Generating Neutrosophic Random Variables Following the Poisson Distribution Using the Composition Method
(The Mixed Method of Inverse Transformation Method and Rejection Method)

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Abstract:
Simulation is a numerical technique used to perform tests on a numerical computer, and involves logical and mathematical relationships interacting with each other to describe the behavior and structure of a complex system in the real world over a period of time. Analysis using simulation is a "natural" and logical extension of the mathematical analytical models inherent in operations research, because most operations research methods depend on building mathematical models that closely approximate the real-world environment and we obtain the optimal solution for them using algorithms appropriate to the type of these models. The importance of the simulation process comes In all branches of science, there are many systems that cannot be studied directly, due to the great difficulty that we may encounter when studying, and the high cost, in addition to the fact that some systems cannot be studied directly. The simulation process depends on generating a series of numbers. Randomness subject to a uniform probability distribution over the domain $[0,1]$, then converting these numbers into random variables subject to the law of probability distribution by which the system to be simulated works, using known transformation methods. In previous research, we presented a neutrosophical vision of the reverse transformation method and the method of rejection and acceptance. Which are used to transform random numbers into random variables that follow probability distributions such as: uniform distribution, exponential distribution, beta distribution..., In this research, we present a neutrosophical vision of the Composition method (the mixed method of inverse transformation method and rejection method)**,** used to generate random variables that follow... To some Poisson distribution, the aim is to obtain neutrosophic random variables that we use when simulating systems that operate according to this distribution in order to obtain more accurate simulation results.

key words:
Simulation; neutrosophic logic; generating neutrosophic random numbers; converting neutrosophic random numbers into neutrosophic random variables; synthesis method (mixed method).

Introduction:

To keep pace with the great scientific development that our contemporary world is witnessing, it was necessary to reformulate operations research methods according to the basic concepts of neutrosophic logic, because the margin of freedom enjoyed by neutrosophic values gives more accurate results, which has prompted many researchers to prepare many researches in various fields of science. Especially in the field of mathematics and its applications [1-19], when performing the simulation process for any system according to classical logic, we begin by generating random numbers that follow a regular probability distribution over the domain $[0,1]$ using one of the known methods, and then we convert these random numbers into variables. Randomness follows the probability distribution in which the system to be simulated operates. The simulation process we conduct produces specific results that do not take into account changes that may occur in the system's operating environment. To obtain more accurate results, we have presented, in previous research, a neutrosophical vision of the following topics:

In the paper [20] we generated neutrosophic random numbers that follow a uniform neutrosophic distribution over the domain $[0,1]$.In research [21] we used the inverse transformation method to convert neutrosophic random numbers into neutrosophic random variables that follow a uniform distribution over the domain $[0,1]$.In research [22], we used the inverse transformation method to convert random numbers into random variables that follow the neutrosophic exponential distribution. In research [23] we used the rejection method to transform random numbers into random variables that follow the probability distribution according to which the system to be simulated operates. In the research [24] using the rejection method to generate random numbers that follow the beta distribution.

In this research, we present a neutrosophical study of transforming random numbers into random variables that follow the Poisson distribution using the composition method (The mixed method of inverse transformation method and rejection method), a distribution that has many uses in practical life. Such as inventory control, queueing theory, quality control, traffic flow, and many other fields of management science.

Discussion:

Classic Composition method: [25-26]

The Composition method is based on the inverse transformation method and the rejection and acceptance method is special for generating random variables that follow complex probability distributions.

Using the conditional distribution of the variable x, we assume that $f(x)$ is the law of the probability distribution to be simulated, and that $g(x|y)$ is the conditional distribution of If y belongs to the cumulative distribution $H(y)$ and $P(x,y)$ is the joint distribution of (x,y), then:

$$P(x,y) = h(y)g(x|y)$$

Thus, we find:

$$f(x) = \int_{-\infty}^{+\infty} P(x,y)dy = \int_{-\infty}^{+\infty} h(y)g(x|y)dy$$

When the time periods between possible events are distributed exponentially, the number of events that occur in one period of time has a Poisson distribution given by the following probability density function:

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad ; x = 0,1,2, \dots , \infty$$

Where λ is the number of expected occurrences in one time, this indicates that the time period between events is exponentially distributed with an average of $\frac{1}{\lambda}$. Using the relationship between the exponential distribution and the Poisson distribution, we can generate random variables that follow the Poisson distribution.

The neutrosophic vision of the method of installation:

The mixed method is based on the neutrosophic countertransference method [21-22] and the neutrosophic rejection and acceptance method [23].

