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Some Characterizations of Linguistic Neutrosophic Topological Spaces

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Abstract. Many properties of linguistic neutrosophic cl-open spaces and linguistic neutrosophic semi spaces are characterized. Furthermore some conditions are studied which are essential for the existence of certain linguistic neutrosophic spaces. As well the linkage among the spaces are examined and exemplified with precise counter cases. Also, the notion of linguistic neutrosophic quasi-semi component and contra-semi closed graph are introduced and some results are discussed.

Keywords: Ultra hausdorff space; cl-open normal space; cl-open regular space; urysohn space; Quasi semi-component space; in Linguistic Neutrosophic Spaces.

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1. Introduction

People's needs have changed with the advancement of technology and topology has become inefficient in real-life situations as a result. Separation axioms played a critical role in different kinds of topological spaces that were later discovered. Chang [2] discovered fuzzy topological spaces based on fuzzy sets [12]. Coker [3] developed a hybrid topological space by utilizing intuitionistic fuzzy sets [1]. A new set called neutrosophic set is described by Smarandache [11] by combining indeterminacy membership functions with truth and falsity memberships. Further neutrosophic topological space has been found by Salama and Alblowi [10]. Meanwhile, Gayathri and Helen [6] instigated the notion linguistic neutrosophic topology. The purpose of this article is to examine the inter-linkage between linguistic neutrosophic cl-open spaces. Studies are also conducted on the properties of linguistic neutrosophic semi spaces.

2. Preliminaries

Definition 2.1. [11] Let S be a space of points (objects), with a generic element in x denoted by S. A neutrosophic set A in S is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is
$T_A : S \rightarrow]0^-, 1^+[, I_A : S \rightarrow]0^-, 1^+[, F_A : S \rightarrow]0^-, 1^+[$
There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2.2. [11] Let S be a space of points (objects), with a generic element in x denoted by S. A single valued neutrosophic set (SVNS) A in S is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point S in S, $T_A(x), I_A(x), F_A(x) \in [0, 1]$.
When S is continuous, a SVNS A can be written as $A = \int \langle T(x), I(x), F(x)\rangle / x \in S$.
When S is discrete, a SVNS A can be written as $A = \sum \langle T(x_i), I(x_i), F(x_i)\rangle / x_i \in S$.

Definition 2.3. [9] Let $S = \{s_\theta | \theta = 0, 1, 2, ..., \tau\}$ be a finite and totally ordered discrete term set, where τ is the even value and s_θ represents a possible value for a linguistic variable.

Definition 2.4. [9] Let $Q = \{s_0, s_1, s_2, ..., s_t\}$ be a linguistic term set (LTS) with odd cardinality $t+1$ and $\overline{Q} = \{s_h / s_0 \leq s_h \leq s_t, h \in [0, t]\}$. Then, a linguistic single valued neutrosophic set A is defined by, $A = \{\langle x, s_\theta(x), s_\psi(x), s_\sigma(x)\rangle | x \in S\}$, where $s_\theta(x), s_\psi(x), s_\sigma(x) \in \overline{Q}$ represent the linguistic truth, linguistic indeterminacy and linguistic falsity degrees of S to A, respectively, with condition $0 \leq \theta + \psi + \sigma \leq 3t$. This triplet $(s_\theta, s_\psi, s_\sigma)$ is called a linguistic single valued neutrosophic number.

Definition 2.5. [6] For a linguistic neutrosophic topology τ_{LN}, the collection of linguistic neutrosophic sets should obey,

(1) $0_{LN}, 1_{LN} \in \tau_{LN}$
(2) $K_1 \bigcap K_2 \in \tau_{LN}$ for any $K_1, K_2 \in \tau_{LN}$
(3) $\bigcup K_i \in \tau_{LN}, \forall \{K_i : i \in J\} \subseteq \tau_{LN}$

We call, the pair (S_{LN}, τ_{LN}), a linguistic neutrosophic topological space.

Definition 2.6. A topological space (S_{LN}, τ_{LN}) is said to be

(1) LN semi-T_0 [7] if for each pair of distinct linguistic neutrosophic points in S_{LN}, there exists a LN semi-open set containing one but not the other.
(2) LN semi-T_1 [7](resp. LN cl-open-T_1 [5]) if for each pair of distinct linguistic neutrosophic points s_1 and s_2 in S_{LN}, there exist LN semi-open(resp. cl-open) sets E_{LN} and F_{LN} containing s_1 and s_2 such that $s_1 \in E_{LN}, s_2 \notin F_{LN}$ and $s_2 \notin E_{LN}, s_2 \in F_{LN}$.

(3) LN semi-T_2 [7](resp. LN ultra-hausdorff [12]) if every two linguistic neutrosophic points can be separated by disjoint LN semi-open(resp. LN cl-open) sets.

(4) LN semi-normal [4](resp. LN cl-open-normal [5]) if for each pair of distinct LN semi-closed(resp. LN cl-open) sets E_{LN} and F_{LN} of S_{LN}, there exist two disjoint LN semi-open(resp. LN open) sets G_{LN} and H_{LN} such that $E_{LN} \subset G_{LN}$ and $F_{LN} \subset H_{LN}$.

(5) LN semi-regular [4](resp. cl-open-regular [5]) if for each LN semi-closed(resp. LN cl-open) set K_{LN} of S_{LN} and each $s \notin K_{LN}$, there exist two disjoint LN semi-open(resp. LN open) sets E_{LN} and F_{LN} such that $K_{LN} \subset E_{LN}$ and $s \in F_{LN}$.

(6) The LN quasi-component [9] of s_1 is such that the set of all linguistic neutrosophic points s_2 in S_{LN} such that s_1 and s_2 cannot be separated by LN semi-separation of S_{LN}.

