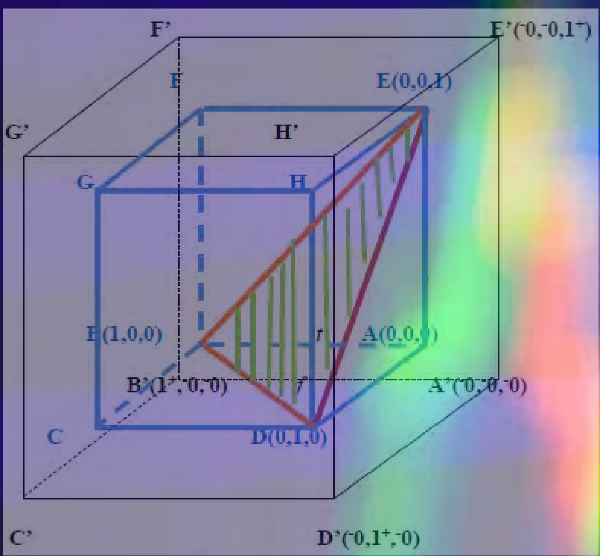
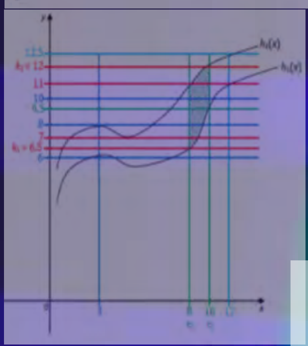
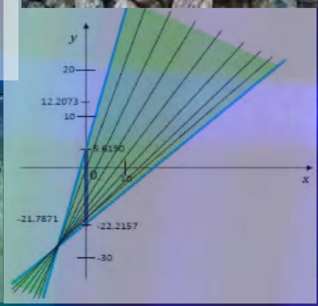
Volume 66,2024

Neutrosophic Sets and Systems

An International Journal in Information Science and Engineering







<A> <neutA> <antiA>

Florentin Smarandache . Mohamed Abdel-Basset . Said Broumi
Editors-in-Chief

ISSN 2331-6055 (Print)
ISSN 2331-608X (Online)

Neutrosophic Science
International Association (NSIA)

ISSN 2331-6055 (print) *ISSN 2331-608X (online)*

Neutrosophic Sets and Systems

An International Journal in Information Science and Engineering

University of New Mexico

ISSN 2331-6055 (print) ISSN 2331-608X (online)

University of New Mexico

NSS

Neutrosophic Sets and Systems

An International Journal in Information Science and Engineering

Copyright Notice

Information for Authors and Subscribers

"Neutrosophic Sets and Systems" has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^-0, 1^+[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

All submissions should be designed in MS Word format using our template file:

http://fs.unm.edu/NSS/NSS-paper-template.doc.

A variety of scientific books in many languages can be downloaded freely from the Digital Library of Science:

http://fs.unm.edu/ScienceLibrary.htm.

To submit a paper, mail the file to the Editor-in-Chief. To order printed issues, contact the Editor-in-Chief. This journal is non-commercial, academic edition. It is printed from private donations.

Information about the neutrosophics you get from the UNM website:

http://fs.unm.edu/neutrosophy.htm. The

home page of the journal is accessed on

http://fs.unm.edu/NSS.

ISSN 2331-6055 (print) ISSN 2331-608X (online)

University of New Mexico

NSS

Neutrosophic Sets and Systems

An International Journal in Information Science and Engineering

**** NSS has been accepted by SCOPUS. Starting with Vol. 19, 2018, the NSS articles are indexed in Scopus.**

NSS ABSTRACTED/INDEXED IN

SCOPUS,
Google Scholar,
Google Plus,
Google Books,
EBSCO,
Cengage Thompson Gale (USA),
Cengage Learning (USA),
ProQuest (USA),
Amazon Kindle (USA),
University Grants Commission (UGC) - India,
DOAJ (Sweden),
International Society for Research Activity (ISRA),
Scientific Index Services (SIS),
Academic Research Index (ResearchBib),
Index Copernicus (European Union),
CNKI (Tongfang Knowledge Network Technology Co.,
Beijing, China),
Baidu Scholar (China),
Redalyc - Universidad Autonoma del Estado de Mexico (IberoAmerica),
Publons,
Scimago, etc.

Google Dictionaries have translated the neologisms "**neutrosophy**" (1) and"**neutrosophic**" (2), coined in 1995 for the first time, into about 100 languages.
FOLDOC Dictionary of Computing (**1, 2**), Webster Dictionary (**1, 2**), Wordnik (**1**),Dictionary.com, The Free Dictionary (**1**), Wiktionary (**2**), YourDictionary (**1, 2**),OneLook Dictionary (**1, 2**), Dictionary / Thesaurus (**1**), Online Medical Dictionary (**1,2**), Encyclopedia (**1, 2**), Chinese Fanyi Baidu Dictionary (**2**), Chinese Youdao Dictionary (**2**) etc. have included these scientific neologisms.

.

ISSN 2331-6055 (print) NSS *ISSN 2331-608X (online)*
University of New Mexico

Contents

Neutrosophic Cognitive Maps for Clinical Decision Making in Mental Healthcare: A Federated Learning Approach

Jagan M. Obbineni [1], Ilanthenral Kandasamy [2,*], Madhumitha Ramesh [2], Florentin Smarandache [3] and Vasantha Kandasamy [2]

[1] VIT School for Agricultural Innovations and Advanced Learning (VAIAL), Vellore Institute of Technology, Katpadi, Vellore, Tamil Nadu, 632014, India; jagan.obbineni@vit.ac.in;
[2] School of Computer Science and Engineering (SCOPE), Vellore Institute of Technology, Katpadi, Vellore, Tamil Nadu, 632014, India; ilanthenral.k@vit.ac.in; madhumitha.r2022@vitstudent.ac.in; vasantha.wb@vit.ac.in;
[3] Dept of Math and Sciences, University of New Mexico, Gallup, NM, United States; smarand@unm.edu

* Correspondence: ilanthenral.k@vit.ac.in;

Abstract: Due to data privacy concerns and a lack of broadly applicable modelling approaches, mental health prediction encounters substantial challenges. This research introduces a pioneering decentralized framework integrating federated learning with Neutrosophic Cognitive Maps (NCMs) to facilitate secure and accurate mental health predictions while preserving data privacy. This innovative approach allows collaborative NCMs training on sensitive patient data across diverse sites without centralizing or transferring the data. The NCMs incorporated into the framework effectively model relationships between various symptoms and mental health states, offering interpretable insights into the complex dynamics of mental health. To address the limitations of local data availability, a multi-task learning methodology is employed, leveraging commonalities between related mental health prediction tasks to enhance modelling. Experiments are done on a synthetic mental health dataset to validate the proposed approach, demonstrating significant improvements. The decentralized nature of the approach ensures robust privacy guarantees by preventing direct access to patient data. The proposed framework contributes to the responsible application of soft computing and AI in the sensitive mental health domain. Furthermore, the interpretability of NCM models facilitates a nuanced analysis of indeterminate interrelationships between various psychological concepts, offering valuable support for data-driven decision-making in mental health contexts.

Keywords: Federated Learning; Neutrosophic Cognitive Maps (NCMs); Mental Health; Psychological Concepts; SDG 3-4;

1. Introduction

Mental health, a critical aspect of overall well-being, requires innovative data analysis and modelling approaches. Traditional methodologies in mental health modelling face significant challenges, particularly when it comes to protecting individual privacy and accurately representing the inherent uncertainty associated with mental health data. This paper explores a pioneering integration of two powerful paradigms, federated learning and Neutrosophic Cognitive Maps (NCMs), to improve the diagnostic results or prediction to handle indeterminacy-related challenges. The rise of digital health records and the growing adoption of mobile mental health apps have created vast datasets. However, using these datasets for meaningful insights raises ethical concerns about the privacy of sensitive information.

Federated learning, a decentralized machine learning approach, represents a promising solution [1]. By enabling modelling across distributed devices without exposing raw data, federated learning ensures the confidentiality of individual records, making it particularly suitable for the sensitive nature of mental health data. At the same time, mental health data often exhibit inherent uncertainty and inaccuracy. NCMs, an extension of traditional cognitive maps, represent a new way of dealing with uncertainty by incorporating neutrosophic logic [2]. This logic accepts indeterminate and inconsistent information and provides a robust framework for modelling the complexity of mental health processes.

By decentralizing the model training process, federated learning addresses privacy concerns, while NCMs improve the representation of uncertainty and imprecision within cognitive modelling. This research work aims to explore these concepts with the overarching goal of advancing mental health modelling methodology. At a time when mental health is an escalating global issue, this research seeks to contribute to a more ethical, efficient and privacy-friendly approach to mental health data analysis and modelling.

The primary objective of this research is to develop and evaluate federated learning approaches that leverage NCMs to enhance healthcare decision support while ensuring the privacy and security of sensitive patient data.

This research addresses the following objectives:

1. To design a federated learning framework that accommodates NCMs and their associated uncertainty, indeterminacy, and partial truth.
2. Integrating NCMs into the federated learning process to model and reason with complex medical data, capturing indeterminate and contradictory information nuances.
3. To develop optimizations and personalization techniques for efficient federated learning of NCMs across heterogeneous medical datasets.
4. To evaluate the feasibility and performance of the proposed federated-NCMs approach using simulations on synthetic medical datasets.
5. To analyze the reasoning capability of the proposed federated-NCMs method.

The paper unfolds in a structured manner, beginning with an introduction that outlines the background and motivation. A comprehensive literature review builds the foundation by examining models of federated learning and NCMs. The theoretical framework represents the principles underlying federated learning and NCMs.

The methodology details the research design, data collection, and implementation of an integrated model. Model development and integration are explored in section three, which provides an overview of the architecture, training processes, and challenges encountered. The results and discussion show the integrated model's performance, compare it with existing approaches, and discuss implications. Practical applications and use cases in mental health scenarios highlight the potential benefits for clinicians, researchers, and patients.

The paper concludes with a comprehensive summary of key findings, contributions, and recommendations for future research. This research contributes to the ongoing discourse on privacy-preserving mental health modelling and provides a promising framework combining federated learning and NCMs.

2. Literature Survey

A survey by Liang et al. [1] offers a comprehensive overview of federated learning techniques in intelligent healthcare. Despite introducing various federated learning algorithms, the paper needs a detailed comparison of their performance and effectiveness in smart healthcare scenarios. The survey contributes to understanding the environment of federated learning applications in healthcare but does not provide insight into the specific nuances and requirements of intelligent healthcare environments. Tedeshini et al. [3] focused on decentralized, federated learning for tumour segmentation. Using methodologies such as FedAvg, FedClus, and FedVote, the study explores their applicability in the context of tumour segmentation. However, the study could be more extensive, especially the specific application of tumour segmentation. It must address broader applications requiring different or more complex models, such as tumour detection, classification or prediction. Gram et al. [4] contributed to this field by implementing and evaluating robust aggregation methods in federated learning for healthcare.

Fuzzy theory was introduced in 1985 by Lofti A. Zadeh [5]. Since then, fuzzy theory has found many real-world applications. Fuzzy sets have been extended and generalized into intuitionistic fuzzy sets (IFS) and neutrosophic sets. Fuzzy Cognitive Maps (FCMs) [6] are graphical models representing the relationships and interactions between different variables within a system. Integrating FCMs into medical decision-making processes has emerged as a remarkable research area that can improve understanding of complex systems characterized by uncertainty. Research by Jayashree et al. [7] applied FCMs for geospatial risk prediction of dengue outbreaks. These studies demonstrate FCM's utility in capturing complex medical decision dynamics. Beyond dengue applications, Evangelia et al. developed time-dependent FCMs to represent the temporal evolution of medical conditions for improved diagnostics. Greeda et al. [8] presented a survey on FCM techniques used in medicine, encompassing disease diagnosis, treatment planning, and medical imaging.

FCMs techniques enhance complex diagnosis under uncertainty by representing the multifaceted relationships between variables affecting disease diagnosis and management. In healthcare, federated learning is paramount for the privacy and security of patient data. Federated learning enables training a collaborative model across decentralized entities without centralizing raw data, ensuring that sensitive medical data remains localized. Integrating FCM and federated learning is valuable in addressing complex medical decision-making processes was studied by Hoyos et al. [9-12]. However, Hoyos et al.'s study falls short of providing a comprehensive assessment of uncertainty and indeterminacy that is inherent in mental health diseases.

Neutrosophy is a branch of philosophy investigating neutralities' origin, nature, and scope and their interactions. Florentin Smarandache introduced neutrosophy in the 1990s [13]. Neutrosophy regards a proposition, hypothesis, concept, event, or entity depending on the modelling. Neutrosophy is the basis of the neutrosophic set, logic, probability, and statistics. Indeterminacy "I" is a concept in neutrosophy that measures the degree of neutrality or uncertainty of a proposition, event, theory, entity or concept. Neutrosophic Cognitive Maps (NCMs) are an extension of FCMs that can handle indeterminate relationships between two concepts, obtaining more significant sensitive results. It was introduced in [14] and has since been used to analyze diverse social issues [15-29]. NCMs have been modelled considerably on the AI focus to mimic the thinking-human approach. Here, it is unsupervised data and has a limited set of features.

Indeterminacy deals with imprecise and incomplete concepts characterized by unknown or neutral elements. A study by Ramalingam et al. [21] demonstrates the superiority of NCMs over FCMs in various scenarios, highlighting their greater accuracy and reliability. This becomes especially important in medical diagnostics, where uncertainties and incomplete information are inherent. The ability of NCM to represent unknown or neutral elements provides a unique advantage, especially when dealing with concepts or relationships where information is missing or uncertain. Zafar et al. [24] propose a mathematical model based on NCMs to analyze uncertain factors' role in spreading pandemics such as COVID-19.

3. Federated Learning using NCM

This paper proposes a novel methodology that synergistically combines federated learning with Neutrosophic Cognitive Maps (NCMs) to enable secure, privacy-preserving artificial intelligence for complex healthcare applications. The proposed approach aligns with the critical need for collaborative and ethical AI solutions in healthcare. The methodology's core is a decentralized, federated learning framework that facilitates model training on sensitive patient data distributed across multiple sites without requiring central data aggregation. This allows collaborative learning on NCMs while ensuring data privacy and security. The adaptive modelling capabilities of NCMs further allow for the capture of dynamic relationships between various medical concepts.

While this paper demonstrates the potential applicability of NCM in intelligent healthcare and decision-making, it does not validate the model with actual data or empirical evidence. Relying on hypothetical values and weights for concepts and associations in the NCM raises concerns about the actual dynamics and accuracy of the proposed model in depicting scenarios. Specific goals include creating privacy-preserving NCM models, formulating guidelines for federated training of NCMs, and evaluating the approach on real-world medical datasets.

3.1 Proposed System:

The proposed system's architecture as given in Figure 1 is the backbone of our innovative approach, seamlessly integrating components to enable federated learning with NCMs in mental health. This section provides detailed insight into the structural design, addressing limitations in existing systems and proposing a novel framework.

Before delving into the proposed architecture, it is essential to acknowledge the limitations of current systems. Existing models may struggle with the intricacies of mental health data, including heterogeneity, privacy concerns, and the dynamic nature of mental health conditions. These limitations set the stage for the innovative solutions proposed in the subsequent sections.

Our proposed system introduces a sophisticated architecture designed to overcome the identified limitations. It embraces a decentralized approach, ensuring enhanced data privacy, personalization, and effective learning.

The system comprises the following key components:

Heterogeneous data collector: The heterogeneous data collector is the entry point, adept at gathering diverse data types such as text, images, audio, and sensor readings. Its versatility allows for a comprehensive collection of heterogeneous mental health data from various sources.

Data handler: Upon data collection, the data handler takes charge of preprocessing tasks, including cleaning, normalization, and feature extraction. This ensures that the data is optimized for subsequent model training, addressing the challenges posed by the heterogeneity of mental health data.

Local database: The local database acts as a central repository, storing processed data for easy access during training. This organized storage facilitates efficient sampling for model training and promotes collaboration among decentralized components.

Multi-Tasker model trainer: At the heart of the proposed system, the multi-tasker model trainer facilitates simultaneous training for multiple mental health tasks. This component optimizes shared representations across tasks, leveraging collective knowledge to enhance overall model performance.

Algorithm refitter: The algorithm refitter complements the training process by fine-tuning models based on task-specific feedback. This ensures continuous improvement in model parameters, adapting to the dynamic nature of mental health conditions.

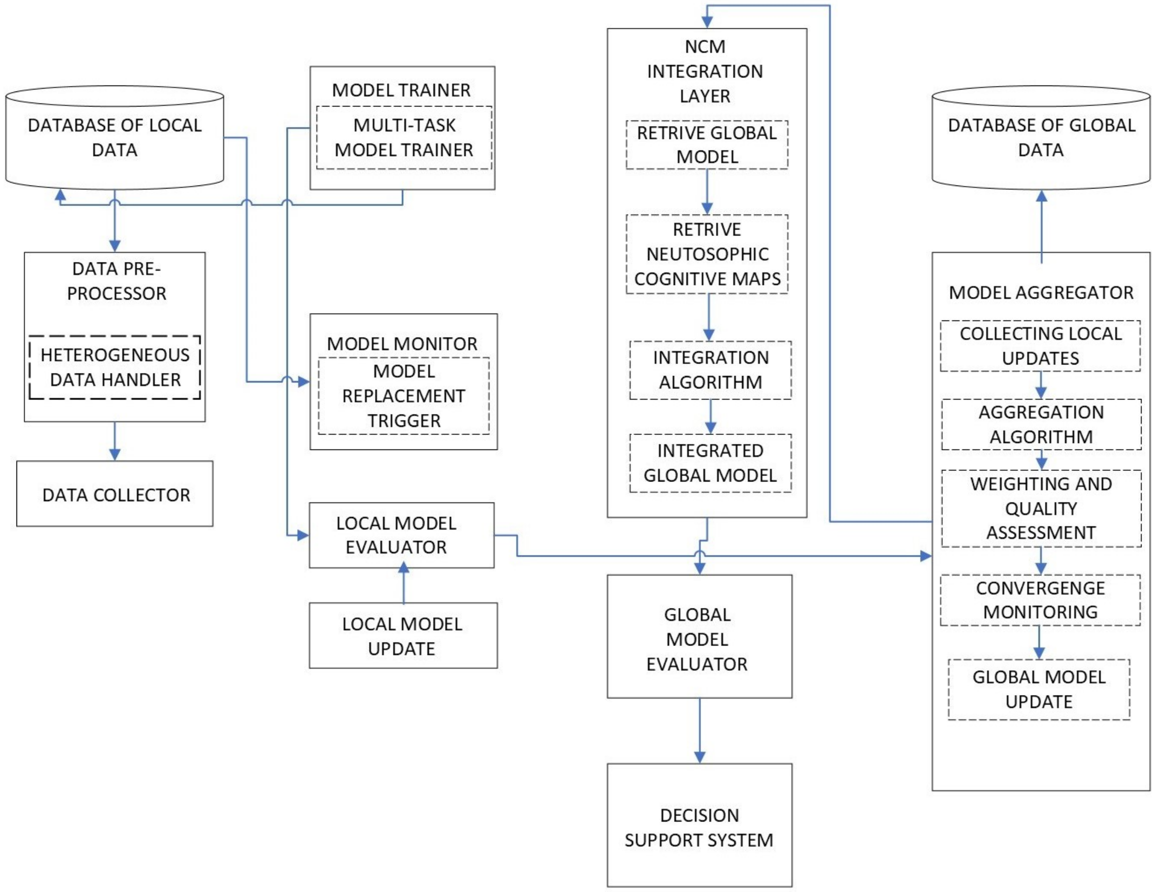


Figure 1. Architecture of the proposed system

Evaluator model: Post-training, the evaluator model assesses the performance of trained models, scoring them on relevant metrics. This critical evaluation provides insights into the effectiveness of the models, guiding further refinements.

Local model update: Each local database has its own local NCM model, which is updated iteratively during training. This decentralized approach allows specialization in understanding specific mental health conditions, improving model accuracy.

Model Aggregator: The local model updates are aggregated and used to update the global model; they are assessed by an expert for quality before the global model is updated.

The proposed system has several advantages: Adopting a federated learning approach prioritizes data privacy, allowing decentralized entities to contribute without compromising sensitive information. The system is designed to adapt and personalize models for various mental health tasks, ensuring tailored diagnoses for diverse conditions. Leveraging shared knowledge across tasks enhances the overall learning efficiency of the system, leading to improved mental health predictions. This proposed system addresses the unique challenges in mental health data analysis, offering a decentralized, privacy-preserving, and efficient solution. Indeterminate and uncertain relationship among the symptoms is handled correctly by the proposed algorithm.

Integrating federated learning and NCMs forms the core of our approach to addressing the complexity of mental health data analysis. Federated learning, with its focus on a collaborative training model and privacy protection, aligns seamlessly with the nuanced representation of uncertainty provided by NCM. On the other hand, because they explicitly reflect unknown or uncertain variables, NCMs excel at modelling the dynamic and complicated nature of mental health disorders. Combining these two methods provides a comprehensive solution that improves the accuracy and adaptability of mental health models while upholding strict privacy standards. By providing a more thorough understanding of intricate relationships and patterns, local models outfitted with NCMs provide the global model with distinctive insights into the dynamic nature of mental health disorders. NCMs' adaptive learning characteristics, which record changes over time, work in combination with FL's decentralized adaptive learning capabilities to guarantee that the model adapts dynamically to each person's unique mental health condition.

Moreover, NCMs' capacity to clearly describe uncertainties complements the ethical issues intrinsic to FL, such as transparency and privacy preservation. This integration improves the model's interpretability by resolving issues with the opacity of various machine-learning techniques.

Training process: In the training process, local models, each specializing in different mental health tasks, are independently trained on different datasets. A federated learning approach facilitates aggregating knowledge from these localized models to create a unified global privacy-preserving model. NCMs enhance this process by appropriately capturing mental health relationships' inherent uncertainty and dynamic nature and providing a comprehensive and nuanced understanding. This algorithmic fusion creates a robust foundation for our research and offers innovative solutions to the complex problems of mental health data analysis.

3.2 Data Generation:

The dataset used in this study was generated to aid in capturing the presence of indeterminate relationships that affect the various aspects of mental health. A dataset was curated using existing literature and basic datasets available. It is designed to gather comprehensive information on mental health, it included more diversity to enhance the generalizability of research findings to broader contexts.

The following attributes were considered *feeling_nervous, panic, breathing_rapidly, sweating, trouble_in_concentration, having_trouble_in_sleeping, having_trouble_with_work, hopelessness, anger, over_react, change_in_eating, suicidal_thought, feeling_tired, close_friend, social_media_addiction, weight_gain, introvert, popping_up_stressful_memory, having_nightmares, avoids_people_or_activities, feeling_negative, trouble_in_attention, blaming_yourself, hallucinations, repetitive_behaviour,* and *increased_energy.*

The data was labelled into *Attention-Deficit/Hyperactivity Disorder (ADHD), anxiety, Autism Spectrum Disorder (ASD), bipolar, eating disorder, loneliness, Major Depressive Disorder (MDD), Obsessive-Compulsive Disorder (OCD), Pervasive Developmental Disorders (PDD), psychotic depression, Post-Traumatic Stress Disorder (PTSD)* and *sleeping disorder.*

Over 600 records were generated based on the available regular mental health dataset, with indeterminacy introduced into the records to enable the creation of NCMs.

3.3 Data Preprocessing:

The collected data underwent a preprocessing phase to enhance its quality and reliability. Data cleaning procedures involved identifying and addressing inconsistencies, outliers, and inaccuracies in the dataset. Through careful validation and verification processes, erroneous entries were rectified or removed, ensuring a robust foundation for subsequent analyses. Normalization techniques were applied to ensure uniformity in the scale of numerical attributes. This step facilitates fair comparisons between different features, preventing any particular attribute from dominating the analysis due to its scale. Standardization ensured that all numerical features contributed equally to the modelling process, preventing bias in subsequent machine-learning algorithms. Efficient strategies were employed to handle missing values in the dataset. Missing data points were identified, and appropriate imputation methods were applied based on the nature of the missing information. This meticulous handling of missing values contributes to the completeness and reliability of the dataset used in the research.

3.4 Integration of Components

The seamless integration of these elements ensures a cohesive and collaborative system. The progression from data collection through preprocessing, collaborative training, iterative refinement, and evaluation establishes a robust foundation for predicting mental health conditions. The decentralized approach, facilitated by local models and a shared database, promotes adaptability and privacy in the learning process. This modular architecture is meticulously designed to tackle the multifaceted challenges in mental health prediction, providing a systematic and organized approach to enhance decision support through the fusion of federated learning and NCMs.

Figure 1 demonstrates a decentralized, federated learning approach for the collaborative development of NCMs while preserving data privacy. Independent participants encode customized NCMs on their local confidential data to model domain-specific concepts and interrelationships. Without sharing raw data, participants broadcast derivative model updates containing only parameters and weights to an aggregation server. Using multi-party computation protocols focused on the representational inclusivity of contributors, the server consolidates the collective model updates into an integrated consensus NCM. This federated NCM further undergoes auditing on various criteria to generate feedback for participants to enhance their localized models. Through iterative coordinated cycles of private simulation, anonymous aggregated orchestration and guided refinement, participants can strategically evolve personalized NCMs trained on their data while aligning with learnings from the collective federation.

4. Results and discussions

The data was separated into training and testing datasets and distributed across four local databases. Since there are over 600 records, 400 records were used for training the NCMs, one hundred at each local database. The remaining 200 records were used for testing, 50 each in the local database, to check if the updating of the local and global NCMs is happening.

Since 26 symptoms and 12 diseases are under consideration, visualization of the complete NCM might not be feasible. Simple NCMs are created at each local database level. If a symptom occurs in most cases of a particular disease, it is mapped to 1; otherwise, if it is indeterminate or has an uncertain effect on the disease, it is mapped to *I* in the local NCM.

Creation of global model: Edge weights in the global NCM are created based on the number of records considered for egde weight creation at the local NCM. The NCMs from the local database are aggregated to create the global NCM according to the weightage (number of cases considered) for each disease to update its NCM; once the NCMs are created, they can be updated. Whenever a local update occurs, it aggregates the updates, and the edge weights are altered accordingly, triggering a global model update.

The global model consists of aggregated values across the map to create a complete system with a mapping of all the symptoms and diseases. Whenever local updates occur, the local model updates are aggregated and used to update the global model; an expert for quality assesses them before the global model is updated.

Since the visualization is too massive to show, samples of the complete graphs are shown in Figures 2 and 3. These samples are obtained from the local database's NCM model, which was created using a part of the sample data. These figures pertain to the training data and are not universal in nature. For example, if anyone has attention span troubles, it does not imply concentration troubles or psychotic depression. They pertain to the representation of the dataset. This model shows that the relationship between sweating and psychotic depression is indeterminate; likewise, the relationship between stressful memory recall and anxiety is indeterminate.

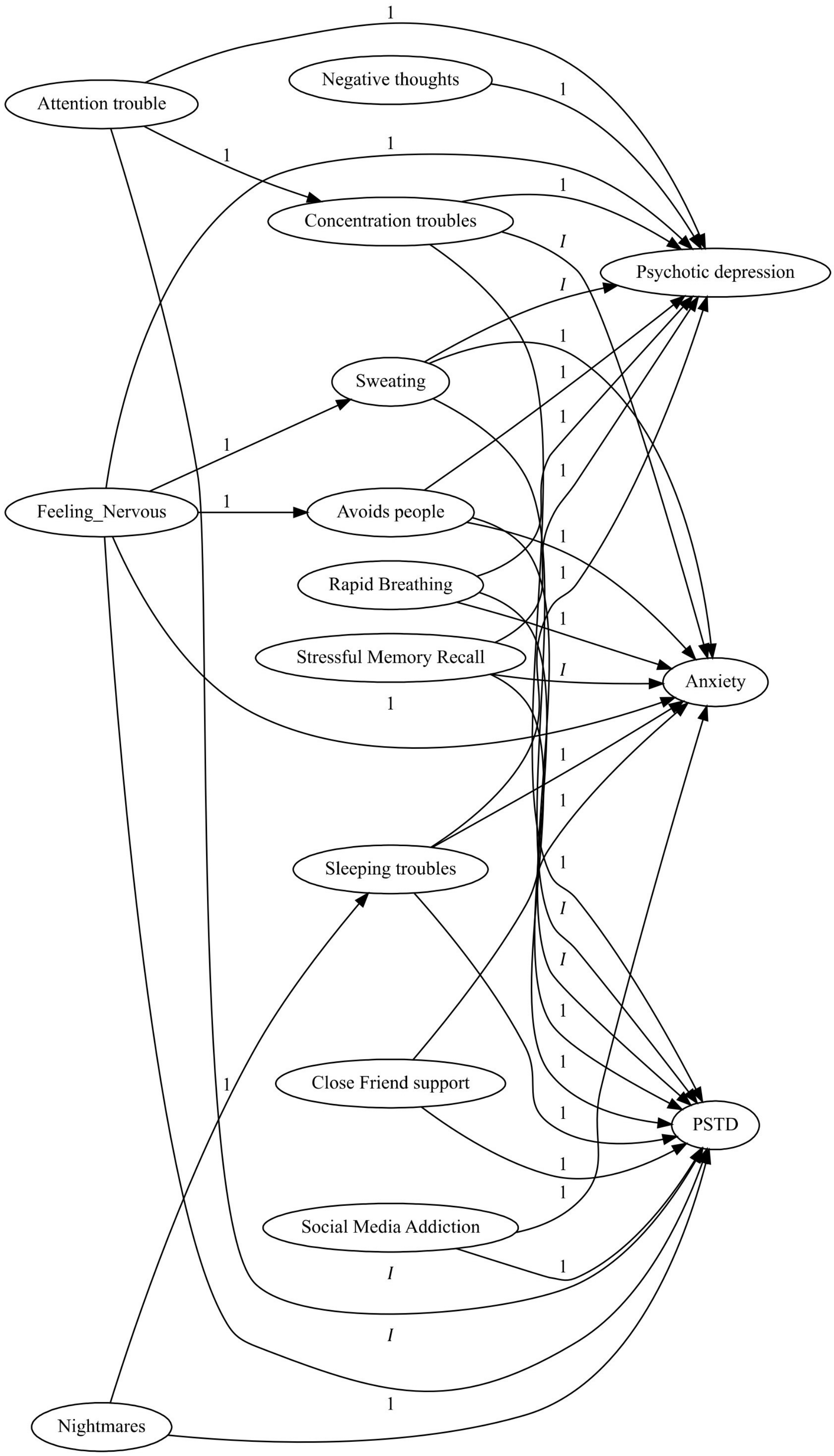


Figure 2: Part of the sample NCMs from the local database related to Anxiety, PSTD and psychotic depression.

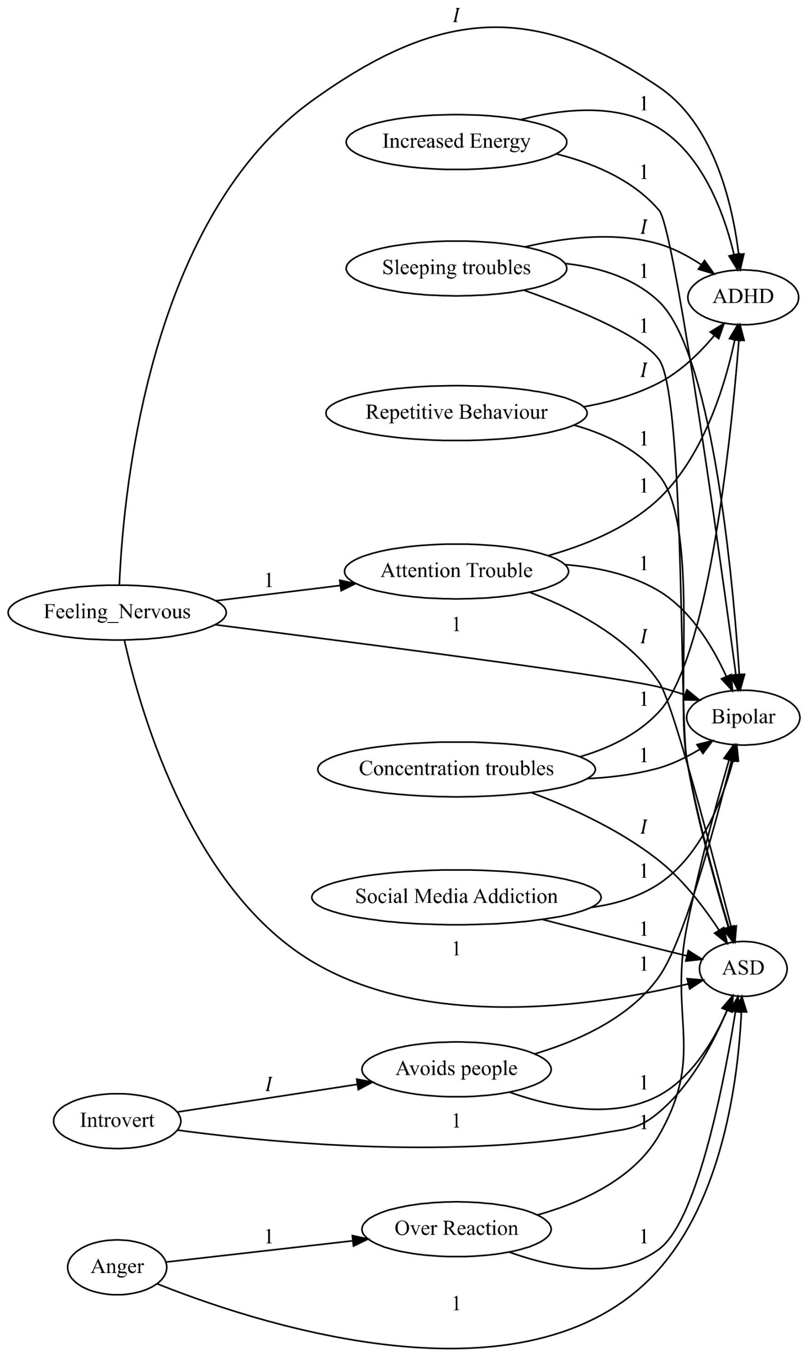


Figure 3 Part of the sample NCMs from the local database related to ADHD, ASD and Bipolar.

While considering symptoms and disease as given in Figure 3, it is seen that feeling nervous has an indeterminate effect on ADHD; similarly, being introverted has an indeterminate effect on avoiding people. Anger in general or for a general person anger can lead to overreaction, but anger → overreaction → ASD is not true, without the other symptoms. The presence of a combination of symptoms only results in a particular disease. The capturing of indeterminate relationships has made the data more sensitive in representation.

5. Conclusion and Future Enhancement

Complex uncertainties and interdependencies in clinical contexts related to mental health are difficult to fully capture. Hence, the training and testing are done on synthetic diagnosis outcome data. Since this data is limited, this model has validity concerns. In essence, federated modelling aligns NCM development with ethical imperatives around decentralization, privacy, security, and accessibility. It propagates collective knowledge from multifaceted domains in an inclusive ecosystem with sound data governance.

The resulting models can map multidimensional interactions underlying psychiatric conditions to inform personalized interventions. However, extensive testing is imperative to validate the effectiveness of noisy real-world applications. While the approach could transform crowdsourced insights for complex healthcare challenges, numerous extensions around security, ethics and engineering robustness are critical - including support for secure computations, blockchain-powered trust mechanisms, enhanced explainability and bias mitigation techniques.

This work presents a novel decentralized architecture that preserves data privacy while enabling safe and reliable mental health forecasts by integrating federated learning with NCM. This method allows for cooperative NCM training on private patient data dispersed among various sites without transferring data. NCMs offer interpretable insights and efficiently model correlations between symptoms and mental health states. This research fosters collaboration and trustworthy AI that respects privacy demands, making substantial progress towards developing ethical and practical AI solutions for mental health treatment. To facilitate data-driven decision-making, interpretable NCM models additionally allow a nuanced investigation of the links between various psychological concepts.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

Data Availability: The authors can be contacted to obtain the data used.

REFERENCES

1. Liang, Y., Zhang, J., & Liu, Y. (2021). Federated Learning for Smart Healthcare: A Survey. IEEE Journal of Biomedical and Health Informatics, 25(6), 1833-1848.
2. Kandasamy, W. B. V., & Smarandache, F. (2003). Fuzzy cognitive maps and neutrosophic cognitive maps. American Research Press, USA.
3. Tedeschini, B. C., Savazzi, S., Stoklasa, R., Barbieri, L., Stathopoulos, I., Nicoli, M., & Serio, L. (2022). Decentralized federated learning for healthcare networks: A case study on tumor segmentation. IEEE access, 10, 8693-8708.
4. Grama, M., Musat, M., Muñoz-González, L., Passerat-Palmbach, J., Rueckert, D., & Alansary, A. (2020). Robust aggregation for adaptive privacy preserving federated learning in healthcare. arXiv preprint arXiv:2009.08294.
5. Zadeh, L. A. (1999). Fuzzy sets as a basis for a theory of possibility. Fuzzy sets and systems, 100, 9-34.
6. Kosko, B. (1986). Fuzzy cognitive maps. International journal of man-machine studies, 24(1), 65-75.
7. Jayashree, L.S., Lakshmi Devi, R., Papandrianos, N., Papageorgiou, E.I., 2018. Application of fuzzy cognitive map for geospatial dengue outbreak risk prediction of tropical regions of Southern India. Intell. Decis. Technol. 12 (2), 231–250. http://dx.doi.org/10.3233/IDT-180330.
8. Greeda,J.; Mageswari,A,; Nithya.R.A. Study on Fuzzy Logic and its Applications in Medicine, International Journal of Pure and Applied Mathematics, 2018, 119(16), pp 1515-1525.
9. Hoyos, W., Aguilar, J., Toro, M., 2022. An autonomous cycle of data analysis tasks for the clinical management of dengue. Heliyon8(10),e10846. http://dx.doi.org/10.1016/j.heliyon.2022.e10846.
10. Hoyos, W., Aguilar, J., Toro, M., 2022b. A clinical decision-support system for dengue based on fuzzy cognitive maps. Health Care Manag. Sci. 1–16. http://dx.doi.org/10.1007/s10729-022-09611-6.
11. Hoyos, W., Aguilar, J., Toro, M., 2022c. PRV-FCM: an extension of fuzzy cognitive maps for prescriptive modeling. Preprint Submitted to a Journal.

12. Hoyos, W., Aguilar, J., & Toro, M. (2023). Federated learning approaches for fuzzy cognitive maps to support clinical decision-making in dengue. Engineering Applications of Artificial Intelligence, 123, 106371. Elsevier. Retrieved from www.elsevier.com/locate/engappai
13. Smarandache, F. (1999). A unifying field in Logics: Neutrosophic Logic. In Philosophy (pp. 1-141). American Research Press.
14. Kandasamy, D. W., & Smarandache, F. (2003). Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps. American Research Press, arXiv preprint math/0311063.
15. Kandasamy, W. B., & Smarandache, F. (2004). Analysis of Social Aspects of Migrant Labourers Living With HIV/AIDS Using Fuzzy Theory and Neutrosophic Cognitive Maps: With Special Reference to Rural Tamil Nadu in India. arXiv preprint math/0406304.
16. Vasantha, W. B., Kandasamy, I., Devvrat, V., & Ghildiyal, S. (2019). Study of Imaginative Play in Children using Neutrosophic Cognitive Maps Model. Neutrosophic Sets and Systems, 241.
17. Kandasamy, I., Arumugam, D., Rathore, A., Arun, A., Jain, M., .W.B, V., & Smarandache, F. (2023). NCMPy: A Modelling Software for Neutrosophic Cognitive Maps based on Python Package. Neutrosophic Systems With Applications, 13, 1-22. https://doi.org/10.61356/j.nswa.2024.114
18. Obbineni, J., Kandasamy, I., Vasantha, W. B., & Smarandache, F. (2023). Combining SWOT analysis and neutrosophic cognitive maps for multi-criteria decision making: a case study of organic agriculture in India. Soft Computing, 27(23), 18311-18332.
19. Murugesan, R., Kumaravel, S. K., Mohan, K. R., Rathinam, N. D., & Rasappan, S. (2023). Analysis of Parameters Related to Malaria with Comparative Study on Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps. Fuzzy Logic Applications in Computer Science and Mathematics, 233-247.
20. Murugesan, R., Parthiban, Y., Devi, R., Mohan, K. R., & Kumaravel, S. K. (2023). A comparative study of fuzzy cognitive maps and neutrosophic cognitive maps on covid variants. Neutrosophic Sets and Systems, 55(1), 20.
21. Ramalingam, S., Govindan, K., & Broumi, S. (2021). Analysis of covid-19 via fuzzy cognitive maps and neutrosophic cognitive maps. Neutrosophic Sets and Systems, 42, 102-116.
22. Naeem, A., Anees, T., Naqvi, R. A., & Loh, W.-K. (2022). A Comprehensive Analysis of Recent Deep and Federated-Learning-Based Methodologies for Brain Tumor Diagnosis. IEEE Access, 10, 1-17.
23. Gonzalez, I. A., Acevedo, G. G., & Morocho Quinchuela, F. B. (2023). Choosing Suppliers for Healthcare Supply Chains under Neutrosophic Multi-Criteria Decision- Making Method. International Journal of Environmental Research and Public Health, 20(17), 1-18.
24. Zafar, A., & Wajid, M. A. (2020). A Mathematical Model to Analyze the Role of Uncertain and Indeterminate Factors in the Spread of Pandemics like COVID-19 Using Neutrosophy: A Case Study of India. (2020). International Journal of Fuzzy Systems, 22(7), 2348-2362.
25. Bhutani, K., Gaur, S., Panwar, P., & Garg, S. (2023, February). A Neutrosophic Cognitive Maps Approach for Pestle Analysis in Food Industry. In International Conference On Emerging Trends In Expert Applications & Security (pp. 349-360). Singapore: Springer Nature Singapore. https://doi.org/10.1007/978- 981-99-1909-3_30.
26. Karadayi-Usta, S. (2022). The Role of the Paper Packaging Industry in the Circular Economy: The Causal Relationship Analysis via Neutrosophic Cognitive Maps. In Handbook of Research on Advances and Applications of Fuzzy Sets and Logic (pp. 605-618). IGI Global. https://doi.org/10.4018/978-1-7998-7979- 4.ch027.
27. Rojas, H. E., Nelson, F. S., & Cabrita Marina, M. (2022). Neutrosophic Cognitive Maps for Violence Cause Analysis. International Journal of Neutrosophic Science (IJNS), 19(1).
28. Rodríguez, J. O. C., Veloz, Á. P. M., & Andi, S. T. T. (2022). Neutrosophic Cognitive Maps for the Analysis of the Factors in the proper Diagnosis of Conversion Disorder. Neutrosophic Sets and Systems, 52(1), 36.
29. Raúl, D. C., Fabricio, T. T., & Elizabeth, D. P. F. (2022). Analyzing the impact of social factors on homelessness with neutrosophic cognitive maps. International Journal of Neutrosophic Science (IJNS), 19(1).

Received: Jan 2, 2024. Accepted: Mar 27, 2024

Neutrosophic Sets and Systems, Vol. 66, 2024
University of New Mexico

A Generalization of Neutrosophic Metric Space and Related Fixed Point Results

Fahim Ud Din [1], Umar Ishtiaq [2,*], Lakhdar Ragoub [3], Khalil Javed [4], Muhammad Arshad [4]

[1]Abdus Salam School of Mathematical Sciences, Government College University, Lahore, Pakistan, Email: fahim@sms.edu.pk

[2]Office of Research, Innovation and Commercialization, University of Management and Technology Lahore, Pakistan, Email: umarishtiaq000@gmail.com

[3] Mathematics Department, University of Prince Mugrin, P.O. Box 41040, Al Madinah 42241, Saudi Arabia, Email: l.ragoub@upm.edu.sa

[4] Department of Math & Stats, International Islamic University Islamabad, Pakistan, Email: khalil.phdma127@iiu.edu.pk (K.J.); marshadzia@iiu.edu.pk (M.A.)

* Correspondence: umarishtiaq000@gmail.com

Abstract: Neutrosophic set is a generalization of classical sets, fuzzy set, intuitionistic sets etc. A mathematical notion neutrosophic set dealing issues containing inconsistent, indeterminate and imprecise data. In this manuscript, we toss the notion of Neutrosophic b-metric-like spaces and obtain some fixed point results in the sense of Neutrosophic b-metric-like spaces. Our results are improvements of recent results in the existing literature. For the validity of these results some non-trivial examples are imparted.

Keywords: Metric-like spaces; Neutrosophic b-metric-like spaces; fixed point; uniqueness

1. Introduction and Preliminaries

The notion of fuzzy sets (FSs) given by Zadeh [9], this auspicious concept gave a new direction of research and this idea has deeply influenced many scientific fields. In this connectedness, Kramosil and Michalek [10] initiated the notion of fuzzy metric spaces (FMSs) by reformulate the notion of probabilistic metric spaces to FMSs. George and Veeramani [11] derived a Hausdorff topology originated by FMS. Afterward, the utility of FMS appeared in applied sciences such as fixed point theory, image and signal processing, medical imaging, decision-making. Later on, the existence theory of fixed point in FMS has been enriched with a number of altered generalizations. Fuzzy version of Banach contraction principle was given by Garbiec [12] in the sense of FMS. For some necessary concepts please see [13, 14].

In recent times, Harandi [7] originated the notion of metric-like spaces (MLS), which generalized the concept of metric spaces in beautiful manners. In this connectedness, Shukla and Abbas [8] reformulated the notion of (MLS) and originated fuzzy metric-like spaces (FMLS). The approach of intuitionistic FMS was tossed by Park in [2]. Kirişci and Simsek [1] generalized the approach of intuitionistic FMS and tossed the approach of neutrosophic metric space (NMS). Simsek, Kirişci [5] and Sowndrarajan et al. [6] proved some fixed point (FP) results in the setting of NMS.

In this article, we tossed the notion of Neutrosophic b-metric-like space (NBMLS) in which self distance may not be equal to 1, 0 and 0. So, our approach is more generalized in the existing literature. Also this article is enriched with fixed point results and non-trivial examples. For some necessary results see [3, 4, 15]. Authors in [16-20] worked on different generalizations of NMSs and proved several fixed point results.

The main objectives of this manuscript are:

1. To introduce the concept of neutrosophic b-metric-like space.
2. To prove some fixed point results in the sense of neutrosophic b-metric-like space.
3. To enhance existing literature of fuzzy metric spaces and fuzzy fixed point theory.

This manuscript is organized with some rudimentary concepts of FMLS and NMS. The concept of NBMLS is discussed in detail and some fixed point results with non-trivial examples are imparted. A conclusion is provided for the obtained results.

In this section, some basic definitions are given that are helpful to understand the main results. Shukla and Abbas introduced the concept of FMLS and utilized this idea to investigate fixed point results. Shukla and Abbas defined the notion of FMLS as follows:

Definition 1.1 [8] A 3-tuple $(Q, \Psi, *)$ is said an FMLS if $Q \neq \Phi$ is a random set, $*$ is a continuous t-norm (CTN) and Ψ is a FS on $Q \times Q \times (0, \infty)$ meet the points below for all $\vartheta, \mathcal{I}, \mu \in Q, \partial, s > 0$:

FL1) $\Psi(\vartheta, \mathcal{I}, \partial) > 0$;

FL2) If $\Psi(\vartheta, \mathcal{I}, \partial) = 1$, then $\vartheta = \mathcal{I}$;

FL3) $\Psi(\vartheta, \mathcal{I}, \partial) = \Psi(\mathcal{I}, \vartheta, \partial)$;

FL4) $\Psi(\vartheta, \mu, \partial + s) \geq \Psi(\vartheta, \mathcal{I}, \partial) * \Psi(\mathcal{I}, \mu, s)$;

FL5) $\Psi(\vartheta, \mathcal{I}, \cdot): (0, \infty) \to [0,1]$ is continuous.

Example 1.2 [8] Assume $Q = \mathbb{R}^+$, $a \in \mathbb{R}^+$ and $n > 0$. Define CTN by $\sigma * \kappa = \sigma\kappa$ and the FS Ψ on $Q \times Q \times (0, \infty)$ by

$$\Psi(\vartheta, \mathcal{I}, \partial) = \frac{a\partial}{a\partial + n(\max\{\vartheta, \mathcal{I}\})}, \forall\, \vartheta, \mathcal{I} \in Q, \partial > 0.$$

Then $(Q, \Psi, *)$ is a FMLS.

The concept of neutrosophic metric spaces was discussed by Kirişci and Simsek in his work and he defined the said concept as follows:

Definition 1.3 [1] Suppose $Q \neq \emptyset$, assume a six tuple $(Q, \Psi, \Phi, \Omega, *, \circ)$ where $*$ is a CTN, $\circ$ is a continuous t-conorm (CTCN), Ψ, Φ and Ω are (Neutrosophic sets) NSs on $Q \times Q \times (0, \infty)$. If $(Q, \Psi, \Phi, \Omega, *, \circ)$ meet the below circumstances for all $\vartheta, \mathcal{I}, \mu \in Q$ and $\Omega, s > 0$:

(NS1) $\Psi(\vartheta, \mathcal{I}, \partial) + \Phi(\vartheta, \mathcal{I}, \partial) + \Omega(\vartheta, \mathcal{I}, \partial) \leq 3$,

(NS2) $0 \leq \Psi(\vartheta, \mathcal{I}, \partial) \leq 1$,

(NS3) $\Psi(\vartheta, \mathcal{I}, \partial) = 1 \Leftrightarrow \vartheta = \mathcal{I}$,

(NS4) $\Psi(\vartheta, \mathcal{I}, \partial) = \Psi(\mathcal{I}, \vartheta, \partial)$,

(NS5) $\Psi\big(\vartheta, \mu, (\partial + s)\big) \geq \Psi(\vartheta, \mathcal{I}, \partial) * \Psi(\mathcal{I}, \mu, s)$,

(NS6) $\Psi(\vartheta, \mathcal{I}, \cdot): [0, \infty) \to [0,1]$ is a continuous,

(NS7) $\lim_{\Omega \to \infty} \Psi(\vartheta, \mathcal{I}, \partial) = 1$,

(NS8) $0 \leq \Phi(\vartheta, \mathcal{I}, \partial) \leq 1$,

(NS9) $\Phi(\vartheta, \mathcal{I}, \partial) = 0 \Leftrightarrow \vartheta = \mathcal{I}$,

(NS10) $\Phi(\vartheta, \mathcal{I}, \partial) = \Phi(\mathcal{I}, \vartheta, \partial)$,

(NS11) $\Phi\big(\vartheta, \mu, b(\partial + s)\big) \leq \Phi(\vartheta, \mathcal{I}, \partial) \circ \Phi(\mathcal{I}, \mu, s)$,

(NS12) $\Phi(\vartheta, \mathcal{I}, \cdot): [0, \infty) \to [0,1]$ is a continuous,

(NS13) $\lim_{\Omega \to \infty} \Phi(\vartheta, \mathcal{I}, \partial) = 0$,

(NS14) $0 \leq \Omega(\vartheta, \mathcal{I}, \partial) \leq 1$,

(NS15) $\Omega(\vartheta, \mathcal{I}, \partial) = 0 \Leftrightarrow \vartheta = \mathcal{I}$,

(NS16) $\Omega(\vartheta, \mathcal{I}, \partial) = \Omega(\mathcal{I}, \vartheta, \partial)$,

(NS17) $\Omega\big(\vartheta, \mu, (\partial + s)\big) \leq \Omega(\vartheta, \mathcal{I}, \partial) \circ \Omega(\mathcal{I}, \mu, s)$, (NS18) $\Omega(\vartheta,\beth,\cdot):[0,\infty)\to[0,1]$ is a continuous,

(NS19) $\lim_{\Omega\to\infty}\Omega(\vartheta,\beth,\partial)=0$,

(NS20) If $\Omega\le 0$ then $\Psi(\vartheta,\beth,\partial)=0, \Phi(\vartheta,\beth,\partial)=1, \Omega(\vartheta,\beth,\partial)=1$.

Then (Q,Ψ,Φ,Ω) Neutrosophic metric on Q and $(Q,\Psi,\Phi,\Omega,*,\circ)$ is an NMS.

2. Main Results

In this section, we introduce the concept of NBMLS and prove some FP results.

