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Logarithmic Similarity Measure of Neutrosophic Z-Number Sets for Undergraduate Teaching Quality Evaluation

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Abstract: In practical decision-making problems, the different reliability levels of decision-makers in their evaluation information are usually ignored by most existing decision-making methods. Then, a neutrosophic Z-number (NZN) is a powerful model for simultaneously describing the restraint and associated reliability of evaluation information in view of truth, indeterminacy, and falsity Z-numbers. This paper first proposed a logarithmic similarity measure (LSM) of NZN sets, which is more flexible and can change its measure result by adjusting the exponential parameters of the restraint and reliability. Then, a multi-attribute decision-making approach is developed based on the presented LSM of NZN sets, and its applicability and flexibility are further illustrated by a case study of undergraduate teaching quality assessment.

Keywords: teaching quality evaluation; neutrosophic Z-number; logarithmic similarity measure; confidence degree

1. Introduction

Teaching quality assessment is an effective way to accelerate the development of universities and to ensure the quality of university education. Resulting from the ambiguity and uncertainty of human thinking, various fuzzy theories have already been widely used for the teaching quality assessment, such as the spherical fuzzy set (FS) [1], the q-rung orthopair FS [2], the triangular Pythagorean FS [3], the fuzzy rough set [4], the interval-valued (IV) Pythagorean FS [5], the IV dual-hesitant FS [6], the IV intuitionistic FS [7], the IV hesitant fuzzy linguistic sets [8], and the Plithogenic cubic vague sets [9]. Especially in recent years, the neutrosophic decision-making (DM) approaches [10-11] were introduced in incomplete, uncertainty and inconsistent environments. For example, researchers successively proposed various DM approaches using the neutrosophic reducible weighted Maclaurin symmetric mean [12], the hyperbolic sine similarity measure for neutrosophic multivalued sets [13], the tangent trigonometric single-valued neutrosophic number (SvNN) aggregation operators [14], the TOPSIS method of simplified neutrosophic indeterminate sets [15], the grey relation analysis (GRA) method of 2-tuple linguistic neutrosophic numbers [16], the improved GRA method of probabilistic simplified neutrosophic sets [17], the combined compromise solution method of double-valued neutrosophic sets(NSs) [18], the GRA method of interval-valued NSs [19], and so on. However, the above methods only emphasize the assessment data, but ignore the fact that the evaluators usually imply the measures/levels of reliability related to their assessment data.

Fortunately, to fully describe human judgments, Zadeh [20] first defined the notion of Z-numbers, where a pair of fuzzy numbers is used to represent the restraint and reliability of evaluation information. Subsequently, Z-numbers had been further studied in both theory and applications, such as the arithmetic for discrete [21] or continuous Z-numbers [22], the approximate reasoning [23] according to Z-number-valued if-then rules, and the DM approaches for supplier selection [24], medicine selection [25], and environmental assessment [26]. Furthermore, to express indeterminate and inconsistent information with reliability measures, a neutrosophic Z-number (NZN) [27] was proposed, which uses three order pairs of fuzzy numbers to demonstrate the restraint and reliability of evaluation information in terms of truth, indeterminacy, and falsity. Weighted aggregation operators such as Dombi [28], Aczel-Alsina [29], and similarity measures on the basis of the generalized distance [30], and the correlation coefficient based on mean, variance, and covariance [31] had been proposed for multi-attribute DM (MADM) in NZN setting. As the extended forms of NZN, the trapezoidal NZN [32], and the linguistic NZN [33] had also been proposed for DM problems.

However, the logarithmic similarity measure (LSM) [34] is an effective tool for MADM. Then, NZN can ensure the levels of reliability of true, false and indeterminate values. Therefore, this paper proposes a generalized LSM of NZN sets (NZNLSM), in which the weights of the restraint and reliability of evaluation information can be adjusted according to the decision preferences of the evaluators. Specifically, as the weight of the reliability measure increases, the outcome of the decision will be more important by the reliability level of the evaluator. Furthermore, based on the proposed LSM of NZNSs, this paper developed a generalized DM method for performing MADM problems, and demonstrates the effectiveness and flexibility of the approach using the undergraduate teaching quality evaluation (UTQE) as an example.

In the rest of the paper, the basic notion of a NZN set and the definition of LSM of NZNSs are introduced in Section 2. Then, the MADM approach applying LSM of NZNSs is presented in Section 3. Furthermore, an example of UTQE is illustrated in Section 4, and both the comparison analysis and the sensitivity analysis are carried out in more detail. Lastly, the conclusion and the relative future research plan are shown in Section 5.

2. Neutrosophic Z-number sets

Du et al [27] put forward the neutrosophic Z-number set (NZNS) and its basic operational laws from the definition of the neutrosophic set [10-11] and Z-number [20].

Definition 1 [27]**.** A NZNS in a universe set S can be expressed as

$$X = \left\{ \left\langle s, \left(T_D(s), T_R(s)\right), \left(I_D(s), I_R(s)\right), \left(V_D(s), V_R(s)\right) \right\rangle | s \in S \right\}. \quad (1)$$

where $(T_D(s), T_R(s))$, $(I_D(s), I_R(s))$, and $(V_D(s), V_R(s))$ are the sequence pairs of truth, indeterminacy and falsity fuzzy values, and each pair is given by a Z-number that consists of the evaluation values such as $T_D(s_k)$, $I_D(s_k)$, $V_D(s_k)$ and the reliability measures such as $T_R(s_k)$, $I_R(s_k)$ and $V_R(s_k)$ corresponding to the evaluation values. Here, the element $x = \langle s, (T_D(s), T_R(s)), (I_D(s), I_R(s)), (V_D(s), V_R(s))\rangle$ in X is a NZN, with the conditions $T_D(s) + I_D(s) + V_D(s) \in [0, 3]$ and $T_R(s) + I_R(s) + V_R(s) \in [0, 3]$. For convenience, the element $\langle s, (T_D(s), T_R(s)), (I_D(s), I_R(s)), (V_D(s), V_R(s))\rangle$ in X can be simply written as $x = \langle(T_D, T_R), (I_D, I_R), (V_D, V_R)\rangle$, which is called NZN.

