Interval quadripartitioned neutrosophic sets

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Abstract
Quadripartitioned neutrosophic set is a mathematical tool, which is the extension of neutrosophic set and n-valued neutrosophic refined logic for dealing with real-life problems. A generalization of the notion of quadripartitioned neutrosophic set is introduced. The new notion is called the Interval Quadripartitioned Neutrosophic set (IQNS). The interval quadripartitioned neutrosophic set is developed by combining the quadripartitioned neutrosophic set and interval neutrosophic set. Several set theoretic operations of IQNSs, namely, inclusion, complement, and intersection are defined. Various properties of set-theoretic operators of IQNS are established.

Keywords: Neutrosophic set, Single valued neutrosophic set, Interval neutrosophic set, quadripartitioned neutrosophic set, Interval quadripartitioned neutrosophic set

1. Introduction


No investigation regarding Interval Quadripartitioned Neutrosophic Set (IQNS) is reported in the literature. The motivation of the present work comes from the works of Chatterjee et al. [1, 5]. The notion of IQNS is developed by combining the concept of QSVNS and Interval Neutrosophic Set (INS) [7]. The proposed structure is a generalization of existing theories of INS and QSVNS.

The organization of the remainder of the paper is presented in table 1.

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Table 1. Outline of the paper
2. Preliminary

**Definition 2.1.** Assume that a set \( W \) is fixed. An NS \([8]\) over \( W \) is defined as:

\[
H = \{ (w, T_H(w), I_H(w), F_H(w)) : w \in W \} \quad \text{where} \quad T_H, I_H, F_H : W \rightarrow [0,1]
\]

\[
\therefore 0 \leq T_H(w) + I_H(w) + F_H(w) \leq 3
\]

**Definition 2.2.** Assume that a set \( W \) is fixed. An SVNS \([2]\) over \( W \) is defined as:

\[
H = \{ (w, T_H(w), I_H(w), F_H(w)) : w \in W \} \quad \text{where} \quad T_H, I_H, F_H : W \rightarrow [0,1]
\]

\[
0 \leq T_H(w) + I_H(w) + F_H(w) \leq 3
\]

**Definition 2.3.** Let a set \( W \) be fixed. An INS \([7]\) over \( W \) is defined as:

\[
H = \{ (w, T_H(w), I_H(w), F_H(w)) : w \in W \}
\]

where for each \( w \in W \), \( T_H(w), I_H(w), F_H(w) \subseteq [0,1] \) are the degrees of membership functions of truth, indeterminacy, and falsity and

\[
T_H(w) = [\inf T_H(w), \sup T_H(w)], I_H(w) = [\inf I_H(w), \sup I_H(w)], F_H(w) = [\inf F_H(w), \sup F_H(w)]
\]

\[
0 \leq \sup T_H(w) + \sup I_H(w) + \sup F_H(w) \leq 3
\]

\( H \) can be expressed as:

\[
H = \{ (w, (T_H(w), I_H(w), F_H(w)) : w \in W \}
\]

2.4. Let a set \( W \) be fixed. A QSVNS \([1]\) over \( W \) is defined as:

\[
H = \{ (w, T_H(w), C_H(w), U_H(w), F_H(w)) : w \in W \}, \quad \text{where for each point} \quad w \in W, \quad T_H(w), C_H(w), U_H(w), F_H(w) \rightarrow [0,1]
\]

are the degrees of membership functions of truth, contradiction, ignorance, and falsity and

\[
0 \leq \sup T_H(w) + \sup C_H(w) + \sup U_H(w) + \sup F_H(w) \leq 4.
\]

3. The Basic Theory of IQNSs

**Definition 3.1. IQNS**

Let \( W \) be a fixed set. Then, an IQNS over \( W \) is denoted by \( H \) and defined as follows:

\[
H = \{ (w, T_H(w), C_H(w), U_H(w), F_H(w)) : w \in W \}, \quad \text{where for each point} \quad w \in W, \quad T_H(w), C_H(w), U_H(w), F_H(w) \subseteq [0,1]
\]

are the degrees of membership functions of truth, contradiction, ignorance, and falsity and

\[
T_H(w) = [\inf T_H(w), \sup T_H(w)], C_H(w) = [\inf C_H(w), \sup C_H(w)], U_H(w) = [\inf U_H(w), \sup U_H(w)],
\]

\[
F_H(w) = [\inf F_H(w), \sup F_H(w)] \subseteq [0,1]
\]

and

\[
0 \leq \sup T_H(w) + \sup C_H(w) + \sup U_H(w) + \sup F_H(w) \leq 4.
\]

