NSS

Neutrosophic Sets and Systems, Vol. 55, 2023

University of New Mexico

A Total Order on Single Valued and Interval Valued Neutrosophic Triplets

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Abstract. L.A.Zadeh (1965) proposed the concept of fuzzy subsets, which was later expanded to include intuitionistic fuzzy subsets by K.Atanassov (1983). We have come across several generalisations of sets since the birth of fuzzy sets theory, one of which is Florentine Smarandache [15] introduced the neutrosophic sets as a major category. Many real-life decision-making problems have been studied in [10], [13], [16]. In multi-criteria decision making (MCDM) situations [1], [2], [6], the ordering of neutrosophic triplets (T; I; F) is crucial. In this study, we define and analyse new membership, non-membership, and average score functions on single-valued neutrosophic triplets (T; I; F). We create a technique for ordering single valued neutrosophic triplets (SVNT) using these three functions, with the goal of achieving a total ordering on neutrosophic triplets. The total ordering on IVNT is then provided by extending these score functions and ranking mechanism to interval valued neutrosophic triplets (IVNT). A comparison is also made between the suggested method and the present ranking method in the literature.

Keywords: Neutrosophic Sets; Interval Valued Neutrosophic Triplets; MCDM

1. Introduction

Our daily life is filled with uncertain situations that require us to make the best decisions possible given the volatility. Despite this, L.A.Zadeh established the concept of fuzzy sets [18] in 1965 to handle such ambiguity. This idea of fuzzy sets, which claims that available data is not necessarily an accurate value but always contains the hand of uncertainty, was reluctantly acknowledged at the time and that analyzing this uncertainty or vagueness might bring a tremendous revolution in the future with real-life MCDM problems. Later, a great progress has been made in the research of fuzzy set generalisations resulting in numerous forms of fuzzy sets such as intuitionistic fuzzy sets, neutrosophic sets, picture fuzzy sets, bi-polar fuzzy sets, and so on [3], [4], [5], [15], [20]. These different versions of fuzzy sets were widely used in a variety of real-world problems.

More specifically, theory of neutrosophic sets is one of the most growing research areas due to its needfulness in various real life situations like Medical diagnosis [27], Supply chain management [25]. Neutrosophic sets are later expanded to many other research areas like Graph theory [28], Optimization [29], Goal Programming [23]. The MCDM is a rising topic of research due to its importance in most real-world challenges [12], [14], [17], [19]. Some MCDM problems have been studied in real-world scenarios using neutrosophic sets. To solve such MCDM problems, we need total ordering on neutrosophic triplets. For each fuzzy MCDM problem, there are several techniques of total ordering on fuzzy numbers available in the literature [7], [8], [9], [11]. Furthermore, the decision maker selects the total ordering strategy that best suits his needs. The total order does not have to be unique in fuzzy MCDM. Various kinds of MCDM and MADM problems have been studied based on neutrosophic sets in literature [21], [22], [26], [30].

Florentine Smarandache [16] defined three score functions on single-valued neutrosophic triplets based on which a total ordering on single-valued neutrosophic triplets has been proposed and the proposed score functions have been extended to interval valued neutrosophic triplets. But the proposed ordering methods in the literature give total ordering only on Single valued neutrosophic triplets but only neutrosophically total ordering / a partial ordering on Interval valued neutrosophic triplets. There is no total ordering method exists in the literature. To overcome this research gap, we construct a new total ordering on neutrosophic triplets in this study which can be extended to a total ordering method on Interval valued neutrosophic triplets, which contributes a lot to IVNT based MCDM problems.

In section 2, we define some key terms that will help us to comprehend the rest of the work. The proposed ranking method's motivation is presented in section 3. In sections 4 and 5, we introduce new scoring functions and suggest a complete total ordering technique for single-valued neutrosophic triplets based on those functions (T, I, F). The scoring functions and ranking approach in sections 4 and 5 are generalised to interval valued neutrosophic triplets in sections 7 and 8. In section 9, we detail our ranking method's algorithm as well as its comparison to other ranking method in literature. To achieve this we have considered an MCDM problem from [21]. In [21] the Similarity measure based MCDM ranking method have been studied, we inherits the method and modified the given data to our SVNT based MCDM data to compute the ranking throughwhich we compare our method with existing methods. In this section 9 the limitations of existing methods and advantages of our proposed method are discussed.

2. Preliminaries

This section contains all of the necessary definitions to move deeper into the concept of total ordering on neutrosophic triplets.

Definition 2.1. [16] Let $\mathcal{M} = \{(T, I, F), \text{where } T, I, F \in [0, 1], 0 \leq T + I + F \leq 3\}$ be the set of single valued neutrosophic triplet (SVNT) numbers. Let $N = (T, I, F) \in \mathcal{M}$ be a SVNT number, where T denotes grade of membership ; I denotes indeterminacy grade ; F denotes grade of non-membership .

Definition 2.2. [16] A SVNT score $s : \mathcal{M} \to [0, 1]$ is given by

$$s(T, I, F) = \frac{T + (1 - I) + (1 - F)}{3}.$$

A SVNT accuracy score $a : \mathcal{M} \to [-1, 1]$ is given by

$$a(T, I, F) = T - F.$$

A SVNT certainty score $c : \mathcal{M} \to [0, 1]$ is given by

$$c(T, I, F) = T.$$

With the foregoing functions, Smarandache created a total ordering in SVNT [16].

3. Motivation

Let us use the ranking method of [16] for following three neutrosophic triplets $n_1 = (1, 0, 0)$ where $t = 1, i = 0, f = 0$; $n_2 = (0, 1, 0)$ where $t = 0, i = 1, f = 0$ and $n_3 = (0, 0, 1)$ where $t = 0, i = 0, f = 1$.

It is natural to assume that the ranking order is $n_1 > n_2 > n_3$. We normally put full membership first and full non-membership last, since n_1, n_2 and n_3 signify absolute membership, hesitant (which is somewhat of membership and somewhat of non-membership), and absolute non-membership, respectively.

