A Novel of Domination in Neutrosophic Over Graphs

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Abstract: The degree of data uncertainty may be measured using a variety of mathematical techniques, but neutrosophic logic is a potent instrument for analysis when compared to fuzzy and intuitionistic fuzzy logics. This article introduces the novel idea of "dominance in neutrosophic over graph (NOG)s" and discusses some of the intriguing characteristics of complete, complete bipartite, and neutrosophic over bridge domination. With the relevant instances, the characterization of neutrosophic over domination set and neutrosophic over minimal domination set is developed, and their dominance numbers are examined.

Keywords: Neutrosophic **o**ver domination set, Neutrosophic **o**ver minimum domination set, Neutrosophic **o**ver domination number.

Introduction:

Graph theory has various uses in a range of fields, including operations research, physics, chemistry, economics, genetic engineering, and computer science, among others. A classical graph cannot accurately represent uncertain issues since there are two choices for each vertex or edge: it is either in the graph or it is not. A generalized form of the classical set known as a fuzzy set [18] is one in which objects have membership degrees ranging from zero to one. On fuzzy graphs, more work has previously been done. Zadeh invented the fuzzy set and presented the degree of membership in 1965. The intuitionistic fuzzy set and the degree of falsity (F) were both proposed by Atanassov [1] in 1983. The neutrosophic set (NS) of components (T, I, F) and the degree of indeterminacy (I) were created by Smarandache [10,11,12,13] in 1995. Prem Kumar [14,15,16] developed three different forms of lattices for neutrosophic graphs. Three sets of neutrosophic over/off/under were introduced by Smarandache [8, 9] and their use in nursing research approaches was discussed. This inspired the creation of the notion of dominance number [17] in the realm of NOG. Later, Narmada Devi worked on a novel neutrosophic off graph and minimum dominance using NOGs and NO- top graphs [2, 3, 4, 5, 6, 7].

Smarandache has created a NS [15]. It is a generalization of intuitionistic fuzzy sets and fuzzy sets. With some ambiguity, consistency, and partial knowledge that is used in daily life, it is a potent instrument to influence. The three elements of membership (M), indeterminacy (I), and non-membership (N) functions are dealt with by NSs. It is a really useful programme for dealing with challenges in everyday life. However, it only applies to three attribute values. Advanced research suggests that in order to increase accuracy, the measurement of data uncertainty needs to be handled with greater attribute value. This is true across many scientific disciplines, including biology, physics, information technology, networks, decision-making analysis, etc.

Recently, several writers have focused on dealing with multi-valued attribute data sets based on Smarandache Plithogenic set [7, 8]. It is regarded as one of the important sets that depict the contradictory multi-valued properties. Finding some of the valuable patterns from data with lithogenic properties and its graphical visualisation, which were inspired by several recent work in this subject, presents a challenge.

1. PRELIMINARIES:

Definition 2.1 [8] Let V be a given non-empty set. Then the set A is said to be Fuzzy set on V such that its membership function $\mu_A: V \to [0,1]$ for each $x \in V$.

Definition 2.2 [5] Let V be a given non-empty set. A N**S** A in V is characterized by a truth membership function $T_A(x)$, an indeterminate membership function $I_A(x)$ and a false membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are fuzzy sets on V. That is, T_A, I_A, F_A : V → [0, 1] and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.3 [6] Let $\mathcal{U}$ be the universe of discourse and the NS $A \subset \mathcal{U}$. A N**eutrosophic over set** A is defined as $A = \{(x, \langle T(x), I(x), F(x) \rangle), x \in \mathcal{U}$, where $T, I, F: V \to [0, \Omega]$ that is, $0 < 1 < \Omega$ and Ω is said to be a over limit, $T(x), I(x), F(x) \in [0, \Omega]$ such that no element has neutrosophic components of <0, and there is at least one element that has at least one neutrosophic component of >1.

