Neutrosophic Sets and Systems, Vol. 65, 2024
University of New Mexico

Applications of sets and functions by using an open sets in Fuzzy neutrosophic topological spaces

Basker P [1], Broumi Said [2] and Vennila J[3,*]

[1] Associate Professer, Department of Mathematics, Chandigarh University, Punjab-140413, India.
E-Mail: mcpdbasker@gmail.com, basker.e11236@cumail.in, mcpdbasker@gmail.com

[2] Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, Casablanca, Morocco;
broumisaid78@gmail.com

[3] Assistant Professer in Statistics, Manipal College of Health Professions, Manipal Academy of Higher Education, Manipal-576104, India

* Correspondence: nilajagan92@gmail.com

Abstract: The definitions provided by the authors of the current study are offered together with a discussion of the recent advances that they have contributed. We begin with an introduction to fn-$\mathrm{Fr}_{\#\mathrm{b}_{\varrho_N}}$, which includes the concepts of closed and open sets. We explore characteristics in fn-$\beta\mathrm{d}^{\#\mathrm{b}_{\varrho_N}}$ and fn-$\mathrm{e}_{\mathrm{b}_{\varrho_N}}(\varrho_N)$, and provide an idea of obtained results by adding the notion of $\mathrm{FNb}_{\varrho_N}\mathrm{OS}$ and analyzing a few of their properties in $fnts$. We've researched the contrasts between the derived, exterior, and frontier notions that are provided. We also looked at the ideas of $\langle \mathcal{T}^{\mathrm{b}_{\varrho_N}} \rangle C$-functions and Γ^S-segregated functions and examined and determined the traits.

Keywords: $\mathrm{Fr}_{\#\mathrm{b}_{\varrho_N}}$, $\beta\mathrm{d}^{\#\mathrm{b}_{\varrho_N}}$, $\mathrm{e}_{\mathrm{b}_{\varrho_N}}(\varrho_N)$, $fn\mathrm{b}_{\varrho_N}\mathrm{OS}$, derived, exterior, and frontier

1. Introduction

Uncertainties are a major source of real-world difficulties in the fields of business, finance, medicine, engineering, and the behavioural sciences. Using conventional mathematical methods to solve the uncertainties for these data presents challenges. To avoid problems while working with ambiguous data, there exist methods like fuzzy sets, rough sets, fuzzy sets with intuitionistic properties, and vague sets that may be used as mathematical tools. Due to the inadequate parametrization tools, all of these techniques implicitly face difficulties when attempting to solve problems involving inconsistent and indeterminate data. The characteristics of n-closed sets, interior operators, closure operators, and open sets determine how neutrosophic is used in topology. Topologists explored sets next to neutrosophic closed and open sets.

L. A. Zadeh [29] proposed fuzzy sets in 1965 sand investigated various aspects of their features, A fuzzy set is a class of elements with an assortment of membership grades. Such a collection is characterised by a membership (or feature) function that assigns a membership grade, ranging from zero to one, to each item. He extended the notions of inclusion, union, intersection, complement, connection, convexity, etc. to these sets and demonstrated various aspects of these notions in relation to fuzzy sets. In particular, a separation theorem for convex fuzzy sets is proved that does not need the fuzzy sets to be disjoint.

Atanassov[14, 15, 16] have created intuitionistic fuzzy sets and looked through numerous outcomes, he presented the concept of the "Generalised Net" and examined its fundamental characteristics along with a few of its uses in the fields of artificial intelligence, systems theory, health, economics, transportation, and the chemical industry.

He spearheaded most of the applied research in the field of generalised nets and was the driving force behind its theoretical investigation. Many of the operations and interactions he has established over generalised nets have parallels in the theory of regular Petri nets. Nevertheless, there is no counterpart in Petri net theory for the topological and logical operators he has presented. Atanassov's other primary area of study is fuzzy sets, originally established by Zadeh, which he developed further by presenting the concept of "Intuitionistic Fuzzy Sets" and investigating the elements that make up its foundation. He is also recognised as a pioneer in the use of intuitionistic fuzzy sets to expert systems, systems theory, decision-making, and other domains.

F.Smarandache [9, 10, 24] examined the idea of using a neutrosophic set as a technique for resolving problems involving persistent, unpredictable, and unreliable data. He also noted the features of the generalisation of intuitionistic fuzzy logic. The study of the nature, origin, and scope of neutralities as well as their interactions with other ideational spectra is done within a branch of neutrosophy called the neutrosophic set. The neutrosophic set is a robust universal formal framework that was introduced lately. However, from a technical point of view, the neutrosophic set has to be specified.

P. Basker and Broumi Said [5, 6, 7] Investigators investigated the idea of $N\psi_{\alpha}^{\#}\ {}^{0}$ and $N\psi_{\alpha}^{\#}\ {}^{1}$-spaces and neutrosophic functions in neutrosophic topological spaces, and neutrosophic homeomorphisms from which the notion of $(\beta_{\rho n})$-OS in pythagorean neutrosophic topological spaces

Neurosophic topological spaces and the resulting neutrosophic set were studied in 2012 by A. A. Salama and S. A. Alblowi [23]. The concepts of fuzzy neutrosophic topological spaces and fuzzy neutrosophic sets were examined in 2014 by I. Arockiarani and J. Martina Jency [4]. In 2018, Fatimah M. Mohammed, Anas A. Hijab, and Shaymaa F. Matar [8] implemented fuzzy neutrosophic weakly-generalized closed sets in fuzzy neutrosophic topological spaces.

The concept of sharp, weakly neutrosophic closed functions was introduced by Ali Hussein Mahmood Al Obaidi, Qays Hatem Imran, and Murtadha Mohammed Abdulkadhim [1]. Hypersoft topological spaces were employed by Sagvan Y. Musa and Baravan A. Asaad [22] to connect the concepts.

In 2023, the neutrosophic soft generalised b-closed sets in neutrosophic soft topological spaces were created by Alkan Özkan, \eyda Yazgan, and Sandeep Kaur [2], Muthumari G et al. [20] the neutrosophic over topologized graphs' homomorphism and isomorphism were derived, Tomasz Witczak [27], Interior and closure of anti-minimal and anti-biminimal areas in the framework of anti-topology. The authors developed and examined a novel class of neutrosophic open and closed maps in neutrosophic topological spaces. P.Anbarasi Rodrigo et al. [3] and P. Thangaraja et al. [28]. Separation Axioms, Neighbourhood and Continuity were discussed in [21, 25, 26]. A few descriptions of both new and Neutrosophic objects were covered in [11, 12, 13]. An application of neutrosophic theory and computation of neutrosophic were generalized in [17, 18, 19].

This paper's Section 1 lists the definitions cited by the authors as well as recent advances that they have provided. We introduce the concept of FNb_{ϱ_N} OS in Section 2 using $fnts$. FNb_{ϱ_N} have determined $Fr_{\#b_{\varrho_N}}, \beta d^{\#b_{\varrho_N}}, e_{b_{\varrho_N}}(\varrho_N)$ and $E^{FNb_{\varrho_N}}(\varrho_N)$ studied some of their properties by using the above concepts we have derived the applications of fn-open and closed sets. In this study, FNS, FNTS, MN and MX stand for fn-set, fn-Topological Spaces, Minimum and Maximum respectively.

The following are the main novelties of this paper.

- fn-open and closed sets
- FNb_{ϱ_N}- point of interior
- fn-b_{ϱ_N}-border
- fn-b_{ϱ_N}-frontier
- fn-b_{ϱ_N}-exterior
- fn-b_{ϱ_N}-derived
- fn-Γ^S-segregated
- fn-b_{ϱ_N}-Totally-Continuous

The essential definitions listed below will aid in understanding this research work.

Definition 1.1.[4] A $\boldsymbol{fn}$-set $\boldsymbol{A}$ on X is defined as A is equal to $\langle \boldsymbol{\varpi}, \boldsymbol{I_A(\varpi)}, \boldsymbol{J_A(\varpi)}, \boldsymbol{K_A(\varpi)} \rangle$, $\boldsymbol{\varpi}$ belongs to $\boldsymbol{X}$ where $\boldsymbol{I}, \boldsymbol{J}, \boldsymbol{K}$ from $\boldsymbol{X}$ to$[\boldsymbol{0}, \boldsymbol{1}]$ and $\boldsymbol{0} \leq \boldsymbol{sum\ \ of}\ \ \{\boldsymbol{I_A(\varpi)},\ \boldsymbol{J_A(\varpi)},\ \boldsymbol{K_A(\varpi)}\} \leq \boldsymbol{3}$.

