Neutrosophic Sets and Systems, Vol. 64, 2024
University of New Mexico

MADM Model Using Einstein Aggregation Operators of Sine Single-Valued Neutrosophic Values and Its Application

Sumin Zhang[1], Jun Ye[2]*

[1] School of Mechanical and Electrical Engineering, Shaoxing University, Shaoxing, 312000, P. R. China
[2] Institute of Rock Mechanics, Ningbo University, Ningbo, Zhejiang, 315211, P.R. China
E-mail: zhangsumin@usx.edu.cn, yejun1@nbu.edu.cn

* Correspondence: yejun1@nbu.edu.cn

Abstract: In various fuzzy multiple attribute decision making (MADM) applications, different information descriptions and aggregation operators (AOs) play a crucial role. However, both the Einstein sum and product can include their typical algebraic operation features, but they lack the characteristics of periodicity operations. To fill the research gap of Einstein AOs for single-valued neutrosophic values (SvNVs), this article aims to propose Einstein AOs of sine SvNVs and their MADM model as their new extension. In this study, we first define a new sine SvNV, which integrates sine functions into the membership functions of indeterminacy, falsehood, and truth, and the Einstein operation laws of sine SvNVs. Then, we present the sine SvNV Einstein weighted average and geometric AOs and their properties. Furthermore, we develop a MADM model based on the proposed Einstein AOs in a SvNV circumstance. Lastly, we apply the developed MADM model to a site selection example of a hydrogen power plant as the verification of its application in a SvNV circumstance. The decision results reveal the rationality and validity of the developed model with respect to the comparison of the related models.

Keywords: decision making problem; sine single-valued neutrosophic value; einstein operation law; sine single-valued neutrosophic value Einstein aggregation operator

1. Introduction

Recently, various fuzzy multiple attribute/criteria decision making (MADM/MCDM) theories and approaches have become research hotspots in uncertain decision applications. In MADM applications, various fuzzy information descriptions and operations/aggregations imply their importance and necessity. Fuzzy sets (FSs) [1] contain only membership degrees, but lack non-membership degrees. Then, intuitionistic or interval-valued FSs (IFSs/IvFSs) [2, 3] can contain both membership and non-membership degrees and the dependent relationship of both, but cannot reflect the independent relationship of indeterminate, false, and true membership degrees in inconsistent and uncertain situations. As a general framework of different FSs, a neutrosophic set (NS) [4] can reflect them. Regarding the subsets of NS, some scholars presented single-valued or interval-valued or simplified neutrosophic values (SvNVs/IvNVs/SNVs) [5-7] and their operation laws and aggregation operators (AOs) to effectively meet scientific and engineering applications. Zhang et al. proposed the improved AOs of IvNVs for MADM [8]. Then, Yang and Li [9] introduced the power AOs of SvNVs for MADM. Liu et al. [10] presented the power Muirhead mean AOs of SvNVs for group decision making (GDM). Liu [11] also introduced the Archimedean AOs of SvNVs for MADM. Garg [12] proposed the Frank norm AOs of SvNVs for MADM. Deli and Subas [13] introduced a

SvNV sorting method for MADM. Liu and Liu [14] put forward a generalized weighted power averaging operator of SvNVs for GDM. Karaaslan and Hayat [15] presented some operations of interval-valued neutrosophic matrices and applied them to GDM. Garg [16] used the multiplicative preference relation of SvNVs for MADM. Giri and Roy [17] introduced a neutrosophic programming approach to solve the transportation problem of green four-dimensional fixed charges. Therefore, the SNVs (SvNVs and/or IvNVs) have also revealed their merits in inconsistent and uncertain MADM applications [18]. Consequently, many scholars have further developed SNV (SvNV and/or IvNV) AOs, such as ordinary weighted arithmetic and geometric AOs, Einstein AOs, generalized AOs [19], Bonferroni mean AOs [20], Hamacher AOs [21], exponential AOs [22], subtraction and division AOs [23], Frank AOs [24], prioritized interactive AOs [25], and fairly AOs [26] for SNVs (SvNVs and/or IvNVs).

Recently, some scholars [27, 28] have proposed logarithmic SvNV operation laws and logarithmic SvNV Einstein AOs for GDM in view of t-conorm and t-norm. However, they reflect some limitations, for example, logx(y) cannot be defined when x = 1 or y = 0. Due to the periodicity feature of the sine function, it implies some merit that satisfies the multiple periodicity MADM needs in real problems. Therefore, the operation laws and AOs of sine SvNVs (S-SvNVs) [29, 30] have been introduced in MADM applications. Then, there are the defects of some membership functions that belong to the range of [0, 0.46) instead of the whole range of [0, 1] in S-SvNV [29, 30]. To overcome this deficiency, AOs of tangent SvNVs (T-SvNVs), where the three membership functions belong to the whole range of [0, 1], were presented for MADM [31]. Based on cosine, sine, arccosine, and arcsine operations, Ye et al. [32] first proposed the single-valued neutrosophic credibility value trigonometric AOs for MADM. However, the Einstein sum and product can include their typical algebraic operation merits [19], but lack periodicity operation features. Furthermore, no Einstein operation laws and AOs of S-SvNVs are presented in the existing literature. Therefore, it is necessary to develop them for MADM issues with S-SvNV information to fill this gap. Motivated by the new ideas, this article will propose the Einstein AOs of S-SvNVs and their MADM model as a new extension to address the defects and research gaps in the existing S-SvNV operation laws and AOs [29, 30]. In this study, the objectives of this paper are to: (1) define a suitable S-SvNV including three membership degrees belonging to the whole range of [0, 1] and Einstein operation laws (EOLs) of S-SvNVs, (2) establish the S-SvNV Einstein weighted average (S-SvNVEWA) and geometric (S-SvNVEWG) AOs, (3) develop a MADM model using the S-SvNVEWA and S-SvNVEWG AOs, and (4) apply the proposed MADM model to a site selection example of a hydrogen power plant (HPP) in a SvNV circumstance. However, the comparison results with the existing related models indicate the rationality and validity of the proposed model.

The remainder of this article is composed of these parts. Section 2 simply reviews the preliminaries of single-valued NSs (SvNSs), including the operation laws and AOs of SvNVs and S-SvNVs. In view of the integration of sine functions into indeterminate, false, and true membership functions, Section 3 defines a new S-SvNV and the EOLs of S-SvNVs, and then presents the S-SvNVEWA and S-SvNVEWG AOs and their properties. In Section 4, a MADM model is developed in terms of the S-SvNVEWA and S-SvNVEWG AOs. Section 5 applies the developed MADM model to a site selection example of HPP in a SvNV circumstance. The comparative results of the existing related models reveal the validity of the developed model. Conclusions and future research are summarized in Section 6.

2. Preliminaries of SvNSs

2.1. Operation laws and AOs of SvNVs

Set X_C as a fixed universe set. Then, the SvNS Φ_N in X_C is represented as $\Phi_N = \{<x_c, \varphi_{Nt}(x_c), \varphi_{Nu}(x_c), \varphi_{Nv}(x_c)>| x_c \in X_C\}$ [6], where $\varphi_{Nv}(x_c)$, $\varphi_{Nu}(x_c)$, $\varphi_{Nt}(x_c) \in [0, 1]$ are the membership functions of falsehood, indeterminacy, and truth subject to $0 \le \varphi_{Nt}(x_c) + \varphi_{Nu}(x_c) + \varphi_{Nv}(x_c) \le 3$ for any $x_c \in X_C$. Then, $<x_c, \varphi_{Nt}(x_c), \varphi_{Nu}(x_c), \varphi_{Nv}(x_c)>$ in Φ_N is represented as the SvNV $\varphi_N = <\varphi_{Nt}, \varphi_{Nu}, \varphi_{Nv}>$ for simplicity.

Set two SvNVs as $\varphi_{Ns} = <\varphi_{Nt(s)}, \varphi_{Nu(s)}, \varphi_{Nv(s)}>$ (s = 1, 2) with $\mu_w > 0$. Then, their operation relationships are presented below [7, 19]:

(1) $\varphi_{N1} \supseteq \varphi_{N2} \Leftrightarrow \varphi_{Nt(1)} \geq \varphi_{Nt(2)}$, $\varphi_{Nu(1)} \leq \varphi_{Nu(2)}$, and $\varphi_{Nv(1)} \leq \varphi_{Nv(2)}$;

(2) $\varphi_{N1} = \varphi_{N2} \Leftrightarrow \varphi_{N1} \subseteq \varphi_{N2}$ and $\varphi_{N1} \supseteq \varphi_{N2}$;

(3) $\varphi_{N1} \cup \varphi_{N2} = \left\langle \varphi_{Nt(1)} \vee \varphi_{Nt(2)}, \varphi_{Nu(1)} \wedge \varphi_{Nu(2)}, \varphi_{Nv(1)} \wedge \varphi_{Nv(2)} \right\rangle$;

(4) $\varphi_{N1} \cap \varphi_{N2} = \left\langle \varphi_{Nt(1)} \wedge \varphi_{Nt(2)}, \varphi_{Nu(1)} \vee \varphi_{Nu(2)}, \varphi_{Nv(1)} \vee \varphi_{Nv(2)} \right\rangle$;

(5) $(\varphi_{N1})^c = \left\langle \varphi_{Nv(1)}, 1-\varphi_{Nu(1)}, \varphi_{Nt(1)} \right\rangle$ (Complement of φ_{N1});

(6) $\varphi_{N1} \oplus \varphi_{N2} = \left\langle \varphi_{Nt(1)} + \varphi_{Nt(2)} - \varphi_{Nt(1)}\varphi_{Nt(2)}, \varphi_{Nu(1)}\varphi_{Nu(2)}, \varphi_{Nv(1)}\varphi_{Nv(2)} \right\rangle$;

(7) $\varphi_{N1} \otimes \varphi_{N2} = \left\langle \varphi_{Nt(1)}\varphi_{Nt(2)}, \varphi_{Nu(1)} + \varphi_{Nu(2)} - \varphi_{Nu(1)}\varphi_{Nu(2)}, \varphi_{Nv(1)} + \varphi_{Nv(2)} - \varphi_{Nv(1)}\varphi_{Nv(2)} \right\rangle$;

(8) $\mu_w \cdot \varphi_{N1} = \left\langle 1-(1-\varphi_{Nt(1)})^{\mu_w}, \varphi_{Nu(1)}^{\mu_w}, \varphi_{Nv(1)}^{\mu_w} \right\rangle$;

(9) $\varphi_{N1}^{\mu_w} = \left\langle \varphi_{Nt(1)}^{\mu_w}, 1-(1-\varphi_{Nu(1)})^{\mu_w}, 1-(1-\varphi_{Nv(1)})^{\mu_w} \right\rangle$.