For two NZNs denoted by $x_a = \langle(T_{Da}, T_{Ra}), (I_{Da}, I_{Ra}), (V_{Da}, V_{Ra})\rangle$ and $x_b = \langle(T_{Db}, T_{Rb}), (I_{Db}, I_{Rb}), (V_{Db}, V_{Rb})\rangle$, there are the following relations:

(1) $x_a \oplus x_b = \left\langle \left(T_{Da} + T_{Db} - T_{Da}T_{Db}, T_{Ra} + T_{Rb} - T_{Ra}T_{Rb}\right), \left(I_{Da}I_{Db}, I_{Ra}I_{Rb}\right), \left(V_{Da}V_{Db}, V_{Ra}V_{Rb}\right) \right\rangle$;

(2) $x_a \otimes x_b = \left\langle \left(T_{Da}T_{Db}, T_{Ra}T_{Rb}\right), \left(I_{Da} + I_{Db} - I_{Da}I_{Db}, I_{Ra} + I_{Rb} - I_{Ra}I_{Rb}\right), \left(V_{Da} + V_{Db} - V_{Da}V_{Db}, V_{Ra} + V_{Rb} - V_{Ra}V_{Rb}\right) \right\rangle$;

(3) $\lambda x_a = \left\langle \left(1-\left(1-V_{Da}\right)^{\lambda}, 1-\left(1-V_{Ra}\right)^{\lambda}\right), \left(I_{Da}^{\lambda}, I_{Ra}^{\lambda}\right), \left(V_{Da}^{\lambda}, V_{Ra}^{\lambda}\right) \right\rangle$;

(4) $x_a^{\lambda} = \left\langle \left(T_{Da}^{\lambda}, T_{Ra}^{\lambda}\right), \left(1-\left(1-I_{Da}\right)^{\lambda}, 1-\left(1-I_{Ra}\right)^{\lambda}\right), \left(1-\left(1-V_{Da}\right)^{\lambda}, 1-\left(1-V_{Ra}\right)^{\lambda}\right) \right\rangle$;

(5) $x_a \supseteq x_b \Leftrightarrow T_{Da} \ge T_{Db}, T_{Ra} \ge T_{Rb}, I_{Da} \le I_{Db}, I_{Ra} \le I_{Rb}, V_{Da} \le V_{Db}$, and $V_{Ra} \le V_{Rb}$;

(6) $x_a = x_b \Leftrightarrow x_a \supseteq x_b$ and $x_a \subseteq x_b$;

(7) $x_a \cup x_b = \left\langle \left(T_{Da} \vee T_{Db}, T_{Ra} \vee T_{Rb}\right), \left(I_{Da} \wedge I_{Db}, I_{Ra} \wedge I_{Rb}\right), \left(V_{Da} \wedge V_{Db}, V_{Ra} \wedge V_{Rb}\right) \right\rangle$;

(8) $x_a \cap x_b = \left\langle \left(T_{Da} \wedge T_{Db}, T_{Ra} \wedge T_{Rb}\right), \left(I_{Da} \vee I_{Db}, I_{Ra} \vee I_{Rb}\right), \left(V_{Da} \vee V_{Db}, V_{Ra} \vee V_{Rb}\right) \right\rangle$;

(9) $\left(x_a\right)^C = \left\langle \left(V_{Da}, V_{Ra}\right), \left(1-I_{Da}, 1-I_{Ra}\right), \left(T_{Da}, T_{Ra}\right) \right\rangle$.

3. Logarithmic Similarity Measure of NZNSs

This section presents LSM of NZNSs as an extension of the LSM of dynamic neutrosophic cubic sets [34].

Assume that there are two NZNs x_a = <(T_{Da}, T_{Ra}), (I_{Da}, I_{Ra}), (V_{Da}, V_{Ra})> and x_b= <(T_{Db}, T_{Rb}), (I_{Db}, I_{Rb}), (V_{Db}, V_{Rb})>. We can define a function $\zeta(x_a, x_b)$ as

$$\zeta\left(x_a, x_b\right) = \frac{\left|T_{Da}^{\lambda} T_{Da}^{(1-\lambda)} - \left(T_{Db}\right)^{\lambda}\left(T_{Rb}\right)^{(1-\lambda)}\right| + \left|I_{Da}^{\lambda} I_{Da}^{(1-\lambda)} - \left(I_{Db}\right)^{\lambda}\left(I_{Rb}\right)^{(1-\lambda)}\right| + \left|V_{Da}^{\lambda} V_{Da}^{(1-\lambda)} - \left(V_{Db}\right)^{\lambda}\left(V_{Rb}\right)^{(1-\lambda)}\right|}{3}, \tag{2}$$

where the exponential parameter $\lambda \in (0, 1]$.

Let $X(s)$ = {<s_κ, ($T_{Dx}(s_\kappa)$, $T_{Rx}(s_\kappa)$), ($I_{Dx}(s_\kappa)$, $I_{Rx}(s_\kappa)$), ($V_{Dx}(s_\kappa)$, $V_{Rx}(s_\kappa)$)>|$s_\kappa \in S$} and $Y(s)$ = {<s_κ, ($T_{Dy}(s_\kappa)$, $T_{Ry}(s_\kappa)$), ($I_{Dy}(s_\kappa)$, $I_{Ry}(s_\kappa)$), ($V_{Dy}(s_\kappa)$, $V_{Ry}(s_\kappa)$)>|$s_\kappa \in S$} be two NZNSs in $S = \{s_1, s_2, \ldots, s_\eta\}$ whose weights are given by a vector $\tau = \{\tau(s_1), \tau(s_2), \ldots, \tau(s_\eta)\}$ with $\sum_{\kappa=1}^{\eta} \tau\left(s_\kappa\right) = 1$. Therefore, the weighted LSM between $X(s)$ and $Y(s)$ can be calculated by