But, according to the ranking method of [16], we get $s(1, 0, 0) = 1$, $s(0, 1, 0) = \frac{1}{3}$, $s(0, 0, 1) = \frac{1}{3}$. Therefore, we get $R(n_1) > R(n_2) = R(n_3)$. So, we go to next step to find ordering between n_2 and n_3. Since $a(0, 1, 0) = 0$, $a(0, 0, 1) = -1$, $R(n_2) > R(n_3)$. Finally, we get the ranking $R(n_1) > R(n_2) > R(n_3)$. In this case, when we intuitively discovered the ranking order, we are unable to rank them using the score function (step 1) alone in the present technique and must rely on the accuracy function (step 2).

We intended to rank these types of triplets using the score function alone, rather than having to move on to the next function. The score function was defined in [16] by summing all the positive quantities ($T, (1 - I)$, and $(1 - F)$) of the triplet (T, I, F), with $1 - I$ and $1 - F$ representing positive triplet quantities. However, various portions of non-indeterminacy

$((1-I)T)$ and $((1-I)F)$ should be recognised. Positive and negative amounts of (T, I, F) may be represented as $1-I$, which is based on positive and negative quantities of neutrosophic information. As a result, we created a new membership score function by combining membership (T) and positive membership quantity from indeterminacy $((1-I)T)$, then subtracting negative membership (F), indeterminacy (I), and positive non-membership quantity from indeterminacy $((1-I)F)$. The proposed new score functions are based on this basic idea and reasoning.

4. Membership, Non-membership and Average score functions on SVNT

Let $\mathcal{M} = \{(T, I, F), \text{where } T, I, F \in [0,1], 0 \leq T + I + F \leq 3\}$. where $T,\ I,\ F$ are single valued. Based on the motivation given in the last paragraph, the following score functions are defined.

Definition 4.1. A SVNT membership score $S^+ : \mathcal{M} \to [0,1]$ is given by

$$S^+(T, I, F) = \frac{2 + T + (1-I)T - F - I - (1-I)F}{4} = \frac{2 + (T-F)(2-I) - I}{4}.$$

Definition 4.2. A SVNT non-membership score $S^- : \mathcal{M} \to [0,1]$ is given by

$$S^-(T, I, F) = \frac{2 + F + (1-I)F - T - I - (1-I)T}{4} = \frac{2 + (F-T)(2-I) - I}{4}.$$

Definition 4.3. A SVNT average score $C : \mathcal{M} \to [0,1]$ is given by

$$C(T, I, F) = \frac{T+F}{2}$$

Definition 4.4. A SVNT indeterminacy score $H : \mathcal{M} \to [0, \frac{1}{2}]$ given by

$$H(T, I, F) = \frac{I}{2}$$

Remark 4.5. We note that $0 \leq S^+ + S^- \leq 1$ because of $S^+ + S^- = 1 - \frac{I}{2}$(which is ≤ 1).

Remark 4.6. We note that $S^+ + S^- + H = 1$, which shows the sum of all membership, non-membership and indeterminacy scores equals to 1.

From the above remark, we note that S^+ and S^- form membership and non-membership functions of IFS with indeterminacy H. So, any neutrosophic set A can be viewed as intuitionistic fuzzy set $IF(A) = (S^+(A), S^-(A))$.

Remark 4.7. When there is no indeterminacy (i.e I=0), we get $S^+ + S^- = 1$, which is the fuzzy form of neutrosophic triplets.

Remark 4.8. As we mentioned in earlier, let us try to rank the following three triplets $n_1 = (1,0,0)$, $n_2 = (0,1,0)$ and $n_3 = (0,0,1)$ When we use membership score, we get $S^+(1,0,0) = 1$, $S^+(0,1,0) = \frac{1}{4}$ and $S^+(0,0,1) = 0$. Thus, we got the ranking as $R(n_1) > R(n_2) > R(n_3)$. As we mentioned in section 3, we have ranked these triplets by using score function itself.

Remark 4.9. We note that we can rank these triplets by using non-membership score as follows $S^-(1,0,0) = 0$, $S^-(0,1,0) = \frac{1}{4}$ and $S^-(0,0,1) = 1$ which gives us $R(1,0,0) > R(0,1,0) > R(0,0,1)$. Thus, again we get $R(n_1) > R(n_2) > R(n_3)$ by using non-membership score only.

5. A total order on SVNT

In this section, we present a new ranking technique for SVNT that preserves total ordering.

5.1. *New Ranking Algorithm for SVNT*

Let $A = (a,b,c)$ and $B = (d,e,f)$ be two SVNT of $\mathcal{M}$, where $T(A) = a$, $I(A) = b$, $F(A) = c$; $T(B) = d$, $I(B) = e$, $F(B) = f$ and $a,b,c,d,e,f \in [0,1]$.

Step 1: Apply proposed new neutrosophic membership score function S^+.

(1) If $S^+(a,b,c) > S^+(d,e,f)$ $(S^+(a,b,c) < S^+(d,e,f))$, then $(a,b,c) > (d,e,f)$ $((a,b,c) < (d,e,f))$.

(2) Suppose $S^+(a,b,c) = S^+(d,e,f)$, go to step 2.

Step 2: Apply proposed new neutrosophic non-membership score function S^-.

(1) If $S^-(a,b,c) > S^-(d,e,f)$ $(S^-(a,b,c) < S^-(d,e,f))$, then $(a,b,c) < (d,e,f)$ $((a,b,c) > (d,e,f))$.

(2) Suppose $S^-(a,b,c) = S^-(d,e,f)$, go to step 3.

Step 3: Apply proposed new neutrosophic average function C.

(1) If $C(a,b,c) > C(d,e,f)$ $(C(a,b,c) < C(d,e,f))$, then $(a,b,c) > (d,e,f)$ $((a,b,c) < (d,e,f))$.

(2) Suppose $C(a,b,c) = C(d,e,f)$, then conclude that $(a,b,c) \equiv (d,e,f)$.

