Neutrosophic Sets and Systems, Vol. 56, 2023
University of New Mexico

An Inventory Control Model in the Framework of COVID-19 Disruptions Considering Overage Items with Neutrosophic Fuzzy Uncertainty

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Abstract: With the goal of profit maximization and overage control, a mathematical model in a single valued triangular neutrosophic fuzzy environment has been designed to fulfil the demands of the industrial sectors during pandemic situations. This research presents that overage is a crucial factor because even in well-arranged businesses, some proportion of the items might be in overage during a prescribed time interval. Well-planned inventory is important to the successful operation of healthiest businesses. The costs of occurrence of overage factors are converted to fuzzy model, and the overall profit is calculated using the signed distance technique and compared using a numerical example. To determine how the overage will influence the overall system, a sensitivity analysis is undertaken. The ideal amount that provides the maximum value of the projected profit per unit of time is found in both crisp and fuzzy models. The suggested maximizing model will undoubtedly aid decision-makers in dealing with overage circumstances induced by pandemic social distance. The new method of this research is to model the merger of profit maximization and overage concept in the framework of COVID-19 to benefit the inventory management and supply chain sectors.

Keywords: EOQ model; Neutrosophic fuzzy; Profit maximization; COVID-19; Signed distance method

1. Introduction

COVID-19 (Coronavirus disease 2019) is a disease which has become a global pandemic and is rapidly spreading [1]. The disease had spread to almost all countries, territories and areas by 8 October 2020. Despite the installation of new production technology, production sectors require human resources to run and oversee the manufacturing process, as well as supervise and exercise inventory management to minimise shortages. Guo et al. [2] describes the status update on COVID-19 origin and transmission. When lockdown is imposed, however, the manufacturing process is slowed and inventory levels are left uncontrolled. Every manufacturing company has many layers of inventory, such as raw items [3]. Economic decline occurs because delivery of these types of items is extremely unlikely. These new comers collect the bare minimum of goods. The issue now is that if the situation is resolved, the inventory replenishment plan would be disrupted. The stock on hand accumulates, when the same system of inventory up gradation is continued, or the inventory police planner, can alter the system after calculating the required inventory level to be purchased while taking into consideration the current extra stock on hand. There is a stumbling block in managerial decision-making process; it is the issue of inventory quality. The manufacturing atmosphere is shattered by a mix of inventories with varying degrees of quality. Following the regular inventory renewal cycle is the best way to avoid such quality impediment situations. However, the next pressing concern is what to do with the surplus [4]. Inventory management strategies of a Mexican industrial goods firm during the COVID 19 epidemic to retain liquidity and improve customer service levels are discussed in [5]. In reference [6], the inventory management at functioning in Polish collective buying organisations during the COVID-19 epidemic were studied.

Relph [7] used the phrase "overage" to characterize it. He highlighted the elements that contribute to overage inventory in his study work on the subject, and said that overage inventory has some beneficial effects on effective supervision. Ritha and Jayanthi [8] expanded the concept to a fuzzy model since it was so compatible. These models focused on single-item inventory, but when multi-item inventory is considered, the single-item overage inventory model becomes an overage inventory management model for multi-item inventory. This study offers an inventory approach to address inventory management errors induced by COVID-19 social distance. Harris [9] was the first to establish an economic order quantity model, and Taft [10] was the first to develop a production inventory model. These two models serve as the foundation for inventory models. Models of inventory with shortfall costs, Backlogs, price breaks, trade discounts, deteriorating products, and environmental sustainability were all formulated to tackle the issues individually or concurrently. There are several shortage inventory models, and they have been coupled with other inventory management issues. Many studies have been published in the previous two decades in which demand is price dependent. Fuzzy EOQ model was developed by many researcher, for instance, [11], [12].

Pakhira et al. [13] during a shortage, applied the effect of memory to an inventory problem describing the demand function varying with price. Under fuzzy choice factors, Garai [14] presented an inventory problem involving time-varying holding cost. Indrajitsingha et al. [15] discussed a two-warehouse problem in partial- backlogging and fuzzy Environment. Many articles have been written about price-dependent demand in [16], [17] and [18]. Indrajitsingha et al. [19] has developed an EOQ model for perishable products during the COVID-19 pandemic. Dey et al. [20] developed a model assuming the discrete setup cost, varying safety factor, and the demand as a function of price. In the midst of a pandemic, a method for managing disruptions in supply chain was developed in

[21]. Afterwards, Deshmukh and Haleem [22] designed a mathematical model for manufacturing in the after-COVID-19 period.

A customer's daily need may vary from day to day in reality. Because of a lack of historical data or an abundance of information, evaluating a demand distribution is useful. Demand and different inventory model elements have lately been viewed as separate forms of fuzzy numbers by certain scholars. The neutrosophic set is concerned with its relationships with other aspects of the conceptual spectrum. A new strategy for resolving completely neutrosophic linear programming problem has been developed Abdel-Basset [23]. The neutrosophic set is a versatile and widely used formal framework for analysing data sets that contain uncertainty. Mondal et al. [24] presented optimal policy for stock problem with restricted storage capacity using Neutrosophic sets. Mullai and Surya [25] formulated a problem with price break using neutrosophic sets, Mullai et al. [26] and Mullai et. al. [27] used a single valued neutrosophic set to construct multiple inventory models. Surya and Mullai [28] originated on the premise of quick return for faulty items in their neutrosophic lot-size model. Pal and Chakraborty *[29-30] developed a triangular neutrosophic-based EOQ model for non-instantaneously decaying items under scarcity conditions. Authors in [32-35] developed many optimization model in neutosophic environment.* In a neutrosophic paradigm, the work could readily manage any company's inventory system. When uncertain and unexpected circumstances arise in the inventory system, this approach offers more accurate results than prior methodologies. To demonstrate the model's outputs, a sensitivity analysis for crisp and neutrosophic sets is provided, and the findings are briefly reviewed. However, overage models are not well-defined, thus this study sheds light on the state of overage inventory of several goods, with a particular focus on the pandemic epidemic. Overage management, trash disposal, and product dispersion are all part of this profit maximization inventory model.

This paper answers key questions as follows:

(1) Why does this paper apply COVID-19 inventory crisis combating model in neutrosophic environment instead of any other models?