$$L_w\left(X(s), Y(s)\right) = \sum_{\kappa=1}^{\eta} \tau\left(s_\kappa\right) \log_\varepsilon \left\{\varepsilon - (\varepsilon-1) \times \zeta\left(X\left(s_\kappa\right), Y\left(s_\kappa\right)\right)\right\}$$
$$= \sum_{\kappa=1}^{\eta} \tau\left(s_\kappa\right) \left\{ \log_\varepsilon \left[\varepsilon - (\varepsilon-1) \times \frac{\begin{array}{c} \left|\left(T_{Dx}(s_\kappa)\right)^{\lambda}\left(T_{Rx}(s_\kappa)\right)^{1-\lambda} - \left(T_{Dy}(s_\kappa)\right)^{\lambda}\left(T_{Ry}(s_\kappa)\right)^{1-\lambda}\right| \\ + \left|\left(I_{Dx}(s_\kappa)\right)^{\lambda}\left(I_{Rx}(s_\kappa)\right)^{1-\lambda} - \left(I_{Dy}(s_\kappa)\right)^{\lambda}\left(I_{Ry}(s_\kappa)\right)^{1-\lambda}\right| \\ + \left|\left(V_{Dx}(s_\kappa)\right)^{\lambda}\left(V_{Rx}(s_\kappa)\right)^{1-\lambda} - \left(V_{Dy}(s_\kappa)\right)^{\lambda}\left(V_{Ry}(s_\kappa)\right)^{1-\lambda}\right| \end{array}}{3} \right] \right\} \text{ for } \varepsilon \geq 2, \lambda \in (0, 1], \tag{3}$$

where ε is an integer, and $s_\kappa \in S$ (κ = 1, 2, …, η). Especially when the parameter λ = 1, the LSM of NZNSs will be reduced to that of SvNSs.

Example 1. Let $X(s) = \left[\left\langle(0.8,0.7),(0.2,0.6),(0.1,0.7)\right\rangle \; \left\langle(0.7,0.6),(0.1,0.7),(0.3,0.8)\right\rangle \; \left\langle(0.7,0.6),(0.2,0.8),(0.2,0.8)\right\rangle\right]$ and $Y(s) = \left[\left\langle(0.7,0.7),(0.2,0.7),(0.2,0.9)\right\rangle \; \left\langle(0.6,0.7),(0.2,0.7),(0.1,0.9)\right\rangle \; \left\langle(0.7,0.8),(0.1,0.7),(0.2,0.6)\right\rangle\right]$ be two NZNSs in $S = \{s_1, s_2, s_3\}$ whose weights are given by the vector $\tau = \{0.3, 0.4, 0.3\}$. If ε = 2 and λ = 0.5 are chosen, the weighted LSM between $X(s)$ and $Y(s)$ is $L_w(X(s),Y(s))$ = 0.3×0.28265 + 0.4*0.37045 + 0.3*0.27861 = 0.9317 according to Eq.(3). While when λ = 1, $L_w(X(s),Y(s))$ = 0.3×0.28533 + 0.4*0.36019 + 0.3*0.29273 = 0.9382.

Theorem 1. Let $X(s)$ = {<s_κ, ($T_{Dx}(s_\kappa)$, $T_{Rx}(s_\kappa)$), ($I_{Dx}(s_\kappa)$, $I_{Rx}(s_\kappa)$), ($V_{Dx}(s_\kappa)$, $V_{Rx}(s_\kappa)$)>|$s_\kappa \in S$} and $Y(s)$ = {<s_κ, ($T_{Dy}(s_\kappa)$, $T_{Ry}(s_\kappa)$), ($I_{Dy}(s_\kappa)$, $I_{Ry}(s_\kappa)$), ($V_{Dy}(s_\kappa)$, $V_{Ry}(s_\kappa)$)>|$s_\kappa \in S$ } be two NZNSs in $S = \{s_1, s_2, \ldots, s_\eta\}$ whose weights are given by a vector $\tau = \{\tau(s_1), \tau(s_2), \ldots, \tau(s_\eta)\}$ with $\sum_{\kappa=1}^{\eta} \tau\left(s_\kappa\right) = 1$. The weighted LSM of NZNSs denoted by $L_w(X(s), Y(s))$ contains the properties as follows:

(R1) $0 \leq L_w(X(s), Y(s)) \leq 1$;

(R2) $L_w(X(s), Y(s)) = 1$ if and only if $X(s) = Y(s)$;

(R3) $L_w(X(s), Y(s)) = L_w(Y(s), X(s))$;

(R4) If $M(s)$ is a NZNS in S, and $X(s) \subseteq Y(s) \subseteq M(s)$, then $L_w(X(s), M(s)) \le L_w(X(s), Y(s))$ and $L_w(X(s), M(s)) \le L_w(Y(s), M(s))$.

Proof. (R1) Since all variable values of $X(s)$ and $Y(s)$ are in the range of 0 to 1, $\zeta(X(s_\kappa), Y(s_\kappa)) \in [0,1]$ can be easily determined from Eq. (2). Then, when $\kappa = 1, 2, \ldots, \eta$, one arrives at $\varepsilon-(\varepsilon-1)\times\zeta(X(s_\kappa), Y(s_\kappa)) \in [1, \varepsilon]$, whose logarithm to the base ε is in [0,1]. Therefore, $L_w(X(s), Y(s)) \in [0,1]$ can be derived by Eq. (3) when the condition $\sum_{\kappa=1}^{\eta}\tau(s_\kappa)=1$ is satisfied.

(R2) If $X(s) = Y(s)$, then for $\kappa = 1, 2, \ldots, \eta$, there is $T_{Dx}(s_\kappa) = T_{Dy}(s_\kappa)$, $T_{Rx}(s_\kappa) = T_{Ry}(s_\kappa)$, $I_{Dx}(s_\kappa) = I_{Dy}(s_\kappa)$, $I_{Rx}(s_\kappa) = I_{Ry}(s_\kappa)$, $V_{Dx}(s_\kappa) = V_{Dy}(s_\kappa)$, and $V_{Rx}(s_\kappa) = V_{Ry}(s_\kappa)$. Thus, $\zeta(X(s_\kappa), Y(s_\kappa)) = 0$ can be further inferred from Eq. (2), and that the logarithm of $\varepsilon-(\varepsilon-1)\times\zeta(X(s_\kappa), Y(s_\kappa))$ to the base ε equals 1 by Eq. (3). Therefore, $L_w(X(s), Y(s)) = 1$ can be gotten for the condition $\sum_{\kappa=1}^{\eta}\tau(s_\kappa)=1$.