Theorem 5.1. *A total order on $\mathcal{M}$ is formed by the single-valued neutrosophic membership, non-membership, and average score functions.*

Proof. Let $n_1 = (t_1, i_1, f_1)$ and $n_2 = (t_2, i_2, f_2)$ be two SVNT of $\mathcal{M}$. We show that for any two SVNT n_1 and n_2 in $\mathcal{M}$, either $n_1 < n_2$ or $n_1 > n_2$ or $n_1 = n_2$. First we apply membership score function S^+. Suppose $S^+(n_1) > S^+(n_2)$(or $S^+(n_1) < S^+(n_2)$), then we have $n_1 > n_2$(or $n_1 < n_2$), which is done. When $S^+(n_1) = S^+(n_2)$, we have to go to step 2. So, Suppose $\frac{2+(t_1-f_1)(2-i_1)-i_1}{4} = \frac{2+(t_2-f_2)(2-i_2)-i_2}{4}$, equivalently, if $(t_1-f_1)(2-i_1)-i_1 = (t_2-f_2)(2-i_2)-i_2$, we apply step 2 using non-membership score. Hence, if $S^-(n_1) > S^-(n_2)$ $(S^-(n_1) < S^-(n_2))$, then $n_1 < n_2 (n_1 > n_2)$, which is done. When $S^-(n_1) = S^-(n_2)$, equivalently, if $(f_1 - t_1)(2 - i_1) - i_1 = (f_2 - t_2)(2 - i_2) - i_2$, we have to go to step 3 using average score function. Hence, suppose $C(n_1) > C(n_2)$ (or $C(n_1) < C(n_2)$), then we have $n_1 > n_2$ (or $n_1 < n_2$), which is done. When $C(n_1) = C(n_2)$, we have $t_1 + f_1 = t_2 + f_2$. At this stage, we have triplets n_1 and n_2 satisfying following system of 3 equations.

$$(t_1 - f_1)(2 - i_1) - i_1 = (t_2 - f_2)(2 - i_2) - i_2 \tag{1}$$

$$(f_1 - t_1)(2 - i_1) - i_1 = (f_2 - t_2)(2 - i_2) - i_2 \tag{2}$$

$$t_1 + f_1 = t_2 + f_2 \tag{3}$$

Now, we solve this system of equations. By adding equations 1 and 2, we get $i_1 = i_2$ which makes equation 1 into

$$t_1 - f_1 = t_2 - f_2$$

now, by adding the above equation with equation 3, we get $f_1 = f_2$ and $t_1 = t_2$.
Thus, we get

$$(t_1, i_1, f_1) = (t_2, i_2, f_2).$$

As a result, we infer that any two SVNT are either bigger than the other or identical. As a result, we have established a total ordering on $\mathcal{M}$. □

The following statement's proofs are direct applications of definitions, hence proofs are omitted.

Proposition 5.2. *Let $n_1 = (t_1, i_1, f_1)$ and $n_2 = (t_1, i_2, f_1)$*

(1) *If $i_1 > i_2$, then $R(n_1) < R(n_2)$.*
(2) *If $i_1 < i_2$, then $R(n_1) > R(n_2)$.*

Proposition 5.3. *Let $n_1 = (t_1, i_1, f_1)$ and $n_2 = (t_1, i_1, f_2)$*

(1) *If $f_1 > f_2$, then $R(n_1) < R(n_2)$.*
(2) *If $f_1 < f_2$, then $R(n_1) > R(n_2)$.*

Proposition 5.4. *Let $n_1 = (t_1, i_1, f_1)$ and $n_2 = (t_2, i_1, f_1)$*

(1) *If $t_1 > t_2$, then $R(n_1) > R(n_2)$.*
(2) *If $t_1 < t_2$, then $R(n_1) < R(n_2)$.*

Remark 5.5. Let $n_1 = (t, 0, f)$ and $n_2 = (t, 1, f)$ in which n_1 and n_2 have same membership and non-membership grades with n_1 has no indeterminacy and n_2 has full indeterminacy. Then $R(n_1) > R(n_2)$ which favors our intuition.

Remark 5.6. Let $n_1 = (t_1, i_1, f_1)$ and $n_2 = (t_2, i_1, f_2)$ i.e., indeterminacy of n_1 = indeterminacy of n_2. If $S^+(n_1) > S^+(n_2)$, then $S^-(n_1) < S^-(n_2)$ which is more logical.

6. **Equivalence of proposed method over existing ranking method**

In this section, we examine the new approach's equivalency to the existing ranking algorithm [16].

Remark 6.1. Our proposed Algorithm and Florentin Smarandache [16] Algorithm for ranking Neutrosophic Triplets are same when the triplets have same indeterminacy value.

Proof. Let $n_1 = (t_1, i_1, f_1)$ and $n_2 = (t_2, i_1, f_2)$ be two SVNT of $\mathcal{M}$, where n_1 and n_2 have same indeterminacy value i_1. In [16], if $s(n_1) > s(n_2)$, then $(n_1) > (n_2)$.
Now, $s(n_1) > s(n_2) \Leftrightarrow \frac{2+t_1-f_1-i_1}{3} > \frac{2+t_2-f_2-i_1}{3} \Leftrightarrow t_1 - f_1 > t_2 - f_2 \Leftrightarrow \frac{2+(t_1-f_1)(2-i_1)-i_1}{4} > \frac{2+(t_2-f_2)(2-i_1)-i_1}{4} \Leftrightarrow S^+(n_1) > S^+(n_2)$. Similarly, $s(n_1) < s(n_2) \Leftrightarrow S^+(n_1) < S^+(n_2)$. Further, $s(n_1) = s(n_2) \Leftrightarrow S^+(n_1) = S^(n_2)$.
Hence, ranking by membership score function by s in [16] is same as ranking by proposed neutrosophic membership score S^+.

By similar argument, we have $a(n_1) > a(n_2) \Leftrightarrow S^-(n_1) < S^-(n_2)$ and $a(n_1) = a(n_2) \Leftrightarrow S^-(n_1) = S^-(n_2)$. Hence, ranking by membership score function by a in [16] is same as ranking by proposed neutrosophic membership score S^-.

Now, we prove that $c(n_1) > c(n_2) \Leftrightarrow C(n_1) > C(n_2)$, $c(n_1) < c(n_2) \Leftrightarrow C(n_1) < C(n_2)$ and $c(n_1) = c(n_2) \Leftrightarrow C(n_1) = C(n_2)$ if $s(n_1) = s(n_2)$ and $a(n_1) = a(n_2)$ (and hence $S^+(n_1) = S^+(n_2)$, $S^-(n_1) = S^-(n_2)$. If $s(n_1) = s(n_2)$ and $a(n_1) = a(n_2)$, then $t_1 - f_1 = t_2 - f_2$. Now, $c(n_1) > c(n_2) \Leftrightarrow t_1 > t_2 \Leftrightarrow f_1 > f_2$ using $t_1 - f_1 = t_2 - f_2) \Leftrightarrow \frac{t_1+f_1}{2} > \frac{t_2+f_2}{2} \Leftrightarrow C(n_1) > C(n_2)$. Similarly, $c(n_1) < c(n_2) \Leftrightarrow C(n_1) < C(n_2)$. Further, $c(n_1) = c(n_2) \Leftrightarrow C(n_1) = C(n_2)$ if $s(n_1) = s(n_2)$ and $a(n_1) = a(n_2)$ (and hence $S^+(n_1) = S^+(n_2)$, $S^-(n_1) = S^-(n_2)$). As a result, if triplets share the same indeterminacy, ranking by membership score function in [16] is the same as ranking by proposed neutrosophic membership score function. □

The proof of the following remarks are immediate applications of definitions, hence they are omitted.