Set $\varphi_{Ns} = <\varphi_{Nt(s)}, \varphi_{Nu(s)}, \varphi_{Nv(s)}>$ (s = 1, 2, …, q) as a collection of SvNVs with their weight vector $\boldsymbol{\mu}_W = (\mu_{w1}, \mu_{w2}, \ldots, \mu_{wq})$ subject to $0 \leq \mu_{ws} \leq 1$ and $\sum_{s=1}^{q} \mu_{ws} = 1$. Then, SvNVWA and SvNVWG are denoted as the SvNV weighted average and geometric AOs and expressed by the two formulae [19]:

$$SvNVWA(\varphi_{N1}, \varphi_{N2}, \ldots, \varphi_{Nq}) = \sum_{s=1}^{q} \mu_{ws}\varphi_{Ns} = \left\langle 1-\prod_{s=1}^{q}\left(1-\varphi_{Nt(s)}\right)^{\mu_{ws}}, \prod_{s=1}^{q}\left(\varphi_{Nu(s)}\right)^{\mu_{ws}}, \prod_{s=1}^{q}\left(\varphi_{Nv(s)}\right)^{\mu_{ws}} \right\rangle, \quad (1)$$

$$SvNVWG(\varphi_{N1}, \varphi_{N2}, \ldots, \varphi_{Nq}) = \prod_{s=1}^{q}\left(\varphi_{Ns}\right)^{\mu_{ws}} = \left\langle \prod_{s=1}^{q}\left(\varphi_{Nt(s)}\right)^{\mu_{ws}}, 1-\prod_{s=1}^{q}\left(1-\varphi_{Nu(s)}\right)^{\mu_{ws}}, 1-\prod_{s=1}^{q}\left(1-\varphi_{Nv(s)}\right)^{\mu_{ws}} \right\rangle. \quad (2)$$

Set two SvNVs as $\varphi_{Ns} = <\varphi_{Nt(s)}, \varphi_{Nu(s)}, \varphi_{Nv(s)}>$ (s = 1, 2) with $\mu_w > 0$. Then, their EOLs are presented as follows [19]:

(1) $\varphi_{N1} \oplus_E \varphi_{N2} = \left\langle \dfrac{\varphi_{Nt(1)} + \varphi_{Nt(2)}}{1+\varphi_{Nt(1)}\varphi_{Nt(2)}}, \dfrac{\varphi_{Nu(1)}\varphi_{Nu(2)}}{1+(1-\varphi_{Nu(1)})(1-\varphi_{Nu(2)})}, \dfrac{\varphi_{Nv(1)}\varphi_{Nv(2)}}{1+(1-\varphi_{Nv(1)})(1-\varphi_{Nv(2)})} \right\rangle$;

(2) $\varphi_{N1} \otimes_E \varphi_{N2} = \left\langle \dfrac{\varphi_{Nt(1)}\varphi_{Nt(2)}}{1+(1-\varphi_{Nt(1)})(1-\varphi_{Nt(2)})}, \dfrac{\varphi_{Nu(1)} + \varphi_{Nu(2)}}{1+\varphi_{Nu(1)}\varphi_{Nu(2)}}, \dfrac{\varphi_{Nv(1)} + \varphi_{Nv(2)}}{1+\varphi_{Nv(1)}\varphi_{Nv(2)}} \right\rangle$;

(3) $\mu_w \cdot \varphi_{N1} = \left\langle \dfrac{(1+\varphi_{Nt(1)})^{\mu_w} - (1-\varphi_{Nt(1)})^{\mu_w}}{(1+\varphi_{Nt(1)})^{\mu_w} + (1-\varphi_{Nt(1)})^{\mu_w}}, \dfrac{2\varphi_{Nu(1)}^{\mu_w}}{(2-\varphi_{Nu(1)})^{\mu_w} + \varphi_{Nu(1)}^{\mu_w}}, \dfrac{2\varphi_{Nv(1)}^{\mu_w}}{(2-\varphi_{Nv(1)})^{\mu_w} + \varphi_{Nv(1)}^{\mu_w}} \right\rangle$;

(4) $\varphi_{N1}^{\mu_w} = \left\langle \dfrac{2\varphi_{Nt(1)}^{\mu_w}}{(2-\varphi_{Nt(1)})^{\mu_w} + \varphi_{Nt(1)}^{\mu_w}}, \dfrac{(1+\varphi_{Nu(1)})^{\mu_w} - (1-\varphi_{Nu(1)})^{\mu_w}}{(1+\varphi_{Nu(1)})^{\mu_w} + (1-\varphi_{Nu(1)})^{\mu_w}}, \dfrac{(1+\varphi_{Nv(1)})^{\mu_w} - (1-\varphi_{Nv(1)})^{\mu_w}}{(1+\varphi_{Nv(1)})^{\mu_w} + (1-\varphi_{Nv(1)})^{\mu_w}} \right\rangle$.

For a collection of SvNVs $\varphi_{Ns} = <\varphi_{Nt(s)}, \varphi_{Nu(s)}, \varphi_{Nv(s)}>$ (s = 1, 2, …, q) with their weight vector $\boldsymbol{\mu}_W = (\mu_{w1}, \mu_{w2}, \ldots, \mu_{wq})$ subject to $0 \leq \mu_{ws} \leq 1$ and $\sum_{s=1}^{q} \mu_{ws} = 1$, SvNVEWA and SvNVEWG are denoted as the SvNV Einstein weighted average and geometric AOs and introduced by the two formulae [19]:

$$SvNVEWA(\varphi_{N1}, \varphi_{N2}, \ldots, \varphi_{Nq}) = \sum_{s=1}^{q}{}_{E}\, \mu_{ws}\varphi_{Ns} = \left(\begin{array}{c} \dfrac{\prod_{s=1}^{q}\left(1+\varphi_{Nt(s)}\right)^{\mu_{ws}} - \prod_{s=1}^{q}\left(1-\varphi_{Nt(s)}\right)^{\mu_{ws}}}{\prod_{s=1}^{q}\left(1+\varphi_{Nt(s)}\right)^{\mu_{ws}} + \prod_{s=1}^{q}\left(1-\varphi_{Nt(s)}\right)^{\mu_{ws}}}, \\ \dfrac{2\prod_{s=1}^{q}\left(\varphi_{Nu(s)}\right)^{\mu_{ws}}}{\prod_{s=1}^{q}\left(2-\varphi_{Nu(s)}\right)^{\mu_{ws}} + \prod_{s=1}^{q}\left(\varphi_{Nu(s)}\right)^{\mu_{ws}}}, \dfrac{2\prod_{s=1}^{q}\left(\varphi_{Nv(s)}\right)^{\mu_{ws}}}{\prod_{s=1}^{q}\left(2-\varphi_{Nv(s)}\right)^{\mu_{ws}} + \prod_{s=1}^{q}\left(\varphi_{Nv(s)}\right)^{\mu_{ws}}} \end{array} \right), \quad (3)$$

$$SvNVEWG(\varphi_{N1},\varphi_{N2},...,\varphi_{Nq})=\prod_{s=1}^{q}{}_{E}\left(\varphi_{Ns}\right)^{\mu_{ws}}=\left\langle \begin{array}{c} \dfrac{2\prod_{s=1}^{q}(\varphi_{Nt(s)})^{\mu_{ws}}}{\prod_{s=1}^{q}(2-\varphi_{Nt(s)})^{\mu_{ws}}+\prod_{s=1}^{q}(\varphi_{Nt(s)})^{\mu_{ws}}}, \\ \dfrac{\prod_{s=1}^{q}(1+\varphi_{Nu(s)})^{\mu_{ws}}-\prod_{s=1}^{q}(1-\varphi_{Nu(s)})^{\mu_{ws}}}{\prod_{s=1}^{q}(1+\varphi_{Nu(s)})^{\mu_{ws}}+\prod_{s=1}^{q}(1-\varphi_{Nu(s)})^{\mu_{ws}}}, \\ \dfrac{\prod_{s=1}^{q}(1+\varphi_{Nv(s)})^{\mu_{ws}}-\prod_{s=1}^{q}(1-\varphi_{Nv(s)})^{\mu_{ws}}}{\prod_{s=1}^{q}(1+\varphi_{Nv(s)})^{\mu_{ws}}+\prod_{s=1}^{q}(1-\varphi_{Nv(s)})^{\mu_{ws}}} \end{array} \right\rangle. \quad (4)$$

To sort SvNVs $\varphi_{Ns} = <\varphi_{Nt(s)}, \varphi_{Nu(s)}, \varphi_{Nv(s)}>$ (s = 1, 2), the score and accuracy equations of SvNVs [19] are presented below:

$$U(\varphi_{Ns})=(2+\varphi_{Nt(s)}-\varphi_{Nu(s)}-\varphi_{Nv(s)})/3 \text{ for } U(\varphi_{Ns})\in[0,1], \quad (5)$$

$$V(\varphi_{Ns})=\varphi_{Nt(s)}-\varphi_{Nv(s)} \text{ for } V(\varphi_{Ns})\in[-1,1]. \quad (6)$$

In terms of the score and accuracy equations, a sorting order of two SvNVs is defined by the following rules:

(1) $\varphi_{N1} > \varphi_{N2}$ for $U(\varphi_{N1}) > U(\varphi_{N2})$;

(2) $\varphi_{N1} > \varphi_{N2}$ for $U(\varphi_{N1}) = U(\varphi_{N2})$ and $V(\varphi_{N1}) > V(\varphi_{N2})$;

(3) $\varphi_{N1} \cong \varphi_{N2}$ for $U(\varphi_{N1}) = U(\varphi_{N2})$ and $V(\varphi_{N1}) = V(\varphi_{N2})$.

2.2 Operation laws and AOs of S-SvNVs

Set SvNV as $\varphi_N = <\varphi_{Nt}, \varphi_{Nu}, \varphi_{Nv}>$. Then, S-SvNV is presented by $\sin(\varphi_N)=\langle\sin(0.5\varphi_{Nt}\pi),1-\sin(0.5\pi-\varphi_{Nu}),1-\sin(0.5\pi-\varphi_{Nv})\rangle$ [29, 30], where the membership degrees of the indeterminacy, falsehood, and truth are $1-\sin(0.5\pi-\varphi_{Nu})\in[0,0.46)$, $1-\sin(0.5\pi-\varphi_{Nv})\in[0,0.46)$, and $\sin(0.5\varphi_{Nt}\pi)\in[0,1]$, respectively.