Definition 2.1 Suppose $Q\neq\emptyset$, assume a six tuple $(Q,\Psi,\Phi,\Omega,*,\circ)$ where $*$ is a CTN, $\circ$ is a CTCN, Ψ,Φ and Ω are NSs on $Q\times Q\times(0,\infty)$. If $(Q,\Psi,\Phi,\Omega,*,\circ)$ meet the below circumstances for all $\vartheta,\beth,\mu\in Q$ and $\Omega,s>0$:

(NL1) $\Psi(\vartheta,\beth,\partial)+\Phi(\vartheta,\beth,\partial)+\Omega(\vartheta,\beth,\partial)\le 3$,

(NL2) $0\le\Psi(\vartheta,\beth,\partial)\le 1$,

(NL3) $\Psi(\vartheta,\beth,\partial)=1$ implies $\vartheta=\beth$,

(NL4) $\Psi(\vartheta,\beth,\partial)=\Psi(\beth,\vartheta,\partial)$,

(NL5) $\Psi(\vartheta,\mu,b(\partial+s))\ge\Psi(\vartheta,\beth,\partial)*\Psi(\beth,\mu,s)$,

(NL6) $\Psi(\vartheta,\beth,\cdot):[0,\infty)\to[0,1]$ is a continuous,

(NL7) $\lim_{\Omega\to\infty}\Psi(\vartheta,\beth,\partial)=1$,

(NL8) $0\le\Phi(\vartheta,\beth,\partial)\le 1$,

(NL9) $\Phi(\vartheta,\beth,\partial)=0$ implies $\vartheta=\beth$,

(NL10) $\Phi(\vartheta,\beth,\partial)=\Phi(\beth,\vartheta,\partial)$,

(NL11) $\Phi\big(\vartheta,\mu,b(\partial+s)\big)\le\Phi(\vartheta,\beth,\partial)\circ\Phi(\beth,\mu,s)$,

(NL12) $\Phi(\vartheta,\beth,\cdot):[0,\infty)\to[0,1]$ is a continuous, (NL13) $\lim_{\Omega\to\infty} \Phi(\vartheta,\mathcal{J},\partial) = 0,$

(NL14) $0 \le \Omega(\vartheta,\mathcal{J},\partial) \le 1,$

(NL15) $\Omega(\vartheta,\mathcal{J},\partial) = 0$ implies $\vartheta = \mathcal{J},$

(NL16) $\Omega(\vartheta,\mathcal{J},\partial) = \Omega(\mathcal{J},\vartheta,\partial),$

(NL17) $\Omega(\vartheta,\mu,b(\partial+s)) \le \Omega(\vartheta,\mathcal{J},\partial)\circ\Omega(\mathcal{J},\mu,s),$

(NL18) $\Omega(\vartheta,\mathcal{J},\cdot):[0,\infty) \to [0,1]$ is a continuous,

(NL19) $\lim_{\Omega\to\infty} \Omega(\vartheta,\mathcal{J},\partial) = 0,$

(NS20) If $\Omega \le 0$ then $\Psi(\vartheta,\mathcal{J},\partial) = 0, \Phi(\vartheta,\mathcal{J},\partial) = 1, \Omega(\vartheta,\mathcal{J},\partial) = 1.$

Then (Q,Ψ,Φ,Ω) is known as NBML on Q and $(Q,\Psi,\Phi,\Omega,*,\circ)$ be an NBMLS.

Remark 2.2 In definition (2.1), a set Q is gifted a NBMLS with a CTN (*) and CTCN (∘). (NL3), (NL9) and (NL15) circumstances of NBMLS, that is, the self-distance may not be equal to 1, 0 and 0, i.e., $\Psi(\vartheta,\vartheta,\partial) \ne 1, \Phi(\vartheta,\vartheta,\partial) \ne 0$ and $\Omega(\vartheta,\vartheta,\partial) \ne 0$ for all $\partial > 0$, for some or may be for all $\vartheta \in Q$.

Proposition 2.3 Let (Q,σ) be any BMLS. Then $(Q,\Psi,\Phi,\Omega,*,\circ)$ is a NBMLS, where '*' and '∘' are defined respectively $\sigma * \kappa = \sigma\kappa$ and $\sigma\circ\kappa = \max\{\sigma,\kappa\}$ and NSs Ψ,Φ and Ω are given by

$$\Psi(\vartheta,\mathcal{J},\partial) = \frac{a\partial^n}{a\partial^n + m\sigma(\vartheta,\mathcal{J})}\ \forall\ \vartheta,\mathcal{J} \in Q, \partial > 0,$$
$$\Phi(\vartheta,\mathcal{J},\partial) = \frac{m\sigma(\vartheta,\mathcal{J})}{a\partial^n + m\sigma(\vartheta,\mathcal{J})}\ \forall\ \vartheta,\mathcal{J} \in Q, \partial > 0,$$
$$\Omega(\vartheta,\mathcal{J},\partial) = \frac{m\sigma(\vartheta,\mathcal{J})}{a\partial^n}\ \forall\ \vartheta,\mathcal{J} \in Q, \partial > 0.$$

Where, $a \in \mathbb{R}^+, m > 0$ and $n \ge 1$.

Then (Q,Ψ,Φ,Ω) be a Neutrosophic b-metric-like on Q and $(Q,\Psi,\Phi,\Omega,*,\circ)$ be an NBMLS.

Remark 2.4 Note that the above proposition also holds for CTN $\sigma * \kappa = \min\{\sigma, \kappa\}$ and CTCN $\sigma \circ \kappa = \max\{\sigma, \kappa\}$.

Remark 2.5 The proposition (2.3) shows that every BMLS induces a NBMLS. For $a = n = m = 1$ the induced NBMLS $(Q, \Psi, \Phi, \Omega, *, \circ)$ is called the standard NBMLS, where $a \in \mathbb{R}^+$

$$\Psi_{bl}(\vartheta, \mathcal{J}, \partial) = \frac{\partial}{\partial + \sigma(\vartheta, \mathcal{J})} \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0,$$
$$\Phi(\vartheta, \mathcal{J}, \partial) = \frac{\sigma(\vartheta, \mathcal{J})}{\partial + \sigma(\vartheta, \mathcal{J})} \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0,$$
$$\Omega(\vartheta, \mathcal{J}, \partial) = \frac{\sigma(\vartheta, \mathcal{J})}{\partial} \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0.$$

Example 2.6 Let $Q = \mathbb{R}^+, a \in \mathbb{R}^+$ and $m > 0$. Define * by $\sigma * \kappa = \sigma\kappa$ and $\circ$ by $\sigma \circ \kappa = \max\{\sigma, \kappa\}$ and NSs Ψ, Φ and Ω in $Q \times Q \times (0, \infty)$ by

$$\Psi(\vartheta, \mathcal{J}, \partial) = \frac{a\partial}{a\partial + m(\max\{\vartheta, \mathcal{J}\}^2\ )} \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0,$$
$$\Phi(\vartheta, \mathcal{J}, \partial) = \frac{m(\max\{\vartheta, \mathcal{J}\}^2\ )}{a\partial + m(\max\{\vartheta, \mathcal{J}\}^2\ )} \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0,$$
$$\Omega(\vartheta, \mathcal{J}, \partial) = \frac{m(\max\{\vartheta, \mathcal{J}\}^2\ )}{a\partial} \ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0.$$

Then, since $\sigma(\vartheta, \mathcal{J}) = \max\{\vartheta, \mathcal{J}\}^2 \ \ \forall\ \vartheta, \mathcal{J} \in Q$ is a BMLS on Q. Therefore, by proposition (2.3) $(Q, \Psi, \Phi, \Omega, *, \circ)$ is a NBMLS, but self-distance not equal to 1, 0 and 0.

As,

$$\Psi(\vartheta, \vartheta, \partial) = \frac{a\partial}{a\partial + m\vartheta^2} \neq 1\ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0,$$
$$\Phi(\vartheta, \vartheta, \partial) = \frac{m\vartheta^2}{a\partial + m\vartheta^2} \neq 0\ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0,$$
$$\Omega(\vartheta, \vartheta, \partial) = \frac{m\vartheta^2}{a\partial} \neq 0\ \forall\ \vartheta, \mathcal{J} \in Q, \partial > 0.$$

Definition 2.7 A sequence $\{\vartheta_n\}$ is NBMLS $(Q, \Psi, \Phi, \Omega, *, \circ)$ is said to be convergent to $\vartheta \in Q$ if

$$\lim_{n\to\infty} \Psi(\vartheta_n, \vartheta, \partial) = \Psi(\vartheta, \vartheta, \partial)\ \forall\ \partial > 0,$$

$$\lim_{n\to\infty} \Phi(\vartheta_n, \vartheta, \partial) = \Phi(\vartheta, \vartheta, \partial)\ \forall\ \partial > 0,$$

and

$$\lim_{n\to\infty} \Omega(\vartheta_n, \vartheta, \partial) = \Omega(\vartheta, \vartheta, \partial) \ \forall\ \partial > 0.$$

Definition 2.8 A sequence $\{\vartheta_n\}$ in a NBMLS $(Q, \Psi, \Phi, \Omega, *, \circ)$ is said to be Cauchy sequence (CS) if

$$\lim_{n\to\infty} \Psi(\vartheta_n, \vartheta_{n+\varepsilon}, \partial),$$

$$\lim_{n\to\infty} \Phi(\vartheta_n, \vartheta_{n+\varepsilon}, \partial),$$

and

$$\lim_{n\to\infty} \Omega(\vartheta_n, \vartheta_{n+\varepsilon}, \partial),$$

for all $\partial \geq 0, \varepsilon \geq 1$ exist and is finite.

Definition 2.9 A NBMLS $(Q, \Psi, \Phi, \Omega, *, \circ)$ is said to be complete if every CS $\{\vartheta_n\}$ in Q converge to some $\vartheta \in Q$ such that

$$\lim_{n\to\infty} \Psi(\vartheta_n, \vartheta, \partial) = \Psi(\vartheta, \vartheta, \partial) = \lim_{n\to\infty} \Psi(\vartheta_n, \vartheta_{n+\varepsilon}, \partial) \text{ for all } \partial \geq 0, \varepsilon \geq 1,$$

$$\lim_{n\to\infty} \Phi(\vartheta_n, \vartheta, \partial) = \Phi(\vartheta, \vartheta, \partial) = \lim_{n\to\infty} \Phi(\vartheta_n, \vartheta_{n+\varepsilon}, \partial) \text{for all } \partial \geq 0, \varepsilon \geq 1,$$

and

$$\lim_{n\to\infty} \Omega(\vartheta_n, \vartheta, \partial) = \Omega(\vartheta, \vartheta, \partial) = \lim_{n\to\infty} \Omega(\vartheta_n, \vartheta_{n+\varepsilon}, \partial) \text{ for all } \partial \geq 0, \varepsilon \geq 1.$$

Remark 2.10 In NBMLS, the limit of a convergent sequence may not be unique for instance, for a NBMLS $(Q, \Psi, \Phi, \Omega, *, \circ)$ given in proposition (2.3) with $\sigma(\vartheta, \mathcal{I}) = \max\{\vartheta, \mathcal{I}\}^2$ and $n = a = m = 1$. Define a sequence $\{\vartheta_n\}$ in Q by $\vartheta_n = 1 - \frac{1}{n}, \forall\ n \in \mathbb{N}$. If $\vartheta \geq 1$ then

$$\lim_{n\to\infty} \Psi(\vartheta_n, \vartheta, \partial) = \lim_{n\to\infty} \frac{\partial}{\partial + \max\{\vartheta_n, \vartheta\}^2} = \frac{\partial}{\partial + \max\{\vartheta, \vartheta\}^2} = \Psi(\vartheta, \vartheta, \partial)\ \forall \partial > 0,$$

$$\lim_{n\to\infty} \Phi(\vartheta_n, \vartheta, \partial) = \lim_{n\to\infty} \frac{\max\{\vartheta_n, \vartheta\}^2}{\partial + \max\{\vartheta_n, \vartheta\}^2} = \frac{\max\{\vartheta, \vartheta\}^2}{\partial + \max\{\vartheta, \vartheta\}^2} = \Phi(\vartheta, \vartheta, \partial)\ \forall \partial > 0,$$

$$\lim_{n\to\infty} \Omega(\vartheta_n, \vartheta, \partial) = \lim_{n\to\infty} \frac{\max\{\vartheta_n, \vartheta\}^2}{\partial} = \frac{\max\{\vartheta, \vartheta\}^2}{\partial} = \Omega(\vartheta, \vartheta, \partial)\ \forall \partial > 0.$$

Therefore, the sequence $\{\vartheta_n\}$ converge to all $\vartheta \in Q$ with $\vartheta \geq 1$.

Remark 2.11 In an NBMLS, a convergent sequence may not be Cauchy. Assume an NBMLS $(Q,\Psi,\Phi,\Omega,*,\circ)$ given in above remark (2.10) Define a sequence $\{\vartheta_n\}$ in Q by $\vartheta_n = 1+(-1)^n, \forall\, n \in \mathbb{N}$. If $\vartheta \geq 2$, then

$$\lim_{n\to\infty} \Psi(\vartheta_n,\vartheta,\partial) = \lim_{n\to\infty} \frac{\partial}{\partial+\max\{\vartheta_n,\vartheta\}^2} = \frac{\partial}{\partial+\max\{\vartheta,\vartheta\}^2} = \Psi(\vartheta,\vartheta,\partial)\ \forall\partial>0,$$

$$\lim_{n\to\infty} \Phi(\vartheta_n,\vartheta,\partial) = \lim_{n\to\infty} \frac{\max\{\vartheta_n,\vartheta\}^2}{\partial+\max\{\vartheta_n,\vartheta\}^2} = \frac{\max\{\vartheta,\vartheta\}^2}{\partial+\max\{\vartheta,\vartheta\}^2} = \Phi(\vartheta,\vartheta,\partial)\ \forall\partial>0,$$

$$\lim_{n\to\infty} \Omega(\vartheta_n,\vartheta,\partial) = \lim_{n\to\infty} \frac{\max\{\vartheta_n,\vartheta\}^2}{\partial} = \frac{\max\{\vartheta,\vartheta\}^2}{\partial} = \Omega(\vartheta,\vartheta,\partial)\ \forall\partial>0.$$

Therefore, a sequence $\{\vartheta_n\}$ converges to all $\vartheta \in Q$ with $\vartheta \geq 2$, but it is not a Cauchy sequence as $\lim_{n\to\infty} \Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)$, $\lim_{n\to\infty} \Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)$ and $\lim_{n\to\infty} \Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial)$ does not exist.

Theorem 2.12 Let $(Q,\Psi,\Phi,\Omega,*,\circ)$ be a complete NBMLS such that

$$\lim_{\partial\to\infty} \Psi(\vartheta,\mathcal{J},\partial) = 1\,, \lim_{\partial\to\infty} \Phi(\vartheta,\mathcal{J},\partial) = 0 \text{ and } \lim_{\partial\to\infty} \Omega(\vartheta,\mathcal{J},\partial) = 0$$

for all $\vartheta,\mathcal{J} \in Q, \partial > 0$ and $\xi: Q \to Q$ be a mapping fulfill the circumstances

$$\Psi(\xi\vartheta,\xi\mathcal{J},\alpha\partial) \geq \Psi(\vartheta,\mathcal{J},\partial), \qquad \Phi(\xi\vartheta,\xi\mathcal{J},\alpha\partial) \leq \Phi(\vartheta,\mathcal{J},\partial)$$

$$\text{and } \Omega(\xi\vartheta,\xi\mathcal{J},\alpha\partial) \leq \Omega(\vartheta,\mathcal{J},\partial), \tag{1}$$

for all $\vartheta,\mathcal{J} \in Q, \partial > 0$, where $\alpha \in (0,1)$. Then ξ has a unique FP $\pi \in Q$ and

$$\Psi(\pi,\pi,\partial) = 1, \Phi(\pi,\pi,\partial) = 0 \text{ and } \Omega(\pi,\pi,\partial) = 0\ \ \forall\, \partial > 0.$$

Proof: Let $(Q,\Psi,\Phi,\Omega,*,\circ)$ be a complete NBMLS. For a random $\vartheta_0 \in Q$, define a sequence $\{\vartheta_n\}$ in Q by

$$\vartheta_1 = \xi\vartheta_0\ ,\ \vartheta_2 = \xi^2\vartheta_0 = \xi\vartheta_1, \dots, \vartheta_n = \xi^n\vartheta_0 = \xi\vartheta_{n-1} \text{ for all } n \in \mathbb{N}.$$

If $\vartheta_n = \vartheta_{n-1}$ for some $n \in \mathbb{N}$ then ϑ_n is a FP of ξ. We suppose that $\vartheta_n \neq \vartheta_{n-1}$ for all $n \in \mathbb{N}$.

For $\partial > 0$ and $n \in \mathbb{N}$, we obtain from (1) that

$$\Psi(\vartheta_n,\vartheta_{n+1},\partial) \geq \Psi(\vartheta_{n+1},\vartheta_n,\alpha\partial) = \Psi(\xi\vartheta_n,\xi\vartheta_{n-1},\alpha\partial) \geq \Psi(\vartheta_n,\vartheta_{n-1},\partial),$$

$$\Phi(\vartheta_n,\vartheta_{n+1},\partial)\leq\Phi(\vartheta_{n+1},\vartheta_n,\alpha\partial)=\Phi(\xi\vartheta_n,\xi\vartheta_{n-1},\alpha\partial)\leq\Phi(\vartheta_n,\vartheta_{n-1},\partial)$$

and

$$\Omega(\vartheta_n,\vartheta_{n+1},\partial)\leq\Omega(\vartheta_{n+1},\vartheta_n,\alpha\partial)=\Omega(\xi\vartheta_n,\xi\vartheta_{n-1},\alpha\partial)\leq\Omega(\vartheta_n,\vartheta_{n-1},\partial),$$

for all $n\in\mathbb{N}$ and $\partial>0.$ Therefore, by using the above inequalities, we obtain that

$$\Psi(\vartheta_{n+1},\vartheta_n,\partial)\geq\Psi(\vartheta_{n+1},\vartheta_n,\alpha\partial)=\Psi(\xi\vartheta_n,\xi\vartheta_{n-1},\alpha\partial)\geq\Psi(\vartheta_n,\vartheta_{n-1},\partial)$$

$$=\Psi(\xi\vartheta_{n-1},\xi\vartheta_{n-2},\partial)\geq\Psi\left(\vartheta_{n-1},\vartheta_{n-2},\frac{\partial}{\alpha}\right)\geq\cdots\geq\Psi\left(\vartheta_1,\vartheta_0,\frac{\partial}{\alpha^n}\right),\qquad(2)$$

$$\Phi(\vartheta_{n+1},\vartheta_n,\partial)\leq\Phi(\vartheta_{n+1},\vartheta_n,\alpha\partial)=\Phi(\xi\vartheta_n,\xi\vartheta_{n-1},\alpha\partial)\leq\Phi(\vartheta_n,\vartheta_{n-1},\partial)$$

$$=\Phi(\xi\vartheta_{n-1},\xi\vartheta_{n-2},\partial)\leq\Phi\left(\vartheta_{n-1},\vartheta_{n-2},\frac{\partial}{\alpha}\right)\leq\cdots\leq\Phi\left(\vartheta_1,\vartheta_0,\frac{\partial}{\alpha^n}\right)\qquad(3)$$

and

$$\Omega(\vartheta_{n+1},\vartheta_n,\partial)\leq\Omega(\vartheta_{n+1},\vartheta_n,\alpha\partial)=\Omega(\xi\vartheta_n,\xi\vartheta_{n-1},\alpha\partial)\leq\Omega(\vartheta_n,\vartheta_{n-1},\partial)$$

$$=\Omega(\xi\vartheta_{n-1},\xi\vartheta_{n-2},\partial)\leq\Omega\left(\vartheta_{n-1},\vartheta_{n-2},\frac{\partial}{\alpha}\right)\leq\cdots\leq\Omega\left(\vartheta_1,\vartheta_0,\frac{\partial}{\alpha^n}\right)\qquad(4)$$

for all $n\in\mathbb{N}$, $\varepsilon\geq 1$ and ∂>0. We obtain that

$$\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\geq\Psi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)*\Psi\left(\vartheta_{n+1},\vartheta_{n+\varepsilon},\frac{\partial}{b}\right),$$

$$\Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\leq\Phi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Phi\left(\vartheta_{n+1},\vartheta_{n+\varepsilon},\frac{\partial}{b}\right)$$

and

$$\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\leq\Omega\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Omega\left(\vartheta_{n+1},\vartheta_{n+\varepsilon},\frac{\partial}{b}\right).$$

Ongoing in this track, we deduce

$$\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\geq\Psi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)*\Psi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right)*\cdots*\Psi\left(\vartheta_{n+\varepsilon-1},\vartheta_{n+\varepsilon},\frac{\partial}{b^{\varepsilon-1}}\right)$$

and

$$\Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\leq\Phi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Phi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right)\circ\cdots\circ\Phi\left(\vartheta_{n+\varepsilon-1},\vartheta_{n+\varepsilon},\frac{\partial}{b^{\varepsilon-1}}\right),$$

and

$$\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\leq\Omega\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Omega\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right)\circ\cdots\circ\Omega\left(\vartheta_{n+\varepsilon-1},\vartheta_{n+\varepsilon},\frac{\partial}{b^{\varepsilon-1}}\right).$$

Using (2), (3) and (4) in the above inequality, we deduce

$$\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\geq\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b\alpha^n}\right)*\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^2\alpha^{n+1}}\right)*\cdots*\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^{\varepsilon-1}\alpha^{n+\varepsilon-1}}\right),\qquad(5)$$

$$\Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial) \leq \Phi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b\alpha^n}\right)\circ\Phi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^2\alpha^{n+1}}\right)\circ\cdots\circ\Phi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^{\varepsilon-1}\alpha^{n+\varepsilon-1}}\right), \tag{6}$$

and

$$\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial) \leq \Omega\left(\vartheta_0,\vartheta_1,\frac{\partial}{b\alpha^n}\right)\circ\Omega\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^2\alpha^{n+1}}\right)\circ\cdots\circ\Omega\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^{\varepsilon-1}\alpha^{n+\varepsilon-1}}\right) \tag{7}$$

We know that $\lim_{n\to\infty}\Psi(\vartheta,\mathcal{J},\partial)=1$, $s\lim_{n\to\infty}\Phi(\vartheta,\mathcal{J},\partial)=0, \forall\, \vartheta,\mathcal{J}\in Q$ and $\partial>0, \alpha\in(0,1)$. So, from (5), (6) and (7) we deduce that

$$\lim_{n\to\infty}\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)=1*1*\cdots*1=1, \forall\, \partial>0, \varepsilon\geq 1,$$

$$\lim_{n\to\infty}\Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)=0\circ 0\circ\cdots\circ 0=0, \forall\, \partial>0, \varepsilon\geq 1,$$

and

$$\lim_{n\to\infty}\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial)=0\circ 0\circ\cdots\circ 0=0, \forall\, \partial>0, \varepsilon\geq 1,$$

Hence, $\{\vartheta_n\}$ is a CS. The completeness of the NBMLS

$(Q,\Psi,\Phi,\Omega,*,\circ)$ agrees that there exists $\pi\in Q$ such that

$$\lim_{n\to\infty}\Psi(\vartheta_n,\pi,\partial)=\lim_{n\to\infty}\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)=\Psi(\pi,\pi,\partial)=1, \forall\, \partial>0, \varepsilon\geq 1, \tag{8}$$

$$\lim_{n\to\infty}\Phi(\vartheta_n,\pi,\partial)=\lim_{n\to\infty}\Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)=\Phi(\pi,\pi,\partial)=0, \forall\, \partial>0, \varepsilon\geq 1, \tag{9}$$

and

$$\lim_{n\to\infty}\Omega(\vartheta_n,\pi,\partial)=\lim_{n\to\infty}\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial)=\Omega(\pi,\pi,\partial)=0, \forall\, \partial>0, \varepsilon\geq 1. \tag{10}$$

Now, we examine that $\pi\in Q$ is a FP of ξ. We have

$$\begin{aligned}
\Psi(\pi,\xi\pi,\partial) &\geq \Psi\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right)*\Psi\left(\vartheta_{n+1},\xi\pi,\frac{\partial}{2b}\right), \forall\, \partial>0,\\
&= \Psi\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right)*\Psi\left(\xi\vartheta_n,\xi\pi,\frac{\partial}{2b}\right)\\
&\geq \Psi\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right)*\Psi\left(\vartheta_n,\pi,\frac{\partial}{2b\alpha}\right),\\
\Phi(\pi,\xi\pi,\partial) &\leq \Phi\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right)\circ\Phi\left(\vartheta_{n+1},\xi\pi,\frac{\partial}{2b}\right), \forall\, \partial>0,\\
&= \Phi\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right)\circ\Phi\left(\xi\vartheta_n,\xi\pi,\frac{\partial}{2b}\right)\\
&\leq \Phi\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right)\circ\Phi\left(\vartheta_n,\pi,\frac{\partial}{2b\alpha}\right)
\end{aligned}$$

and

$$\Omega(\pi,\xi\pi,\partial) \leq \Omega\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right) \circ \Omega\left(\vartheta_{n+1},\xi\pi,\frac{\partial}{2b}\right), \forall\, \partial > 0,$$
$$= \Omega\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right) \circ \Omega\left(\xi\vartheta_{n},\xi\pi,\frac{\partial}{2b}\right)$$
$$\leq \Omega\left(\pi,\vartheta_{n+1},\frac{\partial}{2b}\right) \circ \Omega\left(\vartheta_{n},\pi,\frac{\partial}{2b\alpha}\right).$$

Taking limit as $n \to +\infty$, and by (8), (9) and (10), we get

$$\Psi(\pi,\xi\pi,\partial) = 1 * 1 = 1,$$

$$\Phi(\pi,\xi\pi,\partial) = 0 \circ 0 = 0$$

and

$$\Omega(\pi,\xi\pi,\partial) = 0 \circ 0 = 0.$$

That is, π is a FP of ξ,

$$\Psi(\pi,\pi,\partial) = 1, \Phi(\pi,\pi,\partial) = 0 \text{ and } \Omega(\pi,\pi,\partial) = 0, \forall\, \partial > 0.$$

Now, for proving the uniqueness of FP, assume that γ and π are two FPs of ξ, then by (1), we get

$$\Psi(\pi,\gamma,\partial) = \Psi(\xi\pi,\xi\gamma,\partial) \geq \Psi\left(\pi,\gamma,\frac{\partial}{\alpha}\right)$$
$$\Psi(\pi,\gamma,\partial) \geq \Psi\left(\pi,\gamma,\frac{\partial}{\alpha}\right), \forall\, \partial > 0,$$
$$\Phi(\pi,\gamma,\partial) = \Phi(\xi\pi,\xi\gamma,\partial) \leq \Phi\left(\pi,\gamma,\frac{\partial}{\alpha}\right)$$
$$\Phi(\pi,\gamma,\partial) \leq \Phi\left(\pi,\gamma,\frac{\partial}{\alpha}\right), \forall\, \partial > 0$$

and

$$\Omega(\pi,\gamma,\partial) = \Omega(\xi\pi,\xi\gamma,\partial) \leq \Omega\left(\pi,\gamma,\frac{\partial}{\alpha}\right)$$
$$\Omega(\pi,\gamma,\partial) \leq \Omega\left(\pi,\gamma,\frac{\partial}{\alpha}\right), \forall\, \partial > 0.$$

We get

$$\Psi(\pi,\gamma,\partial) \geq \Psi\left(\pi,\gamma,\frac{\partial}{\alpha^{n}}\right), \forall\, n \in \mathbb{N},$$
$$\Phi(\pi,\gamma,\partial) \leq \Phi\left(\pi,\gamma,\frac{\partial}{\alpha^{n}}\right), \forall\, n \in \mathbb{N}$$

and

$$\Omega(\pi,\gamma,\partial) \leq \Omega\left(\pi,\gamma,\frac{\partial}{\alpha^n}\right), \forall\, n \in \mathbb{N}.$$

Taking limit as $n \to +\infty$ and applying the circumstance $\lim_{\partial\to\infty} \Psi(\vartheta,\mathcal{J},\partial) = 1$ and $\lim_{\partial\to\infty} \Phi(\vartheta,\mathcal{J},\partial) = 0$ and $\lim_{\partial\to\infty} \Omega(\vartheta,\mathcal{J},\partial) = 0$, so $\pi = \gamma$, hence the FP is unique.

Example 2.13 Assume $Q = [0,1]$, CTN and CTCN respectively defined as $\sigma * \kappa = \sigma\kappa$ and $\sigma \circ \kappa = \max\{\sigma,\kappa\}$. Also, Ψ, Φ and Ω are defined as

$$\Psi(\vartheta,\mathcal{J},\partial) = \frac{\partial}{\partial + \max\{\vartheta,\mathcal{J}\}^2}\ \forall\, \vartheta,\mathcal{J} \in Q, \partial > 0,$$
$$\Phi(\vartheta,\mathcal{J},\partial) = \frac{\max\{\vartheta,\mathcal{J}\}^2}{\partial + \max\{\vartheta,\mathcal{J}\}^2}\ \forall\, \vartheta,\mathcal{J} \in Q, \partial > 0,$$
$$\Omega(\vartheta,\mathcal{J},\partial) = \frac{\max\{\vartheta,\mathcal{J}\}^2}{\partial}\ \forall\, \vartheta,\mathcal{J} \in Q, \partial > 0.$$

Then $(Q,\Psi,\Phi,\Omega,*,\circ)$ be a complete NBMLS. Define $\xi: Q \to Q$ by

$$\xi\vartheta = \begin{cases} 0, & \vartheta \in \left[0,\frac{1}{2}\right] \\ \frac{\vartheta}{4}, & \vartheta \in \left(\frac{1}{2},1\right]. \end{cases}$$

Now,

$$\lim_{\partial\to\infty} \Psi(\vartheta,\mathcal{J},\partial) = \lim_{\partial\to\infty} \frac{\partial}{\partial + \max\{\vartheta,\mathcal{J}\}^2} = 1,$$
$$\lim_{\partial\to\infty} \Phi(\vartheta,\mathcal{J},\partial) = \lim_{\partial\to\infty} \frac{\max\{\vartheta,\mathcal{J}\}^2}{\partial + \max\{\vartheta,\mathcal{J}\}^2} = 0,$$
$$\lim_{\partial\to\infty} \Omega(\vartheta,\mathcal{J},\partial) = \lim_{\partial\to\infty} \frac{\max\{\vartheta,\mathcal{J}\}^2}{\partial} = 0.$$

For $\alpha \in \left[\frac{1}{2},1\right)$, we have four cases:

Case.1) If $\vartheta,\mathcal{J} \in \left[0,\frac{1}{2}\right]$, then $\xi\vartheta = \xi\mathcal{J} = 0$.

Case.2) If $\vartheta \in \left[0,\frac{1}{2}\right]$ and $\mathcal{J} \in \left(\frac{1}{2},1\right]$, then $\xi\vartheta = 0$ and $\xi\mathcal{J} = \frac{\mathcal{J}}{4}$.

Case.3) If $\vartheta,\mathcal{J} \in \left(\frac{1}{2},1\right]$, then $\xi\vartheta = \frac{\vartheta}{4}$ and $\xi\mathcal{J} = \frac{\mathcal{J}}{4}$.

Case.4) If $\vartheta \in \left(\frac{1}{2},1\right]$ and $\mathcal{J} \in \left[0,\frac{1}{2}\right]$, then $\xi\vartheta = \frac{\vartheta}{4}$ and $\xi\mathcal{J} = 0$.

From all 4 cases, we obtain that

$$\Psi(\xi\vartheta,\xi\mathcal{J},\alpha\partial)\geq\Psi(\vartheta,\mathcal{J},\partial),$$

$$\Phi(\xi\vartheta,\xi\mathcal{J},\alpha\partial)\leq\Phi(\vartheta,\mathcal{J},\partial),$$

$$\Omega(\xi\vartheta,\xi\mathcal{J},\alpha\partial)\leq\Omega(\vartheta,\mathcal{J},\partial).$$

Hence all circumstances of Theorem 2.12 are fulfilled and 0 is the unique FP of ξ. Also,

$$\Psi(\pi,\pi,\partial)=\Psi(0,0,\partial)=1,\forall\ \partial>0,$$

$$\Phi(\pi,\pi,\partial)=\Phi(0,0,\partial)=0,\forall\ \partial>0,$$

$$\Omega(\pi,\pi,\partial)=\Omega(0,0,\partial)=0,\forall\ \partial>0.$$

Definition 2.14 Let $(Q,\Psi,\Phi,\Omega,*,\circ)$ be an NBMLS. A mapping $\xi:Q\to Q$ is named to be NBML contractive (NBMLC) if $a\in(0,1)$ such that

$$\frac{1}{\Psi(\xi\vartheta,\xi\mathcal{J},\partial)}-1\leq a\left[\frac{1}{\Psi(\vartheta,\mathcal{J},\partial)}-1\right],\qquad \Phi(\xi\vartheta,\xi\mathcal{J},\partial)\leq a\Phi(\vartheta,\mathcal{J},\partial)$$

$$\text{and } \Omega(\xi\vartheta,\xi\mathcal{J},\partial)\leq a\Omega(\vartheta,\mathcal{J},\partial) \qquad (11)$$

for all $\vartheta,\mathcal{J}\in Q$ and $\partial>0$. Here, a is called the NBMLC constant of ξ.

Theorem 2.15 Let $(Q,\Psi,\Phi,\Omega,*,\circ)$ be a complete NBMLS and $\xi:Q\to Q$ be a NBMLC mapping with a NBMLC constant a, then ξ has a unique FP $\pi\in Q$ so that $\Psi(\pi,\pi,\partial)=1$, $\Phi(\pi,\pi,\partial)=0$ and $\Omega(\pi,\pi,\partial)=0$, for all $\partial>0$.

Proof: Let $(Q,\Psi,\Phi,\Omega,*,\circ)$ be a complete NBMLS. For a random $\vartheta_0\in Q$, express a sequence $\{\vartheta_n\}$ in Q by

$$\vartheta_1=\xi\vartheta_0\ ,\ \vartheta_2=\xi^2\vartheta_0=\xi\vartheta_1,\dots,\vartheta_n=\xi^n\vartheta_0=\xi\vartheta_{n-1} \text{ for all } n\in\mathbb{N}.$$

If $\vartheta_n=\vartheta_{n-1}$ for some $n\in\mathbb{N}$, then ϑ_n is a FP of ξ. We suppose that $\vartheta_n\neq\vartheta_{n-1}$ for all $n\in\mathbb{N}$. For $\partial>0$ and $n\in\mathbb{N}$, we obtain from (11)

$$\frac{1}{\Psi(\vartheta_n,\vartheta_{n+1},\partial)}-1=\frac{1}{\Psi(\xi\vartheta_{n-1},\xi\vartheta_n,\partial)}-1\leq a\left[\frac{1}{\Psi(\vartheta_{n-1},\vartheta_n,\partial)}-1\right].$$

We have

$$\frac{1}{\Psi(\vartheta_n,\vartheta_{n+1},\partial)}\leq\frac{a}{\Psi(\vartheta_{n-1},\vartheta_n,\partial)}+(1-a),\forall\,\partial>0,$$

$$=\frac{a}{\Psi(\xi\vartheta_{n-2},\xi\vartheta_{n-1},\partial)}+(1-a)\leq\frac{a^2}{\Psi(\vartheta_{n-2},\vartheta_{n-1},\partial)}+a(1-a)+(1-a).$$

Ongoing in this track, we obtain

$$\frac{1}{\Psi(\vartheta_n,\vartheta_{n+1},\partial)}\leq\frac{a^n}{\Psi(\vartheta_0,\vartheta_1,\partial)}+a^{n-1}(1-a)+a^{n-2}(1-a)+\cdots+a(1-a)+(1-a)$$

$$\leq\frac{a^n}{\Psi(\vartheta_0,\vartheta_1,\partial)}+(a^{n-1}+a^{n-2}+\cdots+1)(1-a)$$

$$\leq\frac{a^n}{\Psi(\vartheta_0,\vartheta_1,\partial)}+(1-a^n).$$

We have

$$\frac{1}{\frac{a^n}{\Psi(\vartheta_0,\vartheta_1,\partial)}+(1-a^n)}\leq\ \Psi(\vartheta_n,\vartheta_{n+1},\partial),\forall\,\partial>0,n\in\mathbb{N}. \tag{12}$$

Now,

$$\Phi(\vartheta_n,\vartheta_{n+1},\partial)=\Phi(\xi\vartheta_{n-1},\xi\vartheta_n,\partial)\leq a\Phi(\vartheta_{n-1},\vartheta_n,\partial)=a\Phi(\xi\vartheta_{n-2},\xi\vartheta_{n-1},\partial)$$

$$\leq a^2\Phi(\vartheta_{n-2},\vartheta_{n-1},\partial)\leq\cdots\leq a^n\Phi(\vartheta_0,\vartheta_1,\partial) \tag{13}$$

and

$$\Omega(\vartheta_n,\vartheta_{n+1},\partial)=\Omega(\xi\vartheta_{n-1},\xi\vartheta_n,\partial)\leq a\Omega(\vartheta_{n-1},\vartheta_n,\partial)=a\Omega(\xi\vartheta_{n-2},\xi\vartheta_{n-1},\partial)$$

$$\leq a^2\Omega(\vartheta_{n-2},\vartheta_{n-1},\partial)\leq\cdots\leq a^n\Omega(\vartheta_0,\vartheta_1,\partial) \tag{14}$$

Now, for $\varepsilon\geq 1$ and $n\in\mathbb{N}$, we get

$$\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\geq\Psi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)*\Psi\left(\vartheta_{n+1},\vartheta_{n+\varepsilon},\frac{\partial}{b}\right)$$

$$\geq\Psi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)*\Psi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right)*\Psi\left(\vartheta_{n+2},\vartheta_{n+\varepsilon},\frac{\partial}{b^2}\right).$$

Ongoing in this track, we derive

$$\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\geq\Psi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)*\Psi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right)*\cdots*\Psi\left(\vartheta_{n+\varepsilon-1},\vartheta_{n+\varepsilon},\frac{\partial}{b^{\varepsilon-1}}\right),$$

$$\Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\leq\Phi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Phi\left(\vartheta_{n+1},\vartheta_{n+\varepsilon},\frac{\partial}{b}\right)$$

$$\leq\Phi\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Phi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right)\circ\Phi\left(\vartheta_{n+2},\vartheta_{n+\varepsilon},\frac{\partial}{b^2}\right)$$

and

$$\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\leq\Omega\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Omega\left(\vartheta_{n+1},\vartheta_{n+\varepsilon},\frac{\partial}{b}\right)$$

$$\leq\Omega\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Omega\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right)\circ\Omega\left(\vartheta_{n+2},\vartheta_{n+\varepsilon},\frac{\partial}{b^2}\right).$$

Ongoing in this track, we deduce that

$$\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\leq\Omega\left(\vartheta_n,\vartheta_{n+1},\frac{\partial}{b}\right)\circ\Omega\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\partial}{b^2}\right)\circ\cdots\circ\Omega\left(\vartheta_{n+\varepsilon-1},\vartheta_{n+\varepsilon},\frac{\partial}{b^{\varepsilon-1}}\right)$$

By using (12), (13) and (14) in the above inequality, we have

$$\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\geq\frac{1}{\frac{a^n}{\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b}\right)}+(1-a^n)}*\frac{1}{\frac{a^{n+1}}{\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^2}\right)}+(1-a^{n+1})}*\cdots$$

$$*\frac{1}{\frac{a^{n+\varepsilon-1}}{\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^{\varepsilon-1}}\right)}+(1-a^{n+\varepsilon-1})},$$

$$\geq\frac{1}{\frac{a^n}{\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b}\right)}+1}*\frac{1}{\frac{a^{n+1}}{\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^2}\right)}+1}*\cdots*\frac{1}{\frac{a^{n+\varepsilon-1}}{\Psi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^{\varepsilon-1}}\right)}+1},$$

$$\Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\leq a^n\Phi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b}\right)\circ a^{n+1}\Phi\left(\vartheta_1,\vartheta_2,\frac{\partial}{b^2}\right)\circ\cdots\circ a^{n+\varepsilon-1}\Phi\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^{\varepsilon-1}}\right),$$

and

$$\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial)\leq a^n\Omega\left(\vartheta_0,\vartheta_1,\frac{\partial}{b}\right)\circ a^{n+1}\Omega\left(\vartheta_1,\vartheta_2,\frac{\partial}{b^2}\right)\circ\cdots\circ a^{n+\varepsilon-1}\Omega\left(\vartheta_0,\vartheta_1,\frac{\partial}{b^{\varepsilon-1}}\right)$$

Here, $a\in(0,1)$, we deduce from the above expression that

$$\lim_{n\to\infty}\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)=1 \text{ for all } \partial>0,\varepsilon\geq 1,$$

$$\lim_{n\to\infty}\Phi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)=0 \text{ for all } \partial>0,\varepsilon\geq 1,$$

and

$$\lim_{n\to\infty}\Omega(\vartheta_n,\vartheta_{n+\varepsilon},\partial)=0 \text{ for all } \partial>0,\varepsilon\geq 1.$$

That is, $\{\vartheta_n\}$ is a CS in $(Q,\Psi,\Phi,\Omega,*,\circ)$. By the completeness of $(Q,\Psi,\Phi,\Omega,*,\circ)$. There is $\pi\in Q$, such that

$$\lim_{n\to\infty}\Psi(\vartheta_n,\pi,\partial)=\lim_{n\to\infty}\Psi(\vartheta_n,\vartheta_{n+\varepsilon},\partial)=\lim_{n\to\infty}\Psi(\pi,\pi,\partial)=1,\forall\,\partial>0,\varepsilon\geq 1. \tag{15}$$

$$\lim_{n\to\infty} \Phi(\vartheta_n, \pi, \partial) = \lim_{n\to\infty} \Phi(\vartheta_n, \vartheta_{n+\varepsilon}, \partial) = \lim_{n\to\infty} \Phi(\pi, \pi, \partial) = 0, \forall\, \partial > 0, \varepsilon \geq 1. \tag{16}$$

and

$$\lim_{n\to\infty} \Omega(\vartheta_n, \pi, \partial) = \lim_{n\to\infty} \Omega(\vartheta_n, \vartheta_{n+\varepsilon}, \partial) = \lim_{n\to\infty} \Omega(\pi, \pi, \partial) = 0, \forall\, \partial > 0, \varepsilon \geq 1. \tag{17}$$

Now, we examine that π is a FP for ξ. By using (11) we get

$$\frac{1}{\Psi(\xi\vartheta_n, \xi\pi, \partial)} - 1 \leq a\left[\frac{1}{\Psi(\vartheta_n, \pi, \partial)} - 1\right] = \frac{a}{\Psi(\vartheta_n, \pi, \partial)} - a,$$

$$\frac{1}{\frac{a}{\Psi(\vartheta_n, \pi, \partial)} + 1 - a} \leq \Psi(\xi\vartheta_n, \xi\pi, \partial).$$

applying the above expression, we deduce

$$\Psi(\pi, \xi\pi, \partial) \geq \Psi\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) * \Psi\left(\vartheta_{n+1}, \xi\pi, \frac{\partial}{2b}\right)$$

$$= \Psi\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) * \Psi\left(\xi\vartheta_n, \xi\pi, \frac{\partial}{2b}\right)$$

$$\geq \Psi\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) * \frac{1}{\frac{a}{\Psi\left(\vartheta_n, \pi, \frac{\partial}{2b}\right)} + 1 - a},$$

$$\Phi(\pi, \xi\pi, \partial) \leq \Phi\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) \circ \Phi\left(\vartheta_{n+1}, \xi\pi, \frac{\partial}{2b}\right)$$

$$= \Phi\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) \circ \Phi\left(\xi\vartheta_n, \xi\pi, \frac{\partial}{2b}\right)$$

$$\leq \Phi\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) \circ a\Phi\left(\vartheta_n, \pi, \frac{\partial}{2b}\right),$$

and

$$\Omega(\pi, \xi\pi, \partial) \leq \Omega\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) \circ \Omega\left(\vartheta_{n+1}, \xi\pi, \frac{\partial}{2b}\right)$$

$$= \Omega\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) \circ \Omega\left(\xi\vartheta_n, \xi\pi, \frac{\partial}{2b}\right)$$

$$\leq \Omega\left(\pi, \vartheta_{n+1}, \frac{\partial}{2b}\right) \circ a\Omega\left(\vartheta_n, \pi, \frac{\partial}{2b}\right)$$

Taking limit as $n \to \infty$ and using (15), (16) and (17) in the above inequalities, we examine $\Psi(\pi, \xi\pi, \partial) = 1$, $\Phi(\pi, \xi\pi, \partial) = 0$ and $\Omega(\pi, \xi\pi, \partial) = 0$, therefore, $\xi\pi = \pi$. That is, π is a FP of ξ and $\Psi(\pi, \pi, \partial) = 1$, $\Phi(\pi, \pi, \partial) = 0$ and $\Omega(\pi, \pi, \partial) = 0$ for all $\partial > 0$.

Now, for proving the uniqueness of the FP π of ξ. Let γ be another FP of ξ, such that $\Psi(\pi,\gamma,$ t)$\neq 1, \Phi(\pi,\gamma,\partial) \neq 0$ and $\Omega(\pi,\gamma,\partial) \neq 0$ for some $\partial > 0$. It monitors from (11) that

$$\frac{1}{\Psi(\pi,\gamma,\partial)} - 1 = \frac{1}{\Psi(\xi\pi,\xi\gamma,\partial)} - 1$$
$$\leq a\left[\frac{1}{\Psi(\pi,\gamma,\partial)} - 1\right] < \frac{1}{\Psi(\pi,\gamma,\partial)} - 1,$$
$$\Phi(\pi,\gamma,\partial) = \Phi(\xi\pi,\xi\gamma,\partial) \leq a\Phi(\pi,\gamma,\partial) < \Phi(\pi,\gamma,\partial),$$

and

$$\Omega(\pi,\gamma,\partial) = \Omega(\xi\pi,\xi\gamma,\partial) \leq a\Omega(\pi,\gamma,\partial) < \Omega(\pi,\gamma,\partial),$$

a contradiction.

That is, we have $\Psi(\pi,\gamma,\partial) = 1, \Phi(\pi,\gamma,\partial) = 0$ and $\Omega(\pi,\gamma,\partial) = 0$, for all $\partial > 0$, and hence $\pi = \gamma$.

Corollary 2.16 Assume $(Q,\Psi,\Phi,\Omega,*,\circ)$ be a complete NBMLS and $\xi: Q \rightarrow Q$ be a mapping satisfying

$$\frac{1}{\Psi(\xi^n\vartheta,\xi^n\mathcal{I},\partial)} - 1 \leq a\left[\frac{1}{\Psi(\vartheta,\mathcal{I},\partial)} - 1\right],$$
$$\Phi(\xi^n\vartheta,\xi^n\mathcal{I},\partial) \leq a\Phi(\vartheta,\mathcal{I},\partial),$$

and

$$\Omega(\xi^n\vartheta,\xi^n\mathcal{I},\partial) \leq a\Omega(\vartheta,\mathcal{I},\partial)$$

for some $n \in \mathbb{N}, \forall\, \vartheta,\mathcal{I} \in Q, \partial > 0$, where $0 < a < 1$. Then ξ has a unique FP $\pi \in Q$ and $\Psi(\pi,\pi,\partial) = 1, \Phi(\pi,\pi,\partial) = 0$ and $\Omega(\pi,\pi,\partial) = 0\ \forall\, \partial > 0$.

Proof: $\pi \in Q$ is the unique FP of ξ^n by applying theorem 2.15, and $\Psi(\pi,\pi,\partial) = 1,\ \Phi(\pi,\pi,\partial) = 0$ and $\Omega(\pi,\pi,\partial) = 0\ \forall\, \partial > 0$. $\xi\pi$ is also a FP of ξ^n as $\xi^n(\xi\pi) = \xi\pi$ and from Theorem 2.15, $\xi\pi = \pi$, π is the unique FP, therefore, the unique FP of ξ is also the unique FP of ξ^n.

Example 2.17 Assume $Q = [0,2]$, CTN and CTCN respectively defined as $\sigma * \kappa = \sigma\kappa$ and $\sigma \circ \kappa = \max\{\sigma, \kappa\}$, given Ψ, Φ and Ω as

$$\Psi(\vartheta, \mathcal{I}, \partial) = \frac{\partial}{\partial + \max\{\vartheta, \mathcal{I}\}^2} \ \forall \, \vartheta, \mathcal{I} \in Q, \partial > 0,$$

$$\Phi(\vartheta, \mathcal{I}, \partial) = \frac{\max\{\vartheta, \mathcal{I}\}^2}{\partial + \max\{\vartheta, \mathcal{I}\}^2} \ \forall \, \vartheta, \mathcal{I} \in Q, \partial > 0,$$

$$\Omega(\vartheta, \mathcal{I}, \partial) = \frac{\max\{\vartheta, \mathcal{I}\}^2}{\partial} \ \forall \, \vartheta, \mathcal{I} \in Q, \partial > 0.$$

for all $\vartheta, \mathcal{I} \in Q$ and $\partial > 0$.Then $(Q, \Psi, \Phi, \Omega, *, \circ)$ is a complete NMLS. Define $\xi: Q \to Q$ as

$$\xi\vartheta = \begin{cases} 0, & \vartheta = 1 \\ \frac{\vartheta}{5}, & \vartheta \in [0,1) \\ \frac{\vartheta}{7}, & \vartheta \in (1,2]. \end{cases}$$

Then we have 8 cases:

Case.1) If $\vartheta = \mathcal{I} = 1$, then $\xi\vartheta = \xi\mathcal{I} = 0$.

Case.2) If $\vartheta = 1$ and $\mathcal{I} \in [0,1)$, then $\xi\vartheta = 0$ and $\xi\mathcal{I} = \frac{\mathcal{I}}{5}$.

Case.3) If $\vartheta = 1$ and $\mathcal{I} \in (1,2]$, then $\xi\vartheta = 0$ and $\xi\mathcal{I} = \frac{\mathcal{I}}{7}$.

Case.4) If $\vartheta \in [0,1)$ and $\mathcal{I} \in (1,2]$, then $\xi\vartheta = \frac{\vartheta}{5}$ and $\xi\mathcal{I} = \frac{\mathcal{I}}{7}$.

Case.5) If $\vartheta \in [0,1)$ and $\mathcal{I} \in [0,1)$, then $\xi\vartheta = \frac{\vartheta}{5}$ and $\xi\mathcal{I} = \frac{\mathcal{I}}{5}$.

Case.6) If $\vartheta \in [0,1)$ and $\mathcal{I} = 1$, then $\xi\vartheta = \frac{\vartheta}{5}$ and $\xi\mathcal{I} = 0$.

Case.7) If $\vartheta \in (1,2]$ and $\mathcal{I} = 1$, then $\xi\vartheta = \frac{\vartheta}{7}$ and $\xi\mathcal{I} = 0$.

Case.8) If $\vartheta \in (1,2]$ and $\mathcal{I} \in (1,2]$, then $\xi\vartheta = \frac{\vartheta}{7}$ and $\xi\mathcal{I} = \frac{\mathcal{I}}{7}$.

All above cases satisfy the NBMLC:

$$\frac{1}{\Psi(\xi\vartheta, \xi\mathcal{I}, \partial)} - 1 \leq a\left[\frac{1}{\Psi(\vartheta, \mathcal{I}, \partial)} - 1\right],$$

$$\Phi(\xi\vartheta, \xi\mathcal{I}, \partial) \leq a\Phi(\vartheta, \mathcal{I}, \partial),$$

$$\Omega(\xi\vartheta, \xi\mathcal{I}, \partial) \leq a\Omega(\vartheta, \mathcal{I}, \partial)$$

with $a \in \left[\frac{1}{2}, 1\right)$ the NBMLC constant. Hence ξ is a NBMLC mapping with $a \in \left[\frac{1}{2}, 1\right)$. All circumstances of theorem 2.15 are fulfilled. Also, 0 is the unique FP of ξ and $\Psi(0,0,\partial) = 1, \Phi(0,0,\partial) = 0$ and $\Omega(0,0,\partial) = 0, \forall\ \partial > 0.$

Theorem 2.18 Let $(Q, \Psi, \Phi, \Omega, *, \circ)$ be a complete NBMLS and $\xi: Q \to Q$ be a NBMLC mapping with an NBMLC constant a . Suppose that their exist $\pi \in Q$, such that $\Psi(\pi, \xi\pi, \partial) \geq \Psi(\vartheta, \xi\vartheta, \partial)$, $\Phi(\pi, \xi\pi, \partial) \leq \Phi(\vartheta, \xi\vartheta, \partial)$ and $\Omega(\pi, \xi\pi, \partial) \leq \Omega(\vartheta, \xi\vartheta, \partial)$ for all $\vartheta \in Q$ and $\partial > 0$, we claim that $\Psi(\pi, \xi\pi, \partial) = 1$, $\Phi(\pi, \xi\pi, \partial) = 0$ and $\Omega(\pi, \xi\pi, \partial) = 0$ for all $\pi \in Q$ and $\partial > 0$, then ξ has a unique FP $\pi \in Q$ so that $\Psi(\pi, \pi, \partial) = 1$, $\Phi(\pi, \pi, \partial) = 0$ and $\Phi(\pi, \pi, \partial) = 0$ for all $\partial > 0$.