Remark 6.2. Let $n_1 = (t_1, i_1, f_1)$ and $n_2 = (t_2, i_2, f_2)$ be two SVNT. When $(t_1 - f_1) > (t_2 - f_2)$ and $i_1 < i_2$, our suggested Algorithm for ranking Neutrosophic Triplets (T, I, F) and Florentin Smarandache's [16] Algorithm for ranking Neutrosophic Triplets are the same.

Remark 6.3. Our proposed Algorithm and Florentin Smarandache [16] Algorithm for ranking of SVNT (T, I, F) are ranking in a same manner when the difference between membership and non-membership values $(T - F)$ of triplets (T, I, F) have same value.

7. Membership, Non-Membership and Average score functions on IVNT

The algorithm for ranking SVNT is expanded to IVNT in this section. We begin by discussing score functions for neutrosophic triplets with interval values.

Definition 7.1. Let $\mathcal{M}_{int} = \{(T, I, F), \text{where } T, I, F \text{ are closed subsets of } [0, 1]\}$ be the set of IVNT. Let $N = (T, I, F) \in \mathcal{M}_{int}$ be a IVNT number. Here $T^L = inf\ T$ and $T^U = sup\ T$; $I^L = inf\ I$ and $I^U = sup\ I$; $F^L = inf\ F$ and $F^U = sup\ F$; where $T^L, T^U, I^L, I^U, F^L, F^U \in [0, 1]$ with $T^L < T^U, I^L < I^U, F^L < F^U$. Then neutrosophic triplet N is of the form $([T^L, T^U], [I^L, I^U], [F^L, F^U])$

Definition 7.2. If two intervals $[a, b]$ and $[c, d]$ have same midpoint, then they are said to be neutrosophically equal and are indicated as $[a, b] =_N [c, d]$.

Definition 7.3. An IVNT membership score function $S^+ : \mathcal{M}_{int} \to [0, 1]$ is defined by

$$S^+(T, I, F) = \frac{8 + (T^L + T^U - F^L - F^U)(4 - I^L - I^U) - 2(I^L + I^U)}{12}.$$

Definition 7.4. An IVNT non-membership score function $S^- : \mathcal{M}_{int} \to [0, 1]$ is defined by

$$S^-(T, I, F) = \frac{8 + (F^L + F^U - T^L - T^U)(4 - I^L - I^U) - 2(I^L + I^U)}{12}.$$

Definition 7.5. An IVNT average score function $C : \mathcal{M}_{int} \to [0, 1]$ is defined by

$$C(T, I, F) = \frac{T^L + T^U + F^L + F^U}{4}.$$

We now provide a new technique for ranking neutrosophic triplets with interval values.

8. A total order on IVNT

In this section, we introduce score functions through which a new algorithm for total ordering on interval valued neutrosophic triplets is aimed.

8.1. *Ranking algorithm on IVNT*

Let $A = ([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$ and $B = ([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$ be two interval valued neutrosophic triplets of $\mathcal{M}_{int}$. Now, by applying the following algorithm, we can rank any two numbers as either one is bigger than other or both are neutrosophically equal.

Step 1: Apply our New Neutrosophic Membership score function S^+.

(1) If $S^+(A) > S^+(B)$ $(S^+(A) < S^+(B))$, then $R(A) > R(B)$ $(R(A) < R(B))$.

(2) Suppose $S^+(A) = S^+(B)$, we go to step 2.

Step 2: Apply our New Neutrosophic non-membership score function S^-.

(1) If $S^-(A) > S^-(B)$ $(S^-(A) < S^-(B))$, then $R(A) < R(B)$ $(R(A) > R(B))$.

(2) Suppose $S^-(A) = S^-(B)$, we go to step 3.

Step 3: Apply our New Neutrosophic Average function C.
(1) If $C(A) > C(B)$ $(C(A) < C(B))$, then $R(A) > R(B)(R(A) < R(B))$.
(2) Suppose $C(A) = C(B)$, then conclude that $([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U]) =_N ([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$. So A and B are neutrosophically equal.

Theorem 8.1. *We prove that the interval valued neutrosophic membership, interval valued neutrosophic non-membership and average score functions together form a neutrosophically total ordering on M, that is either they are greater(lesser) than other numbers or neutrosophically equal.*

Proof. Now, we prove for any two interval valued neutrosophic triplets $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U])$ and $([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$,
either $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) > ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$
or $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) < ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$
or $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) =_N ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$.

Let$([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U])$ and $([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ be two interval valued neutrosophic triplets. First, we apply membership score function S^+.
If $S^+ ([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) > S^+ ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$, then $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) > ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ which is done.
If $S^+ ([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) < S^+ ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$, then $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) < ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ which is also done.
But, when we get the equality

$$S^+([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) = S^+([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U]),$$

we have,

$$\frac{8 + (t_1^L + t_1^U - f_1^L - f_1^U)(4 - i_1^L - i_1^U) - 2(i_1^L + i_1^U)}{12} = \frac{8 + (t_2^L + t_2^U - f_2^L - f_2^U)(4 - i_2^L - i_2^U) - 2(i_2^L + i_2^U)}{12}$$

$\Leftrightarrow$

$$(t_1^L + t_1^U - f_1^L - f_1^U)(4 - i_1^L - i_1^U) - 2(i_1^L + i_1^U) = (t_2^L + t_2^U - f_2^L - f_2^U)(4 - i_2^L - i_2^U) - 2(i_2^L + i_2^U). \tag{4}$$

So, next we go for non-membership score function S^-.
If $S^- ([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) > S^- ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$, then $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) < ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ which is done. If $S^- ([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) < S^- ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$, then $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) > ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ which is also done.

