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# Edge Regular Complex Neutrosophic Graph Structure and it is Application

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**Abstract**. A modified version of a Neutrosophic Set (NS), a Complex Neutrosophic Set (CNS) offers a more accurate description of ambiguous situations than established fuzzy sets (FSs). It is widely applied in uncertain control. This study offers the idea of Single-Valued Complex Neutrophilic Graph Structure (SVCNGS). Further research is done into the relationship between an $\eta_J - edge$ regular SVCNGS degree and the $\eta_J$-degree of a vertex. Also, we introduce the notions of totally $\eta_J - edge$ regular and regular $\eta_J - edge$ SVCNGS. There is an explanation of the conditions in which $\eta_J - edge$ regular SVCNGS and totally $\eta_J - edge$ regular SVCNGS are same. Moreover, this study several $\eta_J - edge$ regular and totally $\eta_J - edge$ regular SVCNGS properties using an example, and we have discussed their application in SVCNGS. Finally, we develop an algorithm that explains the fundamental workings of our application.

**Keywords:** SVCNGS, $\eta_J - edge$ regular, totally $\eta_J - edge$ regular, application

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## 1. **Introduction**

The phrase FSs it initially used by L.A. Zadeh [48] in 1965 to describe a way to show the ambiguity of FSs. The business sector is vital to our daily lives because it helps us see ambiguities and identify them in most fields of science and medicine. T. Atanassov [4] suggested that Intuitionistic FSs (IFSs) may be created by deriving a new component, degree of membership and non-membership, based on the features of the FS. As a result, it can explain more accurately and completely than a FS. However, it can only handle partial and ambiguous information; it cannot manage the ambiguous and contradictory informationthat frequently occurs in real-life situations. It can only handle partial and ambiguous information, not the ambiguous and indeterminate information that often occurs in real-life situations. Therefore, the terms NS, a unifying field in logics and a generalization of the IFSs are introduced by F. Smarandache [ [27], [28], [29], [30], [31]] and is used in many different fields to deal with ambiguous and contradictory data. If the total of these values in the NS is between 0 and 3, the terms of truth membership, indeterminacy membership, and false membership are all done separately, and the indeterminacy value is directly quantified. Neutrosophy: Neutral Probability, Neutral Set, and Neutral Logic Introduce the idea of NS, N probability, and logic in more detail. Due to the broad range of description situations it covers, the NS has quickly drawn the attention of many scholars. This new set also helps to manage the vagueness brought on by the N scope. Furthermore, a thorough evaluation of Xindong Peng and Jingguo Dai [40] citation is provided. A bibliometric analysis of the neutrosophic collection is presented, covering the period from 1998 to 2017. Ramot [18] created the idea of a Complex FS (CFS) in 2012 by changing the range of the membership function for the amount disc for complex and real integers. A helpful generalisation of FS is the membership grade of this concept, which is expressed as $re^{i\theta}$, where r stands for the amplitude term and $\theta$ for the phase term. Only values from the complex plane's unit circle are permitted. The phase term of CFS matters because it can handle cyclical problems or persistently troubling circumstances more successful. Given that this term is a part of CFS, there will undoubtedly be circumstances in which another dimension is required. In contrast to every other type of information that is currently available, CFS is described in this phrase. A detailed investigation of CFS's [43] was performed by Yazdanbakhsh and Dick. Alkouri and Salleh [2] first introduced the ideas of CIFSs in 2012. It is important to familiarize out with the novel forms, such as CIFSs, which significantly expand upon CFSs; useful details regarding these kinds of structures can be discovered in [ [19], [20]]. Recently, Prem Kumar Singh developed the equation of complex vague set idea lattice and its features in his paper [16]. K. Ullah and T. Mahmood [39] presented the concept of CPFSs in 2019 in addition to expanding the range of existing distance measures to take into consideration CPF values. Mumtaz Ali and Florentin Smarandache developed the concept of a CNS in 2016 [32]. A complex-valued NS is one whose real-valued amplitude terms for truth, indeterminacy, and falsehood, along with the phase terms that go along with them, are combined to form its complex-valued membership functions. The NS is expanded upon by the CNS. Further, the establishment of Hypersoft Set Hybrids with CFS, CIFS, and CNS are introduced in 2020 by Atiqe U. R., Muhammad.S, Florentin Smarandache, and Muhammad R. A [6]. In 1975, Rosenfeld [21] developed fuzzy graph theory. Examined the Fuzzy Graphs (FG) for which Kauffmann created the fundamental concept in 1973. He explored a number of basic concepts in graph theory and developed some of their characteristics. In his remarks on FGs, Bhattacharya [7] demonstrated that the conclusions drawn from (crisp) graph theory are not necessarily applicable to FGs. In 1994, Shannon and Atanassov proposed the ideas of IF relations and IFGs. Rashmanlou [15] studied FGs with irregular interval values. Additionally, they defined FGs [17], various features of very irregular interval-valued FGs [17]. The Edge Regular IFG was first proposed by M.G. Karunambigai and K. Palanivel [10] in 2015. CFGs were developed by Thirunavukarasu et al. [38] to manage ambiguous and uncertain relationships with periodic nature. CIFGs were defined by Yaqoob et al. [44]. They looked into the homomorphisms of CIFG and demonstrated a CIFG usage among cellular network supplier companies to test their proposed approach. CNGs were introduced by Yaqoob and Akram to expand the idea of NGs and CIFGs [45]. They addressed various fundamental CNG operations and described them using specific instances. They also demonstrated CNGs' energy. Anam Luqman, Muhammad Akram, and Florentin Smarandache [1] further elaborate on the idea of CN Hypergraphs: New Social Network Models in 2019. Two voting processes are the best instances and source of inspiration for CNS and the example is provided in their introduction to prove the applicability of their suggested model. The research papers Applications of graph's total degree with bipolar fuzzy information and Estimation of most effected cycles and busiest network route based on complexity function of graph in fuzzy environment in 2022 by Soumitra Poulik and Ganesh Ghorai [ [33], [34], [35]] is worth being referred to for more information. Also, in 2021 proposed the idea of Determining the order of journeys based on a graph's Wiener absolute index using bipolar fuzzy information. Sampathkumar [23] introduced Graph Structures (GSs) in 2006 to be a generalization of signed graphs and graphs with colored or labeled edges. The idea of a FGS was first presented by Dinesh [8], and also discussed some relevant properties. Recently, the notions of Operations on IFGSs were introduced by Muhammad Akram [ [12], [13], [14]]. Also, introduce the ideas of simplified Interval-Valued PFGs with applications and a novel decision-making approach under CPF environments further. Later, the idea of complex Pythagorean fuzzy planar graphs was created.

### 1.1. *Framework of this research*

This concept can be restated in an abstract form then applied in SVCNGS. The organization of this work is as follows:

- This study introduces the idea of SVCNGS. In regular SVCNGS, the relationship between vertex degree and edge degree is further investigated.
- We also define total $\eta_J - edge$ regular SVCNGS and $\eta_J - edge$ regular SVCNGS. It is described under which conditions $\eta_J - edge$ regular SVCNGS and total $\eta_J - edge$ regular SVCNGS are comparable.

- Furthermore, Applications and algorithm explaining for SVCNGS were also covered. Finally, an explanation of all these studies is provided in Conclusion and future works.

## 2. Preliminaries

The construction of the research studies will be aided by the discussion of some fundamental definitions and properties in this section.

**Definition 2.1.** [32] An object with the form of a SVCNS $Q$ on a non-void set $X$

$$Q = \{j, T_Q(j)e^{i\alpha_Q(j)}, I_Q(j)e^{i\beta_Q(j)}, F_Q(j)e^{i\gamma_Q(j)} : j \in X\}$$

where $i = \sqrt{-1}$, amplitude terms $T_Q(j), I_Q(j), F_Q(j) \in [0,1]$ and phase terms $\alpha_Q(j), \beta_Q(j), \gamma_Q(j) \in [0, 2\pi]$.

**Definition 2.2.** [39] Let $\chi = \{j, T_\chi(j)e^{i\alpha_\chi(j)}, I_\chi(j)e^{i\beta_\chi(j)}, F_\chi(j)e^{i\gamma_\chi(j)} : j \in X\}, \eta = \{j, T_\eta(j)e^{i\alpha_\eta(j)}, I_\eta(j)e^{i\beta_\eta(j)}, F_\eta(j)e^{i\gamma_\eta(j)} : j \in X\}$ be the two SVCNS in X, then

- $\chi \subseteq \eta$ if and only if $T_\chi(j) \leq T_\eta(j),\ I_\chi(j) \leq I_\eta(j)$ and $F_\chi(j) \leq F_\eta(j)$ for amplitude terms and $\alpha_\chi(j) \leq \alpha_\eta(j),\ \beta_\chi(j) \leq \beta_\eta(j)$ and $\gamma_\chi(j) \leq \gamma_\eta(j)$ for phase terms, for all $j \in X$;
- $\chi = \eta$ if and only if $T_\chi(j) = T_\eta(j),\ I_\chi(j) = I_\eta(j)$ and $F_\chi(j) = F_\eta(j)$ for amplitude terms and $\alpha_\chi(j) = \alpha_\eta(j),\ \beta_\chi(j) = \beta_\eta(j)$ and $\gamma_\chi(j) = \gamma_\eta(j)$ for phase terms, for all $j \in X$;

For simplicity, the $(j, T(j)e^{i\alpha(j)}, I(j)e^{i\beta(j)}, F(j)e^{i\gamma(j)} : j \in X)$ is called the SVCN Number (SVCNN), where $T(j), I(j), F(j) \in [0,1]$ such that $0 \leq T(j) + I(j) + F(j) \leq 3$ and $\alpha, \beta, \gamma \in [0, 2\pi]$.

**Definition 2.3.** [13] On a non-empty set $X$, a SVCNG is a pair $G = (\chi, \eta)$, where $\chi$ and $\eta$ are SVCNSs on X and a SVCN relation on X, respectively, such that:

$$\begin{aligned} T_\eta(rs)e^{i\alpha_\eta(rs)} &\leq \min\{T_\chi(r), T_\chi(s)\}e^{i\min\{\alpha_\chi(r), \alpha_\chi(s)\}} \\ I_\eta(rs)e^{i\beta_\eta(rs)} &\leq \max\{I_\chi(r), I_\chi(s)\}e^{i\max\{\beta_\chi(r), \beta_\chi(s)\}} \\ F_\eta(rs)e^{i\gamma_\eta(rs)} &\leq \max\{F_\chi(r), F_\chi(s)\}e^{i\max\{\gamma_\chi(r), \gamma_\chi(s)\}} \end{aligned}$$

$0 \leq T_\eta(rs) + I_\eta(rs) + F_\eta(rs) \leq 3$ for all $r, s \in X$. We call $\chi$ and $\eta$ the SVCN vertex set and the SVCN edge set of $G$, respectively.

## 3. Some Result on SVCNGS

The concept of SVCNGS is introduced in this section, along with definitions that are useful in understanding the main findings. With examples, we further analyse several SVCNGS characteristics.

**Definition 3.1.** Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ is referred to as an SVCNGS of GS $\tau^* = (M, W_1, W_2, ..., W_k)$ if $\gamma = \{r, \gamma_1(r)e^{i\alpha_1(r)}, \gamma_2(r)e^{i\alpha_2(r)}, \gamma_3(r)e^{i\alpha_3(r)}\}$ is an SVCN set on $Q$ and $\eta_J = \{rs, \eta_{1J}(rs)e^{i\beta_{1J}(rs)}, \eta_{2J}(rs)e^{i\beta_{2J}(rs)}, \eta_{3J}(rs)e^{i\beta_{3J}(rs)}\}$ are SVCN sets on $M$ and $W_J$ such that

$$\begin{aligned}
\eta_{1J}(r,s)e^{i\beta_{1J}(rs)} &\leq \min\{\gamma_1(r), \gamma_1(s)\}e^{i\min\{\alpha_1(r),\alpha_1(s)\}},\\
\eta_{2J}(r,s)e^{i\beta_{2J}(rs)} &\leq \max\{\gamma_2(r), \gamma_2(s)\}e^{i\max\{\alpha_2(r),\alpha_2(s)\}},\\
\eta_{3J}(r,s)e^{i\beta_{3J}(rs)} &\leq \max\{\gamma_3(r), \gamma_3(s)\}e^{i\max\{\alpha_3(r),\alpha_3(s)\}}
\end{aligned}$$

such that $0 \leq \eta_{1J}(r,s) + \eta_{2J}(r,s) + \eta_{3J}(r,s) \leq 3$ and $\beta_{1J}(rs), \beta_{2J}(rs), \beta_{3J}(rs) \in [0, 2\pi]$ for all $(r,s) \in R_J, J = 1, 2, ..., k$.