COVID-19's spread has sparked one of the most devastating pandemics in contemporary human past times. Humanity is still now in the early stages of learning about this terrible sickness, and dealing properly with such a big public disaster is a critical issue. Although there are differences in human natural semantics, the lack of knowledge makes it worse. The Neutrosophic set approaches are a useful tool for representing the ambiguity of genuine human semantic expression, as well as the analysis of partial and uncertain data. For example, a person providing their opinion on a product's quality, with 0.7 indicating "possibility that the quality is good," 0.2 indicating "possibility that he or she is unsure about quality," and 0.5 indicating "possibility that the quality is not good." Then this set is represented by a single neutrosophic number (0.7, 0.2, 0.5), where. $0 \le 0.7 + 0.2 + 0.5 \le 3$. So the benefit of the SVNS is that any indeterminacy updating in your day-to-day life if a few people work on decisions that aren't certain, that set corresponds to the neutrosophic set. Other fuzzy models do not take into account this uncertain fact. Because it parallels human thinking and decision-making, it gives a remarkably efficient solution to difficult issues in all areas of life.

(2) What is the practical implication of this model?

The suggested maximizing model will undoubtedly aid decision-makers in dealing with overage circumstances induced by pandemic social distance.

The present paper is arranged below: Basic definitions involving Neutrosophic sets are provided in section 2. In section 3, we present some assumptions & notations. In section 4, the proposed model as well as methodology is demonstrated. Numerical experimentation is executed in section 5. In sections 6 and 7, graphical representation and sensitivity analysis have been given, respectively. In Section 8, some result and discussion are done based on the numerical study and in section 9 concluding remarks of the whole model and future scope have been discussed.

2. Preliminaries

In this section, the required preliminaries are explained here, and they are particularly relevant for the suggested model.

Definition 2.1. [31] Neutrosophic set

A neutrosophic set A in the universal set U defined by a truth-membership function T_A, an indeterminacy-membership function I_A, and a falsity-membership function F_A $T_A(x), I_A(x)$ and $F_A(x)$ are real members of [0,1]. Then,

$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in U, T_A(x), I_A(x), F_A(x) \in]0^-, 1^+[\}$. No restriction is applied on the sum of $T_A(x), I_A(x)$ $F_A(x)$ and, so $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2.2. *[29]* **Triangular single valued neutrosophic numbers of Type 1.**

A triangular single valued Neutrosophic number of Type 1 is referred as $A_{Neu} = (a_1, a_2, a_3; b_1, b_2, b_3; c_1, c_2, c_3)$ whose truth membership, indeterminacy, and falsity membership is defined as follows:

$$T_{A_{Neu}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & when\, a_1 \le x \le a_2\\ 1 & when \quad x=a_2\\ \dfrac{a_3-x}{a_3-a_1} & when\, a_2 \le x \le a_3\\ 0 & otherwise\end{cases}$$

$$I_{A_{Neu}}(x)=\begin{cases}\dfrac{b_2-x}{b_2-b_1} & when\, b_1 \le x \le b_2\\ 0 & when \quad x=a_2\\ \dfrac{x-a_2}{a_3-a_2} & when\, a_2 \le x \le a_3\\ 1 & otherwise\end{cases}$$

$$F_{A_{Neu}}(x)=\begin{cases}\dfrac{c_2-x}{c_2-c_1} & when\, c_1 \le x \le c_2\\ 0 & when \quad x=c_2\\ \dfrac{x-c_2}{c_3-c_2} & when\, c_2 \le x \le c_3\\ 1 & otherwise\end{cases}$$

Here, $0 \le T_{A_{Neu}}(x)+I_{A_{Neu}}(x)+F_{A_{Neu}}(x) \le 3, x \in A_{Neu}$

Definition 2.3. [29] Alpha cut

The parametric form of the triangular single-valued neutrosophic number A_{Neu} is presented as:

$$\left(A_{Neu}\right)_{\alpha,\beta,\gamma}=\left[A_L(\alpha),A_R(\alpha);A_L(\beta),A_R(\beta);A_L(\gamma),A_R(\gamma)\right].$$

Here, $A_L(\alpha)=a_1+(a_2-a_1)\alpha$, $A_R(\alpha)=a_3-(a_3-a_2)\alpha$ $A_L(\beta)=b_2-(b_2-b_1)\beta$ $A_R(\beta)=b_2+(b_3-b_2)\beta$ $A_L(\gamma)=c_2-(c_2-c_1)\gamma$ $A_R(\gamma)=c_2+(c_3-c_2)\gamma$ is the alpha cut of the A_{Neu}. Also, $\alpha \in [0,1], \beta \in [0,1]\, and\, \gamma \in [0,1]$.

2. Assumptions and Notations

2.1 Assumption

(a) The greatest stock level is exceeded on account of any of the following factors: insufficient transportation services, climatic barriers, natural disasters and lockdown during COVID-19 pandemics.

(b) It is assumed that demand changes as and when the selling price of the product changes. Mathematically, $D(S)=a-bS$. Here, a is a positive constant and varies with selling price (S), and $0 < b < \frac{a}{S}$. Further, a and b are assumed as triangular single valued neutrosophic fuzzy numbers.

(c) The expense of obtaining a new customer is referred to as the acquisition cost.

(d) The products are of the partly degraded kind.

(e) There is no end to the planned horizon.

3.2 Notations

Q : Order Quantity

P : The proportion of Q occurrences that cause overage in coming cycle.

$f(P)$: Density functions of probability for P.

K : Placing an order has a set price.

H : The greatest stock, per unit of time.

H_{SI} : Keeping a unique inventory of cost per unit of time.

S : Items in maximum supply are sold at a unit selling price.

V : Selling price per unit of overage products, v < s.

R : Re-novating an overage item's cost per unit.

W_{dc} : The expense of removing the goods from the landfill.

P_d : The percentage of overage times which is discarded.

$f(P_d)$: The density function.

P_{pc} : Cost of product dissemination.

A : Cost of acquiring a new consumer.

N : Total number of new clients gained.

T : Duration of the cycle.

4. Model Development

4.1. Crisp Model

The proposed model is a progression of the client acquisition and overage management paradigms. In order to restore the overage to the maximum stock level, this model takes into account the inventory levels of different items. Overage occurs when inventory surpluses and deficits exceed the maximum stock level. Consider the current situation: before to the spread of COVID 19, the manufacturing sectors were operating with their inventory levels. When a firm closes at time t, it has three inventory levels, item 1, item 2, and item 3. If the scenario is restarted at time t_1, the company's normal operations begin, which include ordering, manufacturing, and marketing. Assume these three things are the raw ingredients or partially finished goods that will be used to create the final product. The time period $t_1 - t$ influences the quality of three things. If $t_1 - t \le t^*$, then an overage situation can be avoided by using a various inventory ordering procedure. (t^*, the given time for quality verification of the products, assuming that the physical properties of the inputs are uniform) If $t_1 - t \ge t^*$, the inventory replenishment pattern remains unchanged, and overage occurs.