Let $\sin(\varphi_{Ns})=\langle\sin(0.5\varphi_{Nt(s)}\pi),1-\sin(0.5\pi-\varphi_{Nu(s)}),1-\sin(0.5\pi-\varphi_{Nv(s)})\rangle$ for s = 1, 2 be two S-SvNVs with $\mu_w > 0$. Then, their operation laws are introduced below [29, 30]:

(1) $$\sin(\varphi_{N1})\oplus\sin(\varphi_{N2})=\left\langle\begin{array}{l}1-(1-\sin(0.5\varphi_{Nt(1)}\pi))(1-\sin(0.5\varphi_{Nt(2)}\pi)),\\ (1-\sin(0.5\pi-\varphi_{Nu(1)}))(1-\sin(0.5\pi-\varphi_{Nu(2)})),\\ (1-\sin(0.5\pi-\varphi_{Nv(1)}))(1-\sin(0.5\pi-\varphi_{Nv(2)}))\end{array}\right\rangle;$$

(2) $$\sin(\varphi_{N1})\otimes\sin(\varphi_{N2})=\left\langle\begin{array}{l}\sin(0.5\varphi_{Nt(1)}\pi)\sin(0.5\varphi_{Nt(2)}\pi),\\ 1-\sin(0.5\pi-\varphi_{Nu(1)})\sin(0.5\pi-\varphi_{Nu(2)}),\\ 1-\sin(0.5\pi-\varphi_{Nv(1)})\sin(0.5\pi-\varphi_{Nv(2)})\end{array}\right\rangle;$$

(3) $$\mu_w\cdot\sin(\varphi_{N1})=\left\langle\begin{array}{l}1-(1-\sin(0.5\varphi_{Nt(1)}\pi))^{\mu_w},\\ (1-\sin(0.5\pi-\varphi_{Nu(1)}))^{\mu_w},\\ (1-\sin(0.5\pi-\varphi_{Nv(1)}))^{\mu_w}\end{array}\right\rangle;$$

(4) $$(\sin(\varphi_{N1}))^{\mu_w}=\left\langle(\sin(0.5\varphi_{Nt(1)}\pi))^{\mu_w},1-(\sin(0.5\pi-\varphi_{Nu(1)}))^{\mu_w},1-(\sin(0.5\pi-\varphi_{Nv(1)}))^{\mu_w}\right\rangle.$$

For a group of SvNVs $\varphi_{Ns} = <\varphi_{Nt(s)}, \varphi_{Nu(s)}, \varphi_{Nv(s)}>$ (s = 1, 2, …, q) with their weight vector $\mu_w = (\mu_{w1}, \mu_{w2}, \ldots, \mu_{wq})$ subject to $0 \le \mu_{ws} \le 1$ and $\sum_{s=1}^{q} \mu_{ws} = 1$, S-SvNVWA and S-SvNVWG are denoted as the S-SvNV weighted average and geometric AOs and introduced by the two equations [29, 30]:

$$\begin{aligned} &S-SvNVWA(\varphi_{N1}, \varphi_{N2}, ..., \varphi_{Nq}) = \sum_{s=1}^{q} \mu_{ws} \sin(\varphi_{Ns}) \\ &= \left(1 - \prod_{s=1}^{q} \left(1 - \sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}, \prod_{s=1}^{q} \left(1 - \sin\left(0.5\pi - \varphi_{Nu(s)}\right)\right)^{\mu_{ws}}, \prod_{s=1}^{q} \left(1 - \sin\left(0.5\pi - \varphi_{Nv(s)}\right)\right)^{\mu_{ws}} \right) \end{aligned}, \quad (7)$$

$$\begin{aligned} &S-SvNVWG(\varphi_{N1}, \varphi_{N2}, ..., \varphi_{Nq}) = \prod_{s=1}^{q} \left(\sin(\varphi_{Ns})\right)^{\mu_{ws}} \\ &= \left(\prod_{s=1}^{q} \left(\sin(0.5\varphi_{Nt(s)}\pi)\right)^{\mu_{ws}}, 1 - \prod_{s=1}^{q} \left(\sin(0.5\pi - \varphi_{Nu(s)})\right)^{\mu_{ws}}, 1 - \prod_{s=1}^{q} \left(\sin(0.5\pi - \varphi_{Nv(s)})\right)^{\mu_{ws}} \right) \end{aligned}. \quad (8)$$

2.3 Operation laws and AOs of T-SvNVs

Set SvNV as $\varphi_N = <\varphi_{Nt}, \varphi_{Nu}, \varphi_{Nv}>$. Then, T-SvNV is presented by $\tan(\varphi_N) = \left\langle \tan(0.25\varphi_{Nt}\pi), 1 - \tan(0.25\pi(1-\varphi_{Nu})), 1 - \tan(0.25\pi(1-\varphi_{Nv})) \right\rangle$ [31], where the membership degrees of the indeterminacy, falsehood, and truth are $1 - \tan(0.25\pi(1-\varphi_{Nu})) \in [0,1]$, $1 - \tan(0.25\pi(1-\varphi_{Nv})) \in [0,1]$, and $\tan(0.25\varphi_{Nt}\pi) \in [0,1]$, respectively.

Let $\tan(\varphi_{Ns}) = \left\langle \tan(0.25\varphi_{Nt(s)}\pi), 1 - \tan(0.25\pi(1-\varphi_{Nu(s)})), 1 - \tan(0.25\pi(1-\varphi_{Nv(s)})) \right\rangle$ for s = 1, 2 be two T-SvNVs with $\mu_w > 0$. Then, their operation laws are introduced below [31]:

(1) $$\tan(\varphi_{N1}) \oplus \tan(\varphi_{N2}) = \left\langle \begin{array}{l} 1-(1-\tan(0.25\varphi_{Nt(1)}\pi))(1-\tan(0.25\varphi_{Nt(2)}\pi)), \\ (1-\tan(0.25\pi(1-\varphi_{Nu(1)})))(1-\tan(0.25\pi(1-\varphi_{Nu(2)}))), \\ (1-\tan(0.25\pi(1-\varphi_{Nv(1)})))(1-\tan(0.25\pi(1-\varphi_{Nv(2)}))) \end{array} \right\rangle;$$

(2) $$\tan(\varphi_{N1}) \otimes \tan(\varphi_{N2}) = \left\langle \begin{array}{l} \tan(0.25\varphi_{Nt(1)}\pi)\tan(0.25\varphi_{Nt(2)}\pi), \\ 1-\tan(0.25\pi(1-\varphi_{Nu(1)}))\tan(0.25\pi(1-\varphi_{Nu(2)})), \\ 1-\tan(0.25\pi(1-\varphi_{Nv(1)}))\tan(0.25\pi(1-\varphi_{Nv(2)})) \end{array} \right\rangle;$$

(3) $$\mu_w \tan(\varphi_{N1}) = \left\langle \begin{array}{l} 1-(1-\tan(0.25\varphi_{Nt(1)}\pi))^{\mu_w}, \\ (1-\tan(0.25\pi(1-\varphi_{Nu(1)})))^{\mu_w}, \\ (1-\tan(0.25\pi(1-\varphi_{Nv(1)})))^{\mu_w} \end{array} \right\rangle;$$

(4) $$(\tan(\varphi_{N1}))^{\mu_w} = \left\langle \begin{array}{l} (\tan(0.25\varphi_{Nt(1)}\pi))^{\mu_w}, \\ 1-(\tan(0.25\pi(1-\varphi_{Nu(1)})))^{\mu_w}, \\ 1-(\tan(0.25\pi(1-\varphi_{Nv(1)})))^{\mu_w} \end{array} \right\rangle.$$

For a group of SvNVs $\varphi_{Ns} = <\varphi_{Nt(s)}, \varphi_{Nu(s)}, \varphi_{Nv(s)}>$ (s = 1, 2, …, q) with their weight vector $\mu_w = (\mu_{w1}, \mu_{w2}, \ldots, \mu_{wq})$ subject to $0 \le \mu_{ws} \le 1$ and $\sum_{s=1}^{q} \mu_{ws} = 1$, T-SvNVWA and T-SvNVWG are denoted as the T-SvNV weighted average and geometric AOs and introduced by the two equations [31]:

$$\begin{aligned} &T-SvNVWA(\varphi_{N1}, \varphi_{N2}, ..., \varphi_{Nq}) = \sum_{s=1}^{q} \mu_{ws} \tan(\varphi_{Ns}) \\ &= \left(1 - \prod_{s=1}^{q} \left(1 - \tan\left(0.25\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}, \prod_{s=1}^{q} \left(1 - \tan\left(0.25\pi(1-\varphi_{Nu(s)})\right)\right)^{\mu_{ws}}, \prod_{s=1}^{q} \left(1 - \sin\left(0.25\pi(1-\varphi_{Nv(s)})\right)\right)^{\mu_{ws}} \right) \end{aligned}, \quad (9)$$

$$T-SvNVWG(\varphi_{N1},\varphi_{N2},...,\varphi_{Nq})=\prod_{s=1}^{q}\left(\tan(\varphi_{Ns})\right)^{\mu_{ws}}$$
$$=\left(\prod_{s=1}^{q}\left(\tan(0.25\varphi_{Nt(s)}\pi)\right)^{\mu_{ws}},1-\prod_{s=1}^{q}\left(\tan(0.25\pi(1-\varphi_{Nu(s)}))\right)^{\mu_{ws}},1-\prod_{s=1}^{q}\left(\tan(0.25\pi(1-\varphi_{Nv(s)}))\right)^{\mu_{ws}}\right). \quad (10)$$

3. EOLs and Einstein AOs of S-SvNVs

This part presents several EOLs and Einstain AOs of S-SvNVs and their properties.

First, we give a new S-SvNV definition below.

Definition 1. If $\varphi_N = <\varphi_{Nt}, \varphi_{Nu}, \varphi_{Nv}>$ is SvNV, then S-SvNV is defined by $\sin(\varphi_N)=\langle\sin(0.5\varphi_{Nt}\pi),1-\sin(0.5(1-\varphi_{Nu})\pi),1-\sin(0.5(1-\varphi_{Nv})\pi)\rangle$, where the membership degrees of indeterminacy, falsehood, and truth are $1-\sin(0.5(1-\varphi_{Nu})\pi)\in[0,1]$, $1-\sin(0.5(1-\varphi_{Nv})\pi)\in[0,1]$, and $\sin(0.5\varphi_{Nt}\pi)\in[0,1]$, respectively.