Definition 2.7. [7] A LNS $P_{LN} = \{\langle s_1, T_{P_{LN}}(s_1), I_{P_{LN}}(s_1), F_{P_{LN}}(s_1)\rangle : s_1 \in S_{LN}\}$ is called a linguistic neutrosophic point(LNP in short) if and only if for any element $s_2 \in S_{LN}$,
$\langle T_{P_{LN}}(s_1), I_{P_{LN}}(s_1), F_{P_{LN}}\rangle = \langle l_p, l_q, l_r\rangle$ for $s_2 = s_1$,
$\langle T_{P_{LN}}(s_1), I_{P_{LN}}(s_1), F_{P_{LN}}\rangle = \langle 0, 0, 1\rangle$ for $s_2 \neq s_1$.
where $0 < p \leq t, 0 \leq q < t, 0 \leq r < t$.

Definition 2.8. [7] A LNP $P_{LN} = \{\langle s, T_{P_{LN}}(s), I_{P_{LN}}(s), F_{P_{LN}}(s)\rangle : s \in S_{LN}\}$ will be denoted by $s_{\langle l_p, l_q, l_r\rangle}$. The complement of the LNP $s_{\langle l_p, l_q, l_r\rangle}$ will be denoted by ${s^c}_{\langle l_p, l_q, l_r\rangle}$.

Definition 2.9. [7] A LNTS (S_{LN}, τ) is semi-R_0 if for every LNSO set K_{LN}, $s \in K_{LN}$ implies $LNSCl(\{s\}) \subseteq K_{LN}$.

3. Some Characterization of Linguistic Neutrosophic Spaces

Definition 3.1. A LNTS (S_{LN}, τ_{LN}) is said to be

(1) LNCOS-T_1 if for every pair of distinct points in S_{LN}, there exist LNCOSs E_{LN} and F_{LN} containing two points respectively such that $E_{LN} \cap F_{LN} = \phi$.

(2) LN ultra-hausdorff if every two distinct points of S_{LN} can be separated by disjoint LNCOSs.

(3) LNCOS-normal if for each pair of disjoint LNCOS sets E_{LN} and F_{LN} of S_{LN}, there exist two disjoint LNOSs K_{LN} and H_{LN} such that $E_{LN} \subset G_{LN}$ and $F_{LN} \subset H_{LN}$.

(4) LNCOS-regular if for each LNCOS E_{LN} of S_{LN} and each $s \notin E_{LN}$, there exist disjoint LNOSs K_{LN} and H_{LN} such that $E_{LN} \subset G_{LN}$ and $s \in H_{LN}$.

(5) LN locally-indiscrete if each LNOS of S_{LN} is LNCS in S_{LN}.

Definition 3.2. A LNTS (S_{LN}, τ_{LN}) is said to be LNS-regular if for each LNSCS E_{LN} of S_{LN} and each $s \notin E_{LN}$, there exist disjoint LNSOSs K_{LN} and H_{LN} such that $E_{LN} \subset K_{LN}$ and $s \in H_{LN}$.

Example 3.3. Let the universe of discourse be $\mathcal{U} = \{x, y, z, w\}$ and let $S_{LN} = \{x, y, z\}$.
The set of all LTS be L= { very poor (l_0), poor(l_1), very weak(l_2), weak(l_3), below average (l_4), average (l_5), above average (l_6), good (l_7), very good (l_8), excellent (l_9), outstanding (l_{10})}.
And let $A_{LN} = \{\langle(x, l_4, l_5, l_2), (y, l_3, l_2, l_1), (z, l_9, l_6, l_8)\rangle\}$ and
$B_{LN} = \{\langle(x, l_2, l_4, l_5), (y, l_1, l_1, l_2), (z, l_6, l_5, l_8)\rangle\}$.
Let LNP be $s_{\langle(x,l_2,l_4,l_6),(y,l_1,l_4,l_5),(z,l_3,l_3,l_9)\rangle}$.
The LNSCS and the LNSOSs are given by $E_{LN} = \langle(x, l_1, l_3, l_4),\ (y, l_1, l_6, l_5), (z, l_5, l_8, l_9)$ and
$K_{LN} = \langle(x, l_5, l_6, l_1), (y, l_4, l_6, l_1), (z, l_9, l_8, l_5)\rangle$,
$H_{LN} = \langle(x, l_3, l_5, l_4), (y, l_1, l_6, l_2), (z, l_7, l_5, l_8)\rangle$ respectively with $s \notin E_{LN}$.
Now, $s \in H_{LN}$ and $E_{LN} \subset K_{LN}$ and thus S_{LN} is LNS-regular.

Definition 3.4. A LNTS (S_{LN}, τ_{LN}) is said to be LNS-normal if for each pair of disjoint LNCSs A_{LN} and B_{LN}, there exist two distinct LNSOSs K_{LN} and H_{LN} with $A_{LN} \subseteq K_{LN}$, $B_{LN} \subseteq H_{LN}$.

Example 3.5. Let the universe of discourse $\mathcal{U}$ and LTS be as in Example (3.3).
Let the LNCS be $A_{LN} = \langle(x, l_1, l_3, l_4),\ (y, l_1, l_6, l_5),\ (z, l_5, l_8, l_9)\rangle$,
$B_{LN} = \langle(x, l_1, l_1, l_5),\ (y, l_0, l_1, l_6),\ (z, l_6, l_5, l_8)\rangle$.
The LNSOSs are given by $K_{LN} = \langle(x, l_5, l_6, l_1), (y, l_4, l_6, l_1), (z, l_9, l_8, l_5)\rangle$
and $H_{LN} = \langle(x, l_3, l_5, l_4), (y, l_1, l_6, l_2), (z, l_7, l_5, l_8)\rangle$.
Now, $A_{LN} \subseteq K_{LN}, B_{LN} \subseteq H_{LN}$ and hence S_{LN} is LNS-normal.