Proof: Let $\Psi_\vartheta(\partial) = \Psi(\vartheta, \xi\vartheta, \partial)$, $\Phi_\vartheta(\partial) = \Phi(\vartheta, \xi\vartheta, \partial)$ and $\Omega_\vartheta(\partial) = \Omega(\vartheta, \xi\vartheta, \partial)$ for all $\vartheta \in Q$ and $\partial > 0$. Then by the assumption $\Psi_\pi(\partial) \geq \Psi_\vartheta(\partial)$, $\Phi_\pi(\partial) \leq \Phi_\vartheta(\partial)$ and $\Omega_\pi(\partial) \leq \Omega_\vartheta(\partial)$ for all $\vartheta \in Q$ and $\partial > 0$. We claim that $\Psi(\pi, \xi\pi, \partial) = 1$, $\Phi(\pi, \xi\pi, \partial) = 0$ and $\Omega(\pi, \xi\pi, \partial) = 0$ for all $\partial > 0$. Indeed, if $\Psi_\pi(\partial) = \Psi(\pi, \xi\pi, \partial) < 1$, $\Phi_\pi(\partial) = \Phi(\pi, \xi\pi, \partial) > 0$ and $\Omega_\pi(\partial) = \Omega(\pi, \xi\pi, \partial) > 0$ for some $\partial > 0$, then it examine from (11) that

$$\frac{1}{\Psi_{\xi\pi}(\partial)} - 1 = \frac{1}{\Psi(\xi\pi, \xi\,\xi\pi, \partial)} - 1$$

$$\leq a\left[\frac{1}{\Psi(\pi, \xi\pi, \partial)} - 1\right] = a\left[\frac{1}{\Psi_\pi(\partial)} - 1\right] < \left[\frac{1}{\Psi_\pi(\partial)} - 1\right],$$

$$\Phi_{\xi\pi}(\partial) = \Phi(\xi\pi, \xi\,\xi\pi, \partial) \leq a[\Phi(\pi, \xi\pi, \partial)] = a[\Phi_\pi(\partial)] < \Phi_\pi(\partial),$$

$$\Omega_{\xi\pi}(\partial) = \Omega(\xi\pi, \xi\,\xi\pi, \partial) \leq a[\Omega(\pi, \xi\pi, \partial)] = a[\Omega_\pi(\partial)] < \Omega_\pi(\partial).$$

That is $\Psi_\pi(\partial) \leq \Psi_{\xi\pi}(\partial), \xi\pi \in Q$ a contradiction. Therefore, we have $\Psi_\vartheta(\partial) = \Psi(\pi, \xi\pi\ \partial) = 1$, $\Phi_\vartheta(\partial) = \Phi(\pi, \xi\pi\ \partial) = 0$ and $\Omega_\vartheta(\partial) = \Omega(\pi, \xi\pi\ \partial) = 0$ for all

$\partial > 0$, and so $\xi\pi = \pi$. Following the similar argument as in theorem (2.15), uniqueness of FP of ξ follows. If $\Psi(\pi,\pi,\partial) < 1, \Phi(\pi,\pi,\partial) > 0$ and $\Omega(\pi,\pi,\partial) > 0$ for some $\partial > 0$, then from (11), we have

$$\frac{1}{\Psi(\pi,\pi,\partial)} - 1 = \frac{1}{\Psi(\xi\pi,\xi\pi,\partial)} - 1 \leq a\left[\frac{1}{\Psi(\pi,\pi,\partial)} - 1\right] < \left[\frac{1}{\Psi(\pi,\pi,\partial)} - 1\right],$$

$$\Phi(\pi,\pi,\partial) = \Phi(\xi\pi,\xi\pi,\partial) \leq a[\Phi(\pi,\pi,\partial)] < \Phi(\pi,\pi,\partial),$$

$$\Omega(\pi,\pi,\partial) = \Omega(\xi\pi,\xi\pi,\partial) \leq a[\Omega(\pi,\pi,\partial)] < \Omega(\pi,\pi,\partial),$$

a contradiction. Therefore, $\Psi(\pi,\pi,\partial) = 1, \Phi(\pi,\pi,\partial) = 0$ and $\Omega(\pi,\pi,\partial) = 0$.

Remark 2.19 In the above theorem it is shown that in an NBMLS, the self-Neutrosophic distance of the FP of a NBMLC mapping with a NBMLC constant a, is always 1, 0 and 0. That is, the degree of self-nearness of the fixed point of a NBML contractive mapping is perfect.

3. Conclusion

In this article, the concept of neutrosophic b-metric-like spaces is introduced with some fixed point results and non-trivial examples. This work is more generalized in the existing literature. This work can easily extend in the structure of neutrosophic extended b-metric-like space, controlled neutrosophic b-metric-like spaces and many other structures.

Authors' contributions

All authors contributed equally in writing this article. All authors read and approved the final manuscript.

Conflicts of interest

The authors declare no conflict of interest.

Funding

This research received no external funding.

Data availability

The data used to support the findings of this study are available from the corresponding author upon request.

References

[1] M. Kirişci, N. Simsek, Neutrosophic metric spaces, Mathematical Sciences, 14(2020), 241–248.

[2] J. H. Park, Intuitionistic fuzzy metric spaces, Chaos, Solitons and Fractals, 22(2004), 1039–1046.

[3] M. Rafi, M. S. M. Noorani, fixed theorems on intuitionistic fuzzy metric space, Iranian Journal of Fuzzy Systems, 3(2006), 23-29.

[4] W. Sintunavarat and P. Kumam, fixed Theorems for a Generalized Intuitionistic Fuzzy Contraction in Intuitionistic Fuzzy Metric Spaces, Thai Journal of Mathematics, (2012).

[5] S. Sowndrarajan, M. Jeyarama, F. Smarandache, Fixed Point Results for Contraction Theorems in Neutrosophic Metric Spaces, Neutrosophic Sets and Systems, 36(2020).

[6] N. Simsek, M. Kirişci, fixed point theorems in Neutrosophic metric spaces, Sigma Journal of Engineering and Natural Sciences, 10(2019), 221-230.

[7] A. Harandi, Metric-like paces, partial metric spaces and fixed point, Fixed Point Theory and Applications, 204(2012).

[8] S. Shukla, M. Abbas, Fixed point results in fuzzy metric-like spaces, Iranian Journal of Fuzzy Systems, 11(2014), 81-92.

[9] L. A. Zadeh, Fuzzy sets, Information and Control, 3(1965), 338–353.

[10] I. Kramosil, J. Michlek, Fuzzy metric and statistical metric spaces, Kybernetika, 11(1975), 336-344.

[11] A. George, P. Veeramani On some results of analysis for fuzzy metric spaces, Fuzzy Sets and Systems, 90(1997), 365–368.

[12] M. Grabiec, "Fixed points in fuzzy metric spaces," Fuzzy Sets and Systems, 27(1988) 385–389.

[13] T. Došenovic, A. Javaheri, S. Sedghi, N. Shobe, Coupled fixed point theorem in b-fuzzy metric spaces, Novi Sad Journal Mathematics, 47(2017), 77–88.

[14] S. Sedghi, N. Shobe, Common fixed point theorem in b-fuzzy metric space, Nonlinear Functional Analysis and Applications, 17(2012), 349-359.

[15] K. Javed, F. Uddin, H. Aydi, M. Arshad, U. Ishtiaq, H. Alsamir, On Fuzzy b-Metric-Like Spaces, Journal of Function Spaces, (2021).

[16] U. Ishtiaq, N. Saleem, F. Uddin, S. Sessa, K. Ahmad, and F. di Martino, Graphical views of intuitionistic fuzzy double-controlled metric-like spaces and certain fixed-point results with application, Symmetry 14, no. 11 (2022): 2364.

[17] F. Uddin, U. Ishtiaq, A. Hussain, K. Javed, H. Al Sulami, and K. Ahmed, Neutrosophic double controlled metric spaces and related results with application, Fractal and Fractional 6, no. 6 (2022): 318.

[18] N. Saleem, U. Ishtiaq, L. Guran, and M.F. Bota, On graphical fuzzy metric spaces with application to fractional differential equations, Fractal and Fractional 6, no. 5 (2022): 238.

[19] M. Riaz, U. Ishtiaq, C. Park, K. Ahmad, and F. Uddin, Some fixed point results for xi-chainable neutrosophic and generalized neutrosophic cone metric spaces with application, AIMS MATHEMATICS 7, no. 8 (2022): 14756-14784.

[20] N. Saleem, K. Ahmad, U. Ishtiaq and M. De la Sen, Multivalued neutrosophic fractals and Hutchinson-Barnsley operator in neutrosophic metric space, Chaos, Solitons & Fractals 172 (2023): 113607.

Received: Jan 5, 2024. Accepted: Mar 28, 2024

Integration between Bioinformatics Algorithms and Neutrosophic Theory

Romany M. Farag[1]*, Mahmoud Y. Shams[2], Dalia A. Aldawody[3], Huda E. Khalid[4], Hazem M. El-Bakry[5] and Ahmed A. Salama[6]

[1] Dept. of Math and Computer Science, Faculty of Science, Port Said Univ., Egypt; blackfox2100@gmail.com
[2] Dept. of Machine Learning, Faculty of Artificial Intelligence, Kafrelsheikh University, Egypt; mahmoud.yasin@ai.kfs.edu.eg
[3] Dept. of Math and Computer Science, Faculty of Science, Port Said Univ., Egypt; drdaliaawad@yahoo.com
[4] Telafer University, The Administration Assistant for the President of the Telafer University, Telafer, Iraq; https://orcid.org/0000-0002-0968-5611, dr.huda-ismael@uotelafer.edu.iq
[5] Dept. of Information Systems, Faculty of Computer and Information Sciences Mansoura University Egypt; elbakry@mans.edu.eg
[6] Dept. of Math and Computer Science, Faculty of Science, Port Said University, Egypt;drsalama44@gmail.com
Correspondence: blackfox2100@gmail.com

Abstract: This paper presents a neutrosophic inference model for bioinformatics. The model is used to develop a system for accurate comparisons of human nucleic acids, where the new nucleic acid is compared to a database of old nucleic acids. The comparisons are analyzed in terms of accuracy, certainty, uncertainty, neutrality, and bias. The proposed system achieves good results and provides a reliable standard for future comparisons. It highlights the potential of neutrosophic inference models in bioinformatics applications. Data mining and bioinformatics play a crucial role in computational biology, with applications in scientific research and industrial development. Biological analysts rely on specialized tools and algorithms to collect, store, categorize, and analyze large volumes of unstructured data. Data mining techniques are used to extract valuable information from this data, aiding in the development of new therapies and understanding genetic relationships between organisms. Recent advancements in bioinformatics include gene expression tools, Bio sequencing, and Bio databases, which facilitate the extraction and analysis of vital biological information. These technologies contribute to the analysis of big data, identification of key bioinformatics insights, and generation of new biological knowledge. Data collection, analysis, and interpretation in this field involves the use of modern technologies such as cloud computing, machine learning, and artificial intelligence, enabling more efficient and accurate results. Ultimately, data mining and bioinformatics enhance our understanding of genetic relationships, aid in developing new therapies, and improve healthcare outcomes.

Keywords: DNA; Neutrosophic Inference Model; Sequence Analysis; Artificial Intelligence.

1. Introduction

Genomics and proteomics sciences have developed rapidly in recent years, accumulating a huge amount of biological data. Therefore, this data needs complex computational analysis to draw useful conclusions from it. Computational biology or bioinformatics is an interdisciplinary discipline aimed at interpreting and analyzing biological data using computational technology and analytical tools [1-9]. Bio-data management is a crucial research area that has gained significant importance in recent years, primarily due to the rapid expansion of bioinformatics. Bioinformatics encompasses various tasks such as mapping and analyzing DNA and protein sequences, which are utilized to compare and match sequences. Additionally, 3D models can be generated to visualize protein structures. Bioinformatics employs algorithms, databases, information retrieval systems, artificial intelligence techniques, and other tools to effectively manage, analyze, and interpret biological information. The significance of this field stems from the accumulation, transmission, and growth of information within biological systems. Bioinformatics operates in parallel with disciplines like biophysics and biochemistry, focusing on the management, analysis, retrieval, and storage of biological data. Information processing is integrated into highly specialized databases designed to handle this type of data. These databases are maintained to develop and provide researchers with user-friendly interfaces for accessing existing data and contributing new data. The field of bioinformatics aims to analyze and comprehend biological information, transforming it into valuable insights for scientific research and the development of treatments and medicines [1]. The field of bioinformatics provides support for the creation, organization, and updating of databases storing biological data, as well as the necessary tools for analyzing this information. These data can be used to discover and develop gene-based drugs. Gene patterns within the genome are analyzed by comparing biological structures such as DNA and proteins and analyzing sequencing patterns. Indeed, DNA and proteins are fundamental components of every living organism. DNA carries genetic information that is passed from generation to generation, determining individual characteristics such as hair color, eye color, and skin tone, among others. Proteins, on the other hand, provide structure and function to the body, assisting in the formation of tissues and organs and regulating cellular and chemical processes in the body [2]. DNA is composed of a sequence of nucleotides strung together, carrying genetic information in its nucleotide arrangement. When this information is translated in a process called translation, chains of proteins are produced that impact the structure and function of the body. In fact, DNA can be considered a code or guide for protein molecules, as DNA contains instructions for translating and producing different proteins that influence individual traits. Thus, DNA and proteins work together in the creation and maintenance of living organisms [3]. Computer modeling plays a crucial role in studying DNA, proteins, and biological data. Static and dynamic modeling are used to analyze interactions between proteins, nucleic acids, and peptides. Various software and tools are employed to create these models and analyze biological data. The study of DNA and proteins requires collaboration between the fields of Biology, Computer Science, Statistics, Mathematics, and Physics, utilizing different tools and techniques to gather and examine data accurately and efficiently [4].

Traditional sequence alignment techniques face several challenges when dealing with large genomes such as the human genome. They consume significant central processing unit (CPU) and memory resources, as well as time, to perform computational operations. Additionally, the use of multiple sequencing analysis techniques may result in gaps in the results that need to be filled later. To minimize these gaps and handle large genomes more effectively, the shotgun sequencing approach is considered the most efficient [5]. Several sequence comparison packages have been developed for DNA and protein sequence alignment, such as FASTX, FASTY, TFASTX, and TFASTY. A search table is used to identify matching regions between DNA and protein sequences, which are then further examined using matrices like the BLOSUM50 scoring matrix. Similar regions are merged, and gaps are filled, followed by the calculation of the optimal alignment solution. The algorithms used in DNA and protein sequence comparison include those mentioned in the research, such as BLAST and Doplots. There are multiple algorithms used for translation and comparison of sequences between proteins and DNA. Sequences can be compared based on physical similarity or evolutionary relationship. Various algorithms are employed to perform sequence alignment, such as Dotplots and BLAST, which can detect local alignments and find matching sequences in large protein databases. Several programs can be used to perform annotations for sequence analysis, which can identify genes and other biological features within DNA sequences. Dealing with complexities and repetitive regions in DNA sequences can be challenging, thus filtering techniques can be used to reduce noise and increase alignment accuracy. The mentioned research highlights BLAST and Dotplots as examples of algorithms used in sequence comparison [7]. The algorithms used for sequence comparison include local and global algorithms, which are used for sequence alignment. Sequence alignment helps identify identities and similarities between sequences, aiding in determining the relationship between different organisms and predicting the biological functions of genes and proteins. It also helps identify potential mutation sites and variations in sequences, contributing to the understanding of genetic diseases and the development of genomics-based treatments. In general, genome and sequence comparisons are employed in various biological and medical applications and are essential for understanding the genetic relationships between different organisms [8].

2. Previous works

Searching through a database of known modification sites makes it simple to find modification sites. But sometimes using plain database scanning isn't enough, and using neural networks yields superior outcomes. Comparable methods are applied to the prediction of active sites; the neural method makes use of nearest-neighbor classifiers and training data to predict protein modification sites. These methods provide efficient ways to determine locations that are relevant to biology and forecast the functions of proteins that have been changed [10].

2.1 Data mining in proteomics

Protein characteristics including shape, solubility, and stability have all been predicted using neural networks. Protein domain predictions can be achieved by using hierarchical clustering techniques. Numerous developments have been achieved in the field of protein secondary structure prediction through the application of data mining techniques. Furthermore, methods based on information theory, Bayesian theory, nearest neighbors, and neural networks have been developed to provide more precise statistical approaches and methods. For protein interactions, the density-based clustering technique (GDBSCAN) might be employed [11].

It is now possible to predict protein characteristics like stability, solubility, and structure using neural networks. Protein domain predictions are made using hierarchical clustering techniques. Many methods have been developed in the field of protein secondary structure prediction using data mining approaches. Moreover, preliminary statistical methods and improved methods based on neural networks, information theory, Bayesian theory, and nearest neighbors have been described. For protein interactions, the density-based clustering technique (GDBSCAN) can be applied.

Based on both spatial and non-spatial properties, this algorithm can cluster point objects and spatial bodies. It can be applied to the design of biological catalysts, nanomachines, and drug targets' three-dimensional architectures, among other things. An atomic-level computational protocol for modeling and predicting protein structures has been devised.

In terms of protein expression analysis, sequencing and protein expression analysis are two methods used to quantify gene expression. However, because proteins typically serve as the last agents in a cell's activity chain, protein expression is thought to be one of the best measures of true gene activity. High-throughput mass spectrometry (HT-MS) measurements and protein microarray data can shed light on the proteins included in a biological sample. Understanding HT-MS and protein expression analysis data involves computational biology.

The main structure, or amino acid sequence, of a protein can be easily ascertained from the associated gene sequence to predict the protein's structure. Usually, the protein's fundamental structure is found only in its original environment. comprehension protein function requires a comprehension of this structural information. The three main categories of structural information are secondary, tertiary, and quaternary structures. Predicting the structure of proteins is one of the most important uses, along with creating new enzymes and drugs. In terms of protein-protein interactions, X-ray crystallography and Nuclear Magnetic Resonance (NMR) methods have been used to determine the three-dimensional structures of thousands of proteins over the past few decades. Biologists can predict potential interactions between proteins based on these three-dimensional structures without needing to perform actual protein interaction experiments.

Various methods have been developed to address the protein-protein docking problem [12]. Furthermore, several algorithms have been proposed to solve the multiple sequence alignment (MSA) problem in literature. These algorithms are classified based on the methodology they employ, including the Carrillo-Lipman MSA algorithm that relies on dynamic programming (DP). DP is a methodology aimed to solve complex computational problems and optimize multiple sequences in a gradual and efficient manner. There are also other algorithms that rely on heuristics and probabilities to solve the MSA problem. The choice of the appropriate algorithm depends on the nature of the problem and specific application requirements [13]. Generally speaking, the objective of DP algorithms is to break the bigger problem down into smaller subproblems and solve each subproblem independently utilizing the answers to the earlier subproblems. Pairwise alignment subproblems make up the MSA problem in the context of the Carrillo-Lipman MSA algorithm. By resolving these subproblems, the goal is to discover thorough alignment solutions for the whole problem. The majority of contemporary MSA algorithms use a progressive alignment strategy. These algorithms' primary goal is to create a guide tree by gradually adding target sequences to it using the progressive alignment technique. Through this approach, the non-aligned sequences are then sequentially aligned [14]. The alignment tool Clustal-Omega uses the profile hidden Markov model (HMM) approach in conjunction with a guide tree to align a set of sequences. Its main purpose upon development was to align protein sequences. For sequence alignment, the Fast Fourier Transform (MAFFT) method also makes use of a related idea. On the other hand, nucleic acid sequencing has also suggested using it [15]. While MAFFT and Clustal-Omega are thought to be quicker in creating multiple sequence alignments, they are not as accurate as some other programs, such T-Coffee. Although T-Coffee requires more time to align, the precision is higher [16]. Using a position matrix rather than a distance matrix to accurately insert gaps in sequences, the PoMSA (Position-specific Scoring Matrix-based Alignment) technique stands out for increasing alignment efficiency. PoMSA's performance has been assessed with a variety of datasets, including SMART, OXBench, and BAliBASE. Studies have demonstrated that PoMSA outperforms other contemporary algorithms, such as Clustal-Omega, MAFFT, and MUSCLE, in terms of alignment precision [17].

2.2 Neutrosophic Theory in Biomedical informatics:

The proposed Neutrosophic Gaussian Mixture Model (NGMM) is aimed at classifying Breast Ultrasound images. The process involves feature extraction using a Deep Neural Network (DNN), computation of three probability functions using Neutrosophic Logic, and the development of an enhanced Expectation Maximization algorithm that incorporates this logic. The performance of NGMMs is evaluated using a new dataset that combines two public datasets, and the results indicate that NGMMs outperform DNN-based methods and Gaussian Mixture Models (GMMs) in terms of six metrics [18]. In our proposed method, the issue of missing and anomalous data was addressed by utilizing Neutrosophic Logic and inverse Lagrangian interpolation for data processing. The dataset was reshaped using these techniques. Experiments on a breast cancer dataset obtained from Al-Bayrouni Hospital were conducted by employing a Support Vector Machine (SVM) classifier with an orthogonal Legender kernel. The results of this study demonstrated an improved accuracy rate of 97%, surpassing the performance of the classical SVM algorithm [19]. A hybrid neutrosophic set of single values used to measure vector similarity. The measure is applied in the context of multiple-attribute decision-making problems. To validate its effectiveness, the proposed method is compared to existing methods using a numerical example related to medical diagnosis. The results highlight the effectiveness, simplicity, and applicability of the proposed measure. It is demonstrated that the measure can be applied to various decision-making problems in refined neutrosophic environments, including fault diagnosis, cluster analysis, data mining, and investment [20]. The simplification of two complex similarity measures proposed by Ye and Fu into more straightforward distance measures, specifically the maximum norm and arithmetic mean. The study demonstrates that these simplified measures can be effectively used in medical consultations, providing reliable results. Moreover, utilizing these simpler measures also simplifies the computational aspect of the analysis [21].

In this paper, the focus is on estimating the ratio of two means within the framework of neutrosophic theory. The study considers the uncertainty associated with an auxiliary variable, which is modeled as a neutrosophic variable. The bias and variance of the proposed estimator is further derived for a selected sample using Simple Random Sampling. The aim is to provide insights into the estimation process and assess the performance of the proposed estimator within the neutrosophic theory framework [22]. A novel Multi-Criteria Decision-Making (MCDM) method is also proposed, which utilizes single-valued neutrosophic sets in conjunction with the Decision-Making Trial and Evaluation Laboratory (DEMATEL) technique. The method is applied to a subcontractor selection problem, and the results reveal that "Experience" and "Quality" are the most influential criteria, while "Completing on Time" has no effect on the decision-making process [23].

Another aspect of the paper focuses on the lattice structures of neutrosophic theories. It is demonstrated that Zhang-Zhang's Pinang bipolar fuzzy set can be considered as a subclass of the Single-Valued Bipolar Neutrosophic Set. Furthermore, the pair structure is shown to be a specific case of refined neutrosophy, which allows for any finite or infinite number of neutralities or sub-indeterminacies.

In the brain tumor segmentation study, the use of wavelet transform as an auxiliary element in deep learning networks was investigated. The results indicated that the Daubechies1 wavelet function was the most effective in improving network performance while managing computational overload. The choice of wavelet function should be based on specific problem requirements and considerations such as computational load, processing time, and performance [25]. In this work, a Neutrosophic Cognitive Map (NCM) model was presented to aid clinical judgments on the management of pregnant patients with cardiovascular disorders. The use of Triangular Neutrosophic Numbers led to improved diagnosis and treatment accuracy by measuring the degrees of truth, indeterminacy, and falsity in experts' choices. The model was evaluated by professionals based on its accuracy in interpreting results and its capacity to interrelate various concepts. It was validated using data from the Cardiovascular and Pregnancy National Service/Gynecology and Obstetrics Hospital Ramón Gonzales Coros. Future work aims to expand the application of this model to other diseases and construct the map using machine learning techniques [26]. The novel outranking method for bipolar neutrosophic environments: multi-attribute decision-making difficulties. The method consists of defining outranking relations for bipolar neutrosophic numbers based on ELECTRE, going into great depth into the features of these relations, and creating a ranking scheme based on these relations. An actual case is used to demonstrate the approach's efficacy. Furthermore, a straightforward, practical, and efficient multi-criteria decision-making approach based on outranking relations is created for bipolar neutrosophic sets in order to minimize the loss of evaluative information. Subsequent investigations will delve into more effective techniques for making decisions and broaden the utilization of these ideas in the domains of engineering, game theory, multi-agent systems, and decision-making [27]. A diagnostic decision-making strategy for viral disease diagnosis is presented, which utilizes Interval-valued trapezoidal neutrosophic fuzzy numbers (IVTrNFN) and Multiple Attribute Decision Making (MADM). The proposed framework incorporates information entropy to determine attribute weights, grey relational analysis, and projection method to assess the relative closeness of Preferred Interval Sets (PIS) and is verified using an example of viral disease [28]. There are six methods for finding the correlation between fingerprint images using the neutrosophic technique, these methods involve the comparison of fingerprint images with a focus on basic neutrosophic operations, fuzzy concepts, and minutiae matching. The proposed methods demonstrate good accuracy across all image sizes and provide additional information when detecting relationships at high-level images. Future work involves applying machine learning techniques to classify biometric images [29]. Fingerprint matching is a challenging task due to the variations in fingerprint images of the same finger and the similarities between fingerprint images from different fingers. These variations are referred to as intraclass variations and interclass similarity. One of four methods is usually used by fingerprint-matching algorithms: phase matching, minutiae matching, picture correlation, and skeleton matching. Although the fuzzy idea has been applied to fingerprint matching in the past, the neutrosophic model—a sophisticated mathematical framework—expands its application to practical issues. It is feasible to find correlation patterns in fingerprint photos by taking use of their neutrosophic link. This article analyzes fingerprint photos using the neutrosophic technique and suggests six ways to build associations in the image domain using the neutrosophic approach. Three different analysis approaches are used in the paper. The first methodology is the same as that found in [46]. The concept of probabilistic linguistic term sets (PLTSs) is a powerful tool for modeling qualitative assessment information using linguistic terms associated with probabilities or weights. The authors highlighted the lack of research on correlation coefficient and clustering analysis for PLTSs and proposed correlation coefficient formulas to measure the relationship between two PLTSs. These formulas are then utilized to develop two novel clustering algorithms for grouping PLTSs [30]. The proposed methodology put an analogical reasoning framework for mining negative patterns of life events (NLE-LPs) from psychiatric consultation documents. The framework combines word representation approaches (skip-gram and continuous bag-of-words) with pattern inference methods (cosine similarity and cosine multiplication similarity) to extract precise NLE-LPs. Experimental results demonstrate the framework's superiority over traditional methods (positive pairwise mutual information and hyperspace analog to language). The superior results are obtained using CBOW with cosine similarity. The proposed framework's word embedding, and inference engine can also enhance the HAL model. Overall, the framework offers a simple matching function that improves the HAL model's mining performance [31]. The authors of [31] develop a disease risk analysis and prediction model for schizophrenia patients using an automatic Analytic Hierarchy Process (AHP) framework called Auto AHP. The model utilizes over 15 million follow-up records and integrates mental health information and intelligent data processing. Key factors for risk prediction are identified, including changes in mental health policy, public support, regional differences, patient gender, compliance, and social function. The Auto AHP framework achieves high precision, recall, and F1 scores and outperforms general models in risk prediction. It can assist in the clinical analysis of disease risk factors and support decision-making in chronic disease management for schizophrenia patients [49].

Fuzzy systems are widely used in decision-making problems to handle uncertain information. Evolutionary fuzzy systems have been developed using various fuzzy representations such as intuitionistic fuzzy, hesitant fuzzy, and neutrosophic representations. Complex numbers are also utilized to capture compound features and convey multifaceted information in fuzzy and intuitionistic fuzzy sets. However, the existing order relations in these systems have limitations, such as not being total order relations or being defined based on intermediate functions. These limitations make it challenging to build and ensure important properties of logical systems [32]. The introduction of the paper discusses the distinctions between inter-, trans-, multi-, and cross-disciplinary research, both theoretically and in the context of rural and mountain tourism. The main section of the paper presents a detailed presentation, starting with the research methodology and results in various disciplines such as economics, statistics, econometrics, sociology, demography, psychology, anthropology, linguistics, ethnography, folklore, and culture. The section also highlights the specific approaches used in programs, projects, and tourism policies. The central question throughout the main section and the conclusion of the article revolves around the current state of Romanian rural and mountain tourism and proposes solutions to enhance the economic development of Romania through the promotion of rural and mountain tourism [33]. The first algorithm is a fuzzy clustering algorithm, while the second algorithm is an orthogonal clustering algorithm. To validate their proposed clustering algorithm, the authors provide a practical example involving the analysis of general higher education levels in different regions of China. The results demonstrate the usability and effectiveness of their approach. The neutrosophic theory was established in 1998 by Florentin Smarandache, focusing on analyzing the origins of certainty, uncertainty, and neutrality. It uses these ideas to address different intellectual spectra [34, 35]. The neutrosophic theory analyzes the aspects of reality, such as contradiction, compatibility, and incompatibility, by studying the interactions between entities. It identifies and analyzes compatibility, incompatibility, and neutrality between these entities [35]. The theory finds applications in diverse fields, including mathematics, artificial intelligence, image processing, statistics, decision-making, engineering, sciences, and logic [36]. DNA topology refers to the shape and arrangement of DNA molecules and is important for biological processes like transcription and recombination [37]. Topoisomerases are enzymes that alter DNA topology, influencing its function [38] [39]. Neuronal DNA topology applies neutrosophic theory to analyze DNA sequences [40]. It represents DNA sequences as neutrosophic groups, where each nucleotide has degrees of belonging, non-affiliation, and indeterminacy [41] [42]. Neutrosophic crosstalk in neuronal DNA topology helps identify important and stable nucleotides across different DNA sequences [43]. It analyzes the intersection of neutrosophic groups to identify significant nucleotides [44] [45]. The neutrosophic intersection is useful for identifying conserved and critical regions in DNA sequences. Nucleotides with high membership scores indicate their significance [46] [47]. Neutrosophic junction identifies interactions and associations between nucleotides at different positions in the DNA sequence [49]. It helps identify functionally or structurally related nucleotides [50] [51]. Neutrosophic junction is a valuable tool for identifying important properties in DNA sequences, including conserved regions, nucleotide interactions, and functional or structural elements [52]. Neutrosophic junction allows the handling of uncertain and incomplete data in DNA analysis [53]. It uses neutrosophic populations and groups to represent uncertainty and ambiguity [54,55]. Neutral group theory and fuzzy set theory can represent the 3D structures of DNA molecules and accommodate their dynamic nature and uncertain data [56] [57] [58]. Fuzzy phase topological structures, such as nucleosomes and G-quadruplex structures, play vital roles in DNA function. Representing them using neutral and fuzzy groups helps understand their organization, effects of mutations, and potential drug targets [59] [60] [61-67].

3. Results (examples/case studies related to the proposed work)

Through previous studies, an algorithm was designed relying entirely on bioinformatics algorithms with the neutrosophic theory, in a precise sense, the algorithm of integration of bioinformatics techniques with the neutrosophic theory, and the results were quite impressive in relation to the bioinformatics algorithms that were mentioned in previous works.

3.1 Algorithm design method

Where the algorithm was designed to compare nucleic acids, where the nucleic acid is compared to a group of nucleic acids, as shown in Figure 1, where the new DNA is compared to the existing nucleic acids inside the data warehouse.

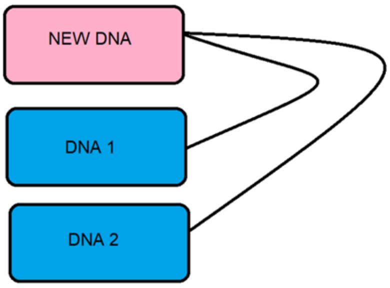


Figure 1. The new DNA from the data warehouse DNA1 and DNA2.

Then the three values are extracted, and they are similarity, neutrality, and dissimilarity (T & I & F) then the results are printed in a table, and then the degrees of accuracy are calculated for the three values, and Figure No. 2 shows the design method.

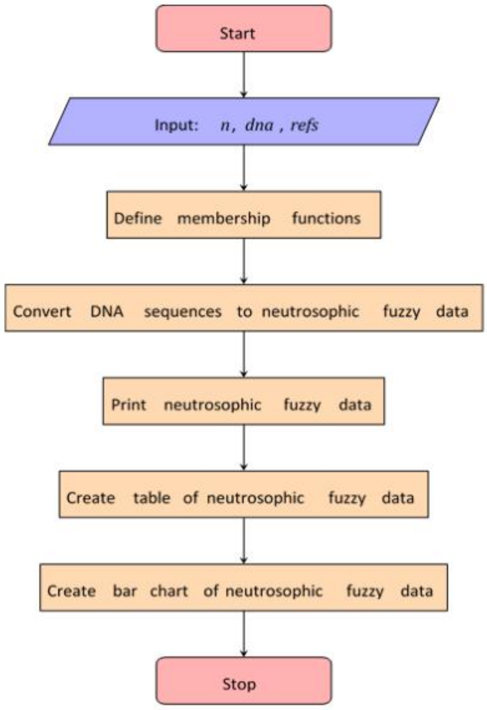


Figure 2. The flow chart of the proposed methodology.

3.2 Algorithm working

DNA Sequence Comparison

Require: new sequence: the new DNA sequence

Require: input file: the path to the text file

1. Procedure Read Sequences from File (file path):
   - Open the file at the specified file path.
   - Read the sequences from the file.
   - Split the sequences into individual strings.
   - Return the sequences.

2. Procedure Insert Gaps (sequences):
   - Create an empty list called modified sequences.
   - Calculate the maximum length as the maximum length of the sequences.
   - For each sequence (seq) in the sequences:
     - If 'C' exists in seq, replace 'C' with 'C-'.
     - If 'G' exists in seq, replace 'G' with 'G-'.
     - Append '-' characters to seq to make its length equal to the maximum length.
     - Append seq to modified sequences.
   - Return modified sequences.

3. Procedure Compare Sequences (seq1, seq2):
   - Initialize match count, mismatch count, and gap count as 0.
   - For each corresponding character (char1, char2) in seq1 and seq2:
     - If char1 is equal to char2, increment the match count by 1.
     - Else if char1 or char2 is '-', increment the gap count by 1.
     - Otherwise, increment the mismatch count by 1.
   - Return the match count, mismatch count, and gap count.

4. Procedure Calculate Similarity (seq1, seq2):
   - Calculate the total length as the sum of the lengths of seq1 and seq2.
   - Invoke the Compare Sequences procedure to get the match count, mismatch count, and gap count.
   - Calculate the similarity percentage as (total length - mismatch count) × 100 / total length.
   - Calculate the mismatch percentage as (total length - match count) × 100 / total length.
   - Calculate the neutrality percentage as (total length - gap count) × 100 / total length.
   - Calculate the similarity accuracy as total length - match count.
   - Calculate the mismatch accuracy as total length - mismatch count.
   - Calculate the neutrality accuracy as total length - gap count.

- Return the similarity percentage, mismatch percentage, neutrality percentage, similarity accuracy, mismatch accuracy, and neutrality accuracy.

5. If the input file exists, perform the steps between lines 46 and 48.

This algorithm is designed to compare DNA sequences. It includes procedures for reading sequences from a file, inserting gaps in sequences, comparing sequences to calculate numbers of matches, numbers of mismatches, and numbers of gaps, and calculating similarity and precision information. The algorithm relies on the basic concepts of DNA sequences and uses mathematical operations to compare and analyze them. Figure 3 shows the flow chart of the pseudo-code of the algorithm.

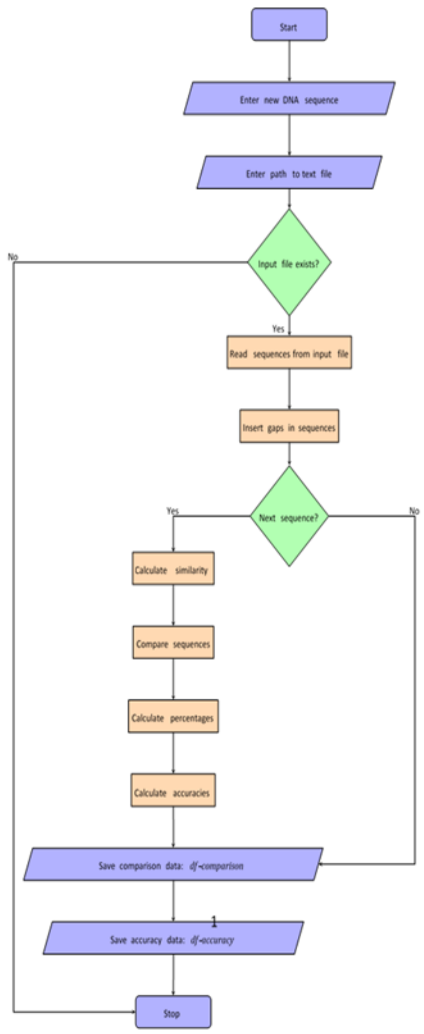


Figure 3. The flow chart steps of the pseudo-code of DNA sequences with neutrosophic.

4. The Results

The data set was obtained from the International Gen Bank (NPCI), where 50 nucleic acids were obtained, and each acid contains 49 letters of DNA. Table 1 shows the data set on which the experiment was conducted, where DNA No. 1 is the new DNA, and the rest of the acids Nuclear are already inside the database.

The new DNA:
TTGCCGACCCATCTGTACAAGAACTTCACTGTCCAGGAGCTGGCCTTG
Ancient nucleic acids:
AAAAATACAAGTGGGTGGTGGCAAGGAGAGTTACAGGCCAGAGGAAAA
AAACTGAAGGGCAAGAATCAGGAGTTCTGCCTGACCGCCTTCATGTCG
AAGCGACAGAAAGGATGGTTTCCTGCCAGTCATGTTAAACTTTTGCTA
AAGGAGAGTGACTGGAACCAGCACAAGGAGCTGGAGAAATGCCGGGGC
AAGGAGGGATATGTTCCACGTAACTTGCTGGGACTGTACTACCAGTAC
AATGGCTATAATGAAACCACAGGGGAAAGGGGGGACTTTCCGGGAACT
ACGCCAGTCGGAGTCATTTATGCGCTTTGGGATTATGAACCTCAGAAT
ACTCCCGGCGATCTGCTCGAACTGCAGATCTGCCCGCTCAACGGATAT
AGAGAGAAGAAAGCTCCCCAGACTATTACAGAACTATGTTCGGAATAT
AGAGCGCTGTATGATTATAAAAAGGAAAGAGAAGAAGATATTGACTTG
AGGGAAGACGAAGATGAAATCGAATGGTGGTGGGCGCGCCTTAATGAT
ATCCAGTCTCTGGAAGTTATCGGTAAAGGCACTCACTGCAACCAGGTT
CAAAAGTCAGAGATTGCTCAGGTAACTTCAGCATATGTTGCTTCTGGT
CAAGTGGTGTATGTCTTCTCCAAGCTGAAGGGCCGTGGGCGGCTCTTC
CACTTGGGTGACATATTGACTGTGAATAAAGGGTCCTTAGTAGCTCTT
CAGAAAAAAGCCATTGAAAGGATGAAGGACACATTAAGAATCACATAT
CAGGACTACATGGCCCCCGACTGCCGATTCCTGACCATTCACCGGGGC
CCAGTTTGGAAAGGACCAGCAAAGCTTCTGTGGAAAGGTGAAGGGGCA
CCCAATTCAATTGCGGCAATCAGTATGAAAAACAACCTTTTCGTTGCA
CCCTTCCAGAACCCGGAGGAGCAGGATGAAGGCTGGCTCATGGGCGTG
CGCAACACACAAATATATACGATAAATGACAAGATACTATCATATACG
CTCAAGAGCCGGATCGCGCTGACGGTGGAAGACTCGCCGTATCCGGGC
CTCCGTTGCCTGTGCATCAAAACTACTTCTGGGATCCACCCGAAAAAC
CTGACCGAGACCAAAATTGATAAATTATGTGTATGGAATAATAAAACC
CTGGGCTATTTCCCCAGTAGCATTGTCCGAGAGGACGAGCCATACGTC
CTGTATGATTTTGTGGCCAGTGGAGATAACACTCTAAGCATAACTAAA
GAAGCCCAAACCAAAAATGGCCAAGGCTGGGTCCCAAGCAACTACATC
GAAGTTATCGCTACTCTGAAAGACGGTCGTAAAATCTGTCTAGATCCG
GAATCGATGGCAGGCAAAAGAGAAATGGTTATCATTACATTTAAGAGC
GACACGGATGAGCTGCAGCTCAAGGCTGGGGATGTGGTGCTGGTGATC
GACACGTGGTTCGACACCATGCTTGGCTTTGCCATATCCGCGTATGCG
GACGCTCCACGTATCAAGAAGATCGTTCAGAAAAAACTGGCTGGTGAC
GAGAAAGATGCTCCAAAAGAATTATTAGACATGTTAGCAAGAGCAGAA
GAGGGTGAAGCTGTTGAGGTCATTCACAAGCTCCTGGACGGCTGGTGG
GATGATGAGCTGCCCATGAAAGAAGGAGACTGCATGACAATCATCCAC
GCAAAAATCATTTTCAAGGTGCAGGCCCAGCACGACTACACGGCCACT
GCCATCAAGGCCTACACTGCTGTGGAGGGGGACGAGGTGTCCCTGCTC
GGATTCAGTGATGGACAGGAAGCCAGGCCTGAAGAAATTGGCTGGTTA
GGCAGAAGCCTGGTCCGGGCGTGCCTGTCCGACGCGGGACACGAGCAC
GGCGAAACATTTCAGGTCGAAGTCCCGGGCAGTCAACATATAGACTCC

GGTGAAAAGCTCCGGGTCTTAGGCTATAATCACAATGGGGAATGGTGT
GTAGACACTTCAAAGATAAAGAAGGTTTGGAGAGTAGGCAAAATGGTG
GTAGTAATACAAGATAATAGTGACATCAAAGTAGTGCCAAGAAGAAAA
GTCATCAGGAAAGACGACGTCACAGGCTACTTCCCGTCCATGTACCTG
GTCTTCCCTGAGAACTTTACCGAGCGAGTATCCATGGCTGTGGCCCTT
TACGTAGAATATATTAATTTTCGGGTTTATTACAGGGACAGCAGAGAT
TCCTTTACCTATGACGACAATGGTAAGACAGGTAGAGGAGCTGTAAGC
TCTGAACAACTTAGCCTTGCACCAGGACAGTTAATATTAATTCTAAAG
TGCGAAATGGTGAAGGTAAAGTTCAAGTATAAGGGTGAAGAGAAAGAA

Following completing the comparison process, the following results were produced. Table 1 presents the results of the comparison process alongside the results of the three ratios. Table 2 shows the results of the accuracy standard.

Table 1. The evaluation of total length with gap amount and determination of neutrality (F).

Total Length	Match Count	Mismatch Count	Gap Count	Similarity (T)%	Mismatch (I)%	Neutrality (F)%
118	9	25	14	78.8135593	92.37288136	88.13559322
121	12	21	15	82.6446281	90.08264463	87.60330579
116	10	22	16	81.0344828	91.37931034	86.20689655
124	7	24	17	80.6451613	94.35483871	86.29032258
119	7	25	16	78.9915966	94.11764706	86.55462185
120	11	22	15	81.6666667	90.83333333	87.5
118	8	23	17	80.5084746	93.22033898	85.59322034
124	8	22	18	82.2580645	93.5483871	85.48387097
115	5	29	14	74.7826087	95.65217391	87.82608696
112	6	30	12	73.2142857	94.64285714	89.28571429
120	9	24	15	80	92.5	87.5
119	11	22	15	81.512605	90.75630252	87.39495798
116	9	25	14	78.4482759	92.24137931	87.93103448
124	3	29	16	76.6129032	97.58064516	87.09677419
116	10	24	14	79.3103448	91.37931034	87.93103448
112	8	27	13	75.8928571	92.85714286	88.39285714
127	12	18	18	85.8267717	90.5511811	85.82677165
121	4	28	16	76.8595041	96.69421488	86.7768595
115	6	29	13	74.7826087	94.7826087	88.69565217
127	8	22	18	82.6771654	93.7007874	85.82677165

111	12	25	11	77.4774775	89.18918919	90.09009009
128	5	24	19	81.25	96.09375	85.15625
120	4	28	16	76.6666667	96.66666667	86.66666667
111	7	29	12	73.8738739	93.69369369	89.18918919
123	7	24	17	80.4878049	94.30894309	86.17886179
114	12	22	14	80.7017544	89.47368421	87.71929825
121	10	21	17	82.6446281	91.73553719	85.95041322
117	5	29	14	75.2136752	95.72649573	88.03418803
114	3	31	14	72.8070175	97.36842105	87.71929825
125	10	20	18	84	92	85.6
123	6	25	17	79.6747967	95.12195122	86.17886179
119	6	27	15	77.3109244	94.95798319	87.39495798
113	14	23	11	79.6460177	87.61061947	90.26548673
124	3	29	16	76.6129032	97.58064516	87.09677419
119	8	24	16	79.8319328	93.27731092	86.55462185
121	10	23	15	80.9917355	91.73553719	87.60330579
120	9	22	17	81.6666667	92.5	85.83333333
131	9	19	20	85.4961832	93.12977099	84.73282443
121	9	21	18	82.6446281	92.56198347	85.12396694
119	8	25	15	78.9915966	93.27731092	87.39495798
115	8	27	13	76.5217391	93.04347826	88.69565217
111	9	28	11	74.7747748	91.89189189	90.09009009
122	10	22	16	81.9672131	91.80327869	86.8852459
122	12	20	16	83.6065574	90.16393443	86.8852459
112	7	30	11	73.2142857	93.75	90.17857143
118	12	22	14	81.3559322	89.83050847	88.13559322
113	13	20	15	82.300885	88.49557522	86.72566372
114	15	20	13	82.4561404	86.84210526	88.59649123
124	10	21	17	83.0645161	91.93548387	86.29032258

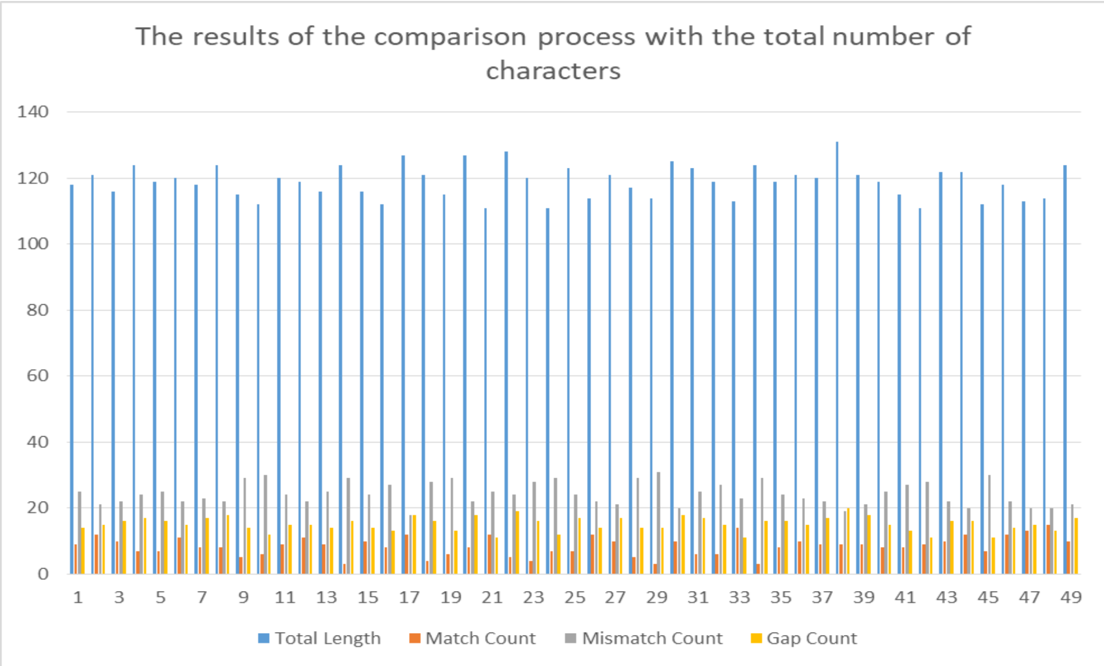


Figure 4. the results of the comparison process with the total number of characters.

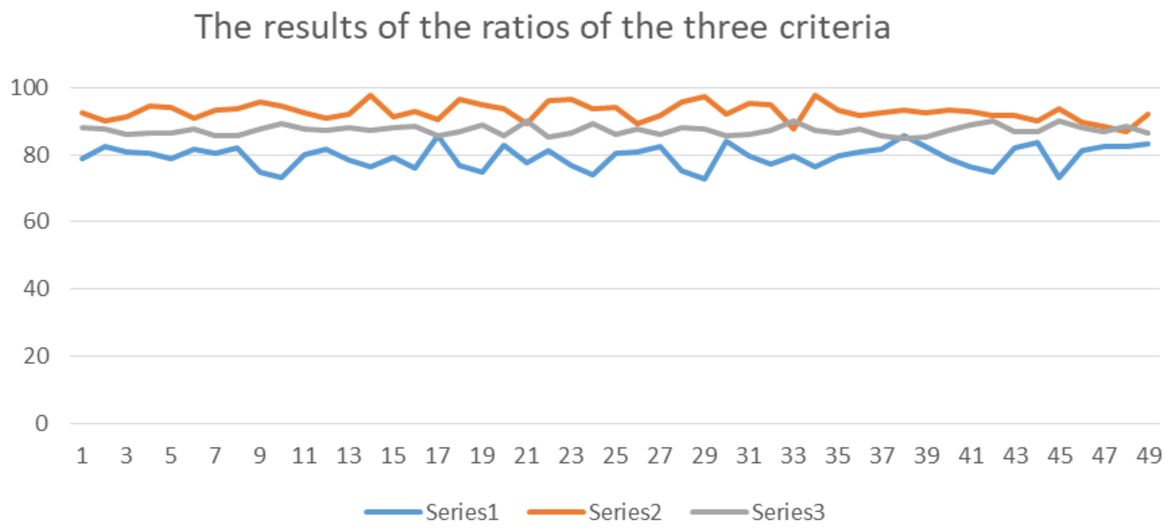


Figure 5. The results of the ratios of the three criteria

Table 2. The similarity accuracy with the mismatch and neutrality determination

Similarity Accuracy%	Mismatch Accuracy	Neutrality Accuracy%
78.81355932	92.37288136	88.13559322
82.6446281	90.08264463	87.60330579
81.03448276	91.37931034	86.20689655

80.64516129	94.35483871	86.29032258
78.99159664	94.11764706	86.55462185
81.66666667	90.83333333	87.5
80.50847458	93.22033898	85.59322034
82.25806452	93.5483871	85.48387097
74.7826087	95.65217391	87.82608696
73.21428571	94.64285714	89.28571429
80	92.5	87.5
81.51260504	90.75630252	87.39495798
78.44827586	92.24137931	87.93103448
76.61290323	97.58064516	87.09677419
79.31034483	91.37931034	87.93103448
75.89285714	92.85714286	88.39285714
85.82677165	90.5511811	85.82677165
76.85950413	96.69421488	86.7768595
74.7826087	94.7826087	88.69565217
82.67716535	93.7007874	85.82677165
77.47747748	89.18918919	90.09009009
81.25	96.09375	85.15625
76.66666667	96.66666667	86.66666667
73.87387387	93.69369369	89.18918919
80.48780488	94.30894309	86.17886179
80.70175439	89.47368421	87.71929825
82.6446281	91.73553719	85.95041322
75.21367521	95.72649573	88.03418803
72.80701754	97.36842105	87.71929825
84	92	85.6
79.67479675	95.12195122	86.17886179
77.31092437	94.95798319	87.39495798
79.6460177	87.61061947	90.26548673
76.61290323	97.58064516	87.09677419
79.83193277	93.27731092	86.55462185
80.99173554	91.73553719	87.60330579
81.66666667	92.5	85.83333333
85.49618321	93.12977099	84.73282443

82.6446281	92.56198347	85.12396694
78.99159664	93.27731092	87.39495798
76.52173913	93.04347826	88.69565217
74.77477477	91.89189189	90.09009009
81.96721311	91.80327869	86.8852459
83.60655738	90.16393443	86.8852459
73.21428571	93.75	90.17857143
81.3559322	89.83050847	88.13559322
82.30088496	88.49557522	86.72566372
82.45614035	86.84210526	88.59649123
83.06451613	91.93548387	86.29032258

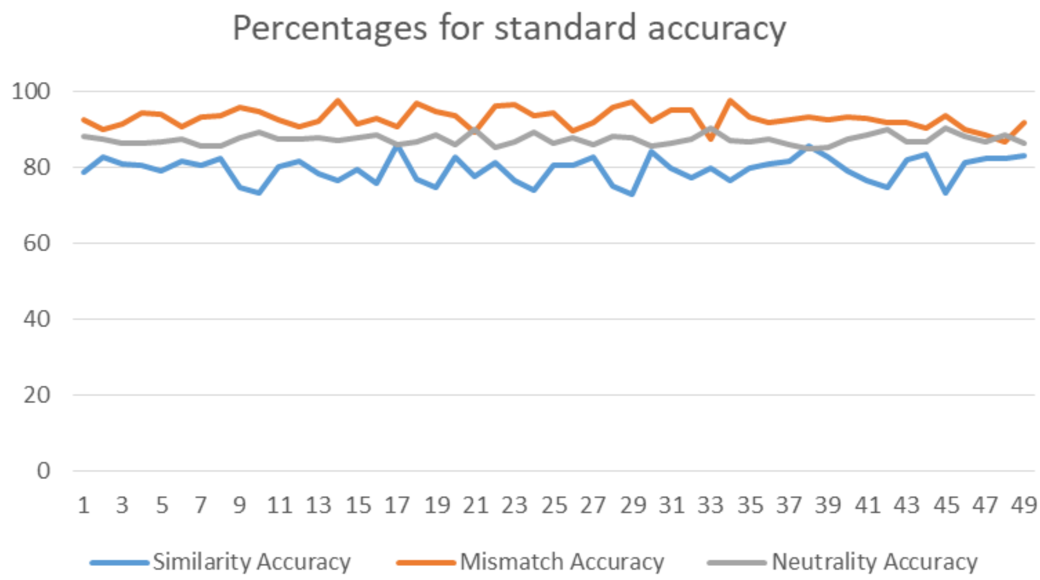


Figure 6. Accuracy standard of similarity, mismatch, and neutrality

5. Conclusion and future work

The neutrosophic has been applied to determine the measurement of similarity, dissimilarity, and neutrality, and the results have been good due to the experiment was highly fruitful since the accuracy standard for similarity and dissimilarity was measured. The similarity was encoded as "T" and dissimilarity as "F". Artificial intelligence techniques, such as deep learning and random forest, could be integrated with neutrosophic for optimal use in biomedical informatics. The paper presents the accuracy of standard similarity of the DNA sequence analysis indicating the mismatch account as well as the neutrality accuracy based on neutrosophic theory. As future work, more standard datasets can be used to represent more patients and cases in order to determine the suitable drug based on DNA sequence analysis and alignment.