Definition 2. Let $\varphi_{Ns} = <\varphi_{Nt(s)}, \varphi_{Nu(s)}, \varphi_{Nv(s)}>$ (s = 1, 2) be two SvNVs and $\mu_w > 0$. Then, the EOLs of S-SvNVs are defined below:

$$(1)\quad \sin(\varphi_{N1})\oplus_E\sin(\varphi_{N2})=\left(\begin{array}{l}\dfrac{\sin(0.5\varphi_{Nt(1)}\pi)+\sin(0.5\varphi_{Nt(2)}\pi)}{1+\sin(0.5\varphi_{Nt(1)}\pi)\sin(0.5\varphi_{Nt(2)}\pi)},\\ \dfrac{(1-\sin(0.5(1-\varphi_{Nu(1)})\pi))(1-\sin(0.5(1-\varphi_{Nu(2)})\pi))}{1+\sin(0.5(1-\varphi_{Nu(1)})\pi)\sin(0.5(1-\varphi_{Nu(2)})\pi)},\\ \dfrac{(1-\sin(0.5(1-\varphi_{Nv(1)})\pi))(1-\sin(0.5(1-\varphi_{Nv(2)})\pi))}{1+\sin(0.5(1-\varphi_{Nv(1)})\pi)\sin(0.5(1-\varphi_{Nv(2)})\pi)}\end{array}\right);$$

$$(2)\quad \sin(\varphi_{N1})\otimes_E\sin(\varphi_{N2})=\left(\begin{array}{l}\dfrac{\sin(0.5\varphi_{Nt(1)}\pi)\sin(0.5\varphi_{Nt(2)}\pi)}{1+(1-\sin(0.5\varphi_{Nt(1)}\pi))(1-\sin(0.5\varphi_{Nt(2)}\pi))},\\ \dfrac{1-\sin(0.5(1-\varphi_{Nu(1)})\pi)+1-\sin(0.5(1-\varphi_{Nu(2)})\pi)}{1+(1-\sin(0.5(1-\varphi_{Nu(1)})\pi))(1-\sin(0.5(1-\varphi_{Nu(2)})\pi))},\\ \dfrac{1-\sin(0.5(1-\varphi_{Nv(1)})\pi)+1-\sin(0.5(1-\varphi_{Nv(2)})\pi)}{1+(1-\sin(0.5(1-\varphi_{Nv(1)})\pi))(1-\sin(0.5(1-\varphi_{Nv(2)})\pi))}\end{array}\right);$$

$$(3)\quad (\sin(\varphi_{N1}))^{\mu_w}=\left(\begin{array}{l}\dfrac{2(\sin(0.5\varphi_{Nt(1)}\pi))^{\mu_w}}{(2-\sin(0.5\varphi_{Nt(1)}\pi))^{\mu_w}+(\sin(0.5\varphi_{Nt(1)}\pi))^{\mu_w}},\\ \dfrac{(2-\sin(0.5(1-\varphi_{Nu(1)})\pi))^{\mu_w}-(\sin(0.5(1-\varphi_{Nu(1)})\pi))^{\mu_w}}{(2-\sin(0.5(1+\varphi_{Nu(1)})\pi))^{\mu_w}+(\sin(0.5(1-\varphi_{Nu(1)})\pi))^{\mu_w}},\\ \dfrac{(2-\sin(0.5(1-\varphi_{Nv(1)})\pi))^{\mu_w}-(\sin(0.5(1-\varphi_{Nv(1)})\pi))^{\mu_w}}{(2-\sin(0.5(1-\varphi_{Nv(1)})\pi))^{\mu_w}+(\sin(0.5(1-\varphi_{Nv(1)})\pi))^{\mu_w}}\end{array}\right);$$

$$(4)\quad \mu_w \cdot \sin(\varphi_{N1}) = \left(\begin{array}{c} \dfrac{(1+\sin(0.5\varphi_{Nt(1)}\pi))^{\mu_w} - (1-\sin(0.5\varphi_{Nt(1)}\pi))^{\mu_w}}{(1+\sin(0.5\varphi_{Nt(1)}\pi))^{\mu_w} + (1-\sin(0.5\varphi_{Nt(1)}\pi))^{\mu_w}}, \\ \dfrac{2(1-\sin(0.5(1-\varphi_{Nu(1)})\pi))^{\mu_w}}{(1+\sin(0.5(1-\varphi_{Nu(1)})\pi))^{\mu_w} + (1-\sin(0.5(1-\varphi_{Nu(1)})\pi))^{\mu_w}}, \\ \dfrac{2(1-\sin(0.5(1-\varphi_{Nv(1)})\pi))^{\mu_w}}{(1+\sin(0.5(1-\varphi_{Nv(1)})\pi))^{\mu_w} + (1-\sin(0.5(1-\varphi_{Nv(1)})\pi))^{\mu_w}} \end{array} \right).$$

In view of EOLs of S-SvNVs, we define the S-SvNVEWA and S-SvNVEWG AOs.

Definition 3. If $\varphi_{Ns} = <\varphi_{Nt(s)}, \varphi_{Nu(s)}, \varphi_{Nv(s)}>$ (s = 1, 2, …, q) are a collection of SvNVs with their weight vector $\boldsymbol{\mu}_W = (\mu_{w1}, \mu_{w2}, \ldots, \mu_{wq})$ for $0 \le \mu_{ws} \le 1$ and $\sum_{s=1}^{q} \mu_{ws} = 1$, the S-SvNVEWA and S-SvNVEWG AOs can be defined as follows:

$$S-SvNVEWA(\varphi_{N1}, \varphi_{N2}, ..., \varphi_{Nq}) = \mu_{w1}\sin(\varphi_{N1}) \oplus_E \mu_{w2}\sin(\varphi_{N2}) \oplus_E ... \oplus_E \mu_{wq}\sin(\varphi_{Nq}) = \sum_{s=1}^{q}{}_E\, \mu_{ws}\sin(\varphi_{Ns}), \quad (11)$$

$$S-SvNVWG(\varphi_{N1}, \varphi_{N2}, ..., \varphi_{Nq}) = \left(\sin(\varphi_{N1})\right)^{\mu_{w1}} \otimes_E \left(\sin(\varphi_{N2})\right)^{\mu_{w2}} \otimes_E ... \otimes_E \left(\sin(\varphi_{Nq})\right)^{\mu_{wq}} = \prod_{s=1}^{q}{}_E \left(\sin(\varphi_{Ns})\right)^{\mu_{ws}}. \quad (12)$$

Theorem 1. If $\varphi_{Ns} = <\varphi_{Nt(s)}, \varphi_{Nu(s)}, \varphi_{Nv(s)}>$ (s = 1, 2, …, q) are a collection of SvNVs with their weight vector $\boldsymbol{\mu}_W = (\mu_{w1}, \mu_{w2}, \ldots, \mu_{wq})$ for $0 \le \mu_{ws} \le 1$ and $\sum_{s=1}^{q} \mu_{ws} = 1$, the aggregated result of the S-SvNVEWA AO is still S-SvNV, which is yielded by the equation:

$$S-SvNVEWA(\varphi_{N1}, \varphi_{N2}, ..., \varphi_{Nq}) = \sum_{s=1}^{q}{}_E\, \mu_{ws}\sin(\varphi_{Ns})$$
$$= \left(\begin{array}{c} \dfrac{\prod_{s=1}^{q}\left(1+\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}} - \prod_{s=1}^{q}\left(1-\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{q}\left(1+\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}} + \prod_{s=1}^{q}\left(1-\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}}, \\ \dfrac{2\prod_{s=1}^{q}\left(1-\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{q}\left(1+\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}} + \prod_{s=1}^{q}\left(1-\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}}, \\ \dfrac{2\prod_{s=1}^{q}\left(1-\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{q}\left(1+\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}} + \prod_{s=1}^{q}\left(1-\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}} \end{array} \right). \quad (13)$$

Proof. In terms of mathematical induction and Definition 2, we can give the proof of Theorem 1.

For q = 2, the operational results are given below:

$$\mu_{w1} \cdot \sin(\varphi_{N1}) = \left(\begin{array}{c} \dfrac{(1+\sin(0.5\varphi_{Nt(1)}\pi))^{\mu_{w1}} - (1-\sin(0.5\varphi_{Nt(1)}\pi))^{\mu_{w1}}}{(1+\sin(0.5\varphi_{Nt(1)}\pi))^{\mu_{w1}} + (1-\sin(0.5\varphi_{Nt(1)}\pi))^{\mu_{w1}}}, \\ \dfrac{2(1-\sin(0.5(1-\varphi_{Nu(1)})\pi))^{\mu_{w1}}}{(1+\sin(0.5(1-\varphi_{Nu(1)})\pi))^{\mu_{w1}} + (1-\sin(0.5(1-\varphi_{Nu(1)})\pi))^{\mu_{w1}}}, \\ \dfrac{2(1-\sin(0.5(1-\varphi_{Nv(1)})\pi))^{\mu_1}}{(1+\sin(0.5(1-\varphi_{Nv(1)})\pi))^{\mu_1} + (1-\sin(0.5(1-\varphi_{Nv(1)})\pi))^{\mu_1}} \end{array} \right),$$

$$\mu_{w2}\cdot\sin(\varphi_{N2})=\left(\begin{array}{c}\dfrac{(1+\sin(0.5\varphi_{Nt(2)}\pi))^{\mu_{w2}}-(1-\sin(0.5\varphi_{Nt(2)}\pi))^{\mu_{w2}}}{(1+\sin(0.5\varphi_{Nt(2)}\pi))^{\mu_{w2}}+(1-\sin(0.5\varphi_{Nt(2)}\pi))^{\mu_{w2}}},\\ \dfrac{2(1-\sin(0.5(1-\varphi_{Nu(2)})\pi))^{\mu_{w2}}}{(1+\sin(0.5(1-\varphi_{Nu(2)})\pi))^{\mu_{w2}}+(1-\sin(0.5(1-\varphi_{Nu(2)})\pi))^{\mu_{w2}}},\\ \dfrac{2(1-\sin(0.5(1-\varphi_{Nv(2)})\pi))^{\mu_{w2}}}{(1+\sin(0.5(1-\varphi_{Nv(2)})\pi))^{\mu_{w2}}+(1-\sin(0.5(1-\varphi_{Nv(2)})\pi))^{\mu_{w2}}}\end{array}\right).$$

Then, there is the following result:

$$S-SvNVEWA(\varphi_{N1},\varphi_{N2})=\sum_{s=1}^{2}{}_E\,\mu_{ws}\sin(\varphi_{Ns})$$
$$=\left(\begin{array}{c}\dfrac{\prod_{s=1}^{2}\left(1+\sin\left(0.5\varphi_{t(s)}\pi\right)\right)^{\mu_{ws}}-\prod_{s=1}^{2}\left(1-\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{2}\left(1+\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}+\prod_{s=1}^{2}\left(1-\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}},\\ \dfrac{2\prod_{s=1}^{2}\left(1-\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{2}\left(1+\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}+\prod_{s=1}^{2}\left(1-\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}},\\ \dfrac{2\prod_{s=1}^{2}\left(1-\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{2}\left(1+\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}+\prod_{s=1}^{2}\left(1-\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}}\end{array}\right). \quad (14)$$