The models are created based on the above premise. The components in this suggested model are somewhat decaying in nature, which is one of the underlying assumptions. We have the following equations:

$$U_{TR}(Q) = S\ (1-P)\ D\ +\ \mathrm{V\ P}D \tag{1}$$

$$U_{TC}(Q) = \frac{KD}{Q} + \frac{NAD}{Q} + RPD + \frac{HQ}{2} + \frac{QP^2}{2}\left(H_{SI} - H\right) + \frac{W_{dc}PP_dD}{Q} + \frac{DP_{pc}}{Q} \tag{2}$$

The total profit per unit time as:

$$U_{TP}(Q) = \left(S\ (1-P)\ D\ +\ \mathrm{V\ P}D\right) - \begin{pmatrix} \frac{KD}{Q} + \frac{NAD}{Q} + RPD + \frac{HQ}{2} \\ + \frac{QP^2}{2}\left(H_{SI} - H\right) + \frac{W_{dc}PP_dD}{Q} + \frac{DP_{pc}}{Q} \end{pmatrix} \tag{3}$$

Since P_d is a random variate, therefore, the expectation of (1), $U_{ETP}(Q)$ is presented as:

$$U_{ETP}(Q) = \left(S\ (1-E(P))\ D\ +\ \mathrm{V\ E(P)}D\right) - \begin{pmatrix} \frac{KD}{Q} + \frac{NAD}{Q} + RE(P)D + \frac{HQ}{2} \\ + \frac{QE(P^2)}{2}\left(H_{SI} - H\right) + \frac{W_{dc}E(P)E(P_d)D}{Q} + \frac{DP_{pc}}{Q} \end{pmatrix} \tag{4}$$

The Optimal order quantity

$$Q = \sqrt{\frac{2D\left(K + NA + W_{dc}E(P)E(P_d) + P_{pc}\right)}{H + E(P^2)\left(H_{SI} - H\right)}} \tag{5}$$

4.2 Fuzzy Model

It is uncommon t to specify each and every parameter exactly due to the ambiguity in the environment. To account for more realistic situations, it is possible to assume that a few of the parameters, such as a, b, D, H, H_S *and* R alters under certain bounds. Let a, b, D, H, H_{SI}, *and* R be fuzzy neutrosophic numbers. Then, we obtain

$$a = \left\langle \left(a_1, a_2, a_3\right), \left(a_1^{'}, a_2^{'}, a_3^{'}\right), \left(a_1^{''}, a_2^{''}, a_3^{''}\right) \right\rangle$$

$$\tilde{b} = \left\langle \left(b_1, b_2, b_3\right), \left(b_1^{'}, b_2^{'}, a_3^{'}\right), \left(b_1^{''}, b_2^{''}, b_3^{''}\right) \right\rangle$$

$$D = \left\langle \left(D_1, D_2, D_3\right), \left(D_1^{'}, D_2^{'}, D_3^{'}\right), \left(D_1^{''}, D_2^{''}, D_3^{''}\right) \right\rangle$$

$$H = \left\langle \left(H_1, H_2, H_3\right), \left(H_1^{'}, H_2^{'}, H_3^{'}\right), \left(H_1^{''}, H_2^{''}, H_3^{''}\right) \right\rangle$$

$$H_{SI} = \left\langle \left(H_{SI1}, H_{SI2}, H_{SI3}\right), \left(H_{SI1}^{'}, H_{SI2}^{'}, H_{SI3}^{'}\right), \left(H_{SI1}^{''}, H_{SI2}^{''}, H_{SI3}^{''}\right) \right\rangle$$

$$R = \left\langle \left(R_1, R_2, R_3\right), \left(R_1^{'}, R_2^{''}, R_3^{'}\right), \left(R_1^{''}, R_2^{''}, R_3^{''}\right) \right\rangle$$

We defuzzify using the signed distance method.

The signed distance of $B = \left\langle \left(B_1, B_2, B_3\right), \left(B_1^{'}, B_2^{'}, B_3^{'}\right)\left(B_1^{''}, B_2^{''}, B_3^{''}\right) \right\rangle$ is presented as

$$d\left(B, 0\right) = \frac{B_1 + 2B_2 + B_3 + B_1^{'} + 2B_2^{'} + B_3^{'} + B_1^{''} + 2B_2^{''} + B_3^{''}}{12}$$

The neutrosophic total cost is given by

$$U_{TP}\left(Q\right) = \left(S\ \left(1-P\right)\ D\ +\ \text{V PD}\right) - \left(\begin{array}{l} \dfrac{KD}{Q} + \dfrac{NAD}{Q} + RPD + \dfrac{HQ}{2} \\ + \dfrac{QP^2}{2}\left(H_{SI} - H\right) + \dfrac{W_{dc} P P_d D}{Q} + \dfrac{DP_{pc}}{Q} \end{array} \right) \tag{6}$$