Using the conditional distribution of the variable x, we assume that $f_N(x)$ is the law of the probability distribution to be simulated, and that $g_N(x|y)$ is the conditional distribution of If y belongs to the cumulative distribution $H_N(y)$ and $P_N(x,y)$ is the joint distribution of (x,y), then:

$$P_N(x,y) = h_N(y) g_N(x|y)$$

Thus, we find:

$$f_N(x) = \int_{-\infty}^{+\infty} P_N(x,y)dy = \int_{-\infty}^{+\infty} h_N(y) g_N(x|y)dy$$

When the time periods between possible events are distributed exponentially, the number of events that occur in one period of time has a neutrosophic Poisson distribution given by the following probability density function:

Where λ_N is a neutrosophic value from reference [27]. We find that what is meant by neutrosophic data are completely indeterminate values written in the following standard formula $N = a + bI$ where $a \, and \, b$ are real or complex coefficients, a represents the specified part and bI the indeterminate part (indeterminacy). For the number N, it could be $[\lambda_1 , \lambda_2]$ or $\{\lambda_1 , \lambda_1 \}$ or...otherwise it is any set close to the real value a, expressing the number of expected occurrences in One time, this indicates that the time period between events is exponentially distributed with an average of $\frac{1}{\lambda_N}$. Using the relationship between the neutrosophic exponential distribution and the neutrosophic Poisson distribution, we can generate neutrosophic random variables that follow the Poisson distribution.

Here we distinguish three cases:

First case: the random numbers are neutrosophic and the probability distribution is classical.

Second case: classical random numbers and neutrosophic probability distribution.

Third case: neutrosophic random numbers and neutrosophic probability distribution.

We start with the first case; the random numbers are neutrosophic and the probability distribution is classical:

In this case the probability density function of the Poisson distribution takes the following form:

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad ; x = 0,1,2, \dots, \infty$$

Where λ is the number of expected occurrences in one time, this indicates that the time period between events is exponentially distributed with an average of $\frac{1}{\lambda}$, since the random numbers must be neutrosophic to obtain them, we follow the following steps:

a. Using the mean square method given by the following relation:

$$R_{i+1} = Mid[R_i^2] \; ; i = 0,1,2,3, - - \qquad (1)$$

Where Mid symbolizes the middle four ranks of R_i^2, and R_i is chosen, any fractional random number composed of four places (called a seed) and does not contain a zero in any of its four places, [25-26], we generate a series of random numbers that follow the distribution regular over the domain $[0,1]$, we get the following series:

$$R_1, R_2, R_3 \dots R_m, \dots \qquad (2)$$

b. Using the study given in reference [20] we convert these random numbers into neutrosophic random numbers and here we distinguish three forms of the field [0,1] with margin of indeterminacy, in the three forms we have $\varepsilon \in [0, n]$ and $0 < n < 1$

The first form: $[\mathbf{0} + \boldsymbol{\varepsilon}, \mathbf{1}]$ indeterminacy at the minimum of the field we find:

$$R_{Ni} \in \left[R_i, \frac{R_i - n}{1 - n}\right] \quad ; \quad 0 < n < 1$$

It is calculated from the following relation:

$$R_{Ni} = \frac{R_i - \varepsilon}{1 - \varepsilon} \quad ; \quad \varepsilon \in [0, n]$$

The second form: $[\mathbf{0}, \mathbf{1} + \boldsymbol{\varepsilon}]$ indeterminacy at the upper limit of the field we find:

$$R_{Ni} \in \left[R_i, \frac{R_i}{1 + n}\right] \quad ; \quad 0 < n < 1$$

It is calculated from the following relation:

$$R_{Ni} = \frac{R_i}{1 + \varepsilon} \quad ; \quad \varepsilon \in [0, n]$$

The third form: $[\mathbf{0} + \boldsymbol{\varepsilon}, \mathbf{1} + \boldsymbol{\varepsilon}]$ Indeterminacy at the upper and lower limits of the field we find:

$$R_{Ni} \in [R_i, R_i - n] \quad ; \quad 0 < n < 1$$

It is calculated from the following relation:

$$R_{Ni} = R_i - \varepsilon \ ; \quad \varepsilon \in [0, n]$$

From each of the previous forms we get the following series of neutrosophic random numbers:

$$R_{N1}, R_{N2}, R_{N3} \dots R_{Nm}, \dots \qquad (3)$$

c. Using the study mentioned in References [21-22], we convert these neutrosophic random numbers into neutrosophic random numbers that follow the exponential distribution defined by the following relation:

$$h(y) = \lambda . e^{-\lambda y}$$

Using the relation:

$$y_{Ni} = -\frac{1}{\lambda} ln R_{Ni}$$

We obtain the series of neutrosophic random numbers that follow the following exponential distribution:

$$y_{N1}, y_{N2}, y_{N3} \dots y_{Nm}, \dots \qquad (4)$$

We apply the accept-reject method given in reference [23]:

We take the cumulative sum of these numbers if the following inequality:

$$\sum_{i=1}^{x} y_{Ni} \leq 1 \pm \varepsilon \leq \sum_{i=1}^{x+1} y_{Ni+1}$$

Then we consider the number x to be subject to the Poisson distribution, where x is the number of random numbers y_{Ni} that are subject to the exponential distribution.