Definition 1.2. [4] A $\boldsymbol{fn}$-set, A belongs to the subset of a $\boldsymbol{fn}$-set B (i.e.,) $\boldsymbol{A \subseteq B}$ $\forall \boldsymbol{\varpi}$ if $\boldsymbol{I_A(\varpi) \leq I_B(\varpi)}$ $\boldsymbol{J_A(\varpi) \leq J_B(\varpi)}$ $\boldsymbol{K_A(\varpi) \geq L_B(\varpi)}$

Definition 1.3. [4] Let $\boldsymbol{X}$ must represent a non-empty set., and $\boldsymbol{A} = \langle \boldsymbol{\varpi}, \boldsymbol{I_A(\varpi)}, \boldsymbol{J_A(\varpi)}, \boldsymbol{K_A(\varpi)} \rangle$, $\boldsymbol{B} = \langle \boldsymbol{\varpi}, \boldsymbol{I_B(\varpi)}, \boldsymbol{J_B(\varpi)}, \boldsymbol{K_B(\varpi)} \rangle$ be two $\boldsymbol{fn}$-set. Then
Union of $\boldsymbol{A}$ and $\boldsymbol{B}$ is $\langle \boldsymbol{\varpi}, \mathbf{MX\ \ of}\{\boldsymbol{I_A(\varpi)}, \boldsymbol{I_B(\varpi)}\}, \mathbf{MN\ \ of}\{\boldsymbol{J_A(\varpi)}, \boldsymbol{J_B(\varpi)}\}, \mathbf{MN\ \ of}\{\boldsymbol{K_A(\varpi)}, \boldsymbol{K_B(\varpi)}\} \rangle$ and Intersection of $\boldsymbol{A}$ and $\boldsymbol{B}$ is $\langle \boldsymbol{\varpi}, \mathbf{MN\ \ of}\{\boldsymbol{I_A(\varpi)}, \boldsymbol{I_B(\varpi)}\}, \mathbf{MN\ \ of}\{\boldsymbol{J_A(\varpi)}, \boldsymbol{J_B(\varpi)}\}, \mathbf{MX\ \ of}\{\boldsymbol{K_A(\varpi)}, \boldsymbol{K_B(\varpi)}\} \rangle$.

Definition 1.4. [4] The difference between two $\boldsymbol{fn}$-set A and B is defined as
Differ from $\boldsymbol{A}$ to $\boldsymbol{B}$ is $\langle \boldsymbol{\varpi}, \mathbf{MN\ \ of}\{\boldsymbol{I_A(\varpi)}, \boldsymbol{K_B(\varpi)}\}, \mathbf{MN\ \ of}\{\boldsymbol{J_A(\varpi)}, \boldsymbol{1} - \boldsymbol{J_B(\varpi)}\}, \mathbf{MN\ \ of}\{\boldsymbol{K_A(\varpi)}, \boldsymbol{I_B(\varpi)}\} \rangle$.

Definition 1.5. [4] A $\boldsymbol{fn}$-set it is said that $\boldsymbol{A}$ over the universe $\boldsymbol{X}$ equals

- Null or empty $\boldsymbol{fn}$-set if $\boldsymbol{0_N} = \langle \boldsymbol{\varpi}, \boldsymbol{0}, \boldsymbol{0}, \boldsymbol{1} \rangle$ $\forall \boldsymbol{\varpi} \in \boldsymbol{X}$.
- Absolute (universe) $\boldsymbol{fn}$-set if $\boldsymbol{1_N} = \langle \boldsymbol{\varpi}, \boldsymbol{1}, \boldsymbol{1}, \boldsymbol{0} \rangle$ $\forall \boldsymbol{\varpi} \in \boldsymbol{X}$.

Definition 1.6. [4] $\boldsymbol{A^c}$ represents the complement of a $\boldsymbol{fn}$-set $\boldsymbol{A}$, which is defined as $\boldsymbol{A^c} = \langle \boldsymbol{\varpi}, \boldsymbol{I_{(A^c)}(\varpi)}, \boldsymbol{J_{(A^c)}(\varpi)}, \boldsymbol{K_{(A^c)}(\varpi)} \rangle$, Where $\boldsymbol{I_{(A^c)}(\varpi)} = \boldsymbol{K_A(\varpi)}, \boldsymbol{J_{(A^c)}(\varpi)} = \boldsymbol{1} - \boldsymbol{J_A(\varpi)}, \boldsymbol{K_{(A^c)}(\varpi)} = \boldsymbol{I_A(\varpi)}$.
Another way to define the complement of a $\boldsymbol{fn}$-set A is as $\boldsymbol{A^c} = \boldsymbol{1_N} - \boldsymbol{A}$.

2. Applications of Fuzzy Neutrosophic open and closed sets

Definition 2.1. A fns, $\varrho_N = \langle H, \zeta_{\varrho_N}, \eta_{\varrho_N}, \theta_{\lambda_N} \rangle$ in a $fnts$ Γ is to be
(i) fn-b_{ϱ_N}-OS (FNbOS), $FNi(FNc(\varrho_N)) \cup FNc(FNi(\varrho_N)) \supseteq \varrho_N$
(ii) fn-b_{ϱ_N}-CS (FNbCS), $FNi(FNc(\varrho_N)) \cap FNc(FNi(\varrho_N)) \subseteq \varrho_N$
We'll utilize shortened versions of FNb_{ϱ_N}-ℕbhd, for the word FNb_{ϱ_N}-neighbourhood

Definition 2.2. Let Γ be an $fnts$ and let $\varrho_n^1 \in \Gamma$. A part of $\mathbb{N}$ of Γ is FNb_{ϱ_N}-ℕbhd of ϱ_n^1, if $\exists$ a FNb_{ϱ_N}-OS, E such that $\varrho_n^1 \in E \subset \mathbb{N}$.

Definition 2.3. Let $\boldsymbol{\varrho_N}$ be a subset of $\boldsymbol{\Gamma}$. Then, if $\boldsymbol{\varrho_N}$ is a $\mathbf{FNb_{\varrho_N}}$-Nbhd of $\boldsymbol{\varrho_n^1}$, then $\boldsymbol{\varrho_n^1 \in \varrho_N}$ is to be $\mathbf{FNb_{\varrho_N}}$-point of interior $\boldsymbol{\varrho_N}$. $\mathbf{FNb_{\varrho_N}}$-interior $\boldsymbol{\varrho_N}$ is the whole set $\mathbf{FNb_{\varrho_N}}$-point of interior $\boldsymbol{\varrho_N}$, and it is $\mathbf{b_{\varrho_N}}$-int($\boldsymbol{\varrho_N}$), $\mathbf{IN_{b_{\varrho_N}}}(\boldsymbol{\varrho_N}) = \cup\{\mathbf{E: E\ \ is\ \ FNb_{\varrho_N}OS,\ \ E \subset \varrho_N}\}$

Let be the part of a space $\boldsymbol{\varrho_N}$ Γ. The meeting point for all $\mathbf{FNb_{\varrho_N}}$-closed sets containing $\boldsymbol{\varrho_N}$ is defined as the $\mathbf{FNb_{\varrho_N}}$-closure of $\boldsymbol{\varrho_N}$, $\mathbf{CL_{b_{\varrho_N}}}(\boldsymbol{\varrho_N}) = \cap\{\mathbf{E: \varrho_N \subset E \in FNb_{\varrho_N}(\Gamma)}\}$

Definition 2.4. An ϱ_N be a space that has a group of individuals. Γ, an element $\varrho_n^1 \in \Gamma$ is to be b_{ϱ_N}-point of ϱ_N if for all b_{ϱ_N}-OS, Γ_1 containing ϱ_n^1, $\Gamma_1 \cap (\varrho_N - \{\varrho_N^1\}) \neq \phi$. The whole set b_{ϱ_N}-point of ϱ_N is b_{ϱ_N}-derived (briefly. $\mathrm{e}_{b_{\varrho_N}}$) of ϱ_N as indicated by $\mathrm{e}_{b_{\varrho_N}}(\varrho_N)$.

Example 2.5. Let $\Gamma = \{\alpha, \beta, \gamma\}$ and $\Upsilon = \{0_N, 1_N, \varrho_{N_1}, \varrho_{N_2}, \varrho_{N_3}, \varrho_{N_4}\}$ where
$\varrho_{N_1} = \{\langle \Gamma(\alpha)0.82, \Gamma(\alpha)0.79, \Gamma(\alpha)0.59\rangle, \langle \Gamma(\beta)0.4, \Gamma(\beta)0.61, \Gamma(\beta)0.4\rangle, \langle \Gamma(\gamma)0.39, \Gamma(\gamma)0.4, \Gamma(\gamma)0.5\rangle\}$,
$\varrho_{N_2} = \{\langle \Gamma(\alpha)0.69, \Gamma(\alpha)0.59, \Gamma(\alpha)0.39\rangle, \langle \Gamma(\beta)0.78, \Gamma(\beta)0.2, \Gamma(\beta)0.3\rangle, \langle \Gamma(\gamma)0.99, \Gamma(\gamma)0.39, \Gamma(\gamma)0.19\rangle\}$,
$\varrho_{N_3} = \{\langle \Gamma(\alpha)0.82, \Gamma(\alpha)0.78, \Gamma(\alpha)0.49\rangle, \langle \Gamma(\beta)0.8, \Gamma(\beta)0.51, \Gamma(\beta)0.4\rangle, \langle \Gamma(\gamma)0.9, \Gamma(\gamma)0.7, \Gamma(\gamma)0.2\rangle\}$,
$\varrho_{N_4} = \{\langle \Gamma(\alpha)0.69, \Gamma(\alpha)0.59, \Gamma(\alpha)0.59\rangle, \langle \Gamma(\beta)0.59, \Gamma(\beta)0.21, \Gamma(\beta)0.4\rangle, \langle \Gamma(\gamma)0.39, \Gamma(\gamma)0.3, \Gamma(\gamma)0.4\rangle\}$.
Here ϱ_{N_3} be a subset of a space Γ and a point $\alpha \in \Gamma$ and Γ_1 a b_{ϱ_N}-OS, then it is a b_{ϱ_N}-point of ϱ_N is $\mathrm{e}_{b_{\varrho_N}}(\{\langle \Gamma(\alpha)0.82, \Gamma(\alpha)0.79, \Gamma(\alpha)0.49\rangle, \langle \Gamma(\beta)0.8, \Gamma(\beta)0.51, \Gamma(\beta)0.4\rangle, \langle \Gamma(\gamma)0.9, \Gamma(\gamma)0.7, \Gamma(\gamma)0.2\rangle\})$.