Definition 2.4 [6] Let $G^* = (V, E)$ be a crisp graph with V is vertex and E is edge. A **NOG** is a pair $G = (A, B)$ of G^* where A is a neutrosophic over set in V and B is a neutrosophic over set in $E \subseteq V \times V$ such that

$$T_B(xy) \leq min\{T_A(x), T_A(y)\},$$

$$I_B(xy) \leq min\{I_A(x), I_A(y)\},$$

$$F_B(xy) \geq max\{F_A(x), F_A(y)\}, \text{ for all } xy \in E.$$

Here A and B are said to be neutrosophic over vertex set and neutrosophic over edge set of G, respectively.

2. **VARIOUS TYPES OF NOGs**

Definition 3.1 Let $G = (A, B)$ be a **NOG** in G^*. G is said to be **complete** if:

$$\mathcal{T}_B(xy) = min\{\mathcal{T}_A(x), \mathcal{T}_A(y)\},$$

$$\mathcal{I}_B(xy) = min\{\mathcal{I}_A(x), \mathcal{I}_A(y)\},$$

$$\mathcal{F}_B(xy) = max\{\mathcal{F}_A(x), \mathcal{F}_A(y)\}, \text{ for all } (x, y) \in E.$$

Example 3.1:

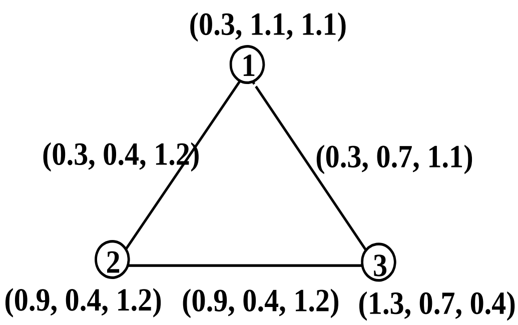

Figure 1: Complete NOG.

Definition 3.2 A NOG $G = (A, B)$ in G^* is a neutrosophic over bipartite (**NOBipar**) if the set V can be partitioned into two non-empty over sets V_1 and V_2 such that $\mathcal{T}_B(xy) = \mathcal{I}_B(xy) = \mathcal{F}_B(xy) = 0$, for all $(x, y) \in E_1$ or $(x, y) \in E_2$.

A **NOBipar** graph is a **complete-NOBipar** if $\mathcal{T}_B(xy) = min\{\mathcal{T}_A(x), \mathcal{T}_A(y)\}$, $\mathcal{I}_B(xy) = min\{\mathcal{I}_A(x), \mathcal{I}_A(y)\}$, $\mathcal{F}_B(xy) = max\{\mathcal{F}_A(x), \mathcal{F}_A(y)\}$, for all $(x, y) \in E$.

A complete-**NOBipar** graph is a **star** NOG if either $|V_1| = 1$ or $|V_2| = 1$.

Example 3.2:

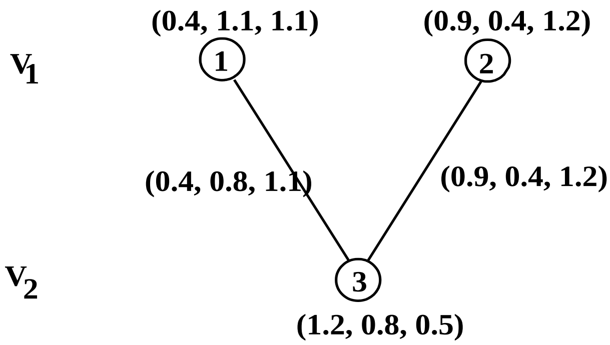

Figure 2: Complete Bi-par and Star NOG.

Definition 3.3 Let $G = (A, B)$ be a NOG in G^*. Then

a) The real number L is said to be the $\mathcal{T}$-order, if $L = \sum_{u \in V} \mathcal{T}_A(u)$.

b) The real number M is said to be the $\mathcal{I}$-order, if $M = \sum_{u \in V} \mathcal{I}_A(u)$.

c) The real number N is said to be the $\mathcal{F}$-order, if $N = \sum_{u \in V} \mathcal{F}_A(u)$.

d) A real number is said to be the order of a NOG, if it is equal to order $= \langle L, M, N \rangle$.

