheta^*_{\{m \to n\}}(c_1, c_2, \ldots, c_m) \leq \vartheta^*_{\{m \to n\}}(\iota, c_1, \ldots, c_{m-1}, c_m)$.

Proof. Occurring by definition. □

Theorem 3.8. *Assume $(X, \vartheta^*_{\{m \to n\}})$ is an $\{m \to n\}$ S.H BE algebra. Afterwards $\iota \in \vartheta^*_{\{m \to n\}}(\underbrace{(\iota, \iota, \ldots, \iota, \iota)}_{(\kappa)-times}, x, \vartheta^*_{\{m \to n\}}(y, x, \ldots, x))$, when $m - 2 = \kappa$*

Proof. Assume $(X, \vartheta^*_{\{m \to n\}})$ is an $\{m \to n\}$ S.H BE algebra and $x, y \in X$. Afterwards

$$\begin{aligned} & \iota \in \vartheta^*_{\{m \to n\}}(\underbrace{(\iota, \iota, \ldots, \iota, \iota)}_{(\kappa)-times}, y, \iota) \\ \subseteq \quad & \vartheta^*_{\{m \to n\}}(\underbrace{(\iota, \iota, \ldots, \iota, \iota)}_{(\kappa)-times}, y, \vartheta^*_{\{m \to n\}}(x, x, \ldots, x)) \\ = \quad & \vartheta^*_{\{m \to n\}}(\underbrace{(\iota, \iota, \ldots, \iota, \iota)}_{(\kappa)-times}, x, \vartheta^*_{\{m \to n\}}(y, x, \ldots, x)). \end{aligned}$$

□

Theorem 3.9. *Suppose $(X, \vartheta^*_{\{m \to n\}})$ is an $\{m \to n\}$ S.H BE algebra. Afterwards for $m - 2 = \kappa$,*

$$\iota \in \vartheta^*_{\{m \to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, x, \vartheta^*_{\{m \to n\}}(\vartheta^*_{\{m \to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, x, y), \underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, y).$$

Proof. Assume $x, y \in X$. Afterwards

$$\begin{aligned}\iota \in\ & \vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},\vartheta^*_{\{m\to n\}}((\underbrace{\iota,\iota,\ldots,\iota,\iota)}_{(\kappa)-times},x,y),\vartheta^*_{\{m\to n\}}((\underbrace{\iota,\iota,\ldots,\iota,\iota)}_{(\kappa)-times},x,y))\\ =\ & \vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y),y)).\end{aligned}$$

It concludes that $\iota \in \vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,\vartheta^*_{\{m\to n\}}(\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y),\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},y)$. □

Definition 3.10. Let $(X, \vartheta^*_{\{m\to n\}})$ be an $\{m \to n\}$ S.H BE algebra. Afterwards $(X, \vartheta^*_{\{m\to n\}})$ known as distributive, provided

$$\begin{aligned}& \vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},y,z))\\ =\ & \vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y),\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,z)),\end{aligned}$$

when $m - 2 = \kappa$.

Theorem 3.11. *Assume $(X, \vartheta^*_{\{m\to n\}})$ is a distributive $\{m \to n\}$ S.H BE algebra. Afterwards for when $m - 2 = \kappa$,*

(*i*) *If $x \le y$, afterwards* $\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},z,x) \le \vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},z,y)$,

(*ii*) $\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},y,z) \le \vartheta^*_{\{m\to n\}}(\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y),\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,z))$,

(*iii*) $\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},y,x) \le \vartheta^*_{\{m\to n\}}(\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},z,y),\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},z,x))$.

Proof. (*i*) Let $m - 2 = \kappa$, $x, y, z \in X$. Afterwards $x \le y$, imples that $\iota \in \vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y)$. Whereof $(X, \vartheta^*_{\{m\to n\}})$ is a distributive $\{m \to n\}$ S.H BE algebra, we get that

$$\begin{aligned}\iota \in\ & \vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},z,\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y))\\ =\ & \vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},z,x),\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},z,y)).\end{aligned}$$

It follows that $\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},z,x) \le \vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},z,y)$.

(ii) Let $x, y, z \in X$. Afterwards for $\beta = \underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}$, we get that

$$\begin{aligned}
&\iota \in \vartheta^*_{\{m\to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, \vartheta^*_{\{m\to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, y, z)\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times})\\
\subseteq\ &\vartheta^*_{\{m\to n\}}(\beta, \vartheta^*_{\{m\to n\}}(\beta, \vartheta^*_{\{m\to n\}}(\beta, y, z), x), \vartheta^*_{\{m\to n\}}(\beta, \vartheta^*_{\{m\to n\}}(\beta, y, z), \vartheta^*_{\{m\to n\}}(\beta, y, z))\\
\subseteq\ &\vartheta^*_{\{m\to n\}}(\beta, \vartheta^*_{\{m\to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, y, z), \vartheta^*_{\{m\to n\}}(\beta, x, \vartheta^*_{\{m\to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, y, z)))\\
\subseteq\ &\vartheta^*_{\{m\to n\}}(\beta, \vartheta^*_{\{m\to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, y, z), \vartheta^*_{\{m\to n\}}(\beta, (\vartheta^*_{\{m\to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, x, y),\\
&\vartheta^*_{\{m\to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, x, z)).
\end{aligned}$$