An IQNS in \( \mathbb{R}^4 \) is illustrated in Figure 1.
Figure 1. Illustration of an IQNS in $R^1$

Example 3.1. Suppose that $W = [w_1, w_2, w_3]$, where $w_1, w_2,$ and $w_3$ present respectively the capability, trustworthiness, and price. The values of $w_1, w_2,$ and $w_3$ are in $[0, 1]$. They are obtained from the questionnaire of some domain experts, their option could be degree of truth (good), degree of contradiction, degree of ignorance, and degree of false (poor). $H_i$ is an IQNS of $W$ defined by

$H_1 = ([0.5, 0.7], [0.15, 0.2], [0.2, 0.4], [0.2, 0.3])/w_1 + ([0.55, 0.85], [0.25, 0.35], [0.15, 0.25], [0.2, 0.3])/w_2 + (0.65, 0.85), [0.2, 0.35], [0.1, 0.25], [0.15, 0.25])/w_3$

$H_2$ is an IPNS of $W$ defined by

$H_2 = ([0.6, 0.8], [0.1, 0.2], [0.1, 0.25], [0.15, 0.3])/w_1 + ([0.6, 0.9], [0.25, 0.3], [0.1, 0.2], [0.1, 0.3])/w_2 + (0.5, 0.7], [0.1, 0.2], [0.15, 0.25], [0.1; 0.2])/w_3$

Definition 3.2. An IQNS $H$ is said to be empty (null) denoted by $\hat{0}$ iff

$\inf T_H(w) = \sup T_H(w) = 0, \inf C_H(w) = \sup C_H(w) = 0, \inf U_H(w) = \sup U_H(w) = 1, \inf F_H(w) = \sup F_H(w) = 1$,

$\hat{0} = \{0, 0\}, [0, 0], [1, 1], [1, 1]\}$

Definition 3.3. An IQNS $H$ is said to be unity denoted by $\hat{1}$ iff

$\inf T_H(w) = \sup T_H(w) = 1, \inf C_H(w) = \sup C_H(w) = 1, \inf U_H(w) = \sup U_H(w) = 0$,

$\inf F_H(w) = \sup F_H(w) = 0$
\[ \hat{1} = \{[1,1],[1,1],[0,0],[0,0]\} \]

Also, we have \( 0 = \langle 0,0,1,1 \rangle \) and \( \overline{1} = \langle 1,1,0,0 \rangle \)

**Definition 3.4. (Containment)** Let \( H_1 \) and \( H_2 \) be any two IQNS over \( W \), \( H_1 \) is said to be contained in \( H_2 \), denoted by \( H_1 \subseteq H_2 \), iff

for any \( w \in W \),

\[
\begin{align*}
&\inf T_{H_1}(w) \leq \inf T_{H_2}(w), \sup T_{H_1}(w) \leq \sup T_{H_2}(w), \\
&\inf C_{H_1}(w) \leq \inf C_{H_2}(w), \sup C_{H_1}(w) \leq \sup C_{H_2}(w), \\
&\inf U_{H_1}(w) \geq \inf U_{H_2}(w), \sup U_{H_1}(w) \geq \sup U_{H_2}(w), \\
&\inf F_{H_1}(w) \geq \inf F_{H_2}(w), \sup F_{H_1}(w) \geq \sup F_{H_2}(w).
\end{align*}
\]

**Definition 3.5.** Any two IQNS \( H_1 \) and \( H_2 \) are equal iff \( H_1 \subseteq H_2 \) and \( H_2 \subseteq H_1 \)

**Definition 3.6. (Complement)** Let \( H = \{(w,T_{H}(w),C_{H}(w),U_{H}(w),F_{H}(w)) : w \in W\} \) be an IQNS.

The complement of \( H \) is denoted by \( H' \) and defined as:

\[
T_{H'}(w) = F_{H}(w), \quad C_{H'}(w) = U_{H}(w), \quad U_{H'}(w) = C_{H}(w), \quad F_{H'}(w) = T_{H}(w)
\]

\[
H' = \{(w,\inf F_{H}(w),\sup F_{H}(w)),\inf U_{H}(w),\sup U_{H}(w)),\inf C_{H}(w),\sup C_{H}(w)),\inf T_{H}(w),\sup T_{H}(w)) : w \in W\}
\]

**Example 3.2.** Consider an IQNS \( H \) of the form:

\[
H = \{(0.35,0.75),(0.2,0.25),(0.2,0.3),(0.2,0.4)/w_{1} + \{(0.55,0.85),(0.2,0.3),(0.15,0.25),(0.2,0.35)/w_{2}\}
\]