Example 3.3:

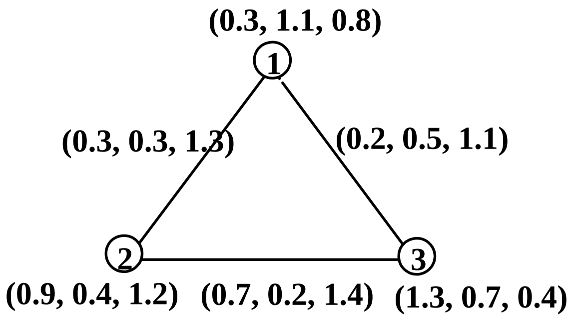

Figure 3: NOG.

In this above Figure 3 of NOG, the $\mathcal{T}$-order, $L = 2.5$, $\mathcal{I}$-order, $M = 2.2$, $\mathcal{F}$-order, $N = 2.4$ and the order is <2.5, 2.2, 2.4>.

Definition 3.4 Let v_0, v_n be two given vertices in a NOG $G = (A, B)$ such that $n \in \mathbb{N}$. Then

a) A $\mathcal{T}$ **-path** is an sequence of distinct vertices $P : v_0, v_1, \dots, v_n$ of length n from v_0 to v_n if $\mathcal{T}_B(v_i v_{i+1}) > 0$, for $i = 0,1, \dots, n-1$.

   The **strength** of that $\mathcal{T}$-path is $min_{i=0}^{n-1}\{\mathcal{T}_B(v_i v_{i+1})\}$ and denoted by $\mu_G(P)_{\mathcal{T}}$

b) A $\mathcal{I}$**-path** is an sequence of distinct vertices $P : v_0, v_1, \dots, v_n$ of length n from v_0 to v_n if $\mathcal{I}_B(v_i v_{i+1}) > 0$, for $i = 0,1, \dots, n-1$.

   The **strength** of that $\mathcal{I}$-path is $min_{i=0}^{n-1}\{\mathcal{I}_B(v_i v_{i+1})\}$ and denoted by $\mu_G(P)_{\mathcal{I}}$.

c) A $\mathcal{F}$**-path** is an sequence of distinct vertices $P : v_0, v_1, \dots, v_n$ of length n from v_0 to v_n if $\mathcal{F}_B(v_i v_{i+1}) > 0$, for $i = 0,1, \dots, n-1$.

   The **strength** of that $\mathcal{F}$-path is $min_{i=0}^{n-1}\{\mathcal{F}_B(v_i v_{i+1})\}$ and denoted by $\mu_G(P)_{\mathcal{F}}$.

d) A **path** is an sequence of distinct vertices $P: v_0, v_1, \dots, v_n$ of length n from v_0 to v_n if it be $\mathcal{T}$-path, $\mathcal{I}$-path, $\mathcal{F}$-path, simultaneously.

   The **strength** of that path is $min\{\mu_G(P)_{\mathcal{T}}, \mu_G(P)_{\mathcal{I}}, \mu_G(P)_{\mathcal{F}}\}$ and denoted by $\mu_G(P)$.

Example 3.4:

In the figure 3 of NOG, the various types of path from 1 - 2 are P_1 : $1 \longrightarrow 2$ and $P_2 : 1 \longrightarrow 3 \longrightarrow 2$ which are $\mathcal{T}$ -path, $\mathcal{I}$-path, $\mathcal{F}$-path respectively. $\mu_G(P_1)$= (0.3, 0.3, 1.3), $\mu_G(P_2)$ = (0.2, 0.2, 1.1).