Theorem 2.6. As for segments ϱ_{N_1}, ϱ_{N_2} of a space Γ, all of the following claims are true::
If $\varrho_{N_2} \supset \varrho_{N_1}$, then

a) $\mathrm{e}_{b_{\varrho_N}}(\varrho_{N_2}) \supset \mathrm{e}_{b_{\varrho_N}}(\varrho_{N_1})$
b) $\mathrm{e}_{b_{\varrho_N}}(\varrho_{N_1} \cup \varrho_{N_2}) \supset \mathrm{e}_{b_{\varrho_N}}(\varrho_{N_1}) \cup \mathrm{e}_{b_{\varrho_N}}(\varrho_{N_2})$
c) $\mathrm{e}_{b_{\varrho_N}}(\varrho_{N_1}) \supset \mathrm{e}_{b_{\varrho_N}}\left(\mathrm{e}_{b_{\varrho_N}}(\varrho_{N_1})\right) - \varrho_{N_1}$
d) $\varrho_{N_1} \cup \mathrm{e}_{b_{\varrho_N}}(\varrho_{N_1}) \supset \mathrm{e}_{b_{\varrho_N}}\left(\varrho_{N_1} \cup \mathrm{e}_{b_{\varrho_N}}(\varrho_{N_1})\right)$.

Proof. (a) It is obvious. (b) It is an immediate consequence of (c).

(c) If $\varrho_n^1 \in \mathrm{e}_{b_{\varrho_N}}\left(\mathrm{e}_{b_{\varrho_N}}(\varrho_{N_1})\right) - \varrho_{N_1}$ and Γ_1 is a b_{ϱ_N}-OS, offering ϱ_n^1, $\Gamma_1 \cap \left(\mathrm{e}_{b_{\varrho_N}}(\varrho_{N_1}) - \{\varrho_n^1\}\right) \neq \phi$. Permit $\varrho_n^2 \in \Gamma_1 \cap \left(\mathrm{e}_{b_{\varrho_N}}(\varrho_{N_1}) - \{\varrho_n^1\}\right)$. Then due to the fact $\varrho_n^2 \in \mathrm{e}_{b_{\varrho_N}}(\varrho_{N_1})$, $\varrho_n^2 \in \Gamma_1$, $\Gamma_1 \cap (\varrho_{N_1} - \{\varrho_n^2\}) \neq \phi$. Permit $\Gamma^\# \in \Gamma_1 \cap (\varrho_{N_1} - \{\varrho_n^2\})$, $\Gamma^\# \neq \varrho_n^1$ to be for $\Gamma^\# \in \varrho_{N_1}$, $\varrho_n^1 \notin \varrho_{N_1}$. Accordingly $\Gamma_1 \cap (\varrho_{N_1} - \{\varrho_n^1\}) \neq \phi$. Consequently $\varrho_n^1 \in \mathrm{e}_{b_{\varrho_N}}(\varrho_{N_1})$.

(d) Let's Take $\varrho_n^1 \in \mathrm{e}_{b_{\varrho_N}}\left(\varrho_N \cup \mathrm{e}_{b_{\varrho_N}}(\varrho_N)\right)$. If $\varrho_n^1 \in \varrho_N$, The ultimate result is clear. Let $\varrho_n^1 \in \mathrm{e}_{b_{\varrho_N}}\left(\varrho_N \cup \mathrm{e}_{b_{\varrho_N}}(\varrho_N)\right) - \varrho_N$, for b_{ϱ_N}-OS, $\Gamma_1 \subset \varrho_n^1$, $\Gamma_1 \cap \left(\varrho_N \cup \mathrm{e}_{b_{\varrho_N}}(\varrho_N) - \{\varrho_n^1\}\right) \neq \phi$. Consequently $\Gamma_1 \cap (\varrho_N - \{\varrho_n^1\}) \neq \phi$ or $\Gamma_1 \cap \left(\mathrm{e}_{b_{\varrho_N}}(\varrho_N) - \{\varrho_n^1\}\right) \neq \phi$. It eventually follows (c) that $\Gamma_1 \cap (\varrho_N - \{\varrho_n^1\}) \neq \phi$. So $\varrho_n^1 \in \mathrm{e}_{b_{\varrho_N}}(\varrho_N)$. So, whatever the circumstance, $\varrho_N \cup \mathrm{e}_{b_{\varrho_N}}(\varrho_N) \supset \mathrm{e}_{b_{\varrho_N}}\left(\varrho_N \cup \mathrm{e}_{b_{\varrho_N}}(\varrho_N)\right)$.

Theorem 2.7. In any subset that exists ϱ_N of a Γ, $b_{\varrho_N}\mathrm{CLof}(\varrho_N) = \varrho_N \cup \mathrm{e}_{b_{\varrho_N}}(\varrho_N)$.

Proof. Since $\mathrm{e}_{b_{\varrho_N}}(\varrho_N) \subset b_{\varrho_N}\mathrm{CLof}(\varrho_N)$, $\varrho_N \cup \mathrm{e}_{b_{\varrho_N}}(\varrho_N) \subset b_{\varrho_N}\mathrm{CLof}(\varrho_N)$. As opposed to that, let $\varrho_n^1 \in b_{\varrho_N}\mathrm{CLof}(\varrho_N)$. If $\varrho_n^1 \in \varrho_N$, then the evidence is conclusive. If $\varrho_n^1 \notin \varrho_N$, then every single b_{ϱ_N}-OS $\Gamma_1 \subset \varrho_n^1 \cap \varrho_N$ at something different from ϱ_n^1 .Consequently $\varrho_n^1 \in \mathrm{e}_{b_{\varrho_N}}(\varrho_N)$. Thus $\varrho_N \cup \mathrm{e}_{b_{\varrho_N}}(\varrho_N) \supset b_{\varrho_N}\mathrm{CLof}(\varrho_N) \Longrightarrow b_{\varrho_N}\mathrm{CLof}(\varrho_N) = \varrho_N \cup \mathrm{e}_{b_{\varrho_N}}(\varrho_N)$. This concludes the evidence to be presented.

Observation 2.8. In any subset that exists $\varrho_{N_1}, \varrho_{N_2}$ of Γ, These statements are all accurate:

a) $\mathrm{IN}_{b_{\varrho_N}}(\varrho_{N_1})$ being the biggest b_{ϱ_N}-OS $\subset \varrho_{N_1}$.
b) ϱ_{N_1} is b_{ϱ_N}-OS $\Leftrightarrow \varrho_{N_1} = \mathrm{IN}_{b_{\varrho_N}}(\varrho_{N_1})$.
c) $\mathrm{IN}_{b_{\varrho_N}}\left(\mathrm{IN}_{b_{\varrho_N}}(\varrho_{N_1})\right) = \mathrm{IN}_{b_{\varrho_N}}(\varrho_{N_1})$.
d) $\Gamma - \mathrm{CL}_{b_{\varrho_N}}(\varrho_{N_1}) = \mathrm{IN}_{b_{\varrho_N}}(\Gamma - \varrho_{N_1})$.

e) $\varrho_{N_1} \subset \varrho_{N_2}$, then $IN_{b_{\varrho N}}(\varrho_{N_2}) \supset IN_{b_{\varrho N}}(\varrho_{N_1})$.
f) $IN_{b_{\varrho N}}(\varrho_{N_1} \cup \varrho_{N_2}) \supset IN_{b_{\varrho N}}(\varrho_{N_1}) \cup IN_{b_{\varrho N}}(\varrho_{N_2})$.

Theorem 2.9. In any of the subsets $\varrho_{N_1}, \varrho_{N_2}$ of Γ, All of these claims are true:
a) $IN_{b_{\varrho N}}(\varrho_{N_1}) = \varrho_{N_1} - \mathrm{e}_{b_{\varrho N}}(\Gamma - \varrho_{N_1})$.
b) $\Gamma - IN_{b_{\varrho N}}(\varrho_{N_1}) = CL_{b_{\varrho N}}(\Gamma - \varrho_{N_1})$.

Proof.

(a) Let $\varrho_n^1 \in \varrho_{N_1} - \mathrm{e}_{b_{\varrho N}}(\Gamma - \varrho_{N_1}) \Rightarrow \varrho_n^1 \notin \mathrm{e}_{b_{\varrho N}}(\Gamma - \varrho_{N_1})$ and so $\exists$ a $b_{\varrho N}$-OS, Γ_1 containing ϱ_n^1 such that $\Gamma_1 \cap (\Gamma - \varrho_{N_1}) = \phi$. Then $\varrho_n^1 \in \Gamma_1 \subset \varrho_{N_1}$ and hence $\varrho_n^1 \in b_{\varrho N}INTof(\varrho_{N_1})$, i.e., $\varrho_{N_1} - \mathrm{e}_{b_{\varrho N}}(\Gamma - \varrho_{N_1}) \subset b_{\varrho N}INTof(\varrho_{N_1})$. As opposed to that, if $\varrho_n^1 \in b_{\varrho N}INTof(\varrho_{N_1}) \Rightarrow \varrho_n^1 \notin \mathrm{e}_{b_{\varrho N}}(\Gamma - \varrho_{N_1})$. Since $b_{\varrho N}INTof(\varrho_{N_1})$ is $b_{\varrho N}$-open and $b_{\varrho N}INTof(\varrho_{N_1}) \cap (\Gamma - \varrho_{N_1}) = \phi$. Hence $b_{\varrho N}INTof(\varrho_{N_1}) = \varrho_{N_1} - \mathrm{e}_{b_{\varrho N}}(\Gamma - \varrho_{N_1})$.