$$= \left\{ \begin{array}{l} \left(S\,(1-P)\,D_1 + V\,PD_1\right) - \left(\begin{array}{l} \frac{KD_1}{Q} + \frac{NAD_1}{Q} + R_1 PD_1 + \frac{H_1 Q}{2} \\ + \frac{QP^2}{2}\left(H_{SI1} - H_1\right) + \frac{W_{dc} PP_d D_1}{Q} + \frac{D_1 P_{pc}}{Q} \end{array} \right), \\ \left(S\,(1-P)\,D_2 + V\,PD_2\right) - \left(\begin{array}{l} \frac{KD_2}{Q} + \frac{NAD_2}{Q} + R_2 PD_2 + \frac{H_2 Q}{2} \\ + \frac{QP^2}{2}\left(H_{SI2} - H_2\right) + \frac{W_{dc} PP_d D_2}{Q} + \frac{D_2 P_{pc}}{Q} \end{array} \right), \\ \left(S\,(1-P)\,D_3 + V\,PD_3\right) - \left(\begin{array}{l} \frac{KD_3}{Q} + \frac{NAD_3}{Q} + R_3 PD_3 + \frac{H_3 Q}{2} \\ + \frac{QP^2}{2}\left(H_{SI3} - H_3\right) + \frac{W_{dc} PP_d D_3}{Q} + \frac{D_3 P_{pc}}{Q} \end{array} \right) \end{array} \right\}, \left\{ \begin{array}{l} \left(S\,(1-P)\,D'_1 + V\,PD'_1\right) - \left(\begin{array}{l} \frac{KD'_1}{Q} + \frac{NAD'_1}{Q} + R'_1 PD'_1 + \frac{H'_1 Q}{2} \\ + \frac{QP^2}{2}\left(H'_{SI1} - H'_1\right) + \frac{W_{dc} PP_d D'_1}{Q} + \frac{D'_1 P_{pc}}{Q} \end{array} \right), \\ \left(S\,(1-P)\,D'_2 + V\,PD'_2\right) - \left(\begin{array}{l} \frac{KD'_2}{Q} + \frac{NAD'_2}{Q} + R'_2 PD'_2 + \frac{H'_2 Q}{2} \\ + \frac{QP^2}{2}\left(H'_{SI2} - H'_2\right) + \frac{W_{dc} PP_d D'_2}{Q} + \frac{D'_2 P_{pc}}{Q} \end{array} \right), \\ \left(S\,(1-P)\,D'_3 + V\,PD'_3\right) - \left(\begin{array}{l} \frac{KD'_3}{Q} + \frac{NAD'_3}{Q} + R'_3 PD'_3 + \frac{H'_3 Q}{2} \\ + \frac{QP^2}{2}\left(H'_{SI3} - H'_3\right) + \frac{W_{dc} PP_d D'_3}{Q} + \frac{D'_3 P_{pc}}{Q} \end{array} \right) \end{array} \right\},$$

$$\left\{ \begin{array}{l} \left(S\,(1-P)\,D''_1 + V\,PD''_1\right) - \left(\begin{array}{l} \frac{KD''_1}{Q} + \frac{NAD''_1}{Q} + R''_1 PD''_1 + \frac{H''_1 Q}{2} \\ + \frac{QP^2}{2}\left(H''_{SI1} - H''_1\right) + \frac{W_{dc} PP_d D''_1}{Q} + \frac{D''_1 P_{pc}}{Q} \end{array} \right), \\ \left(S\,(1-P)\,D''_2 + V\,PD''_2\right) - \left(\begin{array}{l} \frac{KD''_2}{Q} + \frac{NAD''_2}{Q} + R''_2 PD''_2 + \frac{H''_2 Q}{2} \\ + \frac{QP^2}{2}\left(H''_{SI2} - H''_2\right) + \frac{W_{dc} PP_d D''_2}{Q} + \frac{D''_2 P_{pc}}{Q} \end{array} \right), \\ \left(S\,(1-P)\,D''_3 + V\,PD''_3\right) - \left(\begin{array}{l} \frac{KD''_3}{Q} + \frac{NAD''_3}{Q} + R''_3 PD''_3 + \frac{H''_3 Q}{2} \\ + \frac{QP^2}{2}\left(H''_{SI3} - H''_3\right) + \frac{W_{dc} PP_d D''_3}{Q} + \frac{D''_3 P_{pc}}{Q} \end{array} \right) \end{array} \right\}$$

The defuzzified neutrosophic total cost using above signed distance method is given by

$$
d\left(U_{TP}(Q),0\right)=\frac{1}{12}\left[\begin{array}{l}
\left(S\ (1-P)\ \mathrm{D}_1+\mathrm{V\ PD}_1\right)-\left(\begin{array}{l}\frac{KD_1}{Q}+\frac{NAD_1}{Q}+R_1PD_1+\frac{H_1Q}{2}\\+\frac{QP^2}{2}\left(H_{SI1}-H_1\right)+\frac{W_{dc}PP_dD_1}{Q}+\frac{D_1P_{pc}}{Q}\end{array}\right)\\
+2\left\{\left(S\ (1-P)\ \mathrm{D}_2+\mathrm{V\ PD}_2\right)-\left(\begin{array}{l}\frac{KD_2}{Q}+\frac{NAD_2}{Q}+R_2PD_2+\frac{H_2Q}{2}\\+\frac{QP^2}{2}\left(H_{SI2}-H_2\right)+\frac{W_{dc}PP_dD_2}{Q}+\frac{D_2P_{pc}}{Q}\end{array}\right)\right\}\\
+\left(S\ (1-P)\ \mathrm{D}_3+\mathrm{V\ PD}_3\right)-\left(\begin{array}{l}\frac{KD_3}{Q}+\frac{NAD_3}{Q}+R_3PD_3+\frac{H_3Q}{2}\\+\frac{QP^2}{2}\left(H_{SI3}-H_3\right)+\frac{W_{dc}PP_dD_3}{Q}+\frac{D_3P_{pc}}{Q}\end{array}\right)\\
+\left(S\ (1-P)\ \mathrm{D}_1'+\mathrm{V\ PD}_1'\right)-\left(\begin{array}{l}\frac{KD_1'}{Q}+\frac{NAD_1'}{Q}+R_1'PD_1'+\frac{H_1'Q}{2}\\+\frac{QP^2}{2}\left(H_{SI1}'-H_1'\right)+\frac{W_{dc}PP_dD_1'}{Q}+\frac{D_1'P_{pc}}{Q}\end{array}\right)\\
+2\left\{\left(S\ (1-P)\ \mathrm{D}_2'+\mathrm{V\ PD}_2'\right)-\left(\begin{array}{l}\frac{KD_2'}{Q}+\frac{NAD_2'}{Q}+R_2'PD_2'+\frac{H_2'Q}{2}\\+\frac{QP^2}{2}\left(H_{SI2}'-H_2'\right)+\frac{W_{dc}PP_dD_2'}{Q}+\frac{D_2'P_{pc}}{Q}\end{array}\right)\right\}\\
+\left(S\ (1-P)\ \mathrm{D}_3''+\mathrm{V\ PD}_3''\right)-\left(\begin{array}{l}\frac{KD_3''}{Q}+\frac{NAD_3''}{Q}+R_3''PD_3''+\frac{H_3''Q}{2}\\+\frac{QP^2}{2}\left(H_{SI3}''-H_3''\right)+\frac{W_{dc}PP_dD_3''}{Q}+\frac{D_3''P_{pc}}{Q}\end{array}\right)\\
+\left(S\ (1-P)\ \mathrm{D}_1''+\mathrm{V\ PD}_1''\right)-\left(\begin{array}{l}\frac{KD_1''}{Q}+\frac{NAD_1''}{Q}+R_1''PD_1''+\frac{H_1''Q}{2}\\+\frac{QP^2}{2}\left(H_{SI1}''-H_1''\right)+\frac{W_{dc}PP_dD_1''}{Q}+\frac{D_1''P_{pc}}{Q}\end{array}\right)\\
+2\left\{\left(S\ (1-P)\ \mathrm{D}_2''+\mathrm{V\ PD}_2''\right)-\left(\begin{array}{l}\frac{KD_2''}{Q}+\frac{NAD_2''}{Q}+R_2''PD_2''+\frac{H_2''Q}{2}\\+\frac{QP^2}{2}\left(H_{SI2}''-H_2''\right)+\frac{W_{dc}PP_dD_2''}{Q}+\frac{D_2''P_{pc}}{Q}\end{array}\right)\right\}\\
+\left(S\ (1-P)\ \mathrm{D}_3''+\mathrm{V\ PD}_3''\right)-\left(\begin{array}{l}\frac{KD_3''}{Q}+\frac{NAD_3''}{Q}+R_3''PD_3''+\frac{H_3''Q}{2}\\+\frac{QP^2}{2}\left(H_{SI3}''-H_3''\right)+\frac{W_{dc}PP_dD_3''}{Q}+\frac{D_3''P_{pc}}{Q}\end{array}\right)
\end{array}\right] \quad (7)
$$