Then, complement of

\[
H' = \{(0.2,0.4),(0.2,0.3),(0.2,0.25),(0.35,0.75)/w_{1} + \{(0.2,0.35),(0.15,0.25),(0.2,0.3),(0.55,0.85)/w_{2}\}
\]

**Definition 3.7. (Intersection)**

The intersection of any two IQNSs \( H_1 \) and \( H_2 \) is an IQNS, denoted as \( H_3 \) and presented as:

\[
H_3 = H_1 \cap H_2
\]

\[
\{(w,\inf T_{H_1}(w),\sup T_{H_1}(w)),\inf C_{H_1}(w),\sup C_{H_1}(w)),\inf U_{H_1}(w),\sup U_{H_1}(w)),\inf F_{H_1}(w),\sup F_{H_1}(w) : w \in W\}
\]

**Example 3.3.** Let \( H_1 \) and \( H_2 \) be the IQNSs defined in Example 3.1.

Then, \( H_1 \cap H_2 = \{(0.5,0.7),(0.1,0.2),(0.2,0.4),(0.2,0.3)/w_{1} + \{(0.55,0.85),(0.25,0.3),(0.15,0.25),(0.2,0.3)/w_{2}\}
\]

**Definition 3.8. (Union)** The union of any two IQNSs \( H_1 \) and \( H_2 \) is an IQNS denoted as \( H_3 \) and presented as:
H_3 = H_1 \cup H_2
\{\{w, [\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)], [\inf U_{H_1}(w), \sup U_{H_1}(w)],
[\inf F_{H_1}(w), \sup F_{H_1}(w)] : w \in W\}\}.
\{\{w, [\max(\inf T_{H_1}(w), \inf T_{H_1}(w)), \max(\sup T_{H_1}(w), \sup T_{H_1}(w))], [\max(\inf C_{H_1}(w), \inf C_{H_1}(w)), \max(\sup C_{H_1}(w), \sup C_{H_1}(w))],
[\min(\inf U_{H_1}(w), \inf U_{H_1}(w)), \min(\sup U_{H_1}(w), \sup U_{H_1}(w))], [\min(\inf F_{H_1}(w), \inf F_{H_1}(w)), \min(\sup F_{H_1}(w), \sup F_{H_1}(w))]: w \in W\}\}.

Example 3.4. Let H_1 and H_2 be the IQNSs in example 3.1. Then
H_1 \cup H_2 = \{[0.6, 0.8], [0.15, 0.2], [0.1, 0.25], [0.15, 0.3], [0.15, 0.35], [0.1, 0.2], [0.1, 0.3]/w_2+[0.65, 0.85], [0.2, 0.35], [0.1, 0.25], [0.1, 0.2]/w_3

Theorem 3.1 Let H_1 and H_2 be any two IQNSs over W defined by
H_i = \{\{w, T_{H_i}(w), C_{H_i}(w), G_{H_i}(w), U_{H_i}(w), F_{H_i}(w)\} : w \in W\}, i = 1, 2.

Then
(a) H_1 \cup H_2 = H_2 \cup H_1
(b) H_1 \cap H_2 = H_2 \cap H_1

Proof. (a):

H_1 \cup H_2 = \{\{w, [\max(\inf T_{H_1}(w), \inf T_{H_1}(w)), \max(\sup T_{H_1}(w), \sup T_{H_1}(w))], [\max(\inf C_{H_1}(w), \inf C_{H_1}(w)), \max(\sup C_{H_1}(w), \sup C_{H_1}(w))],
[\min(\inf U_{H_1}(w), \inf U_{H_1}(w)), \min(\sup U_{H_1}(w), \sup U_{H_1}(w))], [\min(\inf F_{H_1}(w), \inf F_{H_1}(w)), \min(\sup F_{H_1}(w), \sup F_{H_1}(w))]: w \in W\}\}.

Proof. (b):

H_1 \cap H_2 = \{\{w, [\min(\inf T_{H_1}(w), \inf T_{H_1}(w)), \min(\sup T_{H_1}(w), \sup T_{H_1}(w))], [\min(\inf C_{H_1}(w), \inf C_{H_1}(w)), \min(\sup C_{H_1}(w), \sup C_{H_1}(w))],
[\max(\inf U_{H_1}(w), \inf U_{H_1}(w)), \max(\sup U_{H_1}(w), \sup U_{H_1}(w))], [\max(\inf F_{H_1}(w), \inf F_{H_1}(w)), \max(\sup F_{H_1}(w), \sup F_{H_1}(w))]: w \in W\}\}.