Conversely, if $L_w(X(s), Y(s)) = 1$, by Eq. (3) and the condition $\sum_{\kappa=1}^{\eta}\tau(s_\kappa)=1$, there exists the logarithm of $\varepsilon-(\varepsilon-1)\times\zeta(X(s_\kappa), Y(s_\kappa))$ to the base ε equals 1 for $\kappa = 1, 2, \ldots, \eta$. Then, $\zeta(X(s_\kappa), Y(s_\kappa))$ must be zero for $\kappa = 1, 2, \ldots, \eta$. Thus, by Eq. (2) there is $T_{Dx}(s_\kappa) = T_{Dy}(s_\kappa)$, $T_{Rx}(s_\kappa) = T_{Ry}(s_\kappa)$, $I_{Dx}(s_\kappa) = I_{Dy}(s_\kappa)$, $I_{Rx}(s_\kappa) = I_{Ry}(s_\kappa)$, $V_{Dx}(s_\kappa) = V_{Dy}(s_\kappa)$, and $V_{Rx}(s_\kappa) = V_{Ry}(s_\kappa)$ for $\kappa = 1, 2, \ldots, \eta$, that is $X(s) = Y(s)$.

(R3) By Eq. (3), $L_w(X(s), Y(s)) = L_w(Y(s), X(s))$ can be straightforwardly obtained.

(R4) The condition $X(s) \subseteq Y(s) \subseteq P(s)$ implies that $T_{Dx}(s_\kappa) \le T_{Dy}(s_\kappa) \le T_{Dp}(s_\kappa)$, $T_{Rx}(s_\kappa) \le T_{Ry}(s_\kappa) \le T_{Rp}(s_\kappa)$, $I_{Dx}(s_\kappa) \ge I_{Dy}(s_\kappa) \ge I_{Dp}(s_\kappa)$, $I_{Rx}(s_\kappa) \ge I_{Ry}(s_\kappa) \ge I_{Rp}(s_\kappa)$, $V_{Dx}(s_\kappa) \ge V_{Dy}(s_\kappa) \ge V_{Dp}(s_\kappa)$, and $V_{Rx}(s_\kappa) \ge V_{Ry}(s_\kappa) \ge V_{Rp}(s_\kappa)$ for $\kappa = 1, 2, \ldots, \eta$. Since the power function with a positive base increases monotonically in the range of positive exponents, for $\lambda \in (0,1]$, there is $(T_{Dx}(s_\kappa))^\lambda \le (T_{Dy}(s_\kappa))^\lambda \le (T_{Dp}(s_\kappa))^\lambda$, $(T_{Rx}(s_\kappa))^{(1-\lambda)} \le (T_{Ry}(s_\kappa))^{(1-\lambda)} \le (T_{Rp}(s_\kappa))^{(1-\lambda)}$, $(I_{Dx}(s_\kappa))^\lambda \ge (I_{Dy}(s_\kappa))^\lambda \ge (I_{Dp}(s_\kappa))^\lambda$, $(I_{Rx}(s_\kappa))^{(1-\lambda)} \ge (I_{Ry}(s_\kappa))^{(1-\lambda)} \ge (I_{Rp}(s_\kappa))^{(1-\lambda)}$, $(V_{Dx}(s_\kappa))^\lambda \ge (V_{Dy}(s_\kappa))^\lambda \ge (V_{Dp}(s_\kappa))^\lambda$, $(V_{Rx}(s_\kappa))^{(1-\lambda)} \ge (V_{Ry}(s_\kappa))^{(1-\lambda)} \ge (V_{Rp}(s_\kappa))^{(1-\lambda)}$. Hence, we can obtain the follows:

$$\left|(T_{Dx}(s_\kappa))^\lambda(T_{Rx}(s_\kappa))^{(1-\lambda)}-(T_{Dy}(s_\kappa))^\lambda(T_{Ry}(s_\kappa))^{(1-\lambda)}\right| \le \left|(T_{Dx}(s_\kappa))^\lambda(T_{Rx}(s_\kappa))^{(1-\lambda)}-(T_{Dp}(s_\kappa))^\lambda(T_{Rp}(s_\kappa))^{(1-\lambda)}\right|,$$
$$\left|(T_{Dy}(s_\kappa))^\lambda(T_{Ry}(s_\kappa))^{(1-\lambda)}-(T_{Dp}(s_\kappa))^\lambda(T_{Rp}(s_\kappa))^{(1-\lambda)}\right| \le \left|(T_{Dx}(s_\kappa))^\lambda(T_{Rx}(s_\kappa))^{(1-\lambda)}-(T_{Dp}(s_\kappa))^\lambda(T_{Rp}(s_\kappa))^{(1-\lambda)}\right|,$$
$$\left|(I_{Dx}(s_\kappa))^\lambda(I_{Rx}(s_\kappa))^{(1-\lambda)}-(I_{Dy}(s_\kappa))^\lambda(I_{Ry}(s_\kappa))^{(1-\lambda)}\right| \le \left|(I_{Dx}(s_\kappa))^\lambda(I_{Rx}(s_\kappa))^{(1-\lambda)}-(I_{Dp}(s_\kappa))^\lambda(I_{Rp}(s_\kappa))^{(1-\lambda)}\right|,$$
$$\left|(I_{Dy}(s_\kappa))^\lambda(I_{Ry}(s_\kappa))^{(1-\lambda)}-(I_{Dp}(s_\kappa))^\lambda(I_{Rp}(s_\kappa))^{(1-\lambda)}\right| \le \left|(I_{Dx}(s_\kappa))^\lambda(I_{Rx}(s_\kappa))^{(1-\lambda)}-(I_{Dp}(s_\kappa))^\lambda(I_{Rp}(s_\kappa))^{(1-\lambda)}\right|,$$
$$\left|(V_{Dx}(s_\kappa))^\lambda(V_{Rx}(s_\kappa))^{(1-\lambda)}-(V_{Dy}(s_\kappa))^\lambda(V_{Ry}(s_\kappa))^{(1-\lambda)}\right| \le \left|(V_{Dx}(s_\kappa))^\lambda(V_{Rx}(s_\kappa))^{(1-\lambda)}-(V_{Dp}(s_\kappa))^\lambda(V_{Rp}(s_\kappa))^{(1-\lambda)}\right|,$$
$$\left|(V_{Dy}(s_\kappa))^\lambda(V_{Ry}(s_\kappa))^{(1-\lambda)}-(V_{Dp}(s_\kappa))^\lambda(V_{Rp}(s_\kappa))^{(1-\lambda)}\right| \le \left|(V_{Dx}(s_\kappa))^\lambda(V_{Rx}(s_\kappa))^{(1-\lambda)}-(V_{Dp}(s_\kappa))^\lambda(V_{Rp}(s_\kappa))^{(1-\lambda)}\right|.$$

According to the above conclusion and Eq. (2), it is not difficult to get $\zeta(X(s), Y(s)) \le \zeta(X(s), P(s))$ and $\zeta(Y(s), P(s)) \le \zeta(X(s), P(s))$. And $L_w(X(s), P(s)) \le L_w(X(s), Y(s))$ and $L_w(X(s), P(s)) \le L_w(Y(s), P(s))$ can be further determined for the reason that the logarithm to the base ε increases with the function ζ decreasing.