To find the optimal solution by taking the derivative of $d\left(U_{TP}\left(Q\right),0\right)$ relative to Q and then equating it to zero, we get

$$Q=\sqrt{\frac{2\left(D_1+2D_2+D_3+D_1^{'}+2D_2^{'}+D_3^{'}+D_1^{''}+2D_2^{''}+D_3^{''}\right)\left(K+NA+W_{dc}PP_d+P_{pc}\right)}{\left(H_1+2H_2+H_3+H_1^{'}+2H_2^{'}+H_3^{'}+H_1^{''}+2H_2^{''}+H_3^{''}\right)+P^2\begin{pmatrix}\left(H_{SI1}-H_1\right)+2\left(H_{SI2}-H_2\right)+\left(H_{SI3}-H_3\right)\\+\left(H_{SI1}^{'}-H_1^{'}\right)+2\left(H_{SI2}^{'}-H_2^{'}\right)+\left(H_{SI3}^{'}-H_3^{'}\right)\\+\left(H_{SI1}^{''}-H_1^{''}\right)+2\left(H_{SI2}^{''}-H_2^{''}\right)+\left(H_{SI3}^{''}-H_3^{''}\right)\end{pmatrix}}} \tag{8}$$

5. Numerical Example

Case study: To demonstrate the situation, suppose the demand of a product fall due to insufficient transportation services, climatic barriers, natural disasters and lockdown during COVID-19 pandemics etc. In this situation, business model encounters the fall in the forecast requirement for the item 1, item 2 and item3. And the inventories goes up maximum into overage. A numerical example using the following inputs, as in Table 1, is used to validate the proposed model.

Table 1. Input data

Parameter	Item 1	Item 2	Item 3
a	50,000 unit / year	50,000 unit / year	50,000 unit / year
b	50	50	50
K	200 / cycle	220 / cycle	230 / cycle
H	5 / unit / year	4 / unit / year	6 / unit / year
H_{SI}	6 / unit / year	7 / unit / yea	8 / unit / yea
S	50 / unit	60 / unit	55 / unit
V	35 / unit	40 / unit	35 / unit
R	5 / item	6 / item	5 / item
A	30 / customers	35 / customers	40 / customers
N	2	3	4
W_{dc}	3 / unit	4 / unit	5 / unit
P_{pc}	300 / cycle	320 / cycle	330 / cycle

It is assumed that $P\, and\, P_d$ follows uniform distribution with the below density function:

$$f(P)=\begin{cases}4, & 0\le P\le 0.25\\ 0, & Otherwise\end{cases} \quad \text{and} \quad f(P_d)=\begin{cases}20, & 0\le P_d\le 0.05\\ 0, & Otherwise\end{cases}$$

$$E(P)=0.125, E(P^2)=0.021 and\ E(P_d)=0.1.$$

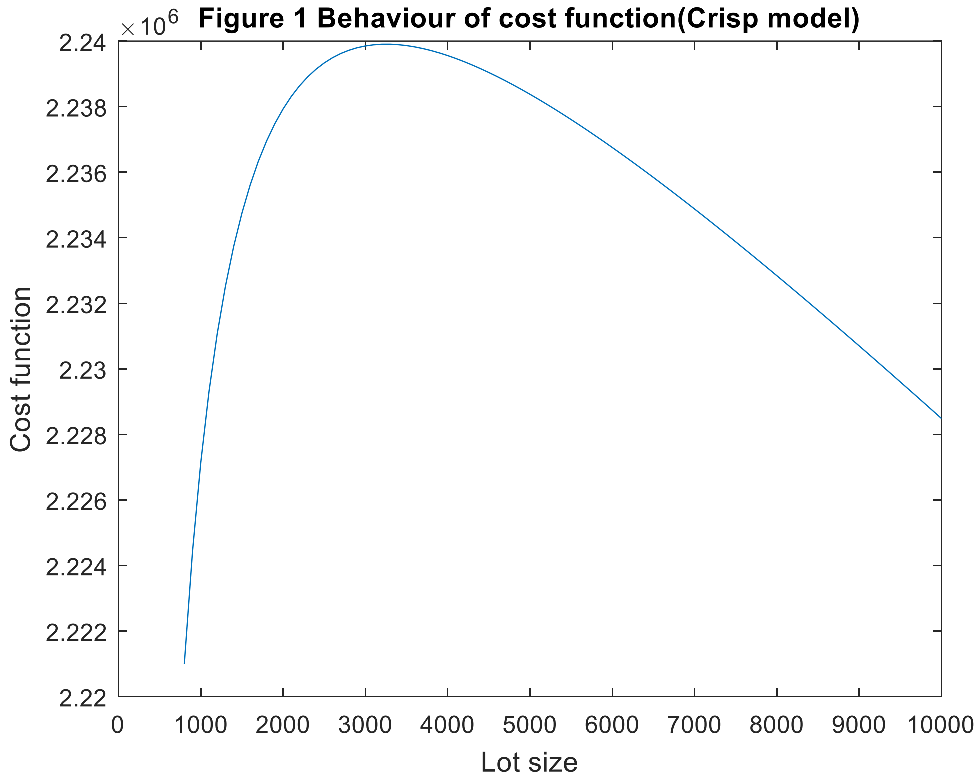
The following Table 2 demonstrates the optimal results.