$h(y) = \lambda . e^{-\lambda y}$, the sum of which we took and the number did not exceed $1 \pm \varepsilon$, but if we added another number y_{Ni+1} the sum became greater than $1 \pm \varepsilon$. However, if the inequality is not met, we return to Step (a), we repeat the work until we obtain the required number of random numbers that follow the Poisson distribution.

The second case: classical random numbers and Poisson-Neutrosophic distribution.

The probability density function of the neutrosophic Poisson distribution is defined by the following relationship: [28]

$$f_N(x) = \frac{\lambda_N^x e^{-\lambda_N}}{x!} \quad ; x = 0,1,2, \dots, \infty$$

Where λ_N is a neutrosophic value that expresses the number of expected occurrences in one time, this indicates that the time period between events is exponentially distributed with an average of $\frac{1}{\lambda_N}$.

a. Since classical random numbers follow a uniform distribution over the interval [0,1], we take the sequence of random numbers from relation (2).

b. Using the inverse transformation method, we convert these random numbers into neutrosophic random numbers that follow the neutrosophic exponential distribution defined by the following relation:

$$h_N(y) = \lambda_N . e^{-\lambda_N\, y}$$

c. Using the conversion relation:

$$y'_{Ni} = -\frac{\ln R_i}{\lambda_N}$$

We obtain the series of neutrosophic random numbers that follow the following exponential distribution:

$$y'_{N1}, y'_{N2}, y'_{N3}, \dots, y'_{Nm}, \dots \qquad (4)$$

We take the cumulative sum of these numbers if the following inequality:

$$\sum_{i=1}^{x} y'_{Ni} \leq 1 \pm \varepsilon \leq \sum_{i=1}^{x+1} y'_{Ni+1}$$

Then we consider the number x to be subject to the Poisson distribution, where x is the number of random numbers y'_{Ni} which is subject to the exponential distribution.

$h(y) = \lambda . e^{-\lambda y}$ whose sum we took and the number did not exceed $1 \pm \varepsilon$, but if we added another number y'_{Ni+1} the sum became greater than $1 \pm \varepsilon$, but if the inequality is not satisfied we return to step (a), repeating the work until we obtain the required number of random numbers that follow the Poisson distribution.

The third case: neutrosophic random numbers and neutrosophic probability distribution.

From the study in the first case, we obtain the series of neutrosophic random numbers as in (3).

Poisson Neutrosophic distribution, i.e., the probability density function is defined as it is in the second case. To convert neutrosophic random numbers $R_{N1}, R_{N2}, R_{N3} \dots R_{Nm}, \dots$, into random numbers that follow the exponential distribution, we use the following relation:

$$y''_{Ni} = -\frac{\ln R_{Ni}}{\lambda_N}$$

We obtain the series of neutrosophic random numbers that follow the following exponential distribution:

$$y''_{N1}, y''_{N2}, y''_{N3}, \dots, y''_{Nm}, \dots \qquad (5)$$

We take the cumulative sum of these numbers if the following inequality:

$$\sum_{i=1}^{x} y''_{Ni} \leq 1 \pm \varepsilon \leq \sum_{i=1}^{x+1} y''_{Ni+1}$$

achieved, then we consider the number x to be subject to the Poisson distribution, where x is the number of random numbers y''_{Ni} that obey the neutrosophic exponential distribution $h_N(y) = \lambda_N . e^{-\lambda_N y}$. The sum of which we took and the number did not exceed $1 \pm \varepsilon$, but if we added another number y''_{Ni+1}, the sum became greater than $1 \pm \varepsilon$. However, if the inequality is not met, we return to step (a), and we repeat the work until we obtain the number the required random numbers follow a Poisson distribution.

Conclusion and results:

In this research, we presented a neutrosophic vision of the composition method used to generate random numbers that follow complex probability distributions from simple distributions. Random numbers that follow them can be generated using the neutrosophic inverse transformation or the neutrosophic rejection and acceptance method, using the relations provided by students and researchers in the field of mathematical statistics that link the probability distributions. Complex with simple probability distributions, such as the following relation between the Poisson distribution and the exponential distribution: When the time periods between possible events are exponentially distributed, the number of events that occur in one period of time has a Poisson distribution, which is relied upon to generate neutrosophic random numbers that follow the distribution Poisson, which has many uses in practical life, Such as inventory control, queueing theory, quality control, traffic flow, and many other fields of management science.

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