**Example 3.2.** An SVCNGS $\tau = (\gamma, \eta_1, \eta_2)$ of a GS $\tau^* = (M, W_1, W_2)$ given figure-1 is a SVCNGS $\tau = (\gamma, \eta_1, \eta_2)$ such that $\gamma = \{u_1(.4e^{i1.6\pi}, .6e^{i1.2\pi}, .3e^{i1.4\pi}),$
$u_2(.5e^{i1.0\pi}, .6e^{i.8\pi}, .4e^{i1.6\pi}),\ u_3(.5e^{i.8\pi}, .4e^{i1.0\pi}, .6e^{i1.4\pi}),\ u_4(.3e^{i.6\pi}, .5e^{i1.8\pi}, .4e^{i1.6\pi})\}$.

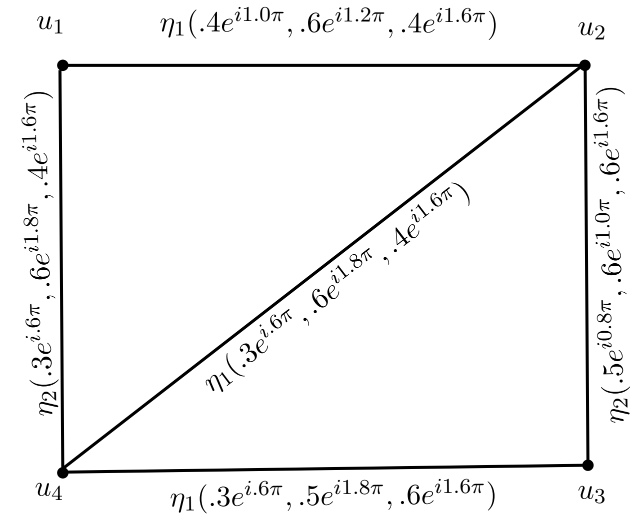


FIGURE 1. $\tau = (\gamma, \eta_1, \eta_2)$ is SVCNGS of $\tau^*$

**Definition 3.3.** Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. Then the vertex $\eta_J - degree$ is defined as

$$\begin{aligned}
&d_{\eta_J}(f) = (d_{\eta_{1J}}(f), d_{\eta_{2J}}(f), d_{\eta_{3J}}(f)),\\
&d_{\eta_{1J}}(f) = \sum_{(f,v)\in W_J} \eta_{1J}(f,v)e^{i\sum_{(f,v)\in R_J}\beta_{1J}(f,v)},\\
&d_{\eta_{2J}}(f) = \sum_{(f,v)\in W_J} \eta_{2J}(f,v)e^{i\sum_{(f,v)\in R_J}\beta_{2J}(f,v)},\\
&d_{\eta_{3J}}(f) = \sum_{(f,v)\in W_J} \eta_{3J}(f,v)e^{i\sum_{(f,v)\in W_J}\beta_{3J}(f,v)},\\
&\forall J = 1, 2, ..., k.
\end{aligned}$$

**Definition 3.4.** A SVCNGS $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ is $\eta_J - strong$ if

$$\begin{aligned}
\eta_{1J}(r,s)e^{i\beta_{1J}(rs)} &= \min\{\gamma_1(r), \gamma_1(s)\}e^{i\min\{\alpha_1(r),\alpha_1(s)\}},\\
\eta_{2J}(r,s)e^{i\beta_{2J}(rs)} &= \max\{\gamma_2(r), \gamma_2(s)\}e^{i\max\{\alpha_2(r),\alpha_2(s)\}},\\
\eta_{3J}(r,s)e^{i\beta_{3J}(rs)} &= \max\{\gamma_3(r), \gamma_3(s)\}e^{i\max\{\alpha_3(r),\alpha_3(s)\}} \text{ for all } (r,s) \in W_J, J = 1, 2, ..., k.
\end{aligned}$$

**Example 3.5.** Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS. Next, for every $J = 1, 2$, the degree of a $\eta_J - strong$ vertex is shown in figure-1. The $\eta_1 - strong$ degree of vertex $u_i$, i=1,2,3,4 is

$$\begin{aligned}
d_{\eta_1}(u_1) &= (d_{\eta_{11}}(u_1), d_{\eta_{21}}(u_1), d_{\eta_{31}}(u_1))\\
d_{\eta_1}(u_1) &= (.4e^{i1.0\pi}, .6e^{i1.2\pi}, .4e^{i1.6\pi}),\\
d_{\eta_1}(u_2) &= (.7e^{i1.6\pi}, 1.2e^{i3.0\pi}, .8e^{i3.2\pi}),\\
d_{\eta_1}(u_3) &= (.3e^{i.6\pi}, .5e^{i1.8\pi}, .6e^{i1.6\pi}),\\
d_{\eta_1}(u_4) &= (.6e^{i1.2\pi}, 1.1e^{i3.6\pi}, 1.0e^{i3.2\pi})
\end{aligned}$$

The $\eta_2 - strong$ degree of vertex $u_i$, i=1,2,3,4 is

$$\begin{aligned}
d_{\eta_2}(u_1) &= (d_{\eta_{12}}(u_1), d_{\eta_{22}}(u_1), d_{\eta_{32}}(u_1))\\
d_{\eta_2}(u_1) &= (.3e^{i.6\pi}, .6e^{i1.8\pi}, .4e^{i1.6\pi}),\\
d_{\eta_2}(u_2) &= (.5e^{i.8\pi}, .6e^{i1.0\pi}, .6e^{i1.6\pi}),\\
d_{\eta_2}(u_3) &= (.5e^{i.8\pi}, .6e^{i1.0\pi}, .6e^{i1.6\pi}),\\
d_{\eta_2}(u_4) &= (.3e^{i.6\pi}, .6e^{i1.8\pi}, .4e^{i1.6\pi})
\end{aligned}$$

**Theorem 3.6.** *Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. Then $\sum_{i=1}^{n} d_{\eta_J}(u_i) = (2\sum_{(u_i,v)\in W_J}\eta_{1J}(u_i,v)e^{i2\sum_{(u_i,v)\in W_J}\beta_{1J}(u_i,v)}, 2\sum_{(u_i,v)\in W_J}\eta_{2J}(u_i,v)$ $e^{i2\sum_{(u_i,v)\in W_J}\beta_{2J}(u_i,v)}, 2\sum_{(u_i,v)\in W_J}\eta_{3J}(u_i,v)e^{i2\sum_{(u_i,v)\in W_J}\beta_{3J}(u_i,v)})$ is $\eta_J - strong$ SVCNGS for all $J = 1, 2, ..., k$.*

**Example 3.7.** Next, we demonstrate the above theorem's example - 3.6. Let us Consider a SVCNGS $\tau = (\gamma, \eta_1, \eta_2)$ as shown in figure-1. Then $\sum_{i=1}^{4} d_{\eta_J}(u_i) =$
$(2\sum_{(u_i,v)\in W_J} \eta_{1J}(u_i,v)e^{i2\sum_{(u_i,v)\in W_J}\beta_{1J}(u_i,v)}, 2\sum_{(u_i,v)\in W_J} \eta 2J(u_i,v)e^{i2\sum_{(u_i,v)\in W_J}\beta_{2J}(u_i,v)},$
$2\sum_{(u_i,v)\in W_J} \eta_{3J}(u_i,v)e^{i2\sum_{(u_i,v)\in W_J}\beta_{3J}(u_i,v)})$ is $\eta_J - strong$ SVCNGS for all $J = 1, 2$.
Twice the degree of sum of $\eta_1 - edges$ in $\tau$ is

$$\begin{aligned}
2\sum_{(u_i,v)\in W_1} \eta_{11}(u_i,v)e^{i2\sum_{(u_i,v)\in W_1}\beta_{11}(u_i,v)} &= 2(\eta_{11}(u_1,u_2)+\eta_{11}(u_2,u_4)+\eta_{11}(u_3,u_4)) \\
&\quad e^{i2(\beta_{11}(u_1,u_2)+\beta_{11}(u_2,u_4)+\beta_{11}(u_3,u_4))} \\
&= 2(.4+.3+.3)e^{i2(1.0\pi+.6\pi+.6\pi)} \\
&= 2.0e^{i4.4\pi}
\end{aligned}$$

$$\begin{aligned}
2\sum_{(u_i,v)\in W_1} \eta_{21}(u_i,v)e^{i2\sum_{(u_i,v)\in W_1}\beta_{21}(u_i,v)} &= 2(\eta_{21}(u_1,u_2)+\eta_{21}(u_2,u_4)+\eta_{21}(u_3,u_4)) \\
&\quad e^{i2(\beta_{21}(u_1,u_2)+\beta_{21}(u_2,u_4)+\beta_{21}(u_3,u_4))} \\
&= 2(.6+.6+.5)e^{i2(1.2\pi+1.8\pi+1.8\pi)} \\
&= 3.4e^{i9.6\pi}
\end{aligned}$$

$$\begin{aligned}
2\sum_{(u_i,v)\in W_1} \eta_{31}(u_i,v)e^{i2\sum_{(u_i,v)\in W_1}\beta_{31}(u_i,v)} &= 2(\eta_{31}(u_1,u_2)+\eta_{31}(u_2,u_4)+\eta_{31}(u_3,u_4)) \\
&\quad e^{i2(\beta_{31}(u_1,u_2)+\beta_{31}(u_2,u_4)+\beta_{31}(u_3,u_4))} \\
&= 2(.4+.4+.6)e^{i2(1.6\pi+1.6\pi+1.6\pi)} \\
&= 2.8e^{i9.6\pi}
\end{aligned}$$

Degree of $\eta_1 - strong$ vertices in SVCNGS is given by Example-3.5.

$$\begin{aligned}
\sum_{i=1}^{4} d_{\eta_1}(u_i) &= (\sum_{i=1}^{4} d_{\eta_{1J}}(u_i), \sum_{i=1}^{4} d_{\eta_{2J}}(u_i), \sum_{i=1}^{4} d_{\eta_{3J}}(u_i)) \\
&= (2.0e^{i4.4\pi}, 3.4e^{i9.6\pi}, 2.8e^{i9.6\pi})
\end{aligned}$$

$$\begin{aligned}
\sum_{i=1}^{4} d_{\eta_1}(u_i) &= (2\sum_{(u_i,v)\in W_1} \eta_{11}(u_i,v)e^{i2\sum_{(u_i,v)\in W_1}\beta_{11}(u_i,v)}, 2\sum_{(u_i,v)\in W_1} \eta_{21}(u_i,v) \\
&\quad e^{i2\sum_{(u_i,v)\in W_1}\beta_{21}(u_i,v)}, 2\sum_{(u_i,v)\in W_1} \lambda_{31}(u_i,v)e^{i2\sum_{(u_i,v)\in W_1}\beta_{31}(u_i,v)})
\end{aligned}$$

Similarly, we calculate

$$\begin{aligned}
\sum_{i=1}^{4} d_{\eta_2}(u_i) &= (2\sum_{(u_i,v)\in W_2} \eta_{12}(u_i,v)e^{i2\sum_{(u_i,v)\in W_2}\beta_{12}(u_i,v)}, 2\sum_{(u_i,v)\in W_2} \eta_{22}(u_i,v) \\
& \quad e^{i2\sum_{(u_i,v)\in W_2}\beta_{22}(u_i,v)}, 2\sum_{(u_i,v)\in W_2} \eta_{32}(u_i,v)e^{i2\sum_{(u_i,v)\in W_2}\beta_{32}(u_i,v)}) \\
&= (1.6e^{i2.8\pi}, 2.4e^{i5.6\pi}, 2.0e^{i6.4\pi})
\end{aligned}$$

**Definition 3.8.** Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. If $d_{\eta_J}(u_i) = (a, b, c)$ for all $u_i \in Q$, then for every vertex with a degree of $\eta_{1J} - degree$, there is an equal degree of $a$; similarly, for every vertex with a degree of $\eta_{2J} - degree$, there is an equal degree of $b$; and for every vertex with a degree of $\eta_{3J} - degree$, there is an equal degree of $c$. For all $J = 1, 2, ..., k$, $\tau$ is then considered to be $\eta_J$ regular SVCNGS.

**Definition 3.9.** Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. The total degree of $\eta_J$ vertex is defined as

$$td_{\eta_J}(f) = (td_{\eta_{1J}}(f), td_{\eta_{2J}}(f), td_{\eta_{3J}}(f))$$

$$\begin{aligned}
td_{\eta_{1J}}(f) &= (\sum_{(f,v)\in W_J} \eta_{1J}(f,v) + \gamma_1(f))e^{i\sum_{(f,v)\in W_J}\beta_{1J}(f,v)+\alpha_1(f)}, \\
td_{\eta_{2J}}(f) &= (\sum_{(f,v)\in W_J} \eta_{2J}(f,v) + \gamma_2(f))e^{i\sum_{(f,v)\in W_J}\beta_{2J}(f,v)+\alpha_2(f)}, \\
td_{\eta_{3J}}(f) &= (\sum_{(f,v)\in W_J} \eta_{3J}(f,v) + \gamma_3(f))e^{i\sum_{(f,v)\in W_J}\beta_{3J}(f,v)+\alpha_3(f)}
\end{aligned}$$

The total degree of every vertex in $\eta_{1J}$ has the same degree. $n_1$ and the total degree of each vertex in $\eta_{2J}$ has the same degree. $n_2$, and the total degree of each vertex in $\eta_{3J}$ has the same degree $n_3$. For all $J = 1, 2, ..., k$, $\tau$ is then considered to be totally $\eta_J$ regular SVCNGS.