Definition 3.5

Let v_i, v_j be two given vertices in a NOG $G = (A, B)$ such that $i > j$ and $i, j \in \mathbb{N}$. Then

a) The **$\mathcal{T}$-strength** between v_i and v_j is $max\{\mu_G(P)_{\mathcal{T}}\}$ and denoted by $\mu_G^\infty(v_i, v_j)_{\mathcal{T}}$

b) The **$\mathcal{I}$-strength** between v_i and v_j is $max\{\mu_G(P)_{\mathcal{I}}\}$ and denoted by $\mu_G^\infty(v_i, v_j)_{\mathcal{I}}$

c) The **$\mathcal{F}$-strength** between v_i and v_j is max$\{\mu_G(P)_{\mathcal{F}}\}$ and denoted by $\mu_G^\infty(v_i, v_j)_{\mathcal{F}}$

d) The **strength** between v_i and v_j is $max\{\mu_G^\infty(v_i, v_j)_{\mathcal{T}}, \mu_G^\infty(v_i, v_j)_{\mathcal{I}}, \mu_G^\infty(v_i, v_j)_{\mathcal{F}}\}$ and denoted by $\mu_G^\infty(v_i, v_j)$.

Example 3.5

In the Figure 3 of NOG, the various type of paths from 1 to 2 are $P_1 : 1 \longrightarrow 2$ and $P_2 : 1 \longrightarrow 3 \longrightarrow 2$ which are $\mathcal{T}$ -path, $\mathcal{I}$-path, $\mathcal{F}$-path respectively. $\mu_G(P_1)$= (0.3, 0.3, 1.3), $\mu_G(P_2)$= (0.2, 0.2, 1.1). Then the $\mathcal{T}$ -strength, $\mathcal{I}$-strength, $\mathcal{F}$-strength are 0.3, 0.3,1.3 and the strength = 1.3.

Example 3.6

Consider $G = (A, B)$ is a NOG on G^* as Figure 4. Various paths of length 3 from v_1 to v_2 are explored.

$P_1 : v_1 \longrightarrow v_3 \longrightarrow v_4 \longrightarrow v_2$

$P_2 : v_1 \longrightarrow v_3 \longrightarrow v_5 \longrightarrow v_2$

$P_3 : v_1 \longrightarrow v_4 \longrightarrow v_3 \longrightarrow v_2$

$P_4 : v_1 \longrightarrow v_4 \longrightarrow v_5 \longrightarrow v_2$

$P_5 : v_1 \longrightarrow v_5 \longrightarrow v_3 \longrightarrow v_2$

$P_6 : v_1 \longrightarrow v_5 \longrightarrow v_4 \longrightarrow v_2$, respectively.

And from following graph, we find the strength of that paths as follows.

For P_1: $\mu_G(P_1)_{\mathcal{T}}$ = 0.3, $\mu_G(P_1)_{\mathcal{I}}$ = 0.3, $\mu_G(P_1)_{\mathcal{F}}$ = 1.2 and $\mu_G(P_1)$ = 0.3

For P_2: $\mu_G(P_2)_{\mathcal{T}}$ = 0.2 , $\mu_G(P_2)_{\mathcal{I}}$ = 0.5, $\mu_G(P_2)_{\mathcal{F}}$ = 1.2 and $\mu_G(P_2)$= 0.2

For P_3: $\mu_G(P_3)_{\mathcal{T}}$ = 0.3 , $\mu_G(P_3)_{\mathcal{I}}$ = 0.4, $\mu_G(P_3)_{\mathcal{F}}$ = 1.2 and $\mu_G(P_3)$= 0.3

For P_4: $\mu_G(P_4)_{\mathcal{T}}$ = 0.4 , $\mu_G(P_4)_{\mathcal{I}}$ = 0.6, $\mu_G(P_4)_{\mathcal{F}}$ = 1.2 and $\mu_G(P_4)$= 0.4

For P_5: $\mu_G(P_5)_{\mathcal{T}}$ = 0.2, $\mu_G(P_5)_{\mathcal{I}}$ = 0.5, $\mu_G(P_5)_{\mathcal{F}}$ = 1.2 and $\mu_G(P_5)$ = 0.2

For P_6: $\mu_G(P_6)_{\mathcal{T}}$ = 0.5 , $\mu_G(P_6)_{\mathcal{I}}$ = 0.3, $\mu_G(P_6)_{\mathcal{F}}$ = 1.2 and $\mu_G(P_6)$= 0.3

Finally, we discuss about the strength between two vertices v_1 and v_2.