Table 2. Optimal results for both models

Types of Models	Items	Optimal order Quantity	Expected Maximum profit
Crisp	Item 1	3255	2239905.732
	Item 2	3863	2651554.179
	Item 3	3355	2430817.326
Fuzzy	Item 1	3262	1913343.196
	Item 2	3893	2263803.700
	Item 3	3367	2105973.452

6. Graphical representation

Figures 1 and 2 demonstrate the cost function behavior for both models. The concavity of Figure demonstrates that it maximizes overall profit. Figure 3 depicts a comparison of both crisp and fuzzy models. Figures 4 and 5 show how different cost functions, such as acquisition and fixed costs, vary.



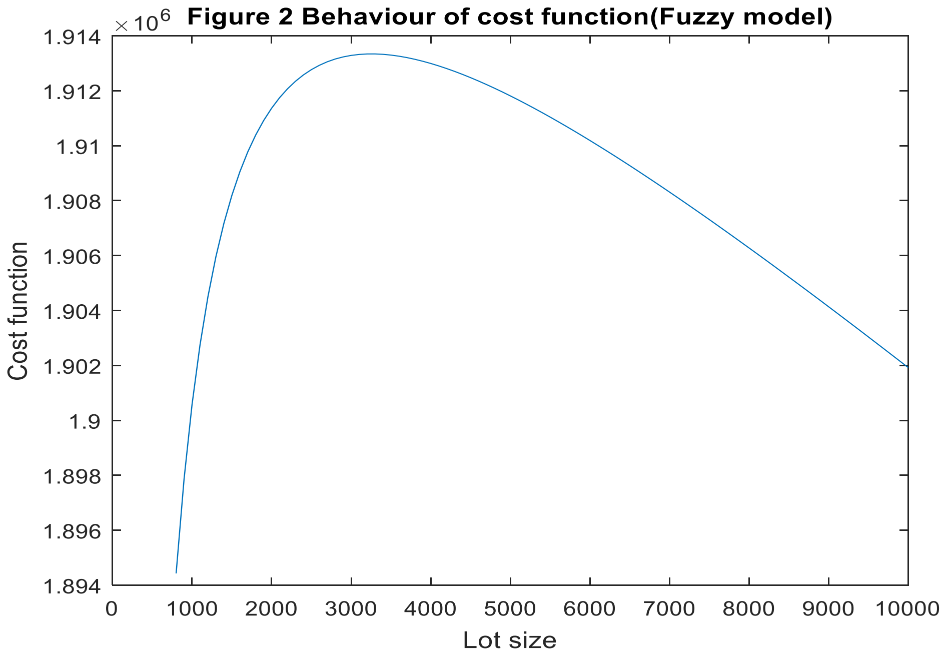


Figure 2 Behaviour of cost function(Fuzzy model)
×10^6
1.914
1.912
1.91
1.908
1.906
1.904
1.902
1.9
1.898
1.896
1.894
Cost function
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Lot size

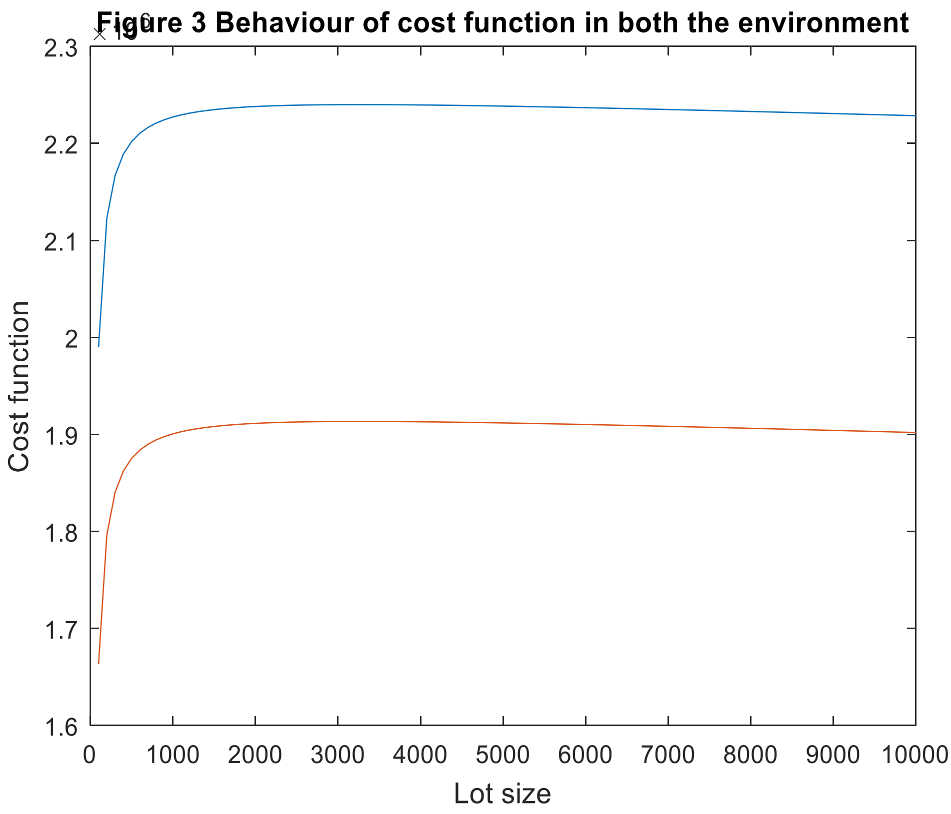


Figure 3 Behaviour of cost function in both the environment
2.3
2.2
2.1
2
1.9
1.8
1.7
1.6
Cost function
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Lot size