Definition 3.6. A LNTS (S_{LN}, τ_{LN}) is said to be LN-urysohn space if there exist two disjoint LNnbds V_{t_1} and V_{t_2}, containing t_1 and t_2 in (T_{LN}, η) such that $LNCl(V_{t_1}) \cap LNCl(V_{t_2}) = \phi$.

Definition 3.7. Let (S_{LN}, τ_{LN}) be a LNTS and $s \in (S_{LN}, \tau_{LN})$. Then the set of all points t in (S_{LN}, τ_{LN}) such that s and t cannot be separated by LNS separation of S_{LN} is called as the LN quasi-semi-component of s.

Remark 3.8. A LN quasi semi-component of s in a LNTS (S_{LN}, τ_{LN}) is the intersection of all LNSO sets containing s.

Theorem 3.9. *If a LNTS is LN semi-regular and LN-T_0, then the space is LN semi-T_2.*

Proof: Since S_{LN} is LN-T_0, there lies a LNO set U_{LN} containing either of the points s_1 or s_2 in S_{LN}. Thus, $S_{LN} \backslash U_{LN}$ is LNC set such that $s_1 \notin S_{LN} \backslash U_{LN}$. Then, there lie disjoint LNSO E_{LN}, F_{LN} with $S_{LN} \backslash U_{LN} \subseteq E_{LN}$, $s_1 \in F_{LN}$.

Remark 3.10. A LN semi-T_2 space need not be LN semi-regular space.

Example 3.11. Let the universe of discourse be $U = \{a, b, c\}$. The set of all linguistic term is, $L = \{$ very salt(l_0), salt(l_1), very sour(l_2), sour(l_3), bitter(l_4), sweet(l_5), very sweet(l_6)$\}$. Let

$S_{LN} = \{a\}$. Let $s_{1\langle a,l_0,l_2,l_6\rangle}$ be a LN point in S_{LN} and let $F_{LN} = \langle a, (l_1, l_0, l_7)\rangle$ be a LNSCS in S_{LN} such that $s_1 \notin F_{LN}$. Also, $A_{LN} = \langle a, (l_1, l_1, l_7)\rangle$ and $B_{LN} = \langle a, (l_2, l_0, l_6)\rangle$ are the LNSOSs in S_{LN}. Then, $s_1 \notin A_{LN}$ but $F_{LN} \subseteq B_{LN}$ respectively, which proves that S_{LN} is not LNS-regular.

Theorem 3.12. *For each LN set E_{LN} such that $s \notin E_{LN}$, where $s \in S_{LN}$, there lies a LNSO set U_{LN} in S_{LN} containing s with $LNSCl(U_{LN}) \cap E_{LN} = \phi$ if and only if the LN space S_{LN} is semi-regular.*

Proof: Necessity Part: Let $s \in S_{LN}$ be arbitrary and $s \notin E_{LN}$, where E_{LN} is any LNC set in S_{LN}. Then, there exists $U_{LN} \in LNSO(S_{LN}, s)$ such that $LNSCl(U_{LN}) \cap E_{LN} = \phi$. Thus, $E_{LN} \subseteq S_{LN} \backslash LNSCl(U_{LN})$.

Sufficiency Part: Let $s \in S_{LN}$ be arbitrary and $s \notin E_{LN}$, where E_{LN} is any LNC set in S_{LN}.Then, $S_{LN} \backslash E_{LN}$ is LNO set containing s. From the hypothesis, there lies a LNSO set U_{LN} containing s with $LNSCl(U_{LN}) \subseteq S_{LN} \backslash E_{LN}$.

Theorem 3.13. *A LN space S_{LN} is semi-regular if and only if for each LNC set E_{LN} and for $s \notin E_{LN}$, there lies a LNSO subsets C_{LN} and D_{LN} in S_{LN} such that $s \in C_{LN}$ and $E_{LN} \subseteq D_{LN}$. Also, $LNCl(C_{LN}) \cap LNCl(D_{LN}) = \phi$.*

Proof: Necessity Part: Let E_{LN} be any LNC set that is not containing the point s in S_{LN}. Then, there exists two disjoint LNSO sets $(C_s)_{LN}$ and F_{LN} in S_{LN} such that $E_{LN} \subseteq F_{LN}, s \in C_{LN}$. Then, there lies two LNSO sets U_{LN} and V_{LN} with zero inter section in S_{LN} such that $LNCl(F_{LN}) \subseteq V_{LN}$ and $s \in U_{LN}$, since $LNCl(F_{LN})$ is a LNC subset in S_{LN} that is not containing s. Then, $LNCl(U_{LN}) \cap V_{LN} = \phi$. Now, $(C_s)_{LN} \cap U_{LN}$ is a LNSO set different from F_{LN} such that $s \in C_{LN}$ and $E_{LN} \subseteq F_{LN}$.

Sufficiency Part: For any LNC set E_{LN} that is not containing the point s of S_{LN}, there exists LNSO subsets C_{LN} and D_{LN} in S_{LN} such that $s \in C_{LN}$ and $E_{LN} \subseteq D_{LN}$. Moreover, $LNCl(C_{LN}) \cap LNCl(D_{LN}) = \phi$.