$\mu_G^{\infty}(v_1,v_2)_{\mathcal{T}}$ = 0.5, $\mu_G^{\infty}(v_1,v_2)_{\mathcal{I}}$ = 0.6, $\mu_G^{\infty}(v_1,v_2)_{\mathcal{F}}$ = 1.2, $\mu_G^{\infty}(v_1,v_2)$ = 1.2.

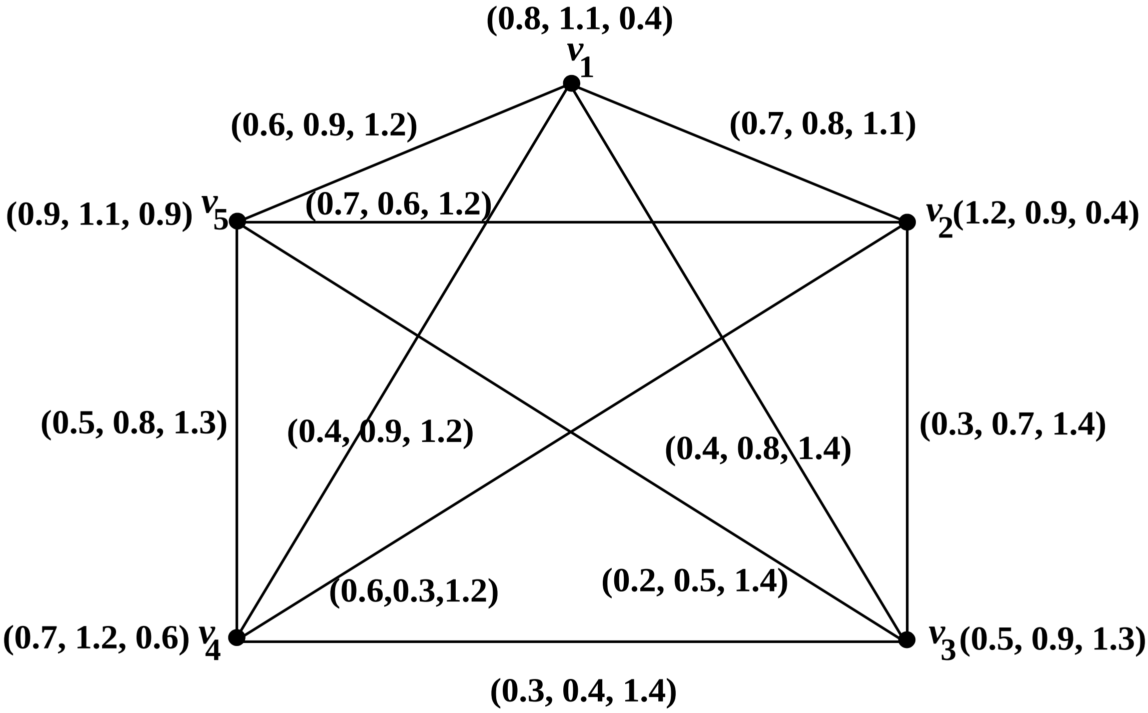

Figure 4 NOG

Definition 3.6 An edge xy in a NOG $G = (A, B)$ is said to be the

a) **$\mathcal{T}$-bridge**, if the strengths of each $\mathcal{T}$ -path P from x to y excluding xy were less than $\mathcal{T}_B(xy)$.

b) **$\mathcal{I}$- bridge**, if the strengths of each $\mathcal{I}$-path P from x to y excluding xy were less than $\mathcal{I}_B(xy)$.

c) **$\mathcal{F}$- bridge**, if the strengths of each $\mathcal{F}$-path P from x to y excluding xy were less than $\mathcal{F}_B(xy)$.

d) **Bridge**, if it is either $\mathcal{T}$ -bridge, $\mathcal{I}$-bridge, $\mathcal{F}$-bridge.

Example 3.7 In the figure 3 of NOG, the edges 12 and 23 are $\mathcal{T}$-bridge and $\mathcal{F}$-bridge respectively

a) The edges 12 and 13 are $\mathcal{I}$-bridge and

b) The edges 12, 13 and 23 is bridge

Notation: $\mu^{\infty}_{G-\{xy\}}(x,y)$ is the strength between x and y obtained from G by deleting the edge xy.