But, when we get the equality

$$S^-([t_1^L,t_1^U],[i_1^L,i_1^U],[f_1^L,f_1^U]) = S^-([t_2^L,t_2^U],[i_2^L,i_2^U],[f_2^L,f_2^U]),$$

we have,

$$\frac{8+(f_1^L+f_1^U-t_1^L-t_1^U)(4-i_1^L-i_1^U)-2(i_1^L+i_1^U)}{12} = \frac{8+(f_2^L+f_2^U-t_2^L-t_2^U)(4-i_2^L-i_2^U)-2(i_2^L+i_2^U)}{12}$$

$\Leftrightarrow$

$$(f_1^L+f_1^U-t_1^L-t_1^U)(4-i_1^L-i_1^U)-2(i_1^L+i_1^U) = (f_2^L+f_2^U-t_2^L-t_2^U)(4-i_2^L-i_2^U)-2(i_2^L+i_2^U). \tag{5}$$

So, we next go for average score function. If C $([t_1^L,t_1^U],[i_1^L,i_1^U],[f_1^L,f_1^U]) > C$ $([t_2^L,t_2^U],[i_2^L,i_2^U],[f_2^L,f_2^U])$, then $([t_1^L,t_1^U],[i_1^L,i_1^U],[f_1^L,f_1^U]) > ([t_2^L,t_2^U],[i_2^L,i_2^U],[f_2^L,f_2^U])$ which is done.
If C $([t_1^L,t_1^U],[i_1^L,i_1^U],[f_1^L,f_1^U]) < C$ $([t_2^L,t_2^U],[i_2^L,i_2^U],[f_2^L,f_2^U])$, then $([t_1^L,t_1^U],[i_1^L,i_1^U],[f_1^L,f_1^U])$ $< ([t_2^L,t_2^U],[i_2^L,i_2^U],[f_2^L,f_2^U])$ which is also done.
But, when we get the equality C $([t_1^L,t_1^U],[i_1^L,i_1^U],[f_1^L,f_1^U]) = C$ $([t_2^L,t_2^U],[i_2^L,i_2^U],[f_2^L,f_2^U])$, we have

$$t_1^L+t_1^U+f_1^L+f_1^U = t_2^L+t_2^U+f_2^L+f_2^U. \tag{6}$$

If these triplets would have not ranked till now, then we have triplets $([t_1^L,t_1^U],[i_1^L,i_1^U],[f_1^L,f_1^U])$ and $([t_2^L,t_2^U],[i_2^L,i_2^U],[f_2^L,f_2^U])$ satisfying following system of 3 equations (from the equations 4, 5 and 6).

$$\begin{cases}(t_1^L+t_1^U-f_1^L-f_1^U)(4-i_1^L-i_1^U)-2(i_1^L+i_1^U) & = (t_2^L+t_2^U-f_2^L-f_2^U)(4-i_2^L-i_2^U)-2(i_2^L+i_2^U)\\ (f_1^L+f_1^U-t_1^L-t_1^U)(4-i_1^L-i_1^U)-2(i_1^L+i_1^U) & = (f_2^L+f_2^U-t_2^L-t_2^U)(4-i_2^L-i_2^U)-2(i_2^L+i_2^U)\\ t_1^L+t_1^U+f_1^L+f_1^U & = t_2^L+t_2^U+f_2^L+f_2^U\end{cases}$$

By adding equations 4 and 5, we get $i_1^L+i_1^U = i_2^L+i_2^U$ which makes equation 4 into

$$t_1^L+t_1^U-f_1^L-f_1^U = t_2^L+t_2^U-f_2^L-f_2^U.$$

Now, by adding the above equation with equation 6, we get $t_1^L+t_1^U = t_2^L+t_2^U$ and hence we get $f_1^L+f_1^U = f_2^L+f_2^U$.
Thus, the system of 3 equations become

$$\begin{cases}t_1^L+t_1^U & = t_2^L+t_2^U\\ i_1^L+i_1^U & = i_2^L+i_2^U\\ f_1^L+f_1^U & = f_2^L+f_2^U\end{cases}$$

$\Longrightarrow$

$$\begin{cases} \frac{t_1^L+t_1^U}{2} & = \frac{t_2^L+t_2^U}{2} \\ \frac{i_1^L+i_1^U}{2} & = \frac{i_2^L+i_2^U}{2} \\ \frac{f_1^L+f_1^U}{2} & = \frac{f_2^L+f_2^U}{2} \end{cases}$$

Hence intervals $[t_1^L, t_1^U]$ and $[t_2^L, t_2^U]$, $[i_1^L, i_1^U]$ and $[i_2^L, i_2^U]$, $[f_1^L, f_1^U]$ and $[f_2^L, f_2^U]$ are neutrosophically equal.

Therefore

$$([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) =_N ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U]).$$

We can therefore conclude that for any interval-valued neutrosophic triplets, one is larger than the other or both are neutrosophically equal..

Note: We did not extract the total ordering between interval valued neutrosophic triplets using our ranking algorithm. Take $A = ([0.2, 0.8], [0.1, 0.3], [0.2, 0.4])$ and $B = ([0.4, 0.6], [0, 0.4], [0.1, 0.5])$ as examples. We find A and B are neutrosophically equal using the preceding procedure, but they are not the same interval valued neutrosophic triplets. As a result, we will add furthermore three score functions along with membership, non-membership, and average score functions, to produce total ordering on neutrosophic interval valued numbers. □

8.2. *Total ordering on IVNT*

As we mentioned, we derive three new score functions through which the total ordering on interval valued neutrosophic triplets is achieved.

Definition 8.2. An IVNT positive range score function $S'^{+} : \mathcal{M}_{int} \to [0, 1]$ is defined by

$$S'^{+}(T, I, F) = \frac{8 + (T^U - T^L - F^U + F^L)(4 - I^U + I^L) - 2(I^U - I^L)}{12}.$$

Definition 8.3. An IVNT negative range score function $S'^{-} : \mathcal{M}_{int} \to [0, 1]$ is defined by

$$S'^{-}(T, I, F) = \frac{8 + (F^U - F^L - T^U + T^L)(4 - I^U + I^L) - 2(I^U - I^L)}{12}.$$

Definition 8.4. An IVNT average range score function $C' : \mathcal{M}_{int} \to [0, 1]$ is defined by

$$C'(T, I, F) = \frac{T^U - T^L + F^U - F^L}{4}.$$

Now, we introduce new algorithm for total ordering the interval valued neutrosophic triplets.