Theorem 3.14. *If $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is injective, LN closed and LN irresolute function and (T_{LN}, η_{LN}) is LN semi-regular space, then S_{LN} is LN semi-regular space.*

Proof: For any $s \in S_{LN}$ and for any LN subset E_{LN}, we have $s \notin E_{LN}$. Thus, there lie LNSO sets U_{LN} and V_{LN} with zero intersection in T_{LN} with $f_{LN}(s) \in U_{LN}$ and $f_{LN}(E_{LN}) \in V_{LN}$. For any LN irresolute function, the LNSO sets $(f_{LN})^{-1}(U_{LN})$ and $(f_{LN})^{-1}(V_{LN})$ are disjoint, so that $s \in (f_{LN})^{-1}(U_{LN})$ and $E_{LN} \subseteq (f_{LN})^{-1}(V_{LN})$.

Theorem 3.15. *For any LN space (S_{LN}, τ_{LN}) the following are equivalent.*

(a) *the space (S_{LN}, τ_{LN}) is LN semi-regular.*

(b) *For each $s \in S_{LN}$ and for each LN U_{LN} containing s, there lies a LNSO set V_{LN} containing s such that $LNSCl(V_{LN}) \subseteq U_{LN}$.*

(c) *For each non-void set A_{LN}, different from a LNO set U_{LN}, there lies a LNSO set V_{LN} such that $A_{LN} \cap V_{LN} \notin \phi$ and $LNSCl(V_{LN}) \subseteq U_{LN}$.*

(d) *For each non-void set A_{LN}, different from a LNC set E_{LN}, there lie two LNSO sets U_{LN} and V_{LN} such that $A_{LN} \cap V_{LN} \notin \phi$ and $E_{LN} \subseteq U_{LN}$.*

Proof: $(a) \Rightarrow (b)$: Let U_{LN} be any LNO set U_{LN} such that $s \in U_{LN}$. Then, $S_{LN} \backslash U_{LN}$ is LNC set that is not containing s. Then, there lie LNSO sets V_{LN} and W_{LN} with zero intersection so that $S_{LN} \backslash U_{LN} \subseteq W_{LN}, s \in V_{LN} \subseteq LNSCl(V_{LN})$. Suppose that $s_1 \in S_{LN}$ such that $s_1 \notin U_{LN}$, then W_{LN} is LNSO set containing s_1 such that $V_{LN} \cap W_{LN} = \phi$. Now, $s_1 \in LNSCl(V_{LN})$ and thus $LNSCl(V_{LN}) \subseteq U_{LN}$.

$(b) \Rightarrow (c)$: Let A_{LN} be a non-void LN set which has zero intersection with a LNO subset U_{LN} of S_{LN}. Let $s \in A_{LN} \cap U_{LN}$. Then, U_{LN} is a LNSO subset V_{LN} of S_{LN} such that $s \in V_{LN} \subseteq LNSCl(V_{LN}) \subseteq U_{LN}$.

$(c) \Rightarrow (d)$: Let A_{LN} be a non-void LN set which has zero intersection with a LNC subset E_{LN} of S_{LN}, then $S_{LN} \backslash E_{LN}$ is a LNC set such that $A_{LN} \cap (S_{LN} \backslash E_{LN}) \neq \phi$. By the assumption, there lies a LNSO set V_{LN} with $A_{LN} \cap V_{LN} \neq \phi$ and $LNSCl(V_{LN}) \subseteq S_{LN} \backslash E_{LN}$.

$(d) \Rightarrow (a)$: Suppose let E_{LN} be any LNC set in S_{LN} and $s \notin S_{LN}$. If $A_{LN} = \{s\}$, then $A_{LN} \cap E_{LN} = \phi$. Then, there lies disjoint LNSO sets U_{LN} and V_{LN} such that $A_{LN} \cap V_{LN} \neq \phi$ and $E_{LN} \subseteq U_{LN}$.

Theorem 3.16. *Every LN semi-regular space (S_{LN}, τ_{LN}) is LN semi-normal.*

Proof: Let A_{LN} and B_{LN} be any two LNC sets that has void intersection and $s \in A_{LN}$, then $s \notin B_{LN}$. Then, there lie two different LNSO sets U_s, V_s with $s \in U_s$, $B_{LN} \subseteq V_s$. Thus, $U_{LN} = \bigcup_{s \in A_{LN}} U_s$ is a LNSO set in S_{LN} such that $A_{LN} \subseteq U_{LN}$. Moreover, $U_{LN} \cap V_s = \phi$, (i.e) (S_{LN}, τ_{LN}) is LN semi normal.

Remark 3.17. A LN semi-normal space need not be LN semi-regular in general, which is clear from the following example.

Example 3.18. Let the universe of discourse be $\mathcal{U} = \{x, y, z, w\}$ and let $S_{LN} = \{x, y, z\}$. And LTS be as in Example (3.3). The space S_{LN} is LNS-normal, by Example (3.5). The LNSCS E_{LN} is given by, $E_{LN} = \langle (x, l_1, l_2, l_5), (y, l_0, l_1, l_6), (z, l_6, l_5, l_9) \rangle$ and let the point s be $s_{\langle (x,l_2,l_6,l_2),(y,l_3,l_4,l_2),(z,l_6,l_4,l_7) \rangle}$. Now, for the LNSOS's $K_{LN} = \langle (x, l_5, l_6, l_1), (y, l_4, l_6, l_1), (z, l_9, l_8, l_5) \rangle$, and $H_{LN} = \langle (x, l_3, l_5, l_4), (y, l_1, l_6, l_2), (z, l_7, l_5, l_8) \rangle$, the inclusion relationship $s \notin H_{LN}$ does not hold. Thus the space is not LNS-regular.

Remark 3.19. A LN semi-normal space is a LN semi-regular if and only if the LN space is LN semi-R_0.