(b) $\Gamma - b_{\varrho N}INTof(\varrho_{N_1}) = \Gamma - \left(\varrho_{N_1} - \mathrm{e}_{b_{\varrho N}}(\Gamma - \varrho_{N_1})\right) = (\Gamma - \varrho_{N_1}) \cup \mathrm{e}_{b_{\varrho N}}(\Gamma - \varrho_{N_1}) = b_{\varrho N}CLof(\Gamma - \varrho_{N_1})$.

Definition 2.10. In any of the subsets ϱ_N of Γ, $\beta d^{\#b_{\varrho N}}(\varrho_N) = \varrho_N - b_{\varrho N}INTof(\varrho_N)$ It has been stated to have $b_{\varrho N}$-border about ϱ_N.

Example 2.11. Let $\Gamma = \{\alpha, \beta, \gamma\}$ and $\Upsilon = \{0_N, 1_N, \varrho_{N_1}, \varrho_{N_2}, \varrho_{N_3}, \varrho_{N_4}\}$ where
$\varrho_{N_1} = \{\langle\Gamma(\alpha)0.71, \Gamma(\alpha)0.69, \Gamma(\alpha)0.5\rangle, \langle\Gamma(\beta)0.3, \Gamma(\beta)0.52, \Gamma(\beta)0.43\rangle, \langle\Gamma(\gamma)0.29, \Gamma(\gamma)0.29, \Gamma(\gamma)0.29\rangle\}$,
$\varrho_{N_2} = \{\langle\Gamma(\alpha)0.59, \Gamma(\alpha)0.61, \Gamma(\alpha)0.36\rangle, \langle\Gamma(\beta)0.76, \Gamma(\beta)0.23, \Gamma(\beta)0.33\rangle, \langle\Gamma(\gamma)0.89, \Gamma(\gamma)0.29, \Gamma(\gamma)0.29\rangle\}$,
$\varrho_{N_3} = \{\langle\Gamma(\alpha)0.62, \Gamma(\alpha)0.68, \Gamma(\alpha)0.39\rangle, \langle\Gamma(\beta)0.18, \Gamma(\beta)0.61, \Gamma(\beta)0.74\rangle, \langle\Gamma(\gamma)0.19, \Gamma(\gamma)0.23, \Gamma(\gamma)0.43\rangle\}$,
$\varrho_{N_4} = \{\langle\Gamma(\alpha)0.39, \Gamma(\alpha)0.49, \Gamma(\alpha)0.39\rangle, \langle\Gamma(\beta)0.62, \Gamma(\beta)0.24, \Gamma(\beta)0.14\rangle, \langle\Gamma(\gamma)0.23, \Gamma(\gamma)0.31, \Gamma(\gamma)0.32\rangle\}$
Here ϱ_{N_2} be a subset of a space Γ and
$\beta d^{\#b_{\varrho N}}(\{\langle\Gamma(\alpha)0.59, \Gamma(\alpha)0.61, \Gamma(\alpha)0.36\rangle, \langle\Gamma(\beta)0.76, \Gamma(\beta)0.23, \Gamma(\beta)0.33\rangle, \langle\Gamma(\gamma)0.89, \Gamma(\gamma)0.29, \Gamma(\gamma)0.29\rangle\}) =$
$\{\langle\Gamma(\alpha)0.59, \Gamma(\alpha)0.61, \Gamma(\alpha)0.36\rangle, \langle\Gamma(\beta)0.76, \Gamma(\beta)0.23, \Gamma(\beta)0.33\rangle, \langle\Gamma(\gamma)0.89, \Gamma(\gamma)0.29, \Gamma(\gamma)0.29\rangle\} -$
$IN_{b_{\varrho N}}(\{\langle\Gamma(\alpha)0.59, \Gamma(\alpha)0.61, \Gamma(\alpha)0.36\rangle, \langle\Gamma(\beta)0.76, \Gamma(\beta)0.23, \Gamma(\beta)0.33\rangle, \langle\Gamma(\gamma)0.89, \Gamma(\gamma)0.29, \Gamma(\gamma)0.29\rangle\})$

Observation 2.12. In any of the subsets ϱ_N of Γ, All of these claims are true:
a) $\varrho_N = IN_{b_{\varrho N}}(\varrho_N) \cup \beta d^{\#b_{\varrho N}}(\varrho_N)$.
b) $IN_{b_{\varrho N}}(\varrho_N) \cap \beta d^{\#b_{\varrho N}}(\varrho_N) = \phi$.
c) ϱ_N a $b_{\varrho N}$-OS $\Leftrightarrow \beta d^{\#b_{\varrho N}}(\varrho_N) = \phi$.
d) $\beta d^{\#b_{\varrho N}}\left(IN_{b_{\varrho N}}(\varrho_N)\right) = \phi$.
e) $IN_{b_{\varrho N}}\left(\beta d^{\#b_{\varrho N}}(\varrho_N)\right) = \phi$.

Theorem 2.13. In any of the subsets ϱ_N of Γ, All of these claims are correct:
a) $\beta d^{\#b_{\varrho N}}\left(\beta d^{\#b_{\varrho N}}(\varrho_N)\right) = \beta d^{\#b_{\varrho N}}(\varrho_N)$.
b) $\beta d^{\#b_{\varrho N}}(\varrho_N) = \varrho_N \cap CL_{b_{\varrho N}}(\Gamma - \varrho_N)$.
c) $\beta d^{\#b_{\varrho N}}(\varrho_N) = \mathrm{e}_{b_{\varrho N}}(\Gamma - \varrho_N)$.

Proof.

(a) If $\varrho_n^1 \in \mathrm{IN}_{b_{\varrho_N}}\left(\beta d^{\#b_{\varrho_N}}(\varrho_N)\right)$, then $\varrho_n^1 \in \beta d^{\#b_{\varrho_N}}(\varrho_N)$. As opposed to that, $\beta d^{\#b_{\varrho_N}}(\varrho_N) \subset \varrho_N$, $\varrho_n^1 \in \mathrm{IN}_{b_{\varrho_N}}\left(\beta d^{\#b_{\varrho_N}}(\varrho_N)\right) \subset \mathrm{IN}_{b_{\varrho_N}}(\varrho_N)$. Hence $\varrho_n^1 \in \mathrm{IN}_{b_{\varrho_N}}(\varrho_N) \cap \beta d^{\#b_{\varrho_N}}(\varrho_N)$ which contradicts (c). Thus $\cap\, of\ \mathrm{IN}_{b_{\varrho_N}}(\varrho_N)$ & $\beta d^{\#b_{\varrho_N}}(\varrho_N)$ is ϕ.

(b) $\beta d^{\#b_{\varrho_N}}(\varrho_N) =$ difference of $\mathrm{IN}_{b_{\varrho_N}}(\varrho_N)$ from $\varrho_N = \varrho_N - \left(\Gamma - \mathrm{CL}_{b_{\varrho_N}}(\Gamma - \varrho_N)\right) = \varrho_N \cap \mathrm{CL}_{b_{\varrho_N}}(\Gamma - \varrho_N)$.

(c) $\beta d^{\#b_{\varrho_N}}(\varrho_N) =$ difference of $\mathrm{IN}_{b_{\varrho_N}}(\varrho_N)$ from $\varrho_N = \varrho_N - \left(\varrho_N - \mathfrak{D}\varepsilon''_{\alpha\delta}(\Gamma - \varrho_N)\right) = \mathbb{e}_{b_{\varrho_N}}(\Gamma - \varrho_N)$.

Definition 2.14. A b_{ϱ_N}-frontier of any of the subsets ϱ_N of Γ is $\mathrm{Fr}_{\#b_{\varrho_N}}(\varrho_N) = \cap\, of\ \mathrm{CL}_{b_{\varrho_N}}(\varrho_N) \& \mathrm{CL}_{b_{\varrho_N}}(\Gamma \backslash \varrho_N)$.

Example 2.15. Let $\Gamma = \{\alpha, \beta, \gamma\}$ and consider the family $\Upsilon = \{0_N, 1_N, \varrho_{N_1}, \varrho_{N_2}\}$ where
$\varrho_{N_1} = \{\langle\Gamma(\alpha)0.6, \Gamma(\alpha)0.5, \Gamma(\alpha)0.3\rangle, \langle\Gamma(\beta)0.3, \Gamma(\beta)0.7, \Gamma(\beta)0.3\rangle, \langle\Gamma(\gamma)0.1, \Gamma(\gamma)0.2, \Gamma(\gamma)0.6\rangle\}$,
$\varrho_{N_2} = \{\langle\Gamma(\alpha)0.9, \Gamma(\alpha)0.1, \Gamma(\alpha)0.3\rangle, \langle\Gamma(\beta)0.6, \Gamma(\beta)0.2, \Gamma(\beta)0.3\rangle, \langle\Gamma(\gamma)0.9, \Gamma(\gamma)0.9, \Gamma(\gamma)0.2\rangle\}$,
Here ϱ_{N_1} be a subset of a space Γ and
$\mathrm{Fr}_{\#b_{\varrho_N}}(\{\langle\Gamma(\alpha)0.6, \Gamma(\alpha)0.5, \Gamma(\alpha)0.3\rangle, \langle\Gamma(\beta)0.3, \Gamma(\beta)0.7, \Gamma(\beta)0.3\rangle, \langle\Gamma(\gamma)0.1, \Gamma(\gamma)0.2, \Gamma(\gamma)0.6\rangle\})$ is equal to
$\mathrm{CL}_{b_{\varrho_N}}(\{\langle\Gamma(\alpha)0.6, \Gamma(\alpha)0.5, \Gamma(\alpha)0.3\rangle, \langle\Gamma(\beta)0.3, \Gamma(\beta)0.7, \Gamma(\beta)0.3\rangle, \langle\Gamma(\gamma)0.1, \Gamma(\gamma)0.2, \Gamma(\gamma)0.6\rangle\})$
$\cap \mathrm{CL}_{b_{\varrho_N}}(\{\langle\Gamma(\alpha)0.6, \Gamma(\alpha)0.5, \Gamma(\alpha)0.3\rangle, \langle\Gamma(\beta)0.3, \Gamma(\beta)0.7, \Gamma(\beta)0.3\rangle, \langle\Gamma(\gamma)0.1, \Gamma(\gamma)0.2, \Gamma(\gamma)0.6\rangle\})$.