References

1. Christine. (March 2011). Bioinformatics–The application of the different technologies for gathering and analyzing information. [Online]. Available: http://www.ismb2008.org/bioinformatics-the-application-of-the-differ ent-technologies-for-gathering-and-analyzing-information
2. P. E. Gary, W. A. Reynolds, and R. Reynolds, "DNA: The key to life," Programmed Biology Series, Educational Methods, Chicago, 1977.
3. Genetic Science Learning Center. (August 6, 2012). Tour of the Basics. Learn. Genetics. [Online]. Available: http://learn.genetics.utah.edu/ content/begin/tour
4. P. H. Searls and B. David, "The roots of bioinformatics in theoretical biology," PLoS Computational Biology, vol. 7, no. 3, 2011.
5. W. R. Pearson, T. Wood, Z. Zhang, and W. Miller, "Comparison of DNA sequences with protein sequences," Genomics, vol. 46, issue 1, pp. 24-36, 1997.
6. A. Elumalai, "An overview of bioinformatics," Journal of Science – Biological Science, vol. 2, Issue 2, pp. 71-80, 2012
7. Shamsi F, Memon ZA, Soomrani AR, Khand QU. Comparative Study of Different Computational Systems for Analysis of DNA and Protein Sequences. International Journal of Bioscience, Biochemistry and Bioinformatics. 2014 Mar 1;4(2):86.
8. Mount, D. W., "Bioinformatics: Sequence and genome analysis". Spring Harbor Press, (2002).
9. Whishart, D.S., "Tools for protein technologies", In: Sensen, C.W. (Ed.), Biotechnology, Vol 5b, Genomics and Bioinformatics, Wiley-VCH. pp. 325-344, 2002
10. Sander, J., Ester, M., Kriegel, P.-H. and Xu, X., "Density-based clustering in spatial databases: The algorithm GDBSCAN and its applications", Data Mining and Knowledge Discovery, Vol. 2, issue 2, pp. 169-194, 1998.
11. Lee, K., "Computational study for protein-protein docking using global optimization and empirical potentials", International Journal of Molecular Science, Vol. 9, pp. 65-77, 2008.
12. Katoh, Kazutaka, and Daron M. Standley. "MAFFT multiple sequence alignment software version 7: improvements in performance and usability." Molecular biology and evolution 30, no. 4 (2013): 772- 780.
13. Thompson, Julie D., Desmond G. Higgins, and Toby J. Gibson. "CLUSTAL W: improving the sensitivity of progressive multiple sequence alignment through sequence weighting, position-specific gap penalties and weight matrix choice." Nucleic acids research 22, no. 22 (1994): 4673-4680.
14. Katoh, Kazutaka, and Daron M. Standley. "MAFFT: iterative refinement and additional methods." Multiple Sequence Alignment Methods (2014): 131-146.
15. Notredame, Cédric, Desmond G. Higgins, and Jaap Heringa. "T-Coffee: A novel method for fast and accurate multiple sequence alignment." Journal of molecular biology 302, no. 1 (2000): 205-217.
16. Shehab S, Shohdy S, Keshk AE. Pomsa: An efficient and precise position-based multiple sequence alignment technique. arXiv preprint arXiv:1708.01508. 2017 Aug 3.
17. Huang K, Xu M, Qi X. NGMMs: Neutrosophic Gaussian Mixture Models for Breast Ultrasound Image Classification. In2021 43rd Annual International Conference of the IEEE Engineering in Medicine & Biology Society (EMBC) 2021 Nov 1 (pp. 3943-3947). IEEE.
18. Alshikho M, Jdid M, Broumi S. A Study of a Support Vector Machine Algorithm with an Orthogonal Legendre Kernel According to Neutrosophic logic and Inverse Lagrangian Interpolation. Journal of Neutrosophic and Fuzzy Systems (JNFS) Vol. 2023;5(01):41-51.
19. Pramanik S, Dey PP, Giri BC. Hybrid vector similarity measure of single valued refined neutrosophic sets to multi-attribute decision making problems. Infinite Study; 2018.
20. Chou JC, Lin YF, Lin SS. A further study on multiperiod health diagnostics methodology under a single-valued neutrosophic set. Computational and Mathematical Methods in Medicine. 2020 Aug 1;2020.

21. Bouza-Herrera CN, Subzar M. Estimating The Ratio Of A Crisp Variable And A Neutrosophic Variable. International Journal of Neutrosophic Science (IJNS) Volume 11, 2020.:9.
22. Abdullah, L., Ong, Z. and Mahali, S.M., 2021. Research Article Single-Valued Neutrosophic DEMATEL for Segregating Types of Criteria: A Case of Subcontractors' Selection.
23. Smarandache F. Refined neutrosophy and lattices vs. pair structures and YinYang bipolar fuzzy set. Mathematics. 2019 Apr 16;7(4):353.
24. Hajiabadi M, Alizadeh Savareh B, Emami H, Bashiri A. Comparison of wavelet transformations to enhance convolutional neural network performance in brain tumor segmentation. BMC Medical Informatics and Decision Making. 2021 Dec;21(1):1-1.
25. Al-subhi SH, Rubio PA, Pérez PP, Papageorgiou EI, Vacacela RG, Mahdi GS. A New Neutrosophic Clinical Decision Support Model for The Treatment of Pregnant Women with Heart Diseases. Infinite Study.
26. Yildiz I, Sahin M. A new approach for multi-attribute decision-making problems in bipolar neutrosophic sets. Neutrosophic Sets and Systems. 2018; 23:142-59.
27. Touqeer M, Rasool E. Multiple Attribute Decision Making Based on Interval-Valued Neutrosophic Trapezoidal Fuzzy Numbers and its Application in the Diagnosis of Viral Flu. International Journal of Emerging Multidisciplinaries: Mathematics. 2022 May 14;1(2):124-44.
28. Devarasan E. Neutrosophic Based Image Relationship Analysis for Fingerprint Image Pattern Recogniton and Matching. Available at SSRN 4047124.
29. Lin M, Wang H, Xu Z, Yao Z, Huang J. Clustering algorithms based on correlation coefficients for probabilistic linguistic term sets. International Journal of Intelligent Systems. 2018 Dec;33(12):2402-24.
30. Wu JL, Xiao X, Yu LC, Ye SZ, Lai KR. Using an analogical reasoning framework to infer language patterns for negative life events. BMC Medical Informatics and Decision Making. 2019 Dec;19(1):1-2.
31. Tan W, Weng H, Lin H, Ou A, He Z, Jia F. Disease risk analysis for schizophrenia patients by an automatic AHP framework. BMC Medical Informatics and Decision Making. 2021 Nov;21(9):1-2.
32. Ngan RT, Ali M, Fujita H, Abdel-Basset M, Giang NL, Manogaran G, Priyan MK. A new representation of intuitionistic fuzzy systems and their applications in critical decision making. IEEE Intelligent Systems. 2019 Aug 30;35(1):6-17.
33. Manea C, SAVOIU G, Siman II, TAICU M. Inter-, trans-and multidisciplinarity, and some ideas or remarks on cultural tourism. Anuarul Institutului de Cercetari Economice" Gheorghe Zane"-Iasi. 2014;23(Supplement):133.
34. Smarandache F. Neutrosophy, a new Branch of Philosophy. Infinite Study; 2002 Jan.
35. Smarandache F. A unifying field in Logics: Neutrosophic Logic. InPhilosophy 1999 (pp. 1-141). American Research Press.
36. Smarandache F. A unifying field in logics: neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability: neutrsophic logic. Neutrosophy, neutrosophic set, neutrosophic probability. Infinite Study; 2005.
37. Luscombe NM, Greenbaum D, Gerstein M. What is bioinformatics? A proposed definition and overview of the field. Methods of information in medicine. 2001;40(04):346-58.
38. Russell PH, Johnson RL, Ananthan S, Harnke B, Carlson NE. A large-scale analysis of bioinformatics code on GitHub. PloS one. 2018 Oct 31;13(10):e0205898.
39. Gaur L, Solanki A, Wamba SF, Jhanjhi NZ, editors. Advanced AI techniques and applications in bioinformatics. CRC Press; 2021 Oct 17.
40. Belal Amin, A. A. Salama, I. M. El-Henawy, Khaled Mahfouz, Mona G. Gafar, "Intelligent Neutrosophic Diagnostic System for Cardiotocography Data", Computational Intelligence and Neuroscience, vol. 2021, Article ID 6656770, 12 pages, 2021.
41. Ibrahim Yasser, Aya A. Abd El-Khalek, Abeer Twakol, Mohy-Eldin Abo-Elsoud, A. A. Salama, Fahmi Khalifa, A hybrid Automated Intelligent COVID-19 Classification System Based on Neutrosophic Logic and Machine Learning Techniques Using Chest X-ray Images, Studies in Systems, Decision and Control,

Advances in Data Science and Intelligent Data Communication Technologies for COVID-19, Vol.378, Springer Nature.

42. Kawther F. Alhasan, A.A. Salama, & Florentin Smarandache. (2021). Introduction to Neutrosophic Reliability Theory. International Journal of Neutrosophic Science, 15(1), 52–61.
43. Ibrahim Yasser, Abeer Twakol, A. A. Abd El-Khalek, Ahmed Samrah and A. A. Salama, COVID-X: Novel Health-Fog Framework Based on Neutrosophic Classifier for Confrontation Covid-19, Neutrosophic Sets and Systems, vol. 35, 2020, pp. 1-21.
44. A.A. Salama, Ahmed Sharaf Al-Din, Issam Abu Al-Qasim, Rafif Alhabib and Magdy Badran, Introduction to Decision Making for Neutrosophic Environment "Study on the Suez Canal Port, Egypt", Neutrosophic Sets and Systems, vol. 35, 2020, pp. 22-44.
45. Alhabib, Rafif, A. A. Salama. "The Neutrosophic Time Series-Study Its Models (Linear-Logarithmic) and test the Coefficients Significance of Its linear model." Neutrosophic Sets and Systems 33.1 (2020) pp105-115.
46. Rafif Alhabib, Ahmad Salama, Studying Neutrosophic Variables, Neutrosophic Theories in Communication, Management and Information Technology, Series: Mathematics Research Developments Mathematics,, ISBN: 978-1-53617-485-4, Categories: Nova, 2020.
47. Salama A.A., Eisa M., ElGhawalby H., Fawzy A.E. (2019) A New Approach in Content-Based Image Retrieval Neutrosophic Domain. In: Kahraman C., Otay İ. (eds) Fuzzy Multi-criteria Decision-Making Using Neutrosophic Sets. Studies in Fuzziness and Soft Computing, vol 369 (pp.361-369),Springer,Cham.
48. Elwahsh, H., Gamal, M., Salama, A., & El-Henawy, I. (2018). A novel approach for classifying MANETs attacks with a neutrosophic intelligent system based on genetic algorithm. Security and Communication Networks, vol.2018, pp1-7.
49. ElWahsh, H., Gamal, M., Salama, A., & El-Henawy, I. (2018). Intrusion detection system and neutrosophic theory for MANETs:A comparative study, Neutrosophic Sets and Systems, 23, pp16-22.
50. Salama, A., Smarandache, F., ElGhawalby, H. (2018). Neutrosophic approach to grayscale images domain. Neutrosophic Sets and Systems, 21, pp13-19.
51. A. A. Salama, Rafif Alhabib.(2018). Some neutrosophic probability distributions. Neutrosophic Sets and Systems, 22, 30-38.
52. Haitham ELwahsha , Mona Gamala, A. A. Salama, I.M. El-Henawy.(2017). Modeling Neutrosophic Data by Self-Organizing Feature Map: MANETs Data Case Study, Procida Computer, Vol.121, pp152-157, 2017.
53. A. Salama, Hewayda ElGhawalby, Shimaa Fathi Ali. (2017). Topological Manifold Space via Neutrosophic Crisp Set Theory, Neutrosophic Sets and Systems, Vol.15, 2017, pp.18-21.
54. A A Salama, Mohamed Eisa and A. E. Fawzy. (2017) A Neutrosophic Image Retrieval Classifier. International Journal of Computer Applications 170 (9):1-6, July 2017.
55. A. A. Salama, I. M. Hanafy, Hewayda Elghawalby, M. S. Dabash. (2016). Neutrosophic Crisp α-Topological Spaces, Neutrosophic Sets and Systems, vol. 12, 2016, pp. 92-96.
56. A. A. Salama. (2015). Basic Structure of Some Classes of Neutrosophic Crisp Nearly Open Sets & Possible Application to GIS Topology, Neutrosophic Sets and Systems, vol. 7, 2015, pp. 18-22.
57. A. A. Salama, F.Smarandache (2015). Neutrosophic Crisp Set Theory, Educational. Education Publishing 1313 Chesapeake, Avenue, Columbus, Ohio 43212, https://digitalrepository.unm.edu/math_fsp/153/
58. A. A. Salama, Florentin Smarandache, Valeri Kroumov.(2014). Neutrosophic Crisp Sets & Neutrosophic Crisp Topological Spaces, Neutrosophic Sets and Systems, vol. 2, 2014, pp.25-30. doi.org/10.5281/zenodo.571502
59. A.A. Salama, Haitham A. El-Ghareeb, Ayman M. Manie, Florentin Smarandache. (2014). Introduction to Develop Some Software Programs for Dealing with Neutrosophic Sets, Neutrosophic Sets and Systems, vol. 3, 2014, pp. 51-52.doi.org/10.5281/zenodo.571453,
60. A. A.Salama and S.A. Alblowi, Neutrosophic Set and Neutrosophic Topological Space, ISOR J. mathematics (IOSR-JM), Vol. (3). Issue (4), (Sep-Oct. 2012). pp 31-35.

61. Huda E. Khalid, "An Original Notion to Find Maximal Solution in the Fuzzy Neutrosophic Relation Equations (FNRE) with Geometric Programming (GP)", Neutrosophic Sets and Systems, vol. 7, 2015, pp. 3-7.
62. Huda E. Khalid, "The Novel Attempt for Finding Minimum Solution in Fuzzy Neutrosophic Relational Geometric Programming (FNRGP) with (max, min) Composition", Neutrosophic Sets and Systems, vol. 11, 2016, pp. 107-111.
63. Huda E. Khalid, F. Smarandache, A. K. Essa, (2018). The Basic Notions for (over, off, under) Neutrosophic Geometric Programming Problems. Neutrosophic Sets and Systems, 22, 50-62.
64. Huda E. Khalid, (2020). Geometric Programming Dealt with a Neutrosophic Relational Equations Under the ($\boldsymbol{max - min}$) Operation. Neutrosophic Sets in Decision Analysis and Operations Research, chapter four. IGI Global Publishing House.
65. Huda E. Khalid, "Neutrosophic Geometric Programming (NGP) with (max-product) Operator, An Innovative Model", Neutrosophic Sets and Systems, vol. 32, 2020.
66. Huda E. Khalid, F. Smarandache, A. K. Essa, (2016). A Neutrosophic Binomial Factorial Theorem with their Refrains. Neutrosophic Sets and Systems, 14, 50-62.
67. Huda E. Khalid, Ahmed K. Essa, The Duality Approach of the Neutrosophic Linear Programming, Neutrosophic Sets and Systems, vol. 46, 2021, pp. 9-23. DOI: 10.5281/zenodo.5553469

Received: Jan 7, 2024. Accepted: Mar 29, 2024

Neutrosophic Sets and Systems, Vol. 66, 2024

University of New Mexico

Neutrosophic Inventory System for Decaying Items with Price Dependent Demand

R. Surya[1], M. Mullai[2,*], G. Vetrivel[3]

[1]Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India; suryarrrm@gmail.com

[2,*]Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India; mullaim@alagappauniversity.ac.in

[3]Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India; menakagovindan@gmail.com

*Correspondence: mullaim@alagappauniversity.ac.in

Abstract. This study presents an empirical investigation of a specific problem in inventory control, notably the management of decaying commodities with demand that is influenced by price. The approach taken in this research incorporates the effective use of trapezoidal neutrosophic numbers. The present model is employed for the objective of finding the neutrosophic optimal total cost and neutrosophic optimal interval of time for the inventory system. Defuzzification process is done with the help of signed distance method. Moreover, a mathematical evaluation is conducted in order to assess the stated model, and the findings are elucidated through the use of a numerical illustration.

Keywords: Neutrosophic sets; Neutrosophic trapezoidal numbers; Neutrosophic total cost; Neutrosophic demand; Neutrosophic purchase cost; Signed distance method.

1. Introduction

Decay refers to the process by which products undergo damage, deterioration, or disintegration over time, resulting in a gradual depletion of value. This phenomenon is particularly relevant in the scenario of materials that are intended for storage and potential future use. So decaying cannot be avoided in any kind of business scenarios. In the vegetable business, it is difficult to predict the quantity that will be purchased. The surplus supply should always be maintained to make sure the customers get everything they ordered. However, there is always a portion of the quantity that will get wasted. This wastage happens by the time the vegetables are bought back to the other retail stores. To avoid this, we can compromise with those to-be wasted vegetables and sell them for 50-60% of their original price. Thus, we are avoiding the vegetables that are made by the exhaustive farming process.

The rate of decay has a positive correlation with the passage of time. Every cycle has shortages that have been partly accumulated. Kundu and Chakrabarti [18] introduced an Economic Order Quantity (EOQ) Model incorporating demand and partial backlogging of fuzzy type in the study they performed. The present model has been specifically developed to cater to the requirements of a continuous review inventory system that manages deteriorating products, while also considering the effects of time-dependent demand. The inventory model put forth by U. Sushil Kumar and S.Rajput [19] examines the management of deteriorating commodities, implementing factors such as time-dependent demand rates and partial backlogging. In their study, Dutta and Kumar [3] examined a fuzzy inventory model that involves the management of degrading products with shortages, specifically under the circumstance of a full backlog.

In a research study performed by M. Maragatham and P.K. Lakshmidevi [5], a fuzzy inventory model was presented to address the issue of deteriorating products with demand which is centered on price. In their study, Palani and Maragatham [10] proposed a fuzzy inventory model to address the management of time-dependent degrading products. The model takes into account factors such as lead time, stock-dependent demand rate, and shortages. In their study, D. Sharmila and R. Uthayakumar [14] developed an inventory model that utilizes fuzzy logic to handle deteriorating products, shortages, and exponential demand. The notions of neutrosophic set and neutrosophic logic were first put forward by Smarandache [15], who addressed it from the perspective of non-standard analysis. In their article, Mullai and Broumi [6] established a novel inventory model involving the concept of neutrosophy and neglects to account for shortages.

This paper is organised as follows:
In section 2 and 3, the basic definitions and a few assumptions and notations that are very beneficial to expand this proposed model are given. Section 4 deals with the development of inventory model for decaying items with price dependent demand using neutrosophic demand and neutrosophic price are represented by trapezoidal neutrosophic numbers. To defuzzify the model, signed distance method is used. Also the neutrosophic optimal total cost and the neutrosophic optimal interval of time for the plan are determined. Our proposed neutrosophic system is illustrated with a numerical example and also sensitivity analysis is taken. The results are observed and compared graphically in section 5.

2. **Preliminaries:**

In this section, the basic definitions involving intuitionistic fuzzy set, intuitionistic fuzzy number, neutrosophic set, single valued neutrosophic sets and trapezoidal neutrosophic number are outlined.

Definition:1 [11] **Intuitionistic fuzzy set**

Let X denotes the universal set. An intuitionistic fuzzy set(IFS) $\bar{A}$ in X is an object having the form $\bar{A} = \{< x, \mu_A(x), \nu_A(x)) > /x \in X\}$, where $\mu_{\bar{A}} : X \to [0,1]$ and $\nu_{\bar{A}} : X \to [0,1]$ represents the membership and non membership degree of the element x∈ X to the set $\bar{A}$(a subset of the set X), respectively. Thus, ∀x∈ X, we have $0 \leq \mu_{\bar{A}}(x) + \nu_{\bar{A}}(x) \leq 1$.

Definition:2 [11] **Intuitionistic fuzzy number**

An intuitionistic fuzzy number $\bar{A}$ preserves the following properties:
i) an intuitionistic fuzzy subset of the real line(R)
ii) normal, i.e., there is any $x_0 \in R$ such that $\mu_{\bar{A}}(x)$ =1
iii) a convex set for the membership function $\mu_{\bar{A}}(x)$
i.e., $\mu_{\bar{A}}[\lambda x_1 + (1-\lambda)x_2] \geq min\{\mu_{\bar{A}}(x_1), \mu_{\bar{A}}(x_2), x_1, x_2 \in R\}, \lambda \in [0,1]$
iv) a concave set for the non membership function $\nu_{\bar{A}}(x)$
i.e., $\nu_{\bar{A}}[\lambda x_1 + (1-\lambda)x_2] \leq max\{\nu_{\bar{A}}(x_1), \nu_{\bar{A}}(x_2), x_1, x_2 \in R\}, \lambda \in [0,1]$

Definition:3 [13] **Trapezoidal intuitionistic fuzzy number**

A trapezoidal intuitionistic fuzzy number $\bar{A} = (a_1, b_1, c_1, d_1; a_1', b_1, c_1, d_1')$ is a subset of IFS on the set of R whose membership and non membership are structured as follows:

$$\mu_{\tilde{A}} = \begin{cases} \frac{x-a_1}{b_1-a_1}, & \text{when } a_1 \leq x \leq b_1 \\ 1, & \text{when } b_1 \leq x \leq c_1 \\ \frac{d_1-x}{d_1-c_1}, & \text{when } c_1 \leq x \leq d_1 \\ 0, & \text{otherwise} \end{cases} \qquad \nu_{\tilde{A}} = \begin{cases} \frac{b_1-x}{b_1-a_1}, & \text{when } a_1' \leq x \leq b_1 \\ 0, & \text{when } c_1 \leq x \leq b_1 \\ \frac{x-c_1}{d_1-c_1}, & \text{when } c_1 \leq x \leq d_1' \\ 1, & \text{otherwise} \end{cases}$$

Definition:4 [4] **(Neutrosophic set)**

Consider X as an universal set. A neutrosophic set A in X is distinguished by a truth T_A, an indeterminacy I_A and falsity-membership function F_A, where $I_A(x)$ and $F_A(x)$ are real standard elements of [0,1]. It is defined as A={⟨ x, T_A(x), $I_A(x)$, $F_A(x)$ ⟩: x∈ X, T_A(x), $I_A(x)$, $F_A(x) \in\ ]0^-, 1^+[$ } and also restriction not allowed on the sum of T_A(x), $I_A(x)$ and $F_A(x)$. So, $0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition:5 [4] **(Single valued neutrosophic set)**

Consider X to be the universal set. A single valued neutrosophic set A in X is distinguished by a truth T_A, indeterminacy I_A and falsity-membership function F_A, where T_A(x), I_A(x) and F_A(x) are real standard elements of [0,1]. It is defined in the following way, A={$\langle$ x, T_A(x), $I_A(x)$, F$_A(x)$ $\rangle$:x∈ X,T_A(x), $I_A(x)$, $F_A(x)$ ∈ [0, 1] } and restriction not defined on the sum of T_A(x), I_A(x) and F_A(x). Here, we have 0≤ T_A(x)+I_A(x) + F_A(x) ≤ 3.

Definition:6 [4] **(Trapezoidal neutrosophic number)**

A single valued trapezoidal neutrosophic number $\tilde{a}$ = $\langle(a_1, b_1, c_1, d_1); \mathrm{w}_{\tilde{a}}, \mathrm{u}_{\tilde{a}}, \mathrm{y}_{\tilde{a}}\rangle$ is a unique set of neutrosophic type on the real number set R, whose truth, indeterminacy, and falsity membership are defined as follows:

$$\mu\tilde{a}(x) = \begin{cases} (x-a_1)\mathrm{w}_{\tilde{a}}/(b_1-a_1), & \text{if } a_1 \le x < b_1 \\ \mathrm{w}_{\tilde{a}}, & \text{if } b_1 \le x \le c_1 \\ (d_1-x)\mathrm{w}_{\tilde{a}}/(d_1-c_1), & \text{if } c_1 < x \le d_1 \\ 0, & \text{if } otherwise \end{cases}$$

$$\nu\tilde{a}(x) = \begin{cases} (b_1-x+(x-a_1)\mathrm{u}_{\tilde{a}})/(b_1-a_1), & \text{if } a_1 \le x < b_1 \\ \mathrm{u}_{\tilde{a}}, & \text{if } b_1 \le x \le c_1 \\ (x-c_1+(d_1-x)\mathrm{u}_{\tilde{a}})/(d_1-c_1), & \text{if } c_1 < x \le d_1 \\ 1, & \text{if } otherwise \end{cases}$$

$$\lambda\tilde{a}(x) = \begin{cases} (b_1-x+(x-a_1)\mathrm{y}_{\tilde{a}})/(b_1-a_1), & \text{if } a_1 \le x < b_1 \\ \mathrm{y}_{\tilde{a}}, & \text{if } b_1 \le x \le c_1 \\ (x-c_1+(d_1-x)\mathrm{y}_{\tilde{a}})/(d_1-c_1), & \text{if } c_1 < x \le d_1 \\ 1, & \text{if } otherwise \end{cases}$$

respectively.

A positive single-valued trapezoidal neutrosophic number, represented by $\tilde{a} > 0$, is defined as $\tilde{a} = \langle(a_1, b_1, c_1, d_1); \mathrm{w}_{\tilde{a}}, \mathrm{u}_{\tilde{a}}, \mathrm{y}_{\tilde{a}}\rangle$, where $a_1 \geq 0$ and at least $d_1 > 0$. Similarly, in the case when d_1 is less than or equal to 0 and at least a_1 is less than 0, the representation $\tilde{a} = \langle(a_1, b_1, c_1, d_1); \mathrm{w}_{\tilde{a}}, \mathrm{u}_{\tilde{a}}, \mathrm{y}_{\tilde{a}}\rangle$ is referred to as a negative single-valued trapezoidal neutrosophic number, which may be expressed as $\tilde{a}$ <0. A trapezoidal neutrosophic number, denoted as $\tilde{a} = \langle(a_1, b_1, c_1, d_1); \mathrm{w}_{\tilde{a}}, \mathrm{u}_{\tilde{a}}, \mathrm{y}_{\tilde{a}}\rangle$, may represent an uncertain amount within a certain range, roughly corresponding to the interval $[b_1, c_1]$.

Definition:7 [3] **(Signed distance method)**

The signed distance of $\tilde{A}$(a fuzzy set defined on R) is defined as:

$$d(\tilde{A}, 0) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha)]d\alpha$$

where

$$A_\alpha = [A_L(\alpha), A_R(\alpha)]$$
$$= [a + (b-a)\alpha, d - (d-c)\alpha], \alpha\epsilon[0,1],$$

is α-cut of fuzzy set $\tilde{A}$, which is a closed interval.

Remark: For every $a, 0 \in R$, the signed distance $d(a,0) = a$. Definition 4's criteria are as follows: $d(a,0) = a$ is the distance between a and 0 if $0 < a$. The distance $-d(a,0) = -a$ exists between a and 0 if $a < 0$. As a result, we state that the signed distance between a and 0 is $d(a,0) = a$.

3. Notations and Assumption

3.1. *Notations:*

$q^I(t)$ = The intuitionistic fuzzy inventory level at time t^I

d^I = The intuitionistic fuzzy demand rate

α_1^I = The intuitionistic fuzzy decaying rate in $[0, t_1^I]$

M^I = The intuitionistic fuzzy shortage level of inventory

R^I = The intuitionistic fuzzy inventory level at t^I=0

Q^I = Lot size of intuitionistic fuzzy type

t_1^I = The interval of time at which the inventory attains zero

t_2^I = The interval of time at which the shortages are allowed

T^I = The intuitionistic fuzzy time length of the plan

C_h^I = Holding cost per unit item of intuitionistic fuzzy type

C_s^I = The shortage cost per unit item of intuitionistic fuzzy type

P^I = Purchase cost per unit item of intuitionistic fuzzy type

C_o^I = The intuitionistic fuzzy ordering cost per order

$F^I(q)$ = Defuzzified intuitionistic fuzzy total cost

$(TC)^I$ = The intuitionistic fuzzy total cost for the period $[0,T^I]$

(TC) = The total cost for the period [0,T]

$(TC)^F$ = The fuzzy total cost for the period $[0,T^F]$

$q^N(t)$ = The neutrosophic inventory level at time t^N

d^N = The neutrosophic demand rate

α_1^N = The neutrosophic decaying rate in $[0,t_1^N]$

M^N = The neutrosophic shortage level of inventory

R^N = The neutrosophic inventory level at t^N=0

Q^N = The neutrosophic lot size

T^N = The neutrosophic time length of the plan

C_h^N = The neutrosophic holding cost per unit item

C_s^N = The neutrosophic shortage cost per unit item

P^N = The neutrosophic purchase cost per unit item

C_o^N = The neutrosophic ordering cost per order

$(TC)^N$ = The neutrosophic total cost for the period $[0,T^N]$

$F^N(q)$ = Defuzzified neutrosophic total cost

3.2. *Assumptions:*

- The neutrosophic demand is related to the unit price as d^N =A(p$^{\beta^N}$) where A > 0, $0 < \beta < 1$.
- The neutrosophic shortages are allowed.
- During the cycle deterioration is not repaired or replaced.
- The neutrosophic purchase cost and neutrosophic demand are considered.
- The neutrosophic lead time is considered to be zero.
- The neutrosophic decay will be instantaneous.

4. Model Description:

This section illustrates the intuitionistic fuzzy model and the neutrosophioc model for finding the optimal total cost and optimal time length for the inventory system.

Intuitionistic fuzzy model:

The model presented in this study demonstrates Intuitionistic fuzzy purchasing cost and Intuitionistic fuzzy demand via trapezoidal Intuitionistic fuzzy numbers.

(ie) P^I = $(p_1^I, p_2^I, p_3^I, p_4^I)(p_1'^I, p_2'^I, p_3'^I, p_4'^I)$

D^I = $(D_1^I, D_2^I, D_3^I, D_4^I)(D_1'^I, D_2'^I, D_3'^I, D_4'^I)$

Based on [5], intuitionistic fuzzy total cost is defined as follows:

$$(TC)^I = P^IQ^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{D^I}{\alpha_1^I})(1 - e^{-\frac{\alpha_1^I(Q^I-M^I)}{(D^I+\alpha_1^I)}}) - \frac{D^I(Q^I-M^I)}{\alpha_1^I(D^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2D^I} + d^I(Q^I - M^I)(1 - \frac{D^I}{D^I+\alpha_1^I})$$

$$= (p_1^I, p_2^I, p_3^I, p_4^I, p_1'^I, p_2'^I, p_3'^I, p_4'^I)Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{D_1^I}{\alpha_1^I}, Q^I - M^I + \frac{D_2^I}{\alpha_1^I}, Q^I - M^I + \frac{D_3^I}{\alpha_1^I}, Q^I - M^I + \frac{D_4^I}{\alpha_1^I}, Q^I - M^I + \frac{D_1'^I}{\alpha_1^I}, Q^I - M^I + \frac{D_2'^I}{\alpha_1^I}, Q^I - M^I + \frac{D_3'^I}{\alpha_1^I}, Q^I - M^I + \frac{D_4'^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}((Q^I - M^I + \frac{D_1^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{(D_1^I+\alpha_1^I)}}),$$

$$(Q^I - M^I + \frac{D_2^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{(D_2^I+\alpha_1^I)}}), (Q^I - M^I + \frac{D_3^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{(D_3^I+\alpha_1^I)}}), (Q^I - M^I + \frac{D_4^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{(D_4^I+\alpha_1^I)}}), (Q^I - M^I + \frac{D_1'^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{(D_1'^I+\alpha_1^I)}}), (Q^I - M^I + \frac{D_2'^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{(D_2'^I+\alpha_1^I)}}), (Q^I - M^I + \frac{D_3'^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{(D_3'^I+\alpha_1^I)}}), (Q^I - M^I + \frac{D_4'^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{(D_4'^I+\alpha_1^I)}}) -$$

$(\frac{D_1^I(Q^I-M^I)}{\alpha_1^I(D_1^I+\alpha_1^I)}, \frac{D_2^I(Q^I-M^I)}{\alpha_1^I(D_2^I+\alpha_1^I)}, \frac{D_3^I(Q^I-M^I)}{\alpha_1^I(D_3^I+\alpha_1^I)}, \frac{D_4^I(Q^I-M^I)}{\alpha_1^I(D_4^I+\alpha_1^I)}, \frac{D_1'^I(Q^I-M^I)}{\alpha_1^I(D_1'^I+\alpha_1^I)}, \frac{D_2'^I(Q^I-M^I)}{\alpha_1^I(D_2'^I+\alpha_1^I)}, \frac{D_3'^I(Q^I-M^I)}{\alpha_1^I(D_3'^I+\alpha_1^I)} \frac{D_4'^I(Q^I-M^I)}{\alpha_1^I(D_4'^I+\alpha_1^I)})]+$
$(\frac{C_s^I M^{2^I}}{2D_1^I},$
$\frac{C_s^I M^{2^I}}{2D_2^I}, \frac{C_s^I M^{2^I}}{2D_3^I}, \frac{C_s^I M^{2^I}}{2D_4^I}, \frac{C_s^I M^{2^I}}{2D_1'^I}, \frac{C_s^I M^{2^I}}{2D_2'^I}, \frac{C_s^I M^{2^I}}{2D_3'^I}, \frac{C_s^I M^{2^I}}{2D_4'^I}) + d^I(Q^I - M^I)(1 -$
$(\frac{D_1^I}{D_1^I+\alpha_1^I}, \frac{D_2^I}{D_2^I+\alpha_1^I}, \frac{D_3^I}{D_3^I+\alpha_1^I}, \frac{D_4^I}{D_4^I+\alpha_1^I}, \frac{D_1'^I}{D_1'^I+\alpha_1^I},$
$\frac{D_2'^I}{D_2'^I+\alpha_1^I}, \frac{D_3'^I}{D_3'^I+\alpha_1^I}, \frac{D_4'^I}{D_4'^I+\alpha_1^I})) = (a^I, b^I, c^I, d^I)(a'^I, b'^I, c'^I, d'^I),$

where

$$a^I = p_1^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_1^\beta)^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_4^\beta)^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_4^\beta)^I+\alpha_1^I)}}) + \frac{(Ap_4^\beta)^I(Q^I-M^I)}{\alpha_1^I((Ap_1^\beta)^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_4^\beta)^I} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_4^\beta)^I}{(Ap_1^\beta)^I+\alpha_1^I}]$$

$$b^I = p_2^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_2^\beta)^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_3^\beta)^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_3^\beta)^I+\alpha_1^I)}}) + \frac{(Ap_3^\beta)^I(Q^I-M^I)}{\alpha_1^I((Ap_2^\beta)^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_3^\beta)^I} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_3^\beta)^I}{(Ap_2^\beta)^I+\alpha_1^I}]$$

$$c^I = p_3^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_3^\beta)^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_2^\beta)^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_2^\beta)^I+\alpha_1^I)}}) + \frac{(Ap_2^\beta)^I(Q^I-M^I)}{\alpha_1^I((Ap_3^\beta)^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_2^\beta)^I} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_2^\beta)^I}{(Ap_3^\beta)^I+\alpha_1^I}]$$

$$d^I = p_4^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_4^\beta)^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_1^\beta)^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_1^\beta)^I+\alpha_1^I)}}) + \frac{(Ap_1^\beta)^I(Q^I-M^I)}{\alpha_1^I((Ap_4^\beta)^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_1^\beta)^I} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_1^\beta)^I}{(Ap_4^\beta)^I+\alpha_1^I}]$$

$$a'^I = p_1'^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_1^\beta)'^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_4^\beta)'^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_4^\beta)'^I+\alpha_1^I)}}) + \frac{(Ap_4^\beta)'^I(Q^I-M^I)}{\alpha_1^I((Ap_1^\beta)'^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_4^\beta)'^I} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_4^\beta)'^I}{(Ap_1^\beta)'^I+\alpha_1^I}]$$

$$b'^I = p_2'^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_2^\beta)'^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_3^\beta)'^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_3^\beta)'^I+\alpha_1^I)}}) + \frac{(Ap_3^\beta)'^I(Q^I-M^I)}{\alpha_1^I((Ap_2^\beta)'^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_3^\beta)'^I} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_3^\beta)'^I}{(Ap_2^\beta)'^I+\alpha_1^I}]$$

$$c'^I = p_3'^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_3^\beta)'^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_2^\beta)'^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_2^\beta)'^I+\alpha_1^I)}}) + \frac{(Ap_2^\beta)'^I(Q^I-M^I)}{\alpha_1^I((Ap_3^\beta)'^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_2^\beta)'^I} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_2^\beta)'^I}{(Ap_3^\beta)'^I+\alpha_1^I}]$$

$$d'^I = p_4'^I Q^I + C_o^I + C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_4^\beta)'^I}{\alpha_1^I})] - C_h^I[\frac{1}{\alpha_1^I}(Q^I - M^I + \frac{(Ap_1^\beta)'^I}{\alpha_1^I})e^{-\frac{\alpha_1^I(Q^I-M^I)}{((Ap_1^\beta)'^I+\alpha_1^I)}}) + \frac{(Ap_1^\beta)'^I(Q^I-M^I)}{\alpha_1^I((Ap_4^\beta)'^I+\alpha_1^I)}] + \frac{C_s^I M^{2^I}}{2(Ap_1^\beta)'^I} + d^I(Q^I - M^I) - d[\frac{(Q^I-M^I)(Ap_1^\beta)'^I}{(Ap_4^\beta)'^I+\alpha_1^I}]$$

The defuzzified intuitionistic fuzzy total cost using signed distance method is given by

$$\text{d}((\text{TC})^I, 0) = \frac{1}{2}[[(p_1^I + p_4^I) + (p_1'^I + p_4'^I)]Q^I + 2C_o^I + C_h^I[\frac{1}{\alpha_1^I}\{(2Q^I - 2M^I + \frac{A[(p_1^{\beta^I} + p_4^{\beta^I}) + (p_1^{\beta'^I} + p_4^{\beta'^I})]}{\alpha_1^I})\}] - C_h^I[\frac{1}{\alpha_1^I}\{(Q^I - M^I + \frac{A}{\alpha_1^I}(p_4^{\beta^I} + p_4^{\beta'^I}))e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_4^{\beta^I}+p_4^{\beta'^I})+\alpha_1^I}} + (Q^I - M^I + \frac{A}{\alpha_1^I}(p_1^{\beta^I} + p_1^{\beta'^I}))e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_1^{\beta^I}+p_1^{\beta'^I})+\alpha_1^I}} + \frac{A(Q^I-M^I)}{\alpha_1^I}\{(\frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I} + \frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I}) + (\frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I} + \frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I})\}\}] + \frac{C_s^I M^{2^I}}{2A}[(\frac{1}{p_4^{\beta^I}} + \frac{1}{p_1^{\beta^I}}) + (\frac{1}{p_4^{\beta'^I}} + \frac{1}{p_1^{\beta'^I}})] - d(Q^I - M^I)A[(\frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I} + \frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I}) + (\frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I} + \frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I})] + 2d^I(Q^I - M^I)]$$
$$+\frac{1}{4}[[(p_2^I - p_1^I) + (p_2'^I + p_1'^I)]Q^I + C_h^I[\frac{1}{\alpha_1^I}\{(\frac{A}{\alpha_1^I}[(p_2^{\beta^I} - p_1^{\beta^I}) + (p_2^{\beta'^I} - p_1^{\beta'^I})])\}] - C_h^I[\frac{1}{\alpha_1^I}\{(Q^I - M^I + \frac{A}{\alpha_1^I}(p_3^{\beta^I} + p_3^{\beta'^I}))e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_3^{\beta^I}+p_3^{\beta'^I})+\alpha_1^I}} - (Q^I - M^I + \frac{A}{\alpha_1^I}(p_4^{\beta^I} + p_4^{\beta'^I}))e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_4^{\beta^I}+p_4^{\beta'^I})+\alpha_1^I}} + \frac{A(Q^I-M^I)}{\alpha_1^I}\{(\frac{p_3^{\beta^I}}{Ap_2^{\beta^I}+\alpha_1^I} - \frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I}) + (\frac{p_3^{\beta'^I}}{Ap_2^{\beta'^I}+\alpha_1^I} - \frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I})\}\}] + \frac{C_s^I M^{2^I}}{2A}[(\frac{1}{p_3^{\beta^I}} - \frac{1}{p_4^{\beta^I}}) + (\frac{1}{p_3^{\beta'^I}} - \frac{1}{p_4^{\beta'^I}})] - d^I(Q^I - M^I)A[(\frac{p_3^{\beta^I}}{Ap_2^{\beta^I}+\alpha_1^I} - \frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I}) + (\frac{p_3^{\beta'^I}}{Ap_2^{\beta'^I}+\alpha_1^I} - \frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I})]$$
$$-\frac{1}{4}[[(p_4^I - p_3^I) + (p_4'^I + p_3'^I)]Q^I + C_h^I[\frac{1}{\alpha_1^I}\{(\frac{A}{\alpha_1^I}[(p_4^{\beta^I} - p_3^{\beta^I}) + (p_4^{\beta'^I} - p_3^{\beta'^I})])\}] - C_h^I[\frac{1}{\alpha_1^I}\{(Q^I - M^I + \frac{A}{\alpha_1^I}(p_1^{\beta^I} + p_1^{\beta'^I}))e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_1^{\beta^I}+p_1^{\beta'^I})+\alpha_1^I}} - (Q^I - M^I + \frac{A}{\alpha_1^I}(p_2^{\beta^I} + p_2^{\beta'^I}))e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_2^{\beta^I}+p_2^{\beta'^I})+\alpha_1^I}} + \frac{A(Q^I-M^I)}{\alpha_1^I}\{(\frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I} - \frac{p_2^{\beta^I}}{Ap_3^{\beta^I}+\alpha_1^I}) + (\frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I} - \frac{p_2^{\beta'^I}}{Ap_3^{\beta'^I}+\alpha_1^I})\}\}] + \frac{C_s^I M^{2^I}}{2A}[(\frac{1}{p_1^{\beta^I}} - \frac{1}{p_2^{\beta^I}}) + (\frac{1}{p_1^{\beta'^I}} - \frac{1}{p_2^{\beta'^I}})] - d^I(Q^I - M^I)A[(\frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I} - \frac{p_2^{\beta^I}}{Ap_3^{\beta^I}+\alpha_1^I}) + (\frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I} - \frac{p_2^{\beta'^I}}{Ap_3^{\beta'^I}+\alpha_1^I})] = \text{F}^I(\text{q})$$

To find the minimum of $D(F^I(q))$ by taking the derivative $D(F^I(q))$, we get

$$\frac{d(F^I(q))}{dM} = \frac{1}{2}[-C_h^I[\frac{2}{\alpha_1^I} + \frac{1}{\alpha_1^I}\{(Q^I - M^I + \frac{A}{\alpha_1^I}(p_4^{\beta^I} + p_4^{\beta'^I}))(\frac{\alpha_1^I}{A(p_4^{\beta^I}+p_4^{\beta''^I})+\alpha_1^I})e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_4^{\beta^I}+p_4^{\beta'^I})+\alpha_1^I}} - e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_4^{\beta^I}+p_4^{\beta'^I})+\alpha_1^I}} + (Q^I - M^I + \frac{A}{\alpha_1^I}(p_1^{\beta^I} + p_1^{\beta'^I}))(\frac{\alpha_1^I}{A(p_1^{\beta^I}+p_1^{\beta'^I})+\alpha_1^I})e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_1^{\beta^I}+p_1^{\beta'^I})+\alpha_1^I}} - e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_1^{\beta^I}+p_1^{\beta'^I})+\alpha_1^I}} -$$

$\frac{A}{\alpha_1^I}\{(\frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I} + \frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I}) + (\frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I} + \frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I})\}] + \frac{C_s^I M^I}{A}[(\frac{1}{p_4^{\beta^I}} + \frac{1}{p_1^{\beta^I}}) + (\frac{1}{p_4^{\beta'^I}} + \frac{1}{p_1^{\beta'^I}})] + dA[(\frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I} + \frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I}) + (\frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I} + \frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I})] - 2d^I]$

$+\frac{1}{4}[-C_h^I(\frac{1}{\alpha_1^I}\{(Q^I - M^I + \frac{A}{\alpha_1^I}(p_3^{\beta^I} + p_3^{\beta'^I}))(\frac{\alpha_1^I}{A(p_3^{\beta^I}+p_3^{\beta''^I})+\alpha_1^I})e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_3^{\beta^I}+p_3^{\beta'^I})+\alpha_1^I}} - e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_3^{\beta^I}+p_3^{\beta'^I})+\alpha_1^I}} - (Q^I - M^I + \frac{A}{\alpha_1^I}(p_4^{\beta^I} + p_4^{\beta'^I}))(\frac{\alpha_1^I}{A(p_4^{\beta^I}+p_4^{\beta'^I})+\alpha_1^I})e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_4^{\beta^I}+p_4^{\beta'^I})+\alpha_1^I}} + e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_4^{\beta^I}+p_4^{\beta'^I})+\alpha_1^I}} - \frac{A}{\alpha_1^I}\{(\frac{p_3^{\beta^I}}{Ap_2^{\beta}+\alpha_1^I} - \frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I}) + (\frac{p_3^{\beta'^I}}{Ap_2^{\beta'^I}+\alpha_1^I} - \frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I})\}] + \frac{C_s^I M^I}{A}[(\frac{1}{p_3^{\beta^I}} - \frac{1}{p_4^{\beta^I}}) + (\frac{1}{p_3^{\beta'^I}} - \frac{1}{p_4^{\beta'^I}})] + d^I A[(\frac{p_3^{\beta^I}}{Ap_2^{\beta^I}+\alpha_1^I} - \frac{p_4^{\beta^I}}{Ap_1^{\beta^I}+\alpha_1^I}) + (\frac{p_3^{\beta'^I}}{Ap_2^{\beta'^I}+\alpha_1^I} - \frac{p_4^{\beta'^I}}{Ap_1^{\beta'^I}+\alpha_1^I})]$

$-\frac{1}{4}[-C_h^I[\frac{1}{\alpha_1^I}\{(Q^I - M^I + \frac{A}{\alpha_1^I}(p_1^{\beta^I} + p_1^{\beta'^I}))(\frac{\alpha_1^I}{A(p_1^{\beta^I}+p_1^{\beta''^I})+\alpha_1^I})e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_1^{\beta^I}+p_1^{\beta'^I})+\alpha_1^I}} - e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_1^{\beta^I}+p_1^{\beta'^I})+\alpha_1^I}} - (Q^I - M^I + \frac{A}{\alpha_1^I}(p_2^{\beta^I} + p_2^{\beta'^I}))(\frac{\alpha_1^I}{A(p_2^{\beta^I}+p_2^{\beta'^I})+\alpha_1^I})e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_2^{\beta^I}+p_2^{\beta'^I})+\alpha_1^I}} + e^{\frac{-\alpha_1^I(Q^I-M^I)}{A(p_2^{\beta^I}+p_2^{\beta'^I})+\alpha_1^I}} - \frac{A}{\alpha_1^I}\{(\frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I} - \frac{p_2^{\beta^I}}{Ap_3^{\beta^I}+\alpha_1^I}) + (\frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I} - \frac{p_2^{\beta'^I}}{Ap_3^{\beta'^I}+\alpha_1^I})\}] + \frac{C_s^I M^I}{A}[(\frac{1}{p_2^{\beta^I}} - \frac{1}{p_4^{\beta^I}}) + (\frac{1}{p_1^{\beta'^I}} - \frac{1}{p_2^{\beta'^I}})] + d^I A[(\frac{p_1^{\beta^I}}{Ap_4^{\beta^I}+\alpha_1^I} - \frac{p_2^{\beta^I}}{Ap_3^{\beta^I}+\alpha_1^I}) + (\frac{p_1^{\beta'^I}}{Ap_4^{\beta'^I}+\alpha_1^I} - \frac{p_2^{\beta'^I}}{Ap_3^{\beta'^I}+\alpha_1^I})]$

Neutrosophic model:

In this specific type of model, the trapezoidal neutrosophic numbers have been used to describe the neutrosophic purchasing cost and neutrosophic demand.