Thus, we notate for $\mathcal{T}$-strength, $\mathcal{I}$-strength, $\mathcal{F}$-strength as $\mu^{\infty}_{G-\{xy\}}(x,y)_{\mathcal{T}}$, $\mu^{\infty}_{G-\{xy\}}(x,y)_{\mathcal{I}}$ and $\mu^{\infty}_{G-\{xy\}}(x,y)_{\mathcal{F}}$ respectively.

Definition 3.7 An edge xy in $G = (A,B)$ is said to be

a) **$\mathcal{T}$-effective**, if $\mathcal{T}_B(xy) > \mu^{\infty}_{G-\{xy\}}(x,y)_{\mathcal{T}}$.

b) **$\mathcal{I}$-effective**, if $\mathcal{I}_B(xy) > \mu^{\infty}_{G-\{xy\}}(x,y)_{\mathcal{I}}$.

c) **$\mathcal{F}$- effective**, if $\mathcal{F}_B(xy) > \mu^{\infty}_{G-\{xy\}}(x,y)_{\mathcal{F}}$.

d) **Effective** if it is either of $\mathcal{T}$-effective, $\mathcal{I}$-effective and $\mathcal{F}$-effective.

Example 3.8

Consider $G = (A,B)$ is a NOG. In the below Table 1, effectiveness of edges of G is established.

Edges	$\mathcal{T}$-effective	$\mathcal{I}$-effective	$\mathcal{F}$-effective	Effective
v_1v_2	√	√	×	√
v_1v_3	√	√	√	√
v_1v_4	×	√	×	√
v_1v_5	×	√	×	√

v_2v_3	×	×	√	√
v_2v_4	√	×	×	√
v_2v_5	√	×	×	√
v_3v_4	×	×	√	√
v_3v_5	×	×	√	√
v_4v_5	×	×	×	×

Table 1 Effectiveness of edges

For example, v_4v_5 have neither of $\mathcal{T}$- effective, $\mathcal{I}$- effective, $\mathcal{F}$- effective and effective property. v_1v_2 have $\mathcal{T}$-effective and $\mathcal{I}$-effective property.

Therefore, it is an effective edge. The collection of edges $\{v_1v_2, v_1v_3, v_2v_4, v_2v_5\}$, $\{v_1v_2, v_1v_3, v_1v_4, v_1v_5\}$, $\{v_1v_3, v_2v_3, v_3v_4, v_3v_5\}$, $\{v_1v_2, v_1v_3, v_1v_4, v_1v_5, v_2v_3, v_2v_4, v_2v_5, v_3v_4, v_3v_5\}$ have $\mathcal{T}$-effective, $\mathcal{I}$-effective, $\mathcal{F}$-effective and effective property, respectively.

3. DOMINATION ON NOGs.

Definition 4.1

a) Let $G = (A, B)$ be a NOG, let $x, y \in V$, we say x **dominates** y in G if there exist an $\mathcal{T}$-effective edge between them.

b) A subset S of V is said to be a **$\mathcal{T}$-effective dominating** set in G if for every $v \in V - S$, there exists $u \in S$ such that u dominates v as $\mathcal{T}$-eff.

c) The **$\mathcal{T}$-weight of** x is defined by $w(x)_{\mathcal{T}} = \mathcal{T}_A(x) + \frac{\sum_{xy \text{ is a } \mathcal{T}-eff\ edge} \mathcal{T}_B(xy)}{\sum_{xy \text{ is a } edge} \mathcal{T}_B(xy)}$.

d) For any $S \subseteq V$, **$\mathcal{T}$-weight of** S is defined by $w(S)_{\mathcal{T}} = \sum_{u \in S}(w(u)_{\mathcal{T}})$.

e) Let A be the set of all $\mathcal{T}$-effective dominating sets in G. The **$\mathcal{T}$-domination number** of G is given by $\gamma(G)_{\mathcal{T}} = \min_{D \in U}(w(D)_{\mathcal{T}})$. Then the $\mathcal{T}$-effective dominating set that correspond to $\gamma(G)_{\mathcal{T}}$ is known as **$\mathcal{T}$-dominating set**.

f) Further, in the similar manner we define $\mathcal{I}$-dominating set and $\mathcal{F}$-dominating set of G, respectively.