8.3. *Total ordering algorithm on IVNT*

Let $A = ([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$ and $B = ([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$ be two interval valued neutrosophic triplets of $\mathcal{M}_{int}$. Now, by applying the following algorithm, we derive a total ordering.

Step 1: Apply our interval valued neutrosophic membership score function S^+.

(1) If $S^+(A) > S^+(B)$ $(S^+(A) < S^+(B))$, then $R(A) > R(B)$ $(R(A) < R(B))$.

(2) Suppose $S^+(A) = S^+(B)$, we go to step 2.

Step 2: Apply our interval valued neutrosophic non-membership score function S^-.

(1) If $S^-(A) > S^-(B)$ $(S^-(A) < S^-(B))$, then $R(A) < R(B)$ $(R(A) > R(B))$.

(2) Suppose $S^-(A) = S^-(B)$, we go to step 3.

Step 3: Apply our interval valued neutrosophic average function C.

(1) If $C(A) > C(B)$ $(C(A) < C(B))$, then $R(A) > R(B)$ $(R(A) < R(B))$.

(2) Suppose $C(A) = C(B)$, then we go to step 4.

Step 4: Apply our interval valued neutrosophic positive range score function S'^+.

(1) If $S'^+(A) > S'^+$ (B) $(S'^+(A) < S'^+(B))$, then $R(A) > R(B)$ $(R(A) < R(B))$.

(2) Suppose $S'^+(A) = S'^+(B)$, we go to step 5.

Step 5: Apply our interval valued neutrosophic negative range score function S'^-.

(1) If $S'^-(A) > S'^-(B)$ $(S'^-(A) < S'^-(B))$, then $R(A) < R(B)$ $(R(A) > R(B))$.

(2) Suppose $S'^-(A) = S'^-(B)$, we go to step 6.

Step 6: Apply our interval valued neutrosophic average range score function C'.

(1) If $C'(A) > C'(B)$ $(C'(A) < C'(B))$, then $R(A) > R(B)$ $(R(A) < R(B))$.

(2) Suppose $C'(A) = C'(B)$, then we can conclude that $A = B$.

Theorem 8.5. *We prove that given algorithm preserves total ordering on interval valued neutrosophic triplets*

Proof. We prove for any two interval valued neutrosophic triplets $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U])$ and $([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$, either $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) > ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ or $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) < ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ or $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U])$ and $([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ are same.

Let $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U])$ and $([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ be two interval valued neutrosophic triplets. By applying step 1, step 2 and step 3, if we would get either $R(A) > R(B)$ or $R(A) < R(B)$, then we are done. Suppose, we get $S^+([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U])) = S^+([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$, $S^-([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U])) = S^-([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ and $C([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U])) = C([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$, then we have $t_1^L + t_1^U = t_2^L + t_2^U$,

$i_1^L + i_1^U = i_2^L + i_2^U$ and $f_1^L + f_1^U = f_2^L + f_2^U$ by 8.1. Now we go to step 4.

Further we apply interval valued neutrosophic positive range score function S'^{+}. If S'^{+} $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) > S'^{+}$ $([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$, then $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U])$ $> ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ which is done. If S'^{+} $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) < S'^{+}$ $([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$, then $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) < ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ which is also done.

But, when we get the equality

$$S'^{+}([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) = S'^{+}([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U]),$$

we have,

$$\frac{8 + (t_1^U - t_1^L - f_1^U + f_1^L)(4 - i_1^U + i_1^L) - 2(i_1^U - i_1^L)}{12} = \frac{8 + (t_2^U - t_2^L - f_2^U + f_2^L)(4 - i_2^U + i_2^L) - 2(i_2^U - i_2^L)}{12}$$

$\Leftrightarrow$

$$(t_1^U - t_1^L - f_1^U + f_1^L)(4 - i_1^U + i_1^L) - 2(i_1^U - i_1^L) = (t_2^U - t_2^L - f_2^U + f_2^L)(4 - i_2^U - i_2^L) - 2(i_2^U - i_2^L). \tag{7}$$

So, next, we go for interval valued neutrosophic negative range score function S'^{-}. If S'^{-} $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) > S'^{-}$ $([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$, then $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U])$ $< ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ which is done. If S'^{-} $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) < S'^{-}$ $([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$, then $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) > ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ which is also done.

But, when we get the equality

$$S'^{-}([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) = S'^{-}([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U]),$$

we have,

$$\frac{8 + (f_1^U - f_1^L - t_1^U + t_1^L)(4 - i_1^U + i_1^L) - 2(i_1^U - i_1^L)}{12} = \frac{8 + (f_2^U - f_2^L - t_2^U + t_2^L)(4 - i_2^U + i_2^L) - 2(i_2^U - i_2^L)}{12}$$

$\Leftrightarrow$

$$(f_1^U - f_1^L - t_1^U + t_1^L)(4 - i_1^U + i_1^L) - 2(i_1^U - i_1^L) = (f_2^U - f_2^L - t_2^U + t_2^L)(4 - i_2^U + i_2^L) - 2(i_2^U - i_2^L). \tag{8}$$

So we next go for average range score function. If C' $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U])$ $> C'$ $([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$, then $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) > ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ which is done. If C' $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) < C'$ $([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$, then