Theorem 2.16. In any of the subsets ϱ_N of Γ, All of these claims are true:

a) $\mathrm{CL}_{b_{\varrho_N}}(\varrho_N) = \mathrm{IN}_{b_{\varrho_N}}(\varrho_N) \cup \mathrm{Fr}_{\#b_{\varrho_N}}(\varrho_N)$

b) $\mathrm{Fr}_{\#b_{\varrho_N}}(\varrho_N) = \beta d^{\#b_{\varrho_N}}(\varrho_N) \cup \mathbb{e}_{b_{\varrho_N}}(\varrho_N)$.

c) $\mathrm{Fr}_{\#b_{\varrho_N}}(\varrho_N) = \mathrm{CL}_{b_{\varrho_N}}(\varrho_N) \cap \mathrm{CL}_{b_{\varrho_N}}(\Gamma \backslash \varrho_N)$.

d) $\mathrm{Fr}_{\#b_{\varrho_N}}(\varrho_N)$ is b_{ϱ_N}-closed

Theorem 2.17. In any of the subsets ϱ_N of Γ, All of these claims are correct:

a) $\mathrm{IN}_{b_{\varrho_N}}(\varrho_N) \cap \mathrm{Fr}_{\#b_{\varrho_N}}(\varrho_N) = \phi$.

b) $\mathrm{Fr}_{\#b_{\varrho_N}}(\varrho_N) \supset \beta d^{\#b_{\varrho_N}}(\varrho_N)$.

c) ϱ_N is b_{ϱ_N}-open set iff $\mathrm{Fr}_{\#b_{\varrho_N}}(\varrho_N) = \mathbb{e}_{b_{\varrho_N}}(\varrho_N)$

d) $\mathrm{Fr}_{\#b_{\varrho_N}}(\varrho_N) = \mathrm{Fr}_{\#b_{\varrho_N}}(\Gamma \backslash \varrho_N)$

e) $\mathrm{Fr}_{\#b_{\varrho_N}}(\varrho_N) \supset \mathrm{CL}_{b_{\varrho_N}}\left(\mathrm{Fr}_{\#b_{\varrho_N}}(\varrho_N)\right)$.

f) $\mathrm{Fr}_{\#b_{\varrho_N}}(\varrho_N) \supset \mathrm{Fr}_{\#b_{\varrho_N}}\left(\mathrm{Fr}_{\#b_{\varrho_N}}(\varrho_N)\right)$.

g) $\mathrm{Fr}_{\#b_{\varrho_N}}(\varrho_N) \supset \mathrm{Fr}_{\#b_{\varrho_N}}\left(\mathrm{CL}_{b_{\varrho_N}}(\varrho_N)\right)$

h) $IN_{b_{\varrho_N}}(\varrho_N) = \varrho_N - Fr_{\#b_{\varrho_N}}(\varrho_N)$.

Proof.

(a) $IN_{b_{\varrho_N}}(\varrho_N) \cup Fr_{\#b_{\varrho_N}}(\varrho_N) = IN_{b_{\varrho_N}}(\varrho_N) \cup \left(CL_{b_{\varrho_N}}(\varrho_N) - IN_{b_{\varrho_N}}(\varrho_N)\right) = CL_{b_{\varrho_N}}(\varrho_N)$.

(b) $IN_{b_{\varrho_N}}(\varrho_N) \cap Fr_{\#b_{\varrho_N}}(\varrho_N) = IN_{b_{\varrho_N}}(\varrho_N) \cap \left(CL_{b_{\varrho_N}}(\varrho_N) - IN_{b_{\varrho_N}}(\varrho_N)\right) = \phi$.

(c) Since $IN_{b_{\varrho_N}}(\varrho_N) \cup Fr_{\#b_{\varrho_N}}(\varrho_N) = IN_{b_{\varrho_N}}(\varrho_N) \cup \beta d^{\#b_{\varrho_N}}(\varrho_N) \cup e_{b_{\varrho_N}}(\varrho_N)$,
$Fr_{\#b_{\varrho_N}}(\varrho_N) = \beta d^{\#b_{\varrho_N}}(\varrho_N) \cup e_{b_{\varrho_N}}(\varrho_N)$

(d) $Fr_{\#b_{\varrho_N}}(\varrho_N) = CL_{b_{\varrho_N}}(\varrho_N) - IN_{b_{\varrho_N}}(\varrho_N) = CL_{b_{\varrho_N}}(\varrho_N) \cap CL_{b_{\varrho_N}}(\Gamma \backslash \varrho_N)$.

(e) $CL_{b_{\varrho_N}}\left(Fr_{\#b_{\varrho_N}}(\varrho_N)\right) = CL_{b_{\varrho_N}}\left(CL_{b_{\varrho_N}}(\varrho_N) \cap CL_{b_{\varrho_N}}(X \backslash \varrho_N)\right)$
$\subset CL_{b_{\varrho_N}}\left(CL_{b_{\varrho_N}}(\varrho_N)\right) \cap CL_{b_{\varrho_N}}\left(CL_{b_{\varrho_N}}(\Gamma \backslash \varrho_N)\right) = Fr_{\#b_{\varrho_N}}(\varrho_N)$.
Hence $Fr_{\#b_{\varrho_N}}(\varrho_N)$ is b_{ϱ_N}-closed.

(f) $Fr_{\#b_{\varrho_N}}\left(Fr_{\#b_{\varrho_N}}(\varrho_N)\right) = CL_{b_{\varrho_N}}$ of $Fr_{\#b_{\varrho_N}}(\varrho_N) \cap CL_{b_{\varrho_N}}\left(\Gamma - Fr_{\#b_{\varrho_N}}(\varrho_N)\right)$
$\subset CL_{b_{\varrho_N}}$ of $Fr_{\#b_{\varrho_N}}(\varrho_N) = Fr_{\#b_{\varrho_N}}(\varrho_N)$

(g) $Fr_{\#b_{\varrho_N}}\left(CL_{b_{\varrho_N}}(\varrho_N)\right) = CL_{b_{\varrho_N}}\left(CL_{b_{\varrho_N}}(\varrho_N)\right) - IN_{b_{\varrho_N}}\left(CL_{b_{\varrho_N}}(\varrho_N)\right) = CL_{b_{\varrho_N}}(\varrho_N) - IN_{b_{\varrho_N}}\left(CL_{b_{\varrho_N}}(\varrho_N)\right) =$
$CL_{b_{\varrho_N}}(\varrho_N) - IN_{b_{\varrho_N}}(\varrho_N) = Fr_{\#b_{\varrho_N}}(\varrho_N)$.

(h) $\varrho_N - Fr_{\#b_{\varrho_N}}(\varrho_N) = \varrho_N - \left(CL_{b_{\varrho_N}}(\varrho_N) - IN_{b_{\varrho_N}}(\varrho_N)\right) = IN_{b_{\varrho_N}}(\varrho_N)$.

Within the ensuing theorem $FNb_{\varrho_N}^{(C)}$ indicate the group of points ϱ_n^1 of Γ which a function is used $q: (\Gamma_1, \xi_1) \to (\Gamma_2, \xi_2)$ is not FNb_{ϱ_N}-C.

Theorem 2.18. The $\cup(FNb_{\varrho_N})$-frontiers of the mirror reflections of FNb_{ϱ_N}-OS that includes $q(\varrho_n^1)$ is $\Leftrightarrow$ to $FNb_{\varrho_N}^{(C)}$.

Proof. Proceed to consider q is not FNb_{ϱ_N}-at a point, continuous ϱ_n^1 of $\Gamma_1 \Longrightarrow \exists$ an OS, $J \subset \Gamma_2$ containing $q(\varrho_n^1) \mid q(I)$ is not a portion of J $\forall I \in FNb_{\varrho_N}O(\Gamma_1)$ containing ϱ_n^1. Hence we've $I \cap \left(\Gamma_1 - q^{-1}(J)\right) \neq \phi$, $\forall$ $I \in FNb_{\varrho_N}O(\Gamma_1)$ containing ϱ_n^1. It follows that $\varrho_n^1 \in CL_{b_{\varrho_N}}\left(\Gamma_1 - q^{-1}(\varrho_N)\right)$. Additionally, we have $\varrho_n^1 \in q^{-1}(J) \subset CL_{b_{\varrho_N}}\left(q^{-1}(\varrho_N)\right)$. Thus, it follows that $\varrho_n^1 \in Fr_{\#b_{\varrho_N}}\left(q^{-1}(J)\right)$. Now, let q be FNb_{ϱ_N}-Cont. at $\varrho_n^1 \in \Gamma_1$ and $J \subset \Gamma_2$ be any OS containing $q(\varrho_n^1)$. Then $\varrho_n^1 \in q^{-1}(J)$ is a FNb_{ϱ_N}-open set of Γ_1. Thus $\varrho_n^1 \in IN_{b_{\varrho_N}}\left(q^{-1}(J)\right)$ and therefore $\varrho_n^1 \notin Fr_{\#b_{\varrho_N}}\left(q^{-1}(J)\right)$ for every OS, J containing $q(\varrho_n^1)$.