Note: If $\sum_{xy \text{ is a edge}} \mathcal{T}_B(xy)$ equals 0, for some $x \in V$. Then $\frac{\sum_{xy \text{ is a } \mathcal{T}-eff \text{ edge}} \mathcal{T}_B(xy)}{\sum_{xy \text{ is a edge}} \mathcal{T}_B(xy)}$ equal with 0.

Definition 4.2

a) We say x **dominates** y in G, if there exist an effective edge between them.

b) A subset S of V is said to be the **effective dominating** set in G, if for every $v \in V - S$, there exists $u \in S$ such that u dominates v as eff.

c) The **weight of S** is defined by $w(S) = \min\{w(D)_T, w(D)_I, w(D)_F\}$.

d) Let A be the set of all eff dominating sets in G. The **domination number** of G is given by $\gamma(G) = \min_{D \in U}(w(D))$. Then the effective dominating set that correspond to $\gamma(G)$ is known as **dominating set**.

4. PROPERTIES OF EFFECTIVE EDGES IN NOGs

Proposition 5.1

Let $G = (A, B)$ be a complete NOG in G^* which has exactly one path between two given vertices and has

a) $\mathcal{T}$-strength. Then $\gamma(G)_{\mathcal{T}} \leq \min_{u \in V} \mathcal{T}_A(u) + 2$.

b) $\mathcal{I}$- strength. Then $\gamma(G)_{\mathcal{I}} \leq \min_{u \in V} \mathcal{I}_A(u) + 2$.

c) $\mathcal{F}$- strength. Then $\gamma(G)_{\mathcal{F}} \leq \min_{u \in V} \mathcal{F}_A(u)$+ 2.

d) strength. Then $\gamma(G) \leq \min_{u \in V}(\mathcal{T}_A(u), \mathcal{I}_A(u), \mathcal{F}_A(u)) + 2$.

Proof

(a) Let $G = (A, B)$ be a NOG on G^*. Then the $\mathcal{T}$-strength of path P from u to v will be $\mathcal{T}_A(u) \wedge \ldots \wedge \mathcal{T}_A(v) \leq \mathcal{T}_A(u) \wedge \mathcal{T}_A(v) = \mathcal{T}_B(uv)$. So $\mu_G^{\infty}(u,v)_{\mathcal{T}} < \mathcal{T}_B(uv)$. uv is a path from u to v such that $\mathcal{T}_B(uv) = \mathcal{T}_A(u) \wedge \mathcal{T}_A(v)$. Therefore $\mu_G^{\infty}(u,v)_{\mathcal{T}} \geq \mathcal{T}_B(uv)$. Hence $\mu_G^{\infty}(u,v)_{\mathcal{T}} = \mathcal{T}_B(uv)$. Then $\mathcal{T}_B(uv) > \mu_{G-\{xy\}}^{\infty}(u,v)_{\mathcal{T}}$ which means the edge uv is $\mathcal{T}$-effective. Therefore, all edges are $\mathcal{T}$-effective and each vertex is adjacent to all other vertices. So $D = \{u\}$ will be the $\mathcal{T}$-effective dominating set and $\sum_{xy \text{ is a } \mathcal{T}-eff \text{ edge}} \mathcal{T}_B(xy) = \sum_{xy \text{ is a edge}} \mathcal{T}_B(xy)$ for each $u \in V$. The result follows.

Similarly, we can prove for (b), (c) and (d).