## 4. **Edge Regular in SVCNGS**

This section introduces the idea of $\eta_J - edge$ regular SVCNGS. Moreover, some properties of the $\eta_J - edge$ regular SVCNGS are explained with examples.

**Definition 4.1.** Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$ and let $e_{ij} \in W_J$ be an edge in $\tau$. Then the degree of an $\eta_J - edge$ $e_{ij} \in W_J$ is defined as

$$d_{\eta_J}(e_{ij}) = (d_{\eta_{1J}}(e_{ij}), d_{\eta_{2J}}(e_{ij}), d_{\eta_{3J}}(e_{ij}))$$

$$
\begin{aligned}
d_{\eta_{1J}}(e_{ij}) &= d_{\eta_{1J}}(u_i) + d_{\eta_{1J}}(u_j) - 2\eta_{1J}(u_i,u_j)e^{i2\beta_{1J}(u_i,u_j)} \ (or)\\
d_{\eta_{1J}}(e_{ij}) &= (\sum_{\ell^r}\eta_{1J}(u_i,u_k) + \sum_{\ell^s}\eta_{1J}(u_k,u_j))\\
&\quad e^{i\sum_{\ell^r}\beta_{1J}(u_i,u_k)+\sum_{\ell^s}\beta_{1J}(u_k,u_j)}
\end{aligned}
$$

$$
\begin{aligned}
d_{\eta_{2J}}(e_{ij}) &= (d_{\eta_{2J}}(u_i) + d_{\eta_{2J}}(u_j) - 2\eta_{2J}(u_i,u_j)e^{i2\beta_{2J}(u_i,u_j)} \ (or)\\
d_{\eta_{2J}}(e_{ij}) &= \sum_{\ell^r}\eta_{2J}(u_i,u_k) + \sum_{\ell^s}\eta_{2J}(u_k,u_j))\\
&\quad e^{i\sum_{\ell^r}\beta_{2J}(u_i,u_k)+\sum_{\ell^s}\beta_{2J}(u_k,u_j)}
\end{aligned}
$$

$$
\begin{aligned}
d_{\eta_{3J}}(e_{ij}) &= (d_{\eta_{3J}}(u_i) + d_{\eta_{3J}}(u_j) - 2\eta_{3J}(u_i,u_j)e^{i2\beta_{3J}(u_i,u_j)} \ (or)\\
d_{\eta_{3J}}(e_{ij}) &= (\sum_{\ell^r}\eta_{3J}(u_i,u_k) + \sum_{\ell^s}\eta_{3J}(u_k,u_j))\\
&\quad e^{i\sum_{\ell^r}\beta_{3J}(u_i,u_k)+\sum_{\ell^s}\beta_{3J}(u_k,u_j)},\\
&\quad \forall\ \ell^r = (u_i,u_k) \in W_J, k \neq j, \ell^s = (u_k,u_j) \in W_J, k \neq i \text{and } J = 1,2,...,k.
\end{aligned}
$$

Notation: An $\eta_J - edge$ of an SVCNGS is denoted by $e_{ij} \in W_J$ or $u_iu_j \in W_J$.

**Note:**

$$
\begin{aligned}
d_{\eta_{lJ}}(e_{ij}) &= (\sum_{(u_i,u_j)\in W_J}\eta_{1J}(u_i,u_j) + \sum_{(u_j,u_k)\in W_J}\eta_{1J}(u_j,u_k) - 2\eta_{1J}(u_i,u_j))\\
&\quad e^{i\sum_{(u_i,u_j)\in W_J}\beta_{lJ}(u_i,u_j)+\sum_{(u_j,u_k)\in W_J}\beta_{1J}(u_j,u_k)-2\beta_{1J}(u_i,u_j)},\ l = 1,2,3. \text{ and } J = 1,2,...,k.
\end{aligned}
$$

**Definition 4.2.** Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. The minimum $\eta_J - edge$ degree of $\tau$ is $\delta_{\eta_J}(G) = (\delta_{\eta_{1J}}(G), \delta_{\eta_{2J}}(G), \delta_{\eta_{3J}}(G))$, where

$$
\begin{aligned}
\delta_{\eta_{1J}}(G) &= \wedge\{d_{\eta_{1J}}(e_{ij})/e_{ij} \in W_J\}\\
\delta_{\eta_{2J}}(G) &= \wedge\{d_{\eta_{2J}}(e_{ij})/e_{ij} \in W_J\}\\
\delta_{\eta_{3J}}(G) &= \wedge\{d_{\eta_{3J}}(e_{ij})/e_{ij} \in W_J\},\ \forall\ J = 1,2,...k.
\end{aligned}
$$

The maximum $\eta_J - edge$ degree of $\tau$ is $\Delta_{\eta_J}(G) = (\Delta_{\eta_{1J}}(G), \Delta_{\eta_{2J}}(G), \Delta_{\eta_{3J}}(G))$, where

$$
\begin{aligned}
\Delta_{\eta_{1J}}(G) &= \vee\{d_{\eta_{1J}}(e_{ij})/e_{ij} \in W_J\}\\
\Delta_{\eta_{2J}}(G) &= \vee\{d_{\eta_{2J}}(e_{ij})/e_{ij} \in W_J\}\\
\Delta_{\eta_{3J}}(G) &= \vee\{d_{\eta_{3J}}(e_{ij})/e_{ij} \in W_J\},\ \forall\ J = 1,2,...k.
\end{aligned}
$$

**Definition 4.3.** Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS of GS $G^* = \{M, W_1, W_2, ..., W_k\}$ and let $e_{ij} \in W_J$ be an edge in $\tau$. Then the total degree of an $\eta_J - edge$ $e_{ij} \in W_J$ is defined as

$$
td_{\eta_J}(e_ij) = (td_{\eta_{1J}}(e_{ij}), td_{\eta_{2J}}(e_{ij}), td_{\eta_{3J}}(e_{ij})),
$$

$$
\begin{aligned}
td_{\eta_{1J}}(e_{ij}) &= \sum_{\ell^r} \eta_{1J}(u_i,u_k) + \sum_{\ell^s} \eta_{1J}(u_k,u_j) + \eta_{1J}(e_{ij}) \\
&\quad e^{i \sum_{\ell^r} \beta_{1J}(u_i,u_k) + \sum_{\ell^s} \beta_{1J}(u_k,u_j) + \beta_{1J}(e_{ij})}, \\
td_{\eta_{2J}}(e_{ij}) &= \sum_{\ell^r} \eta_{2J}(u_i,u_k) + \sum_{\ell^s} \eta_{2J}(u_k,u_j) + \eta_{2J}(e_{ij}) \\
&\quad e^{i \sum_{\ell^r} \beta_{2J}(u_i,u_k) + \sum_{\ell^s} \beta_{2J}(u_k,u_j) + \beta_{2J}(e_{ij})}, \\
td_{\eta_{3J}}(e_{ij}) &= \sum_{\ell^r} \eta_{3J}(u_i,u_k) + \sum_{\ell^s} \eta_{3J}(u_k,u_j) + \eta_{3J}(e_{ij}) \\
&\quad e^{i \sum_{\ell^r} \beta_{3J}(u_i,u_k) + \sum_{\ell^s} \beta_{3J}(u_k,u_j) + \beta_{3J}(e_{ij})}, \\
&\quad \forall\ \ell^r = (u_i,u_k) \in W_J, k \neq j, \ell^s = (u_k,u_j) \in W_J, k \neq i \text{and } J = 1,2,...,k.
\end{aligned}
$$

**Definition 4.4.** Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. If $d_{\eta_J}(e_{ij}) = (p, q, r)$ for all $e_{ij} \in W_J$ for each edge of $\eta_{1J}$ has the same degree $p$ and for each edge of $\eta_{2J}$ has the same degree $q$ and for each edge of $\eta_{3J}$ has the same degree $r$. Then $\tau$ is said to be $\eta_J - edge$ regular SVCNGS for all $J = 1, 2, ..., k$.

**Example 4.5.** Consider an SVCNGS
$tau = (\gamma, \eta_1, \eta_2)$ of GS $\tau^* = (M, W_1, W_2)$ given Figure-2 is $\eta_J - edge$ regular SVCNGS such that $\gamma = \{u_1(.4e^{i.5\pi}, .3e^{i.4\pi}, .5e^{i.6\pi})$,
$u_2(.4e^{i.5\pi}, .4e^{i.5\pi}, .6e^{i.6\pi}), u_3(.5e^{i.5\pi}, .3e^{i.4\pi}, .5e^{i.6\pi}),\ u_4(.4e^{i.5\pi}, .4e^{i.5\pi}, .6e^{i.6\pi})\}$. The degree of

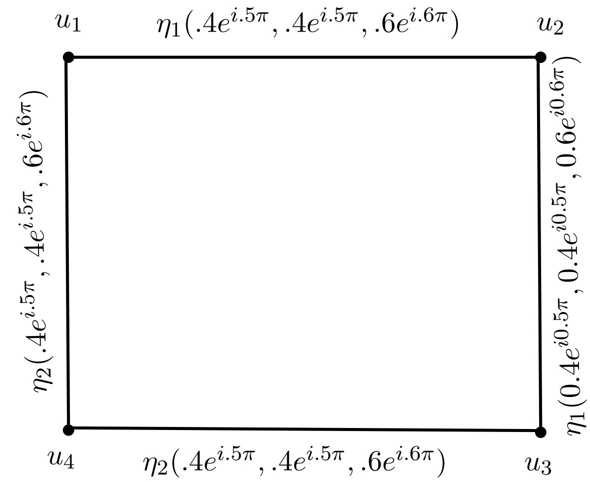


FIGURE 2. $\tau = (\gamma, \eta_1, \eta_2)$ is regular SVCNGS of $\tau^*$

an $\eta_1 - edge$.

$$d_{\eta_1}(e_{12}) = (d_{\eta_{11}}(e_{12}), d_{\eta_{21}}(e_{12}), d_{\eta_{31}}(e_{12}))$$

$$\begin{aligned} d_{\eta_{11}}(e_{12}) &= d_{\eta_{11}}(u_1) + d_{\eta_{11}}(u_2) - 2\eta_{11}(u_1,u_2)e^{i2\beta_{11}(u_1,u_2)} \ (or) \\ d_{\eta_{11}}(e_{12}) &= (\sum_{(u_2,u_4)\in W_1, u_4\neq u_1} \eta_{11}(u_2,u_4))e^{i\sum_{(u_2,u_4)\in W_1,u_4\neq u_1}\beta_{11}(u_2,u_4)} \\ &= (.4+.8-2(.4))e^{i(.5\pi+1.0\pi-2(.5\pi))} \\ &= .4e^{i.5} \end{aligned}$$

$$\begin{aligned} d_{\eta_{21}}(e_{12}) &= d_{\eta_{21}}(u_1) + d_{\eta_{21}}(u_2) - 2\eta_{21}(u_1,u_2)e^{i2\beta_{21}(u_1,u_2)} \\ &= (.4+.8-2(0.4))e^{i(.5\pi+1.0\pi-2(.5\pi))} \\ &= .4e^{i.5} \end{aligned}$$

$$\begin{aligned} d_{\eta_{31}}(e_{12}) &= d_{\eta_{31}}(u_1) + d_{\eta_{31}}(u_2) - 2\eta_{31}(u_1,u_2)e^{i2\beta_{31}(u_1,u_2)} \\ &= (.6+1.2-2(.6))e^{i(.6\pi+1.2\pi-2(.6\pi))} \\ &= .6e^{i.6\pi} \\ d_{\eta_1}(e_{12}) &= (.4e^{i.5\pi}, .4e^{i.5\pi}, .6e^{i.6\pi}) \end{aligned}$$

Similarly, we calculate
$d_{\eta_1}(e_{12}) = d_{\eta_1}(e_{23}) = d_{\eta_1}(e_{34}) = d_{\eta_1}(e_{14}) = (.4e^{i.5\pi}, .4e^{i.5\pi}, .6e^{i.6\pi})$
The degree of an $\eta_2 - edge$.
$d_{\eta_2}(e_{12}) = d_{\eta_2}(e_{23}) = d_{\eta_2}(e_{34}) = d_{\eta_2}(e_{14}) = (.4e^{i.5\pi}, .4e^{i.5\pi}, .6e^{i.6\pi})$
In the above example-4.5 is $\eta_J - edge$ regular SVCNGS for all $J = 1, 2$.

**Definition 4.6.** Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. If $td_{\eta_J}(e_{ij}) = (x, y, z)$ for all $e_{ij} \in W_J$ for each edge of $\eta_{1J}$ has the same total degree $x$ and for each edge of $\eta_{2J}$ has the same total degree $y$ and for each edge of $\eta_{3J}$ has the same total degree $z$. Then $\tau$ is said to be totally $\eta_J - edge$ regular SVCNGS for all $J = 1, 2, ..., k$.