Example 3.20. Let the universe of discourse $\mathcal{U}$ and LTS be as in Example (3.3). Let LNP be $s_{\langle(x,l_1,l_0,l_2),\ (y,l_1,l_3,l_3),\ (z,l_4,l_5,l_6)\rangle}$, which is different from the LNSCS $E_{LN} = \langle(x,l_1,l_3,l_4),\ (y,l_1,l_6,l_5),\ (z,l_5,l_8,l_9)\rangle$.
The LNSOSs are given by $K_{LN} = \langle(x,l_5,l_6,l_1),\ (y,l_4,l_6,l_1),\ (z,l_9,l_8,l_5)\rangle$, $H_{LN} = \langle(x,l_3,l_5,l_4),\ (y,l_1,l_6,l_2),\ (z,l_7,l_5,l_8)\rangle$.
Now, $s \notin H_{LN}$ and $E_{LN} \subset K_{LN}$ and thus S_{LN} is LNS-regular.

Theorem 3.21. *If (S_{LN},τ_{LN}) is LN semi-R_0 and semi-normal then LN space is LN semi-regular.*

Proof: Let $s \notin K_{LN} \in LNC(S_{LN},\tau_{LN})$. As the space is LN semi-R_0, we have $LNCl(\{s\}) \subseteq S_{LN}\backslash K_{LN}$ and $LNCl(\{s\}) \cap K_{LN} = \phi$. Also, there lie LNSO sets U_{LN} and V_{LN} with $LNCl(\{s\}) \subseteq U_{LN}$, $K_{LN} \subseteq V_{LN}$ with $s \in U_{LN}, K_{LN} \subseteq V_{LN}$ and $U_{LN} \cap V_{LN} = \phi$.

Theorem 3.22. *For any two LNC sets C_{LN}, D_{LN} of (S_{LN},τ_{LN}), there lies a LNSO set $U_{LN} \subseteq S_{LN}$ containing A_{LN} and $LNCl(U_{LN}) \cap D_{LN} = \phi$ holds if and only if the space (S_{LN},τ_{LN}) is LN semi-normal.*

Proof: Necessity Part: Let S_{LN} be a LN semi-normal space and suppose that C_{LN} and D_{LN} be any two disjoint LNC sets in S_{LN}, then $C_{LN} \subseteq S_{LN}\backslash D_{LN}$. Then, there lies a LNSO set U_{LN} with $A_{LN} \subseteq U_{LN} \subseteq LNCl(U_{LN}) \subseteq S_{LN}\backslash D_{LN}$.
Sufficiency Part: Suppose C_{LN} and D_{LN} be any two disjoint LNC sets in S_{LN}. From the hypothesis, there lies a LNSO set U_{LN} in S_{LN} containing C_{LN} and $LNScl(U_{LN}) \cap D_{LN} = \phi$.

Theorem 3.23. *If $f_{LN} : (S_{LN},\tau_{LN}) \to (T_{LN},\eta_{LN})$ is injective, closed and LN irresolute function and (T_{LN},η_{LN}) is LN semi-normal space, then S_{LN} is LN semi-normal space.*

Proof: Let A_{LN} and B_{LN} be any two LNC sets in S_{LN} such that $A_{LN} \cap B_{LN} = \phi$. Now, $f_{LN}(A_{LN})$ and $f_{LN}(B_{LN})$ are also LNC in T_{LN}. Moreover, f_{LN} is injective, $f(A_{LN})$ and $f(B_{LN})$ are disjoint LNC in T_{LN}. Now, there lies a LNSO $U_{LN} \subseteq S_{LN}$ with $f_{LN}(A_{LN}) \subseteq U_{LN}$ and $f_{LN}(B_{LN}) \subseteq V_{LN}$, as the space T_{LN} is LN semi-normal. As the function f_{LN} is LN irresolute, the reverse images $(f_{LN})^{-1}(U_{LN})$ and $(f_{LN})^{-1}(V_{LN})$ are disjoint LNSO in S_{LN} with $A_{LN} \subseteq (f_{LN})^{-1}(U_{LN})$ and $B_{LN} \subseteq (f_{LN})^{-1}(V_{LN})$ respectively.

Definition 3.24. A function $f_{LN} : (S_{LN},\tau_{LN}) \to (T_{LN},\eta_{LN})$ is LN semi-totally continuous if the reverse image of every LNSO set is a LNCO subset of (S_{LN},τ_{LN}).

Theorem 3.25. *Let $f_{LN} : (S_{LN},\tau_{LN}) \to (T_{LN},\eta_{LN})$ be LN semi-totally continuous and f_{LN} be injective. Also, the LN space (T_{LN},η_{LN}) LN semi-T_1, then the LN space (S_{LN},τ_{LN}) is LNCO-T_1.*

Proof: As f_{LN} is injective, $f_{LN}(s_1) \neq f_{LN}(s_2)$, where $f_{LN}(s_1), f_{LN}(s_2) \in T_{LN}$. As T_{LN} is LN semi-T_1, there lie LNSOS's E_{LN} and F_{LN} with $f_{LN}(s_1) \in E_{LN}, f_{LN}(s_2) \notin E_{LN}$ and $f_{LN}(s_2) \in F_{LN}, f_{LN}(s_1) \notin F_{LN}$. Thus, $s_1 \in (f_{LN})^{-1}(E_{LN}), s_2 \notin f^{-1}(E_{LN})$ and $s_2 \in (f_{LN})^{-1}(F_{LN}), s_1 \notin (f_{LN})^{-1}(F_{LN})$, where $(f_{LN})^{-1}(E_{LN})$ and $(f_{LN})^{-1}(F_{LN})$ are LNCO subsets of S_{LN}.