Theorem 3.2. For any two IPNS, H_1, and H_2:

(a) H_1 \cup (H_1 \cap H_2) = H_1
(b) H_1 \cap (H_1 \cup H_2) = H_1

Proof. (a):
\[ H_1 \cup (H_1 \cap H_2) = \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)], [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W \} \cup \{w, ([\inf T_{H_1}(w), \inf T_{H_2}(w)], \min(\sup T_{H_1}(w), \sup T_{H_2}(w)), \min(\inf C_{H_1}(w), \inf C_{H_2}(w)), \min(\sup U_{H_1}(w), \sup U_{H_2}(w))], [\inf F_{H_1}(w), \sup F_{H_1}(w))]) : w \in W \} \]

Proof (b):

\[ H_1 \cap (H_1 \cup H_2) = \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)], [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W \} \cap \{w, ([\max(\inf T_{H_1}(w), \sup T_{H_1}(w)], \max(\sup T_{H_2}(w), \inf T_{H_2}(w)), \max(\inf C_{H_1}(w), \inf C_{H_2}(w)), \max(\sup U_{H_1}(w), \sup U_{H_2}(w))), [\max(\inf F_{H_1}(w), \sup F_{H_1}(w)], [\max(\inf F_{H_1}(w), \sup F_{H_1}(w))]) : w \in W \}

\[ \text{Theorem 3.3.} \quad \text{For any IPNS } H_3: \]

(a) \( H_1 \cup H_3 = H_1 \)

(b) \( H_1 \cap H_3 = H_1 \)

Proof. (a):

\[ H_1 \cup H_3 = \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)], [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W \} \cup \{w, ([\inf T_{H_3}(w), \sup T_{H_3}(w)], [\inf C_{H_3}(w), \sup C_{H_3}(w)], [\inf U_{H_3}(w), \sup U_{H_3}(w)], [\inf F_{H_3}(w), \sup F_{H_3}(w)]) : w \in W \}

Proof. (b):
Theorem 3.4 For any IQNS $H_1$, 

(a) $H_1 \cap \hat{0} = \hat{0}$

(b) $H_1 \cup \hat{1} = \hat{1}$

Proof. (a):

$H_1 \cap \hat{0}$
$= \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)], [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)] : w \in W) \}$
$\cap \{[0,0],[0,0],[1,1],[1,1],[1,1]\}$
$= \{w, ([\inf T_{H_1}(w), 0], [\sup T_{H_1}(w), 0])\}$

Proof. (b):

$H_1 \cup \hat{1}$
$= \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)], [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)] : w \in W) \}$
$\cup \{[1,1],[1,1],[0,0],[0,0]\}$
$= \{w, ([\max T_{H_1}(w), 1], [\max T_{H_1}(w), 1]), [\min C_{H_1}(w), 1], [\max C_{H_1}(w), 1], [\min U_{H_1}(w), 0], [\min U_{H_1}(w), 0], [\max U_{H_1}(w), 0], [\max U_{H_1}(w), 0], [\min F_{H_1}(w), 0], [\min F_{H_1}(w), 0]) : w \in W\}$

$= \hat{1}$
Theorem 3.6. For any IQNS $H_1$, $(H_1')' = H_1$

Let $H_1 = \{(w, \min(T(w), T(w)), \min(C(w), C(w)), \min(U(w), U(w)), \min(F(w), F(w))) : w \in W\}$

$H_1 = \{(w, \min(T(w), T(w)), \min(C(w), C(w)), \min(U(w), U(w)), \min(F(w), F(w))) : w \in W\}$

Theorem 3.7. For any two IQNSs, $H_1$ and $H_2$:

(a) $(H_1 \cup H_2)' = H_1' \cap H_2'$

(b) $(H_1 \cap H_2)' = H_1' \cup H_2'$

Proof. (a):

$H_1 \cup H_2 = \{(w, \max(T(w), T(w)), \max(C(w), C(w)), \max(U(w), U(w)), \max(F(w), F(w))) : w \in W\}$

$H_1' \cap H_2' = \{(w, \min(T(w), T(w)), \min(C(w), C(w)), \min(U(w), U(w)), \min(F(w), F(w))) : w \in W\}$