Thus, the above properties are completely proved. □

3. MADM Approach Applying the Proposed LSM in NZN Setting

To solve the DM problems in NZN setting, this section developed a MADM approach based on the presented NZNLSM. Assume that the decision makers need to evaluate α alternatives represented by $U = \{U_1, U_2, \ldots, U_\alpha\}$ on the η attributes represented by $S = \{s_1, s_2, \ldots, s_\eta\}$ to obtain the best option, and the attribute importance is given by the weight vector $\tau = \{\tau_1, \tau_2, \ldots, \tau_\eta\}$, where τ_κ is the weight of the attribute s_κ for $\kappa = 1, 2, \ldots, \eta$. Then, the evaluation value of the option U_ι for the attribute s_κ can be expressed as the NZN $x_{\iota\kappa} = \langle(T_{D\iota\kappa}, T_{R\iota\kappa}), (I_{D\iota\kappa}, I_{R\iota\kappa}), (V_{D\iota\kappa}, V_{R\iota\kappa})\rangle$, where $T_{D\iota\kappa}, I_{D\iota\kappa}, V_{D\iota\kappa} \in [0, 1]$ and $T_{R\iota\kappa}, I_{R\iota\kappa}, V_{R\iota\kappa} \in [0, 1]$. Therefore, for all attributes s_κ ($\kappa = 1, 2, \ldots, \eta$), the evaluated values of all options U_ι ($\iota = 1, 2, \ldots, \alpha$) can be constructed by the NZN matrix $E = (x_{\iota\kappa})_{\alpha\times\eta}$.

Then, the decision process of using the weighted LSM of NZNSs can be given as follows.

Step 1. Since the ideal one of all evaluated NZNs on the attribute s_κ(κ = 1, 2, …, η) can be obtained by

$$\begin{aligned} x_\kappa^* &= \left\langle s_\kappa, \left\langle \left(T_{D\kappa}^*, T_{R\kappa}^*\right), \left(I_{D\kappa}^*, I_{R\kappa}^*\right), \left(V_{D\kappa}^*, V_{R\kappa}^*\right) \right\rangle \right\rangle \\ &= \left\langle s_\kappa, \left\langle \left(\max_\iota\left(T_{D\iota\kappa}\right), \max_\iota\left(T_{R\iota\kappa}\right)\right), \left(\min_\iota\left(I_{D\iota\kappa}\right), \min_\iota\left(I_{R\iota\kappa}\right)\right), \left(\min_\iota\left(V_{D\iota\kappa}\right), \min_\iota\left(V_{R\iota\kappa}\right)\right) \right\rangle \right\rangle, \end{aligned} \tag{4}$$

where ι = 1, 2, …, α. The ideal NZNS considering all attributes can be derived from the formula

$$X^* = \left\{ \left\langle s_\kappa, \left\langle \left(T_{D\kappa}^*, T_{R\kappa}^*\right), \left(I_{D\kappa}^*, I_{R\kappa}^*\right), \left(V_{D\kappa}^*, V_{R\kappa}^*\right) \right\rangle \right\rangle \middle| s_\kappa \in S, \kappa = 1, 2, \ldots, \eta \right\}. \tag{5}$$

Step 2. By Eq. (3), the weighted LSM between X_ι and X^* is calculated as

$$L_w\left(X_\iota, X^*\right) = \sum_{\kappa=1}^{\eta} \tau_\kappa \log_\varepsilon \left[\varepsilon - (\varepsilon - 1) \times \frac{\begin{array}{c} \left| T_{D\iota\kappa}^{\lambda} T_{R\iota\kappa}^{(1-\lambda)} - \left(T_{D\kappa}^*\right)^{\lambda} \left(T_{R\kappa}^*\right)^{(1-\lambda)} \right| + \left| I_{D\iota\kappa}^{\lambda} I_{R\iota\kappa}^{(1-\lambda)} - \left(I_{D\kappa}^*\right)^{\lambda} \left(I_{R\kappa}^*\right)^{(1-\lambda)} \right| \\ + \left| V_{D\iota\kappa}^{\lambda} V_{R\iota\kappa}^{(1-\lambda)} - \left(V_{D\kappa}^*\right)^{\lambda} \left(V_{R\kappa}^*\right)^{(1-\lambda)} \right| \end{array}}{3} \right]. \tag{6}$$

Step 3. According to the largest value of $L_w(X_\iota, X^*)$ for ι = 1, 2, …, α, the optimal alternative can be determined.

Step 4. End.

4. Example and Analysis

The applicability and effectiveness of the presented DM approach are demonstrated through a case of UTQE in this section. Then, its robustness and sensitivity are further analyzed by comparing with the existing methods [27-30] in NZN setting.