Definition 2.19. In any of the subsets ϱ_N of a Γ, $E^{FNb_{\varrho_N}}$ of ϱ_N is b_{ϱ_N} INT of $\Gamma - \varrho_N$ this will eventually take place. FNb_{ϱ_N}-exterior regarding ϱ_N.

Example 2.20. Let $\Gamma = \{\alpha, \beta, \gamma\}$ and consider the family $\Upsilon = \{0_N, 1_N, \varrho_{N_1}, \varrho_{N_2}\}$ where
$\varrho_{N_1} = \{\langle\Gamma(\alpha)0.4, \Gamma(\alpha)0.5, \Gamma(\alpha)0.4\rangle, \langle\Gamma(\beta)0.6, \Gamma(\beta)0.6, \Gamma(\beta)0.4\rangle, \langle\Gamma(\gamma)0.3, \Gamma(\gamma)0.4, \Gamma(\gamma)0.7\rangle\}$,
$\varrho_{N_2} = \{\langle\Gamma(\alpha)0.8, \Gamma(\alpha)0.3, \Gamma(\alpha)0.2\rangle, \langle\Gamma(\beta)0.4, \Gamma(\beta)0.3, \Gamma(\beta)0.2\rangle, \langle\Gamma(\gamma)0.3, \Gamma(\gamma)0.2, \Gamma(\gamma)0.3\rangle\}$,

Here ϱ_{N_1} be a subset of a space Γ and
$E^{FNb_{\varrho_N}}(\{\langle\Gamma(\alpha)0.8,\Gamma(\alpha)0.3,\Gamma(\alpha)0.2\rangle,\langle\Gamma(\beta)0.4,\Gamma(\beta)0.3,\Gamma(\beta)0.2\rangle,\langle\Gamma(\gamma)0.3,\Gamma(\gamma)0.2,\Gamma(\gamma)0.3\rangle\}) =$
$IN_{b_{\varrho_N}}(\Gamma-\{\langle\Gamma(\alpha)0.8,\Gamma(\alpha)0.3,\Gamma(\alpha)0.2\rangle,\langle\Gamma(\beta)0.4,\Gamma(\beta)0.3,\Gamma(\beta)0.2\rangle,\langle\Gamma(\gamma)0.3,\Gamma(\gamma)0.2,\Gamma(\gamma)0.3\rangle\})$.

Observation 2.21. In any of the subsets ϱ_N of Γ, All of these claims are true:

a) $E^{FNb_{\varrho_N}}(\varrho_N)$ is FNb_{ϱ_N}-OS.

b) $E^{FNb_{\varrho_N}}(\varrho_N) = IN_{b_{\varrho_N}}(\Gamma-\varrho_N) = \Gamma - CL_{b_{\varrho_N}}(\varrho_N)$.

c) If $\binom{\varrho_N}{1} \subset \binom{\varrho_N}{2} \Rightarrow E^{FNb_{\varrho_N}}\binom{\varrho_N}{1} \supset E^{FNb_{\varrho_N}}\binom{\varrho_N}{2}$.

d) $E^{FNb_{\varrho_N}}\left(\binom{\varrho_N}{1} \cup \binom{\varrho_N}{2}\right) \subset E^{FNb_{\varrho_N}}\binom{\varrho_N}{1} \cup E^{FNb_{\varrho_N}}\binom{\varrho_N}{2}$.

e) $E^{FNb_{\varrho_N}}(\Gamma) = \phi$.

f) $E^{FNb_{\varrho_N}}(\phi) = \Gamma$.

g) $\Gamma = IN_{b_{\varrho_N}}(\varrho_N) \cup E^{FNb_{\varrho_N}}(\varrho_N) \cup Fr_{\#b_{\varrho_N}}(\varrho_N)$.

Theorem 2.22. In any of the subsets ϱ_N of Γ, All of these claims are correct:

a) $E^{FNb_{\varrho_N}}\left(E^{FNb_{\varrho_N}}(\varrho_N)\right) = IN_{b_{\varrho_N}}\left(CL_{b_{\varrho_N}}(\varrho_N)\right)$.

b) $E^{FNb_{\varrho_N}}(\varrho_N) = E^{FNb_{\varrho_N}}\left(\Gamma - E^{FNb_{\varrho_N}}(\varrho_N)\right)$.

c) $IN_{b_{\varrho_N}}(\varrho_N) \subset E^{FNb_{\varrho_N}}\left(E^{FNb_{\varrho_N}}(\varrho_N)\right)$.

Proof.

(a) $E^{FNb_{\varrho_N}}\left(E^{FNb_{\varrho_N}}(\varrho_N)\right) = E^{FNb_{\varrho_N}}\left(\Gamma - CL_{b_{\varrho_N}}(\varrho_N)\right)$
$= IN_{b_{\varrho_N}}\left(\Gamma-\left(\Gamma - CL_{b_{\varrho_N}}(\varrho_N)\right)\right) = IN_{b_{\varrho_N}}\left(CL_{b_{\varrho_N}}(\varrho_N)\right)$.

(b) $E^{FNb_{\varrho_N}}\left(\Gamma - E^{FNb_{\varrho_N}}(\varrho_N)\right) = E^{FNb_{\varrho_N}}\left(\Gamma - IN_{b_{\varrho_N}}(\Gamma-\varrho_N)\right)$
$= IN_{b_{\varrho_N}}\left(\Gamma-\left(\Gamma - IN_{b_{\varrho_N}}(\Gamma-\varrho_N)\right)\right) = IN_{b_{\varrho_N}}\left(IN_{b_{\varrho_N}}(\Gamma-\varrho_N)\right) = IN_{b_{\varrho_N}}(\Gamma-\varrho_N) = E^{FNb_{\varrho_N}}(\varrho_N)$.

(c) $IN_{b_{\varrho_N}}(\varrho_N) \subset IN_{b_{\varrho_N}}\left(CL_{b_{\varrho_N}}(\varrho_N)\right) = IN_{b_{\varrho_N}}\left(\Gamma - IN_{b_{\varrho_N}}(\Gamma-\varrho_N)\right)$
$= IN_{b_{\varrho_N}}\left(\Gamma - E^{FNb_{\varrho_N}}(\varrho_N)\right) = E^{FNb_{\varrho_N}}\left(E^{FNb_{\varrho_N}}(\varrho_N)\right)$

Definition 2.23. Γ be an $fnts$ and let $\varrho_n^1 \in \Gamma$. A subset $\mathbb{N}$ of Γ is fn-b_{ϱ_N}-$\mathbb{N}$bhd of ϱ_n^1, if $\exists$ a fn-b_{ϱ_N}-OS, E $\mid$ $\varrho_n^1 \in E \subset \mathbb{N}$.

Definition 2.24. An ϱ_N be a $\subset \Gamma$, $\varrho_n^1 \in \varrho_N$ meant to be fn-b_{ϱ_N}- innermost point ϱ_N if ϱ_N is a fn-b_{ϱ_N}-$\mathbb{N}$bhd of ϱ_n^1. The entire set fn-b_{ϱ_N}- point of interior ϱ_N is fn-b_{ϱ_N}-interior ϱ_N and it is $IN_{b_{\varrho_N}}(\varrho_N)$, $IN_{b_{\varrho_N}}(\varrho_N)$ is union of $\{L: L \text{ is } fnb_{\varrho_N}OS,\ L \subset \varrho_N\}$

A ϱ_N be a section of a space. Γ We define FNb_{ϱ_N}-closure of ϱ_N to serve as a junction for all FNb_{ϱ_N}-closed sets made of ϱ_N, b_{ϱ_N} CL of $\varrho_N = \cap\{L: \varrho_N \subset L \in fnb_{\varrho_N}(\Gamma)\}$

Definition 2.25. ϱ_N an area where a number of elements are present. Γ, an element $\varrho_n^1 \in \Gamma$ is to be b_{ϱ_N}-point of ϱ_N if $\forall$ b_{ϱ_N}-OS, Γ_1 containing ϱ_n^1, $\Gamma_1 \cap (\varrho_N - \{\varrho_N^1\}) \neq \phi$. The whole set b_{ϱ_N}-point of ϱ_N is b_{ϱ_N}-derived (briefly. $\mathrm{e}_{b_{\varrho_N}}$) a bunch of ϱ_N as indicated by $\mathrm{e}_{b_{\varrho_N}}(\varrho_N)$.

Definition 2.26. In any subset $\exists$ ϱ_N of a Γ, $E^{FNb_{\varrho_N}}(\varrho_N)$ is b_{ϱ_N} Int of $\Gamma - \varrho_N$ this will occur FNb_{ϱ_N}-exterior regarding ϱ_N.