Proposition 5.2 Let $G = (A, B)$ be any **complete-NOBipar** graph in G^* which has exactly one path between two given vertices and has

a) $\mathcal{T}$-strength. Then $\gamma(G)_{\mathcal{T}}$ is either $\mathcal{T}_A(u) + 1$ or $\min_{u \in V_1, v \in V_2}(\mathcal{T}_A(u) + \mathcal{T}_A(v)) + 2$.

b) $\mathcal{I}$-strength. Then $\gamma(G)_{\mathcal{I}}$ is either $\mathcal{I}_A(u) + 1$ or $\min_{u \in V_1, v \in V_2}(\mathcal{I}_A(u) + \mathcal{I}_A(v)) + 2$.

c) $\mathcal{F}$- strength. Then $\gamma(G)_{\mathcal{F}}$ is either $\mathcal{F}_A(u) + 1$ or $\min_{u \in V_1, v \in V_2}(\mathcal{F}_A(u) + \mathcal{F}_A(v)) + 2$.

d) Then $\gamma(G)$ is either $\min(\mathcal{T}_A(u), I_A(u), \mathcal{F}_A(u)) + 1$ or $\min_{u \in V_1, v \in V_2}(\mathcal{T}_A(u) + \mathcal{T}_A(v), \mathcal{I}_A(u) + \mathcal{I}_A(v), \mathcal{F}_A(u) + \mathcal{F}_A(v)) + 2$. for all $u, v \in V$.

Proof

(a) Let $G = (A, B)$ be any **complete-NOBipar** graph on G^* which has exactly one path and has $\mathcal{T}$-strength between two given vertices. By the proof of Proposition 5.1, all the edges are $\mathcal{T}$-effective.

Case (i): Consider, if G is the star NOG with $V = \{u, v_1, v_2, \ldots v_n\}$ in which u and v_i are the center and the leaves of G, for $1 \leq i \leq n$, respectively. Then $\{u\}$ is the $\mathcal{T}$-dominating set of G. Hence $\gamma(G)_T = T_A(u) + 1$.

Case (ii): Let both of V_1 and V_2 include more than one vertex. Every vertex in V_1 is dominated by every vertex in V_2, as $\mathcal{T}$-effective and conversely every vertex in V_2 is dominated by every vertex in V_1, as $\mathcal{T}$-effective. Thus the $\mathcal{T}$-effective dominating sets are V_1 and V_2 and any set containing 2 vertices one in V_1 and the other in V_2. Therefore, $\gamma(G)_{\mathcal{T}} = \min_{u \in V_1, v \in V_2} (\mathcal{T}_A(u) + \mathcal{T}_A(v)) + 2$. The result follows.

Similarly, we can prove for (b), (c) and (d).

Proposition 5.3 Let $G = (A, B)$ be a NOG in G^*. Then $xy \in E$ is a

a) $\mathcal{T}$-effective edge $\Leftrightarrow$ xy is a $\mathcal{T}$-bridge.

b) $\mathcal{I}$- effective edge $\Leftrightarrow$ xy is a $\mathcal{I}$-bridge.

c) $\mathcal{F}$-effective edge $\Leftrightarrow$ xy is a $\mathcal{F}$-bridge.

d) Effective edge $\Leftrightarrow$ xy is a bridge.

Proof

(a) $\Leftrightarrow$ Suppose xy is a $\mathcal{T}$-effective edge.

$\Leftrightarrow$ By Definition 3.7(a), $\mathcal{T}_B(xy) > \mu^{\infty}_{G-\{xy\}}(x, y)_{\mathcal{T}}$.

$\Leftrightarrow$ Since, $\mathcal{T}_B(xy) = \mu^{\infty}_{G}(x, y)_{\mathcal{T}}$. Therefore $\mu^{\infty}_{G}(x, y)_{\mathcal{T}} > \mu^{\infty}_{G-\{xy\}}(x, y)_{\mathcal{T}}$.

$\Leftrightarrow$ xy is a $\mathcal{T}$-bridge. Therefore, the result follows.

Similarly, we can prove for (b), (c) and (d).

5. CONCLUSION:

Both theoretical studies and practical applications for the idea of dominance in graphs are fairly extensive. In this study, we used strength of path to construct the NOG dominance number and explain it with relevant instances. Numerous applications of expert systems, image processing, computer networks, and social systems can make use of the NOG notion. Additionally, we look at several noteworthy characteristics of the product NOGs with the dominance number, and the suggested ideas are explained with useful examples.

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