(ie) $P^N = (p_1^N, p_2^N, p_3^N, p_4^N)(p_1'^N, p_2'^N, p_3'^N, p_4'^N)(p_1''^N, p_2''^N, p_3''^N, p_4''^N)$

$D^N = (D_1^N, D_2^N, D_3^N, D_4^N)(D_1'^N, D_2'^N, D_3'^N, D_4'^N)(D_1''^N, D_2''^N, D_3''^N, D_4''^N)$

Based on [5], neutrosophic total cost is defined as follows:

$(TC)^N = P^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{D^N}{\alpha_1^N})(1 - e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D^N+\alpha_1^N)}}) - \frac{D^N(Q^N-M^N)}{\alpha_1^N(D^N+\alpha_1^N)}] + \frac{C_s^N M^{2N}}{2D^N} + d^N(Q^N - M^N)(1 - \frac{D^N}{D^N+\alpha_1^N})$

$= (p_1^N, p_2^N, p_3^N, p_4^N, p_1'^N, p_2'^N, p_3'^N, p_4', p_N''^N, p_2''^N, p_3''^N, p_4''^N)Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{D_1^N}{\alpha_1^N}, Q^N - M^N + \frac{D_2^N}{\alpha_1^N}, Q^N - M^N + \frac{D_3^N}{\alpha_1^N}, Q^N - M^N + \frac{D_4^N}{\alpha_1^N}, Q^N - M^N + \frac{D_1'^N}{\alpha_1^N}, Q^N - M^N + \frac{D_2'^N}{\alpha_1^N}, Q^N - M^N + \frac{D_3'^N}{\alpha_1^N}, Q^N - M^N + \frac{D_4'^N}{\alpha_1^N} + \frac{D_1''^N}{\alpha_1^N}, Q^N - M^N + \frac{D_2''^N}{\alpha_1^N}, Q^N - M^N + \frac{D_3''^N}{\alpha_1^N}, Q^N - M^N + \frac{D_4''^N}{\alpha_1^N})] -$

$C_h^N[\frac{1}{\alpha_1^N}((Q^N - M^N + \frac{D_1^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_1^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_2^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_2^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_3^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_3^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_4^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_4^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_1'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_1'^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_2'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_2'^N+\alpha_1^N)}}),$

$(Q^N - M^N + \frac{D_3'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_3'^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_4'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_4'^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_1''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_1''^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_2''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_2''^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_3''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_3''^N+\alpha_1^N)}}), (Q^N - M^N + \frac{D_4''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{(D_4''^N+\alpha_1^N)}}) - (\frac{D_1^N(Q^N-M^N)}{\alpha_1^N(D_1^N+\alpha_1^N)}, \frac{D_2^N(Q^N-M^N)}{\alpha_1^N(D_2^N+\alpha_1^N)},$
$\frac{D_3^N(Q^N-M^N)}{\alpha_1^N(D_3^N+\alpha_1^N)} \frac{D_4^N(Q^N-M^N)}{\alpha_1^N(D_4^N+\alpha_1^N)}, \frac{D_1'^N(Q^N-M^N)}{\alpha_1^N(D_1'^N+\alpha_1^N)}, \frac{D_2'^N(Q^N-M^N)}{\alpha_1^N(D_2'^N+\alpha_1^N)}, \frac{D_3'^N(Q^N-M^N)}{\alpha_1^N(D_3'^N+\alpha_1^N)}, \frac{D_4'^N(Q^N-M^N)}{\alpha_1^N(D_4'^N+\alpha_1^N)}, \frac{D_1''^N(Q^N-M^N)}{\alpha_1^N(D_1''^N+\alpha_1^N)}, \frac{D_2''^N(Q^N-M^N)}{\alpha_1^N(D_2''^N+\alpha_1^N)}, \frac{D_3''^N(Q^N-M^N)}{\alpha_1^N(D_3''^N+\alpha_1^N)},$
$\frac{D_4''^N(Q^N-M^N)}{\alpha_1^N(D_4''^N+\alpha_1^N)})] +$
$(\frac{C_s^N M^{2^N}}{2D_1^N}, \frac{C_s^N M^{2^N}}{2D_2^N}, \frac{C_s^N M^{2^N}}{2D_3^N}, \frac{C_s^N M^{2^N}}{2D_4^N}, \frac{C_s^N M^{2^N}}{2D_1'^N}, \frac{C_s^N M^{2^N}}{2D_2'^N}, \frac{C_s^N M^{2^N}}{2D_3'^N}, \frac{C_s^N M^{2^N}}{2D_4'^N}, \frac{C_s^N M^{2^N}}{2D_1''^N}, \frac{C_s^N M^{2^N}}{2D_2''^N}, \frac{C_s^N M^{2^N}}{2D_3''^N}, \frac{C_s^N M^{2^N}}{2D_4''^N})+$
$d^N(Q^N - M^N)(1 -$
$(\frac{D_1^N}{D_1^N+\alpha_1^N}, \frac{D_2^N}{D_2^N+\alpha_1^N}, \frac{D_3^N}{D_3^N+\alpha_1^N}, \frac{D_4^N}{D_4^N+\alpha_1^N}, \frac{D_1'^N}{D_1'^N+\alpha_1^N}, \frac{D_2'^N}{D_2'^N+\alpha_1^N}, \frac{D_3'^N}{D_3'^N+\alpha_1^N}, \frac{D_4'^N}{D_4'^N+\alpha_1^N}, \frac{D_1''^N}{D_1''^N+\alpha_1^N}, \frac{D_2''^N}{D_2''^N+\alpha_1^N}, \frac{D_3''^N}{D_3''^N+\alpha_1^N}, \frac{D_4''^N}{D_4''^N+\alpha_1^N}))$
$= (a^N, b^N, c^N, d^N)(a'^N, b'^N, c'^N, d'^N)(a''^N, b''^N, c''^N, d''^N),$

where

$$a^N = p_1^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_1^\beta)^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_4^\beta)^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_4^\beta)^N+\alpha_1^N)}}) + \frac{(Ap_4^\beta)^N(Q^N-M^N)}{\alpha_1^N((Ap_1^\beta)^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_4^\beta)^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_4^\beta)^N}{(Ap_1^\beta)^N+\alpha_1^N}]$$

$$b^N = p_2^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_2^\beta)^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_3^\beta)^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_3^\beta)^N+\alpha_1^N)}}) + \frac{(Ap_3^\beta)^N(Q^N-M^N)}{\alpha_1^N((Ap_2^\beta)^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_3^\beta)^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_3^\beta)^N}{(Ap_2^\beta)^N+\alpha_1^N}]$$

$$c^N = p_3^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_3^\beta)^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_2^\beta)^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_2^\beta)^N+\alpha_1^N)}}) + \frac{(Ap_2^\beta)^N(Q^N-M^N)}{\alpha_1^N((Ap_3^\beta)^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_2^\beta)^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_2^\beta)^N}{(Ap_3^\beta)^N+\alpha_1^N}]$$

$$d^N = p_4^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_4^\beta)^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_1^\beta)^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_1^\beta)^N+\alpha_1^N)}}) + \frac{(Ap_1^\beta)^N(Q^N-M^N)}{\alpha_1^N((Ap_4^\beta)^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_1^\beta)^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_1^\beta)^N}{(Ap_4^\beta)^N+\alpha_1^N}]$$

$$a'^N = p_1'^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_1^\beta)'^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_4^\beta)'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_4^\beta)'^N+\alpha_1^N)}}) + \frac{(Ap_4^\beta)'^N(Q^N-M^N)}{\alpha_1^N((Ap_1^\beta)'^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_4^\beta)'^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_4^\beta)'^N}{(Ap_1^\beta)'^N+\alpha_1^N}]$$

$$b'^N = p_2'^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_2^\beta)'^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_3^\beta)'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_3^\beta)'^N+\alpha_1^N)}}) + \frac{(Ap_3^\beta)'^N(Q^N-M^N)}{\alpha_1^N((Ap_2^\beta)'^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_3^\beta)'^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_3^\beta)'^N}{(Ap_2^\beta)'^N+\alpha_1^N}]$$

$$c'^N = p_3'^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_3^\beta)'^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_2^\beta)'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_2^\beta)'^N+\alpha_1^N)}}) + \frac{(Ap_2^\beta)'^N(Q^N-M^N)}{\alpha_1^N((Ap_3^\beta)'^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_2^\beta)'^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_2^\beta)'^N}{(Ap_3^\beta)'^N+\alpha_1^N}]$$

$$d'^N = p_4'^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_4^\beta)'^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_1^\beta)'^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_1^\beta)'^N+\alpha_1^N)}}) + \frac{(Ap_1^\beta)'^N(Q^N-M^N)}{\alpha_1^N((Ap_4^\beta)'^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_1^\beta)'^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_1^\beta)'^N}{(Ap_4^\beta)'^N+\alpha_1^N}]$$

$$a''^N = p_1''^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_1^\beta)''^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_4^\beta)''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_4^\beta)''^N+\alpha_1^N)}}) + \frac{(Ap_4^\beta)''^N(Q^N-M^N)}{\alpha_1^N((Ap_1^\beta)''^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_4^\beta)''^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_4^\beta)''^N}{(Ap_1^\beta)''^N+\alpha_1^N}]$$

$$b''^N = p_2''^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_2^\beta)''^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_3^\beta)''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_3^\beta)''^N+\alpha_1^N)}}) + \frac{(Ap_3^\beta)''^N(Q^N-M^N)}{\alpha_1^N((Ap_2^\beta)''^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_3^\beta)''^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_3^\beta)''^N}{(Ap_2^\beta)''^N+\alpha_1^N}]$$

$$c''^N = p_3''^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_3^\beta)''^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_2^\beta)''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_2^\beta)''^N+\alpha_1^N)}}) + \frac{(Ap_2^\beta)''^N(Q^N-M^N)}{\alpha_1^N((Ap_3^\beta)''^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_2^\beta)''^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_2^\beta)''^N}{(Ap_3^\beta)''^N+\alpha_1^N}]$$

$d''^N = p_4''^N Q^N + C_o^N + C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_4^\beta)''^N}{\alpha_1^N})] - C_h^N[\frac{1}{\alpha_1^N}(Q^N - M^N + \frac{(Ap_1^\beta)''^N}{\alpha_1^N})e^{-\frac{\alpha_1^N(Q^N-M^N)}{((Ap_1^\beta)''^N+\alpha_1^N)}}) + \frac{(Ap_1^\beta)''^N(Q^N-M^N)}{\alpha_1^N((Ap_4^\beta)''^N+\alpha_1^N)}] + \frac{C_s^N M^{2^N}}{2(Ap_1^\beta)''^N} + d^N(Q^N - M^N) - d[\frac{(Q^N-M^N)(Ap_1^\beta)''^N}{(Ap_4^\beta)''^N+\alpha_1^N}]$

The defuzzified neutrosophic total cost using signed distance method is given by

d((TC)^N, 0) $=\frac{1}{2}[[(p_1^N + p_4^N) + (p_1''^N + p_4''^N)]Q^N + 2C_o^N + C_h^N[\frac{1}{\alpha_1^N}\{(2Q^N - 2M^N + \frac{A[(p_1^{\beta^N}+p_4^{\beta^N})+(p_1^{\beta''^N}+p_4^{\beta''^N})]}{\alpha_1^N})\}] - C_h^N[\frac{1}{\alpha_1^N}\{(Q^N - M^N + \frac{A}{\alpha_1^N}(p_4^{\beta^N}+p_4^{\beta''^N}))e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N}} + (Q^N - M^N + \frac{A}{\alpha_1^N}(p_1^{\beta^N}+p_1^{\beta''^N}))e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N}} + \frac{A(Q^N-M^N)}{\alpha_1^N}\{(\frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N} + \frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N}) + (\frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N} + \frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N})\}] + \frac{C_s^N M^{2^N}}{2A}[(\frac{1}{p_4^{\beta^N}} + \frac{1}{p_1^{\beta^N}}) + (\frac{1}{p_4^{\beta''^N}} + \frac{1}{p_1^{\beta''^N}})] - d(Q^N - M^N)A[(\frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N} + \frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N}) + (\frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N} + \frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N})] + 2d^N(Q^N - M^N)]$
$+\frac{1}{4}[[(p_2^N - p_1^N) + (p_2''^N + p_1''^N)]Q^N + C_h^N[\frac{1}{\alpha_1^N}\{(\frac{A}{\alpha_1^N}[(p_2^{\beta^N} - p_1^{\beta^N}) + (p_2^{\beta''^N} - p_1^{\beta''^N})])\}] - C_h^N[\frac{1}{\alpha_1^N}\{(Q^N - M^N + \frac{A}{\alpha_1^N}(p_3^{\beta^N}+p_3^{\beta''^N}))e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_3^{\beta^N}+p_3^{\beta''^N})+\alpha_1^N}} - (Q^N - M^N + \frac{A}{\alpha_1^N}(p_4^{\beta^N}+p_4^{\beta''^N}))e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N}} + \frac{A(Q^N-M^N)}{\alpha_1^N}\{(\frac{p_3^{\beta^N}}{Ap_2^{\beta^N}+\alpha_1^N} - \frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N}) + (\frac{p_3^{\beta''^N}}{Ap_2^{\beta''^N}+\alpha_1^N} - \frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N})\}] + \frac{C_s^N M^{2^N}}{2A}[(\frac{1}{p_3^{\beta^N}} - \frac{1}{p_4^{\beta^N}}) + (\frac{1}{p_3^{\beta''^N}} - \frac{1}{p_4^{\beta''^N}})] - d^N(Q^N - M^N)A[(\frac{p_3^{\beta^N}}{Ap_2^{\beta^N}+\alpha_1^N} - \frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N}) + (\frac{p_3^{\beta''^N}}{Ap_2^{\beta''^N}+\alpha_1^N} - \frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N})]$
$-\frac{1}{4}[[(p_4^N - p_3^N) + (p_4''^N + p_3''^N)]Q^N + C_h^N[\frac{1}{\alpha_1^N}\{(\frac{A}{\alpha_1^N}[(p_4^{\beta^N} - p_3^{\beta^N}) + (p_4^{\beta''^N} - p_3^{\beta''^N})])\}] - C_h^N[\frac{1}{\alpha_1^N}\{(Q^N - M^N + \frac{A}{\alpha_1^N}(p_1^{\beta^N}+p_1^{\beta''^N}))e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N}} - (Q^N - M^N + \frac{A}{\alpha_1^N}(p_2^{\beta^N}+p_2^{\beta''^N}))e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_2^{\beta^N}+p_2^{\beta''^N})+\alpha_1^N}} + \frac{A(Q^N-M^N)}{\alpha_1^N}\{(\frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N} - \frac{p_2^{\beta^N}}{Ap_3^{\beta^N}+\alpha_1^N}) + (\frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N} - \frac{p_2^{\beta''^N}}{Ap_3^{\beta''^N}+\alpha_1^N})\}] + \frac{C_s^N M^{2^N}}{2A}[(\frac{1}{p_1^{\beta^N}} - \frac{1}{p_2^{\beta^N}}) + (\frac{1}{p_1^{\beta''^N}} - \frac{1}{p_2^{\beta''^N}})] - d^N(Q^N - M^N)A[(\frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N} - \frac{p_2^{\beta^N}}{Ap_3^{\beta^N}+\alpha_1^N}) + (\frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N} - \frac{p_2^{\beta''^N}}{Ap_3^{\beta''^N}+\alpha_1^N})] = F^N(M)$

To find the minimum of $D(F^N(q))$ by taking the derivative $D(F^N(q))$, we get

$\frac{d(F^N(q))}{dM} = \frac{1}{2}[-C_h^N[\frac{2}{\alpha_1^N} + \frac{1}{\alpha_1^N}\{(Q^N - M^N + \frac{A}{\alpha_1^N}(p_4^{\beta^N}+p_4^{\beta''^N}))(\frac{\alpha_1^N}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N})e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N}} - e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N}} + (Q^N - M^N + \frac{A}{\alpha_1^N}(p_1^{\beta^N}+p_1^{\beta''^N}))(\frac{\alpha_1^N}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N})e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N}} - e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N}} - \frac{A}{\alpha_1^N}\{(\frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N} + \frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N}) + (\frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N} + \frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N})\}] + \frac{C_s^N M^N}{A}[(\frac{1}{p_4^{\beta^N}} + \frac{1}{p_1^{\beta^N}}) + (\frac{1}{p_4^{\beta''^N}} + \frac{1}{p_1^{\beta''^N}})] + dA[(\frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N} + \frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N}) + (\frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N} + \frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N})] - 2d^N]$

$+\frac{1}{4}[-C_h^N(\frac{1}{\alpha_1^N}\{(Q^N - M^N + \frac{A}{\alpha_1^N}(p_3^{\beta^N}+p_3^{\beta''^N}))(\frac{\alpha_1^N}{A(p_3^{\beta^N}+p_3^{\beta''^N})+\alpha_1^N})e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_3^{\beta^N}+p_3^{\beta''^N})+\alpha_1^N}} - e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_3^{\beta^N}+p_3^{\beta''^N})+\alpha_1^N}} - (Q^N - M^N + \frac{A}{\alpha_1^N}(p_4^{\beta^N} + p_4^{\beta''^N}))(\frac{\alpha_1^N}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N})e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N}} + e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_4^{\beta^N}+p_4^{\beta''^N})+\alpha_1^N}} - \frac{A}{\alpha_1^N}\{(\frac{p_3^{\beta^N}}{Ap_2^{\beta}+\alpha_1^N} - \frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N}) + (\frac{p_3^{\beta''^N}}{Ap_2^{\beta''^N}+\alpha_1^N} - \frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N})\}] + \frac{C_s^N M^N}{A}[(\frac{1}{p_3^{\beta^N}} - \frac{1}{p_4^{\beta^N}}) + (\frac{1}{p_3^{\beta''^N}} - \frac{1}{p_4^{\beta''^N}})] + d^N A[(\frac{p_3^{\beta^N}}{Ap_2^{\beta^N}+\alpha_1^N} - \frac{p_4^{\beta^N}}{Ap_1^{\beta^N}+\alpha_1^N}) + (\frac{p_3^{\beta''^N}}{Ap_2^{\beta''^N}+\alpha_1^N} - \frac{p_4^{\beta''^N}}{Ap_1^{\beta''^N}+\alpha_1^N})]$

$-\frac{1}{4}[-C_h^N[\frac{1}{\alpha_1^N}\{(Q^N - M^N + \frac{A}{\alpha_1^N}(p_1^{\beta^N}+p_1^{\beta''^N}))(\frac{\alpha_1^N}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N})e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N}} - e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_1^{\beta^N}+p_1^{\beta''^N})+\alpha_1^N}} - (Q^N - M^N + \frac{A}{\alpha_1^N}(p_2^{\beta^N} + p_2^{\beta''^N}))(\frac{\alpha_1^N}{A(p_2^{\beta^N}+p_2^{\beta''^N})+\alpha_1^N})e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_2^{\beta^N}+p_2^{\beta''^N})+\alpha_1^N}} + e^{\frac{-\alpha_1^N(Q^N-M^N)}{A(p_2^{\beta^N}+p_2^{\beta''^N})+\alpha_1^N}} - \frac{A}{\alpha_1^N}\{(\frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N} - \frac{p_2^{\beta^N}}{Ap_3^{\beta^N}+\alpha_1^N}) + (\frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N} - \frac{p_2^{\beta''^N}}{Ap_3^{\beta''^N}+\alpha_1^N})\}] + \frac{C_s^N M^N}{A}[(\frac{1}{p_2^{\beta^N}} - \frac{1}{p_4^{\beta^N}}) + (\frac{1}{p_1^{\beta''^N}} - \frac{1}{p_2^{\beta''^N}})] + d^N A[(\frac{p_1^{\beta^N}}{Ap_4^{\beta^N}+\alpha_1^N} - \frac{p_2^{\beta^N}}{Ap_3^{\beta^N}+\alpha_1^N}) + (\frac{p_1^{\beta''^N}}{Ap_4^{\beta''^N}+\alpha_1^N} - \frac{p_2^{\beta''^N}}{Ap_3^{\beta''^N}+\alpha_1^N})]$

5. **Numerical example:**

The relationship between the supply of a commodity at a constant rate of 10 units per day and its unit price may be expressed using the variables A and β. In this case, A represents 100 units and β is equal to 0.02. Supplies of varying quantities can be obtained as needed, with each order incurring a cost of 1000 units. The inventory is managed in lots of 250 units, with a purchase cost of Rs.45 per unit. Additionally, the rate of decay is 0.2. Determine the optimum total cost of the inventory.

Solution:

Let α_1^N = 0.2, C_o^N = 1000 per order, Q^N = 250, d^N = 10, A = 100,

β = 0.02, P^N = (40, 43, 47, 49)(36, 39, 50, 54)(33, 35, 55, 57),

D^N = (85, 95, 109, 112,)(65, 72, 115, 119)(45, 55, 120, 122).

Table 4.1: Analysis of a neutrosophic model as holding and shortage costs increases

S.No	C_h^N	C_s^N	Q$^{N^*}$	(TC)	(TC)^F	(TC)^I	(TC)^N
1	(11,12,13) (10,12,14) (9,12,15)	(11,14,17) (9,14,18) (8,14,19)	134.61	10068943.8	20135637.6	7934942.9	5033909.4
2	(13,14,15) (12,14,16) (11,14,17)	(12,16,19) (11,16,20) (10,16,21)	133.33	11762410.9	23522389.08	9269053.4	5880597.27
3	(15,16,17) (14,16,18) (13,16,19)	(13,18,20) (11,18,21) (10,18,22)	132.35	13454174.6	26905733.96	10601822	6726433.47
4	(17,18,19) (16,18,20) (15,18,21)	(14,20,23) (13,20,23) (12,20,24)	131.57	15144874.7	30286951.97	11933752.72	7571737.99
5	(19,20,21) (18,20,22) (17,20,23)	(15,22,25) (14,22,26) (13,22,27)	130.95	16834859.8	33666740.03	13265120.08	8469470.75

Table 4.2: Evaluation of the neutrosophic model as the shortage cost rises

S.No	C_h^N	C_s^N	Q^{N*}	(TC)	$(TC)^F$	$(TC)^I$	$(TC)^N$
1	(11,12,13) (10,12,14) (9,12,15)	(11,14,17) (9,14,18) (8,14,19)	134.61	10091627.4	20188962.3	7934942.9	5047240.57
2	(11,12,13) (10,12,14) (9,12,15)	(12,16,19) (11,16,20) (10,16,21)	142.85	9944768.5	19894995.01	7819537.9	4973748.75
3	(11,12,13) (10,12,14) (9,12,15)	(13,18,20) (11,18,21) (10,18,22)	150	9746243.2	19497670.67	7663514.21	4874417.65
4	(11,12,13) (10,12,14) (9,12,15)	(14,20,23) (13,20,23) (12,20,24)	156.25	9518236.6	1904134.63	7484311.84	4760343.65
5	(11,12,13) (10,12,14) (9,12,15)	(15,22,25) (14,22,26) (13,22,27)	161.76	9274965.4	18554548.8	7293106.85	4638637.2

Table 4.3: Analysis for neutrosophic model with increase of holding cost

S.No	C_h^N	C_s^N	Q^N	(TC)	(TC)^F	(TC)^I	(TC)^N
1	(11,12,13) (10,12,14) (9,12,15)	(11,14,17) (9,14,18) (8,14,19)	134.61	10112447.15	20220784.4	7934942.9	5055196.1
2	(13,14,15) (12,14,16) (11,14,17)	(11,14,17) (9,14,18) (8,14,19)	124.66	11865307.06	23726156.3	9310188.89	5931539.07
3	(15,16,17) (14,16,18) (13,16,19)	(11,14,17) (9,14,18) (8,14,19)	116.67	13497261.3	26989754.3	10590591.44	6747438.57
4	(17,18,19) (16,18,20) (15,18,21)	(11,14,17) (9,14,18) (8,14,19)	109.37	15012882.6	30020718.7	11779679.57	7505179.67
5	(19,20,21) (18,20,22) (17,20,23)	(11,14,17) (9,14,18) (8,14,19)	102.94	16416561.1	32823899.9	12880818	8205974.98

6. Sensitivity Analysis

In this section, the neutrosophic total cost values are compared graphically.

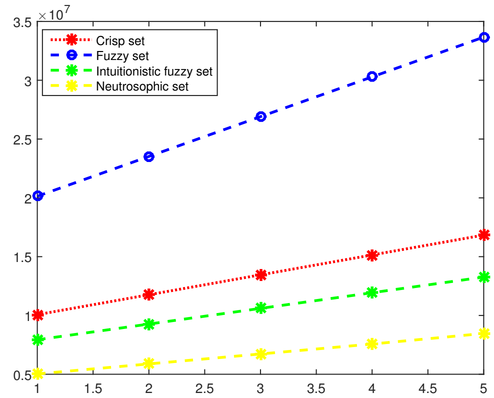


FIGURE 1. A graphical representation of the neutrosophic model as holding and shortages costs increase

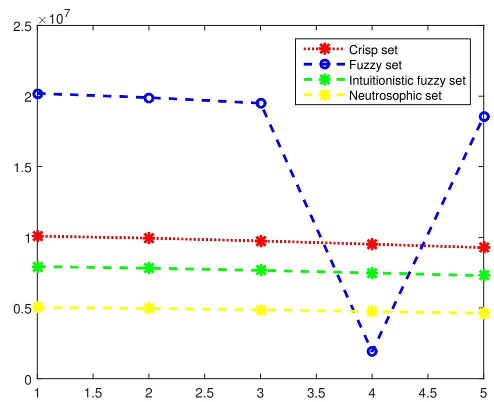


FIGURE 2. Graphical representation for neutrosophic model with increase of shortage cost

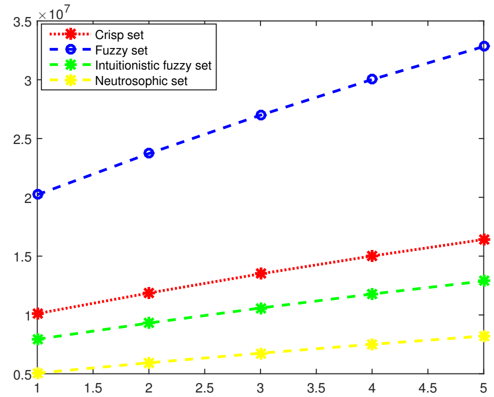


FIGURE 3. Graphical representation for neutrosophic model with increase of holding cost

Observations:

From Tables 1, 2 and 3 in the above descriptive model, it should be noted that compared to the crisp model, the neutrosophic model is very effective. The average total cost obtained in the neutrosophic inventory model is less than the crisp model. Thus, it is possible to claim that making use of neutrosophic sets delivers a more optimal approach to resolving inventory models compared to the conventional crisp, fuzzy, and intuitionistic fuzzy models.

- As neutrosophic holding costs and neutrosophic shortage costs decrease, the total cost of neutrosophics gradually increases.
- Neutrosophic total cost decreases if neutrosophic shortage cost increases.
- There is a slight change in the neutrosophic total cost compared to the neutrosophic holding cost in Table 1.

Conclusion:

A neutrosophic inventory model should be implemented to resolve the administration of decomposing products whose demand is determined by cost. The solution to shortages can be determined by applying the concept of fuzzification to both neutrosophic demand and neutrosophic purchasing cost, using trapezoidal neutrosophic numbers. The signed distance method is used for defuzzification process, to provide a unique optimal solution which will minimize the total cost of the item. Therefore, it can possibly concluded that the neutrosophic inventory method yields improved results compared to other readily accessible models. In future, this study could be extended for shortages and partial backlogging in neutrosophic inventory models.

Acknowledgement

The article has been written with the joint financial support of RUSA-Phase 2.0 grant sanctioned vide letter No.F 24-51/2014-U, Policy (TN Multi-Gen), Dept. of Edn. Govt. of India, Dt.09.10.2018, UGC-SAP (DRS-I) vide letter No.F.510/8/DRS-I/2016(SAP-I) Dt. 23.08.2016 and DST (FST - level I) 657876570 vide letter No.SR/FIST/MS-I/2018/17 Dt. 20.12.2018.

References

[1] Atanassov, K.T. Intuitionistic fuzzy sets. Fuzzy Sets Syst 1986, Vol. 20, page 87-96.

[2] Banerjee, D.; Pramanik, S. Single-objective linear goal programming problem with neutrosophic numbers. International Journal of Engineering Science and Research Technology 2018, Vol.7, No.5, page 454-469.

[3] Dutta, D.; Pavan Kumar. Fuzzy inventory model for deteriorating items with shortages under fully backlogged condition. International Journal of Soft Computing and Engineering (IJSCE) 2013, Vol.3, No.2, page 393-398.

[4] Irfan Deli; Yusuf Subas. Single valued neutrosophic numbers and their applications to multicriteria decision making problem. Journal of Intelligent and Fuzzy Systems 2014.

[5] Maragatham, M.; Lakshmidevi, P.K. A Fuzzy inventory model for deteriorating items with price dependent demand. International Journal of Fuzzy Mathematical Archive 2014, Vol.5, No.1, page 39-47.

[6] Mullai, M.; Broumi, S. Neutrosophic inventory model without shortages. Asian Journal of Mathematics and Computer Research 2018, Vol.23, No.4, page 214-219.

[7] Mullai, M.; Surya, R. Neutrosophic EOQ model with price breaks. Neutrosophic sets and systems 2018, Vol.19, page 24-28.

[8] Nabeeh, N.A.; Abdel-Basset, M.; EL-Ghareeb, H.A.; Aboelfetouh, A. Neutrosophic multi-criteria decision making approach for lot based enterprises. IEEE Access 2019, Vol.7, page 59559-59574.

[9] Nabeeh, N.A.; Smarandache, F.; Abdel-Basset, M.; EL-Ghareeb, H.A.; Aboelfetouh, A. An integrated neutrosophic-topsis approach and its application to personnel selection: A new trend in brain processing and analysis. IEEE Access 2019, Vol.7, page 29734-29744.

[10] Palani, R.; Maragatham, M. Fuzzy inventory model for time dependent deteriorating items with lead time stock dependent demand rate and shortages. International Journal of Development Research 2017, Vol.7, No.10, page 15988-15995.

[11] Prabjot Kaur; Mahuya Deb. An Intuitionistic Approach to an Inventory Model without Shortages. International Journal of Pure and Applied Sciences and Technology 2014, Vol.22, No.2, page 25-35.

[12] Pranab Biswas; Surapati Pramanik; Bibhas C. Giri. Multi-attribute group decision making based on expected value of neutrosophic trapezoidal numbers. New Trends in Neutrosophic Theory and Applications , Vol.2, page 105-124.

[13] Salim Rezvani. Ranking method of trapezoidal intuitionistic fuzzy numbers. Annals of Fuzzy Mathematics and Informatics 2013, Vol.5, No.3, page 515-523.

[14] Sharmila, D.; Uthayakumar, R. Inventory model for deteriorating items involving fuzzy with shortages and exponential demand. International Journal of Supply and Operations Management IJSOM 2015, Vol.2, No.3, page 888-904.

[15] Smarandache, F. Neutrosophic set - a generalization of the intuitionistic fuzzy set. IEEE International Conference 2006, page 38-42.

[16] Smarandache, F.; Pramanik, S. New trends in neutrosophic theory and applications. Brussels: Pons Editions 2018, Vol.2, page 1-425.

[17] Smarandache, F. Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets-Revisited. Neutrosophic Sets and Systems 2018, Vol.21, page 153-166.

[18] Soumita Kundu; Tripti Chakrabarti. An EOQ model for deteriorating items with fuzzy demand and fuzzy partial backlogging. IOSR Journal of Mathematics (IOSRJM) ISSN: 2278-5728 2012, Vol.2, No.3, page 13-20.

[19] Sushil Kumar, U.; Rajput, S. Fuzzy inventory model for deteriorating items with time dependent demand rate and partial backlogging. Applied Mathematics 2006, Vol.6, page 496-509.

[20] Wang, H.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. Single valued neutrosophic sets. Multispace and Multistructure 2010, Vol.4, page 410-413.

Received: Jan 9, 2024. Accepted: Mar 30, 2024

NSS

Neutrosophic Sets and Systems, Vol. 66, 2024

University of New Mexico

Decision Making By Neutrosophic Over Soft Topological Space

R.Narmada Devi[1,*], Yamini Parthiban [2]

[1,2]Department of Mathematics,Vel Tech Rangarajan Dr.Sagunthala R&D Institute of Science and Technology,Avadi,Chennai,Tamil Nadu,India.;narmadadevi23@gmail.com,yaminiparthiban.msc@gmail.com

*Correspondence: narmadadevi23@gmail.com.

Abstract. The empirical correlation system serves as a crucial tool for unveiling the linear interconnections between two variables. Its significance lies in providing a prominent approach to depict a straightforward relationship without explicitly indicating a causal link between the sets involved. In the current research, an innovative concept of correlations is introduced specifically for Neutrosophic Over Soft Sets ($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-sets). This novel framework involves a meticulous examination of basic definitions and operations associated with Neutrosophic Over Soft Sets. Furthermore, the study extends to the introduction of a groundbreaking concept: a topological space integrated with Neutrosophic Over Soft Sets ($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-sets). This addition aims to broaden the scope of understanding and application in mathematical contexts.The research does not merely establish theoretical foundations; it also explores various properties and theorems related to the introduced concepts. This is complemented by a series of numerical examples designed to provide clarity and facilitate a comprehensive grasp of the material. To demonstrate the practical application of these concepts, the research utilizes the correlation framework to present a numerical illustration. Specifically, it is applied to determine the top-performing student at GFC School for the academic year 2022-2023, showcasing the real-world relevance and applicability of the proposed methodologies.

Keywords: Neutrosophic Over Soft Set and Neutrosophic Over Soft Topological Space.

1. **Introduction**

In the course of daily life, uncertainty is a common experience. For instance, when rolling a die or tossing a coin onto an uneven surface, uncertainties emerge. The inception of fuzzy sets was credited to Zadeh [23] (1965), who introduced the notion of membership degrees. Zadeh also laid the foundation for a theory of possibility [24], whereas Bellman et al. [2] delved into decision-making within contexts influenced by fuzziness. Expanding on Zadeh's contributions, Atanassov introduced intuitionist fuzzy sets, concentrating on both degrees of membership and non-membership [1].

Smarandache [19] is attributed with the discovery of neutrosophic sets and the exploration of novel trends and applications within neutrosophic theory. In 1995, Bustince and Burillo [5] investigated the correlation of intuitionist fuzzy sets in scenarios involving interval values. Three potential utilities of neutrosophic sets were postulated by Christianto [6]. In 1999, Molodtsov [14] brought to light the primary result concerning soft sets, with a subsequent finding contributed by Maji et al. [12]. The year 2002 marked the introduction of neutrosophic soft sets by Broumi [3]. Neutrosophic sets have found practicality in medical contexts, as pursued by researchers [17, 18]and also in variouse field [7, 10, 16].Correlation measures are harnessed to discern connections between pairs of variables.

In 2015, Broumi and Deli embarked on an exploration into correlation measures for neutrosophic sets [4]. Radha et al. ushered in the concept of neutrosophic Pythagorean sets and their elevated correlation in the year 2021 [15]. An alternative facet of correlation was ushered into the spotlight by Ye, J., back in 2013 [22]. Wang et al. [21] delved into the realm of single-valued neutrosophic sets. The year 2020 saw Mallick, R., and Pramanik, S. [13] delve into discussions about pentapartitioned neutrosophic sets and their inherent properties. In 2019, Jansi et al. [11] stumbled upon the concept of neutrosophic pythagorean sets featuring both dependent and independent components. Smarandache [20] brought forth the innovative notion of neutrosophic sets replete with over, under, and off limits in 2016. Murugesan et al. [10] (2023) undertake a comparative analysis between Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps to unravel the intricacies of Covid variants. RN Devi and G Muthumari [8] conducted an in-depth study on the realm of topologized domination concerning NOver top graphs in 2021. Moreover, in the subsequent year, 2022, RN Devi etal., introduced a pioneering study on Digital Neutrosophic Topological Spaces, pushing the boundaries of mathematical modeling [9] .

The manuscript introduces innovative concepts: $\mathcal{N}\mathfrak{s}^{\mathfrak{o}}$-set and $\mathcal{N}\mathfrak{s}^{\mathfrak{o}}$-topological space. It also presents measures of correlations for neutrosophic over soft sets, elucidating foundational definitions, operations, and theorems supported by concrete numerical examples. The inclusion of numerical illustrations, drawn from a survey involving five teachers at GFC School, adds practical relevance. This survey aims to identify the top-performing student for the academic year 2022-2023. The manuscript, thus, seamlessly integrates theoretical developments, illustrative examples, and real-world applications, contributing comprehensively to the field of study.

2. Preliminary

This section contains basic definition for Neutrosophic Set(NS),Neutrosophic Over Set(NOS),Neutrosophic Soft Set(NSS),Neutrosophic Topological Space(NTS) and Soft Topological Space(STS).

Definition 2.1. Let $\mathcal{H}$ be an non empty set and $\mathcal{J}$ is said to be an NS.Then

$$\mathcal{J} = \{\langle \mathsf{h}, \aleph(\mathsf{h}), \eth(\mathsf{h}), \Upsilon(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}$$

where $\aleph, \eth, \Upsilon : \mathcal{H} \to [0,1]$ and $0 \leq \aleph(\mathsf{h}) + \eth(\mathsf{h}) + \Upsilon(\mathsf{h}) \leq 3$. Here $\aleph(\mathsf{h}), \eth(\mathsf{h})$ and $\Upsilon(\mathsf{h})$ are degree of true membership,degree of indeterminacy and degree of falsity.

Definition 2.2. Let $\mathcal{J}$ be an NS in $\mathcal{H}$.If $\mathcal{J}$ is said to be an NOS in an non-empty set $\mathcal{H}$ then it has at-least one neutrosophic component is > 1 and no other component are < 0 is defined as,

$$\mathcal{J} = \{\langle \mathsf{h}, \aleph(\mathsf{h}), \eth(\mathsf{h}), \Upsilon(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}$$

Where $\aleph, \eth, \Upsilon : \mathcal{H} \to [0, \Omega]$, $0 \leq \aleph(\mathsf{h}) + \eth(\mathsf{h}) + \Upsilon(\mathsf{h}) \leq 3$ and Ω is said to be over-limit of NOS

Note: $\rho(\mathcal{H})$ is a set of all the $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ subset of an non-empty set $\mathcal{H}$

Definition 2.3. Let an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set $\odot = \{e, \{\langle \mathsf{h}, 0, 0, \Omega\rangle : \mathsf{h} \in \mathcal{H}\} : e \in \mathcal{E}\}$ is said to be a Null $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ and $\circledast = \{e, \{\langle \mathsf{h}, \Omega, \Omega, 0\rangle : \mathsf{h} \in \mathcal{H}\} : e \in \mathcal{E}\}$ is said to be an universal $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$.

Definition 2.4. Let $\mathcal{H}$ be an non-empty set and $\mathcal{E}$ be a set of parameters on $\mathcal{H}$.Consider $\mathcal{A} \subset \mathcal{E}$.The collection $(\lambda, \mathcal{A})$ is an NSS then it is defined as,$\mathfrak{h}$

$$\lambda : \mathcal{A} \to \rho(\mathcal{H})$$

where λ is a mapping and $\rho(\mathcal{H})$ is a collection of all the subsets of NS in $\mathcal{H}$.

Definition 2.5. A neutrosophic topology(NT) τ_{NT} is a collection of subset of an NS $\mathcal{W}$ such that

(i) $\odot, \circledast \in \tau_{NT}$.

(ii) The union of an arbitrary collection τ_{NT} is in τ_{NT}.

(iii) The finite intersection of subsets τ_{NT} is in τ_{NT}.

Then $(\mathcal{W}, \tau_{NT})$ is called neutrosophic topological space(NTS).An element of τ_{NOS} is called an neutrosophic open set and τ_{NCS} is called an neutrosophic closed set.

Definition 2.6. A soft topology(ST) τ_{ST} is a collection of subset of an soft set(SS) $\mathcal{W}$ such that

(i) $\odot, \circledast \in \tau_{ST}$.

(ii) The union of an arbitrary collection τ_{ST} is in τ_{ST}.

(iii) The finite intersection of subsets τ_{ST} is in τ_{ST}.

Then $(\mathcal{W}, \tau_{ST})$ is called soft topological space(STS).An element of τ_{SOS} is called an soft open set and τ_{SCS} is called an soft closed set.

3. Neutrosophic Over Soft Topological Space

Definition 3.1. Let $\mathcal{H}$ be an non-empty set and $\mathcal{E}$ be a set of parameter on $\mathcal{H}$.Then $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set is defined by a set valued function

$$\lambda_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} : \mathcal{E} \to \rho(\mathcal{H})$$

where $\rho(\mathcal{H})$ is an set of all $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set on $\mathcal{H}$.$\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set is an valued function from the set of parameter $\mathcal{E}$ on $\mathcal{H}$ is defined as

$$\mathcal{J} = (\lambda_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}, \mathcal{E}) = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

Definition 3.2. Let $\mathcal{J} = (\mathcal{J}_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}, \mathcal{E})$ and $\mathcal{W} = (\mathcal{W}_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}, \mathcal{E})$ be a two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set.If $\mathcal{J}$ is said to be a subset of $\mathcal{W}$ i.e.,$\mathcal{J} \subseteq \mathcal{W}$ then

$$\aleph_{\mathcal{J}}(\mathsf{h}) \leq \aleph_{\mathcal{W}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}) \leq \eth_{\mathcal{W}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h}) \geq \Upsilon_{\mathcal{W}}(\mathsf{h})$$

In other words $\mathcal{W}$ is an super set of $\mathcal{J}$

Definition 3.3. Let $\mathcal{J} \subset \mathcal{W}$ and $\mathcal{W} \subset \mathcal{J}$ then $\mathcal{J} = \mathcal{W}$

Definition 3.4. Let $\mathcal{J}$ and $\mathcal{W}$ be two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set, Then the union,intersection and compliment are defined by

(i)$\mathcal{J} \mho \mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), max(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$

(ii)$\mathcal{J} \Omega \mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), min(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$

(iii)$\mathcal{J}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$

Proposition 3.5. *Let $\mathcal{J}$ be an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set on $\mathcal{H}$. Then*

(i). $\odot^{☾} = \circledast$

(ii). $\circledast^{☾} = \odot$

(iii). $(\mathcal{J}^{☾})^{☾} = \mathcal{J}$

Proof. 1. $\odot^{☾} = \circledast$

$$\odot = \{e, \{\langle \mathsf{h}, 0, 0, \Omega\rangle : \mathsf{h} \in \mathcal{H}\} : e \in \mathcal{E}\}$$

$$\odot^{☾} = \{\langle \mathsf{h}, \Omega, \Omega, 0\rangle : \mathsf{h} \in \mathcal{H}\} = \circledast$$

$$\implies \odot^{☾} = \circledast$$

2. $\circledast^{☾} = \odot$

$$\circledast = \{\langle \mathsf{h}, \Omega, \Omega, 0\rangle : \mathsf{h} \in \mathcal{H}\}$$

$$\circledast^{☾} = \{\langle \mathsf{h}, \Omega, \Omega, 0\rangle : \mathsf{h} \in \mathcal{H}\} = \odot$$

$$\Longrightarrow \circledast^{\complement} = \odot$$

3. $(\mathcal{J}^{\complement})^{\complement} = \mathcal{J}$

$$\mathcal{J}^{\complement} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$(\mathcal{J}^{\complement})^{\complement} = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \Omega - (\Omega - \eth_{\mathcal{J}}(\mathsf{h})), \Upsilon_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} = \mathcal{J}$$

$$\Longrightarrow (\mathcal{J}^{\complement})^{\complement} = \mathcal{J}$$

□

Proposition 3.6. *Let $\mathcal{J}$ and $\mathcal{W}$ be an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set on $\mathcal{H}$. Then*

(i). $\mathcal{J} \mho \mathcal{J} = \mathcal{J} \varOmega \mathcal{J} = \mathcal{J}$

(ii). $\mathcal{J} \mho \mathcal{W} = \mathcal{W} \mho \mathcal{J}$

(iii). $\mathcal{J} \varOmega \mathcal{W} = \mathcal{W} \varOmega \mathcal{J}$

(iv). $\mathcal{J} \mho \odot = \mathcal{J}$ *and* $\mathcal{J} \mho \circledast = \circledast$

(v). $\mathcal{J} \varOmega \odot = \odot$ *and* $\mathcal{J} \varOmega \circledast = \mathcal{J}$

Proof. The proof is obvious from the definition. □

Theorem 3.7. *Let $\mathcal{J}$ and $\mathcal{W} \in \mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set. Then*

(i). $(\mathcal{J} \mho \mathcal{W})^{\complement} = \mathcal{J}^{\complement} \mho \mathcal{W}^{\complement}$

(ii). $(\mathcal{J} \varOmega \mathcal{W})^{\complement} = \mathcal{J}^{\complement} \varOmega \mathcal{W}^{\complement}$

Proof. (i).By the union definition,

$$\mathcal{J} \mho \mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), max(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$(\mathcal{J} \mho \mathcal{W})^{\complement} = \{(\mathsf{e}, \{\langle \mathsf{h}, min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), max(\Omega - \eth_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h})), max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} \quad (1)$$

By the definition of compliment

$$\mathcal{J}^{\complement} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{W}^{\complement} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{W}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{J}^{\complement} \mho \mathcal{W}^{\complement} = \{(\mathsf{e}, \{\langle \mathsf{h}, min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), max(\Omega - \eth_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h})), max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} \quad (2)$$

From (1) and (2) we get,

$$(\mathcal{J} \mho \mathcal{W})^{\complement} = \mathcal{J}^{\complement} \mho \mathcal{W}^{\complement}$$

(ii). By the union definition we know that,

$$\mathcal{J} \varOmega \mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), min(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$(\mathcal{J} \Omega \mathcal{W})^{\complement} = \{(\mathsf{e}, \{\langle \mathsf{h}, max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), min(\Omega - \eth_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h})), min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} \quad (3)$$

By the definition of compliment

$$\mathcal{J}^{\complement} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{W}^{\complement} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{W}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{J}^{\complement} \Omega \mathcal{W}^{\complement} = \{(\mathsf{e}, \{\langle \mathsf{h}, max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), min(\Omega - \eth_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h})), min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} \quad (4)$$

From (3) and (4) we get,

$(\mathcal{J} \Omega \mathcal{W})^{\complement} = \mathcal{J}^{\complement} \Omega \mathcal{W}^{\complement}$ □

Definition 3.8. Let $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ be a neutrosophic over soft topology($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topology) in $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set $\mathcal{J}$ is a collection of subset of an non-empty set $\mathcal{H}$ such that

(i) $\odot, \circledast \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$.

(ii) The union of an arbitrary collection $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ is in $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$.

(iii) The finite intersection of subsets $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ is in $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$.

Then $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ is called neutrosophic over soft topological space($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space).An element of τ_{NOSOS} is called an neutrosophic over soft open set and τ_{NOSCS} is named an neutrosophic over soft closed set.

Example 3.9. Let $\mathcal{H} = \{\mathsf{r}_1, \mathsf{r}_2\}$ be the two students, $\mathcal{A} = \{Puntuality(\mathsf{q})\}$ and $\mathcal{G} \in \tau_{NOSCS}$ such that,

$$\mathcal{Q}(\mathsf{q}) = \{\langle \mathsf{r}_1, 1.2, 0.6, 0.5\rangle, \langle \mathsf{r}_2, 1.1, 0.3, 0.5\rangle\}$$

$(\mathcal{Q}, \mathcal{A}) = \{\mathsf{q} = \{\langle \mathsf{r}_1, 1.2, 0.6, 0.5\rangle, \langle \mathsf{r}_2, 1.1, 0.3, 0.5\rangle\}\}$.

Then,$\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} = \{\odot, \circledast, (\mathcal{Q}, \mathcal{A})\}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topology on $\mathcal{W}$.

Theorem 3.10. *Let $(\mathcal{H}, \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ and $(\mathcal{H}, \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ be two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space on $\mathcal{H}$, then $(\mathcal{H}, \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} \Omega \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space over $\mathcal{H}$.*

Proof. Let $(\mathcal{H}, \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ and $(\mathcal{H}, \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ be $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space over $\mathcal{H}$.

$\implies \odot, \circledast \in \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ and $\odot, \circledast \in \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$

$\implies \odot, \circledast \in \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} \Omega \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ $\therefore$ $(\mathcal{H}, \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} \Omega \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space over $\mathcal{H}$. □

Remark 3.11. In the theorem 2.2 instead of the intersection operation if we use union operation the claim may not be true. It can be seen following example.

Example 3.12. Let $\mathcal{H} = \{\mathsf{r}_1, \mathsf{r}_2\}$ be the two mobile phone and $\mathcal{A} = \{batterydurability(\mathsf{q}_1), workingspeed(\mathsf{q}_2)\}$.

Then $(\mathcal{R}_1, \mathcal{A}), (\mathcal{R}_2, \mathcal{A}) \in \tau_{NOSCS}$ such that,

$$\mathcal{R}_1(\mathsf{q}_1) = \{\langle \mathsf{r}_1, 1.2, 0.4, 0.6\rangle, \langle \mathsf{r}_2, 1.1, 0.3, 0.5\rangle\}\}$$

$$\mathcal{R}_1(\mathsf{q}_2) = \{\langle \mathsf{r}_1, 1.3, 0.5, 0.6\rangle, \langle \mathsf{r}_2, 1.2, 0.5, 0.6\rangle\}\}$$

$$(\mathcal{R}_1, \mathcal{A}) = \{\{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.2, 0.4, 0.6\rangle, \langle \mathsf{r}_2, 1.1, 0.3, 0.5\rangle\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, 1.3, 0.5, 0.6\rangle, \langle \mathsf{r}_2, 1.2, 0.5, 0.6\rangle\}\}\}$$

$$\mathcal{R}_2(\mathsf{q}_1) = \{\langle \mathsf{r}_1, 1.3, 0.4, 0.5\rangle, \langle \mathsf{r}_2, 1.2, 0.3, 0.3\rangle\}$$

$$\mathcal{R}_2(\mathsf{q}_2) = \{\langle \mathsf{r}_1 1.4, 0.5, 0.7\rangle, \langle \mathsf{r}_2 1.1, 0.6, 0.6\rangle\}$$

$$(\mathcal{R}_2, \mathcal{A}) = \{\{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.3, 0.4, 0.5\rangle, \langle \mathsf{r}_2, 1.2, 0.3, 0.3\rangle\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, 1.4, 0.5, 0.7\rangle, \langle \mathsf{r}_2, 1.1, 0.6, 0.6\rangle\}\}\}$$

Then, $\tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} = \{\odot, \circledast, (\mathcal{R}_1, \mathcal{A})\}$ and $\tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} = \{\odot, \circledast, (\mathcal{R}_2, \mathcal{A})\}$ are two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space on $\mathcal{W}$.

But $\tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} \mho \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} = \{\odot, \circledast, (\mathcal{R}_1, \mathcal{A}), (\mathcal{R}_2, \mathcal{A})\}$.

Because $(\mathcal{R}_1, \mathcal{A}) \Omega (\mathcal{R}_2, \mathcal{A}) \notin \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} \mho \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$.So, $\tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} \mho \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ is not $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space on $\mathcal{H}$.

Definition 3.13. An operators of $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ $\mathcal{R} \in \tau_{NOSCS}$, then neutrosophic over soft topological interior and closure are $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$ and $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$ is defined as:

$int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}) = \mho\{\mathcal{N} : \mathcal{N} \subseteq \mathcal{H} \text{and} \mathcal{N} \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}\}$ and

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}) = \Omega\{\mathcal{O} : \mathcal{H} \subseteq \mathcal{O} \text{and} \mathcal{O}^{\complement} \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}\}$.

Proposition 3.14. *Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space and $\mathcal{R}$ is a subset of $\langle$, then*

(i) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$ is the largest NOS open set contained in $\mathcal{R}$.

(ii) $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$ is the smallest NOS closed set containing $\mathcal{R}$.

Proof. (i) By the definition of interior, $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$.Let $\mathcal{N}$ be an open set such that $\mathcal{N} \subset \mathcal{R}$. $\because \mathcal{N}$ is open and $\mathcal{N} \subset \mathcal{R}$,then

$\mathcal{N} \subset int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}) \implies int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$ is the largest open set contained in $\mathcal{R}$.

(ii) By the closure definition,

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}) = \Omega\{\mathcal{O} : \mathcal{H} \subseteq \mathcal{O} \text{and} \mathcal{O}^{\complement} \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}\}$

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$ is the smallest closed set containing $\mathcal{R}$. □

Theorem 3.15. *Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space on $\mathcal{H}$.Let $\mathcal{R}$ and $\mathcal{Q}$ in τ_{NOSCS}.Then,*

(i) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\odot) = \odot$ and $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\circledast) = \circledast$.

(ii) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}) \subseteq \mathcal{R}$.

(iii) $\mathcal{Q}$ is a NOSOS iff $\mathcal{Q} = int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{Q})$.

(iv) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})) = int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$

(v) $\mathcal{R} \subseteq \mathcal{Q} \implies int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}) \subseteq int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{Q})$

(vi) $int_{\mathcal{N}_s^o}(\mathcal{R}) \mho int_{\mathcal{N}_s^o}(\mathcal{Q}) \subseteq int_{\mathcal{N}_s^o}(\mathcal{R} \mho \mathcal{Q})$

(vii) $int_{\mathcal{N}_s^o}(\mathcal{R} \Omega \mathcal{Q}) = int_{\mathcal{N}_s^o}(\mathcal{R}) \Omega int_{\mathcal{N}_s^o}(\mathcal{Q})$

Proof. (i) and (ii) are obviously true.