Theorem 3.26. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be LN semi-totally continuous and f_{LN} be injective. Also, the LN space (T_{LN}, η_{LN}) LN semi-T_0, then the LN space (S_{LN}, τ_{LN}) is LN ultra-hausdorff.*

Proof: Let s_1 and s_2 be any two points in S_{LN}. As f_{LN} is injective, $f_{LN}(s_1) \neq f_{LN}(s_2)$, where $f_{LN}(s_1), f_{LN}(s_2) \in T_{LN}$. As T_{LN} is LN semi-T_0, there lies a LNSO set E_{LN} containing $f_{LN}(s_1)$ but not $f_{LN}(s_2)$. Then, $s_1 \in (f_{LN})^{-1}(E_{LN})$ and $s_2 \notin (f_{LN})^{-1}(E_{LN})$. As f_{LN} is LN semi-totally continuous, $(f_{LN})^{-1}(E_{LN})$ is LNCOs in S_{LN}. Moreover, $s_1 \in (f_{LN})^{-1}(E_{LN})$ and $s_2 \in S_{LN} \backslash (f_{LN})^{-1}(E_{LN})$.

Theorem 3.27. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be LN semi-totally continuous and f_{LN} be injective. Also, the LN space (T_{LN}, η_{LN}) LN semi-T_2, then the LN space (S_{LN}, τ_{LN}) is LN ultra-hausdorff.*

Proof: Let $s_1, s_2 \in S_{LN}$ with $s_1 \neq s_2$. As f_{LN} is injective, we have, $f_{LN}(s_1) \neq f_{LN}(s_2)$. In addition, (T_{LN}, η_{LN}) is LN semi T_2, there lie LNSOS's K_{LN} and H_{LN} with $f_{LN}(s_1) \in K_{LN}, f_{LN}(s_2) \in H_{LN}$ and $K_{LN} \cap H_{LN} = \phi$. Then, $s_1 \in (f_{LN})^{-1}(K_{LN})$ and $s_2 \in (f_{LN})^{-1}(HLN)$. As f_{LN} is LN semi-totally continuous, $(f_{LN})^{-1}(K_{LN})$ and $(f_{LN})^{-1}(H_{LN})$ are LNCO in S_{LN}. Moreover, $(f_{LN})^{-1}(K_{LN}) \cap (f_{LN})^{-1}(H_{LN}) = (f_{LN})^{-1}(K_{LN} \cap H_{LN}) = \phi$.

Theorem 3.28. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be LN semi-totally continuous, injective and LN semi-open mapping from LNCO regular topological space (S_{LN}, τ_{LN}) into a LN space (T_{LN}, η_{LN}). Then (T_{LN}, η_{LN}) is LN semi-regular.*

Proof: Let K_{LN} be a LNSC set in T_{LN} and $t \notin K_{LN}$. Since f is LN semi-totally continuous, $(f_{LN})^{-1}(K_{LN})$ is LNCO set in S_{LN}. Then, $(f_{LN})^{-1}(t) \notin (f_{LN})^{-1}(K_{LN})$. As S_{LN} is LNCO regular, there lie distinct LNO sets A_{LN} and B_{LN} such that ${f^{-1}}_{LN}(K_{LN}) \subset A_{LN}$ and $(f_{LN})^{-1}(t) \in B_{LN}$. Thus, $K_{LN} \subset f_{LN}(A_{LN})$ and $t \in f_{LN}(B_{LN})$. Also, as the map f_{LN} is LNSO and injective, we have, $f_{LN}(A_{LN})$ and $f_{LN}(B_{LN})$ are LNSO sets and $f_{LN}(A_{LN}) \cap f_{LN}(B_{LN}) = f_{LN}(A_{LN} \cap B_{LN}) = \phi$.

Theorem 3.29. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be LN semi-totally continuous, injective and LN semi-closed function. If T_{LN} is LN semi-regular, then (S_{LN}, η_{LN}) is LN ultra-regular.*

Proof: Let H_{LN} be a LNC set and $s \notin H_{LN}$ in (S_{LN}, τ_{LN}). As f_{LN} is LNSC, $f_{LN}(H_{LN})$ is LNSC set in T_{LN}, not containing $f_{LN}(s)$. As T_{LN} is LN semi-regular, there lie distinct LNSOS's

A_{LN}, B_{LN} with $f_{LN}(s) \in A_{LN}$, $f_{LN}(H_{LN}) \subset B_{LN}$. Then, we have, $s \in (f_{LN})^{-1}(A_{LN})$ and $H_{LN} \subset (f_{LN})^{-1}(B_{LN})$. Because the function f_{LN} is LN semi-totally continuous, the LN sets $(f_{LN})^{-1}(A_{LN})$ and $(f_{LN})^{-1}(B_{LN})$ are LNCOS's. As f_{LN} is injective, $(f_{LN})^{-1}(A_{LN}) \cap (f_{LN})^{-1}(B_{LN}) = (f_{LN})^{-1}(A_{LN} \cap B_{LN}) = (f_{LN})^{-1}(\phi) = \phi$.

Theorem 3.30. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be LN semi-totally continuous, injective and LNSO function from LNCO normal topological space (S_{LN}, τ_{LN}) into a LN space (T_{LN}, η_{LN}), then (T_{LN}, η_{LN}) is LN semi-normal.*

Proof: As f_{LN} is LN semi-totally continuous, $(f_{LN})^{-1}(U_{LN})$ and $(f_{LN})^{-1}(V_{LN})$ are LNCOS's in S_{LN}, where U_{LN} and V_{LN} be two different LNSC sets in T_{LN}. As f_{LN} is injective, $(f_{LN})^{-1}(U_{LN}) \cap (f_{LN})^{-1}(V_{LN}) = (f_{LN})^{-1}(\phi) = \phi$. Then, there lies disjoint LNO's A_{LN} and B_{LN} with $U_{LN} \subset A_{LN}$ and $V_{LN} \subset B_{LN}$, (i.e)$f_{LN}(U_{LN}) \subset f_{LN}(A_{LN})$ and $f_{LN}(V_{LN}) \subset f_{LN}(B_{LN})$. As f_{LN} is injective and LNSO, $f_{LN}(A_{LN})$ and $f_{LN}(B_{LN})$ are disjoint LNSOS's.