Suppose that Eq. (13) holds for $q = p$. Then, there is the equation:

$$S-SvNVEWA(\varphi_{N1},\varphi_{N2},...,\varphi_{Np})=\sum_{s=1}^{p}{}_E\,\mu_{ws}\sin(\varphi_{Ns})$$
$$=\left(\begin{array}{c}\dfrac{\prod_{s=1}^{p}\left(1+\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}-\prod_{s=1}^{p}\left(1-\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{p}\left(1+\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}+\prod_{s=1}^{p}\left(1-\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}},\\ \dfrac{2\prod_{s=1}^{p}\left(1-\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{p}\left(1+\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}+\prod_{s=1}^{p}\left(1-\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}},\\ \dfrac{2\prod_{s=1}^{p}\left(1-\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{p}\left(1+\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}+\prod_{s=1}^{p}\left(1-\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}}\end{array}\right). \quad (15)$$

Based on Eqs. (14) and (15) for $q = p+1$, we have

$$S-SvNVEWA(\varphi_{N1},\varphi_{N2},...,\varphi_{Np},\varphi_{Np+1})=\sum_{s=1}^{p+1}{}_{E}\,\mu_{ws}\sin(\varphi_{Ns})$$

$$=\left\langle \begin{array}{l} \dfrac{\prod_{s=1}^{p}\left(1+\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}-\prod_{s=1}^{p}\left(1-\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{p}\left(1+\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}+\prod_{s=1}^{p}\left(1-\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}}, \\ \dfrac{2\prod_{s=1}^{p}\left(1-\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{p}\left(1+\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}+\prod_{s=1}^{p}\left(1-\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}}, \\ \dfrac{2\prod_{s=1}^{p}\left(1-\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{p}\left(1+\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}+\prod_{s=1}^{p}\left(1-\sin\left(\frac{\pi}{2}(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}} \end{array}\right\rangle$$

$$\oplus_{E}\left\langle \begin{array}{l} \dfrac{(1+\sin(0.5\varphi_{Nt(p+1)}\pi))^{\mu_{wp+1}}-(1-\sin(\frac{\pi}{2}\varphi_{Nt(p+1)}\pi))^{\mu_{wp+1}}}{(1+\sin(\frac{\pi}{2}\varphi_{Nt(p+1)}\pi))^{\mu_{wp+1}}+(1-\sin(\frac{\pi}{2}\varphi_{Nt(p+1)}\pi))^{\mu_{wp+1}}}, \\ \dfrac{2(1-\sin(0.5(1-\varphi_{Nu(p+1)})\pi))^{\mu_{wp+1}}}{(1+\sin(0.5(1-\varphi_{Nu(p+1)})\pi))^{\mu_{wp+1}}+(1-\sin(0.5(1-\varphi_{Nu(p+1)})\pi))^{\mu_{wp+1}}}, \\ \dfrac{2(1-\sin(0.5(1-\varphi_{Nv(p+1)})\pi))^{\mu_{wp+1}}}{(1+\sin(0.5(1-\varphi_{Nv(p+1)})\pi))^{\mu_{wp+1}}+(1-\sin(0.5(1-\varphi_{Nv(p+1)})\pi))^{\mu_{wp+1}}} \end{array}\right\rangle$$

$$=\left\langle \begin{array}{l} \dfrac{\prod_{s=1}^{p+1}\left(1+\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}-\prod_{s=1}^{p+1}\left(1-\sin\left(0.5\varphi_{t(s)}\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{p+1}\left(1+\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}+\prod_{s=1}^{p+1}\left(1-\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}}, \\ \dfrac{2\prod_{s=1}^{p+1}\left(1-\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{p+1}\left(1+\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}+\prod_{s=1}^{p+1}\left(1-\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}}, \\ \dfrac{2\prod_{s=1}^{p+1}\left(1-\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{p+1}\left(1+\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}+\prod_{s=1}^{p+1}\left(1-\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}} \end{array}\right\rangle.$$

Since Eq. (13) can hold for $q = p+1$, it can exist for all q.

Then, this S-SvNVEWA AO reveals the features below.

Theorem 2. The S-SvNVEWA AO reveals some features in view of the sine function below.

(1) Idempotency: If $\varphi_{Ns} = <\varphi_{Nt(s)}, \varphi_{Nu(s)}, \varphi_{Nv(s)}> = <\varphi_{Nt}, \varphi_{Nu}, \varphi_{Nv}> = \varphi_N$ (s = 1, 2, …, q), then there is $S-SvNVEWA(\varphi_{N1},\varphi_{N2},...,\varphi_{Nq})=\sin(\varphi_N)$.

(2) Boundedness: Set $\varphi_N^- = \left\langle \min_s(\varphi_{Nt(s)}), \max_s(\varphi_{Nu(s)}), \max_s(\varphi_{Nv(s)})\right\rangle$ and $\varphi_N^+ = \left\langle \max_s(\varphi_{Nt(s)}), \min_s(\varphi_{Nu(s)}), \min_s(\varphi_{Nv(s)})\right\rangle$ as the minimum and maximum SvNVs. Then, there exists $\sin(\varphi_N^-) \le S-SvNVEWA(\varphi_{N1},\varphi_{N2},...,\varphi_{Nq}) \le \sin(\varphi_N^+)$.

(3) Monotonicity: Let $\varphi_{Ns} = <\varphi_{Nt(s)}, \varphi_{Nu(s)}, \varphi_{Nv(s)}>$ and $\varphi_{Ns}^{*} = \left\langle \varphi_{Nt(s)}^{*}, \varphi_{Nu(s)}^{*}, \varphi_{Nv(s)}^{*} \right\rangle$ (s = 1, 2, …, q) be two collections of SvNVs. Then $S-SvNVEWA(\varphi_{N1}, \varphi_{N2}, \cdots, \varphi_{Nq}) \le S-SvNVEWA(\varphi_{N1}^{*}, \varphi_{N2}^{*}, \cdots, \varphi_{Nq}^{*})$ exists if $\varphi_{Ns} \le \varphi_{Ns}^{*}$.

Proof. (1) Applying Eq. (13) for $\varphi_{Ns} = \varphi_{N}$, we obtain

$$S-SvNVEWA(\varphi_{N1}, \varphi_{N2}, ..., \varphi_{Nq}) = \sum_{s=1}^{q}{}_{E}\, \mu_{ws} \sin(\varphi_{Ns})$$

$$= \left(\begin{array}{c} \dfrac{\prod_{s=1}^{q}\left(1+\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}} - \prod_{s=1}^{q}\left(1-\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{q}\left(1+\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}} + \prod_{s=1}^{q}\left(1-\sin\left(0.5\varphi_{Nt(s)}\pi\right)\right)^{\mu_{ws}}}, \\ \dfrac{2\prod_{s=1}^{q}\left(1-\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{q}\left(1+\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}} + \prod_{s=1}^{q}\left(1-\sin\left(0.5(1-\varphi_{Nu(s)})\pi\right)\right)^{\mu_{ws}}}, \\ \dfrac{2\prod_{s=1}^{q}\left(1-\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{q}\left(1+\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}} + \prod_{s=1}^{q}\left(1-\sin\left(0.5(1-\varphi_{Nv(s)})\pi\right)\right)^{\mu_{ws}}} \end{array} \right)$$

$$= \left(\begin{array}{c} \dfrac{(1+\sin(0.5\varphi_{Nt}\pi))^{\sum_{s=1}^{q}\mu_{ws}} - (1-\sin(0.5\varphi_{Nt}\pi))^{\sum_{s=1}^{q}\mu_{ws}}}{(1+\sin(0.5\varphi_{Nt}\pi))^{\sum_{s=1}^{q}\mu_{ws}} + (1-\sin(0.5\varphi_{Nt}\pi))^{\sum_{s=1}^{q}\mu_{ws}}}, \\ \dfrac{2(1-\sin(0.5(1-\varphi_{Nu})\pi))^{\sum_{s=1}^{q}\mu_{ws}}}{(1+\sin(0.5(1-\varphi_{Nu})\pi))^{\sum_{s=1}^{q}\mu_{ws}} + (1-\sin(0.5(1-\varphi_{Nu})\pi))^{\sum_{s=1}^{q}\mu_{ws}}}, \\ \dfrac{2(1-\sin(0.5(1-\varphi_{Nv})\pi))^{\sum_{s=1}^{q}\mu_{ws}}}{(1+\sin(0.5(1-\varphi_{Nv})\pi))^{\sum_{s=1}^{q}\mu_{ws}} + (1-\sin(0.5(1-\varphi_{Nv})\pi))^{\sum_{s=1}^{q}\mu_{ws}}} \end{array} \right)$$

$$= \left(\begin{array}{c} \dfrac{(1+\sin(0.5\varphi_{Nt}\pi)) - (1-\sin(0.5\varphi_{Nt}\pi))}{(1+\sin(0.5\varphi_{Nt}\pi)) + (1-\sin(0.5\varphi_{Nt}\pi))}, \\ \dfrac{2(1-\sin(0.5(1-\varphi_{Nu})\pi))}{(1+\sin(0.5(1-\varphi_{Nu})\pi)) + (1-\sin(0.5(1-\varphi_{Nu})\pi))}, \\ \dfrac{2(1-\sin(0.5(1-\varphi_{Nv})\pi))}{(1+\sin(0.5(1-\varphi_{Nv})\pi)) + (1-\sin(0.5(1-\varphi_{Nv})\pi))} \end{array} \right) = \sin(\varphi_{N}).$$

(2) For $\varphi_{N}^{-} \le \varphi_{Ns} \le \varphi_{N}^{+}$, $\sin(\varphi_{N}^{-}) \le \sin(\varphi_{Ns}) \le \sin(\varphi_{N}^{+})$ exists since sin(z) for 0 ≤ z ≤ π/2 is an increasing function. Then, $\sum_{s=1}^{q}{}_{E}\, \mu_{ws} \sin(\varphi_{N}^{-}) \le \sum_{s=1}^{q}{}_{E}\, \mu_{ws} \sin(\varphi_{Ns}) \le \sum_{s=1}^{q} \mu_{ws} \sin(\varphi_{N}^{+})$ is held. In view of the feature (1), $\sin(\varphi_{N}^{-}) \le S-SvNVEWA(\varphi_{N1}, \varphi_{N2}, ..., \varphi_{Nq}) \le \sin(\varphi_{N}^{+})$ can be also held.

(3) For $\varphi_{Ns} \le \varphi_{Ns}^{*}$, $\sin(\varphi_{Ns}) \le \sin(\varphi_{Ns}^{*})$ is held since sin(z) for 0 ≤ z ≤ π/2 is an increasing function. $\sum_{s=1}^{q} \mu_{ws} \sin(\varphi_{Ns}) \le \sum_{s=1}^{q} \mu_{ws} \sin(\varphi_{Ns}^{*})$ can be held in view of the feature (2). Thus, $S-SvNVEWA(\varphi_{N1}, \varphi_{N2}, \cdots, \varphi_{Nq}) \le S-SvNVEWA(\varphi_{N1}^{*}, \varphi_{N2}^{*}, \cdots, \varphi_{Nq}^{*})$ can also hold.