Definition 2.27. Let (F, Γ_F) be an FNTS. Two never empty FNS's ϱ_{N_1} and ϱ_{N_2} of Γ are regarded as Γ^S-segregated if $\varrho_{N_1} \cap b_{\varrho_N}$ CL of$(\varrho_{N_2}) = \phi_N$ $\varrho_{N_1} \cap b_{\varrho_N}$ CL of$(\varrho_{N_2}) = \phi_N$ and b_{ϱ_N} CL of$(\varrho_{N_1}) \cap \varrho_{N_2} = \phi_N$. Both of these circumstances are comparable to the one condition. $\left(\varrho_{N_1} \cap b_{\varrho_N}\ CL\ \text{of}(\varrho_{N_2})\right) \cup \left(b_{\varrho_N}\ CL\ \text{of}(\varrho_{N_1}) \cap \varrho_{N_2}\right) = \phi_N$.

Definition 2.28. Let a FNTS be (F, Γ_F). If G is a FN subset of F, then the collection Γ^S of G is $\{G \cap U: U \in \Gamma\}$ G is referred to be a FN subspace topology on F if is a FNT on G.

Observation 2.29. FN disjoint is any two FN separated sets. FN, however, does not necessarily divide two independent sets of FN.

Theorem 2.30. A $(G, \Gamma^S(G))$ be a FNTS's FN subspace. (F, Γ_F), $\varrho_{N_1}, \varrho_{N_2}$ be 2 NF sets of G. Then $\varrho_{N_1}, \varrho_{N_2}$ a FN Γ^S-segregated $\Leftrightarrow$ they are FN $\Gamma^S(G)$-segregated.
Proof: By concept, $CL_{b_{\varrho_N}}G(\varrho_{N_1}) = CL_{b_{\varrho_N}}F(\varrho_{N_1}) \cap G$ and $CL_{b_{\varrho_N}}G(\varrho_{N_2}) = CL_{b_{\varrho_N}}F(\varrho_{N_2}) \cap G$.
Now $\left(CL_{b_{\varrho_N}}G(\varrho_{N_1}) \cap \varrho_{N_2}\right) \cup \left(\varrho_{N_1} \cap CL_{b_{\varrho_N}}G(\varrho_{N_2})\right) = \left(CL_{b_{\varrho_N}}F(\varrho_{N_1}) \cap G \cap \varrho_{N_2}\right) \cup \left(\varrho_{N_1} \cap CL_{b_{\varrho_N}}F(\varrho_{N_2}) \cap G\right) = \left(CL_{b_{\varrho_N}}F(\varrho_{N_1}) \cap \varrho_{N_2}\right) \cup \left(\varrho_{N_1} \cap CL_{b_{\varrho_N}}F(\varrho_{N_2})\right)$.

Hence $\left(CL_{b_{\varrho_N}}G(\varrho_{N_1}) \cap \varrho_{N_2}\right) \cup \left(\varrho_{N_1} \cap CL_{b_{\varrho_N}}G(\varrho_{N_2})\right) = \phi_N$
$\Leftrightarrow \left(CL_{b_{\varrho_N}}F(\varrho_{N_1}) \cap \varrho_{N_2}\right) \cup \left(\varrho_{N_1} \cap CL_{b_{\varrho_N}}F(\varrho_{N_2})\right) = \phi_N$, because $\varrho_{N_1}, \varrho_{N_2} \subset G$.
It follows that $\varrho_{N_1}, \varrho_{N_2}$ are FN Γ^S-segregated if and only if they are FN $\Gamma^S(G)$-segregated.

Theorem 2.31. If ϱ_{N_1} and ϱ_{N_2} are Γ^S-segregated sets of an FNTS (F, Γ_F) and $C_1 \subset \varrho_{N_1}$ and $C_2 \subset \varrho_{N_2}$, then C_1 and C_2 are also $\Gamma^S(G)$-segregated.

Proof: Given $C_1 \subset \varrho_{N_1} \Rightarrow b_{\varrho_N}$ CL of$(C_1) \subset b_{\varrho_N}$ CL of(ϱ_{N_1}) and $C_2 \subset \varrho_{N_2} \Rightarrow b_{\varrho_N}$ CL of$(C_2) \subset b_{\varrho_N}$ CL of(ϱ_{N_2}). Since $\varrho_{N_1} \cap b_{\varrho_N}$ CL of$(\varrho_{N_2}) = \phi_N$ and b_{ϱ_N} CL of$(\varrho_{N_1}) \cap \varrho_{N_2} = \phi_N$. It follows that $C_1 \cap b_{\varrho_N}$ CL of$(C_2) = \phi_N$ and b_{ϱ_N} CL of$(C_1) \cap C_2 = \phi_N$. Hence C_1 and C_2 are $\Gamma^S(G)$-segregated.

Theorem 2.32. Two FNC(FNO) sets ϱ_{N_1} and ϱ_{N_2} of an FNTS are Γ^S-segregated $\Leftrightarrow$ They don't make appropriate.

Proof: Given that any 2 Γ^S-segregated sets don't match. If ϱ_{N_1} and ϱ_{N_2} are both disjoint and FN closed, then $\varrho_{N_1} \cap \varrho_{N_2} = \phi_N$, b_{ϱ_N} CL of $(\varrho_{N_1}) = \varrho_{N_1}$ and b_{ϱ_N} CL of $(\varrho_{N_2}) = \varrho_{N_2}$. So b_{ϱ_N} CL of$(\varrho_{N_1}) \cap \varrho_{N_2} = \phi_N$ and b_{ϱ_N} CL of$(\varrho_{N_2}) \cap \varrho_{N_1} = \phi_N$ implies ϱ_{N_1} and ϱ_{N_2} are Γ^S-segregated. If ϱ_{N_1} and ϱ_{N_2} are both disjoint and FN open, then $\varrho_{N_1}(c)$ and $\varrho_{N_2}(c)$ are both FN closed so that b_{ϱ_N} CL of$\left(\varrho_{N_1}(c)\right)$ and b_{ϱ_N} CL of$\left(\varrho_{N_2}(c)\right)$.

Also $\varrho_{N_1} \cap \varrho_{N_2} = \phi_N \Rightarrow \varrho_{N_1} \subset \varrho_{N_2}(c)$ and $\varrho_{N_2} \subset \varrho_{N_1}(c) \Rightarrow b_{\varrho_N}\ CL$ of$(\varrho_{N_1}) \subset b_{\varrho_N}\ CL$ of$\left(\varrho_{N_2}(c)\right) = \varrho_{N_2}(c)$ and $b_{\varrho_N}\ CL$ of$(\varrho_{N_2}) \subset b_{\varrho_N}\ CL$ of$\left(\varrho_{N_1}(c)\right) = \varrho_{N_1}(c) \Rightarrow b_{\varrho_N}\ CL$ of$(\varrho_{N_1}) \cap \varrho_{N_2} = \phi_N$ and $b_{\varrho_N}\ CL$ of$(\varrho_{N_2}) \cap \varrho_{N_1} = \phi_N \Rightarrow \varrho_{N_1}$ and ϱ_{N_2} are Γ^S-segregated.

Theorem 2.33. Two FN disjoint sets ϱ_{N_1} and ϱ_{N_2} are Γ^S-segregated in an FNTS(F, Γ_F) $\Leftrightarrow$ they are both FNO & FNC in the FN subspace $\varrho_{N_1} \cup \varrho_{N_2}$.

Proof: Let the disjoint FN sets ϱ_{N_1} and ϱ_{N_2} be Γ^S-segregated in Γ, so that $\varrho_{N_1} \cap \mathrm{CL}_{b_{\varrho_N}}\Gamma(\varrho_{N_2}) = \phi_N$ and $\varrho_{N_2} \cap \mathrm{CL}_{b_{\varrho_N}}\Gamma(\varrho_{N_1}) = \phi_N$. Let $L = \varrho_{N_1} \cup \varrho_{N_2}$, $\mathrm{CL}_{b_{\varrho_N}}L(\varrho_{N_1}) = \mathrm{CL}_{b_{\varrho_N}}\Gamma(\varrho_{N_1}) \cap L = \mathrm{CL}_{b_{\varrho_N}}\Gamma(\varrho_{N_1}) \cap (\varrho_{N_1} \cup \varrho_{N_2}) = \left[\mathrm{CL}_{b_{\varrho_N}}\Gamma(\varrho_{N_1}) \cap \varrho_{N_1}\right] \cup \left[\mathrm{CL}_{b_{\varrho_N}}\Gamma(\varrho_{N_1}) \cap \varrho_{N_2}\right] = \varrho_{N_1} \cup \phi_N = \varrho_{N_1}$ [because $\varrho_{N_1} \subset \mathrm{CL}_{b_{\varrho_N}}\Gamma(\varrho_{N_1})$ and $\mathrm{CL}_{b_{\varrho_N}}\Gamma(\varrho_{N_1}) \cap \varrho_{N_2} = \phi_N$]. A is FNC in the FN subspace $\varrho_{N_1} \cup \varrho_{N_2}$, by the definition of FNC. Similarly ϱ_{N_2} is FNC in $\varrho_{N_1} \cup \varrho_{N_2}$. Again $\varrho_{N_1} \cap \varrho_{N_2} = \phi_N$, they are complements of each other in L and hence they are both FNO in L. Conversely, let the disjoint FN sets ϱ_{N_1} and ϱ_{N_2} be both FNO and FNC in L. So $\varrho_{N_1} = b_{\varrho_N}\ CL$ of $L(\varrho_{N_1}) = [b_{\varrho_N}\ CL$ of $\Gamma(\varrho_{N_1}) \cap L] = b_{\varrho_N}\ CL$ of $\Gamma(\varrho_{N_1}) \cap (\varrho_{N_1} \cup \varrho_{N_2}) = [b_{\varrho_N}\ CL$ of $\Gamma(\varrho_{N_1}) \cap \varrho_{N_1}] \cup [b_{\varrho_N}\ CL$ of $\Gamma(\varrho_{N_1}) \cap \varrho_{N_2}] = \varrho_{N_1} \cup [b_{\varrho_N}\ CL$ of $\Gamma(\varrho_{N_1}) \cap \varrho_{N_2}]$ because $\varrho_{N_1} \subset b_{\varrho_N}\ CL$ of $\Gamma(\varrho_{N_1})$ $\longrightarrow$ (1). Since $\varrho_{N_1} \cap \varrho_{N_2} = \phi_N \Rightarrow \varrho_{N_1} \cap \left(b_{\varrho_N}\ CL \text{ of } \Gamma(\varrho_{N_1}) \cap \varrho_{N_2}\right) = \phi_N$, it follows from (1) that is $\left(\mathrm{CL}_{b_{\varrho_N}}\Gamma(\varrho_{N_1}) \cap \varrho_{N_2}\right) = \phi_N$. Similarly $(\mathrm{CL}_{b_{\varrho_N}}\Gamma(\varrho_{N_2}) \cap \varrho_{N_1}) = \phi_N$. Hence ϱ_{N_1} and ϱ_{N_2} are Γ^S-segregated in Γ.