4.1 Example of UTQE

Suppose that there are four universities represented by $U = \{U_1, U_2, U_3, U_4\}$ to participate in UTQE. Experts are required to assess them in terms of five aspects denoted by $S = \{s_1, s_2, s_3, s_4, s_5\}$, in which s_1 is the adaptability degree; s_2 is the achievement degree; s_3 is the guarantee degree; s_4 is the satisfied degree; and s_5 is the effective degree, with the corresponding weight vector given by $\tau = \{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5\} = \{0.2, 0.3, 0.15, 0.15, 0.2\}$. Then, all decision information of the UTQE can be finally established as the following NZN matrix:

$$E = \left(x_{\iota\kappa}\right)_{4\times 5} = \begin{bmatrix} \left\langle \begin{array}{l}(0.7,0.8),(0.1,0.7),\\(0.2,0.8)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.7,0.8),(0.1,0.7),\\(0.2,0.8)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.7,0.7),(0.2,0.7),\\(0.2,0.9)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.8,0.6),(0.2,0.8),\\(0.3,0.8)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.6,0.8),(0.2,0.9),\\(0.2,0.8)\end{array}\right\rangle \\ \left\langle \begin{array}{l}(0.8,0.7),(0.2,0.6),\\(0.1,0.7)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.6,0.7),(0.2,0.7),\\(0.1,0.9)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.6,0.8),(0.3,0.8),\\(0.3,0.8)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.8,0.7),(0.2,0.7),\\(0.2,0.6)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.7,0.8),(0.2,0.8),\\(0.1,0.8)\end{array}\right\rangle \\ \left\langle \begin{array}{l}(0.9,0.8),(0.1,0.8),\\(0.1,0.7)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.7,0.6),(0.1,0.7),\\(0.3,0.8)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.7,0.7),(0.4,0.7),\\(0.6,0.7)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.9,0.8),(0.1,0.7),\\(0.1,0.8)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.8,0.7),(0.1,0.7),\\(0.1,0.7)\end{array}\right\rangle \\ \left\langle \begin{array}{l}(0.7,0.8),(0.1,0.7),\\(0.2,0.6)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.6,0.7),(0.1,0.8),\\(0.2,0.7)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.7,0.6),(0.2,0.8),\\(0.2,0.8)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.7,0.8),(0.2,0.6),\\(0.3,0.7)\end{array}\right\rangle & \left\langle \begin{array}{l}(0.5,0.6),(0.2,0.8),\\(0.1,0.8)\end{array}\right\rangle \end{bmatrix}.$$

Thus, the novel MADM approach based upon the NZNLSM can be applied to the UTQE problem. First, according to Eqs. (4) and (5), the ideal option can be calculated from the decision matrix X as

X^* = {<(0.9,0.8), (0.1,0.6), (0.1,0.6)>, <(0.7,0.8), (0.1,0.7), (0.1,0.7)>, <(0.7,0.8), (0.2,0.7), (0.2,0.7)>, <(0.9,0.8), (0.1,0.6), (0.1,0.6)>, <(0.8,0.8), (0.1,0.7), (0.1,0.7)>}.

Then, using Eq. (6), we calculate the weighted LSM between the alternative X_ι (ι = 1, 2, 3, 4) and the ideal option X^*. To verify whether the alternative ranking changes with the parameter value of λ, the LSM values between X_ι (ι = 1, 2, 3, 4) and X^* are calculated with λ changing from 0.1 to 1 for ε = 2, which are shown in Table 1.

Table 1. The MADM results applying the proposed approach for ε = 2 with $\lambda \in (0, 1]$

	$L_w(X_\iota, X^*)$ for ι = 1, 2, 3, 4	Ranking order	The optimal one
λ=0.1	0.9278, 0.9408, 0.9445, 0.9379	$U_3 > U_2 > U_4 > U_1$	U_3
λ=0.2	0.9277, 0.9390, 0.9458, 0.9350	$U_3 > U_2 > U_4 > U_1$	U_3
λ=0.3	0.9285, 0.9381, 0.9471, 0.9330	$U_3 > U_2 > U_4 > U_1$	U_3
λ=0.4	0.9300, 0.9379, 0.9487, 0.9318	$U_3 > U_2 > U_4 > U_1$	U_3
λ=0.5	0.9320, 0.9382, 0.9506, 0.9311	$U_3 > U_2 > U_1 > U_4$	U_3
λ=0.6	0.9341, 0.9388, 0.9526, 0.9308	$U_3 > U_2 > U_1 > U_4$	U_3
λ=0.7	0.9364, 0.9398, 0.9548, 0.9308	$U_3 > U_2 > U_1 > U_4$	U_3
λ=0.8	0.9387, 0.9409, 0.9572, 0.9311	$U_3 > U_2 > U_1 > U_4$	U_3
λ=0.9	0.9409, 0.9422, 0.9598, 0.9314	$U_3 > U_2 > U_1 > U_4$	U_3
λ=1.0	0.9431, 0.9436, 0.9625, 0.9318	$U_3 > U_2 > U_1 > U_4$	U_3

From Table 1, the use of different values of λ affects the ranking results of the alternatives. In this case, although the best alternative keeps the same as U_3, the ranking of the alternatives is $U_3 > U_2 > U_4 > U_1$ for $\lambda < 0.5$, while it changes into $U_3 > U_2 > U_1 > U_4$ for $\lambda \geq 0.5$. This is because the exponent of the restraint in Eq. (6) is λ and the exponent of reliability is $1-\lambda$. Therefore, the ranking order is more sensitive to the reliability measure when λ is smaller than 0.5, while it seems to be more sensitive to the restraint when λ is bigger than 0.5, which illustrates the flexibility of the proposed LSM-based MADM. In practice, the value of λ can be made based on the decision maker's preferences and DM requirements.

4.2 Comparative Analysis

A comparison of the presented MADM approach with the published MADM approaches was carried out in NZN setting. Table 2 lists the DM results using the published MADM methods based on the operators including the NZN weighted arithmetic average (NZNWAA) and the NZN weighted geometric average (NZNWGA) [27], the NZN Dombi weighted arithmetic average (NZNDWAA) and the NZN Dombi weighted geometric average (NZNDWGA) [28], the NZN Aczel-Alsina weighted arithmetic average (NZNAAWAA) and the NZN Aczel-Alsina weighted geometric average (NZNAAWGA) [29], or based on the similarity measures including the NZN generalized distance-based similarity measure (NZNDSM) [30], the NZN cosine similarity measure (NZNCSM) [30], and the NZN cotangent similarity measure (NZNCTSM) [30].