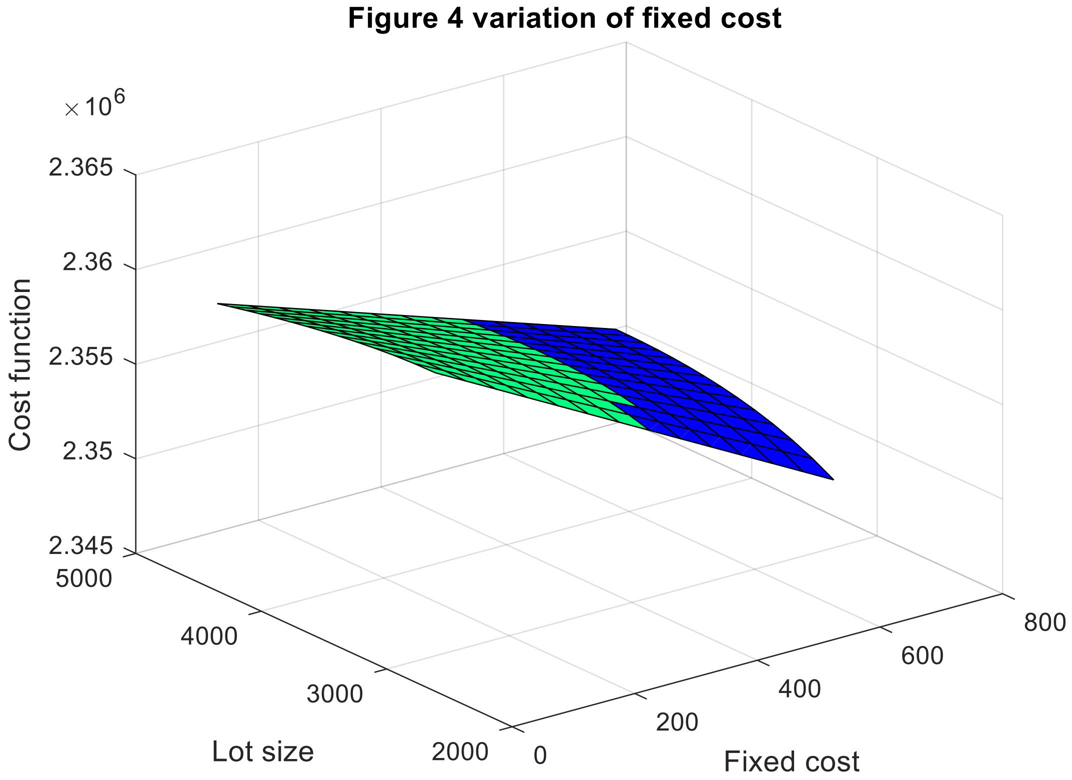


Figure 4 variation of fixed cost
×10^6
2.365
2.36
2.355
2.35
2.345
Cost function
5000
4000
3000
2000
Lot size
0
200
400
600
800
Fixed cost

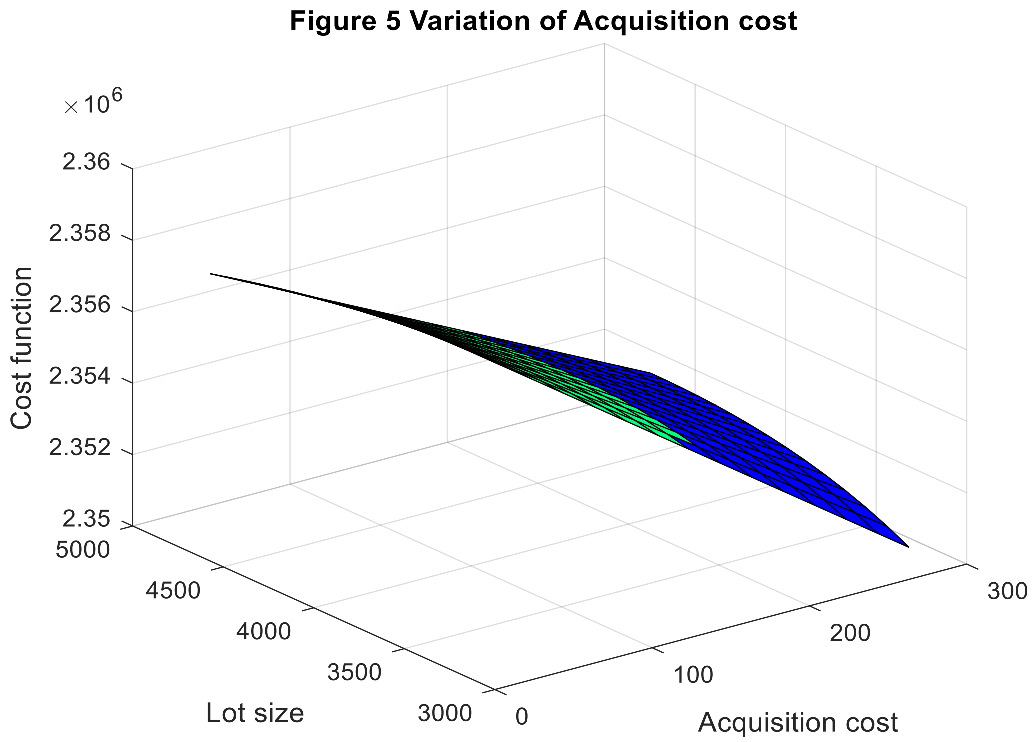


Figure 5 Variation of Acquisition cost
×10^6
2.36
2.358
2.356
2.354
2.352
2.35
Cost function
5000
4500
4000
3500
3000
Lot size
0
100
200
300
Acquisition cost

7. Sensitivity Analysis

We were able to observe some changes in the final cost by gently changing the parameters in this section. By changing each of the parameters by -15 %, -10 %, -5 %, 0 %, +5 %, +10 %, and +15 % this study does a sensitivity analysis. We may do a sensitivity analysis for the given situation. We changed one of the parameters' percent while leaving the rest alone. We've also looked into the whole cost's influence. The findings are summarized in Table 3.

Table 3: Percentage change in parameter

Variation of Parameters	-15	-10	-5	0	5	10	15
a	1885001	2003289	2121591	2239905	2358231	2476567	2594912
b	2257653	2251737	2245821	2239905	2233989	2228073	2222157
K	2240349	2240200	2240052	2239905	2239760	2239616	2239473
H	2241148	2240722	2240309	2239905	2239512	2239127	2238751
H_{SI}	2239936	2239926	2239915	2239905	2239895	2239885	2239874
S	1943474	2042831	2141642	2239905	2337622	2434792	2531415
V	2208733	2219124	2229515	2239905	2250296	2260686	2271077
R	2244358	2242874	2241390	2239905	2238421	2236936	2235452
A	2240037	2239993	2239949	2239905	2239862	2239818	2239774
W_{dc}	2239905	2239905	2239905	2239905	2239905	2239905	2239905
P_{pc}	2240576	2240349	2240126	2239905	2239688	2239473	2239261