Example 1. Suppose that three SvNVs are φ_{N1} = <0.6, 0.2, 0.3>, φ_{N2} = <0.8, 0.1, 0.1>, and φ_{N3} = <0.7, 0.3, 0.3> with their weight vector μ_W = (0.4, 0.3, 0.3). Using Eq. (13), we give the following aggregation result of the S-SvNVEWA AO:

$$S-SvNVEWA(\varphi_{N1},\varphi_{N2},\varphi_{N3})=\sum_{s=1}^{3}{}_E\,\mu_{ws}\sin(\varphi_{Ns})$$

$$=\left(\begin{array}{c}\dfrac{\left(\begin{array}{l}\left(1+\sin\left(0.5\times0.6\pi\right)\right)^{0.4}\left(1+\sin\left(0.5\times0.8\pi\right)\right)^{0.3}\left(1+\sin\left(0.5\times0.7\pi\right)\right)^{0.3}\\-\left(1-\sin\left(0.5\times0.6\pi\right)\right)^{0.4}\left(1-\sin\left(0.5\times0.8\pi\right)\right)^{0.3}\left(1-\sin\left(0.5\times0.7\pi\right)\right)^{0.3}\end{array}\right)}{\left(\begin{array}{l}\left(1+\sin\left(0.5\times0.6\pi\right)\right)^{0.4}\left(1+\sin\left(0.5\times0.8\pi\right)\right)^{0.3}\left(1+\sin\left(0.5\times0.7\pi\right)\right)^{0.3}\\+\left(1-\sin\left(0.5\times0.6\pi\right)\right)^{0.4}\left(1-\sin\left(0.5\times0.8\pi\right)\right)^{0.3}\left(1-\sin\left(0.5\times0.7\pi\right)\right)^{0.3}\end{array}\right)},\\ \dfrac{2\left(1-\sin\left(0.5(1-0.2)\pi\right)\right)^{0.4}\left(1-\sin\left(0.5(1-0.1)\pi\right)\right)^{0.3}\left(1-\sin\left(0.5(1-0.3)\pi\right)\right)^{0.3}}{\left(\begin{array}{l}\left(1+\sin\left(0.5(1-0.2)\pi\right)\right)^{0.4}\left(1+\sin\left(0.5(1-0.1)\pi\right)\right)^{0.3}\left(1+\sin\left(0.5(1-0.3)\pi\right)\right)^{0.3}\\+\left(1-\sin\left(0.5(1-0.2)\pi\right)\right)^{0.4}\left(1-\sin\left(0.5(1-0.1)\pi\right)\right)^{0.3}\left(1-\sin\left(0.5(1-0.3)\pi\right)\right)^{0.3}\end{array}\right)},\\ \dfrac{2\left(1-\sin\left(0.5(1-0.3)\pi\right)\right)^{0.4}\left(1-\sin\left(0.5(1-0.1)\pi\right)\right)^{0.3}\left(1-\sin\left(0.5(1-0.3)\pi\right)\right)^{0.3}}{\left(\begin{array}{l}\left(1+\sin\left(0.5(1-0.3)\pi\right)\right)^{0.4}\left(1+\sin\left(0.5(1-0.1)\pi\right)\right)^{0.3}\left(1+\sin\left(0.5(1-0.3)\pi\right)\right)^{0.3}\\+\left(1-\sin\left(0.5(1-0.3)\pi\right)\right)^{0.4}\left(1-\sin\left(0.5(1-0.1)\pi\right)\right)^{0.3}\left(1-\sin\left(0.5(1-0.3)\pi\right)\right)^{0.3}\end{array}\right)}\end{array}\right)$$

$$=<0.8918,\ 0.0414,\ 0.0573>.$$

Theorem 3. Set φ_{Ns} = <$\varphi_{Nt(s)}$, $\varphi_{Nu(s)}$, $\varphi_{Nv(s)}$> (s = 1, 2, …, q) as a collection of SvNVs with their weight vector μ_W = (μ_{w1}, μ_{w2}, …, μ_{wq}) for $0 \le \mu_{ws} \le 1$ and $\sum_{s=1}^{q}\mu_{ws}=1$. Then, the aggregated result of the S-SvNVEWG AO is still S-SvNV, which is yielded by the equation:

$$S-SvNVEWG(\varphi_{N1},\varphi_{N2},...,\varphi_{Nq})=\prod_{s=1}^{q}{}_E\left(\sin(\varphi_{Ns})\right)^{\mu_{ws}}$$

$$=\left\langle\begin{array}{c}\dfrac{2\prod_{s=1}^{q}(\sin(0.5\varphi_{Nt(s)}\pi))^{\mu_{ws}}}{\prod_{s=1}^{q}(2-\sin(0.5\varphi_{Nt(s)}\pi))^{\mu_{ws}}+\prod_{s=1}^{q}(\sin(0.5\varphi_{Nt(s)}\pi))^{\mu_{ws}}},\\ \dfrac{\prod_{s=1}^{q}(2-\sin(0.5(1-\varphi_{Nu(s)})\pi))^{\mu_{ws}}-\prod_{s=1}^{q}(\sin(0.5(1-\varphi_{Nu(s)})\pi))^{\mu_{ws}}}{\prod_{s=1}^{q}(2-\sin(0.5(1-\varphi_{Nu(s)})\pi))^{\mu_{ws}}+\prod_{s=1}^{q}(\sin(0.5(1-\varphi_{Nu(s)})\pi))^{\mu_{ws}}},\\ \dfrac{\prod_{s=1}^{q}(2-\sin(0.5(1-\varphi_{Nv(s)})\pi))^{\mu_{ws}}-\prod_{s=1}^{q}(\sin(0.5(1-\varphi_{Nv(s)})\pi))^{\mu_{ws}}}{\prod_{s=1}^{q}(2-\sin(0.5(1-\varphi_{v(s)})\pi))^{\mu_{ws}}+\prod_{s=1}^{q}(\sin(0.5(1-\varphi_{v(s)})\pi))^{\mu_{ws}}}\end{array}\right\rangle. \tag{16}$$

However, the proof of Theorem 3 can be given based on a similar proof of Theorem 1, which is omitted.

Similarly, the S-SvNVEWG AO also indicates some features by the following theorem.

Theorem 4. The S-SvNVEWG AO reveals some features in view of the sine function below:

(1) Idempotency: If φ_{Ns} = <$\varphi_{Nt(s)}$, $\varphi_{Nu(s)}$, $\varphi_{Nv(s)}$> = <φ_{Nt}, φ_{Nu}, φ_{Nv}> = φ_N (s = 1, 2, …, q), then there is $S-SvNVEWG(\varphi_{N1},\varphi_{N2},...,\varphi_{Nq})=\sin(\varphi_N)$.

(2) Boundedness: Set $\varphi_N^- = \left\langle \min_s(\varphi_{Nt(s)}), \max_s(\varphi_{Nu(s)}), \max_s(\varphi_{Nv(s)}) \right\rangle$ and $\varphi_N^+ = \left\langle \max_s(\varphi_{Nt(s)}), \min_s(\varphi_{Nu(s)}), \min_s(\varphi_{Nv(s)}) \right\rangle$ as the minimum and maximum SvNVs. Then, there exists $\sin(\varphi_N^-) \le S-SvNVEWG(\varphi_{N1},\varphi_{N2},...,\varphi_{Nq}) \le \sin(\varphi_N^+)$.

(3) Monotonicity: Let φ_{Ns} = <$\varphi_{Nt(s)}$, $\varphi_{Nu(s)}$, $\varphi_{Nv(s)}$> and $\varphi_{Ns}^* = \left\langle \varphi_{Nt(s)}^*, \varphi_{Nu(s)}^*, \varphi_{Nv(s)}^* \right\rangle$ (s = 1, 2, …, q) be two collections of SvNVs. Then, $S-SvNVEWG(\varphi_{N1},\varphi_{N2},\cdots,\varphi_{Nq}) \le S-SvNVEWG(\varphi_{N1}^*,\varphi_{N2}^*,\cdots,\varphi_{Nq}^*)$ exists if $\varphi_{Ns} \le \varphi_{Ns}^*$.

However, the proof of Theorem 4 can also be given in terms of a similar proof of Theorem 2, which is omitted.

Example 2. Suppose that three SvNVs are φ_{N1} = <0.8, 0.3, 0.1>, φ_{N2} = <0.7, 0.2, 0.2>, and φ_{N3} = <0.9, 0.4, 0.4> with their weight vector $\boldsymbol{\mu}_w$ = (0.5, 0.3, 0.2). Using Eq. (16), we give the following aggregation result of the S-SvNVEWG AO:

$$S-SvNVEWG(\varphi_{N1},\varphi_{N2},\varphi_{N3}) = \prod_{s=1}^{3}{}_E\left(\sin(\varphi_{Ns})\right)^{\mu_{ws}}$$

$$= \left\langle \begin{array}{l} \dfrac{2\times(\sin(0.5\times0.8\pi))^{0.5}(\sin(0.5\times0.7\pi))^{0.3}(\sin(0.5\times0.9\pi))^{0.2}}{\left(\begin{array}{l}(2-\sin(0.5\times0.8\pi))^{0.5}(2-\sin(0.5\times0.7\pi))^{0.3}(2-\sin(0.5\times0.9\pi))^{0.2}\\ +(\sin(0.5\times0.8\pi))^{0.5}(\sin(0.5\times0.7\pi))^{0.3}(\sin(0.5\times0.9\pi))^{0.2}\end{array}\right)}, \\ \dfrac{\left(\begin{array}{l}(2-\sin(0.5(1-0.3)\pi))^{0.5}(2-\sin(0.5(1-0.2)\pi))^{0.3}(2-\sin(0.5(1-0.4)\pi))^{0.2}\\ -(\sin(0.5(1-0.3)\pi))^{0.5}(\sin(0.5(1-0.2)\pi))^{0.3}(\sin(0.5(1-0.4)\pi))^{0.2}\end{array}\right)}{\left(\begin{array}{l}(2-\sin(0.5(1-0.3)\pi))^{0.5}(2-\sin(0.5(1-0.2)\pi))^{0.3}(2-\sin(0.5(1-0.4)\pi))^{0.2}\\ +(\sin(0.5(1-0.3)\pi))^{0.5}(\sin(0.5(1-0.2)\pi))^{0.3}(\sin(0.5(1-0.4)\pi))^{0.2}\end{array}\right)}, \\ \dfrac{\left(\begin{array}{l}(2-\sin(0.5(1-0.1)\pi))^{0.5}(2-\sin(0.5(1-0.2)\pi))^{0.3}(2-\sin(0.5(1-0.4)\pi))^{0.2}\\ -(\sin(0.5(1-0.1)\pi))^{0.5}(\sin(0.5(1-0.2)\pi))^{0.3}(\sin(0.5(1-0.4)\pi))^{0.2}\end{array}\right)}{\left(\begin{array}{l}(2-\sin(0.5(1-0.1)\pi))^{0.5}(2-\sin(0.5(1-0.2)\pi))^{0.3}(2-\sin(0.5(1-0.4)\pi))^{0.2}\\ +(\sin(0.5(1-0.1)\pi))^{0.5}(\sin(0.5(1-0.2)\pi))^{0.3}(\sin(0.5(1-0.4)\pi))^{0.2}\end{array}\right)} \end{array} \right\rangle$$

$$=<0.9403,\ 0.1077,\ 0.0595>.$$

4. MADM model

This part develops a MADM model in view of the S-SvNVEWA and S-SvNVEWG AOs in the circumstance of SvNVs.