(iii) If $\mathcal{Q}$ is a NOSOS over $\mathcal{H}$, then $\mathcal{Q}$ is itself a NOSOS over $\mathcal{H}$ which contains $\mathcal{Q}$.

So,$\mathcal{Q}$ is the largest $\mathcal{N}_s^o$ contained in $\mathcal{Q}$

$\implies int_{\mathcal{N}_s^o}(\mathcal{Q}) = \mathcal{Q}$.

Conversely, suppose that $int_{\mathcal{N}_s^o}(\mathcal{Q}) = \mathcal{Q}$.then $\mathcal{Q} \in \tau_{\mathcal{N}_s^o}$.

(iv) Let $int_{\mathcal{N}_s^o}(\mathcal{R}) = \mathcal{Q}$.

Then, $int_{\mathcal{N}_s^o}(\mathcal{Q}) = \mathcal{Q}$ from (iii).

$\implies int_{\mathcal{N}_s^o}(int_{\mathcal{N}_s^o}(\mathcal{R})) = int_{\mathcal{N}_s^o}(\mathcal{R})$

(v) Suppose that $\mathcal{R} \subseteq \mathcal{Q}$.As $int_{\mathcal{N}_s^o}(\mathcal{R}) \subseteq \mathcal{R} \subseteq \mathcal{Q}.int_{\mathcal{N}_s^o}(\mathcal{R})$ is a Neutrosophic over soft subset of $\mathcal{Q}$

From definition (3.2) we get, $int_{\mathcal{N}_s^o}(\mathcal{R}) \subseteq int_{\mathcal{N}_s^o}(\mathcal{Q})$.

(vi) It is clear that $\mathcal{R} \subseteq \mathcal{R} \mho \mathcal{Q}$ and $\mathcal{Q} \subseteq \mathcal{R} \mho \mathcal{Q}$.

Thus,

$int_{\mathcal{N}_s^o}(\mathcal{R}) \subseteq int_{\mathcal{N}_s^o}(\mathcal{R}) \mho int_{\mathcal{N}_s^o}(\mathcal{Q})$ and

$int_{\mathcal{N}_s^o}(\mathcal{Q}) \subseteq int_{\mathcal{N}_s^o}(\mathcal{R}) \mho int_{\mathcal{N}_s^o}(\mathcal{Q})$

$\implies int_{\mathcal{N}_s^o}(\mathcal{R}) \mho int_{\mathcal{N}_s^o}(\mathcal{Q}) \subseteq int_{\mathcal{N}_s^o}(\mathcal{R} \mho \mathcal{Q})$ [By (v)].

(vii) Clearly w.k.t.

$int_{\mathcal{N}_s^o}(\mathcal{R} \Omega \mathcal{Q}) \subseteq int_{\mathcal{N}_s^o}((R))$ and $int_{\mathcal{N}_s^o}(\mathcal{R} \Omega \mathcal{Q}) \subseteq int_{\mathcal{N}_s^o}((Q))$[By (v)].

So, that $int_{\mathcal{N}_s^o}(\mathcal{R} \Omega \mathcal{Q}) \subseteq int_{\mathcal{N}_s^o}(\mathcal{R}) \Omega int_{\mathcal{N}_s^o}(\mathcal{Q})$

Also,$int_{\mathcal{N}_s^o}(\mathcal{R}) \subseteq \mathcal{R}$ and $int_{\mathcal{N}_s^o}(\mathcal{Q}) \subseteq \mathcal{Q}$ we have

$int_{\mathcal{N}_s^o}(\mathcal{R}) \Omega int_{\mathcal{N}_s^o}(\mathcal{Q}) \subseteq \mathcal{R} \Omega \mathcal{Q}$.

$\implies int_{\mathcal{N}_s^o}(\mathcal{R} \Omega \mathcal{Q}) = int_{\mathcal{N}_s^o}(\mathcal{R}) \Omega int_{\mathcal{N}_s^o}(\mathcal{Q})$ □

Example 3.16. Let $\mathcal{H} = \{\mathsf{r}_1, \mathsf{r}_2\}$ be the two team in an company and

$\mathcal{A} = \{\text{punctuality}(\mathsf{q}_1), \text{accuracy of target}(\mathsf{q}_2)\}$. Then

$$(\mathcal{R}_1, \mathcal{A}) = \{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.3, 0.3, 0.1\rangle, \langle \mathsf{r}_2, 1.3, 0.5, 0.3\rangle\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, 1.1, 0.3, 0.1\rangle, \langle \mathsf{r}_2, 1.2, 0.5, 0.4\rangle\}\}, \{\mathsf{q}_3 = \{\langle \mathsf{r}_1, 1.2, 0.3, 0.1\rangle, \langle \mathsf{r}_2, 1.1, 0.5, 0.2\rangle\}\}\}$$

$$(\mathcal{R}_2, \mathcal{A}) = \{\{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.2, 1.2, 0.1\rangle, \langle \mathsf{r}_2, 1.6, 0.6, 0.5\rangle\}\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, 1.1, 1.1, 0.1\rangle, \langle \mathsf{r}_2, 1.5, 0.5, 0.5\rangle\}\}, \{\mathsf{q}_3 = \{\langle \mathsf{r}_1, 1.1, 1.2, 0.1\rangle, \langle \mathsf{r}_2, 1.4, 0.6, 0.4\rangle\}\}\}$$

$$(\mathcal{R}_3, \mathcal{A}) = \{\{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.4, 1.4, 0.2\rangle, \langle \mathsf{r}_2, 1.2, 0.4, 0.2\rangle\}\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, 1.3, 1.1, 0.2\rangle, \langle \mathsf{r}_2, 1.1, 0.4, 0.1\rangle\}\}, \{\mathsf{q}_3 =, \{\langle \mathsf{r}_1, 1.3, 1.4, 0.1\rangle, \langle \mathsf{r}_2, 1.1, 0.3, 0.2\rangle\}\}\}$$

Then, $\tau_{\mathcal{N}_s^o} = \{\odot, \circledast, (\mathcal{R}_1, \mathcal{A})\}$.

$int_{\mathcal{N}_s^o}(\mathcal{R}_2, \mathcal{A}) = \odot$

$int_{\mathcal{N}_s^o}(\mathcal{R}_3, \mathcal{A}) = \odot$

Then, $int_{\mathcal{N}_s^o}(\mathcal{R}_2, \mathcal{A}) \mho int_{\mathcal{N}_s^o}(\mathcal{R}_3, \mathcal{A}) = \odot$

$int_{\mathcal{N}_s^o}((\mathcal{R}_2, \mathcal{A}) \mho (\mathcal{R}_3, \mathcal{A})) = (\mathcal{R}_1, \mathcal{A})$

$\therefore int_{\mathcal{N}_s^o}((\mathcal{R}_2, \mathcal{A}) \mho (\mathcal{R}_3, \mathcal{A})) \neq int_{\mathcal{N}_s^o}(\mathcal{R}_2, \mathcal{A}) \mho int_{\mathcal{N}_s^o}(\mathcal{R}_3, \mathcal{A})$

Theorem 3.17. *Let $(\mathcal{H}, \tau_{\mathcal{N}_s^o})$ be a $\mathcal{N}_s^o$-topological space on $\mathcal{H}$.Let $\mathcal{R}$ and $\mathcal{Q}$ in τ_{NOSCS}.Then,*

(i) $cl_{\mathcal{N}_s^o}(\odot) = \odot$ and $cl_{\mathcal{N}_s^o}(\circledast) = \circledast$.

(ii) $cl_{\mathcal{N}_s^o}(\mathcal{R}) \supseteq \mathcal{R}$.

(iii) $\mathcal{Q}$ is a NOSCS iff $\mathcal{Q} = cl_{\mathcal{N}_s^o}(\mathcal{Q})$.

(iv) $cl_{\mathcal{N}_s^o}(cl_{\mathcal{N}_s^o}(\mathcal{R})) = cl_{\mathcal{N}_s^o}(\mathcal{R})$

(v) $\mathcal{R} \subseteq \mathcal{Q} \implies cl_{\mathcal{N}_s^o}(\mathcal{R}) \subseteq cl_{\mathcal{N}_s^o}(\mathcal{Q})$

(vi) $cl_{\mathcal{N}_s^o}(\mathcal{R}) \mho cl_{\mathcal{N}_s^o}(\mathcal{Q}) = cl_{\mathcal{N}_s^o}(\mathcal{R} \mho \mathcal{Q})$

(vii) $cl_{\mathcal{N}_s^o}(\mathcal{R} \Omega \mathcal{Q}) \subseteq cl_{\mathcal{N}_s^o}(\mathcal{R}) \Omega cl_{\mathcal{N}_s^o}(\mathcal{Q})$

Proof. (i) and (ii) are obviously true.

Proof of (vi) and (vii) similar to the Theorem 2.3 (vi) and (vii)

(iii) If $\mathcal{R}$ is a NOSCS on $\mathcal{H}$ then $\mathcal{R}$ is itself a NOSCS over $\mathcal{H}$ which contains $\mathcal{R}$.

$\therefore \mathcal{R}$ is a smallest NOSCS containing $\mathcal{R}$. and $\mathcal{R} = cl_{\mathcal{N}_s^o}(\mathcal{R})$.

Conversely, Suppose that $\mathcal{R} = cl_{\mathcal{N}_s^o}(\mathcal{R})$. As. $\mathcal{R}$ is a NOSCS, so $\mathcal{R}$ is a NOSCS over $\mathcal{H}$.

(vi) $\mathcal{R}$ is a NOSCS then by the proof (iii)

$\mathcal{R} = cl_{\mathcal{N}_s^o}(\mathcal{R})$..

(v) Suppose $\mathcal{R} \subseteq \mathcal{Q}$.Then every neutrosophic over soft closed super-set of $\mathcal{Q}$ also contained in $\mathcal{R}$.

$\implies$ super-sets of $\mathcal{Q}$ is also a NOSCS. Thus,

$cl_{\mathcal{N}_s^o}(\mathcal{R}) = cl_{\mathcal{N}_s^o}(\mathcal{Q})$. □

Example 3.18. Let $\mathcal{W} = \{\mathsf{r}_1, \mathsf{r}_2\}$ be the two team in an company and $\mathcal{A} = \{\text{punctuality}(\mathsf{q}_1), \text{accuracy of target}(\mathsf{q}_2)\}$. Then $\mathcal{R}$,$\mathcal{Q}$ and $\mathcal{V} \in \tau_{NOSOS}$ such that

$$(\mathcal{R}_1, \mathcal{A}) = \{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.3, 0.3, 0.1\rangle, \langle \mathsf{r}_2, 1.3, 0.5, 0.3\rangle, \{\mathsf{q}_2, \{\langle \mathsf{r}_1, 1.1, 0.3, 0.1\rangle, \langle \mathsf{r}_2, 1.2, 0.5, 0.4\rangle\}\}\}, \{\mathsf{q}_3 = \{\langle \mathsf{r}_1, 1.2, 0.3, 0.1\rangle, \langle \mathsf{r}_2, 1.1, 0.5, 0.2\rangle\}\}\}$$

$$(\mathcal{R}_2, \mathcal{A}) = \{\{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.2, 1.2, 0.1\rangle, \langle \mathsf{r}_2, 1.6, 0.6, 0.5\rangle\}\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, 1.1, 1.1, 0.1\rangle, \langle \mathsf{r}_2, 1.5, 0.5, 0.5\rangle\}\}, \{\mathsf{q}_3 = \{\langle \mathsf{r}_1, 1.1, 1.2, 0.1\rangle, \langle \mathsf{r}_2, 1.4, 0.6, 0.4\rangle\}\}\}$$

$(\mathcal{R}_3, \mathcal{A}) = \{\{\mathsf{q}_1 = \{\langle \mathsf{r}_1, 1.4, 1.4, 0.2\rangle, \langle \mathsf{r}_2, 1.2, 0.4, 0.2\rangle\}\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, 1.3, 1.1, 0.2\rangle, \langle \mathsf{r}_2, 1.1, 0.4, 0.1\rangle\}\},$
$\{\mathsf{q}_3 =, \{\langle \mathsf{r}_1, 1.3, 1.4, 0.1\rangle, \langle \mathsf{r}_2, 1.1, 0.3, 0.2\rangle\}\}\}$

Then, $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} = \{\odot, \circledast, (\mathcal{R}_1, \mathcal{A})\}$.

$\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}^{☾} = \{\circledast, \odot, \mathcal{R}^{☾}\}$

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}_2, \mathcal{A}) = \circledast$

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}_3, \mathcal{A}) = \circledast$

Then, $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}_2, \mathcal{A}) \mho cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}_3, \mathcal{A}) = \circledast$

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}((\mathcal{R}_2, \mathcal{A}) \mho (\mathcal{R}_3, \mathcal{A})) = \circledast$

$\therefore cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}((\mathcal{R}_2, \mathcal{A}) \mho (\mathcal{R}_3, \mathcal{A})) = cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}_2, \mathcal{A}) \mho cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R}_3, \mathcal{A})$

4. Measure Of Correlation for Neutrosophic Over Soft Set

Definition 4.1. Let $\mathcal{J} = (\mathcal{J}_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}, \mathcal{E})$ and $\mathcal{W} = (\mathcal{W}_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}, \mathcal{E})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set over an non-empty set $\mathcal{H}$ is of the form

$$\mathcal{J} = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{W}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

Then the $\aleph_{\mathfrak{s}}$ correlation coefficient of $\mathcal{J}$ and $\mathcal{W}$ is

$$\varphi(\mathcal{J}, \mathcal{W}) = \frac{\mathcal{K}(\mathcal{J}, \mathcal{W})}{n(\sqrt{\mathcal{K}(\mathcal{J}, \mathcal{J}).\mathcal{K}(\mathcal{W}, \mathcal{W})})} \tag{5}$$

Where

$$\mathcal{K}(\mathcal{J}, \mathcal{W}) = \sum_{\varrho=1}^{n} \left((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\right)$$

$$\mathcal{K}(\mathcal{J}, \mathcal{J}) = \sum_{\varrho=1}^{n} \left((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2\right)$$

$$\mathcal{K}(\mathcal{W}, \mathcal{W}) = \sum_{\varrho=1}^{n} \left((\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\right)$$

Proposition 4.2. *Let $\mathcal{J}$ and $\mathcal{W}$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set on an non-empty set $\mathcal{H}$. Then it satisfies the following condition*

1. $0 \leq \varphi(\mathcal{J}, \mathcal{W}) \leq 1$
2. $\varphi(\mathcal{J}, \mathcal{W}) = \frac{1}{n}$ *iff* $\mathcal{J} = \mathcal{W}$
3. $\varphi(\mathcal{J}, \mathcal{W}) = 1$ *iff* $\mathcal{J} = \mathcal{W}$ *and* $n = 1$
4. $\varphi(\mathcal{J}, \mathcal{W}) = \varphi(\mathcal{W}, \mathcal{J})$

Proof. 1. $0 \leq \varphi(\mathcal{J}, \mathcal{W}) \leq 1$

The definition of $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set is conclude that $\aleph, \eth, \Upsilon : \mathcal{H} \to [0, \Omega]$ so $0 \leq \varphi(\mathcal{J}, \mathcal{W})$

Now we have to prove $\varphi(\mathcal{J}, \mathcal{W}) \leq 1$

$$\mathcal{K}(\mathcal{J}, \mathcal{W}) = \sum_{\varrho=1}^{n} \left((\aleph_{\mathcal{J}}(\mathsf{h}_{\varrho}))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_{\varrho}))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_{\varrho}))^2.(\eth_{\mathcal{W}}(\mathsf{h}_{\varrho}))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_{\varrho}))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_{\varrho}))^2 \right)$$
$$= [(\aleph_{\mathcal{J}}(\mathsf{h}_1))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_1))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_1))^2.(\eth_{\mathcal{W}}(\mathsf{h}_1))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^2]$$
$$+[(\aleph_{\mathcal{J}}(\mathsf{h}_2))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_2))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_2))^2.(\eth_{\mathcal{W}}(\mathsf{h}_2))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^2]$$
$$+ \cdots + [(\aleph_{\mathcal{J}}(\mathsf{h}_n))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_n))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_n))^2.(\eth_{\mathcal{W}}(\mathsf{h}_n))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^2]$$

By Cauchy-Schwartz inequality, we get

$$(\mathcal{K}(\mathcal{J}, \mathcal{W}))^2 \leq \Big([(\aleph_{\mathcal{J}}(\mathsf{h}_1))^4 + (\eth_{\mathcal{J}}(\mathsf{h}_1))^4 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^4] + [(\aleph_{\mathcal{J}}(\mathsf{h}_2))^4 + (\eth_{\mathcal{J}}(\mathsf{h}_2))^4 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^4] + \cdots$$
$$+[(\aleph_{\mathcal{J}}(\mathsf{h}_n))^4 + (\eth_{\mathcal{J}}(\mathsf{h}_n))^4 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^4]\Big).\Big([(\aleph_{\mathcal{W}}(\mathsf{h}_1))^4 + (\eth_{\mathcal{W}}(\mathsf{h}_1))^4 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^4]$$
$$+[(\aleph_{\mathcal{W}}(\mathsf{h}_2))^4 + (\eth_{\mathcal{W}}(\mathsf{h}_2))^4 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^4] + \cdots + [(\aleph_{\mathcal{W}}(\mathsf{h}_n))^4 + (\eth_{\mathcal{W}}(\mathsf{h}_n))^4 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^4]\Big)$$

$$\leq \Big([(\aleph_{\mathcal{J}}(\mathsf{h}_1))^2(\aleph_{\mathcal{J}}(\mathsf{h}_1))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_1))^2(\eth_{\mathcal{J}}(\mathsf{h}_1))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^2(\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^2] + [(\aleph_{\mathcal{J}}(\mathsf{h}_2))^2(\aleph_{\mathcal{J}}(\mathsf{h}_2))^2$$
$$+(\eth_{\mathcal{J}}(\mathsf{h}_2))^2(\eth_{\mathcal{J}}(\mathsf{h}_2))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^2(\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^2] + \cdots + [(\aleph_{\mathcal{J}}(\mathsf{h}_n))^2(\aleph_{\mathcal{J}}(\mathsf{h}_n))^2$$
$$+(\eth_{\mathcal{J}}(\mathsf{h}_n))^2(\eth_{\mathcal{J}}(\mathsf{h}_n))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^2(\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^2]\Big)\Big([(\aleph_{\mathcal{W}}(\mathsf{h}_1))^2(\aleph_{\mathcal{W}}(\mathsf{h}_1))^2 + (\eth_{\mathcal{W}}(\mathsf{h}_1))^2(\eth_{\mathcal{W}}(\mathsf{h}_1))^2$$
$$+(\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^2(\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^2] + [(\aleph_{\mathcal{W}}(\mathsf{h}_2))^2(\aleph_{\mathcal{W}}(\mathsf{h}_2))^2 + (\eth_{\mathcal{W}}(\mathsf{h}_2))^2(\eth_{\mathcal{W}}(\mathsf{h}_2))^2 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^2(\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^2]$$
$$+ \cdots + [(\aleph_{\mathcal{W}}(\mathsf{h}_n))^2(\aleph_{\mathcal{W}}(\mathsf{h}_n))^2 + (\eth_{\mathcal{W}}(\mathsf{h}_n))^2 + (\eth_{\mathcal{W}}(\mathsf{h}_n))^2 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^2(\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^2]\Big)$$
$$= (\mathcal{K}(\mathcal{J}, \mathcal{J}))(\mathcal{K}(\mathcal{W}, \mathcal{W}))$$
$$\implies (\mathcal{K}(\mathcal{J}, \mathcal{W}))^2 \leq [(\mathcal{K}(\mathcal{J}, \mathcal{W}))(\mathcal{K}(\mathcal{J}, \mathcal{W}))]$$
$$\implies \mathcal{K}(\mathcal{J}, \mathcal{W}) \leq \sqrt{[(\mathcal{K}(\mathcal{J}, \mathcal{W}))(\mathcal{K}(\mathcal{J}, \mathcal{W}))]}$$

Then,

$$\mathcal{K}(\mathcal{J}, \mathcal{W}) \leq n\sqrt{[(\mathcal{K}(\mathcal{J}, \mathcal{W}))(\mathcal{K}(\mathcal{J}, \mathcal{W}))]}$$
$$\therefore 0 \leq \varphi(\mathcal{J}, \mathcal{W}) \leq 1$$

2. $\varphi(\mathcal{J}, \mathcal{W}) = \frac{1}{n}$ iff $\mathcal{J} = \mathcal{W}$

given that, $\mathcal{J} = \mathcal{W}$

$$\implies \mathcal{K}(\mathcal{J}, \mathcal{W}) = \mathcal{K}(\mathcal{W}, \mathcal{W}) \tag{6}$$

$$\mathcal{K}(\mathcal{J}, \mathcal{J}) = \mathcal{K}(\mathcal{W}, \mathcal{W}) \tag{7}$$

From (6) and (7), we get

$$\varphi(\mathcal{J},\mathcal{W}) = \frac{\mathcal{K}(\mathcal{J},\mathcal{W})}{n(\sqrt{\mathcal{K}(\mathcal{J},\mathcal{J}).\mathcal{K}(\mathcal{W},\mathcal{W})})}$$

$$\varphi(\mathcal{J},\mathcal{W}) = \frac{\mathcal{K}(\mathcal{W},\mathcal{W})}{n(\sqrt{\mathcal{K}(\mathcal{W},\mathcal{W}).\mathcal{K}(\mathcal{W},\mathcal{W})})}$$

$$\varphi(\mathcal{J},\mathcal{W}) = \frac{1}{n} \tag{8}$$

3. $\varphi(\mathcal{J},\mathcal{W}) = 1$ iff $\mathcal{J} = \mathcal{W}$ and $n = 1$

Put $n = 1$ in (8) we get,

$$\varphi(\mathcal{J},\mathcal{W}) = 1$$

4. $\varphi(\mathcal{J},\mathcal{W}) = \varphi(\mathcal{W},\mathcal{J})$

$$\varphi(\mathcal{J},\mathcal{W}) = \frac{\mathcal{K}(\mathcal{J},\mathcal{W})}{n(\sqrt{\mathcal{K}(\mathcal{J},\mathcal{J}).\mathcal{K}(\mathcal{W},\mathcal{W})})}$$

$$\varphi(\mathcal{W},\mathcal{J}) = \frac{\mathcal{K}(\mathcal{W},\mathcal{J})}{n(\sqrt{\mathcal{K}(\mathcal{W},\mathcal{W}).\mathcal{K}(\mathcal{J},\mathcal{J})})}$$

Only we have to prove $\mathcal{K}(\mathcal{J},\mathcal{W}) = \mathcal{K}(\mathcal{W},\mathcal{J})$

$$\mathcal{K}(\mathcal{J},\mathcal{W}) = \sum_{\varrho=1}^{n} \left((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\right)$$

$$= \sum_{\varrho=1}^{n} \left((\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2\right) = \mathcal{K}(\mathcal{W},\mathcal{J})$$

$$\implies \mathcal{K}(\mathcal{J},\mathcal{W}) = \mathcal{K}(\mathcal{W},\mathcal{J})$$

□

Definition 4.3. Let $\mathcal{J}$ and $\mathcal{W}$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set on an non-empty set $\mathcal{H}$ is of the form

$$\mathcal{J} = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{W}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

Then the $\ddot{\Upsilon}_{\mathfrak{s}}$ correlation coefficient of $\mathcal{J}$ and $\mathcal{W}$ is

$$\varphi^*(\mathcal{J},\mathcal{W}) = \frac{\mathcal{K}(\mathcal{J},\mathcal{W})}{n\big[min(\mathcal{K}(\mathcal{J},\mathcal{J}), \mathcal{K}(\mathcal{W},\mathcal{W}))\big]} \tag{9}$$

Where

$$\mathcal{K}(\mathcal{J},\mathcal{W}) = \sum_{\varrho=1}^{n} \left((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\right)$$

$$\mathcal{K}(\mathcal{J},\mathcal{J})=\sum_{\varrho=1}^{n}\left((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2+(\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2\right)$$

$$\mathcal{K}(\mathcal{W},\mathcal{W})=\sum_{\varrho=1}^{n}\left((\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2+(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2+(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\right)$$

Proposition 4.4. *Let $\mathcal{J}$ and $\mathcal{W}$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set on an non-empty set $\mathcal{H}$. Then it satisfies the following condition*

1. $0 \leq \varphi^*(\mathcal{J},\mathcal{W}) \leq 1$
2. $\varphi^*(\mathcal{J},\mathcal{W}) = \frac{1}{n}$ *iff* $\mathcal{J}=\mathcal{W}$
3. $\varphi^*(\mathcal{J},\mathcal{W}) = 1$ *iff* $\mathcal{J}=\mathcal{W}$ *and* $n=1$
4. $\varphi^*(\mathcal{J},\mathcal{W}) = \varphi^*(\mathcal{W},\mathcal{J})$

Proof. 1. $0 \leq \varphi^*(\mathcal{J},\mathcal{W}) \leq 1$

By the definition of $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set we know that $\aleph,\eth,\Upsilon:\mathcal{H}\to[0,\Omega]$ so $0 \leq \varphi^*(\mathcal{J},\mathcal{W})$

Now we have to prove $\varphi(\mathcal{J},\mathcal{W}) \leq 1$

$$\mathcal{K}(\mathcal{J},\mathcal{W})=\sum_{\varrho=1}^{n}\left((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2+(\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\right)$$

$$=[(\aleph_{\mathcal{J}}(\mathsf{h}_1))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_1))^2+(\eth_{\mathcal{J}}(\mathsf{h}_1))^2.(\eth_{\mathcal{W}}(\mathsf{h}_1))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^2]$$
$$+[(\aleph_{\mathcal{J}}(\mathsf{h}_2))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_2))^2+(\eth_{\mathcal{J}}(\mathsf{h}_2))^2.(\eth_{\mathcal{W}}(\mathsf{h}_2))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^2]$$
$$+\cdots+[(\aleph_{\mathcal{J}}(\mathsf{h}_n))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_n))^2+(\eth_{\mathcal{J}}(\mathsf{h}_n))^2.(\eth_{\mathcal{W}}(\mathsf{h}_n))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^2]$$

By Cauchy-Schwartz inequality, we get

$$(\mathcal{K}(\mathcal{J},\mathcal{W}))^2 \leq \Big([(\aleph_{\mathcal{J}}(\mathsf{h}_1))^4+(\eth_{\mathcal{J}}(\mathsf{h}_1))^4+(\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^4]+[(\aleph_{\mathcal{J}}(\mathsf{h}_2))^4+(\eth_{\mathcal{J}}(\mathsf{h}_2))^4+(\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^4]+\cdots$$
$$+[(\aleph_{\mathcal{J}}(\mathsf{h}_n))^4+(\eth_{\mathcal{J}}(\mathsf{h}_n))^4+(\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^4]\Big).\Big([(\aleph_{\mathcal{W}}(\mathsf{h}_1))^4+(\eth_{\mathcal{W}}(\mathsf{h}_1))^4+(\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^4]$$
$$+[(\aleph_{\mathcal{W}}(\mathsf{h}_2))^4+(\eth_{\mathcal{W}}(\mathsf{h}_2))^4+(\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^4]+\cdots+[(\aleph_{\mathcal{W}}(\mathsf{h}_n))^4+(\eth_{\mathcal{W}}(\mathsf{h}_n))^4+(\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^4]\Big)$$

$$\leq \Big([(\aleph_{\mathcal{J}}(\mathsf{h}_1))^2(\aleph_{\mathcal{J}}(\mathsf{h}_1))^2+(\eth_{\mathcal{J}}(\mathsf{h}_1))^2(\eth_{\mathcal{J}}(\mathsf{h}_1))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^2(\Upsilon_{\mathcal{J}}(\mathsf{h}_1))^2]+[(\aleph_{\mathcal{J}}(\mathsf{h}_2))^2(\aleph_{\mathcal{J}}(\mathsf{h}_2))^2$$
$$+(\eth_{\mathcal{J}}(\mathsf{h}_2))^2(\eth_{\mathcal{J}}(\mathsf{h}_2))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^2(\Upsilon_{\mathcal{J}}(\mathsf{h}_2))^2]+\cdots+[(\aleph_{\mathcal{J}}(\mathsf{h}_n))^2(\aleph_{\mathcal{J}}(\mathsf{h}_n))^2$$
$$+(\eth_{\mathcal{J}}(\mathsf{h}_n))^2(\eth_{\mathcal{J}}(\mathsf{h}_n))^2+(\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^2(\Upsilon_{\mathcal{J}}(\mathsf{h}_n))^2]\Big)\Big([(\aleph_{\mathcal{W}}(\mathsf{h}_1))^2(\aleph_{\mathcal{W}}(\mathsf{h}_1))^2+(\eth_{\mathcal{W}}(\mathsf{h}_1))^2(\eth_{\mathcal{W}}(\mathsf{h}_1))^2$$
$$+(\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^2(\Upsilon_{\mathcal{W}}(\mathsf{h}_1))^2]+[(\aleph_{\mathcal{W}}(\mathsf{h}_2))^2(\aleph_{\mathcal{W}}(\mathsf{h}_2))^2+(\eth_{\mathcal{W}}(\mathsf{h}_2))^2(\eth_{\mathcal{W}}(\mathsf{h}_2))^2+(\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^2(\Upsilon_{\mathcal{W}}(\mathsf{h}_2))^2]$$
$$+\cdots+[(\aleph_{\mathcal{W}}(\mathsf{h}_n))^2(\aleph_{\mathcal{W}}(\mathsf{h}_n))^2+(\eth_{\mathcal{W}}(\mathsf{h}_n))^2+(\eth_{\mathcal{W}}(\mathsf{h}_n))^2+(\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^2(\Upsilon_{\mathcal{W}}(\mathsf{h}_n))^2]\Big)$$

$$=(\mathcal{K}(\mathcal{J},\mathcal{J}))(\mathcal{K}(\mathcal{W},\mathcal{W}))$$

$$\implies (\mathcal{K}(\mathcal{J},\mathcal{W}))^2 \leq [(\mathcal{K}(\mathcal{J},\mathcal{W}))(\mathcal{K}(\mathcal{J},\mathcal{W}))]$$

Then,

$$\mathcal{K}(\mathcal{J},\mathcal{W}) \le n\big[min[(\mathcal{K}(\mathcal{J},\mathcal{W})),(\mathcal{K}(\mathcal{J},\mathcal{W}))]\big]$$

2. $\varphi(\mathcal{J},\mathcal{W}) = \frac{1}{n}$ iff $\mathcal{J} = \mathcal{W}$

given that, $\mathcal{J} = \mathcal{W}$

$$\implies \mathcal{K}(\mathcal{J},\mathcal{W}) = \mathcal{K}(\mathcal{W},\mathcal{W}) \tag{10}$$

$$\mathcal{K}(\mathcal{J},\mathcal{J}) = \mathcal{K}(\mathcal{W},\mathcal{W}) \tag{11}$$

From (10) and (11), we get

$$\varphi^*(\mathcal{J},\mathcal{W}) = \frac{\mathcal{K}(\mathcal{J},\mathcal{W})}{n\big[min[\mathcal{K}(\mathcal{J},\mathcal{J}).\mathcal{K}(\mathcal{W},\mathcal{W})]\big]}$$

$$\varphi^*(\mathcal{J},\mathcal{W}) = \frac{\mathcal{K}(\mathcal{W},\mathcal{W})}{n\big[min[\mathcal{K}(\mathcal{W},\mathcal{W}).\mathcal{K}(\mathcal{W},\mathcal{W})]\big]}$$

$$\varphi^*(\mathcal{J},\mathcal{W}) = \frac{1}{n} \tag{12}$$

3. $\varphi^*(\mathcal{J},\mathcal{W}) = 1$ iff $\mathcal{J} = \mathcal{W}$ and $n = 1$

Put $n = 1$ in (12) we get,

$$\varphi^*(\mathcal{J},\mathcal{W}) = 1$$

4. $\varphi^*(\mathcal{J},\mathcal{W}) = \varphi(\mathcal{W},\mathcal{J})$

$$\varphi^*(\mathcal{J},\mathcal{W}) = \frac{\mathcal{K}(\mathcal{J},\mathcal{W})}{n\big[min[\mathcal{K}(\mathcal{J},\mathcal{J}).\mathcal{K}(\mathcal{W},\mathcal{W})]\big]}$$

$$\varphi^*(\mathcal{W},\mathcal{J}) = \frac{\mathcal{K}(\mathcal{W},\mathcal{J})}{n\big[min[\mathcal{K}(\mathcal{W},\mathcal{W}).\mathcal{K}(\mathcal{J},\mathcal{J})]\big]}$$

Only we have to prove $\mathcal{K}(\mathcal{J},\mathcal{W}) = \mathcal{K}(\mathcal{W},\mathcal{J})$

$$\mathcal{K}(\mathcal{J},\mathcal{W}) = \sum_{\varrho=1}^{n}\big((\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2\big)$$

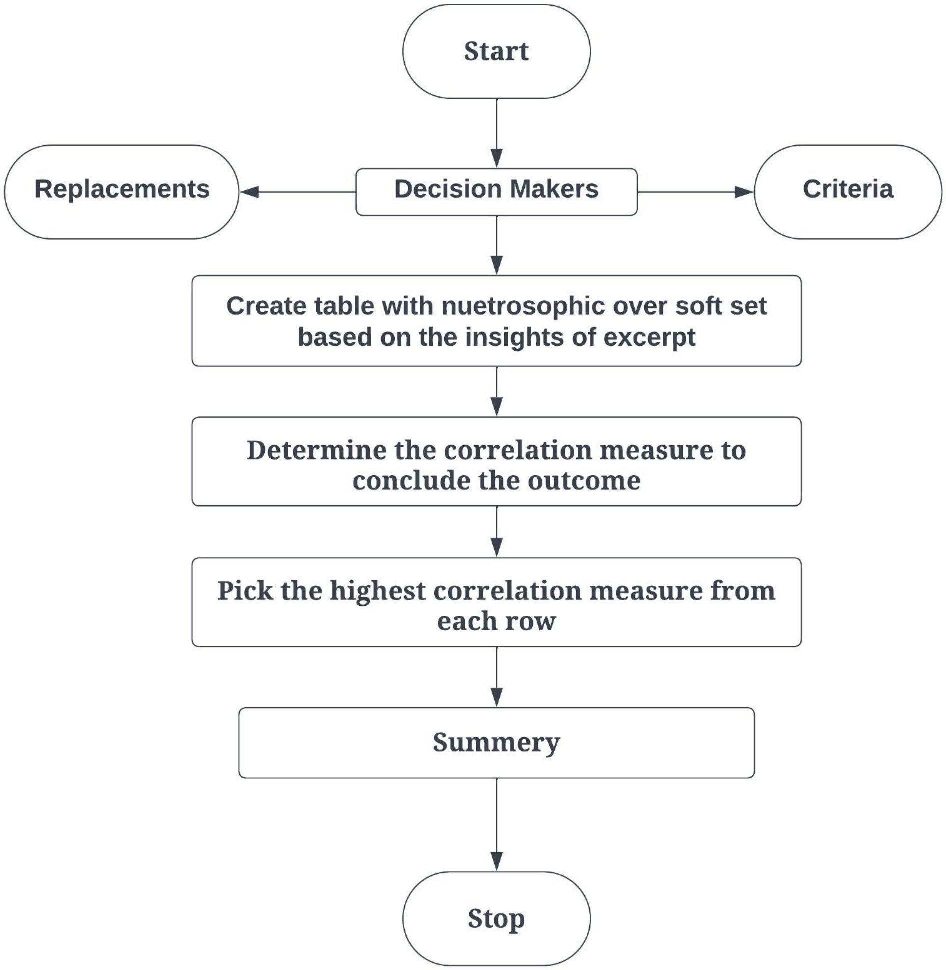
$$= \sum_{\varrho=1}^{n}\big((\aleph_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\aleph_{\mathcal{J}}(\mathsf{h}_\varrho))^2 + (\eth_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\eth_{\mathcal{J}}(\mathsf{h}_\varrho))^2 + (\Upsilon_{\mathcal{W}}(\mathsf{h}_\varrho))^2.(\Upsilon_{\mathcal{J}}(\mathsf{h}_\varrho))^2\big) = \mathcal{K}(\mathcal{W},\mathcal{J})$$

$$\implies \mathcal{K}(\mathcal{J},\mathcal{W}) = \mathcal{K}(\mathcal{W},\mathcal{J})$$

$$\therefore \varphi^*(\mathcal{J},\mathcal{W}) = \varphi^*(\mathcal{W},\mathcal{J})$$

□

5. Flow Chart To Solving $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set Using Correlation Measure



6. Numerical Illustration

Assume an effective instance that helps the awarding committee to make a decision to find top-performing student of the year 2022-2023.top-performing students are selected not only by their education also with many criteria.

Similar situation arises for GFC school they are conducting an competition to select a top-performing student. Now they have to superior one student out of three. So for this situation we are applying $\aleph_{\mathfrak{s}}$ and $\ddot{Y}_{\mathfrak{s}}$ correlation for the set $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set.

Replacement and Criteria:

Let us take three students as S, R, Y .Required Qualities as Education, Self Discipline, Honesty and Awards as First Place, Second Place and Third Place.[Replacement={Students,Awards} and Criteria={Education, Self Discipline, Honesty}]

Where, Education={clearance of all subject,knowledge over subject,general knowledge,team work,creativity in project work,involvement in school educational and sports programs}

Self Discipline= {acceptance,willpower,hard work,persistence,punctuality,regular practice,behaviour}

Honesty={truthfulness,trust towards the student,sincere in following rules and regulation}

Analized Data:

X	Education	Self Discipline	Honesty
S	(1.8,0.7,0.6)	(1.5,0.3,0.8)	(1.6,0.9,0.4)
Y	(1.2,0.7,0.9)	(1.6,0.7,0.6)	(1.4,0.6,0.3)
R	(1.3,1.1,0.5)	(0.8,1.5,0.6)	(1.4,0.8,0.8)

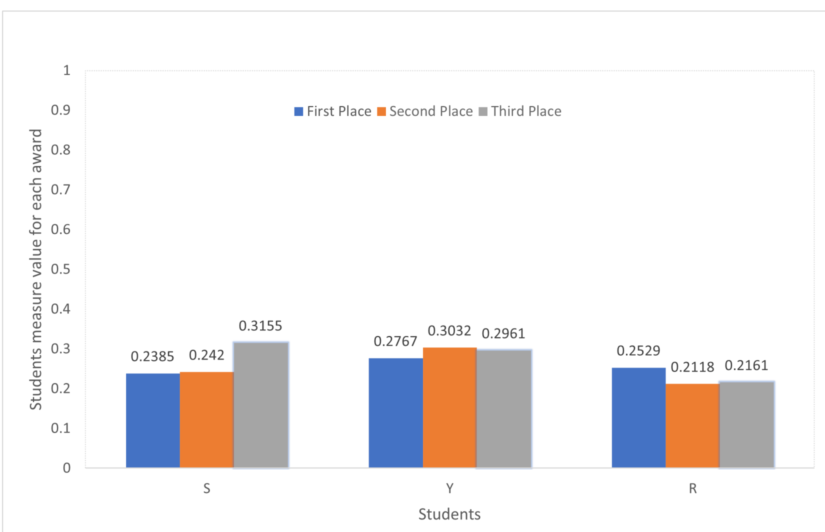
TABLE 1. Relation between Students and Required Qualities

Z	First Place	Second Place	Third Place
Education	(0.9,1.5,0.4)	(0.5,1.3,0.6)	(1.5,0.5,0.6)
Self Discipline	(1.1,0.5,0.3)	(1.5,0.6,0.7)	(1.4,0.3,0.7)
Honesty	(1.3,0.7,0.8)	(1.1,0.7,0.4)	(0.9,0.8,0.6)

TABLE 2. Relation between Required Qualities and Places

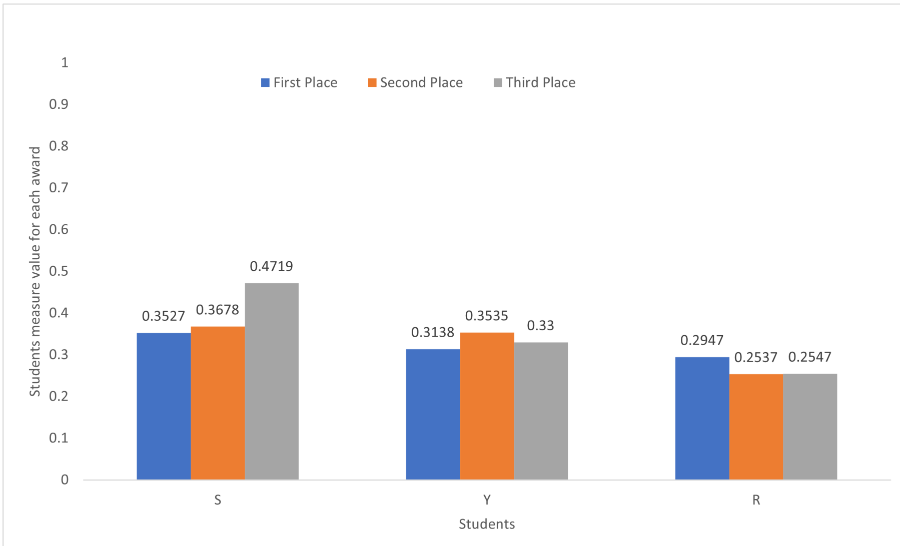
φ	First Place	Second Place	Third Place
S	0.2385	0.2420	**0.3155**
Y	0.2767	**0.3032**	0.2961
R	**0.2529**	0.2118	0.2161

TABLE 3. $\aleph_5$ Correlation between Students and Places



φ^*	**First Place**	**Second Place**	**Third Place**
S	0.3527	0.3678	**0.4719**
Y	0.3138	**0.3535**	0.3300
R	**0.2947**	0.2537	0.2547

TABLE 4. $\ddot{Y}_{\mathfrak{s}}$ Correlation between Students and Places



Student	**Place**
S	Third Place
Y	Second Place
R	First Place

TABLE 5. Summery

Thus the student **R** got first place and awarded for the top-performing student of the year 2022-2023.Also, Student **Y** and **S** got second and third place.

7. Conclusion

This manuscript breaks new ground by offering a novel perspective on correlation within the realm of Neutrosophic Over Soft Sets, contributing to the advancement of theoretical frameworks in this specialized mathematical field. In addition to presenting a comprehensive exploration of various operational characteristics inherent to Neutrosophic Over Soft Sets, the manuscript introduces a fresh analytical approach by proposing a novel formula to quantify correlation. The $\aleph_{\mathfrak{s}}$ correlation and $\ddot{Y}_{\mathfrak{s}}$ correlation extend beyond traditional measures, providing a nuanced understanding of the relationships within this mathematical framework.

Furthermore, the manuscript demonstrates the practical applicability of the introduced correlation measures through an illustrative scenario. In this scenario, Students **R**, **Y**, and **S** secure the first, second, and third positions, respectively, culminating in the designation of Student **R** as the top-performing student for the academic year 2022-2023. This synthesis of theoretical innovation and practical application not only adds depth to the study of Neutrosophic Over Soft Sets but also underscores the real-world relevance of the proposed measures. As such, this manuscript holds significant implications for both theoretical researchers and practitioners seeking advanced analytical tools within this mathematical domain.

In a similar vein, the utilization of the $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set correlation measure extends across a variety of domains, encompassing fields like medicine, industry, and construction.

References

[1] Atanassov, K.T., 2012. On intuitionistic fuzzy sets theory (Vol. 283). *Springer*.

[2] Bellman, R.E. and Zadeh, L.A., 1970. Decision-making in a fuzzy environment.*Management science*, 17(4), pp.B-141.

[3] Broumi, S., 2013. Generalized neutrosophic soft set. *Infinite Study*.

[4] Broumi, S. and Deli, I., 2015. Correlation measure for neutrosophic refined sets and its application in medical diagnosis. *Infinite Study*.

[5] Bustince, H. and Burillo, P., 1995. Correlation of interval-valued intuitionistic fuzzy sets. *Fuzzy sets and systems*, 74(2), pp.237-244.

[6] Christianto, V., Boyd, R.N. and Smarandache, F., 2020. Three possible applications of neutrosophic logic in fundamental and applied sciences. *Infinite Study*.

[7] Devi, R.N. and Parthiban, Y., 2023. Decision making process over neutrosophic pythagorean soft sets using mesure of correlation. *Southeast Europe Journal of Soft Computing*, 12(2), pp.09-09.

[8] Devi, R.N. and Muthumari, G., 2021. View On Neutrosophic Over Topologized Domination Graphs. *Neutrosophic Sets and Systems*, 47(1), p.33.

[9] Narmada Devi, R., Rasappan, S. and Obaid, A.J., 2022, June. Novel on Digital Neutrosophic Topological Spaces. In Proceedings of 2nd International Conference on Mathematical Modeling and Computational Science: ICMMCS 2021 (pp. 213-219). *Singapore: Springer Nature Singapore*.

[10] Murugesan, R., Parthiban, Y., Devi, R., Mohan, K.R. and Kumarave, S.K., 2023. A Comparative Study of Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps on Covid Variants. *Neutrosophic Sets and Systems*, 55(1), p.20.

[11] Jansi, R., Mohana, K. and Smarandache, F., 2019. Correlation measure for Pythagorean neutrosophic sets with T and F as dependent neutrosophic components. *Infinite Study*.

[12] Maji, P.K., Biswas, R. and Roy, A.R., 2003. Soft set theory. *Computers & mathematics with applications*, 45(4-5), pp.555-562.

[13] Mallick, R. and Pramanik, S., 2020. Pentapartitioned neutrosophic set and its properties (Vol. 36). *Infinite Study*.

[14] Molodtsov, D., 1999. Soft set theory?first results.*Computers & mathematics with applications*, 37(4-5), pp.19-31.

[15] Radha, R., Mary, A.S.A., Prema, R. and Broumi, S., 2021. Neutrosophic pythagorean sets with dependent neutrosophic pythagorean components and its improved correlation coefficients. *Neutrosophic Sets and Systems*, 46, pp.77-86.

[16] Rodrigo, P.A. and Maheswari, S., 2023. Neutrosophic gsα*-Open and Closed Maps in Neutrosophic Topological Spaces. Neutrosophic Systems with Applications, 8, pp.42-49.

[17] Saeed, M., Saqlain, M., Mehmood, A. and Yaqoob, S., 2020. Multi-polar neutrosophic soft sets with application in medical diagnosis and Decision-making. *Neutrosophic Sets and Systems*, 33, pp.183-207.

[18] Saqlain, M., Jafar, N., Moin, S., Saeed, M. and Broumi, S., 2020. Single and multi-valued neutrosophic hypersoft set and tangent similarity measure of single valued neutrosophic hypersoft sets. *Neutrosophic Sets and Systems*, 32(1), pp.317-329.

[19] Smarandache, F. and Pramanik, S., 2016. New trends in neutrosophic theory and applications (Vol. 1). *Infinite Study*.

[20] Smarandache, F., 2016. Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset. Similarly for Neutrosophic Over-/Under-/Off-Logic, Probability, and Statistics. *Infinite Study*.

[21] Wang, H., Smarandache, F., Zhang, Y. and Sunderraman, R., 2010. Single valued neutrosophic sets. *Infinite study*, 12.

[22] Ye, J., 2013. Another form of correlation coefficient between single valued neutrosophic sets and its multiple attribute decision-making method. *Neutrosophic Sets and systems*, 1(1), pp.8-12.

[23] Zadeh, L.A., 1965. Fuzzy sets. *Information and control*, 8(3), pp.338-353.

[24] Zadeh, L.A., 1978. Fuzzy sets as a basis for a theory of possibility. *Fuzzy sets and systems*, 1(1), pp.3-28.

Received: Jan 8, 2024. Accepted: Mar 31, 2024

Empowering Artificial Intelligence Techniques with Soft Computing of Neutrosophic Theory in Mystery Circumstances for Plant Diseases

Ahmed El-Massry[1], Florentin Smarandache[2], Mona Mohamed[3]

[1]Department of Computer Science, Faculty of Computers and Informatics, Zagazig University, Zagazig 44519, Egypt; a.elmasry24@fci.zu.edu.eg.

[2]University of New Mexico, 705 Gurley Ave., Gallup, NM 87301, USA; smarand@unm.edu.

[3]Higher Technological Institute, 10th of Ramadan City 44629, Egypt; mona.fouad@hti.edu.eg.

Abstract

Plant diseases are one of the factors that lead to yield and economic losses, which have a direct effect on national and international food production systems. One of the most essential ways to avoid agricultural product loss or reduction in amount is to diagnose plant diseases promptly and accurately. Hence, the diagnosis process for plants is crucial and should be conducted accurately. Moreover, this study focuses on this process by constructing an Artificiality Diagnostics Framework (ADF) to serve the study's objectives which entailed conducting diagnosis for plants in a professional and precise manner over uncertain environments. Thus, neutrosophic theory is considered the principal ingredient in our ADF. Due to the ability of neutrosophic to divide images into Truth (T), Falsity(F), and Indeterminacy (I). Also, deep learning (DL) is considered another principal ingredient in treating vast samples of datasets. Our comparative analysis of the leaves of potatoes is conducted whether leveraging neutrosophic and without utilizing Neutrosophic. ResNet50, ResNet152, and Mobile Net are the principal ingredients for the training dataset. The findings of implementing these networks indicated that ResNet50 achieved the highest accuracy of 0.915 in the T domain, ResNet152 achieved the highest accuracy of 0.905 in the True(T) domain, and Mobile Net achieved the highest accuracy of 0.915 in Truth(T) domain. Accuracy of 0.863 in Indeterminate(I).

Keywords: Neutrosophic theory, Deep Learning, plant disease

1. Introduction

Economic losses arise from yield losses caused by plant diseases, which have a direct effect on national and international food production systems. Oerke et al. [1] estimated that 13% of the loss in agricultural productivity worldwide is attributed to plant diseases. From the perspective of [2] comprehension of the causes of plant diseases is mandatory. A conducive habitat, the pathogen, and the host are the three elements that help diseases develop in plants. Also, drought and plant disease[3] are factors that impact agricultural productivity. As a result of the spread of many plant diseases throughout the crop after infection, [4] regular crop monitoring is necessary since prompt disease control will stop the disease's progress.

In this evolving context [5], the need to accurately and promptly identify illnesses, including early impediments, has never been higher. For decades[6], computer vision technology has been used in agriculture to identify weeds, assess crop geometric sizes, diagnose nutritional deficiencies, and predict crop yields.

Overall, plant phenotyping and precision agriculture need the diagnosis of plant diseases. While diagnosing and monitoring plant diseases is important [7], conventional approaches that require a human visual inspection are costly, time-consuming, dependent on specialists, and unsuitable for precision agriculture. Furthermore, human prejudice is likely to have an impact on these methods, reducing their accuracy. Hence [2], solved these issues by examining the utilization of image processing methods with images of plants.in the same vein [8] that employed image processing techniques to measure corn stripe disease, and it was shown that computer-based approaches outperformed traditional visual analysis in terms of accuracy.

Nevertheless [9] decided that the degree of uncertainty in the image's data increases when two-dimensional images are converted from three-dimensional ones through image processing. Furthermore, many of the concepts in the image are also vague and ambiguous. Whilst a crucial part of image processing and pattern recognition [10], image segmentation is one of the trickiest processes which has many uses and uncertainties [9] that contribute to the difficulty of segmenting images.

Wherefore fuzzy theory is used to the field of image segmentation[11] , to effectively express fuzzy concepts and information. In the discipline of image segmentation [12] , fuzzy image segmentation has grown in importance and popularity as a study area. Subsequently [13] established neutrosophic fuzzy clustering algorithm within the conventional fuzzy C-means clustering techniques. In the same vein, the study of [14] confusing data in the diagnosis of skin cancer are grouped using neutrophilic c-means clustering (NCMC). On the other hand, the notion of Neutrosophic theory is highlighted by [15] which proposed by Smarandache, where the basic distinction between fuzzy and intuitionistic fuzzy logic and set is represented by the neutral concept that is known as Neutrosophy [16].

In the realms of artificial intelligence (AI) and deep learning (DL), Neutrosophic theory (NTh)[17] offers the necessary ability to serve as a universal framework for uncertainty analysis in data sets, particularly with images.

Overall, all of these studies are considering catalyst for implementing neutrosophic theory in field of diagnosis the plant and classify it [18] into three sets as Truth (T), Falsity (F),and Indeterminacy (I).Although, we are leveraging ML especially DL [19] for the identification of diseases as a result of enhanced computing power, larger storage capabilities, and the accessibility of enormous datasets.