**Example 4.7.** Consider an SVCNGS $\tau = (\gamma, \eta_1, \eta_2)$ of GS $\tau^* = (M, W_1, W_2)$ is given Figure-2 in example-4.5 is totally $\eta_J - edge$ regular SVCNGS for all $J = 1, 2$. The total degree of an $\eta_1 - edge$ is
$td_{\eta_1}(e_{12}) = td_{\eta_1}(e_{23}) = td_{\eta_1}(e_{34}) = td_{\eta_1}(e_{14}) = (.8e^{i1.0\pi}, .8e^{i1.0\pi}, 1.2e^{i1.2\pi})$
The total degree of an $\eta_2 - edge$ is
$td_{\eta_2}(e_{12}) = td_{\eta_2}(e_{23}) = td_{\eta_2}(e_{34}) = td_{\eta_2}(e_{14}) = (.8e^{i1.0\pi}, .8e^{i1.0\pi}, 1.2e^{i1.2\pi})$
Hence, $\tau$ is totally $\eta_J - edge$ regular SVCNGS for all $J = 1, 2$.

**Theorem 4.8.** *Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$ and $\tau^*$ is a cycle. Then $\sum_{i=1}^{n} d_{\eta_J}(u_i) = \sum_{i=1}^{n} d_{\eta_J}(e_{ij})$ for all $J = 1, 2, ..., k$ and $j = i + 1$.*

*Proof.* Given that $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$ and $\tau^*$ is a cycle $u_1u_2u_3...u_n$. Then

$$\sum_{i=1}^{n} d_{\eta_J}(e_{ij}) = (\sum_{i=1}^{n} d_{\eta_{1J}}(e_{ij}), \sum_{i=1}^{n} d_{\eta_{2J}}(e_{ij}), \sum_{i=1}^{n} d_{\eta_{3J}}(e_{ij}))$$

$$\forall\ J = 1, 2, ..., k \text{ and } j = i + 1.$$

Consider

$$\begin{aligned}
&\sum_{i=1}^{n} d_{\eta_{1J}}(e_{ij})\\
&= d_{\eta_{1J}}(e_{12}) + d_{\eta_{1J}}(e_{23}) + ... + d_{\eta_{1J}}(e_{n1}), \text{ where } u_{n+1} = u_1\\
&= d_{\eta_{1J}}(u_1) + d_{\eta_{1J}}(u_2) - 2\eta_{1J}(u_1, u_2)e^{i2\beta_{1J}(u_1,u_2)} + d_{\eta_{1J}}(u_2) + d_{\eta_{1J}}(u_3)\\
&\quad -2\eta_{1J}(u_2, u_3)e^{i2\beta_{1J}(u_2,u_3)} + ... + d_{\eta_{1J}}(u_n) + d_{\eta_{1J}}(u_1) - 2\eta_{1J}(u_n, u_1)e^{i2\beta_{1J}(u_n,u_1)}\\
&= 2d_{\eta_{1J}}(u_1) + 2d_{\eta_{1J}}(u_2) + ... + 2d_{\eta_{1J}}(u_n)\\
&\quad -2(\eta_{1J}(u_1, u_2)e^{i2\beta_{1J}(u_1,u_2)} + \eta_{1J}(u_2, u_3)e^{i2\beta_{1J}(u_2,u_3)} + ... + \eta_{1J}(u_n, u_1)e^{i2\beta_{1J}(u_n,u_1)})\\
&= 2\sum_{u_i\in M} d_{\eta_{1J}}(u_i) - 2\sum_{i=1}^{n} \eta_{1J}(u_i, u_{i+1})e^{i2\sum_{i=1}^{n}\beta_{1J}(u_i,u_{i+1})}\\
&= \sum_{u_i\in M} d_{\eta_{1J}}(u_i) + \sum_{u_i\in M} d_{\eta_{1J}}(u_i) - 2\sum_{i=1}^{n} \eta_{1J}(u_i, u_{i+1})e^{i2\sum_{i=1}^{n}\beta_{1J}(u_i,u_{i+1})}\\
&= \sum_{u_i\in M} d_{\eta_{1J}}(u_i) + 2\sum_{i=1}^{n} \eta_{1J}(u_i, u_{i+1})e^{i2\sum_{i=1}^{n}\beta_{1J}(u_i,u_{i+1})} - 2\sum_{i=1}^{n} \eta_{1J}(u_i, u_{i+1})e^{i2\sum_{i=1}^{n}\beta_{1J}(u_i,u_{i+1})}\\
&= \sum_{u_i\in M} d_{\eta_{1J}}(u_i)
\end{aligned}$$

Similarly, we derive the equation
$\sum_{i=1}^{n} d_{\eta_{2J}}(e_{ij}) = \sum_{u_i\in M} d_{\eta_{2J}}(u_i)$,
$\sum_{i=1}^{n} d_{\eta_{3J}}(e_{ij}) = \sum_{u_i\in M} d_{\eta_{3J}}(u_i)$.
Hence, $\sum_{i=1}^{n} d_{\eta_J}(u_i) = \sum_{i=1}^{n} d_{\eta_J}(e_{ij})$ for all $J = 1, 2, ..., k$ and $j = i + 1$. □

**Theorem 4.9.** *Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS on a crisp graph $\tau^S$ of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. Then $\sum_{e_{ij}\in W_J} d_{\eta_J}(e_{ij}) = (\sum_{e_{ij}\in W_J} d^*_{\eta_J}(e_{ij})\eta_{1J}(u_iu_j)e^{i\beta_{1J}(u_iu_j)}$, $\sum_{e_{ij}\in W_J} d^*_{\eta_J}(e_{ij})\eta_{2J}(u_iu_j)e^{i\beta_{2J}(u_iu_j)}, \sum_{e_{ij}\in W_J} d^*_{\eta_J}(e_{ij})\eta_{3J}(u_iu_j)e^{i\beta_{3J}(u_iu_j)})$ where $d^*_{\eta_J}(e_{ij}) = d^*_{\eta_J}(u_i) + d^*_{\eta_J}(u_j) - 2$ for all $e_{ij} \in W_J$ and $J = 1, 2, ..., k$.*

*Proof.* Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS on a crisp graph $\tau^S$ of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. We know that $d_{\eta_J}(e_{ij}) = (d_{\eta_{1J}}(e_{ij}), d_{\eta_{2J}}(e_{ij}), d_{\eta_{3J}}(e_{ij}))$.

Therefore, in $\sum_{e_{ij}\in W_J} d_{\eta_{1J}}(e_{ij})$, every $\eta_{1J}e^{i\beta_{1J}} - edge$ contributes it's truth membership values exactly number of $\eta_{1J}e^{i\beta_{1J}} - edges$ adjacent to that $\eta_{1J}e^{i\beta_{1J}} - edge$ times.
Thus, in $\sum_{e_{ij}\in W_J} d_{\eta_{1J}}(e_{ij})$, each $\eta_{1J}(u_iu_j)e^{i\beta_{1J}(u_iu_j)}$ appears $d^*_{\eta_{1J}}(e_{ij})$ times.

$$\text{Hence, } \sum_{e_{ij}\in W_J} d_{\eta_{1J}}(e_{ij}) = \sum_{e_{ij}\in W_J} d^*_{\eta_{1J}}(e_{ij})\eta_{1J}(u_iu_j)e^{i\beta_{1J}(u_iu_j)}$$

Similarly, we solve the equation

$$\begin{aligned}
\sum_{e_{ij}\in W_J} d_{\eta_{2J}}(e_{ij}) &= \sum_{e_{ij}\in W_J} d^*_{\eta_{2J}}(e_{ij})\eta_{2J}(u_iu_j)e^{i\beta_{2J}(u_iu_j)} \\
\sum_{e_{ij}\in W_J} d_{\eta_{3J}}(e_{ij}) &= \sum_{e_{ij}\in W_J} d^*_{\eta_{3J}}(e_{ij})\eta_{3J}(u_iu_j)e^{i\beta_{3J}(u_iu_j)}
\end{aligned}$$

Hence, $\sum_{e_{ij}\in W_J} d_{\eta_J}(e_{ij}) = (\sum_{e_{ij}\in W_J} d^*_{\eta_J}(e_{ij})\eta_{1J}(u_iu_j)e^{i\beta_{1J}(u_iu_j)}$,
$\sum_{e_{ij}\in W_J} d^*_{\eta_J}(e_{ij})\eta_{2J}(u_iu_j)e^{i\beta_{2J}(u_iu_j)}, \sum_{e_{ij}\in W_J} d^*_{\eta_J}(e_{ij})\eta_{3J}(u_iu_j)e^{i\beta_{3J}(u_iu_j)})$ □

**Theorem 4.10.** *Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS on a $\eta_J$ regular crisp graph $\tau^S$ of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. Then $\sum_{e_{ij}\in W_J} d_{\eta_J}(e_{ij}) = ((k-1)\sum_{u_i\in M} d_{\eta_{1J}}(u_i), (k-1)\sum_{u_i\in M} d_{\eta_{2J}}(u_i), (k-1)\sum_{u_i\in M} d_{\eta_{3J}}(u_i))$*

*Proof.* By Theorem-4.9,

$$\begin{aligned}
\sum_{e_{ij}\in W_J} d_{\eta_J}(e_{ij}) &= (\sum_{e_{ij}\in W_J} d^*_{\eta_1}(e_{ij})\eta_{1J}(u_iu_j), \sum_{e_{ij}\in W_J} d^*_{\eta_J}(e_{ij})\eta_{2J}(u_iu_j), \\
&\quad \sum_{e_{ij}\in W_J} d^*_{\eta_J}(e_{ij})\eta_{3J}(u_iu_j)) \\
&= (\sum_{u_i,u_j\in W_J} (d^*_{\eta_J}(u_i) + d^*_{\eta_J}(u_j) - 2)\eta_{1J}(u_i,u_j)e^{i\beta_{1J}(u_i,u_j)}, \\
&\quad \sum_{u_i,u_j\in W_J} (d^*_{\eta_J}(u_i) + d^*_{\eta_J}(u_j) - 2)\eta_{2J}(u_i,u_j)e^{i\beta_{2J}(u_i,u_j)}, \\
&\quad \sum_{u_i,u_j\in W_J} (d^*_{\eta_J}(u_i) + d^*_{\eta_J}(u_j) - 2)\eta_{3J}(u_i,u_j)e^{i\beta_{3J}(u_i,u_j)})
\end{aligned}$$

□

Since, $\tau^S$ is a $\eta_J$ regular crisp graph of GS, $d^*_{\eta_J}(u_i) = k$ for all $u_i \in M$.

$$\begin{aligned}
\sum_{e_{ij}\in W_J} d_{\eta_J}(e_{ij}) &= ((k+k-2)\sum_{u_i,u_j\in W_J} \lambda_{1J}(u_i,u_j)e^{i\sum_{u_i,u_j\in W_J}\lambda_{1J}(u_i,u_j)}, \\
&\quad (k+k-2)\sum_{u_i,u_j\in W_J} \eta_{2J}(u_i,u_j)e^{i\sum_{u_i,u_j\in W_J}\eta_{2J}(u_i,u_j)}, \\
&\quad (k+k-2)\sum_{u_i,u_j\in W_J} \eta_{3J}(u_i,u_j)e^{i\sum_{u_i,u_j\in W_J}\eta_{3J}(u_i,u_j)}) \\
&= (2(k-1)\sum_{u_i,u_j\in W_J} \eta_{1J}(u_i,u_j)e^{i\sum_{u_i,u_j\in W_J}\eta_{1J}(u_i,u_j)}, \\
&\quad 2(k-1)\sum_{u_i,u_j\in W_J} \eta_{2J}(u_i,u_j)e^{i\sum_{u_i,u_j\in W_J}\eta_{2J}(u_i,u_j)}, \\
&\quad 2(k-1)\sum_{u_i,u_j\in W_J} \eta_{3J}(u_i,u_j)e^{i\sum_{u_i,u_j\in W_J}\eta_{3J}(u_i,u_j)}) \\
&= ((k-1)\sum_{u_i\in M} d_{\eta_{1J}}(u_i), (k-1)\sum_{u_i\in M} d_{\eta_{2J}}(u_i), \\
&\quad (k-1)\sum_{u_i\in Q} d_{\eta_{3J}}(u_i))
\end{aligned}$$

**Theorem 4.11.** *Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS on a crisp graph $\tau^S$ of GS $\tau^* = \{Q, R_1, R_2, ..., R_k\}$. Then* $\sum_{e_{ij}\in W_J} td_{\eta_J}(e_{ij}) = (\sum_{e_{ij}\in W_J} d^*_{\eta_J}(e_{ij})\eta_{1J}(u_iu_j)e^{i\beta_{1J}(u_iu_j)} + \sum_{u_iu_j\in W_J}\eta_{1J}(u_iu_j)e^{i\sum_{u_iu_j\in W_J}\beta_{1J}(u_iu_j)}, \sum_{e_{ij}\in W_J} d^*_{\eta_J}(e_{ij})\eta_{2J}(u_iu_j)e^{i\beta_{2J}(u_iu_j)} + \sum_{u_iu_j\in R_J}\eta_{2J}(u_iu_j)e^{i\sum_{u_iu_j\in W_J}\beta_{2J}(u_iu_j)}, \sum_{e_{ij}\in W_J} d^*_{\eta_J}(e_{ij})\eta_{3J}(u_iu_j)e^{i\beta_{3J}(u_iu_j)} + \sum_{u_iu_j\in R_J}\eta_{3J}(u_iu_j)e^{i\sum_{u_iu_j\in W_J}\beta_{3J}(u_iu_j)})$