A MADM issue commonly includes a set of several alternatives Y_H = {Y_{H1}, Y_{H2}, …, Y_{Hp}} and a set of several attributes X_C = {x_{c1}, x_{c2}, …, x_{cq}}. In the MADM process, the alternatives must meet the requirements of the attributes, and then their SvNV assessment results are represented as their decision matrix $Q_N = (\varphi_{Nrs})_{p\times q}$, where φ_{Nrs} (r = 1, 2, …, p; s = 1, 2, …, q) are SvNVs provided by decision makers (DMs) according to the satisfactory assessment of an alternative Y_{Hr} over attributes x_{cs}. The weight vector of the attributes is specified by $\boldsymbol{\mu}_w$ = (μ_{w1}, μ_{w2}, …, μ_{wq}) for $0 \le \mu_{ws} \le 1$ and $\sum_{s=1}^{q}\mu_{ws}=1$. Thus, the algorithm of the MADM model in the circumstance of SvNVs is described in detail below.

Step 1: The aggregated values of φ_{Nr} for Y_{Hr} (r = 1, 2, …, p) are yielded by one of the S-SvNVEWA and S-SvNVEWG AOs:

$$\varphi_{Nr} = S-SvNVEWA(\varphi_{Nr1},\varphi_{Nr2},...,\varphi_{Nrq}) = \sum_{s=1}^{q}{}_{E}\,\mu_{ws}\sin(\varphi_{Nrs})$$
$$= \left(\begin{array}{c} \dfrac{\prod_{s=1}^{q}\left(1+\sin\left(0.5\varphi_{Nt(rs)}\pi\right)\right)^{\mu_{ws}} - \prod_{s=1}^{q}\left(1-\sin\left(0.5\varphi_{Nt(rs)}\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{q}\left(1+\sin\left(0.5\varphi_{Nt(rs)}\pi\right)\right)^{\mu_{ws}} + \prod_{s=1}^{q}\left(1-\sin\left(0.5\varphi_{Nt(rs)}\pi\right)\right)^{\mu_{ws}}}, \\ \dfrac{2\prod_{s=1}^{q}\left(1-\sin\left(0.5(1-\varphi_{Nu(rs)})\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{q}\left(1+\sin\left(0.5(1-\varphi_{Nu(rs)})\pi\right)\right)^{\mu_{ws}} + \prod_{s=1}^{q}\left(1-\sin\left(0.5(1-\varphi_{Nu(rs)})\pi\right)\right)^{\mu_{ws}}}, \\ \dfrac{2\prod_{s=1}^{q}\left(1-\sin\left(0.5(1-\varphi_{Nv(rs)})\pi\right)\right)^{\mu_{ws}}}{\prod_{s=1}^{q}\left(1+\sin\left(0.5(1-\varphi_{Nv(rs)})\pi\right)\right)^{\mu_{ws}} + \prod_{s=1}^{q}\left(1-\sin\left(0.5(1-\varphi_{Nv(rs)})\pi\right)\right)^{\mu_{ws}}} \end{array} \right), \tag{17}$$

$$\varphi_{Nr} = S-SvNVEWG(\varphi_{Nr1},\varphi_{Nr2},...,\varphi_{Nrq}) = \prod_{s=1}^{q}{}_{E}\left(\sin(\varphi_{Nrs})\right)^{\mu_{ws}}$$
$$= \left\langle \begin{array}{c} \dfrac{2\prod_{s=1}^{q}(\sin(0.5\varphi_{Nt(rs)}\pi))^{\mu_{ws}}}{\prod_{s=1}^{q}(2-\sin(0.5\varphi_{Nt(rs)}\pi))^{\mu_{ws}} + \prod_{s=1}^{q}(\sin(0.5\varphi_{Nt(rs)}\pi))^{\mu_{ws}}}, \\ \dfrac{\prod_{s=1}^{q}(2-\sin(0.5(1-\varphi_{Nu(rs)})\pi))^{\mu_{ws}} - \prod_{s=1}^{q}(\sin(0.5(1-\varphi_{Nu(rs)})\pi))^{\mu_{ws}}}{\prod_{s=1}^{q}(2-\sin(0.5(1-\varphi_{Nu(rs)})\pi))^{\mu_{ws}} + \prod_{s=1}^{q}(\sin(0.5(1-\varphi_{Nu(rs)})\pi))^{\mu_{ws}}}, \\ \dfrac{\prod_{s=1}^{q}(2-\sin(0.5(1-\varphi_{Nv(rs)})\pi))^{\mu_{ws}} - \prod_{s=1}^{q}(\sin(0.5(1-\varphi_{Nv(rs)})\pi))^{\mu_{ws}}}{\prod_{s=1}^{q}(2-\sin(0.5(1-\varphi_{Nv(rs)})\pi))^{\mu_{ws}} + \prod_{s=1}^{q}(\sin(0.5(1-\varphi_{Nv(rs)})\pi))^{\mu_{ws}}} \end{array} \right\rangle. \tag{18}$$

Step 2: The score (accuracy) values of $U(\varphi_{Nr})$ ($V(\varphi_{Nr})$) (r = 1, 2, …, p) are yielded by Eq. (5) (Eq. (6)).

Step 3: Alternatives are sorted in the descending order of the score values (the accuracy values), and then the best alternative is decided.

Step 4: End.

5. MADM application

5.1 Site selection example of HPP

Since hydrogen is one of the most efficient and clean energy sources, its share of world energy has increased significantly. Then, it is important to choose the most suitable location for a HPP project, which is influenced by many factors, such as social, environmental, and economic factors. Hence, the site selection problem of HPP is a MADM problem. To apply the proposed MADM model to the actual MADM problem, this section adopts a site selection example of HPP in [30] for convenient comparison.

In this site selection example of HPP, experts and DMs preliminarily provide five potential locations, which are represented as a set of the five alternatives $Y_H = \{Y_{H1}, Y_{H2}, Y_{H3}, Y_{H4}, Y_{H5}\}$. Then they must satisfy the five main factors/attributes: the economic factor (x_{c1}), the technical factor (x_{c2}), the social factor (x_{c3}), the location factor (x_{c4}), and the environmental factor (x_{c5}). The weight vector of the five factors is given by μ_W = (0.22, 0.2, 0.15, 0.15, 0.28). In terms of the satisfactory degrees of each location corresponding to the five main factors, experts/DMs provide the SvNVs, which are composed of the indeterminate, false, and true degrees due to incompleteness, inconsistency, and uncertainty, including the judgements/opinions of the experts/DMs, and then their decision matrix of SvNVs $Q_N = (\varphi_{Nrs})_{5\times5}$ is presented as follows [30]:

$$Q_N = \begin{bmatrix} <0.3,0.2,0.4> & <0.2,0.2,0.6> & <0.5,0.3,0.4> & <0.3,0.3,0.4> & <0.5,0.2,0.3> \\ <0.6,0.4,0.2> & <0.6,0.3,0.2> & <0.7,0.1,0.3> & <0.7,0.1,0.2> & <0.7,0.2,0.3> \\ <0.5,0.1,0.3> & <0.6,0.1,0.2> & <0.5,0.3,0.4> & <0.6,0.4,0.3> & <0.6,0.2,0.4> \\ <0.5,0.2,0.2> & <0.4,0.5,0.2> & <0.7,0.3,0.2> & <0.4,0.5,0.4> & <0.7,0.2,0.2> \\ <0.4,0.3,0.6> & <0.4,0.1,0.5> & <0.4,0.1,0.3> & <0.3,0.2,0.4> & <0.5,0.1,0.2> \end{bmatrix}.$$

In this site selection problem of HPP, we give its MADM algorithm below.

Step 1: Applying Eq. (17) or Eq. (18), the aggregated results of the S-SvNVEWA or S-SvNVEWG AO are given below:

φ_{N1} = <0.5540, 0.0624, 0.1928>, φ_{N2} = <0.8616, 0.0520, 0.0693>, φ_{N3} = <0.7756, 0.0384, 0.1191>, φ_{N4} = <0.7792, 0.1055, 0.0604>, and φ_{N5} = <0.6073, 0.0247, 0.1594>.

Or φ_{N1} = <0.5174, 0.0670, 0.2154>, φ_{N2} = <0.8563, 0.0817, 0.0748>, φ_{N3} = <0.7707, 0.0642, 0.1326>, φ_{N4} = <0.7393, 0.1455, 0.0705>, and φ_{N5} = <0.5979, 0.0392, 0.2126>.

Step 2: By Eq. (5), the score values of $U(\varphi_{Nr})$ are yielded below:

$U(\varphi_{N1})$ = 0.7663, $U(\varphi_{N2})$ = 0.9134, $U(\varphi_{N3})$ = 0.8727, $U(\varphi_{N4})$ = 0.8711, and $U(\varphi_{N5})$ = 0.8077.

Or $U(\varphi_{N1})$ = 0.7450, $U(\varphi_{N2})$ = 0.8999, $U(\varphi_{N3})$ = 0.8579, $U(\varphi_{N4})$ = 0.8411, and $U(\varphi_{N5})$ = 0.7820.

Step 4: The sorting order of the five selection locations is $Y_{H2} > Y_{H3} > Y_{H4} > Y_{H5} > Y_{H1}$ and then the best one is Y_{H2}.

It is obvious that the sorting orders corresponding to the S-SvNVEWA and S-SvNVEWG AOs are the same.

5.2 Comparative analysis

In view of the above example, this part conducts a comparative investigation with existing related MADM models in the circumstances of SvNVs.

Based on the decision making methods of the existing MADM models [19, 30, 31], we can obtain all the decision results by different AOs of Eqs. (1)–(4) and Eqs. (7)–(10) and the score function of Eq. (5), which are tubulated in Table 1. For easy comparison, the decision results of the proposed MADM model are also shown in Table 1.