Herein, Plant diseases can be diagnosed by merging NTh with DL toward establishing Artificiality Diagnostics Framework (ADF). We applied the established ADF to real problem through implementing the framework on dataset consisting of around 4000 images of potato. The findings of ADF are recorded and analyzed into the results and discussion section.

2. Realm of Neutrosophic Theory

In the context of vague and ambiguity about data and circumstances, the notion of neutrosophic was founded by Smarandache[15] which extended to fuzzy logic.

2.1 Preliminary

This theory described in field of image processing [9, 16] as :

- Assum $\mathfrak{R}$ be a universe in this study. A group of image pixels represent as $\vartheta = \omega * \omega$.whilst $\omega \subseteq \mathfrak{R}$, and ω is an argument.
- Herein, neutrosophic image can be described as ∂ , F, ℓ are symbols of truth, false and indeterminacy.
- Each pixel $p(\chi, \nu)$ in the neutrosophic image can be described as $p_{Neu}(\chi, \nu) = \{\partial(\chi, \nu), \ell(\chi, \nu), F(\chi, \nu)\}$.

$$\partial(\chi, \nu) = \frac{\bar{\kappa}(\chi, \nu) - \bar{\kappa}\min}{\bar{\kappa}\max - \bar{\kappa}\min} \quad (1)$$

Where: $\bar{\kappa}(\chi, \nu)$is the region mean value of $\kappa(\chi, \nu)$

$$\bar{\kappa}(\chi, \nu) = \frac{1}{\omega * \omega} \sum_{m=i-\omega/2}^{i+\omega/2} \sum_{n=i-\omega/2}^{i+\omega/2} \kappa(m, n) \quad (2)$$

$$\ell(\chi, \nu) = \frac{\wp(\chi, \nu) - \wp_{\min}}{\wp_{\max} - \wp_{\min}} \quad (3)$$

where: $\wp(\chi, \nu)$is the absolute value of the difference between intensity $\kappa(\chi, \nu)$ and its local mean value at $\kappa(\chi, \nu)$.whilst ℓ is indeterminacy degree of p_{Neu}

$$\wp(\chi, \nu) = \mathrm{abs}\big(\kappa(\chi, \nu) - \bar{\kappa}(\chi, \nu)\big) \quad (4)$$

where: $\kappa(\chi, \nu)$ is the gray value of $p(\chi, \nu)$

$$F(\chi, \nu) = 1 - \partial(\chi, \nu) \quad (5)$$

- whereas, in interval neutrosophic, p_{INeu} described according to interval number set as:

$$p_{INeu} = \{[\partial_1(\chi, \nu), \partial_2(\chi, \nu)], [\ell_1(\chi, \nu), \ell_2(\chi, \nu)], [F_1(\chi, \nu), F_2(\chi, \nu)\} \ (6)$$

2.2 Applications of Neutrosophic

According to bibliometrics analysis [20], we are conducted analysis based on web of science (WoS) database for prior studies and terms which related to our scope. VoS viewer software is utilized for showcases the findings of queries conducted on WoS for applying neutrosophic in various domains.

2.2.1 Neutrosophic in Agriculture

We conducted bibliometric analysis based on certain keywords such as (“Agriculture” AND “Neutrosophic”) The findings of this process showcase as following Figure 1 where query conducted for Co-citation based on Co-Sources for mentioned key words. The findings indicated that 14 items which classified into two clusters. Cluster 1 has seven items with red color and cluster 2 has seven items with green color.

2.2.2 Neutrosophic in Healthcare

Figure 2 illustrated bibliometric analysis which conducted certain keywords such as (“Healthcare” AND “Neutrosophic”) for Co-citation based on Co-Sources for mentioned key words. The findings indicated that 12 items which classified into two clusters. Cluster 1 has six items with red color and cluster 2 has six items with green color.

2.2.3 Neutrosophic in Climate

The conducted bibliometric analysis for Co-citation based on Co-Sources for utilizing neutrosophic in climate has been represented in Figure 3. According to this Figure, there are two clusters. Each cluster has 6 items. Overall, there are 12 items resulted of query ("Climate" AND "Neutrosophic") for Co-citation based on Co-Sources.

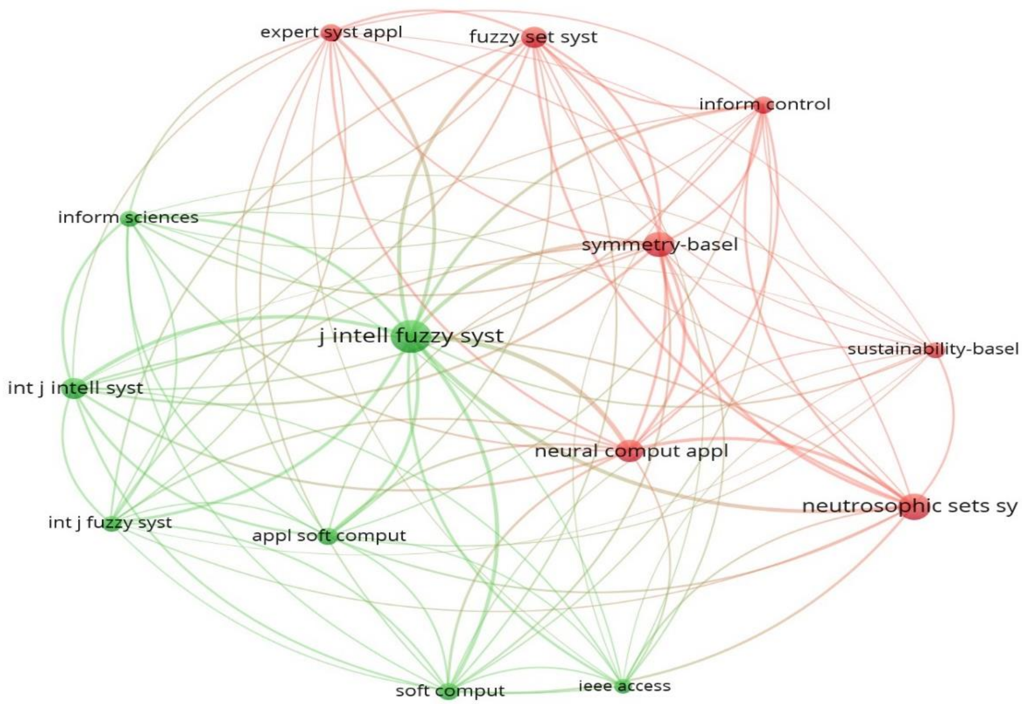


Figure 1. Visualization network for Co-citation based on Co-Sources in agriculture.

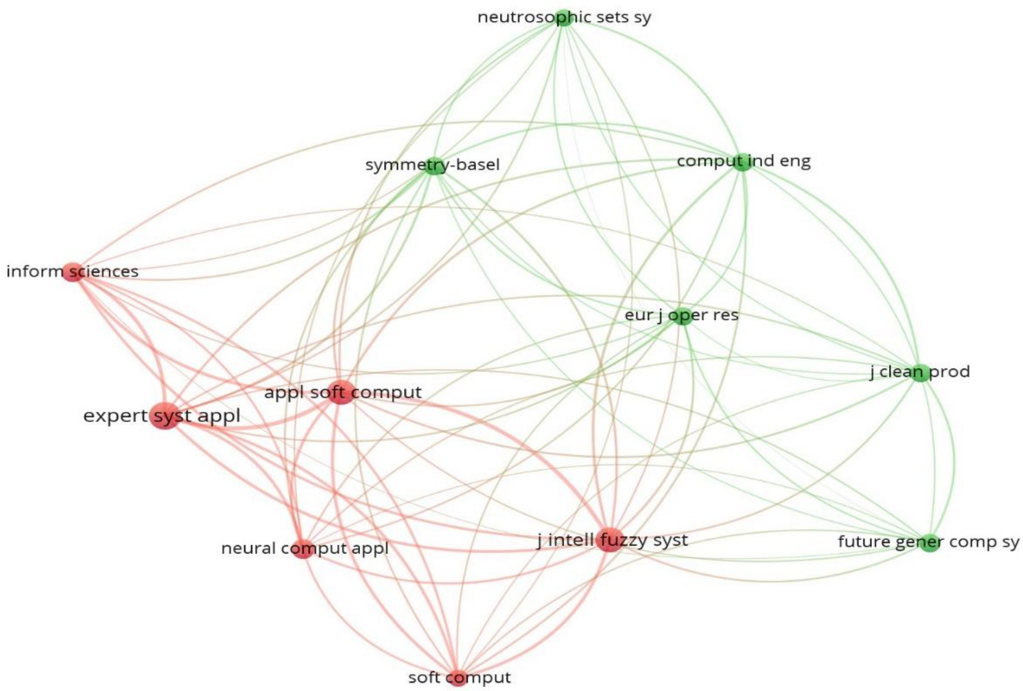


Figure 2. Visualization network for Co-citation based on Co-Sources in healthcare.

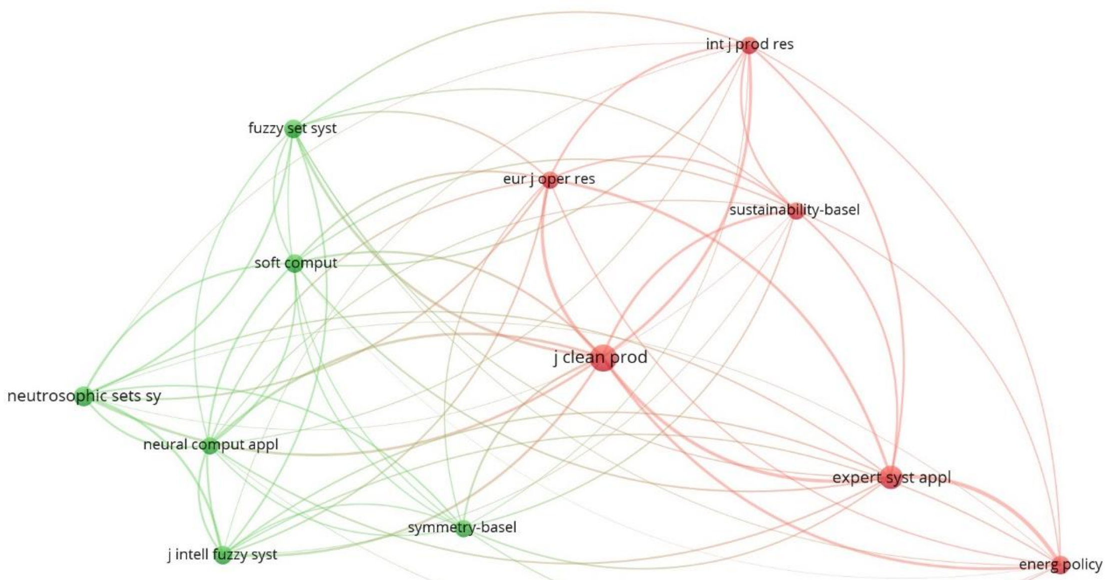


Figure 3. Visualization network for Co-citation based on Co-Sources in climate.

3. Related Work

Herein, we attempted to ensemble the prior studies which related to our scope; through surveys have been conducted. The surveys' findings indicated that not many literary works treat uncertainty and incomplete data. Thereby, this study covered this issue through merging Neutrosophic with DL and established ADF. Table 1 illustrates a set of related studies.

Table 1. previous literary studies

Ref #	Methodology	Objectives	Dataset	Findings
Paul et al. [21]	In this study, the author built a custom lightweight CNN model and compared it with some CNN models based on the transfer learning principle.	Tomato disease detection and classification	The dataset, called multi-source tomato disease, was collected from a single source consisting of 11 varieties and containing 32,535 images.	The proposed model achieved the best with data augmentation accuracy 95% and without 89%.
Memon et al.[22]	Build Meta DL Model and Compare Accuracy it With Custom CNN Model, Vgg16 Transfer learning and ResNet50 Model	Identify and detect cotton leaf diseases	The cotton data set with 2384 samples. The utilized data divided into seven classes: nutrient deficiency, healthy, leafspot, powdery mildew, target spot, verticillium, and leaf curl)	The Proposed Model achieve 98.53%
Rangarajan et al.[23]	Train two Deep learning Model Vgg16 and Alex Net based of Transfer Learning and Compare accuracy of them	Detect Tomato Leaves Disease	The dataset consists of seven categories, including the health category, which consists of 13,262 images extracted from the plant Village Dataset.	Alex Net achieved best Accuracy 97.49 With better performance
Bi et al.[24]	Scholars build mobile phone-based models based on Mobile Net and compare it with InceptionV3 and ResNet152 in terms of speed in image prediction.	Detecting diseases of apple leaves by low-cost model	The data set consists of two categories (Alternaria leaf spots and rust leaves) and contains 334 images. The data set was collected by a group of experts.	The best model in terms of image processing speed is Mobile Net with the fastest speed of .22 seconds
Dahiya et al.[25]	In this research, the author trained a group of the most famous types of deep learning models on plant diseases Dataset such as Google Net and ResNet18 and compared them in terms of accuracy.	Analysis of some deep learning models in terms of accuracy in detecting plant leaf diseases	Plant Village data set consists of 18 classes of 2064 images, divided into 70% for training, 20% for validation, and 10% for testing	The best models achieved Accuracy is ResNet50 and ResNet101
Wei et al.[26]	In this work, the author made a comparison between a group of models such as Alex Net, VGG16, Res Net50, and DenseNet121) on more than one endpoint device (CPU, GPU, VPU).	plant leaf disease identification	The Plant Village dataset, which consists of 55,446 images, is divided into 38 classes	The Best Model archive accuracy is DenseNet121 96.4

Rao et al.[27]	authors implemented pre trained Model Alex Net	Detect Diseases Grape and Mango Leaves	The dataset consists of 7,222 grape leaves from plant Village and 1,216 self-acquired mango leaves	Model archive accuracy in Grape Dataset 99.03% and 89% for Mango Dataset
Belay et al.[28]	proposed model has been built by combining CNN with LSTM and Compare it with server models such a InceptionV3 s VGGNet16 and	Chickpea disease detection	Chickpea Data Set which it consists of 3 Class and contain 1399 image and after augmenting 8391	The Proposed Model achieved highest with 92.55 accuracy

4. Data and Methods

In this study, Potato Dataset has been utilized to compare the performance of a group of models before and after applying the neutrospheric technique to the data. Potato Dataset consists of around 4000 images divided into 3 we used Sample of It in training Model. Table 2 show Statistics of Sample.

The methodology which is employed in this study is to correctly classify potato plant diseases, and this is done in two steps. The first step is to apply the Neutrosophic Domains as an image processing step. The second step is to train the data on deep learning models using the Transfer learning principle. Establised ADF is highlighted in Figure 4.

We used three models for training data ResNet50, ResNet152, and Mobile Net. We trained models with Adam optimizer and lr = .0001.Model training with 50 epochs and evaluate models after each epoch with metrics.

Table 2. Statistics of utilized samples

Classes		Images	Percentage	Total
Diseased	Early Blight	325	0.349%	638
	Late_Blight	313	0.336%	
Healthy	Healthy leaf (C2)	292	0.313 %	930

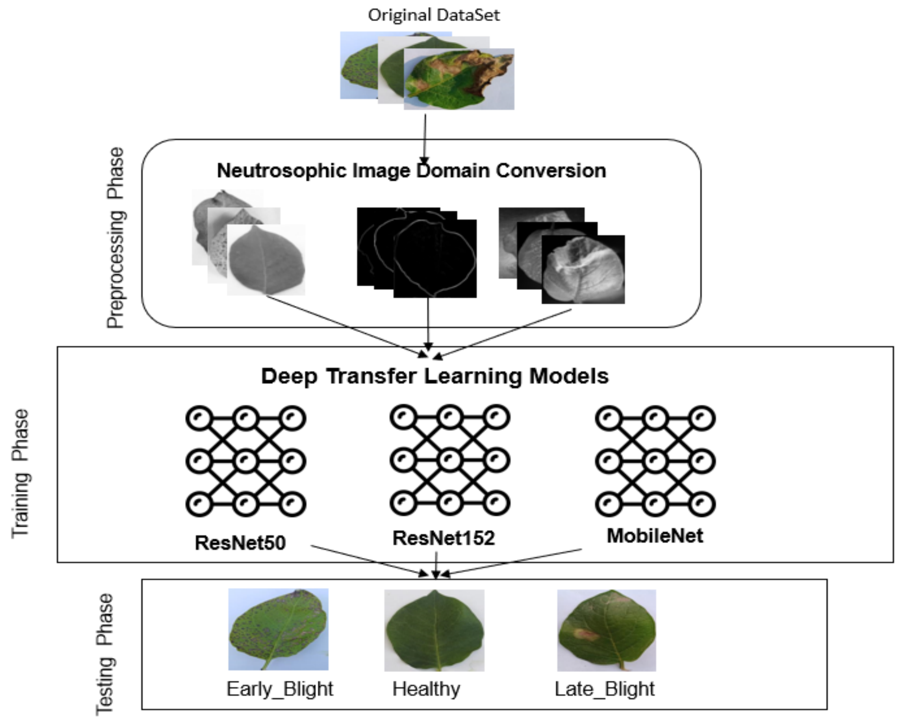


Figure 4. Proposed ADF

5. Result and Discussion

Herein, we illustrate the ADF'S findings and discuss it.

5.1 Performance parameter

Comparison Analysis between models with some matrices has been conducted. We evaluated the model's performance through recall, F1Score, Precision, and Accuracy

5.1.1 Accuracy

This metric is calculated from the number of correct predictions for all categories to the total number of predictions according to Eq.(7).

$$\text{Accuracy} = \frac{(TP+TN)}{(TP+FP+TN+FN)} \qquad (7)$$

5.1.2 Precision

This measure is calculated from the number of correct predictions for a category to the total number of predictions in the same category through implementing Eq.(8).

$$\text{Precision} = \frac{TP}{(TP+FP)} \qquad (8)$$

5.1.3 Recall

This metric is calculated by harmonic mean based on Eq.(9).

$$\text{F1 Score } = 2 \times \frac{\text{recall} \times \text{precision}}{\text{recall} + \text{precision}} \quad (9)$$

We ustilized Kaggle to train all Models that has GPU Nvidia Tesla P100 With Ram 16 GB.We use Python Version 3.7.6 and Keras Version 2.3.1.

5.2 Hyperparameters For Models

All Models train with Adam optimizer through learning rate .0001, batch size 32, number of epoch 50.

5.2.1 Original Dataset Experiment

Summary performance of models for original dataset are listed in Table 3 without Neutrosophic Domain. We observed that ResNet 50 achieved highest accuracy with .873.

Table 3. Performance o various models based on original dataset

Model	Accuracy	Precision	Recall	F1 Score
MobileNet	0.842	0.842	0.841	0.840
ResNet50	0.873	0.871	0.870	0.870
ResNet152	0.831	0.841	0.841	0.826

5.2.2 Neutrosophic Domains Experimental Results

- In table 4, summary performance of models for True(T) NS Domain has been showcased. whilst ResNet152 achieved highest accuracy with .0.905.
- Whereas performance of models for Falsity(F)has been showcased and listed in Table 5. Therefore, ResNet50 achieved highest accuracy with 0.905.
- Table 6 summary performance of models for Indeterminate (I) has been summarized. Moreover, NS Domain Mobile Net achieved highest accuracy with .863%.
-

Table 4. Dataset True(T)

Model	Accuracy	Precision	Recall	F1 Score
MobileNet	0.852	0.854	0.852	0.853
ResNet50	0.852	0.850	0.848	0.846
ResNet152	0.905	0.907	0.901	0.901

Table 5. Dataset Falsity (F)

Model	Accuracy	Precision	Recall	F1 Score
MobileNet	0.694	0.701	0.693	0.694
ResNet50	0.915	0.914	0.913	0.9134
ResNet152	0.905	0.912	0.900	0.899

Table 6. Dataset (I)

Model	Accuracy	Precision	Recall	F1 Score
MobileNet	0.863	0.862	0.861	0.861
ResNet50	0.778	0.777	0.776	0.776
ResNet152	0.831	0.841	0.827	0.826

5.3 Comparative Result of Neutrosophic Domain with the Original Dominant

Table 7 shows best accuracy of models achieved Mobile Net achieved best accuracy in Indeterminate (I) domain with accuracy .863. ResNet50 and ResNet152 achieved best accuracy in True(T) domain with accuracy 91.5 and 0.905. Figure 5 summarized comparison of models' accuracy.

Table 7. Accuracy of various Models

Model	Accuracy	Precision	Recall	F1 Score
MobileNet	0.863	0.862	0.861	0.861
ResNet50	0.915	0.914	0.913	0.9134
ResNet152	0.905	0.912	0.900	0.899

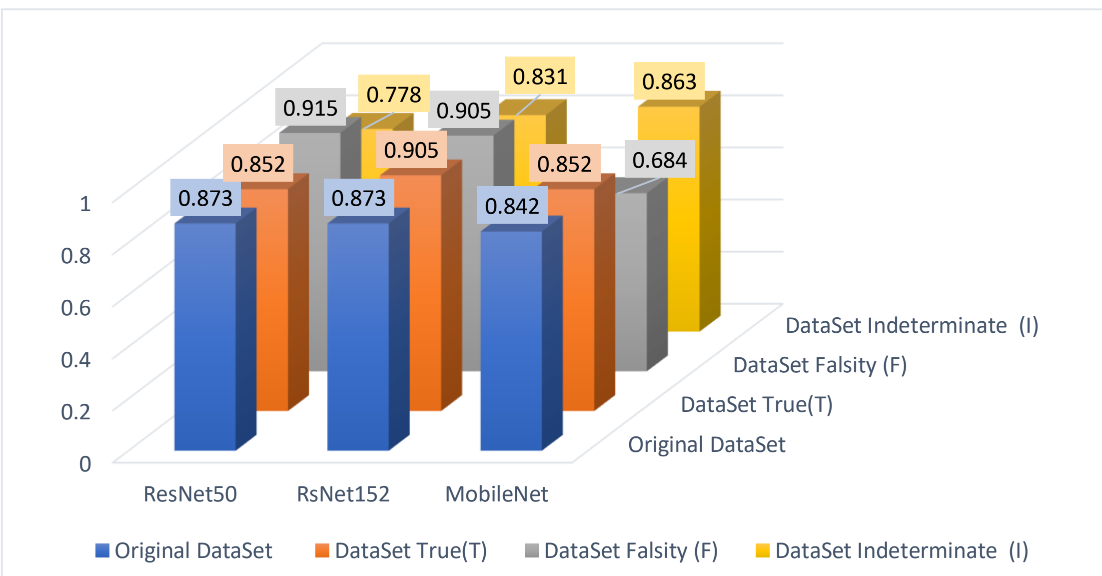
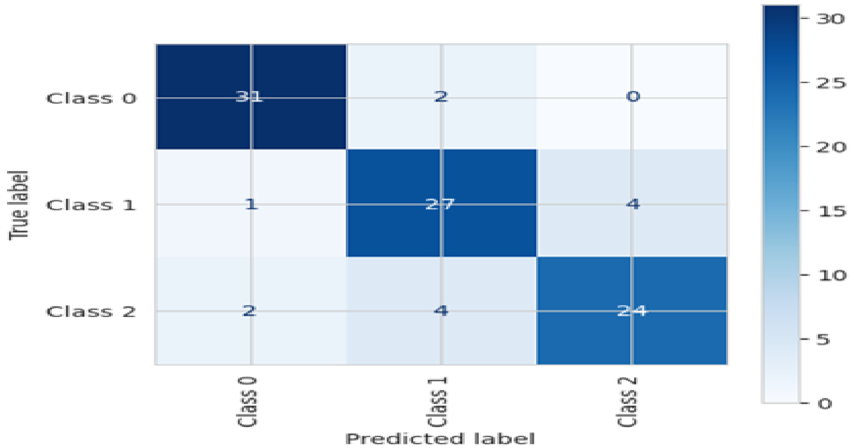


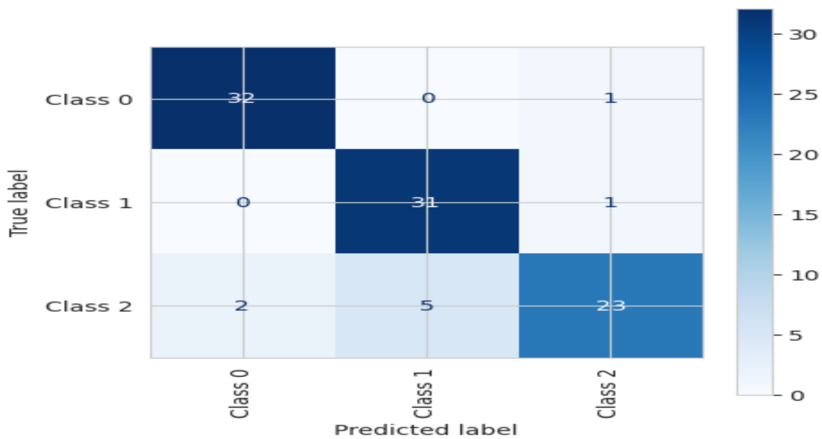
Figure 5. Accuracy of comparative models

5.4 Confusion Matrix of MobileNet

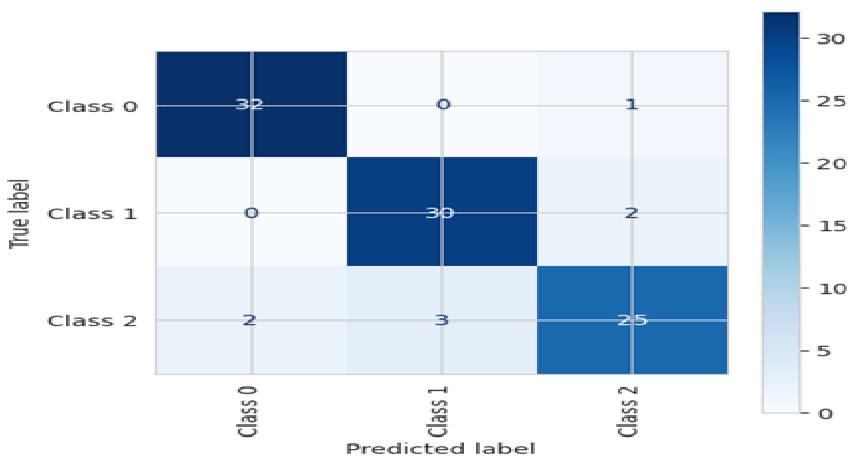
Figure 6. aggregated the confusion matrix for each measurement for truth (T), Indeterminate (I), and Falsity (F).



Confusion Matrix of MobileNet in domain Indeterminate (I)



Confusion Matrix of ResNet152 in domain True(T)



Confusion Matrix of ResNet50 in domain False (F)

Figure 6. Confusion Matrix of Mobile Net for True, False, and Indeterminate

6. Conclusion

This study attempted to cover some of the issues mentioned and determined in the prior studies. One of these issues was incorrect diagnosis for plant diseases. A misdiagnosis might shorten the crop's life or even eradicate it. Moreover, agriculture is diminishing.

Another issue entailed in not many literary studies discuss the uncertainty and incomplete data.

Thereby, these issues are catalysts for developing our ADF which depends on theory which characterized with uncertainty and has ability to treat with ambiguity circumstances. This theory is Neutrosophic which treats with image through three possibilities are T, I, and F. Also, we combined this theory with ML especially DL. Due to the ability of DL to treat with large volume of data. Hence, commenced acquiring an interest in diagnosing diseases especially plants in agriculture domain.

Overall, we applied our constructed ADF potato dataset. Whereas the utilized dataset trained by ResNet50, ResNet152, and Mobile Net models Also, we conducted comparison between the performance of the models before and after applying Neutrosophic theory. The findings of this experiment indicated that Neutrosophic theory proved very effective in improving the accuracy of the models.

References

[1] Oerke, E.-C., & Dehne, H.-W. (2004). Safeguarding production—losses in major crops and the role of crop protection. *Crop protection*, *23*(4), 275–285.

[2] Ahmad, A., Saraswat, D., & El Gamal, A. (2023). A survey on using deep learning techniques for plant disease diagnosis and recommendations for development of appropriate tools. *Smart agricultural technology*, *3*, 100083.

[3] Pantazi, X. E., Moshou, D., & Tamouridou, A. A. (2019). Automated leaf disease detection in different crop species through image features analysis and One Class Classifiers. *Computers and electronics in agriculture*, *156*, 96–104.

[4] DeChant, C., Wiesner-Hanks, T., Chen, S., Stewart, E. L., Yosinski, J., Gore, M. A., … Lipson, H. (2017). Automated identification of northern leaf blight-infected maize plants from field imagery using deep learning. *Phytopathology*, *107*(11), 1426–1432.

[5] Jackulin, C., & Murugavalli, S. (2022). A comprehensive review on detection of plant disease using machine learning and deep learning approaches. *Measurement: sensors*, *24*, 100441.

[6] Senthilkumar, T. P., & Prabhusundhar, P. (n.d.). ENSEMBLE MACHINE LEARNINGAND HYBRID NEUTROSOPHIC COGNITIVE MAPS BASED FEATURE SELECTION FOR RICE DISEASE PREDICTION.

[7] Barbedo, J. G. A. (2019). A review on the use of unmanned aerial vehicles and imaging sensors for monitoring and assessing plant stresses. *Drones*, *3*(2), 40.

[8] Martin, D. P., & Rybicki, E. P. (1998). Microcomputer-based quantification of maize streak virus symptoms in Zea mays. *Phytopathology*, *88*(5), 422–427.

[9] Yuan, Y., Ren, Y., Liu, X., & Wang, J. (2020). *Approach to image segmentation based on interval neutrosophic set*. Infinite Study.

[10] Smarandache, F., Quiroz-Martínez, M. A., Ricardo, J. E., Hernández, N. B., & Vázquez, M. Y. L. (2020). *Application of neutrosophic offsets for digital image processing*. Infinite Study.

[11] Bloch, I. (2015). Fuzzy sets for image processing and understanding. *Fuzzy sets and systems*, *281*, 280–

291.
[12] Zhao, J., Wang, X., & Li, M. (2020). A novel Neutrosophic image segmentation based on improved fuzzy C-means algorithm (NIS-IFCM). *International journal of pattern recognition and artificial intelligence*, *34*(05), 2055011.
[13] Alsmadi, M. K. (2018). A hybrid Fuzzy C-Means and Neutrosophic for jaw lesions segmentation. *Ain shams engineering journal*, *9*(4), 697–706.
[14] Abdelhafeez, A., & Mohamed, H. K. (2023). Skin cancer detection using neutrosophic c-means and fuzzy c-means clustering algorithms. *Journal of intelligent systems and internet of things*, *8*(1), 33–42. DOI:10.54216/JISIoT.080103
[15] Abouhawwash, M., Bacanin, N., & Venkatachalam , K. (2024). A Global Analysis of the Policies and Regulations in Place to Address the Impacts of Climate Change. Multicriteria Algorithms With Applications, 2, 1-28. https://doi.org/10.61356/j.mawa.2024.26461
[16] Smarandache, F. (2019). Neutrosophic set is a generalization of intuitionistic fuzzy set, inconsistent intuitionistic fuzzy set (picture fuzzy set, ternary fuzzy set), pythagorean fuzzy set, spherical fuzzy set, and q-rung orthopair fuzzy set, while neutrosophication is a genera. *Journal of new theory*, (29), 1–31.
[17] Ali, A. M., & Broumi, S. (2024). Machine Learning with Multi-Criteria Decision Making Model for Thyroid Disease Prediction and Analysis. Multicriteria Algorithms With Applications, 2, 80-88. https://doi.org/10.61356/j.mawa.2024.26961
[18] Deli, I., Ali, M., & Smarandache, F. (2015). Bipolar neutrosophic sets and their application based on multi-criteria decision making problems. *2015 international conference on advanced mechatronic systems (icamechs)* (pp. 249–254). Ieee.
[19] Krizhevsky, A., Sutskever, I., & Hinton, G. E. (2017). ImageNet classification with deep convolutional neural networks. *Communications of the acm*, *60*(6), 84–90.
[20] Okoli, C., & Schabram, K. (2015). A guide to conducting a systematic literature review of information systems research.
[21] Paul, S. G., Biswas, A. A., Saha, A., Zulfiker, M. S., Ritu, N. A., Zahan, I., … Islam, M. A. (2023). A real-time application-based convolutional neural network approach for tomato leaf disease classification. *Array*, *19*, 100313.
[22] Memon, M. S., Kumar, P., & Iqbal, R. (2022). Meta deep learn leaf disease identification model for cotton crop. *Computers*, *11*(7), 102.
[23] Rangarajan, A. K., Purushothaman, R., & Ramesh, A. (2018). Tomato crop disease classification using pre-trained deep learning algorithm. *Procedia computer science*, *133*, 1040–1047.
[24] Bi, C., Wang, J., Duan, Y., Fu, B., Kang, J.-R., & Shi, Y. (2022). MobileNet based apple leaf diseases identification. *Mobile networks and applications*, 1–9.
[25] Dahiya, S., Gulati, T., & Gupta, D. (2022). Performance analysis of deep learning architectures for plant leaves disease detection. *Measurement: sensors*, *24*, 100581.
[26] Wei, S. J., Al Riza, D. F., & Nugroho, H. (2022). Comparative study on the performance of deep learning implementation in the edge computing: Case study on the plant leaf disease identification. *Journal of agriculture and food research*, *10*, 100389.
[27] Rao, U. S., Swathi, R., Sanjana, V., Arpitha, L., Chandrasekhar, K., & Naik, P. K. (2021). Deep learning precision farming: grapes and mango leaf disease detection by transfer learning. *Global transitions proceedings*, *2*(2), 535–544.
[28] Belay, A. J., Salau, A. O., Ashagrie, M., & Haile, M. B. (2022). Development of a chickpea disease detection and classification model using deep learning. *Informatics in medicine unlocked*, *31*, 100970.

Received: Jan 10, 2024. Accepted: Mar 30, 2024

NSS

Neutrosophic Sets and Systems, Vol. 66, 2024

University of New Mexico

N_δ-Closure and N_δ-Interior in Neutrosophic Topological Spaces

K.Damodharan[1] and M.Vigneshwaran[2]

[1]Department of Mathematics, KPR Institute of Engineering and Technology(Autonomous), Coimbatore - 641407, India.; catchmedamo@gmail.com

[2]Department of Mathematics, Kongunadu Arts and Science College(Autonomous), Coimbatore - 641029, India.; vignesh.mat@gmail.com

Abstract. Toplology greatly benefits from the concept of δ-cloure. Its quiet nature to extended its properties in other topological spaces. So, with the concept of quasi-coincidence Ganguly and Saha pioneered and extensively examined the notion of δ-closure within the domain of fuzzy topological spaces.(F_{TS}). The theory of δ-closure in intuitionistic fuzzy topological spaces (IF_{TS}) was further extended by Seok Jong Lee and Yeon Seok Eom. In this work, the notion of N_δ-closure in Neutrosophic Topological Spaces (N_{TS}) is put forward and discussed.

Keywords: neutrosophic; δ-Closure; δ-Interior

1. Introduction

In 1965, Zadeh pioneered the concept of fuzzy sets (F_S). In various areas of our daily lives, uncertainty is handled using this innovative mathematical framework. A membership function with the range of 0 to 1 characterizes a F_S. Over the last few decades, F_S is substantially used and applied in many domains, such as computer vision [45], pattern recognition [22], control [44], and others. Researchers in the fields of engineering [22], social sciences [45], and medical diagnosis [45] have all found this idea to be very useful. There is a tonne of information on F_S theory in [22, 34, 45]. A specific value contained within the unit interval [0,1] indicates the F_S's membership function. There is some hesitation as a result, therefore it's not always true that an element's non-membership function equals 1. In order to clarify this scenario, Atanassov [2] created IF_S in 1986 by including the hesitation degree known as the hesitation margin. The definition of the hesitancy margin is 1 . Thus, a membership function and non-membership function for an intuitionistic fuzzy setIF_S have a range of [0,1] with the additional condition that 0 1. As a result, F_S theory was generalized to include IF_S theory.

Decision-making [20], pattern recognition [33], social sciences [5], medical diagnostics [33], and other domains have all benefited from the application of the IF_S theory. It is impossible for fuzzy sets and IF_S to handle data that is unclear, inconsistent, partial, or uncertain. Consequently, Smarandache (Smarandache, 1999) formulated neutrosophic logic in 1998, drawing inspiration from Neutrosophy, a philosophical paradigm that scrutinizes the origin, composition, and application of neutral elements, alongside their interplay with diverse conceptual spectrums. A Neutrosophic set N_S encompasses three distinct membership functions: 'T' for truth, 'I' for indeterminate membership, and 'F' for falsehood. The 'I' component embodies a notable degree of indeterminacy, a key attribute associated with mediocrity. The theoretical frameworks of classical set theory, fuzzy sets (F_S) theory, intuitionistic fuzzy sets (IF_S) theory, interval-valued fuzzy sets theory, paraconsistent theory, dialetheist theory, paradoxist theory, and tautological theory are all encompassed and extended by the overarching N_S theory. This theoretical construct proves itself to be a robust instrument for grappling with the intricate tapestry of ambiguous and contradictory information that pervades our real-world context. Scholars from a multitude of disciplines have effectively harnessed N_S theory to navigate their respective domains. Notably, Wang et al. (2010) given the application of singular-valued N_S in the realms of science and engineering, providing an additional avenue for describing uncertain, partial, imprecise, and inconsistent data. The correlation coefficient of N_S found its investigation in the works of Hanafy et al. (2012 and 2013), while Ye (2013) explored the correlation coefficient within the context of singular-valued N_S. Further exploration of the correlation coefficient in the interval N_S was undertaken by Broumi and Smaradache (2013). In their discussion of N_{TS}, Salama et al. [27] You can find additional research on the N_S in [21, 27, 29, 37, 40–42]. In the decision-making theory [4, 39–42], data base [37], medical diagnosis [42], pattern recognition [11, 23], and other fields, N_S have been successfully employed.

In the realm of conventional topology, when delving into subjects like H-closed spaces, Katetov's and H-closed extensions, the generalizations of the Stone Weierstrass theorem, and other related topics, the ideas of θ-closure and δ-closure emerge as valuable tools [8, 9, 24, 35, 36, 43]. Given the substantial importance of these concepts, it becomes almost inevitable to seek their extension into the context of fuzzy topological spaces (F_{TS}). Thus, by harnessing the notion of quasi-coincidence within F_{TS}, Saha and Ganguly introduced and conducted a thorough investigation into the innovative concept of fuzzy δ-closure. [10]. Furthermore, within the context of intuitionistic fuzzy topological spaces (IF_{TS}), extensive research efforts have been directed towards examining the characteristics of continuous mappings and closure operators. [12,17–19,32]. A generalisation of the δ- closure, the idea of δ-closure in IF_{TS} is introduced by Ganguly and Saha [10]. N_{TS} were first introduced in 2012 by Salama and Alblowi [26]. As an advancement beyond the framework of intuitionistic fuzzy topological spaces (IF_{TS}), they introduced the concept of neutrosophic topological spaces (N_{TS}), along with a corresponding neutrosophic set (N_S), which encapsulates the degrees of membership, indeterminacy, and non-membership for each individual element. In 2016, P. Iswarya and Dr. K. Bageerathi [16] contributed to this exploration by proposing the novel concepts of neutrosophic semiopen sets, neutrosophic semiclosed sets, neutrosophic semi-interior, and neutrosophic semi-closure within the context of neutrosophic topological spaces (N_{TS}). In the subsequent year, Parimala M et al(2018). elaborated on some new notions of homeomorphism within the same neutrosophic topological framework (N_{TS}) [25]. This evolutionary trajectory continued into the year 2022, when Shuker Mahmood Khalil delved into the realm of Neutrosophic Delta Generated Per-Continuous Functions in neutrosophic topological spaces (N_{TS}) [30]. Seok Jong Lee and Yeon Seok Eom [43] developed the concepts of δ-closure and δ-Interior in IF_{TS} in 2012. We are extending the aforementioned ideas to N_S in this study. With the help of examples, we discuss some of the fundamental characteristics of $\mathbf{N}_\delta$-Closure and $\mathbf{N}_\delta$-Interior in N_{TS}.

2. Perliminaries

This part of the study gives an insight to the pertinent and basic preparatory operations about N_S's

Definition 2.1. [26] Consider a non-empty fixed set S. A neutrosophic set I (N_S) can be characterized as an entity taking the following structure: $I = \{\langle s, \mu_m(I(s)), \sigma_i(I(s)), \nu_{nm}(I(s))\rangle \forall s \in S\}$, where $\mu_m(I(s))$, $\sigma_i(I(s))$and $\nu_{nm}(I(s))$ represents the degrees of membership function, indeterminacy function and nonmembership function of each element $s \in S$ to the set I.

Remark 2.2. [26] A N_S
$I = \{\langle s, \mu_m(I(s)), \sigma_i(I(s)), \nu_{nm}(I(s))\rangle \forall s \in S\}$ can be represented by an ordered triple $\langle \mu_m(I(s)), \sigma_i(I(s)), \nu_{nm}(I(s))\rangle$ within the interval $]^-0, 1^+[$ defined over the set S.

Definition 2.3. [26] Consider I as a neutrosophic set N_S in the format
$I = \{\langle s, \mu_m(I(s)), \sigma_i(I(s)), \nu_{nm}(I(s))\rangle \forall s \in S\}$, Subsequently, the complement of I, denoted as I^c, can be stipulated as $I^c = \{\langle s, \nu_{nm}(I(s)), \sigma_i(I(s)), \mu_m(I(s))\rangle \forall s \in S\}$

Definition 2.4. [26] Suppose there are two N_Ss with the structure, I and J.

$I = \{\langle s, \mu_m(I(s)), \sigma_i(I(s)), \nu_{nm}(I(s))\rangle \forall s \in S\}$ and
$J = \{\langle s, \mu_m(J(s)), \sigma_i(J(s)), \nu_{nm}(J(s))\rangle \forall s \in S\}$.
Then,

i) Subsets ($I \subseteq J$) may be defined as follows $I \subseteq J$ if and only if
$\mu_m(I(s)) \leq \mu_m(J(s)), \sigma_i(I(s)) \geq \sigma_i(J(s)), \nu_{nm}(I(s)) \geq \nu_{nm}(J(s))$

ii) Subsets $I = J$ if and only if $I \subseteq J$ and $J \subseteq I$

iii) The union of subsets $I \cup J$ can be defined in the following manner:

$$I \cup J = \{s, max\left[\mu_m\left(I\left(s\right), \mu_m\left(J\left(s\right)\right)\right)\right], min\left[\sigma_i\left(I\left(s\right)\right), \sigma_i\left(J\left(s\right)\right)\right], min\left[\nu_{nm}\left(I\left(s\right)\right), \nu_{nm}\left(J\left(s\right)\right)\right] \forall s \in S\},$$

iv) The intersection of subsets $I \cap J$ can be defined in the following manner:

$$I \cap J = \{s, min\left[\mu_m\left(I\left(s\right), \mu_m\left(J\left(s\right)\right)\right)\right], max\left[\sigma_i\left(I\left(s\right)\right), \sigma_i\left(J\left(s\right)\right)\right], max\left[\nu_{nm}\left(I\left(s\right)\right), \nu_{nm}\left(J\left(s\right)\right)\right] \forall s \in S\},$$

Definition 2.5. [26] A N_T $(S, \dot{\tau})$ that meets the axioms listed below

i) $0_N, 1_N \in \dot{\tau}$,

ii) $H_1 \cap H_2 \in \dot{\tau}$ for any $H_1, H_2 \in \dot{\tau}$,

iii) $\cup H_i \in \dot{\tau} \ \ \forall \{H_i : i \in J\} \subseteq \dot{\tau}$ Then the pair $(S, \dot{\tau})$ or simply S is called a N_{TS}.

Definition 2.6. [7] Let I be a N_S contained in a N_{TS}, representing $(S, \dot{\tau})$. Then

i) $Nint\left(I\right) = \bigcup\left\{J/J \text{ is a} N_{OS} \text{in} \left(S, \dot{\tau}\right) \text{ and } J \subseteq I\right\}$ is termed as the neutrosophic interior of I;

ii) $Ncl\left(I\right) = \bigcap\left\{J/J \text{ is a} N_{CS} \text{in} \left(S, \dot{\tau}\right) \text{ and } J \supseteq I\right\}$ is termed as the neutrosophic closure of I.;

Theorem 2.7. *[6] For any N_S I in a N_{TS} $(S, \dot{\tau})$, we have*

i) $Ncl\left(I^c\right) = \left(Nint\left(I\right)\right)^c$ *and*

ii) $Nint\left(I^c\right) = \left(Ncl\left(I\right)\right)^c$

Definition 2.8. [15] Let $\upsilon, \omega, \xi \in [0, 1]$ and $\upsilon + \omega + \xi \leq 3$. A neutrosophic point(NP) $s_{(\upsilon,\omega,\xi)}$of S is a NP of S, defined as

$$s_{(\upsilon,\omega,\xi)}(t) = \begin{cases} (\upsilon, \omega, \xi), & \text{if } t = s; \\ (0, 1, 1), & \text{if } t \neq s. \end{cases}$$

In this context, 's' is referred to as the support of $s_{(\upsilon,\omega,\xi)}$ and υ, ω and ξ, respectively. A NP $s_{(\upsilon,\omega,\xi)}$ is considered to be a member of a N_S
$I = \langle \mu_m\left(I\left(s\right)\right), \sigma_i\left(I\left(s\right)\right), \nu_{nm}\left(I\left(s\right)\right) \rangle$ in the set S, shown by $s_{(\upsilon,\omega,\xi)} \in I$ if $\upsilon \leq \mu_m\left(I\left(s\right)\right), \omega \leq \sigma_i\left(I\left(s\right)\right)$ and $\xi \geq \nu_{nm}\left(I\left(s\right)\right)$.

Definition 2.9. [1] Let A be a N_S in a N_{TS} $(S, \dot{\tau})$. A is said to be

i) a neutrosophic semi-open set of S, if there exists a N_{OS} B of S such that $B \leq A \leq cl\left(B\right)$.

ii) a N_{ROS} of S, if $Nint\left(Ncl\left(A\right)\right) = A$. The complement of a N_{ROS} is said to be a N_{RCS}.

3. Neutrosophic δ-Closure and δ-Interior

Definition 3.1. Let $(S, \dot{\tau})$ be a N_{TS}. Let I be a N_S and let $s_{(\upsilon,\omega,\xi)}$ be a NP. $s_{(\upsilon,\omega,\xi)}$ is considered to be neutrosophically quasi-coincident with I [denoted by $s_{(\upsilon,\omega,\xi)}qI$] if $\upsilon + \mu_m(I(s)) > 1; \omega + \sigma_i(I(s)) < 1$ and $\xi + \nu_{nm}(I(s)) < 1$.

Definition 3.2. Let I and J be two N_S's. I is said to be neutrosophic quasi coincident with J [denoted by IqJ] if $\mu_m(I(s)) + \mu_m(J(s)) > 1; \sigma_i(I(s)) + \sigma_i(J(s)) < 1$ and $\nu_{nm}(I(s)) + \nu_{nm}(J(s)) < 1$. The term 'not quasi-coincident' will be abbreviated as $\tilde{q}$.

Proposition 3.3. *Consider two N_S, I and J, and an NP in S, $s_{(\upsilon,\omega,\xi)}$. Then*

i) $I\tilde{q}J^c \Leftrightarrow I \subseteq J$
ii) $IqJ \Leftrightarrow I \nsubseteq J^c$
iii) $s_{(\upsilon,\omega,\xi)} \subseteq I \Leftrightarrow s_{(\upsilon,\omega,\xi)}\tilde{q}I^c$
iv) $s_{(\upsilon,\omega,\xi)}qI \Leftrightarrow s_{(\upsilon,\omega,\xi)} \nsubseteq I^c$

Theorem 3.4. *Let $s_{(\upsilon,\omega,\xi)}$ be a NP in S, and*
$I = \langle \mu_m(I(s)), \sigma_i(I(s)), \nu_{nm}(I(s)) \rangle$ a N_S in S. Then $s_{(\upsilon,\omega,\xi)} \in Ncl(I)$ if and only if IqN, for any N^q-nhd N of $s_{(\upsilon,\omega,\xi)}$.

Proof. Consider that $I\tilde{q}N$ exists for every $N \in N_\epsilon^q\left(s_{(\upsilon,\omega,\xi)}\right)$. In this case, $s_{(\upsilon,\omega,\xi)}qG \leq N$ and $G\tilde{q}I$ exist for a set $G \in \dot{\tau}$. since G^c is a N_{CS} and by Proposition 3.3, we have $Ncl(I) \leq G^c$. Also since $s_{(\upsilon,\omega,\xi)} \notin G^c$, we have $s_{(\upsilon,\omega,\xi)} \notin Ncl(I)$. Since, which is contradiction.
Conversely, suppose $s_{(\upsilon,\omega,\xi)} \notin Ncl(I)$. Then, $s_{(\upsilon,\omega,\xi)} \notin V$ and $I \leq V$ exist for a N_{CS} V. Hence by Proposition 3.3, $V^c \in \dot{\tau}$ such that $s_{(\upsilon,\omega,\xi)}qV^c$ and $I\tilde{q}V^c$. Since, which is a contradiction. □

Example 3.5. Consider (X, τ) as a N_{TS} with X as $X = \{p, q, r\}$ and D_1, D_2, D_3, D_4 as N_S's
$D_1 = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right) \right\rangle$
$D_2 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right) \right\rangle$
$D_3 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right) \right\rangle$
$D_4 = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right) \right\rangle$
now the complement of D_1, D_2, D_3, D_4 are
$D_1^c = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right) \right\rangle$
$D_2^c = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right) \right\rangle$
$D_3^c = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right) \right\rangle$
$D_4^c = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right) \right\rangle$

Let the neutrosophic point be

$$s_{(\upsilon,\omega,\xi)} = \begin{cases} (0.7, 0.4, 0.3, \text{ if } x = p \\ (0,1,1)\,, \text{ if } x \neq p. \end{cases}$$

,

where $D_2 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6} \right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4} \right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2} \right) \right\rangle \rightarrow N_{OS}$
Let $N = \left\langle \left(\frac{p}{0.7}, \frac{q}{0.6}, \frac{r}{0.6} \right), \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4} \right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2} \right) \right\rangle$
$D_2 \subseteq N$ therefore N is N^q-nhd N of $s_{(\upsilon,\omega,\xi)}$
Let $I = \left\langle \left(\frac{p}{0.7}, \frac{q}{0.6}, \frac{r}{0.6} \right), \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4} \right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2} \right) \right\rangle$, also IqN
$\Rightarrow s_{(\upsilon,\omega,\xi)} \in Ncl\,(I)$

In N_{TS}, we put forward the idea of neutrosophic δ-closure.

Definition 3.6. Consider $(S, \dot{\tau})$ as a (N_{TS}). A NP $s_{(\upsilon,\omega,\xi)}$ is said to be a neutroscophic δ-cluster point of a N_S I if AqI for each N^q_{RO}-nhd A of $s_{(\upsilon,\omega,\xi)}$. The set of all neutroscophic δ-cluster point of I is called the neutrosophic δ-closure of I and denoted by $Ncl_\delta\,(I)$. A N_S I is said to be a $N_{\delta-CS}$ if $I = Ncl_\delta\,(I)$. A $N_{\delta-OS}$ is considered to be the exact opposite of a $N_{\delta-CS}$.

Definition 3.7. Given a N_{TS} $(S, \dot{\tau})$, let I be a N_S in S. $Nint_\delta\,(I) = (Ncl_\delta\,(I^c))^c$ is the notation and definition of the neutrosophic δ-interior of I.

Remark 3.8. The following relations can be obtained from the definition above:

i) $Ncl_\delta\,(I^c) = (Nint_\delta\,(I))^c$,
ii) $(Ncl_\delta\,(I))^c = Nint_\delta\,(I^c)$.

Remark 3.9. Let I be a $N_{\delta-OS}$ if and only if $Nint_\delta\,(I) = I$ because I is $N_{\delta-OS}$ if and only if I^c is $N_{\delta-CS}$ if and only if $I^c = Ncl_\delta\,(I^c)$ if and only if $I = (Ncl_\delta\,(I^c))^c = Nint_\delta\,(I)$.