*Proof.* By Definition-4.3 of total degree of $\eta_J - edge$ of $\tilde{G}$.

$$td_{\eta_J}(e_ij) = (td_{\eta_{1J}}(e_{ij}), td_{\eta_{2J}}(e_{ij}), td_{\eta_{3J}}(e_{ij}))$$

$$\begin{aligned}
\sum_{e_{ij}\in W_J} td_{\eta_J}(eij) &= (\sum_{u_iu_j\in W_J} td_{\eta_{1J}}(e_{ij}), \sum_{u_iu_j\in W_J} td_{\eta_{2J}}(e_{ij}), \sum_{u_iu_j\in W_J} td_{\eta_{3J}}(e_{ij})) \\
&= (\sum_{e_{ij}\in W_J} (d_{\eta_{1J}}(e_{ij}) + \eta_{1J}(u_iu_j)e^{i\beta_{1J}(u_iu_j)}), \\
&\quad \sum_{e_{ij}\in W_J} (d_{\eta_{2J}}(e_{ij}) + \eta_{2J}(u_iu_j)e^{i\beta_{2J}(u_iu_j)}), \\
&\quad \sum_{e_{ij}\in W_J} (d_{\eta_{3J}}(e_{ij}) + \eta_{3J}(u_iu_j)e^{i\beta_{3J}(u_iu_j)})) \\
&= (\sum_{e_{ij}\in W_J} d_{\eta_{1J}}(e_{ij}) + \sum_{u_iu_j\in W_J} \eta_{1J}(u_iu_j)e^{i\sum_{u_iu_j\in W_J}\beta_{1J}(u_iu_j)}, \\
&\quad \sum_{e_{ij}\in W_J} d_{\eta_{2J}}(e_{ij}) + \sum_{u_iu_j\in W_J} \eta_{2J}(u_iu_j)e^{i\sum_{u_iu_j\in W_J}\beta_{2J}(u_iu_j)}, \\
&\quad \sum_{e_{ij}\in W_J} d_{\eta_{3J}}(e_{ij}) + \sum_{u_iu_j\in W_J} \eta_{3J}(u_iu_j)e^{i\sum_{u_iu_j\in W_J}\beta_{3J}(u_iu_j)})
\end{aligned}$$

By Theorem-4.9, we get

$$\begin{aligned}
\sum_{e_{ij}\in W_J} td_{\eta_J}(e_{ij}) &= (\sum_{e_{ij}\in W_J} d^*_{\eta_J}(e_{ij})\eta_{1J}(u_iu_j)e^{i\beta_{1J}(u_iu_j)} + \sum_{u_iu_j\in W_J} \eta_{1J}(u_iu_j)e^{i\sum_{u_iu_j\in W_J}\beta_{1J}(u_iu_j)}, \\
&\quad \sum_{e_{ij}\in W_J} d^*_{\eta_J}(e_{ij})\eta_{2J}(u_iu_j)e^{i\beta_{2J}(u_iu_j)} + \sum_{u_iu_j\in W_J} \eta_{2J}(u_iu_j)e^{i\sum_{u_iu_j\in W_J}\beta_{2J}(u_iu_j)}, \\
&\quad \sum_{e_{ij}\in W_J} d^*_{\eta_J}(e_{ij})\eta_{3J}(u_iu_j)e^{i\beta_{3J}(u_iu_j)} + \sum_{u_iu_j\in W_J} \eta_{3J}(u_iu_j)e^{i\sum_{u_iu_j\in W_J}\beta_{3J}(u_iu_j)})
\end{aligned}$$

□

**Theorem 4.12.** *Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. If and only if the subsequent statements are equivalent, then $\eta_J$ is a constant functional.*
*(i) $\tau$ is an $\eta_J - edge$ regular SVCNGS.*
*(ii) $\tau$ is a totally $\eta_J - edge$ regular SVCNGS.*

*Proof.* Let us assume that $\eta_J$ is a function that is constant. Then $\eta_{1J}(u_iu_j)e^{i\beta_{1J}(u_iu_j)} = c_1$, $\eta_{2J}(u_iu_j)e^{i\beta_{2J}(u_iu_j)} = c_2$ and $\eta_{3J}(u_iu_j)e^{i\beta_{3J}(u_iu_j)} = c_3$ for every $u_iu_j \in W_J$, where $c_1, c_2, c_3$ are constants. (1)
Assume that $\tau$ is $\eta_J - edge$ regular SVCNGS. Then $d_{\eta_J}(e_{ij}) = (p, q, r)$ for all $e_{ij} \in W_J$. (2)
Consider

$$\begin{aligned}
td_{\eta_J}(eij) &= (d_{\eta_{1J}}(e_{ij}) + \eta_{1J}(u_iu_j)e^{i\beta_{1J}(u_iu_j)}, \\
&\quad d_{\eta_{2J}}(e_{ij}) + \eta_{2J}(u_iu_j)e^{i\beta_{2J}(u_iu_j)}, \\
&\quad d_{\eta_{3J}}(e_{ij}) + \eta_{3J}(u_iu_j)e^{i\beta_{3J}(u_iu_j)}) \\
&= (p + c_1, q + c_2, r + c_3) \text{ for all } u_iu_j \in W_J \text{ by (1) and (2)}
\end{aligned}$$

which implies $\tau$ is totally $\eta_J - edge$ regular SVCNGS.

Therefore, $(i) \Rightarrow (ii)$.

Let $\tau$ be totally $\eta_J - edge$ regular SVCNGS. Then $td_{\eta_J}(e_{ij}) = (x, y, z)$ for all $e_{ij} \in W_J$.

$$\begin{aligned} td_{\eta_J}(e_ij) &= (d_{\eta_{1J}}(e_{ij}) + \eta_{1J}(u_iu_j)e^{i\beta_{1J}(u_iu_j)}, d_{\eta_{2J}}(e_{ij}) + \eta_{2J}(u_iu_j)e^{i\beta_{2J}(u_iu_j)}, \\ &\quad d_{\eta_{3J}}(e_{ij}) + \eta_{3J}(u_iu_j)e^{i\beta_{3J}(u_iu_j)}). \end{aligned}$$

Now,

$$\begin{aligned} d_{\eta_J}(e_{ij}) &= (d_{\eta_{1J}}(e_{ij}), d_{\eta_{2J}}(e_{ij}), d_{\eta_{3J}}(e_{ij})) \\ &= (x - \eta_{1J}(u_iu_j)e^{i\beta_{1J}(u_iu_j)}, y - \eta_{2J}(u_iu_j)e^{i\beta_{2J}(u_iu_j)}, z - \eta_{3J}(u_iu_j)e^{i\beta_{3J}(u_iu_j)}) \\ &= (x - c_1, y - c_2, z - c_3) \text{ by}(1) \end{aligned}$$

Hence, $\tau$ is $\eta_J - edge$ regular SVCNGS.

Thus, $(ii) \Rightarrow (i)$.

Conversely, suppose that (i) and (ii) are equivalent.

As a result$\tau$ is $\eta_J - edge$ regular SVCNGS if and only if $\tau$ is totally $\eta_J - edge$ regular SVCNGS.

We have to prove that $\eta_J$ is a constant function.

Let us assume that $\eta_J$ is not a constant function. (3)

Then

$\eta_{1J}(u_i, u_j)e^{i\beta_{1J}(u_i,u_j)} = \eta_{1J}(u_r, u_s)e^{i\eta_{1J}(u_r,u_s)}$, $\eta_{2J}(u_i, u_j)e^{i\beta_{2J}(u_i,u_j)} = \lambda_{2J}(u_r, u_s)e^{i\lambda_{2J}(u_r,u_s)}$ and $\lambda_{3J}(u_i, u_j)e^{i\beta_{3J}(u_i,u_j)} = \eta_{3J}(u_r, u_s)e^{i\eta_{3J}(u_r,u_s)}$ for at least one pair of $u_iu_j, u_ru_s \in R_J$. Let $\tau$ is $\eta_J - edge$ regular SVCNGS. Then $d_{\eta_J}(e_{ij}) = d_{\eta_J}(e_{rs}) = (p, q, r)$ (4)

$$\begin{aligned} \Rightarrow td_{\eta_J}(e_{ij}) &= (d_{\eta_{1J}}(e_{ij}) + \eta_{1J}(u_i, u_j)e^{i\beta_{1J}(u_i,u_j)}, d_{\eta_{2J}}(e_{ij}) + \eta_{2J}(u_i, u_j)e^{i\beta_{2J}(u_i,u_j)}, \\ &\quad d_{\eta_{3J}}(e_{ij}) + \eta_{3J}e^{i\beta_{1J}(u_i,u_j)}) \\ &= (p + \eta_{1J}(u_i, u_j)e^{i\beta_{1J}(u_i,u_j)}, q + \eta_{2J}(u_i, u_j)e^{i\beta_{2J}(u_i,u_j)}, \\ &\quad r + \eta_{3J}(u_i, u_j)e^{i\beta_{3J}(u_i,u_j)}) \ \forall \ u_iu_j \in W_J. \end{aligned}$$

and

$$\begin{aligned} td_{\eta_J}(e_{rs}) &= (d_{\eta_{1J}}(e_{rs}) + \eta_{1J}(u_ru_s)e^{i\beta_{1J}(u_ru_s)}, d_{\eta_{2J}}(e_{rs}) + \eta_{2J}(u_ru_s)e^{i\beta_{2J}(u_ru_s)}, \\ &\quad d_{\eta_{3J}}(e_{rs}) + \eta_{3J}(u_ru_s)e^{i\beta_{3J}(u_ru_s)}) \\ &= (p + \eta_{1J}(u_ru_s)e^{i\beta_{1J}(u_ru_s)}, q + \eta_{2J}(u_ru_s)e^{i\beta_{2J}(u_ru_s)}, \\ &\quad r + \eta_{3J}(u_ru_s)e^{i\beta_{3J}(u_ru_s)}), \ \forall \ u_ru_s \in W_J. \end{aligned}$$

Since,

$\eta_{1J}(u_i, u_j)e^{i\beta_{1J}(u_i,u_j)} \neq \eta_{1J}(u_r, u_s)e^{i\beta_{1J}(u_r,u_s)}$, $\eta_{2J}(u_i, u_j)e^{i\beta_{2J}(u_i,u_j)} \neq \eta_{2J}(u_r, u_s)e^{i\beta_{2J}(u_r,u_s)}$ and $\eta_{3J}(u_i, u_j)e^{i\beta_{3J}(u_i,u_j)} \neq \eta_{3J}(u_r, u_s)e^{i\beta_{3J}(u_r,u_s)}$

$\Rightarrow td_{\eta_J}(e_{ij}) \neq td_{\eta_J}(e_{rs})$

$\Rightarrow$ Not all of $\tau$ is a totally $\eta_J - edge$ regular SVCNGS
$\Rightarrow$ it is a contradiction.
Hence, $\eta_J$ is a constant function. □

**Theorem 4.13.** *Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS is both $\eta_J - edge$ regular and totally $\eta_J - edge$ regular of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. Then $\eta_J$ is a constant function.*

*Proof.* The result is trivial according to Theorem-4.12.
**Note:** The above theorem-4.12 does not hold in its converse. □

**Theorem 4.14.** *Let $\eta_J$ be constant functions in an SVCNGS $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$ and if $\tau$ is $\eta_J$ regular, Then totally $\eta_J - edge$ regular.*

*Proof.* Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be a $\eta_J$ regular SVCNGS. Then $d_{\eta_J}(u_i) = (a, b, c)$ for all $u_i \in M$. Given that $\eta_J$ are constants. That is, $\eta_J(u_i, u_j) = (c_1, c_2, c_3)$ for all $u_i u_j \in W_J$ where $c_1, c_2, c_3$ are constant
We have to prove that $\tau$ is totally $\eta_J - edge$ regular SVCNGS.
By Definition-4.3 of totally $\eta_J - edge$ degree, we have

$$td_{\eta_J}(e_ij) = (td_{\eta_{1J}}(e_{ij}), td_{\eta_{2J}}(e_{ij}), td_{\eta_{3J}}(e_{ij}))$$

where

$$\begin{aligned} td_{\eta_{1J}}(e_{ij}) &= d_{\eta_{1J}}(u_i) + d_{\eta_{1J}}(u_j) - \eta_{1J}(u_i, u_j)e^{i\beta_{1J}(u_i,u_j)}, \ \forall\ u_iu_j \in W_J \\ &= a + a - c_1, \ \forall\ u_iu_j \in W_J \ (\therefore\ \tau \text{ is regular}) \\ &= 2a + c_1 = constant, \ \forall\ u_iu_j \in W_J. \end{aligned}$$