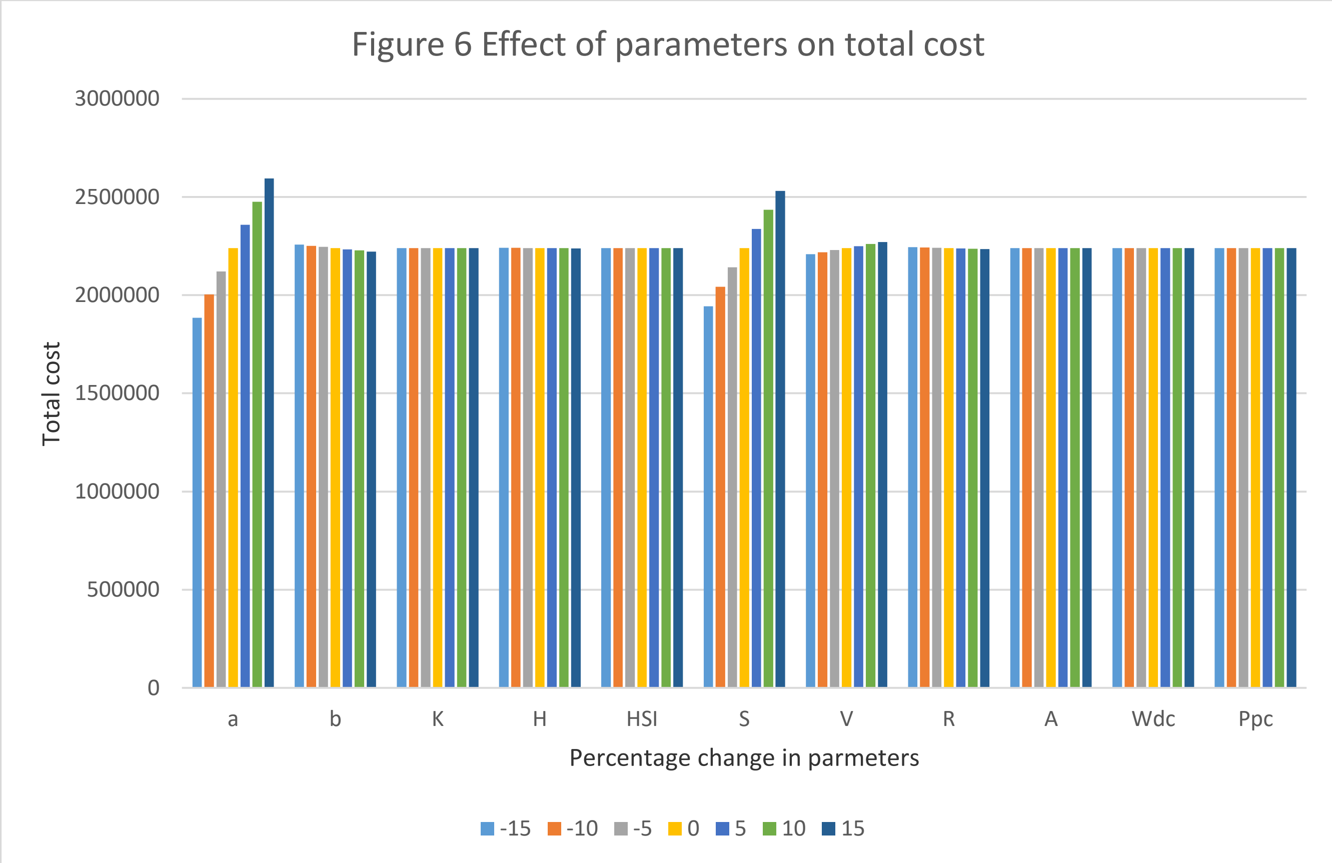
The below mentioned observations are found with the support of Table 3.

(i) An increase in the total cost is caused by a percentage change (PC) in demand parameters ***a***, selling price in maximum stock, and selling price in overage.

(ii) Percentage change (PC) in demand parameter ***b***, K, H, H_{SI}, R, A and P_{pc} lead a decrease in the total cost.

(iii) Percentage change (PC) in Waste disposal cost of the items not effect of the total cost.

Furthermore, we have illustrated below in Figure 6.



8. Results and Discussion

The difference between collective earnings and costs each cycle is the projected aggregate profit per unit of time. Table 2 shows the optimum order quantity for items 1, 2, and 3 to maximize the corresponding projected aggregate profit per unit of time. The optimum order quantity Q for each item is determined, as well as the predicted maximum profit per unit of time. In the above table 2 we also observed that the order quantity of crisp and fuzzy model partially equal but the profit decreases in fuzzy model. The total expenses include the costs of upgrading the overage goods, as well as disposal and product propagation charges. The entire expenses and income related with overage management are represented in this model. The building of inventory is a crucial challenge for the manufacturing industries. The proposed model is not specific to any type of manufacturing company since it includes all of the basic cost characteristics associated with handling excess inventory. The inclusion of environmental costs represents the model's societal relevance in encouraging environmental sustainability; moreover, including environmental costs will make the model compatible with environmental accounting procedures.

9. Conclusions

COVID-19 is a novel experience around the globe. The great nations are running out of endurance in their fight against this terrible health disaster, and the only weapon they have left is an absurd medicine called "Social Distancing." The introduction of quarantine has wreaked havoc on the manufacturing sector. The abrupt curfew interruption disrupted product manufacturing, and the pandemic effect on output resulted in overproduction. In this research, a more generic overage model is formulated to address problems of overage during pandemic outbreaks. This model is for restoring overstock of several goods to maximum stock levels. The suggested maximizing model will undoubtedly aid decision-makers in dealing with overage circumstances induced by pandemic social distance. The goal of this work was to look at price dependent linear demand inventory models in a fuzzy setting. Because the demand and quality of the overage times is not always consistent, it possesses an impact on the overall system. And, in the middle of current variations, our fuzzy model aids in estimating the best projected total profit each cycle. The model is also compared in two different environments, crisp and neutrosophic fuzzy, in the study. This hazy model stresses that customers are more valuable than money and also provides overage control measures. In addition, in the middle of current volatility, our fuzzy model aids in estimating the ideal predicted total profit each cycle. In this study, we did a sensitivity analysis in a crisp setting to show our case. The proposed model may be used to help companies establish production plans in response to pandemics and other unanticipated disasters that interrupt the manufacturing process.

The future scope of this research comprises the investigation of various planning models which allows inclusive planning of overage with different environments such as sustainable and fuzzy. The proposed model reveals a number of ways to extend the model in other dimensions such as demand inversely related to the selling price of the product [37], parabolic holding cost [38], etc. In addition, the developed model can be in multi criteria decision and supply chain problems.

Conflicts of Interest: The authors declare no conflict of interest.

Funding: This research received no external funding.

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Received: March 16, 2023. Accepted: July 18, 2023