Theorem 3.31. *Let any collection of LNSO sets be a LNSO set. And let (T_{LN}, η_{LN}) be a LN urysohn space. If for two different points s_1 and s_2 of (S_{LN}, τ_{LN}), there exists a LN function $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ with $f_{LN}(s_1) \neq f_{LN}(s_2)$ and the map f_{LN} is LN contra-semi continuous at s_1, s_2, ergo (S_{LN}, τ_{LN}) is LN semi-T_2 space.*

Proof: Let (T_{LN}, η_{LN}) be a LN urysohn space and s_1 and s_2 be two distinct points in S_{LN}. Then $f_{LN}(s_1) \neq f_{LN}(s_2)$ in T_{LN}. Also, since (T_{LN}, η_{LN}) is a LN urysohn space, there lie open neighborhoods $V_{f_{LN}(s_1)}$ and $V_{f_{LN}(s_2)}$ in (T_{LN}, η_{LN}) containing $f_{LN}(s_1)$ and $f_{LN}(s_2)$ such that $LNCl(V_{f_{LN}(s_1)}) \cap LNCl(V_{f_{LN}(s_2)}) = \phi$. There lie LNSOS's U_{s_1}, U_{s_2} containing respectively s_1, s_2 with $f_{LN}(U_{s_1}) \subseteq LNCl(V_{f_{LN}(s_1)})$ and $f_{LN}(U_{s_2}) \subseteq LNCl(V_{f_{LN}(s_2)})$. Then, $f_{LN}(U_{s_1} \cap U_{s_2}) \subseteq f_{LN}(U_{s_1}) \cap f_{LN}(U_{s_2}) \subseteq LNCl(V_{f_{LN}(s_1)}) \cap LNCl(V_{f_{LN}(s_2)}) = \phi$, (i.e) $f_{LN}(U_{s_1} \cap U_{s_2}) = \phi$.

Theorem 3.32. *Let the collection of any number of LNSO's be a LNSO set. Then, if the map f_{LN} is LN contra-semi continuous and injective in (S_{LN}, τ_{LN}) and (T_{LN}, η_{LN}) is a LN ultra-hausdorff space, then (S_{LN}, τ_{LN}) is LN semi-T_2 space.*

Proof: For any two points s_1 and s_2 in S_{LN}, we have $f_{LN}(s_1) \neq f_{LN}(s_2)$ as the map f_{LN} is injective. Then, there lie two LNCOS's E_1, E_2 with $f_{LN}(s_1) \in E_1$, $f_{LN}(s_2) \in E_2$ and $E_1 \cap E_2 = \phi$ and there lie LNOS's H_1, H_2 with $f_{LN}(H_1) \subseteq E_1$ and $f_{LN}(H_2) \subseteq E_2$ respectively. Then, $H_1 \subseteq (f_{LN})^{-1}(E_1)$ and $H_2 \subseteq (f_{LN})^{-1}(E_2)$,(i.e) $H_1 \cap H_2 \subseteq (f_{LN})^{-1}(E_1) \cap (f_{LN})^{-1}(E_2) = (f_{LN})^{-1}(E_1 \cap E_2) = (f_{LN})^{-1}(\phi)$. Therefore, $H_1 \cap H_2 = \phi$.

Theorem 3.33. *Let LN function $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be LN contra-semicontinuous, injective and LN closed function. Then the LN space (S_{LN}, τ_{LN}) is LN semi-normal if (T_{LN}, η_{LN}) is LN-ultra normal.*

Proof: Let B_1 and B_2 be two different LNC sets in (S_{LN}, τ_{LN}), then $f_{LN}(B_1)$ and $f_{LN}(B_2)$ are different LNC sets in (T_{LN}, η_{LN}), as the mapping f_{LN} is injective and LN closed. There lie two LNCOS's U_1, U_2 which separates $f_{LN}(B_1)$ and $f_{LN}(B_2)$ in T_{LN} respectively. Thus, $f_{LN}(B_1) \subseteq U_1$ and $f_{LN}(B_2) \subseteq U_2$,(i.e)$B_1 \subseteq (f_{LN})^{-1}(U_1)$ and $B_2 \subseteq (f_{LN})^{-1}(U_2)$ such that $(f_{LN})^{-1}(U_1) \cap (f_{LN})^{-1}(U_2) = \phi$. It is shown that $(f_{LN})^{-1}(U_1)$ and $(f_{LN})^{-1}(U_2)$ are two different LNSO with $B_1 \subseteq (f_{LN})^{-1}(U_1)$ and $B_2 \subseteq (f_{LN})^{-1}(U_2)$ in S_{LN}.

Definition 3.34. A LN graph $LNGR(f_{LN})$ of a function $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is LN contra-semi closed graph if a LNSO set E_{LN} and a LNC set K_{LN} lie with the property $(E_{LN} \cap K_{LN}) \cap LNGR(f_{LN}) = \phi$ for all $(s,t) \in (S_{LN} \times T_{LN})\ LNGR(f_{LN})$.