Table 1. Decision results corresponding to different AOs

AO	Sorting result	Optimal location
SvNVEWA [19]	$Y_{H2} > Y_{H3} > Y_{H4} > Y_{H5} > Y_{H1}$	Y_{H2}
SvNVEWG [19]	$Y_{H2} > Y_{H3} > Y_{H4} > Y_{H5} > Y_{H1}$	Y_{H2}
S-SvNVWA [30]	$Y_{H2} > Y_{H4} > Y_{H3} > Y_{H5} > Y_{H1}$	Y_{H2}
S-SvNVWG [30]	$Y_{H2} > Y_{H3} > Y_{H4} > Y_{H5} > Y_{H1}$	Y_{H2}
T-SvNVWA [31]	$Y_{H2} > Y_{H3} > Y_{H4} > Y_{H5} > Y_{H1}$	Y_{H2}
T-SvNVWG [31]	$Y_{H2} > Y_{H3} > Y_{H4} > Y_{H5} > Y_{H1}$	Y_{H2}
Proposed S-SvNVEWA	$Y_{H2} > Y_{H3} > Y_{H4} > Y_{H5} > Y_{H1}$	Y_{H2}
Proposed S-SvNVEWG	$Y_{H2} > Y_{H3} > Y_{H4} > Y_{H5} > Y_{H1}$	Y_{H2}

In the decision results of Table 1, we see that the sorting results based on the proposed S-SvNVEWA and S-SvNVEWG AOs are the same as those based on the SvNVEWA and SvNVEWG AOs [19], the T-SvNVWA and T-SvNVWG AOs [31], and the S-SvNVWG AO [30], but different from the ranking order based on the S-SvNVWA AO [30]. Then, the optimal site selection is always Y_{H2} among the five alternatives. In addition, in the representation of decision information, the newly defined S-SvNVs can overcome the flaws of the existing S-SvNVs [30], which include the [0, 0.46) range of membership degrees. In the aggregation operations of decision information, the proposed S-SvNVEWA and S-SvNVEWG AOs can overcome the defects of the existing SvNVEWA and SvNVEWG AOs without including periodicity features [19]. In terms of algebraic operation performance, the proposed S-SvNVEWA and S-SvNVEWG AOs are superior to the existing T-SvNVWA and T-SvNVWG AOs. Hence, the proposed model can satisfy the real needs of DMs in the multi-stage decision process. In general, the proposed model reveals obvious superiority over the existing models [19, 30, 31]. The decision results reveal the validity and rationality of the proposed MADM model and can help us to find the best solution in the practical decision application.

The obvious advantages of this study are presented below:

The defined S-SvNV concept contains the superiority of the membership functions belonging to [0, 1], which can overcome the defects in the existing S-SvNV concept with the membership functions belonging to [0, 0.46) [29, 30].

The proposed EOLs and Einstein AOs of S-SvNVs can reflect their typical algebraic operations and compensate for the insufficiencies of the existing AOs [19, 30, 31].

(c) The developed MADM model using the proposed S-SvNVEWA and S-SvNVEWG AOs reveals its superiority over the existing MADM models using the SvNVEWA and SvNVEWG AOs [19], the S-SvNVWA and S-SvNVWG AOs [30], and the T-SvNVWA and T-SvNVWG AOs [31].

6. Conclusions

In this study, the defined S-SvNV EOLs and the proposed S-SvNVEWA and S-SvNVEWG AOs based on the monotonic membership functions of indeterminacy, falsehood, and truth can overcome the insufficiencies of the existing S-SvNV representation, operation laws, and AOs. In view of the presented S-SvNVEWA and S-SvNVEWG AOs, the developed MADM model can effectively improve the MADM models based on the existing SvNVEWA, SvNVEWG, S-SvNVWA, S-SvNVWG, and T-SvNVWA, and T-SvNVWG AOs in the SvNV circumstance. Then, the validity of the developed model was investigated by the actual site selection example of HPP and examined by comparative analysis with the existing related MADM models in the setting of SvNVs.

In this paper, the presented S-SvNVEWA and S-SvNVEWG AOs and their MADM model were used only for single-valued neutrosophic aggregations and MADM problems, which shows their limitations. Furthermore, the presented S-SvNVEWA and S-SvNVEWG AOs are only based on EOLs of S-SvNVs, but cannot imply the trigonometric EOLs of SvNVs based on trigonometric Einstein t-norm and t-conorm, which show their disadvantages. Therefore, in the future work, we need to develop the trigonometric EOLs and AOs of SNVs (SvNVs and IvNVs) and their MADM models. Then, the developed models will be used for decision making problems in the fields of engineering management, economic management, and medical management.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Zadeh, L. A. Fuzzy sets. *Information and Control*, **1965**, 8(3), 338–353.
2. Atanassov, K. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, **1986**, 20, 87–96.
3. Atanassov, K.; Gargov, G. Interval-valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, **1989**, 31, 343–349.
4. Smarandache, F. *Neutrosophy: neutrosophic probability, set, and logic*. American Research Press, Rehoboth, USA,

1998.

5. Wang, H.; Smarandache, F.; Zhang, Y. Q.; Sunderraman, R. Single valued neutrosophic sets. *Multispace Multistructure*, **2010**, 4, 410–413.
6. Wang, H.; Smarandache, F.; Zhang, Y. Q.; Sunderraman, R. *Interval neutrosophic sets and logic: Theory and applications in computing*. Hexis, Phoenix, AZ, **2005**.
7. Ye, J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *Journal of Intelligent & Fuzzy Systems*, **2014**, 26, 2459–2466.
8. Zhang, H. Y.; Wang, J. Q., Chen, X. H. Interval neutrosophic sets and their application in multicriteria decision making problems. *The Scientific World Journal*, **2014**, 2014, 645953.
9. Yang, L. H.; Li, B. L. A multi-criteria decision-making method using power aggregation operators for single-valued neutrosophic sets. *International Journal of Database Theory and Application*, **2016**, 9(2), 23–32.
10. Liu, P.; Khan, Q.; Mahmood, T. Some single-valued neutrosophic power Muirhead mean operators and their application to group decision-making. *Journal of Intelligent & Fuzzy Systems*, **2019**, 37, 2515–2537.
11. Liu, P. The aggregation operators based on Archimedean t-conorm and t-norm for single-valued neutrosophic numbers and their application to decision-making. *International Journal of Fuzzy Systems*, **2016**, 18, 849–863.
12. Garg, H. Novel single-valued neutrosophic aggregated operators under Frank norm operation and its application to decision-making process. *International Journal of Uncertainty and Quantification*, **2016**, 6, 361–375.
13. Deli, I.; Subas, Y. A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems. *International Journal of Machine Learning and Cybernetics*, **2017**, 8, 1309–1322.
14. Liu, P.; Liu, X. The neutrosophic number generalized weighted power averaging operator and its application in multiple attribute group decision-making. *International Journal of Machine Learning and Cybernetics*, **2018**, 9, 347–358.
15. Karaaslan F.; Hayat, K. Some new operations on single-valued neutrosophic matrices and their applications in multi-criteria group decision making. *Applied Intelligence*, **2018**, 48, 4594–4614.
16. Garg, H. SVNMPR - A new single-value neutrosophic multiplicative preference relation and their application to decision-making process. *International Journal of Intelligent Systems*, **2022**, 37(3), 2089–2130.
17. Giri, B. K.; Roy, S. K. Neutrosophic multi-objective green four-dimensional fixed-charge transportation problem. *International Journal of Machine Learning and Cybernetics*, **2022**, 13, 3089–3112.
18. Peng, X.; Dai, J. A bibliometric analysis of neutrosophic set: two decades review from 1998 to 2017. *Artificial Intelligence Review*, **2020**, 53, 199–255.
19. Peng, J. J.; Wang, J. Q.; Wang, J.; Zhang, H. Y.; Chen, X. H. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. *International Journal of Systems Science*, **2016**, 47(10), 2342–2358.
20. Liu, P. D.; Wang, Y. M. Multiple attribute decision making method based on single-valued neutrosophic normalized weighted Bonferroni mean. *Neural Computing and Applications*, **2014**, 25(7-8), 2001–2010.
21. Liu, P.; Chu, Y.; Li, Y.; Chen, Y. Some generalized neutrosophic number Hamacher aggregation operators and their application. *International Journal of Fuzzy Systems*, **2014**, 16(2), 242–255.
22. Ye, J. Exponential operations and aggregation operators of interval neutrosophic sets and their decision making methods. *SpringerPlus*, **2016**, 5, 1488.

23. Ye, J. Subtraction and division operations of simplified neutrosophic sets. *Information*, **2017**, 8, 51.
24. Zhou, L. P.; Dong, J. Y.; Wan, S. P. Two new approaches for multi-attribute group decision-making with interval-valued neutrosophic Frank aggregation operators and incomplete weights. *IEEE Access*, **2019**, 7, 102727–102750.
25. Riaz, M.; Farid, H. M. A.; Antucheviciene, J.; Demir, G. Efficient decision making for sustainable energy using single-valued neutrosophic prioritized interactive aggregation operators. *Mathematics*, **2023**, 11(9), 2186.
26. Riaz, M.; Farid, H. M. A.; Ashraf, S.; Kamacı, H. Single-valued neutrosophic fairly aggregation operators with multi-criteria decision-making. *Computational and Applied Mathematics*, **2023**, 42, 104.
27. Garg, H. New logarithmic operational laws and their applications to multiattribute decision-making for single-valued neutrosophic numbers. *Cognitive Systems Research*, **2018**, 52, 931–946.
28. Ashraf, S.; Abdullah, S.; Smarandache, F. Logarithmic hybrid aggregation operators based on single valued neutrosophic sets and their applications in decision support systems. *Symmetry*, **2019**, 11, 364.
29. Garg, H. Decision making analysis based on sine trigonometric operational laws for single-valued neutrosophic sets and their applications. *Applied and Computational Mathematics*, **2020**, 19(2), 255–276.
30. Ashraf, S.; Abdullah, S.; Zeng, S.; Jin, H.; Ghani, F. Fuzzy decision support modeling for hydrogen power plant selection based on single valued neutrosophic sine trigonometric aggregation operators. *Symmetry*, **2020**, 12, 298.
31. Zhao, M.; Ye, J. MADM technique using tangent trigonometric SVNN aggregation operators for the teaching quality assessment of teachers. *Neutrosophic Sets and Systems*, **2022**, 50, 651–662.
32. Ye, J.; Du, S.; Yong, R. Multi-criteria decision-making model using trigonometric aggregation operators of single-valued neutrosophic credibility numbers. *Information Sciences*, **2023**, 644, 118968.

Received: 14 Dec ,2023 Accepted:18 Feb,2024