Similarly, we solve the equation

$$\begin{aligned} td_{\eta_{2J}}(e_{ij}) &= d_{\eta_{2J}}(u_i) + d_{\eta_{2J}}(u_j) - \eta_{2J}(u_i, u_j)e^{i\beta_{2J}(u_i,u_j)}, \ \forall\ u_iu_j \in W_J \\ &= b + b - c_2, \ \forall\ u_iu_j \in W_J \ (\therefore\ \tau \text{ is regular}) \\ &= 2b + c_2 = constant, \ \forall\ u_iu_j \in W_J. \end{aligned}$$

$$\begin{aligned} td_{\eta_{3J}}(e_{ij}) &= d_{\eta_{3J}}(u_i) + d_{\eta_{3J}}(u_j) - \eta_{3J}(u_i, u_j)e^{i\beta_{3J}(u_i,u_j)}, \ \forall\ u_iu_j \in W_J \\ &= c + c - c_3, \ \forall\ u_iu_j \in W_J \ (\therefore\ \tau \text{ is regular}) \\ &= 2c + c_2 = \text{constant}, \ \forall\ u_iu_j \in W_J. \end{aligned}$$

$(ie)$ $td_{\eta_J}(e_ij) = (2a + c_1, 2b + c_2, 2c + c_3)$
$\Rightarrow$ $\tau$ is a totally $\eta_J - edge$ regular SVCNGS. □

**Theorem 4.15.** *Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS on a regular crisp graph $\tau^S$ of GS $\tau^* = \{Q, R_1, R_2, ..., R_k\}$. Then $\eta_J$ is a constant if and only if $\tau$ is both $\eta_J$ regular and $\eta_J-edge$ regular SVCNGS.*

*Proof.* Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS on a regular crisp GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$.
Assume that $\eta_J$ are constant functions, that is $\eta_J(u_i, u_j) = (c_1, c_2, c_3)$ for all $u_iu_j \in W_J$ where $c_1, c_2, c_3$ are constant
**To prove:** $\tau$ is both $\eta_J$ regular and totally $\eta_J - edge$ regular SVCNGS. By Definition-3.4 of $\eta_J - degree$ of a vertex,

$$\begin{aligned}
d_{\eta_J}(u_i) &= (d_{\eta_{1J}}(u_i), d_{\eta_{2J}}(u_i), d_{\eta_{3J}}(u_i)) \\
&= (\sum_{(u_i,v_j)\in W_J} \eta_{1J}(u_i, v_j)e^{i\sum_{(u_i,v_j)\in W_J}\beta_{1J}(u_i,v_j)}, \sum_{(u_i,v_j)\in W_J} \eta_{2J}(u_i, v_j)e^{i\sum_{(u_i,v_j)\in W_J}\beta_{2J}(u_i,v_j)}, \\
&\quad \sum_{(u_i,v_j)\in W_J} \eta_{3J}(u_i, v_j)e^{i\sum_{(u_i,v_j)\in W_J}\beta_{3J}(u_i,v_j)}),\ \forall\ u_i \in M \\
&= (\sum_{(u_i,v_j)\in W_J} c_1, \sum_{(u_i,v_j)\in W_J} c_2, \sum_{(u_i,v_j)\in W_J} c_3) \\
&= (xc_1, yc_2, zc_3)
\end{aligned}$$

Hence, $\tau$ is $\eta_J$ regular SVCNGS.
Now,

$$td_{\eta_J}(e_{ij}) = (td_{\eta_{1J}}(e_{ij}), td_{\eta_{2J}}(e_{ij}), td_{\eta_{3J}}(e_{ij})), \text{where}$$

$$\begin{aligned}
td_{\eta_{1J}}(e_{ij}) &= \sum_{u_iu_k\in W_J, k\neq j} \eta_{1J}(u_i, u_k)e^{i\sum_{u_iu_k\in W_J,k\neq j}\beta_{1J}(u_i,u_k)} + \\
&\quad \sum_{u_ku_j\in W_J, k\neq i} \eta_{1J}(u_k, u_j)e^{i\sum_{u_ku_j\in W_J,k\neq i}\beta_{1J}(u_k,u_j)} + \eta_{1J}(u_i, u_j)e^{i\beta_{1J}(u_i,u_j)} \\
&= \sum_{u_iu_k\in W_J, k\neq j} c_1 + \sum_{u_ku_j\in W_J, k\neq i} c_1 + c_1 \\
&= c_1(x-1) + c_1(x-1) + c_1,\ \forall\ u_iu_j \in W_J \\
&= c_1(2x-1),\ \forall\ u_iu_j \in W_J.
\end{aligned}$$

Similarly, we solve the equation

$$\begin{aligned}
td_{\lambda_{2J}}(e_{ij}) &= c_2(2y-1),\ \forall\ u_iu_j \in W_J \\
td_{\lambda_{3J}}(e_{ij}) &= c_3(2z-1),\ \forall\ u_iu_j \in W_J
\end{aligned}$$

Hence, $\tau$ is also totally $\eta_J$ regular SVCNGS.
Conversely, assume that $\tau$ is both $\eta_J$ regular and $\eta_J - edge$ regular SVCNGS.
**To prove:** $\eta_J$ is a constant function.

Since, $\tau$ is $\eta_J$ regular, $d_{\eta_J}(u_i) = (a, b, c), \ \forall \ u_i \in M$.
Also, $\tau$ is totally $\eta_J - edge$ regular.
Then $td_{\eta_J}(e_ij) = (x, y, z,), \ \forall \ u_iu_j \in W_J$.
By Definition-4.3 of totally $\eta_J - edge$ degree,

$$td_{\eta_J}(e_ij) = (td_{\eta_{1J}}(e_{ij}), td_{\eta_{2J}}(e_{ij}), td_{\eta_{3J}}(e_{ij})), \text{where}$$

$$\begin{aligned} td_{\eta_J}(e_{ij}) &= d_{\eta_{1J}}(u_i) + d_{\eta_{1J}}(u_j) - \eta_{1J}(u_i, u_j)e^{i\beta_{1J}(u_i,u_j)}, \ \forall \ u_iu_j \in W_J \\ x &= a + a - \eta_{1J}(u_i, u_j)e^{i\beta_{1J}(u_i,u_j)}, \ \forall \ u_iu_j \in W_J \\ \eta_{1J}(u_i, u_j)e^{i\beta_{1J}(u_i,u_j)} &= 2a - x, \ \forall \ u_iu_j \in W_J. \end{aligned}$$

Similarly, we solve the equation

$$\begin{aligned} \eta_{2J}(u_i, u_j)e^{i\beta_{2J}(u_i,u_j)} &= 2b - y, \ \forall \ u_iu_j \in W_J. \\ \eta_{3J}(u_i, u_j)e^{i\beta_{3J}(u_i,u_j)} &= 2c - z, \ \forall \ u_iu_j \in W_J. \end{aligned}$$

Hence, $\eta_J$ is a constant function. □

**Theorem 4.16.** *Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be an SVCNGS on a crisp graph $\tau^S$ of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. If $\eta_J$ is constant functions, then $\tau$ is an $\eta_J - edge$ regular SVCNGS if and only if $\tau^S$ is an $\eta_J - edge$ regular.*

*Proof.* Given that $\eta_J$ is constant functions. That is, $\eta_J(u_i, u_j)e^{i\beta_J(u_i,u_j)} = (c_1, c_2, c_3)$ for all $u_iu_j \in W_J$ where $c_1, c_2, c_3$ are constants.
Assume that $\tau$ is an $\eta_J - edge$ regular.
**To Prove:** $\tau^S$ is an $\eta_J - edge$ regular.
Suppose that $\tau^S$ is not an $\eta_J - edge$ regular. Then $d_{\eta_J}(e_{ij}) \neq d_{\eta_J}(e_{rs})$ for at least one pair of $e_{ij}, e_{rs} \in W_J$.
By Definition-4.1 of an $\eta_J - edge$ degree of an SVCNGS,

$$d_{\eta_J}(e_{ij}) = (d_{\eta_{1J}}(e_{ij}), d_{\eta_{2J}}(e_{ij}), d_{\eta_{3J}}(e_{ij})),$$

where

$$\begin{aligned} d_{\eta_{1J}}(e_{ij}) &= \sum_{u_iu_k\in W_J, k\neq j} \eta_{1J}(u_i,u_k)e^{i\sum_{u_iu_k\in W_J,k\neq j}\beta_{1J}(u_i,u_k)} + \\ &\quad \sum_{u_ku_j\in W_J, k\neq i} \eta_{1J}(u_k,u_j)e^{i\sum_{u_ku_j\in W_J,k\neq i}\beta_{1J}(u_k,u_j)} \\ &= \sum_{u_iu_k\in W_J, k\neq j} c_1 + \sum_{u_ku_j\in W_J, k\neq i} c_1 \\ &= c_1(d^*_{\eta_J}(u_i)-1)+c_1(d^*_{\eta_J}(u_j)-1), \\ &= c_1(d^*_{\eta_J}(u_i)+d^*_{\eta_{1J}}(u_j)-2) \\ &= c_1(d^*_{\eta_J}(e_{ij})) \end{aligned}$$

Similarly, we solve the equation

$$\begin{aligned} d_{\eta_{2J}}(e_{ij}) &= c_2(d^*_{\eta_J}(e_{ij})) \\ d_{\eta_{3J}}(e_{ij}) &= c_3(d^*_{\eta_J}(e_{ij})) \end{aligned}$$

$$\begin{aligned} \therefore d_{\eta_J}(e_{ij}) &= (c_1(d^*_{\eta_J}(e_{ij})), c_2(d^*_{\eta_J}(e_{ij})), c_3(d^*_{\eta_J}(e_{ij}))), \\ d_{\eta_J}(e_{jk}) &= (c_1(d^*_{\eta_J}(e_{jk})), c_2(d^*_{\eta_J}(e_{jk})), c_3(d^*_{\eta_J}(e_{jk}))) \end{aligned}$$

Since, $d^*_{\eta_J}(e_{ij}) \neq d^*_{\eta_J}(e_{jk}) \Rightarrow d_{\eta_J}(e_{ij}) \neq d_{\eta_J}(e_{jk})$. Thus, $\tau$ is not an $\eta_J - edge$ regular. Our assumption is contradicted by this.
Hence, $\tau^S$ is an $\eta_J - edge$ regular.
Conversely, assume that $\eta_J$ are constant functions and $\tau^S$ is an $\eta_J - edge$ regular.
**To prove that:** $\tau$ is an $\eta_J - edge$ regular SVCNGS.
Suppose that $\tau$ is not an $\eta_J - edge$ regular SVCNGS. Then $d_{\eta_J}(e_{ij}) \neq d_{\eta_J}(e_{rs})$ for at least one pair of $u_iu_j,\ u_ru_s \in R_J$

$$(d_{\eta_{1J}}(e_{ij}), d_{\eta_{2J}}(e_{ij}), d_{\eta_{3J}}(e_{ij})) \neq (d_{\eta_{1J}}(e_{rs}), d_{\eta_{2J}}(e_{rs}), d_{\eta_{3J}}(e_{rs}))$$

Now,

$$d_{\eta_{1J}}(e_{ij}) \neq d_{\eta_{1J}}(e_{rs}),$$

$$\sum_{u_iu_k\in W_J, k\neq j} \eta_{1J}(u_i,u_k)e^{i\sum_{u_iu_k\in W_J,k\neq j}\beta_{1J}(u_i,u_k)} + \sum_{u_ku_j\in W_J, k\neq i} \eta_{1J}(u_k,u_j)e^{i\sum_{u_ku_j\in W_J,k\neq i}\beta_{1J}(u_k,u_j)} \neq$$
$$\sum_{u_ru_t\in W_J, t\neq s} \eta_{1J}(u_r,u_t)e^{i\sum_{u_ru_t\in W_J,t\neq s}\eta_{1J}(u_r,u_t)} + \sum_{u_tu_s\in W_J, t\neq r} \eta_{1J}(u_t,u_s)e^{i\sum_{u_tu_s\in W_J,t\neq r}\beta_{1J}(u_t,u_s)},$$

$$\begin{aligned} c_1(d_{\eta_{1J}}(u_i)-1)+c_1(d_{\eta_{1J}}(u_j)-1) &\neq c_1(d_{\eta_{1J}}(u_r)-1)+c_1(d_{\eta_{1J}}(u_s)-1), \\ c_1(d_{\eta_{1J}}(u_i)+d_{\eta_{1J}}(u_j)-2) &\neq c_1(d_{\eta_{1J}}(u_r)+d_{\eta_{1J}}(u_s)-2), \\ c_1 d_{\eta_{1J}}(e_{ij}) &\neq c_1 d_{\eta_{1J}}(e_{rs}) \\ d_{\eta_{1J}}(e_{ij}) &\neq d_{\eta_{1J}}(e_{rs}). \end{aligned}$$