Definition 2.34. Let Γ be a FNTS. A set $Y \subset \Gamma$ is said to be b_{ϱ_N}-Sat if for every $\gamma \in Y$ it follows $b_{\varrho_N}\ CL\ of(\{\gamma\}) \subset Y$. The grouping of all b_{ϱ_N}-saturated sets in Γ, we indicate by $\mathcal{S}at^{b_{\varrho_N}}(\Gamma)$.

Theorem 2.35. Let Γ, a FNTS. Then $\delta^{b_{\varrho_N}}(\Gamma)$ is a whole algebraic Boolean set.

Proof. We'll demonstrate that every combination and complement of each element in $\delta^{b_{\varrho_N}}(\Gamma)$ are members of $\delta^{b_{\varrho_N}}(\Gamma)$. Of course, the only proof that is not trivial is the one using the complements. Let $Y \in \delta^{b_{\varrho_N}}(\Gamma)$ and suppose that $b_{\varrho_N}\ CL\ of(\{\gamma_1\})$ does not contained in $\Gamma - Y$ for some $\gamma_1 \in \Gamma - Y$. Then there exists $\gamma_2 \in Y$ such that $\gamma_2 \in b_{\varrho_N}\ CL\ of(\{\gamma_1\})$. It follows that γ_1, γ_2 possess no disjoint neighbourhoods. Then $\gamma_1 \in b_{\varrho_N}\ CL\ of(\{\gamma_2\})$. However, this is in conflict with the notion of $\delta^{b_{\varrho_N}}(\Gamma)$ we have $b_{\varrho_N}\ CL\ of(\{\gamma_2\}) \subset Y$. Hence, $b_{\varrho_N}\ CL\ of(\{\gamma_1\}) \subset \Gamma - Y$ for every $\gamma_1 \in \Gamma - Y$, which implies $\Gamma - Y \in \delta^{b_{\varrho_N}}(\Gamma)$.

Corollary 2.36. $\delta^{b_{\varrho_N}}(\Gamma)$ includes each intersection and union of b_{ϱ_N}-CS and b_{ϱ_N}-OS's in Γ.

Definition 2.37. A function $\alpha: (\Gamma_1, \varrho_1) \to (\Gamma_2, \varrho_2)$ is referred to as

a) $b_{\varrho_N}(C\#)$ if $\alpha^{-1}(Q_2)$is b_{ϱ_N}-CS in (Γ_1, ϱ_1) for every CS Q_2 of (Γ_2, ϱ_2).
b) b_{ϱ_N}-Totally-Continuous (briefly. $\langle \mathcal{T}^{b_{\varrho_N}} \rangle C$) at a point $\gamma_1 \in \Gamma_1$ if for each open subset Q_2 in Γ_2 containing $\alpha(\gamma_1)$, there exists a b_{ϱ_N}-clopen subset Q_1 in Γ_1 containing γ_1 such that $\alpha(Q_1) \subset Q_2$
c) $\langle \mathcal{T}^{b_{\varrho_N}} \rangle C$ if it has this property at each point of Γ_1.

Theorem 2.38. The following statements are equivalent for a function $\alpha: (\Gamma_1, \varrho_1) \to (\Gamma_2, \varrho_2)$:

a) α is $\langle \mathcal{T}^{b_{\varrho N}} \rangle C$;
b) ∀OS, Q_2 of Γ_2 , $\alpha^{-1}(Q_2)$ is $b_{\varrho_N} CLOS$ in Γ_1;

Proof. $(a) \Rightarrow (b)$ Let Q_2 be an OS of a Γ_2 and let $\gamma \in \alpha^{-1}(Q_2)$. Since $(\gamma) \in Q_2$, by (a), ∃ a b_{ϱ_N}-CLOS Q_{1_γ} in Γ_1 containing γ such that $Q_{1_\gamma} \subset \alpha^{-1}(Q_2)$. We obtain $\alpha^{-1}(Q_2) = \cup_{\gamma \in \alpha^{-1}(Q_2)} Q_{1_\gamma}$. Thus, $\alpha^{-1}(Q_2)$ is b_{ϱ_N}-CLOS in Γ_1.

$(b) \Rightarrow (a)$ Clear.

Remark 2.39. Every $\langle \mathcal{T}^{b_{\varrho N}} \rangle C \Longrightarrow b_{\varrho_N}(C\#)$.

Definition 2.40. A space (Γ_1, ϱ_1) is said to be $b_{\varrho_N} < \sim S >$ if every b_{ϱ_N}-OS of Q_1 is OS in Q_1.

Remark 2.41. If a function $\alpha: (\Gamma_1, \varrho_1) \to (\Gamma_2, \varrho_2)$ is totally continuous and Q_1 is a $b_{\varrho_N} < \sim S >$, then α is $\langle \mathcal{T}^{b_{\varrho N}} \rangle C$.

Definition 2.42. An FNTS (Γ_1, ϱ_1) is said to be $b_{\varrho_N} \ll \mathfrak{Con}$ if the combination of two nonempty disjoint b_{ϱ_N}-OS cannot be expressed in writing.

Theorem 2.43. If α is a $\langle \mathcal{T}^{b_{\varrho N}} \rangle C$-function from a $b_{\varrho_N} \ll \mathfrak{Con}$-space Q_1 onto any space Q_2, then Q_2 is an indiscrete space.

Proof. If possible, suppose that Q_2 is not indiscrete. Let L be a valid OS of Γ_2 that isn't empty. Then $\alpha^{-1}(L)$ is a valid non-empty b_{ϱ_N}-CLOS of (Γ_1, ϱ_1), it is a contradiction to the fact that Γ_1 is $b_{\varrho_N} \ll \mathfrak{Con}$-space.

Theorem 2.44. The set of all points $\gamma \in X$ wherein a function $\alpha: (\Gamma_1, \varrho_1) \to (\Gamma_2, \varrho_2)$ is not $\langle \mathcal{T}^{b_{\varrho N}} \rangle C$ is the ∪ of $Fr_{\#b_{\varrho_N}}$ of the open sets' inverted images that include $\alpha(\gamma)$.

Proof. Suppose that α is not $\langle \mathcal{T}^{b_{\varrho N}} \rangle C$ at $\gamma \in Q_1$ $\Longrightarrow$ ∃ an OS Q_2 of Γ_2 containing $\alpha(\gamma)$ such that $\alpha(Q_1)$ is not contained in Q_2 for each $Q_1 \in b_{\varrho_N} O(\Gamma_1)$ containing γ and hence $\gamma \in b_{\varrho_N}$ CL $of(\Gamma_1 \backslash \alpha^{-1}(Q_2))$. On the other hand, $\Gamma_1 \in \alpha^{-1}(Q_2) \subset b_{\varrho_N}$ CL $of(f^{-1}(Q_2))$ and hence $\Gamma_1 \in Fr_{\#b_{\varrho_N}}(\alpha^{-1}(Q_2))$.

Conversely, suppose that α is $\langle \mathcal{T}^{b_{\varrho N}} \rangle C$ at $\gamma \in \Gamma_1$ and let Q_2 be an OS of Γ_2 containing $\alpha(\gamma) \Longrightarrow$ ∃ $Q_1 \in b_{\varrho_N} O(\Gamma_1)$ containing γ such that $Q_1 \subset \alpha^{-1}(Q_2)$. Hence $\gamma \in b_{\varrho_N}$ INT $of(\alpha^{-1}(Q_2))$. Therefore, $\Gamma_1 \in Fr_{\#b_{\varrho_N}}(\alpha^{-1}(Q_2))$ for each open set Q_2 of Γ_2 containing $\alpha(\gamma)$.

Conclusion: We have given an introduction to fn-$Fr_{\#b_{\varrho_N}}$, including the ideas of closed and open sets. We examined features in fn-$\beta d^{\#b_{\varrho N}}$ and fn-$\mathrm{e}_{b_{\varrho_N}}(\varrho_N)$, and we evaluated some of their features in fn-topological spaces to provide an idea of the findings we gained by adding the concept of fn-b_{ϱ_N}OS. We have produced a comparisons between the provided concepts of border, exterior, and derived. Additionally, we studied and identified the features of $\langle \mathcal{T}^{b_{\varrho N}} \rangle C$-functions and Γ^S-

segregated functions. In the future, we want to investigate more findings derived from the aforementioned principles and endeavour to provide applications.

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Received: 2 Dec, 2023 Accepted: 10 Mar, 2024