$([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) < ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ which is also done. But, when we get the equality C' $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) = C'$ $([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$, we have

$$t_1^U - t_1^L + f_1^U - f_1^L = t_2^U - t_2^L + f_2^U - f_2^L. \tag{9}$$

If these triplets would have not ranked till now, then we have triplets $([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U])$ and $([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U])$ satisfying following system of 6 equations (From the equations 4, 5, 6, 7, 8 and 9)

$$\begin{cases} (t_1^L + t_1^U - f_1^L - f_1^U)(4 - i_1^L - i_1^U) - 2(i_1^L + i_1^U) & = (t_2^L + t_2^U - f_2^L - f_2^U)(4 - i_2^L - i_2^U) - 2(i_2^L + i_2^U) \\ (f_1^L + f_1^U - t_1^L - t_1^U)(4 - i_1^L - i_1^U) - 2(i_1^L + i_1^U) & = (f_2^L + f_2^U - t_2^L - t_2^U)(4 - i_2^L - i_2^U) - 2(i_2^L + i_2^U) \\ t_1^L + t_1^U + f_1^L + f_1^U & = t_2^L + t_2^U + f_2^L + f_2^U \\ (t_1^U - t_1^L - f_1^U + f_1^L)(4 - i_1^U + i_1^L) - 2(i_1^U - i_1^L) & = (t_2^U - t_2^L - f_2^U + f_2^L)(4 - i_2^U - i_2^L) - 2(i_2^U - i_2^L) \\ (f_1^U - f_1^L - t_1^U + t_1^L)(4 - i_1^U + i_1^L) - 2(i_1^U - i_1^L) & = (f_2^U - f_2^L - t_2^U + t_2^L)(4 - i_2^U + i_2^L) - 2(i_2^U - i_2^L) \\ t_1^U - t_1^L + f_1^U - f_1^L & = t_2^U - t_2^L + f_2^U - f_2^L \end{cases}$$

Now we solve these system of equations. By adding equations 7 and 8, we get $i_1^U - i_1^L = i_2^U - i_2^L$, which makes equation 7 into

$$t_1^U - t_1^L - f_1^U + f_1^L = t_2^U - t_2^L - f_2^U + f_2^L.$$

Now, by adding the above equation with equation 9, we get $t_1^U - t_1^L = t_2^U - t_2^L$ and by substituting in the above equation, we get $f_1^U - f_1^L = f_2^U - f_2^L$.
Thus the system of 6 equations become

$$\begin{cases} t_1^L + t_1^U & = t_2^L + t_2^U \\ i_1^L + i_1^U & = i_2^L + i_2^U \\ f_1^L + f_1^U & = f_2^L + f_2^U \\ i_1^U - i_1^L & = i_2^U - i_2^L \\ t_1^U - t_1^L & = t_2^U - t_2^L \\ f_1^U - f_1^L & = f_2^U - f_2^L \end{cases}$$

By solving the above system of equations, we get $t_1^L = t_2^L; t_1^U = t_2^U; i_1^L = i_2^L; i_1^U = i_2^U; f_1^L = f_2^L; f_1^U = f_2^U$. Therefore

$$([t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U]) = ([t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U]).$$

We can therefore conclude that for any interval-valued neutrosophic triplet, either one is larger than the other or both are equal. Alternatively, we have demonstrated that our technique achieves total ordering on neutrosophic triplets with interval values. □

TABLE 1. Assessment of companies corresponding to the criteria. [21]

	Risk	**Availability of raw material**	**Availability of labor**	**Market demand**	**Production quantity**
Automobile company(I_1)	(0.7,0.4,0.3)	(0.27,0.4,0.7)	(0.5,0.1,0.2)	(0.5,0.9,0.4)	(0.1,0.3,0.8)
Food company(I_2)	(0.5,0.8,0.2)	(0.15,0.36,0.78)	(0.9,0,0.2)	(0.6,0.96,0.45)	(0.3,0.5,0.75)
Electronics company(I_3)	(0.9,0.1,0.1)	(0.3,0.6,0.9)	(0.25,0.4,0.5)	(0.72,0.85,0.3)	(0.3,0.45,0.87)
Oil company (I_4)	(0.8,0.6,0.3)	(0.1,0.8,0.2)	(1,0.5,0)	(0.57,0.8,0.35)	(0.1,0.8,0.6)
Parmacutical company(I_5)	(0.65,0.2,0.8)	(0.2,0.45,0.65)	(0.7,0.4,0.6)	(0.4,0.7,0.6)	(0.7,0.2,0.3)

9. Results and Discussions

9.1. *Comparative study with existing methods*

In this section, we consider the same Neutrosophic set based MCDM problem given in [21]. The total ordering methods given in [16], [21] and our proposed method are applied for this MCDM problem. Then, we analyze and compare the ranking order obtained according to each method.

MCDM Problem [21]: Consider an investor who wants to invest into a business. The investor has initially chosen five companies from which one is chosen based on a number of variables which include risk (c_1), raw material availability (c_2), labor availability (c_3), market demand (c_4), and production quantity (c_5). Let us denote those five businesses as automobile company (I_1), food manufacturing (I_2), electronic manufacturing (I_3), oil (I_4), and pharmaceutical (I_5). Table 1 shows the single-valued neutrosophic fuzzy values of each company with respect to each criterion. Since SVNT are denoted as (μ, T, I, F) in [21] and we denote SVNT as (T, I, F), the table given in [21] and the similarity measure method used in [21] have been modified accordingly by applying $\mu = 1$,

When we use first similarity method based ordering method for SVNT in [21], we get $S_1(a^*, I_1) = 0.82, S_1(a^*, I_2) = 0.80, S_1(a^*, I_3) = 0.80, S_1(a^*, I_4) = 0.81, S_1(a^*, I_5) = 0.82$ which gives a ranking $I_5 = I_1 > I_4 > I_2 = I_3$, that leads to a state that not able to make a concrete decision.

When we use second similarity method based ordering method for SVNT in [21], we get $S_1(a^*, I_1) = 0.71, S_1(a^*, I_2) = 0.70, S_1(a^*, I_3) = 0.67, S_1(a^*, I_4) = 0.69, S_1(a^*, I_5) = 0.71$ which gives a ranking $I_5 = I_1 > I_2 > I_4 > I_3$, that again leads to a same problem.

TABLE 2. Score values of companies corresponding to the criteria. [16]

	Risk	Availability of raw material	Availability of labor	Market demand	Production quantity
Automobile company(I_1)	0.66	0.39	0.73	0.4	0.33
Food company(I_2)	0.5	0.34	0.9	0.4	0.35
Electronics company(I_3)	0.9	0.27	0.45	0.52	0.33
Oil company (I_4)	0.63	0.37	0.83	0.47	0.23
Parmacutical company(I_5)	0.55	0.37	0.56	0.37	0.73

TABLE 3. Score values(S^+) of companies corresponding to the criteria.

	Risk	Availability of raw material	Availability of labor	Market demand	Production quantity
Automobile company(I_1)	0.55	0.28	0.63	0.24	0.24
Food company(I_2)	0.32	0.24	0.86	0.25	0.25
Electronics company(I_3)	0.86	0.19	0.43	0.37	0.46
Oil company (I_4)	0.51	0.14	0.75	0.32	0.11
Parmacutical company(I_5)	0.44	0.25	0.46	0.25	0.64

Now we are going to compute this problem by method in [16] .