Similarly, we solve the equation.

$$\begin{aligned} d_{\eta_{2J}}(e_{ij}) &\neq d_{\eta_{2J}}(e_{rs}), \\ d_{\eta_{3J}}(e_{ij}) &\neq d_{\eta_{3J}}(e_{rs}) \end{aligned}$$

$$\therefore (d_{\eta_{1J}}(e_{ij}), d_{\eta_{2J}}(e_{ij}), d_{\eta_{3J}}(e_{ij})) \neq (d_{\eta_{1J}}(e_{rs}), d_{\eta_{2J}}(e_{rs}), d_{\eta_{3J}}(e_{rs}))$$

Our assumption is contradicted by this. $\tau^S$ is an $\eta_J - edge$ regular.
Hence, $\tau$ is an $\eta_J - edge$ regular SVCNGS. □

**Theorem 4.17.** *Let $\tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be a $\eta_J$ regular SVCNGS of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. Then $\tau$ is an $\eta_J - edge$ regular SVCNGS if and only if $\eta_J$ is constant functions.*

*Proof.* Let
$tau = (\gamma, \eta_1, \eta_2, ..., \eta_k)$ be a $\eta_J$ regular SVCNGS of GS $\tau^* = \{M, W_1, W_2, ..., W_k\}$. Then $d_{\eta_J}(u_i) = (a, b, c)$ for all $u_i \in M$. Assume that $\eta_J$ is constant functions, that is $\eta_J(u_i, u_j) = (c_1, c_2, c_3)$, $\forall\ u_iu_j \in W_J$ where $c_1, c_2, c_3$ are constants.
By Definition-4.1 of an $\eta_J - edge$ degree,

$$d_{\eta_J}(e_{ij}) = (d_{\eta_{1J}}(e_{ij}), d_{\eta_{2J}}(e_{ij}), d_{\eta_{3J}}(e_{ij})),$$

$$\begin{aligned} d_{\eta_{1J}}(e_{ij}) &= d_{\eta_{1J}}(u_i) + d_{\eta_{1J}}(u_j) - 2\eta_{1J}(u_i, u_j)e^{i2\beta_{1J}(u_i,u_j)} \\ &= a + a - 2c_1 \\ &= 2(a - c_1) \end{aligned}$$

Similarly, we solve the equation

$$\begin{aligned} d_{\eta_{2J}}(e_{ij}) &= 2(b - c_2) \\ d_{\eta_{3J}}(e_{ij}) &= 2(c - c_3) \end{aligned}$$

$\therefore d_{\eta_J}(e_{ij}) = (2(a - c_1), 2(b - c_2), 2(c - c_3))$.
Hence, $\tau$ is an $\eta_J - edge$ regular SVCNGS.
Conversely, we assume that $\tau$ is an $\eta_J - edge$ regular SVCNGS.
**To prove that** $\eta_J$ is constant functions.

$d_{\eta_J}(e_{ij}) = (p, q, r)$ for all $e_{ij} \in W_J$

Now,

$$\begin{aligned} d_{\eta_{1J}}(e_{ij}) &= d_{\eta_{1J}}(u_i) + d_{\eta_{1J}}(u_j) - 2\eta_{1J}(u_i, u_j)e^{i2\beta_{1J}(u_i,u_j)} \\ p &= a + a - 2\eta_{1J}(u_i, u_j)e^{i2\beta_{1J}(u_i,u_j)} \\ \eta_{1J}(u_i, u_j)e^{i2\beta_{1J}(u_i,u_j)} &= \frac{(2a - p)}{2} \end{aligned}$$

Similarly, we solve the equation

$$\begin{aligned} \eta_{2J}(u_i, u_j)e^{i\beta_{2J}(u_i,u_j)} &= \frac{(2b - q)}{2} \\ \eta_{3J}(u_i, u_j)e^{i\beta_{3J}(u_i,u_j)} &= \frac{(2c - p)}{2} \end{aligned}$$

$\therefore \lambda_J$ is constant functions. □

## 5. Application

Applications are used in this article to find ambiguities in all facets of human existence. This article discusses the developments in all countries around the world, as well as the reasons for their growth. We will compute the growth and value of fundamental needs across the nations of the world. We will determine the value of a country based on how much education its citizens have access to and how much the government helps the country's poor residents. The medical facilities provided by the government for its citizens as well as the contribution it provides to global health, are also taken into account. Through the contribution of military security in that country, we can ascertain the level of security that the people get. We can also find out how much both the government and the inhabitants of that country contribute to the development of its economy. A country's government measures its progress based on how well it upholds the country's laws and works in the best interests of the people. We can determine a country's progress and strength using all the aforementioned variables. We regard a country's strength and development to be calculated as $\gamma_1 e^{i\alpha_1}$, its weakness and underdevelopment to be calculated as $\gamma_3 e^{i\alpha_3}$, and we consider a country's strength and weakness that we cannot predict, ie., indeterminacy to be calculated as $\gamma_2 e^{i\alpha_2}$. We're going to use an ambiguous value to quantify it. A set $M$ is considered to show nations with the highest rates of strength and development. $M$={United States, China, Russia, Germany, United Kingdom, Japan, France, South Korea}. We can determine the development correlation between the United States and other countries using our definition-3.1 (see Table-2). We can determine the development correlation between Japan and other countries (see Table-3). We can determine the development correlation between China and other countries (see Table-4). We can determine the development correlation between Russia and other countries (see Table-5). We can determine the

TABLE 1

| Country | $\gamma_1 e^{i\alpha_1}$ | $\gamma_2 e^{i\alpha_2}$ | $\gamma_3 e^{i\alpha_3}$ |
|---|---|---|---|
| United States (US) | $.8e^{i.7\pi}$ | $.4e^{i.6}$ | $.6e^{i0.71\pi}$ |
| Japan (J) | $.7e^{i.6\pi}$ | $.60e^{i.7\pi}$ | $.51e^{i.6\pi}$ |
| China (C) | $.8e^{i.8\pi}$ | $.52e^{i.4\pi}$ | $.4e^{i.5\pi}$ |
| Russia (R) | $.62e^{i.5\pi}$ | $.5e^{i.4\pi}$ | $.5e^{i.6\pi}$ |
| Germany (G) | $.5e^{i.6\pi}$ | $.6e^{i.7\pi}$ | $.6e^{i.7\pi}$ |
| United Kingdom (U) | $.7e^{i.5\pi}$ | $.5e^{i.6\pi}$ | $.4e^{i.7\pi}$ |
| France (F) | $.6e^{i.4\pi}$ | $.7e^{i.5\pi}$ | $.5e^{i.7\pi}$ |
| South Korea (S) | $.7e^{i.6\pi}$ | $.4e^{i.7\pi}$ | $.6e^{i.5\pi}$ |

TABLE 2. United States and other countries

| □ | (US, C) | (US,G) | (US,S) |
|---|---|---|---|
| □ | $(.8e^{i.7\pi}, .4e^{i.5\pi}, .4e^{i.5\pi})$ | $(.5e^{i.6\pi}, .6e^{i.7\pi}, .6e^{i.7\pi})$ | $(.7e^{i.5\pi}, .4e^{i.7\pi}, .6e^{i.7\pi})$ |
| □ | $(.7e^{i.6\pi}, .4e^{i.6\pi}, .6e^{i.7\pi})$ | $(.5e^{i.6\pi}, .4e^{i.5\pi}, .5e^{i.6\pi})$ | $(.6e^{i.5\pi}, .4e^{i.6\pi}, .5e^{i.5\pi})$ |
| □ | $(.5e^{i.4\pi}, .5e^{i.6\pi}, .5e^{i.5\pi})$ | $(.5e^{i.6\pi}, .5e^{i.6\pi}, .4e^{i.5\pi})$ | $(.7e^{i.6\pi}, .3e^{i.5\pi}, .3e^{i.5\pi})$ |
| □ | $(.7e^{i.7\pi}, .5e^{i.5\pi}, .5e^{i.6\pi})$ | $(.4e^{i.5\pi}, .5e^{i.6\pi}, .4e^{i.5\pi})$ | $(.6e^{i.4\pi}, .5e^{i.6\pi}, .4e^{i.5\pi})$ |
| □ | $(.7e^{i.7\pi}, .5e^{i.6\pi}, .6e^{i.5\pi})$ | $(.5e^{i.5\pi}, .5e^{i.6\pi}, .4e^{i.5\pi})$ | $(.7e^{i.6\pi}, .4e^{i.6\pi}, .5e^{i.6\pi})$ |

TABLE 3. Japan and other countries

| □ | (J, R) | (J,U) | (J,F) |
|---|---|---|---|
| □ | $(.6e^{i.3\pi}, .6e^{i.7\pi}, .5e^{i.6\pi})$ | $(.7e^{i.5\pi}, .5e^{i.5\pi}, .4e^{i.5\pi})$ | $(.6e^{i.4\pi}, .7e^{i.7\pi}, .5e^{i.7\pi})$ |
| □ | $(.5e^{i.4\pi}, .5e^{i.6\pi}, .5e^{i.3\pi})$ | $(.6e^{i.4\pi}, .5e^{i.5\pi}, .5e^{i.7\pi})$ | $(.6e^{i.3\pi}, .6e^{i.6\pi}, .5e^{i.3\pi})$ |
| □ | $(.5e^{i.4\pi}, .4e^{i.6\pi}, .4e^{i.5\pi})$ | $(.6e^{i.5\pi}, .5e^{i.6\pi}, .5e^{i.4\pi})$ | $(.5e^{i.3\pi}, .6e^{i.5\pi}, .5e^{i.2\pi})$ |
| □ | $(.5e^{i.4\pi}, .5e^{i.5\pi}, .6e^{i.5\pi})$ | $(.4e^{i.5\pi}, .5e^{i.6\pi}, .4e^{i.7\pi})$ | $(.6e^{i.4\pi}, .4e^{i.3\pi}, .4e^{i.5\pi})$ |
| □ | $(.6e^{i.5\pi}, .4e^{i.5\pi}, .4e^{i.3\pi})$ | $(.5e^{i.5\pi}, .5e^{i.6\pi}, .5e^{i.5\pi})$ | $(.6e^{i.4\pi}, .7e^{i.6\pi}, .5e^{i.4\pi})$ |

TABLE 4. China and other countries

| □ | (C, G) | (C, U) | (C, S) |
|---|---|---|---|
| □ | $(.5e^{i.5\pi}, .6e^{i.7\pi}, .6e^{i.7\pi})$ | $(.6e^{i.5\pi}, .5e^{i.6\pi}, .4e^{i.7\pi})$ | $(.7e^{i.6\pi}, .4e^{i.5\pi}, .4e^{i.7\pi})$ |
| □ | $(.5e^{i.4\pi}, .5e^{i.6\pi}, .5e^{i.5\pi})$ | $(.7e^{i.5\pi}, .4e^{i.5\pi}, .3e^{i.7\pi})$ | $(.6e^{i.4\pi}, .5e^{i.6\pi}, .6e^{i.5\pi})$ |
| □ | $(.4e^{i.4\pi}, .5e^{i.6\pi}, .4e^{i.7\pi})$ | $(.6e^{i.5\pi}, .5e^{i.5\pi}, .4e^{i.5\pi})$ | $(.6e^{i.3\pi}, .5e^{i.5\pi}, .5e^{i.4\pi})$ |
| □ | $(.5e^{i.4\pi}, .5e^{i.5\pi}, .6e^{i.5\pi})$ | $(.4e^{i.5\pi}, .5e^{i.6\pi}, .4e^{i.7\pi})$ | $(.6e^{i.4\pi}, .5e^{i.6\pi}, .5e^{i.7\pi})$ |
| □ | $(.5e^{i.6\pi}, .3e^{i.4\pi}, .3e^{i.7\pi})$ | $(.5e^{i.5\pi}, .5e^{i.6\pi}, .4e^{i.4\pi})$ | $(.6e^{i.4\pi}, .5e^{i.6\pi}, .5e^{i.7\pi})$ |

TABLE 5. Russia and other countries

| $\square$ | (R, G) | (R, F) | (R, S) |
|---|---|---|---|
| $\square$ | $(.5e^{i.5\pi}, .6e^{i.7\pi}, .6e^{i.7\pi})$ | $(.6e^{i.4\pi}, .7e^{i.5\pi}, .5e^{i.7\pi})$ | $(.6e^{i.3\pi}, .5e^{i.7\pi}, .6e^{i.5\pi})$ |
| $\square$ | $(.5e^{i.4\pi}, .5e^{i.6\pi}, .5e^{i.7\pi})$ | $(.5e^{i.4\pi}, .5e^{i.5\pi}, .5e^{i.5\pi})$ | $(.6e^{i.5\pi}, .4e^{i.5\pi}, .4e^{i.7\pi})$ |
| $\square$ | $(.4e^{i.4\pi}, .5e^{i.6\pi}, .6e^{i.5\pi})$ | $(.6e^{i.4\pi}, .4e^{i.3\pi}, .4e^{i.5\pi})$ | $(.6e^{i.3\pi}, .5e^{i.5\pi}, .5e^{i.7\pi})$ |
| $\square$ | $(.5e^{i.4\pi}, .5e^{i.5\pi}, .6e^{i.5\pi})$ | $(.4e^{i.4\pi}, .5e^{i.5\pi}, .5e^{i.4\pi})$ | $(.6e^{i.4\pi}, .5e^{i.6\pi}, .5e^{i.5\pi})$ |
| $\square$ | $(.5e^{i.4\pi}, .5e^{i.6\pi}, .5e^{i.7\pi})$ | $(.5e^{i.4\pi}, .5e^{i.4\pi}, .5e^{i.7\pi})$ | $(.6e^{i.4\pi}, .5e^{i.6\pi}, .6e^{i.4\pi})$ |

development correlation between United Kingdom and other countries (see Table-5).