It follows that

$$\vartheta^*_{\{m\to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, y, z) \leq \vartheta^*_{\{m\to n\}}(\vartheta^*_{\{m\to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, x, y), \vartheta^*_{\{m\to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, x, z)).$$

(iii) It is similar to (ii). □

Definition 3.12. Assume $(X, \vartheta^*_{\{m\to n\}})$ is an $\{m \to n\}$ S.H BE algebra. Afterwards $(X, \vartheta^*_{\{m\to n\}})$ known as commutative, provided

$$\begin{aligned}
&\vartheta^*_{\{m\to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, \vartheta^*_{\{m\to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, x, y), y)\\
=\ &\vartheta^*_{\{m\to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, \vartheta^*_{\{m\to n\}}(\underbrace{\iota, \iota, \ldots, \iota, \iota}_{(\kappa)-times}, y, x), x),
\end{aligned}$$

when $m - 2 = \kappa$.

Example 3.13. Let $X = \{-1, 1, 2, 3, \ldots\}$.

(i) Afterwards (X, ϑ^*) is a commutative $\{3 \to 3\}$ S.H BE subalgebra what comes next:

$$\vartheta^*_{(3,3)}(x, y, z) = \begin{cases}
\mathcal{P}^3_*(\{-1\}) & \text{provided } z \leq y \leq x\\
\mathcal{P}^3_*(\{-1, y-z, x-z\}) & \text{provided } z < y \leq x\\
\mathcal{P}^3_*(\{-1, y-z, x-z\}) & \text{provided } z < x \leq y\\
\mathcal{P}^3_*(\{-1, z-x, y-x\}) & \text{provided } x < z \leq y\\
\mathcal{P}^3_*(\{-1, z-x, y-x\}) & \text{provided } x < y \leq z\\
\mathcal{P}^3_*(\{-1, z-y, x-y\}) & \text{provided } y < z \leq x\\
\mathcal{P}^3_*(\{-1, z-y, x-y\}) & \text{provided } y < x \leq z\\
\mathcal{P}^3_*(\{x-y, x-z, y-z\}) & o.w
\end{cases},$$

Theorem 3.14. *Assume* $(X,\vartheta^*_{\{m\to n\}})$ *is a commutative* $\{m\to n\}$ *S.H BE algebra. Afterwards for when* $m-2=\kappa$,

$$\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y)$$
$$\subseteq \vartheta^*_{\{m\to n\}}\big(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y),y),y\big) \text{ and } m-2=\kappa.$$

Proof. Let $x,y\in X$. Afterwards for $\alpha=m-1$, we get

$$\begin{aligned}
&\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y)\\
\subseteq\ &\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\alpha)-times},\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y))\\
\subseteq\ &\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\epsilon,\iota}_{(\kappa)-times},\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\lambda)-times},y,\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y),\\
&\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y)))\\
\subseteq\ &\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y),y),y).
\end{aligned}$$

Thus

$$\begin{aligned}
&\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y)\\
\subseteq\ &\vartheta^*_{\{m\to n\}}\big(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\kappa)-times},x,y),y),y\big).
\end{aligned}$$

□

Corollary 3.15. *Assume* $(X,\vartheta^*_{\{m\to n\}})$ *is a commutative* $\{m\to n\}$ *S.H BE algebra. Afterwards* $(X,\vartheta^*_{\{m\to n\}})$ *is distributive iff for every* $x,y\in X$,

$$\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\lambda)-times},x,y)=\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\lambda)-times},x,\vartheta^*_{\{m\to n\}}(\underbrace{\iota,\iota,\ldots,\iota,\iota}_{(\lambda)-times},x,y)) \text{ and } m-1=.$$

4. Conclusion and discussion

One of the aims of this article is to consider the relationship between some arbitrary elements that are dependent on the principles of axiom. Investigating this important issue is a more complicated task compared to the connection of only two elements, wherethrough creates a certain limitation for our article. Of course, the advantage of this work compared to the structural mode is that we have no restrictions on the selection of elements, and for any number of elements you can create a targeted connection. Indeed, the significance of $\{m\to n\}$ S.H BE algebras can cover the defects of hyper BE algebras and so can be applied to more actual problems. We try to define it in such a way that it outcomes in BE algebras and hyper BE algebras significances and at the same time we can connect more elements and eliminate the limitation of two elements in application. In the next investigations, attempt to achieve more outcomes concerning Neutro super (hyper)EQ subalgebras and utilizations in actual-universe problems. We demand to expand the significance of EQ algebras concerning networks and complex hypernetworks.

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