(1)
\[
H'_1 \cap H'_2 = \{ w, ([\inf F_{H_1}(w), \sup F_{H_1}(w)], [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)], [\inf T_{H_1}(w), \sup T_{H_1}(w)] : w \in W \}
\]
\[
\bigcap \{ w, ([\inf F_{H_2}(w), \sup F_{H_2}(w)], [\inf U_{H_2}(w), \sup U_{H_2}(w)], [\inf C_{H_2}(w), \sup C_{H_2}(w)], [\inf T_{H_2}(w), \sup T_{H_2}(w)] : w \in W \} = \{ w, ([\min(\inf F_{H_1}(w), \inf F_{H_2}(w)), \min(\sup F_{H_1}(w), \sup F_{H_2}(w))], [\min(\inf U_{H_1}(w), \inf U_{H_2}(w)), \min(\sup U_{H_1}(w), \sup U_{H_2}(w))], [\max(\inf C_{H_1}(w), \inf C_{H_2}(w)), \max(\sup C_{H_1}(w), \sup C_{H_2}(w))], [\min(\inf T_{H_1}(w), \inf T_{H_2}(w)), \min(\sup T_{H_1}(w), \sup T_{H_2}(w))] : w \in W \}
\]

Therefore from (1) and (2), \( (H_1 \cup H_2)' = H'_1 \cap H'_2 \)

**Proof. (b):**

\[
(H_1 \cap H_2) = \{ w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)], [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W \}
\]
\[
\bigcap \{ w, ([\inf C_{H_2}(w), \sup C_{H_2}(w)], [\inf U_{H_2}(w), \sup U_{H_2}(w)], [\inf F_{H_2}(w), \sup F_{H_2}(w)]) : w \in W \} = \{ w, ([\max(\inf F_{H_1}(w), \inf F_{H_2}(w)), \max(\sup F_{H_1}(w), \sup F_{H_2}(w))], [\max(\inf U_{H_1}(w), \inf U_{H_2}(w)), \max(\sup U_{H_1}(w), \sup U_{H_2}(w))], [\min(\inf C_{H_1}(w), \inf C_{H_2}(w)), \min(\sup C_{H_1}(w), \sup C_{H_2}(w))], [\max(\inf T_{H_1}(w), \inf T_{H_2}(w)), \max(\sup T_{H_1}(w), \sup T_{H_2}(w))] : w \in W \}
\]

Therefore, from (3) and (4), \( (H_1 \cap H_2)' = H'_1 \cup H'_2 \)

**Theorem 3.9.** For any two IPNS \( H_1, H_2 \),

\[
H_1 \subseteq H_2 \iff H'_2 \subseteq H'_1.
\]

**Proof.**

\[
H_1 \subseteq H_2 \iff \inf T_{H_1}(w) \leq \inf T_{H_1}(w), \sup T_{H_1}(w) \leq \sup T_{H_1}(w),
\]
\[
\inf C_{H_1}(w) \leq \inf C_{H_1}(w), \sup C_{H_1}(w) \leq \sup C_{H_1}(w),
\]
\[
\inf U_{H_1}(w) \geq \inf U_{H_1}(w), \sup U_{H_1}(w) \geq \sup U_{H_1}(w),
\]
\[
\inf F_{H_1}(w) \geq \inf F_{H_1}(w), \sup F_{H_1}(w) \geq \sup F_{H_1}(w),
\]
\[
\Rightarrow \inf F_{H_2}(w) \leq \inf F_{H_1}(w), \sup F_{H_2}(w) \leq \sup F_{H_1}(w),
\]
\[
\inf U_{H_2}(w) \leq \inf U_{H_1}(w), \sup U_{H_2}(w) \leq \sup U_{H_1}(w),
\]
\[
\inf C_{H_2}(w) \leq \inf C_{H_1}(w), \sup C_{H_2}(w) \leq \sup C_{H_1}(w),
\]
\[
\inf T_{H_2} \geq \inf T_{H_1}, \sup T_{H_2} \geq \sup T_{H_1}(w)
\]
\[
\Rightarrow H'_2 \subseteq H'_1.
\]

Note: Proposed IQNS can also be called as Interval Quadripartitioned Single Valued Neutrosophic Set (IQSVNS).

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4. Conclusions

In this paper, the notion of IQNS is introduced by combining the QSVNS and INS. The notion of inclusion, complement, intersection, union of IQNSs are defined. Some of the properties of IQNSs, are established. In the future, the logic system based on the truth-value based IQNSs will be investigated and the theory can be used to solve real-life problems in the areas such as information fusion, bioinformatics, web intelligence, etc. Further it is hoped that the proposed IQNS is applicable in neutrosophic decision making [9-11] and graph theory dealing with uncertainty [12-14], etc.

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