From table 2, the aggregated score values for $I_1 = 0.50, I_2 = 0.50, I_3 = 0.49, I_4 = 0.51, I_5 = 0.51$. Hence, $I_5 = I_4 > I_1 = I_2 > I_3$. To differentiate I_5 and I_4, we go for accuracy scores of $I_5 = 0.14, I_4 = -0.186$, which gives $I_5 > I_4$. Similarly accuracy score of $I_1 = -0.06, I_2 = -0.034$, which gives $I_2 > I_1$. Now we get an ordering $I_5 > I_4 > I_2 > I_1 > I_3$.
Now we compute the same problem by the proposed total ordering method.

From Table 3, we get scores of $I_1 = 0.39, I_2 = 0.38, I_3 = 0.46, I_4 = 0.36, I_5 = 0.41$. Therefore the ranking order will be as $I_3 > I_5 > I_1 > I_2 > I_4$.
By the above results, we came to know that the existing method in [21] may not be helpful in

some MCDM situations and the existing method [16] and our proposed method give ranking order which may not be coincide. From the above discussions, we can conclude that our proposed method and total ordering method in [16] are better than the existing similarity based ranking method [21]. Now, in the next subsection, we are analyzing the limitations of existing total ordering [16] and advantages of our proposed total ordering method.

9.2. *Limitations of existing method and advantage of our proposed method*

Let us compare existing method [16] and proposed method with an basic example as follows. Consider two neutrosophic triplets $A = (0.9, 0.5, 0.3)$ and $B = (0.8, 0.6, 0.1)$. By Smarandache method [16], we have $s(A) = 0.7$ and $s(B) = 0.7$. Thus, we get $s(A) = s(B)$. So, we go to next score $a(A) = 0.6$ and $a(B) = 0.7$ so we get $A < B$. But, in proposed method, we have $S^+(A) = 0.65$ and $S^+(B) = 0.595$. Thus, we get $A > B$. Here our ranking is different from existing ranking and we can find ranking in less steps compared to existing method. Consider $A = (0.5, 0.3, 0.2)$ and $B = (0, 0, 0)$, by Smarandache method [16], we have $s(A) = 0.667$ and $s(B) = 0.667$. Thus, we get $s(A) = s(B)$. So, we go to next score $a(A) = 0.3$ and $a(B) = 0$ and therefore we get $A > B$. But, in proposed method, we have $S^+(A) = 0.54$ and $S^+(B) = 0.5$. Thus we get $A > B$. Now proposed ranking is same from existing ranking and we can find ranking in less steps compared to existing method.

Consider $A = (0.5, 0.2, 0.3)$ and $B = (0.4, 0.2, 0.2)$. By Smarandache method [16], we have $s(A) = 0.667$ and $s(B) = 0.667$. Thus we get $s(A) = s(B)$. So, we go to next score $a(A) = 0.2$ and $a(B) = 0.2$ and therefore we get $a(A) = a(B)$. Since still we are unable to rank A and B, so we are going to next score $c(A) = 0.5$ and $c(B) = 0.4$. so we get $A > B$.
But, in proposed method, we have $S^+(A) = 0.54$ and $S^+(B) = 0.54$, Thus we get $S^+(A) = S^+(B)$. So, we go to next score $S^-(A) = 0.36$ and $S^-(B) = 0.36$. We go for next score $C(A) = 0.4$ and $C(B) = 0.3$. Thus, we get $A > B$, in this example both the method needs all three score functions and both ranking were same. These are some of examples to understand that both the ranking method may need not to be similar for single valued neutrosophic triplets. Consider the example given in the previous section. Let $A = ([0.2, 0.8], [0.1, 0.3], [0.2, 0.4])$ and $B = ([0.4, 0.6], [0, 0.4], [0.1, 0.5])$. By using the existing algorithm, we get A and B are neutrosophically equal. But when we apply our method, we get $S'^+(A) = 0.76$, $S'^+(B) = 0.536$. Hence, we get $R(A) > R(B)$. This is one of the example that the existing method failed to rank as it will conclude both of them were neutrosophically equal, but our method rank them in a better way.

Our proposed ranking method involves not only membership, non-membership and indeterminacy values alone, it also consider the part of membership and non-membership value which lying inside the hesitance value. So the formation of our score functions were different from existing score functions, which results there are some difference in the ranking between our method and existing method. Proposed method will be very useful and easy to rank within step 1 itself in many cases, whereas in existing method we may need to go for further steps. Further, in interval valued neutrosophic ranking method, the existing method gives only neutrosophically total ordering, but our method gives total ordering and as well as neutrosophically total ordering.

10. **Conclusion and future scope**

The proposed ranking approach takes into account not only membership, non-membership, and indeterminacy values, but also the portion of membership and non-membership value that is contained within the hesitance value. As a result, the development of our score functions differed from that of current score functions resulting in some differences in ranking between our approach and that of existing methods. In many circumstances, the proposed method will be very beneficial and straightforward to rank within step 1, whereas the old method may require additional stages. Furthermore, the existing interval valued neutrosophic ranking approach delivers neutrosophically total ordering only, but our method gives both total ordering and neutrosophically total ordering. Thus a new algorithm for total ordering both single and interval valued neutrosophic triplets has been derived which will be a beneficial tool for decision makers in MCDM problems.

In this paper, a ranking approach for IVNT is developed as a generalisation of ranking approach for SVNT. By comparing our proposed work with the existing work, we have come to a conclusion that our proposed method involves less steps compared to previous method in some stages. Further, proposed method gives a reliable ordering on alternatives of the MCDM problems due its total ordering nature compared to other methods. In near future, total ordering to triangular and trapezoidal neutrosophic numbers will be studied and hence this opens a new study in the field of neutrosophic sets.

Funding: This research was funded by Council of Scientific and Industrial Research (CSIR-HRDG) India, grant number 09/895(0014)/2019-EMR-I.

Conflicts of Interest: The authors declare that there is no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results. As mentioned earlier the corresponding author thank the Council of Scientific and Industrial Research (CSIR-HRDG) India, for funding the research work under CSIR-JRF.

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Received: August 08, 2022. Accepted: January 11, 2023