Theorem 3.35. *Let any union of LNSO sets be a LNSO set and* $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ *be a function and* $g_{LN} : (S_{LN}, \tau_{LN}) \to (S_{LN}, \tau_{LN}) \times (T_{LN}, \eta_{LN})$ *be the LN graph function given by* $g(s) = (s, f_{LN}(s))$ *for each* $s \in S_{LN}$*. Then* g_{LN} *is LN contra-semi continuous if and only if the map* f_{LN} *is LN contra-semi continuous.*

Proof: Necessity Part: Let V_{LN} be any LNC set of $(S_{LN}, \tau_{LN}) \times (T_{LN}, \eta_{LN})$ containing $g_{LN}(s)$ where $s \in S_{LN}$. Then $V_{LN} \cap (\{s\} \times \{T_{LN}\})$ containing $g_{LN}(s)$. Moreover $\{s\} \times \{T_{LN}\}$ is homeomorphic to T_{LN} and hence $\{t : (s,t) \in V_{LN}\}$ is a LNC subset of T_{LN}. As f_{LN} is LN contra-semi continuous, $\bigcup\{(f_{LN})^{-1}(t) : (s,t) \in V_{LN}\}$ is LNSO in S_{LN} such that $s \in \bigcup\{(f_{LN})^{-1}(t) : (s,t) \in V_{LN}\} \subseteq (g_{LN})^{-1}(V_{LN})$. Thus, $(g_{LN})^{-1}(V_{LN})$ is LNSO.
Sufficiency Part: Let U_{LN} be any LNC subset of T_{LN}. Then, $S_{LN} \times U_{LN} = S_{LN} \times LNCl(U_{LN}) = LNCl(S_{LN} \times U_{LN}) \subseteq S_{LN} \times T_{LN}$ and $S_{LN} \times U_{LN}$ is LNC. Then, $(g_{LN})^{-1}(S_{LN} \times U_{LN})$ is LNSO in S_{LN} as LN contra- semicontinuous. Moreover, $(g_{LN})^{-1}(S_{LN} \times U_{LN}) = (f_{LN})^{-1}(U_{LN})$.

Lemma 3.36. *A LN graph* $LNGR(f_{LN})$ *of a function* $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ *is LN contra-semi closed graph if there exist a LNSO set* E_{LN} *and a LN closed set* K_{LN} *such that* $f_{LN}(E_{LN}) \cap K_{LN} = \phi$.

Theorem 3.37. *Let* $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ *be an injective function and LN contra-semi closed graph, then* S_{LN} *is LN semi-*T_1 *space.*

Proof: Since f_{LN} is LN contra-semi closed graph, $(s, f(t)) \in (S_{LN} \times T_{LN})\ LNGR(f_{LN})$, where s and t are different points of S_{LN}. Then by lemma, (3.25), a LNSO set U_{LN} lies in S_{LN} that contain s and a LNC set V_{LN} lies in T_{LN} that contain $f_{LN}(t)$ with $f_{LN}(U_{LN}) \cap V_{LN} = \phi$. Ergo, S_{LN} is LN semi-T_1 space, as $t \notin U_{LN}$.

Theorem 3.38. *Let* $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ *be LN contra-semi continuous where* (T_{LN}, η_{LN}) *is LN-urysohn space, then the graph of* f_{LN} *is LN contra-semi closed in* $(S_{LN} \times T_{LN})$.

Proof: Let $(s, t) \in (S_{LN} \times T_{LN})\ LNGR(f_{LN})$, then $f_{LN}(s) \neq t$. Now, there lie LNO sets E_{LN} and F_{LN} in T_{LN} such that $f_{LN}(s) \in E_{LN}$ and $t \in F_{LN}$ such that $LNCl(E_{LN}) \cap LNCl(F_{LN}) = \phi$. Now, there exists a LNSO set $K_{LN} \in (S_{LN}, s)$ such that $f_{LN}(K_{LN}) \subseteq LNCl(E_{LN})$, as the function f_{LN} is LN contra-semi continuous. Thus, $f_{LN}(E_{LN}) \cap LNCl(F_{LN}) \subseteq LNCl(E_{LN}) \cap LNCl(F_{LN}) = \phi$. Then, $LNGR(f_{LN})$ is LN contra-semi closed, by lemma (3.25).

Conclusion:

In this study, the characterization of linguistic neutrosophic spaces and cl-open spaces are discussed. The inter connections among these also have studied. Appropriate examples are given to explicate the results and connections. We hope that these inception works will be useful for scholars to progress the research in linguistic neutrosophic topology.

References

[1] Atanassov, K.T. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20, 87-96.

[2] Chang, C.L. (1968). Fuzzy topological spaces, J Math.Anal.Appl.24, 182-190.

[3] Coker, D. (1997). An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems,88,81-89.

[4] Dorsett,C. (1985). Semi-normal spaces, Kyungpook Math. J., 25, 173-180.

[5] Ekici.E, Caldas.M. (2004). Slightly -continuous functions, Bol. Soc. Paran. Mat., 22(2), 63-74.

[6] Gayathri.N, Helen.M. (2021). Linguistic Neutrosophic Topology, Neutrosophic Sets and Systems, 46, 254-267.

[7] Gayathri.N, Helen.M. (2022). Semi-Separation axioms and Semi-regularity Axioms in Linguistic Neutrosophic Topological Spaces, Neutrosophic Sets and Systems, 51, 861-874.

[8] Fang, Zebo and Te, Jun. (2017). Multiple Attribute Group Decision-Making Method Based on Linguistic Neutrosophic Numbers.Symmetry,9(7), 111.

[9] Nour.T. M. (1995). Totally semi-continuous functions, Indian J. Pure Appl.Math., 26(7), 675 - 678.

[10] Salama.A.A, Alblowi.S.A. (2012). Neutrosophic set and Neutrosophic topological spaces, IOSR journal of Mathematics, 3(4), 31-35.

[11] Smarandache F.(1999). A unifying field in logics. Neutrosophy: Neutrosophic probability, set and logic. American Research Press, Rehoboth.

[12] Zadeh, L.A. (1965). Fuzzy Sets. Information and Control, 8, 338-353.

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