Table 2. The MADM results applying published approaches in NZN setting

DM Method	Parameter value	Score value	Ranking order	The optimal one
NZNWAA [27]		0.7521,0.7561,0.7859,0.7390	$U_3>U_2>U_1>U_4$	U_3
NZNWGA [27]		0.7420,0.7421,0.7478,0.7255	$U_3>U_2>U_1>U_4$	U_3
NZNDWAA [28]	q = 1	0.7582,0.7642,0.8051,0.7484	$U_3>U_2>U_1>U_4$	U_3
NZNDWGA [28]	q = 1	0.7381,0.7373,0.7343,0.7213	$U_1>U_2>U_3>U_4$	U_1
NZNAAWAA [29]		0.7570,0.7633,0.7991,0.7466	$U_3>U_2>U_1>U_4$	U_3
NZNAAWGA [29]		0.7329,0.7298,0.7094,0.7149	$U_1>U_2>U_4>U_3$	U_1
NZNDSM [30]	ρ = 1	0.9142, 0.9233, 0.9367, 0.9150	$U_3>U_2>U_4>U_1$	U_3
NZNCSM [30]	ρ = 1	0.9909, 0.9928, 0.9951, 0.9911	$U_3>U_2>U_4>U_1$	U_3
NZNCTSM [30]	ρ = 1	0.8735, 0.8863, 0.9052, 0.8747	$U_3>U_2>U_4>U_1$	U_3

Compared to the outcomes of the proposed MADM approaches in Table 1, the nine published approaches yield four different sorting orders and two different best alternatives. Then, the methods based on the operators of NZNWAA, NZNWGA, NZNDWAA, and NZNAAWAA yield a uniform ranking result of $U_3 > U_2 > U_1 > U_4$, which is as similar as that of the proposed one for $\lambda \geq 0.5$ as shown in Table 1. The existing MADM methods applying the similarity measures of the NZNDSM, the NZNCSM, and the NZNCTSM for $\rho = 1$ yield the ranking result of $U_3 > U_2 > U_1 > U_4$, which is as the same as that of the proposed one with $\lambda < 0.5$.

In a word, the LSM of NZNSs-based MADM method then demonstrates more possible ranking results than the existing methods, and is more flexible as the parameter value of λ changes. Furthermore, LSM of NZNSs with $\lambda = 1$ can deal with DM problems in SvNN setting, which cannot be handled by the existing MADM methods of NZNs. And LSM of NZNSs with $\lambda < 1$ can handle the MADM problem in NZN setting, which cannot be solved by the existing MADM methods of the SvNN. In short, the MADM methods based on LSM of NZNSs is very useful and flexible.

4.3 Sensitivity Analysis

4.3.1 Sensitivity Analysis to Logarithmic Base

To verify whether the ranking order is affected by the value of the logarithmic base ε, the LSM values between X_ι (ι = 1, 2, 3, 4) and X^* with ε in the range of 2 to 100 for λ = 0.1, 0.4, 0.6, 0.9 was calculated by Eq. (6), as shown in Fig.1.

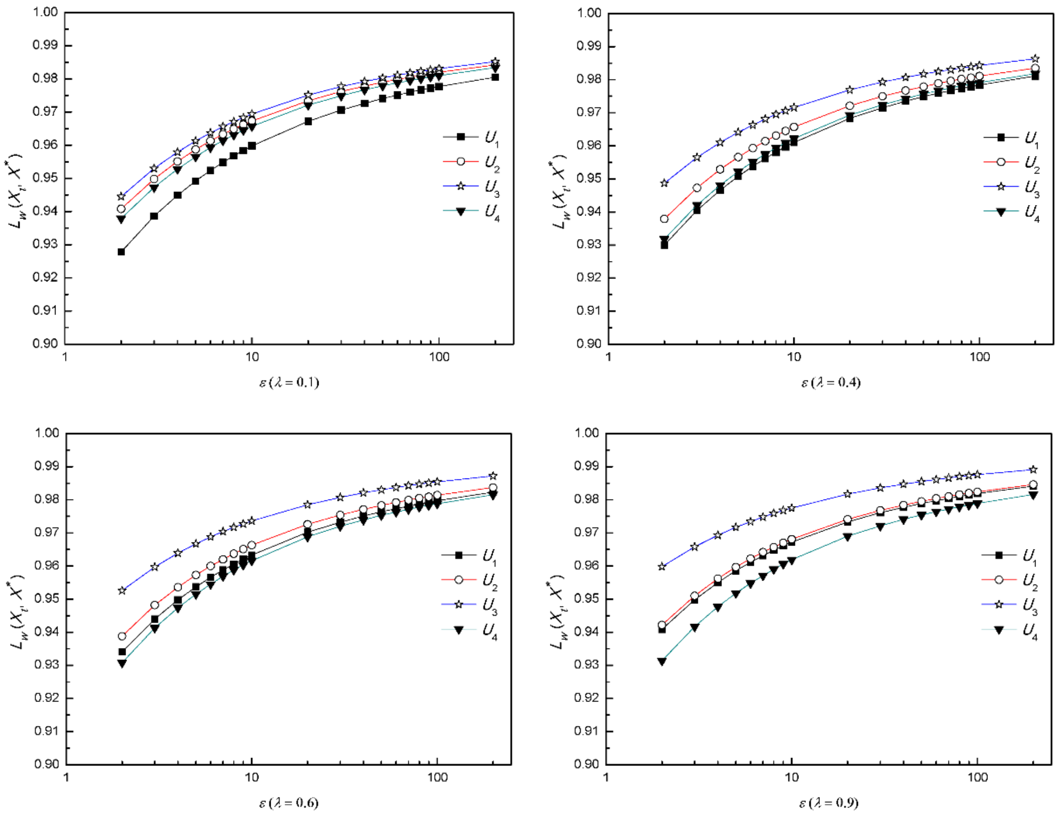

Figure 1. Values of LSM of NZNSs for all alternatives with $\varepsilon \in [2, 200]$ for λ = 0.1, 0.4, 0.6, 0.9

It is clear from Fig.1 that for the same value of λ, the sorting results are consistent over the entire range of ε. In details, when λ = 0.1 or 0.4, the sort result keeps as $U_3 > U_2 > U_4 > U_1$, while when λ = 0.6 or 0.9, it is as consistent as $U_3 > U_2 > U_1 > U_4$. Therefore, the proposed LSM of NZNSs is a generalized

LSM of NZNSs, in which the decision maker can choose a LSM of NZNSs with any valid ε value, since ε has little effect on the final ranking result.