Using these SVCNGS, we illustrate the severity of the development between each pair of

TABLE 6. United Kingdom and other countries

| $\square$ | (U, G) | (U, F) | (U, S) |
|---|---|---|---|
| $\square$ | $(.5e^{i.5\pi}, .6e^{i.7\pi}, .6e^{i.7\pi})$ | $(.6e^{i.4\pi}, .7e^{i.6\pi}, .5e^{i.7\pi})$ | $(.7e^{i.5\pi}, .4e^{i.5\pi}, .4e^{i.7\pi})$ |
| $\square$ | $(.5e^{i.4\pi}, .5e^{i.6\pi}, .6e^{i.5\pi})$ | $(.6e^{i.2\pi}, .6e^{i.5\pi}, .5e^{i.5\pi})$ | $(.6e^{i.4\pi}, .5e^{i.7\pi}, .6e^{i.7\pi})$ |
| $\square$ | $(.4e^{i.4\pi}, .5e^{i.6\pi}, .5e^{i.7\pi})$ | $(.5e^{i.3\pi}, .6e^{i.5\pi}, .5e^{i.5\pi})$ | $(.6e^{i.3\pi}, .5e^{i.5\pi}, .5e^{i.7\pi})$ |
| $\square$ | $(.5e^{i.4\pi}, .5e^{i.5\pi}, .6e^{i.7\pi})$ | $(.6e^{i.4\pi}, .4e^{i.5\pi}, .4e^{i.3\pi})$ | $(.6e^{i.4\pi}, .5e^{i.6\pi}, .5e^{i.5\pi})$ |
| $\square$ | $(.5e^{i.4\pi}, .5e^{i.6\pi}, .5e^{i.6\pi})$ | $(.5e^{i.4\pi}, .5e^{i.4\pi}, .5e^{i.7\pi})$ | $(.6e^{i.4\pi}, .5e^{i.6\pi}, .6e^{i.7\pi})$ |

nations. On set $M$, numerous relations can be defined. Let's explain the relationships on $M$ as follows: $W_1$=education, $W_2$=medical science, $R_3$= military, $W_4$= economic growth, $W_5$ = effected government, such that $\tau^* = (Q, R_1, R_2, R_3, R_4, R_5)$ is a GS. Each element of the relationship exemplifies a certain stage of growth between those two countries. Due to the fact that the GS is $\tau^* = (M, W_1, W_2, W_3, W_4, W_5)$, only one relationship can exist between two countries. Thus, it would be considered a part of that relationship, whose false membership amount is relatively low in comparison to various other relationships, and whose truth-membership amount is relatively high in comparison to other connections. When measured against other relationships, its truth-membership the amount is relatively high, while its indeterminacy-membership amount is relatively low. Using the previously provided data, the SVCNGS on $W_1, W_2, W_3, W_4, W_5$ are formed by matching items in relations with the truth-membership, indeterminacy, and false-membership. They are $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5$, respectively, of these SVCNGS.

$W_1$={(US, C), (J, U),(C, S),(U, S)}, $W_2$={(C, U),(R, S)}, $W_3$= {(US, S),(R, F)}, $W_4$={(J, F),(U, F)}, $W_5$={(J, R),(C, G)}.

The Corresponding SVCNGS are follows:

$\eta_1 = \{(US, C)(.8e^{i.7\pi}, .4e^{i.5\pi}, .4e^{i.5\pi}), (J, U)(.7e^{i.5\pi}, .5e^{i.5\pi}, .4e^{i.5\pi}),$
$(C, S)(.7e^{i.6\pi}, .4e^{i.5\pi}, .4e^{i.7\pi}), (U, S)(.7e^{i.5\pi}, .4e^{i.5\pi}, .4e^{i.7\pi})\},$

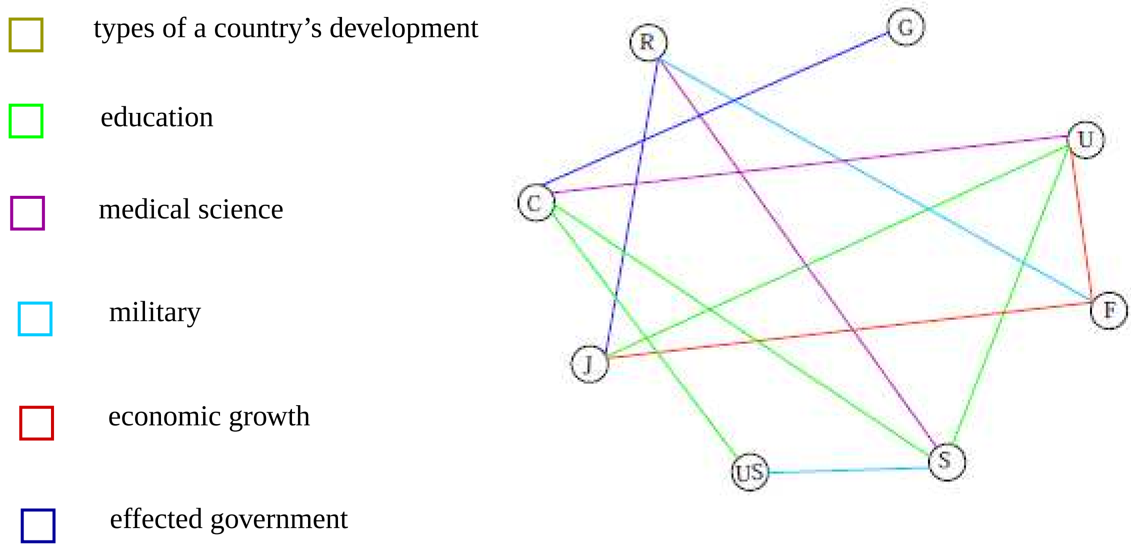


FIGURE 3. SVCNGS

$\eta_2 = \{(C,U)(.7e^{i.5\pi}, .4e^{i.5\pi}, .3e^{i.7\pi}), (R,S)(.6e^{i.5\pi}, .4e^{i.5\pi}, .4e^{i.7\pi})\}$,
$\eta_3 = \{(US,S)(.7e^{i.6\pi}, .3e^{i.5\pi}, .3e^{i.5\pi}), (R,F)(.6e^{i.4\pi}, .4e^{i.3\pi}, .4e^{i.5\pi})\}$,
$\eta_4 = \{(J,F)(.6e^{i.4\pi}, .4e^{i.3\pi}, .4e^{i.5\pi}), (U,F)(.6e^{i.4\pi}, .4e^{i.5\pi}, .4e^{i.3\pi})\}$,
$\eta_5 = \{(J,R)(.6e^{i.5\pi}, .4e^{i.5\pi}, .4e^{i.3\pi}), (C,G)(.5e^{i.6\pi}, .3e^{i.4\pi}, .3e^{i.7pi})\}$.
Therefore, the SVCNGS are represented in Figure-3 is $(\gamma, \eta_1, \eta_2, \eta_3, \eta_4, \eta_5)$. The country with the greatest level of development is represented by each edge of the SVCNGS in Figure-3. As an illustration, the expansion of education, with values for truth-membership, indeterminacy-membership, and false-membership of $.8e^{i.7\pi}$, $.4e^{i.5\pi}$ and $.4e^{i.5\pi}$, respectively, is what contributes to the most powerful and developing relationship between the United States and China. It should be noted that the United States has the lowest vertex degree of indeterminacy-membership, false-membership, and the highest vertex degree of truth-membership for the relation proliferation of education. This shows that the United States has a proliferation of education and is developing alongside other countries. The purpose of this is article is to identify the most developed nations in the world by examining the growth and development of every nation in the world. This opens the way for the growth of all the nations in the world.

### 5.1. *Algorithm*

We now present the stepwise for calculation of our method which is used in this application in the following algorithm.

**Algorithm**

step 1: Input the set $Q = \{q_1, q_2, ..., q_n\}$ of countries (vertices) and put the membership values $\gamma = (\gamma_1 e^{i\alpha_1}, \gamma_2 e^{i\alpha_2}, \gamma_3 e^{i\alpha_3})$ of the nodes $q_i^{\prime}$s, $i = 1, 2, ..., n$, $\gamma_1, \gamma_2, \gamma_3 \in [0, 1]$ and $\alpha_1, \alpha_2, \alpha_3 \in [0, 2\pi]$.

step 2: Input the membership values $\eta_J = (\eta_{1J}(q_iq_j)e^{i\beta_{1J}(q_iq_j)}, \eta_{2J}(q_iq_j)e^{i\beta_{2J}(q_iq_j)}, \eta_{3J}(q_iq_j)e^{i\beta_{3J}(q_iq_j)})$ of the edges $q_iq_j \in W_J$ such that

$$\begin{aligned}
\eta_{1J}(q_iq_j)e^{i\beta_{1J}(q_iq_j)} &\leq \min\{\gamma_1(q_i), \gamma_1(q_j)\}e^{i\min\{\alpha_1(q_i),\alpha_1(q_j)\}},\\
\eta_{2J}(q_iq_j)e^{i\beta_{2J}(q_iq_j)} &\leq \max\{\gamma_2(q_i), \gamma_2(q_j)\}e^{i\max\{\alpha_2(q_i),\alpha_2(q_j)\}},\\
\eta_{3J}(q_iq_j)e^{i\beta_{3J}(q_iq_j)} &\leq \max\{\gamma_3(q_i), \gamma_3(q_j)\}e^{i\max\{\alpha_3(q_i),\alpha_3(q_j)\}}
\end{aligned}$$

such that $0 \leq \eta_{1J}(q_iq_j) + \eta_{2J}(q_iq_j) + \eta_{3J}(q_iq_j) \leq 3$ and $\beta_{1J}(q_iq_j), \beta_{2J}(q_iq_j), \beta_{3J}(q_iq_j) \in [0, 2\pi]$ for all $(q_iq_j) \in W_J$, $J = 1, 2, ..., k$.

step 3: Develop mutually disjoint, irreflexive and symmetric relations $W_1, W_2, ..., W_k$ on the set of countries M and give the name each relation as exemplifies a certain stage of growth between those two countries.

step 4: Select a countries as greatest level of development from one countries to other, whose membership value is superior to that of other nations.

step 5: Construct a graph structure on set of countries with relations, select those pairs of countries having same kind of the highest level of development as elements of same relation.

step 6: Write all elements of resulting relations $\eta_1, \eta_2, ..., \eta_k$ are CNSs on $W_1, W_2, ..., W_k$, respectively and $(\gamma, \eta_1, \eta_2, ..., \eta_k)$ is a SVCNGS.

step 7: Draw the SVCNGS, each of whose edges indicates the best level of development for the related Countries.

## 6. Conclusion and future works

The idea of an SVCNGS has been developed in this study article by the authors. In comparison to traditional fuzzy sets, the Set SVCNS, an extension of the NS, provides a more realistic description of uncertainty. Through fuzzy control, it can be used in a variety of ways. In this research study, the idea of SVCNGS is introduced. Further research is done on the relationship between the degree of a vertex and the degree of an $\eta_J - edge$ in regular SVCNGS. We also define totally $\eta_J - edge$ regular SVCNGS and $\eta_J - edge$ regular SVCNGS. It is described under what conditions $\eta_J - edge$ regular SVCNGS and totally $\eta_J - edge$ regular SVCNGS are comparable. We also investigated various $\eta_J - edge$ regular and totally $\eta_J - edge$ regular SVCNGS properties using an example. Furthermore, we have presented an application of SVCNGS in decision-making, that is, identification of best level of development Countries.

There are several potential areas for future research in this area, if it is possible to use the adjacency matrix SVCNGS. Further, for developing future solutions, analyze the isomorphic adjacency matrix, edge regular adjacency matrix, totally edge regular adjacency matrix, etc. Future research areas include Complex Pythagorean fuzzy graph structures, Complex bipolar fuzzy graph structures, and Complex bipolar neutrosophic graph structures, all of which are based on the various properties of the nodes and edges in GS. The following are some of this work's limitations:

- This research and related network systems were mostly focused on SVCNGS.
- This approach can only be used when there are symmetric, irreflexive, and mutually disjoint relations on the CNS.
- The SVCNGS idea is not relevant if the membership values of the characters are provided in distinct environments.
- Sometimes it may not be possible to get real data.

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