4.3.2 Sensitivity Analysis to Reliability Measure

The sensitivity of the presented MADM approach to the level of reliability is demonstrated by using four different cases, which are all adapted from the matrix E, changing only the parameter values of U_1. As shown in Table 3, all T_Rs for U_1 in Case 1, I_Rs for U_1 in Case 2, and V_Rs for U_1 in Case 3 are increased by 0.1 compared to the original U_1 data in matrix E represented by Case 0. Table 4 shows the DM results for different cases by using the proposed method and existing methods.

Table 3. U_1 data for different cases

		s_1	s_2	s_3	s_4	s_5
Case 0	U_1	<(0.7,0.8),(0.1,0.7),(0.2,0.8)>	<(0.7,0.8),(0.1,0.7),(0.2,0.8)>	<(0.7,0.7),(0.2,0.7),(0.2, 0.9)>	<(0.8,0.6),(0.2,0.8),(0.3,0.8)>	<(0.6,0.8),(0.2,0.9),(0.2,0.8)>
Case 1	U_1	<(0.7,0.9),(0.1,0.7),(0.2,0.8)>	<(0.7,0.9),(0.1,0.7),(0.2,0.8)>	<(0.7,0.8),(0.2,0.7),(0.2, 0.9)>	<(0.8,0.7),(0.2,0.8),(0.3,0.8)>	<(0.6,0.9),(0.2,0.9),(0.2,0.8)>
Case 2	U_1	<(0.7,0.8),(0.1,0.8),(0.2,0.8)>	<(0.7,0.8),(0.1,0.8),(0.2,0.8)>	<(0.7,0.7),(0.2,0.8),(0.2, 0.9)>	<(0.8,0.6),(0.2,0.9),(0.3,0.8)>	<(0.6,0.8),(0.2,1.0),(0.2,0.8)>
Case 3	U_1	<(0.7,0.8),(0.1,0.7), (0.2,0.9)>	<(0.7,0.8),(0.1,0.7),(0.2,0.9)>	<(0.7,0.7),(0.2,0.7),(0.2,1.0)>	<(0.8,0.6),(0.2,0.8),(0.3,0.9)>	<(0.6,0.8),(0.2,0.9),(0.2,0.9)>

Table 4. Ranking orders of different MADM methods

DM method	Parameter value	Ranking order for case 0	Ranking order for case 1	Ranking order for case 2	Ranking order for case 3
The proposed NZNLSM	λ=0.1	$U_3>U_2>U_4>U_1$	$U_1>U_3>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
	λ=0.2	$U_3>U_2>U_4>U_1$	$U_1>U_3>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
	λ=0.3	$U_3>U_2>U_4>U_1$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
	λ=0.4	$U_3>U_2>U_4>U_1$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
	λ=0.5	$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
	λ=0.6	$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
	λ=0.7	$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
	λ=0.8	$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
	λ=0.9	$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
	λ=1.0	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
NZNWAA [27]		$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
NZNWGA [27]		$U_3>U_2>U_1>U_4$	$U_1>U_3>U_2>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
NZNDWAA [28]	q=1	$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
NZNDWGA [28]	q=1	$U_1>U_2>U_3>U_4$	$U_1>U_2>U_3>U_4$	$U_3>U_2>U_1>U_4$	$U_2>U_3>U_1>U_4$
NZNAAWAA [29]		$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
NZNAAWGA [29]		$U_1>U_2>U_4>U_3$	$U_1>U_2>U_4>U_3$	$U_2>U_1>U_4>U_3$	$U_2>U_1>U_4>U_3$
NZNDSM [30]	ρ=1	$U_3>U_2>U_4>U_1$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
NZNCSM [30]	ρ=1	$U_3>U_2>U_4>U_1$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
NZNCTSM [30]	ρ=1	$U_3>U_2>U_4>U_1$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$

From Table 4, compared with the results for Case 0, the sorting position of U_1 in Case 1 is shifted forward, and backward in both Case 2 and Case 3. This is because the proposed LSM value of U_1 increases with the increase of T_R and decreases with the increase of I_R or V_R. Moreover, the smaller the value of the parameter λ, the more obvious the shift of the position of U_1. For example, in Case 1, when λ is less than or equal to 0.2, U_1 is shifted forward from the fourth position to the first position in the sequence, whereas when λ = 0.3 or 0.4, U_1 moves forward to the second position; when λ is in the range of 0.5 to 0.9, U_1 only moves to the third position in the sequence; and when λ = 1, the position of U_1 in the sequence even remains unchanged. The reason can be deduced from Eq. (6) that the exponent of the confidence level is $1 - \lambda$. Therefore, the smaller the parameter value of λ, the more sensitive the proposed LSM is to the reliability level. And when λ = 1, the proposed LSM is reduced to LSM of SvNSs without considering the reliability level, and the related MADM method can process the SvNN information.

In addition, the DM results of the proposed approach at each parameter value of λ almost cover most of the DM results of the published ones. In details, the ranking results with λ= 0.3 or 0.4 are identical to the similarity measures of NZNDSM, NZNCSM and NZNCTSM [30], while the ranking results at λ = 0.7, 0.8 and 0.9 are the same as those of the NZNWAA [27], NZNDWAA [28] and NZNAAWAA [29] operators.

In summary, a smaller value of λ can be used if the decision result is desired to be more sensitive to the reliability measure. On the contrary, a larger value of λ should be chosen if the decision outcome requires more consideration of the restraint. In order to balance the restraint and reliability, an intermediate value of λ, such as 0.5, can be used.

5. Conclusions

In this study, a generalized LSM of NZNSs was presented and its properties were investigated. After that, a MADM approach was put forward based on the LSM of NZNSs to deal with DM problems in NZN setting. At last, the application of the presented MADM method was illustrated by a UTQE example, and the effectiveness and flexibility of the DM method was further verified by comparative analysis and sensitivity analysis. The UTQE results showed that the LSM of NZNSs-based MADM approach demonstrates more possible ranking results by determining the index parameter of the restraint and reliability in terms of the evaluator's preferences. The proposed NZNLSM is more of a generalized LSM because the logarithmic base ε hardly affects the ranking results. However, the logarithmic base ε of the proposed NZNLSM is finite under the condition of $\varepsilon \geq 2$. Therefore, future research can focus on more reliable similarity measures or aggregation operators of NZNSs to overcome the above limitations and develop more reliable DM methods of NZNSs for applications in UTQE, medical diagnosis, and other areas.

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