Lemma 3.10. *For any N_{OS} I in a N_{TS} $(S, \dot{\tau})$ such that $s_{(\upsilon,\omega,\xi)}qI$, $Nint\,(Ncl\,(I))$ is a N^q_{RO}-nhd of $s_{(\upsilon,\omega,\xi)}$.*

Proof. Clearly $Nint\,(I) \subseteq Nint\,(Ncl\,(I))$. Since I is a N_{OS}, we have $I = Nint\,(I) \subseteq Nint\,(Ncl\,(I))$. By definition 2.9, $Nint\,(Ncl\,(I))$ is a N_{ROS}. Therefore $Nint\,(Ncl\,(I))$ is a N^q_{RO}-nhd of $s_{(\upsilon,\omega,\xi)}$. □

Corollary 3.11. *I is a N_{CS} if it is a $N_{\delta-CS}$ in N_{TS} $(S, \dot{\tau})$. The Corollary's counterpart is not true. Example 3.18*

Theorem 3.12. *$Ncl\,(I) = Ncl_\delta\,(I)$ exists if I corresponds to N_{OS} in N_{TS} $(S, \dot{\tau})$.*

Proof. Ensuring that $Ncl_{\delta}(I) \subseteq Ncl(I)$ is sufficient. Take any $s_{(\upsilon,\omega,\xi)} \in Ncl_{\delta}(I)$. Suppose that $s_{(\upsilon,\omega,\xi)} \notin Ncl(I)$. By Theorem 3.4, there exists a N^{q}-nhd G of $s_{(\upsilon,\omega,\xi)}$ such that $G\tilde{q}I$. Since $G\tilde{q}I$, we have $G \subseteq I^{c}$. Since I^{c} is a N_{CS}, $Ncl(G) \subseteq Ncl(I^{c}) = I^{c}$. Therefore, $Nint(Ncl(G)) \subseteq Nint(I^{c}) \subseteq I^{c}$, i.e. $Nint(Ncl(G))\tilde{q}I$. By Lemma 3.10, $Nint(Ncl(I))$ is a N^{q}_{RO}-nhd A of $s_{(\upsilon,\omega,\xi)}$ such that $Nint(Ncl(I))\tilde{q}I$. Hence $s_{(\upsilon,\omega,\xi)} \notin Ncl_{\delta}(I)$. □

Theorem 3.13. *In N_{TS}, if P is a semi-open set, then $Ncl(P) = Ncl_{\delta}(P)$.*

Proof. Enough to show that $Ncl_{\delta}(P) \subseteq Ncl(P)$. Take any $s_{(\upsilon,\omega,\xi)} \in Ncl_{\delta}(P)$. Suppose that $s_{(\upsilon,\omega,\xi)} \notin Ncl(P)$. Then there exists a N^{q}_{O}-nhd Q of $s_{(\upsilon,\omega,\xi)}$ such that $Q\tilde{q}P$. As per the definition of a semi-open set, there is a N_{OS} R such that $R \subseteq P \subseteq Ncl(R)$. Thus $Q \subseteq P^{c} \subseteq R^{c}$. Hence $Ncl(Q) \subseteq Ncl(P^{c}) \subseteq Ncl(R^{c}) = R^{c}$. Also, $Nint(Ncl(Q)) \subseteq Nint(Ncl(P^{c})) \subseteq Nint(Ncl(R^{c})) = Nint(R^{c}) \subseteq R^{c}$, i.e. $Nint(Ncl(Q)) \subseteq R^{c}$. Therefore $R \subseteq (Nint(Ncl(Q)))^{c}$. Hence $P \subseteq Ncl(R) \subseteq (Ncl(Nint(Ncl(Q)))^{c}) = (Nint(Ncl(Q)))^{c}$ because $(Nint(Ncl(Q)))^{c}$ is a N_{CS}. Thus $Nint(Ncl(Q))\tilde{q}P$. Hence $s_{(\upsilon,\omega,\xi)} \notin Ncl_{\delta}(P)$. □

Theorem 3.14. *Given a N_{TS} $(S, \dot{\tau})$, let I and J be two N_{S}. Following that, we have the subsequent characteristics:*

i) $Ncl_{\delta}(0_{N}) = 0_{N}$
ii) $I \subseteq Ncl_{\delta}(I)$
iii) $I \subseteq J \Rightarrow Ncl_{\delta}(I) \subseteq Ncl_{\delta}(J)$
iv) $Ncl_{\delta}(I) \cup Ncl_{\delta}(J) = Ncl_{\delta}(I \cup J)$
v) $Ncl_{\delta}(I \cap J) \subseteq Ncl_{\delta}(I) \cap Ncl_{\delta}(J)$.

Proof. i) Obvious

ii) Since $I \subseteq Ncl(I) \subseteq Ncl_{\delta}(I), I \subseteq Ncl_{\delta}(I)$.

iii) Let $s_{(\upsilon,\omega,\xi)}$ be a NP in S such that $s_{(\upsilon,\omega,\xi)} \notin Ncl_{\delta}(J)$. Then there is a N^{q}_{RO}-nhd A of $s_{(\upsilon,\omega,\xi)}$ such that $A\tilde{q}J$. Since $I \subseteq J$, we have $A\tilde{q}I$. Therefore $s_{(\upsilon,\omega,\xi)} \notin Ncl_{\delta}(I)$.

iv) Since $I \subseteq I \cup J, Ncl_{\delta}(I) \subseteq Ncl_{\delta}(I \cup J)$. Similarily, $Ncl_{\delta}(J) \subseteq Ncl_{\delta}(I \cup J)$. Hence $Ncl_{\delta}(I) \cup Ncl_{\delta}(J) \subseteq Ncl_{\delta}(I \cup J)$. Take any $s_{(\upsilon,\omega,\xi)} \in Ncl_{\delta}(I \cup J)$ for evidence that $Ncl_{\delta}(I \cup J) \subseteq Ncl_{\delta}(I) \cup Ncl_{\delta}(J)$. Then for any N^{q}_{RO}-nhd A of $s_{(\upsilon,\omega,\xi)}$, $Aq(I \cup J)$. Hence, AqI or AqJ. Therefore $s_{(\upsilon,\omega,\xi)} \in Ncl_{\delta}(I)$ or $s_{(\upsilon,\omega,\xi)} \in Ncl_{\delta}(J)$. Hence $s_{(\upsilon,\omega,\xi)} \in Ncl_{\delta}(I) \cup Ncl_{\delta}(J)$.

v) Since $I \cap J \subseteq I, Ncl_{\delta}(I \cap J) \subseteq Ncl_{\delta}(I)$. Similarly, $Ncl_{\delta}(I \cap J) \subseteq Ncl_{\delta}(J)$. Therefore $Ncl_{\delta}(I \cap J) \subseteq Ncl_{\delta}(P) \cap Ncl_{\delta}(J)$.

0.1cm□

Theorem 3.15. *Considering $(S, \dot{\tau})$ to represent a N_{TS}, the following remains true:*

i) *Finite union of $N_{\delta-CS}$ in S is an $N_{\delta-CS}$ in S*

ii) *Arbitrary intersection of $N_{\delta-CS}$s in S is a $N_{\delta-CS}$ in S.*

Proof. i) Let T_1 and T_2 be $N_{\delta-CS}$s. Then $Ncl_\delta (T_1 \cup T_2) = Ncl_\delta (T_1) \cup Ncl_\delta (T_2) = T_1 \cup T_2$. Thus $T_1 \cup T_2$ is a $N_{\delta-CS}$.

ii) Let T_i be a $N_{\delta-CS}$, for each $i \in I$. To show that $Ncl_\delta (\cap T_i) \subseteq \cap T_i$, take any $s_{(\upsilon,\omega,\xi)} \in Ncl_\delta (\cap T_i)$. Suppose that $s_{(\upsilon,\omega,\xi)} \notin \cap T_i$. Then there exists an $i_0 \in I$ such that $s_{(\upsilon,\omega,\xi)} \notin T_{i0}$. Since T_{i0} is a $N_{\delta-CS}$, $s_{(\upsilon,\omega,\xi)} \notin Ncl_\delta (T_{i0})$. Therefore there exists a N^q_{RO}-nhd A of $s_{(\upsilon,\omega,\xi)}$ such that $A\tilde{q}T_{i0}$. Since $A\tilde{q}T_{i0}$ and $\cap T_i \subseteq T_{i0}$, we have $A\tilde{q} (\cap T_{i0})$. Thus $s_{(\upsilon,\omega,\xi)} \notin Ncl_\delta (\cap T_i)$. This is a contradiction. Hence $Ncl_\delta (\cap T_i) \subseteq \cap T_i$.

0.1cm□

Theorem 3.16. *Let R be a N_S in a N_{TS} $(S, \dot{\tau})$, then $Ncl_\delta (R)$ is the intersection of all N_{RCS}s of R or*

$Ncl_\delta (R) = \bigcap \{H/R \subseteq H = Ncl (Nint (H))\}$.

Proof. Suppose that $s_{(\upsilon,\omega,\xi)} \notin \bigcap \{H/R \subseteq H = Ncl (Nint (H))\}$. Then there exists a N_{RCS} H such that $s_{(\upsilon,\omega,\xi)} \notin H$ and $R \subseteq H$. Since $s_{(\upsilon,\omega,\xi)} \notin H$, $s_{(\upsilon,\omega,\xi)} q H^c$. Note that $R \subseteq H$ if and only if $R\tilde{q}H^c$. Thus H^c is a N^q_{RO}-nhd of $s_{(\upsilon,\omega,\xi)}$ such that $R\tilde{q}H^c$. Hence $s_{(\upsilon,\omega,\xi)} \notin Ncl_\delta (R)$.

Let $s_{(\upsilon,\omega,\xi)} \in \bigcap \{H/R \subseteq H = Ncl (Nint (H))\}$. Suppose that $s_{(\upsilon,\omega,\xi)} \notin Ncl_\delta (R)$. Then there exists a N^q_{RO}-nhd I of $s_{(\upsilon,\omega,\xi)}$ such that $R\tilde{q}I$. So, $R \subseteq I^c$. Since $s_{(\upsilon,\omega,\xi)} qI, s_{(\upsilon,\omega,\xi)} \notin I^c$. Therefore there exists a N_{RCS} I^c such that $s_{(\upsilon,\omega,\xi)} \notin I^c$ and $R \subseteq I^c$. Hence $s_{(\upsilon,\omega,\xi)} \notin \bigcap \{H/R \subseteq H = Ncl (Nint (H))\}$. This is a contradiction. Thus $s_{(\upsilon,\omega,\xi)} \in Ncl_\delta (R)$. □

Remark 3.17. From the above theorem, for any N_S R, $Ncl_\delta (R)$ is a N_{CS}. Moreover, $Ncl_\delta (R)$ becomes $N_{\delta-CS}$, which will be shown in the Theorem 3.20.

Example 3.18. Let $S = \{a, b\}$, and R be the N_S defined by
$R = \langle (0.5, 0.3), (0.2, 0.2), (0.3, 0.5) \rangle$ Let $\dot{\tau} = \{0_N, 1_N, R\}$. Then $\dot{\tau}$ is a N_T. Since $Ncl (Nint (R^c)) = Ncl (0_N) = 0_N \neq R^c$, R^c is not a N_{RCS}. Hence 0_N and 1_N are the only regular closed sets. thus $Ncl_\delta (R^c) = \bigcap \{H/R^c \subseteq H = Ncl (Nint (H))\} = 1_N \neq R^c$. Hence R^c is not $N_{\delta-CS}$. Therefore, R^c is a N_{CS} which is not $N_{\delta-CS}$.

Theorem 3.19. *If I is a N_{RCS}, then I is a $N_{\delta-CS}$.*

Proof. Let I be a N_{RCS}. Then $Ncl (Nint (I)) = I$. By Theorem 3.16, $Ncl_\delta (I) = \bigcap \{H/I \subseteq H = Ncl (Nint (H))\} = I$. Thus I is $N_{\delta-CS}$. □

Theorem 3.20. *For any N_S I, $Ncl_\delta(I)$ is a $N_{\delta-CS}$.*

Proof. By Theorem 3.15,3.16,3.19. □

The following properties of neutrosophic δ-interior are the results obtained from neutrosophic δ-closure.

Theorem 3.21. *For a N_{TS} $(S, \dot{\tau})$, let I and J be two N_S. Following that, we have the subsequent characteristics:*

i) $Nint_\delta(1_N) = 1_N$

ii) $Nint_\delta(I) \subseteq I$

iii) $I \subseteq J \Rightarrow Nint_\delta(I) \subseteq Nint_\delta(J)$

iv) $Nint_\delta(I \cap J) = Nint_\delta(I) \cap Nint_\delta(J)$

v) $Nint_\delta(I) \cup Nint_\delta(J) \subseteq Nint_\delta(I \cup J)$.

Theorem 3.22. *Considering $(S, \dot{\tau})$ to represent a N_{TS}, the following remains true:*

i) *Finite intersection of $N_{\delta-OS}$ in S is a $N_{\delta-OS}$ in S*

ii) *Arbitrary union of $N_{\delta-OS}$s in S is a $N_{\delta-OS}$ in S.*

Theorem 3.23. *Given an I of type N_S in the set $(S, \dot{\tau})$, we have $Nint_\delta(I) = \bigcup\{G/Nint(Ncl(G)) = G \subseteq I\}$. It follows that $Nint_\delta(I)$ is a N_{OS}.*

Corollary 3.24. *I is a N_{OS} if and only if I belong to a $N_{\delta-OS}$ in a N_{TS} $(S, \dot{\tau})$.*

Corollary 3.25. *If I is a N_{ROS}, then I is a $N_{\delta-OS}$.*

Corollary 3.26. *For any N_S I, $Nint_\delta(I)$ is a $N_{\delta-OS}$.*

4. Conclusion

This paper covered the fascinating natural subject of $\mathbf{N}_\delta$-Closure and $\mathbf{N}_\delta$-Interior in N_{TS}. It will provide many new opportunities for research into N_{TS}, allowing us to expand on and further analyze the ideas we presented in this paper.

References

[1] I.Arokiarani, R.Dhavaseelan, S.Jafari and M.Parimala., On Some New Notions and Functions in NTS, *Neutrosophic Sets and Systems,* **16**, (2017) 16 – 19.

[2] T. K. Atanassov, Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems*. 20, (1986) 87-96.

[3] S. Broumi, F. Smarandache, Correlation Coefficient of Interval Neutrosophic set, *Periodical of Applied Mechanics and Materials*, Vol. 436, (2013).

[4] P. Chi, and P. Liu. An Extended TOPSIS Method for the Multiple Attribute Decision Making Problems Based on Interval Neutrosophic Set, *Neutrosophic Sets and Systems*, Vol. 01, 2014, 63-71.

[5] G. Deschrijive and E. E. Kerre, On the Position of Intuitionistic Fuzzy Set Theory in the Framework of Theories Modelling Imprecision, *Information Sciences*, Vol. 177, No. 8, 2007, pp. 1860-1866.

[6] R.Dhavaseelan and S.Jafari., Generalized neutrosophic closed sets, *New Trends in Neutrosophic Theory and Applications,* **2** (2017), 261 – 273.

[7] R.Dhavaseelan, S.Jafari, C.Ozel and M.A.Al-Shumrani., Generalized Neutrosophic Contra-Continuity, *New Trends in Neutrosophic Theory and Applications,* **2**, (2017), 1 – 17.

[8] Dickman. Jr. R. F. and Porter. J. R., θ-closed subsets of Hausdorff spaces, *Pacific J. Math.,* **59**(2), (1975), pp. 407 – 415.

[9] M.S.Espelie; J.E.Joseph. Some properties of θ-closure, *Canad. J. Math.,* **33**(1), (1981), pp. 142 – 149. [Online]. Available: http://dx.doi.org/10.4153/CJM-1981-013-8.

[10] S. Ganguly and S. Saha., A note on δ-continuity and δ-connected sets in fuzzy set theory, *Simon Stevin,* **62**, (1988), pp. 127 – 141.

[11] Y. Guo and D. H. Cheng, New neutrosophic approach to image segmentation. *Pattern Recognition*, 42, (2009), 587595.

[12] I.M.Hanafy, A.M.Abd El-Aziz and T.M.Salman., Intuitionistic fuzzy θ-closure operator, *Bol. Asoc. Mat. Venez.,* **13**(1), (2006), pp. 27 – 39.

[13] M. I. Hanafy, A. A. Salama, and K. Mahfouz, Correlation of neutrosophic Data, *International Refereed Journal of Engineering and Science*, 1(2) (2012) 39-43.

[14] M. I. Hanafy, A. A. Salama, and K. M. Mahfouz, Correlation Coefficients of Neutrosophic Sets by Centroid Method, *International Journal of Probability and Statistics*, 2(1) (2013) 9-12.

[15] P.Iswarya and Dr K.Bageerathi., A Study on Neutrosophic Frontier and Neutrosophic Semi-froutier in NTS, *Neutrosophic Sets and Systems,* **16**, (2017), pp. 6 – 15.

[16] P.Iswarya and Dr K.Bageerathi., On neutrosophic semi-open sets in NTS, *International Journal of Mathematics Trends and Technology (IJMTT),* **37**(3), (2016), 24 – 33.

[17] S.J.Lee and S.M.Yun., Fuzzy delta separation axioms, *IEEE International Conference on Fuzzy Systems,* (2011), pp. 592 – 602.

[18] S.J.Lee and J.T.Kim., Properties of fuzzy (r, s)- semi-irresolute mappings in IF_{TS}, *International Journal of Fuzzy Logic and Intelligent Systems,* **11**(3), (2011), pp. 190 – 196.

[19] S.J.Lee and J.T.Kim., Fuzzy strongly (r, s)- precontinuous mappings, *IEEE International Conference on Fuzzy Systems,* (2009), pp. 581 – 586.

[20] L. Lin, X. H. Yuan and Q. Z. Xia, Multicriteria Fuzzy Decision-Making Methods Based on Intuitionistic Fuzzy Sets, *Journal of Computer and System Science*, Vol. 73, No. 1, 2007, pp. 84-88.

[21] P. Majumdar and K. S. Samanta, On similarity and entropy of neutrosophic sets. *Journal of Intelligent and Fuzzy Systems*, 26(3) (2014), 1245-1252.

[22] J. M. Mendel, Fuzzy logic systems for engineering: A tutorial, *Proc. IEEE*, vol. 83, (1995) 345377.

[23] A. Mukherjee, and S. Sarkar. Several Similarity Measures of Interval Valued Neutrosophic Soft Sets and Their Application in Pattern Recognition Problems, *Neutrosophic Sets and Systems*, 06, 2015,55-61.

[24] M.N.Mukherjee and S.P.Sinha., Fuzzy θ-closure operator on fuzzy topological spaces, *Internat. J. Math. Math. Sci.,* **14**(2), (1991), pp. 309 – 314.

[25] Parimala, M., Jeevitha,R., Smarandache, F., Jafari, S. and Udhayakumar, R., Neutrosophic $\alpha\psi$ Homeomorphism in Neutrosophic Topological Spaces, Information, 2018, 9(187), 1-10

[26] A.A.Salama and S.A.Alblowi., NS and neutrosophic topological space, *ISOR J. mathematics,* **3**(4), (2012), pp-31 – 35.

[27] A. A. Salama and A. S. Alblowi, "Generalized Neutrosophic Set and Generalized Neutrousophic Topological Spaces ", *Journal of computer Sci. Engineering*, Vol. 2, No. 7, (2012) 29-32.

[28] A. A. Salama and A. S. Alblowi, Neutrosophic set and Neutrosophic Topological Space, *ISORJ. Mathematics*, Vol. 3(4), (2012) 31-35.

[29] A, A, Salama, A. H. El-Ghareeb, A. M. Manie, and F. Smarandache, Introduction to Develop Some Software Programs for Dealing with Neutrosophic Sets, *Neutrosophic Sets and Systems*, Vol. 3, (2014), 53-54.

[30] Shuker Mahmood Khalil, On Neurosophic Delta Generated Per-Continuous Functions in Neutrosophic Topological Spaces, *Neutrosophic Sets and Systems*, Vol. 48, (2022), 122 – 141.

[31] F. Smarandache, A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic, *American Research Press*, Rehoboth, 1999.

[32] D.Sivaraj., Semi open set characterizations of almost-regular spaces, *Glas. Mat. Ser. III,* **21(41)**(2), (1986), pp. 437 – 440.

[33] E. Szmidt, J. Kacprzyk, Medical diagnostic reasoning using a similarity measure for intuitionistic fuzzy sets, *Note on Intuitionistic Fuzzy Sets* 10 (4) (2004) 61 – 69.

[34] I. Turksen, Interval valued fuzzy sets based on normal forms, *Fuzzy Sets and Systems* 20, (1986) 191-210.

[35] H.V.Velichko., On the theory of H-closed topological spaces, *Sibirsk. Mat. Z.* **8**, (1967), pp. 754 – 763.

[36] N.V.Velicko., H-closed topological spaces, *Mat. Sb. (N.S.),* **70**(112), (1966), pp. 98 – 12.

[37] H. Wang , F. Smarandache, Q. Y. Zhang, and R. Sunderraman, Interval neutrosophic sets and logic: *Theory and applications in computing*, Hexis, Phoenix, AZ, 2005.

[38] H. Wang, F. Smarandache, Q. Y. Zhang, and R. Sunderraman. Single valued neutrosophic sets, *Multispace and Multistructure* 4 (2010) 410-413.

[39] J. Ye, Multi-criteria decision-making method using the correlation coefficient under single-valued neutrosophic environment, *International Journal of General Systems*, 42(4) (2013) 386-394.

[40] J. Ye, Similarity measures between interval neutrosophic sets and their applications in multi-criteria decision making, *Journal of Intelligent and Fuzzy Systems*, 26(1) (2013) 165-172.

[41] J. Ye, Single valued neutrosophic cross-entropy for multi-criteria decision making problems, *Applied Mathematical Modelling*, 38(3) (2014) 1170-1175.

[42] S. Ye, and J. Ye, Dice Similarity Measure between Single Valued Neutrosophic Multisets and Its Application in Medical Diagnosis, *Neutrosophic Sets and Systems*, Vol. 06, 2015, 49-54.

[43] Yeon Seok Eom and Seok Jong Lee., Delta Closure and Delta Interior in Intuitionistic Fuzzy Topological Spaces, *International Journal of Fuzzy Logic and Intelligent Systems,* **12**(4), (2012), pp. 290 – 295.

[44] L. A. Zadeh, Fuzzy sets, *Inform. Control*, vol. 8, (1965) 338353.

[45] A. L. Zimmermann, Fuzzy Set Theory and Its Applications, 2nd ed. Boston, MA: Kluwer, 1991.

Received: Jan 9, 2024. Accepted: Mar 28, 2024

NSS Neutrosophic Sets and Systems, Vol. 66, 2024
University of New Mexico

Translation of Neutrosophic INK-Algebra

Wadei Faris AL-Omeri[1*], M Kaviyarasu[2*] and M Rajeshwari [3]
[1]Department of Mathematics, Faculty of Science and Information Technology Jadara University, Irbid, Jordan; wadeimoon1@hotmail.com
[2]Department of Mathematics, Vel Tech Rangarajan Dr Sagunthala R & D Institute of Science and Technology, Chennai, Tamilnadu-600062,India;kavitamilm@gmail.com
[3] School of Engineering, Presidency University, Bangalure,India; rajeakila@gmail.com
*Correspondence: wadeimoon1@hotmail.com and kavitamilm@gmail.com;

Abstract. This study investigates the philosophical foundations of the neutrosophic set($\mathbb{N}_S$), as first proposed by Smarandache. It clarifies the connection between single-valued $\mathbb{N}_s$s and their function as a specialized subset in the larger context of $\mathbb{N}_s$s, particularly in the fields of science and engineering. This paper investigates neutrosophic INK-ideals(NINK-Is) within INK-algebras(INK-A) by using the notion of translation, which is positioned as an extension of intuitionistic fuzzy sets. The notion of translation neutrosophic INK-algebras(NTINK-A) is introduced and their fundamental characteristics are explored. In addition, properties associated with the translation of INK-subalgebra(INK-Ss) and INK-ideals(INK-I) are investigated, along with the dynamics of their multiplications, unions, and intersections in the context of neutrosophic INK-ideals(NINK-I). Further definitions and theorems are added in the article, providing comprehensive insights complexities of NINK-A.

Keywords: INK-algebra; Translation INK-ideal; Neutrosophic translation INK-ideal; Neutrosophic translation INK-subalgebra.

1. **Introduction**

Zade [9] in 1965 introduced, the idea of fuzzy sets has shown to be useful in addressing uncertainties in a variety of real-world applications, then Imai [31] and Iseki [32], [30] applied the fuzzy set in BCI/BCK-algebras. Atanassov [2] developed the intuitionistic fuzzy set in 1983 as an extension of this concept, and it has since been used in a variety of industries, including financial services, sales analysis, marketing of new products, negotiation techniques, and psychology research. In Mathematica Japonica, an essay by Jun and Meng from 1994 [28] delves into the subject of fuzzy p-ideals in BCI-algebras. The main goal is to comprehend these fuzzy p-ideals' features and attributes about BCI-algebras, algebraic structures having uses in computer science and mathematical logic. Furthermore, neutrosophic probability, set, and logic are included in the notion of neutrosophic, which was first introduced by Smarandache [5]. A philosophical framework called neutrosophic addresses contradiction, indeterminacy, and imperfect knowledge. Jun and Eun Hwan Roh's [7] study, published in Open Mathematics, extends the investigation of neutrosophic ideals in a particular kind of algebraic structure by delving into MBJ-neutrosophic ideals of BCK/BCI-algebras. Kaviyarasu et a. explored fuzzy p-ideals in INK-Algebra and $\mathbb{N}_s$s in the same algebraic structure [10]. The comprehension of fuzzy and neutrosophic notions in the context of INK-Algebra is probably improved by these investigations [11].

The concept of $\mathbb{N}_s$s is presented as a more comprehensive framework that generalizes intuitionistic fuzzy sets in Smarandache's [4]. In [8], Lee, Jun, and Doh published a study on fuzzy multiplications and translations in BCK/BCI-algebras. It probably looks at these operations' characteristics and ramifications to these algebraic structures. Agboola and Davvaz gave an introduction to neutrosophic BCI/BCK algebras [1]. Senapati [20] explores fuzzy translations of fuzzy H-ideals in BCK/BCI-algebras, presumably investigating the connections and implications of fuzzy translations and fuzzy H-ideals in these algebraic structures.

Senapati et a. [21] also published a paper in the Eurasian Mathematical Journal that probably focuses on Atanassov's intuitionistic fuzzy translations of ideals in BCK/BCI-algebras and intuitionistic fuzzy subalgebras, thus expanding the literature on fuzzy translations to intuitionistic fuzzy sets. The 2016 work by Wadei F. Al-Omeri [27] and colleagues investigate the "Ra" operator in ideal topological spaces, presumably looking at its characteristics and uses in the setting of these spaces. Kaviyarasu and Indhira [13]presented on intuitionistic fuzzy INK-ideals of INK-algebras.The study by Mohseni Takallo et al. [16] presents MBJ-neutrosophic structures and their uses in BCK/BCI-algebras, probably investigating the characteristics and importance of these structures to these algebraic systems. The 2018 study by Al-Omeri et al. [26] examines the degree of (L, M)-fuzzy semi-precontinuous and (L, M)-fuzzy semi-preirresolute functions, most likely with an emphasis on their continuity qualities and fuzzy mathematics consequences. The 2019 work by Manokaran et al. [14] addresses MBJNeutrosophic B-Subalgebras, probably examining their characteristics and uses in the context of neutrosophic algebraic structures.

Khalid et al. [15] investigate MBJ-neutrosophic T-ideals on B-algebras, presumably looking into the characteristics and uses of these ideals inside the context of B-algebras.In their 2020 work, Kaviyarasu et al. [12] address the direct product of neutrosophic INKAlgebras, presumably investigating the structure and characteristics of these products with neutrosophic algebraic systems.

Al-Omeri et al. [23] is devoted to cone metric spaces and neutrosophic fixed point theorems; they most likely contain conclusions about the uniqueness and existence of fixed points in

$\mathbb{N}_s$tings. Mixed b-fuzzy topological spaces are covered in Al-Omeri's [24] work, which probably explores the characteristics and uses of these spaces in fuzzy topology and later examines virtually e-I-continuous functions in another article, most likely to examine their continuity qualities and potential uses in mathematical analysis [25]. In their 2021 work, Song et al. [19] use MBJ-Neutrosophic structures to study commutative ideals of BCI-Algebras. They probably investigate these ideals' characteristics and uses to BCI-Algebras. In the Hacettepe Journal of Mathematics and Statistics, Jun's [29] work addresses fuzzy ideal translations in BCK/BCI algebras. This study probably investigates the transformation of fuzzy ideals inside the algebraic structures. Bordbar et al. [3] investigate positive implicative ideals of BCK-algebras based on falling shadows and $\mathbb{N}_s$s. They most likely look into these ideas' characteristics and uses, paying close attention to their implicative elements. The SuperHyperSoft Set and the Fuzzy Extension Super HyperSoft Set are concepts that are introduced in Smarandache's [22], offering a fresh take on set theory with applications in neutrosophic systems. New forms of soft sets, such as HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set, are presented in another work by Smarandache [6]. These new types of soft sets give expansions and enhancements to the theories of current soft sets. The direct product of neutrosophic h-ideal in INK-Algebra is examined in Kaviyarasu and Rajeshwari [17]. It probably looks at the creation and characteristics of such products inside this algebraic structure. Pura Vida Neutrosophic Algebra is the topic of Ranulfo et al. 2023 article, which is expected to present a novel method of algebraic structures with applications in neutrosophic systems [18]. They extended their study to intuitionistic fuzzy translations by examine the interplay between fuzzy translations, extensions, and multiplications. The present paper uses $\mathbb{N}_s$s to study TINK-Is in INK-algebras. The writers investigate the characteristics of translation INK-subalgebra and present the idea of NINK-I. The following aspects are addressed in this discussion:

- Using translations to set up NINK-Ss.
- To discuss NTINK-Is in relation to INK-subalgebras.
- Investigating the relationship between translations of NINK-I and NINK-Ss.
- Presenting the conditions under which a translations of NINK-I can be made from an neutrosophic INK-algebra.
- constructing the translation property for NINK-I.

A summary of the fundamental ideas of INK-algebra and $\mathbb{N}_s$s that are necessary for understanding the discussions in this paper is given in the second section. $\mathbb{N}_s$s are used to study TINK-Ss and INK-I in the third section. This investigation is continued in Section four.

2. **Basic Definitions**

The basic components required to understand this paper are included in this section.

Definition 2.1. [11] In algebra $(\daleth, \star, 0)$ is known as a INK-algebra if it fulfils the following criteria for any $\check{l}, \check{m}, \check{n} \in \daleth$.

(1) $((\check{l} \star \check{m}) \star (\check{l} \star \check{n})) \star (\check{n} \star \check{m}) = 0$
(2) $(((\check{l} \star \check{n}) \star (\check{m} \star \check{n})) \star (\check{l} \star \check{m}) = 0$
(3) $\check{l} \star 0 = \check{l}$
(4) $\check{l} \star \check{m} = 0$ and $\check{m} \star \check{l} = 0 \Longleftrightarrow \check{l} = \check{m}$

where $\star$ is a binary operation and the 0 is a constant of $\daleth$.

Example 2.2. Consider INK-algebra $\daleth = \{0, 1, 2, 3, 4\}$ with the following cayclay table.

·	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	0	0
b	b	b	0	b	0
c	c	a	c	0	a
d	d	d	d	d	0

Table 1. INK-algebra

Definition 2.3. [11] If $\check{l} \star \check{m}$ in $\c{S}$, a non-empty subset $\c{S}$ of an INK-algebra $(\daleth, \star, 0)$ is said to be an INK-s of $\daleth$.

Definition 2.4. [12] Consider the INK-algebra $(\daleth, \star, 0)$. In INK-I of $\daleth$ is a non-empty subset I of $\daleth$ that satisfies,

(1) $0 \in \c{I}$
(2) $((\check{n} \star \check{l}) \star (\check{n} \star \check{m})) \in \c{I}$ and $\check{m} \in \c{I}$ imply $\check{l} \in \c{I}$, $\forall\ \check{l}, \check{m}, \check{n} \in \daleth$.

Definition 2.5. [4] In a non-void set $\daleth$, a $\mathbb{N}_s$ A is the form's structure $A = \left\{\left(\daleth, \Bbbk_A^{T_r}(\check{l}), \Bbbk_A^{I_d}(\check{l}), \Bbbk_A^{F_s}(\check{l})\right) | \check{l} \in \daleth\right\}$, where $\Bbbk_A^{T_r} : \daleth \to [0,1]$ is a truth membership function $\Bbbk_A^{I_d} : \daleth \to [0,1]$ is a indeterminate membership function and $\Bbbk_A^{F_s} : \daleth \to [0,1]$ is a false membership function.

Definition 2.6. [12] A $\mathbb{N}_s$ $A = \left\langle \Bbbk^T, \Bbbk^I, \Bbbk^F \right\rangle$ is known as a NINK-S if it meets all the requirements of INK-algebra,

1. $\left(\Bbbk_A^{T_r}\right)(\check{l} \star \check{m}) \geq \wedge \left\{\left(\Bbbk_A^{T_r}\right)(\check{l}), \left(\Bbbk_A^{T_r}\right)(\check{m})\right\}$
2. $\left(\Bbbk_A^{I_d}\right)(\check{l} \star \check{m}) \geq \wedge \left\{\left(\Bbbk_A^{I_d}\right)(\check{l}), \left(\Bbbk_A^{I_d}\right)(\check{m})\right\}$
3. $\left(\Bbbk_A^{F_s}\right)(\check{l} \star \check{m}) \leq \vee \left\{\left(\Bbbk_A^{F_s}\right)(\check{l}), \left(\Bbbk_A^{F_s}\right)(\check{m})\right\}, \ \forall\ \check{l}, \check{m}, \in \daleth$.

A	0	1	2	3	4
$\Bbbk^{T_r}$	0.5	0.4	0.2	0.5	0.1
$\Bbbk^{I_d}$	0.6	0.3	0.1	0.4	0.1
$\Bbbk^{F_s}$	0.26	0.35	0.45	0.55	0.59

Table 2. NINK-S

Example 2.7. Define $A = \langle \Bbbk^{Tr}, \Bbbk^{I_d}, \Bbbk^{F_s} \rangle$ be an neutrosophic subset of $\beth$ in Table 1.

Then A is a neutrosophic INK-S of $\beth$.

Definition 2.8. [12] Let $\beth$ be a INK-algebra. $\mathbb{N}_s$ $A = \left\{ \left(\beth, \Bbbk_A^{T_r}(\check{l}), \Bbbk_A^{I_d}(\check{l}), \Bbbk_A^{F_s}(\check{l})\right) | \check{l} \in \beth \right\}$ in $\beth$ is referred to as N-I of $\beth$, if it meets the following criteria

(1) $\Bbbk_A^{T_r}(0) \geq \Bbbk_A^{T_r}(\check{l}), \Bbbk_A^{I_d}(0) \geq \Bbbk_A^{I_d}(\check{l})$ and $\Bbbk_A^{F_s}(0) \leq \Bbbk_A^{F_s}(\check{l})$
(2) $\Bbbk_A^{T_r}(\check{l}) \geq \wedge \left\{ \Bbbk_A^{T_r}(\check{l} \star \check{m}), \Bbbk_A^{T_r}(\check{m}) \right\}$
(3) $\Bbbk_A^{I_d}(\check{l}) \geq \wedge \left\{ \Bbbk_A^{I_d}(\check{l} \star \check{m}), \Bbbk_A^{I_d}(\check{m}) \right\}$
(4) $\Bbbk_A^{F_s}(\check{l}) \leq \vee \left\{ \Bbbk_A^{F_s}(\check{l} \star \check{m}), \Bbbk_A^{F_s}(\check{m}) \right\}$. $\forall\ \check{l}, \check{m} \in \beth$.

Definition 2.9. [12] Let $\beth$ be a INK-A. $\mathbb{N}_S A = \left\{ \left(\beth, \Bbbk_A^{T_r}(\check{l}), \Bbbk_A^{I_d}(\check{l}), \Bbbk_A^{F_s}(\check{l})\right) | \check{l} \in \beth \right\}$ in $\beth$ is referred to as NINK-I of $\beth$, if it meets the following criteria

(1) $\Bbbk_A^{T_r}(0) \geq \Bbbk_A^{T_r}(\check{l}), \Bbbk_A^{I_d}(0) \geq \Bbbk_A^{I_d}(\check{l})$ and $\Bbbk_A^{F_s}(0) \leq \Bbbk_A^{F_s}(\check{l})$
(2) $\Bbbk_A^{T_r}(\check{l}) \geq \wedge \left\{ \Bbbk_A^{T_r}((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \Bbbk_A^{T_r}(\check{m}) \right\}$
(3) $\Bbbk_A^{I_d}(\check{l}) \geq \wedge \left\{ \Bbbk_A^{I_d}((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \Bbbk_A^{I_d}(\check{m}) \right\}$
(4) $\Bbbk_A^{F_s}(\check{l}) \leq \vee \left\{ \Bbbk_A^{F_s}((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \Bbbk_A^{F_s}(\check{m}) \right\}$, $\forall\ \check{l}, \check{m}, \check{n} \in \beth$.

3. NTINK-S

For the sake of simplicity, we shall use the symbol $A = \left(\Bbbk_A^{T_r}, \Bbbk_A^{I_d}, \Bbbk_A^{F_s}\right)$ for the neutrosophic subset $A = \left(\check{l}, \Bbbk_A^{T_r}, \Bbbk_A^{I_d}, \Bbbk_A^{F_s}; \check{l} \in \beth\right)$. Throughout this paper, we take $T_r, Id = 1 - sup\left\{\Bbbk_A^{T_r}(\check{l}), \Bbbk_A^{I_d}(\check{l}) | \check{l} \in \beth\right\}$ and $Fs = inf\left\{\Bbbk_A^{F_s}(\check{l}) | \check{l} \in \beth\right\}$ for any $\mathbb{N}_s$ $A = \left(\Bbbk_A^{T_r}, \Bbbk_A^{I_d}, \Bbbk_A^{F_s}\right)$ of $\beth$.

Definition 3.1. A $\mathbb{N}_s$ $A = \left(\Bbbk_A^{T_r}, \Bbbk_A^{I_d}, \Bbbk_A^{F_s}\right)$ be a $\mathbb{N}_s$ of $\beth$ and $\varrho, \zeta, \nu \in [0, Ts]$. An object having the form $\left(A_{\varrho,\zeta,\nu}^{T_r, Id, Fs}\right)^{Ts} = \left\langle \left(\Bbbk_A^{T_r}\right)_{\varrho}^{Ts}, \left(\Bbbk_A^{I_d}\right)_{\zeta}^{Ts}, \left(\Bbbk_A^{F_s}\right)_{\nu}^{Ts} \right\rangle$ is called a neutrosophic ϱ, ζ, ν translations of $\mathbb{N}_s$, if it satisfies $\left(\Bbbk_A^{T_r}\right)_{\varrho}^{Ts}(\check{l}) = \Bbbk_A^{T_r}(\check{l}) + \varrho$, $\left(\Bbbk_A^{I_d}\right)_{\zeta}^{Ts}(\check{l}) = \Bbbk_A^{I_d}(\check{l}) + \zeta$ and $\left(\Bbbk_A^{F_s}\right)_{\nu}^{Ts}(\check{l}) = \Bbbk_A^{F_s}(\check{l}) - \nu$.

Definition 3.2. A $\mathbb{N}_s$ $A = \langle \Bbbk^{Tr}, \Bbbk^{I_d}, \Bbbk^{F_s} \rangle$ is known as a NT INK-S if it meets all the requirements of INK-algebra,

1. $\left(\Bbbk_A^{T_r}\right)_\varrho^{Ts}(\check{l} \star \check{m}) = \wedge \left\{ \left(\Bbbk_A^{T_r}\right)_\varrho^{Ts}(\check{l}), \left(\Bbbk_A^{T_r}\right)_\varrho^{Ts}(\check{m}) \right\}$
2. $\left(\Bbbk_A^{I_d}\right)_\zeta^{Ts}(\check{l} \star \check{m}) = \wedge \left\{ \left(\Bbbk_A^{I_d}\right)_\zeta^{Ts}(\check{l}), \left(\Bbbk_A^{I_d}\right)_\zeta^{Ts}(\check{m}) \right\}$
3. $\left(\Bbbk_A^{F_s}\right)_\nu^{Ts}(\check{l} \star \check{m}) = \vee \left\{ \left(\Bbbk_A^{F_s}\right)_\nu^{Ts}(\check{l}), \left(\Bbbk_A^{F_s}\right)_\nu^{Ts}(\check{m}) \right\}$.

Example 3.3. Consider INK-algebra in Table 2. Let $T_s = 0.5$ and we take $\varrho = 0.4, \zeta = 0.3$ *and* $\nu = 0.2$, then the neutrosophic ϱ, ζ ν translation is define by $(\Bbbk_A^{T_r})_\varrho, (\Bbbk_A^{I_d})_\zeta$ *and* $(\Bbbk_A^{F_s})_\nu$ of A in $\daleth$.

A	0	1	2	3	4
$(\Bbbk_A^{T_r})_\varrho$	0.9	0.8	0.6	0.9	0.5
$(\Bbbk_A^{I_d})_\zeta$	0.9	0.6	0.4	0.7	0.4
$(\Bbbk_A^{F_s})_\nu$	0.16	0.15	0.25	0.35	0.39

TABLE 3. NT INK-Algebra

Then A is a NT INK-S of $\daleth$.

Definition 3.4. Let A be a $\mathbb{N}_s$ of $\daleth$ and $\varrho, \zeta, \nu \in [0, 1]$. An object having the form $\left(A^{T_r,Id,Fs}\right)_{\varrho,\zeta,\nu}^M = \left\langle \left(\Bbbk_A^{T_r}\right)_\varrho^M, \left(\Bbbk_A^{I_d}\right)_\zeta^M, \left(\Bbbk_A^{F_s}\right)_\nu^M \right\rangle$ is called a N-M of A if, $\left(\Bbbk_A^{T_r}\right)_\varrho^M(\check{l}) = (\Bbbk_A^{T_r})^M \cdot \varrho$, $\left(\Bbbk_A^{I_d}\right)_\zeta^M(\check{l}) = (\Bbbk_A^{I_d})^M \cdot \zeta$ and $\left(\Bbbk_A^{F_s}\right)_\nu^M(\check{l}) = (\Bbbk_A^{F_s})^M \cdot \nu$, for all $\check{l} \in A$.

Example 3.5. Consider INK-algebra in Table 2. Let $T_s = 0.4$ and we take $\varrho = 0.3, \zeta = 0.2$ *and* $\nu = 0.1$, then the neutrosophic ϱ, ζ, ν multiplication is define by $(\Bbbk_A^{T_r})_\varrho^M, (\Bbbk_A^{I_d})_\zeta^M, (\Bbbk_A^{F_s})_\nu^M)$ of A in $\daleth$.

A	0	1	2	3	4
$(\Bbbk_A^{T_r})_\varrho^M$	0.15	0.12	0.06	0.15	0..03
$(\Bbbk_A^{I_d})_\zeta^M$	0.12	0.06	0.02	0.08	0.02
$(\Bbbk_A^{F_s})_\nu^M$	0.036	0.035	0.045	0.055	0.059

TABLE 4. N-M INK-A

Then A is a NTINK-S of $\daleth$.

Theorem 3.6. *Let A be a $\mathbb{N}_s$ of $\daleth$ such that the NT $\left(A^{T_r,Id,Fs}\right)_{\varrho,\zeta,\nu}^{Ts}$ of A is a nINK-S of $\daleth$, for some $\varrho, \zeta, \nu \in [0, T]$. Then A is a NINK-S of $\daleth$.*

Proof. Let $\left(A^{T_r,Id,Fs}\right)^{Ts}_{\varrho,\zeta,\nu}$ is a NINK-S of ℷ. For some $\varrho,\zeta,\nu \in [0,Ţ]$.

$$\begin{aligned}
\Bbbk^{T_r}_A(\check{l}\star\check{m})+\varrho &= (\Bbbk^{T_r}_A)^{Ts}_\varrho(\check{l}\star\check{m}) \\
&\geq \wedge\left\{(\Bbbk^{T_r}_A)^{Ts}_\varrho(\check{l}),(\Bbbk^{T_r}_A)^{Ts}_\varrho(\check{m})\right\} \\
&= \wedge\left\{\Bbbk^{T_r}_A(\check{l})+\varrho,\Bbbk^{T_r}_A(\check{m})+\varrho\right\} \\
&= \wedge\left\{\Bbbk^{T_r}_A(\check{l}),\Bbbk^{T_r}_A(\check{m})\right\}+\varrho \\
&= \wedge\left\{\Bbbk^{T_r}_A(\check{l}),\Bbbk^{T_r}_A(\check{m})\right\} \\
\Bbbk^{I_d}_A(\check{l}\star\check{m})+\zeta &= (\Bbbk^{I_d}_A)^{Ts}_\zeta(\check{l}\star\check{m}) \\
&\geq \wedge\left\{(\Bbbk^{I_d}_A)^{Ts}_\zeta(\check{l}),(\Bbbk^{I_d}_A)^{Ts}_\zeta(\check{m})\right\} \\
&= \wedge\left\{\Bbbk^{I_d}_A(\check{l})+\zeta,\Bbbk^{I_d}_A(\check{m})+\zeta\right\} \\
&= \wedge\left\{\Bbbk^{I_d}_A(\check{l}),\Bbbk^{I_d}_A(\check{m})\right\}+\zeta \\
&= \wedge\left\{\Bbbk^{I_d}_A(\check{l}),\Bbbk^{I_d}_A(\check{m})\right\} \\
\Bbbk^{F_s}_A(\check{l}\star\check{m})-\nu &= (\Bbbk^{F_s}_A)^{Ts}_\nu(\check{l}\star\check{m}) \\
&\leq \vee\left\{(\Bbbk^{F_s}_A)^{Ts}_\nu(\check{l}),(\Bbbk^{F_s}_A)^{Ts}_\nu(\check{m})\right\} \\
&= \vee\left\{\Bbbk^{F_s}_A(\check{l})-\nu,\Bbbk^{F_s}_A(\check{m})-\nu\right\} \\
&= \vee\left\{\Bbbk^{F_s}_A(\check{l}),\Bbbk^{F_s}_A(\check{m})\right\}-\nu \\
&= \vee\left\{\Bbbk^{F_s}_A(\check{l}),\Bbbk^{F_s}_A(\check{m})\right\}
\end{aligned}$$

□

Theorem 3.7. *If A be a NINK-S of ℷ,then the N-M of A is a NINK-S a of ℷ for all* $\varrho,\zeta,\nu \in [0,1]$.

Proof. Assume that $A=\left\langle \Bbbk^{T_r}_A,\Bbbk^{I_d}_A,\Bbbk^{F_s}_A\right\rangle$ be a NINK-S of ℷ, $\forall\varrho,\zeta,\nu\in[0,1]$.

$$\begin{aligned}
(\Bbbk^{T_r}_A)^M_\varrho(\check{l}\star\check{m}) &= \varrho\cdot\Bbbk^{T_r}_A(\check{l}\star\check{m}) \\
&= \varrho\cdot\wedge\left\{\Bbbk^{T_r}_A(\check{l}),\Bbbk^{T_r}_A(\check{m})\right\}
\end{aligned}$$

$$
\begin{aligned}
&= \wedge\left\{\varrho \cdot \Bbbk_A^{T_r}(\check{l}), \varrho \cdot \Bbbk_A^{T_r}(\check{m})\right\} \\
&\geq \wedge\left\{(\Bbbk_A^{T_r})_\varrho^M(\check{l}), (\Bbbk_A^{T_r})_\varrho^M(\check{m})\right\} \\
(\Bbbk_A^{I_d})_\zeta^M(\check{l} \star \check{m}) &= \zeta \cdot \Bbbk_A^{I_d}(\check{l} \star \check{m}) \\
&= \zeta \cdot \wedge\left\{\Bbbk_A^{I_d}(\check{l}), \Bbbk_A^{I_d}(\check{m})\right\} \\
&= \wedge\left\{\zeta \cdot \Bbbk_A^{I_d}(\check{l}), \zeta \cdot \Bbbk_A^{I_d}(\check{m})\right\} \\
&\geq \wedge\left\{(\Bbbk_A^{I_d})_\zeta^M(\check{l}), (\Bbbk_A^{I_d})_\zeta^M(\check{m})\right\} \\
(\Bbbk_A^{F_s})_\nu^M(\check{l} \star \check{m}) &= \nu \cdot \Bbbk_A^{F_s}(\check{l} \star \check{m}) \\
&= \nu \cdot \vee\left\{\Bbbk_A^{F_s}(\check{l}), \Bbbk_A^{F_s}(\check{m})\right\} \\
&= \vee\left\{\nu \cdot \Bbbk_A^{F_s}(\check{l}), \nu \cdot \Bbbk_A^{F_s}(\check{m})\right\} \\
&\leq \vee\left\{(\Bbbk_A^{F_s})_\nu^M(\check{l}), (\Bbbk_A^{F_s})_\nu^M(\check{m})\right\}
\end{aligned}
$$

Hence $(\Bbbk_A^{T_r})_\varrho^M, (\Bbbk_A^{I_d})_\zeta^M$ and $(\Bbbk_A^{F_s})_\nu^M$ is a multilication of neutrosophic INK-S of $\beth$. □

4. **Translation of NINK-ideal**

In this section, we define NTINK-Is in INK-algebra and investigate its properties.

Definition 4.1. A neutrosophic set $A = \langle \Bbbk^{T_r}, \Bbbk^{I_d}, \Bbbk^{F_s} \rangle$ is called a NINK-I of INK-A if,

(1) $\left(\Bbbk_A^{T_r}\right)_\varrho^{Ts}(0) \geq \left\{\left(\Bbbk_A^{T_r}\right)_\varrho^{Ts}(\check{l})\right\}, \left(\Bbbk_A^{I_d}\right)_\zeta^{Ts}(0) \geq \left\{\left(\Bbbk_A^{I_d}\right)_\zeta^{Ts}(\check{l})\right\}$ *and* $\left(\Bbbk_A^{F_s}\right)_\nu^{Ts}(0) \leq \left\{\left(\Bbbk_A^{F_s}\right)_\nu^{Ts}(\check{l})\right\}$

(2) $\left(\Bbbk_A^{T_r}\right)_\varrho^{Ts}(\check{l}) \geq \wedge\left\{\left(\Bbbk_A^{T_r}\right)_\varrho^{Ts}((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \left(\Bbbk_A^{T_r}\right)_\varrho^{Ts}(\check{m})\right\}$

(3) $\left(\Bbbk_A^{I_d}\right)_\zeta^{Ts}(\check{l}) \geq \wedge\left\{\left(\Bbbk_A^{I_d}\right)_\zeta^{Ts}((\check{n} \star \check{l}) \star (\check{n} \star \check{m}), \left(\Bbbk_A^{I_d}\right)_\zeta^{Ts}(\check{m})\right\}$

(4) $\left(\Bbbk_A^{F_s}\right)_\nu^{Ts}(\check{l}) \leq \vee\left\{\left(\Bbbk_A^{F_s}\right)_\nu^{Ts}((\check{n} \star \check{l}) \star (\check{n} \star \check{m}), \left(\Bbbk_A^{F_s}\right)_\nu^{Ts}(\check{m})\right\}$

Theorem 4.2. *If the nutrosophic translation $\left(A^{T_r,Id,Fs}\right)_{\varrho,\zeta,\nu}^{Ts}$ of A is a NINK-I of $\beth$ for some $\varrho, \zeta, \nu \in [0, T]$, it must be a neutrosophic ideal of $\beth$.*

Proof. Let $\left(A^{T_r,Id,Fs}\right)_{\varrho,\zeta,\nu}^{Ts}$ be NT of $\beth$.Then we have

$$
\begin{aligned}
(\Bbbk_A^{Tr})_\varrho^{Ts}(\check{l}) \quad &\geq \quad \wedge\left\{\left(\Bbbk_A^{T_r}\right)_\varrho^{Ts}((\check{n} \star \check{l}) \star (\check{n} \star \check{m})), \left(\Bbbk_A^{T_r}\right)_\varrho^{Ts}(\check{m})\right\} \\
&\textit{put } \check{n} = 0
\end{aligned}
$$

$$
\begin{aligned}
\left(\mathbb{k}_A^{T_r}\right)_\varrho^{Ts}(\check{l}) &\geq \wedge\left\{\left(\mathbb{k}_A^{T_r}\right)_\varrho^{Ts}(0\star\check{l})\star(0\star\check{m}),\left(\mathbb{k}_A^{T_r}\right)_\varrho^{Ts}(\check{m})\right\}\\
\left(\mathbb{k}_A^{T_r}\right)_\varrho^{Ts}(\check{l}) &\geq \wedge\left\{\left(\mathbb{k}_A^{T_r}\right)_\varrho^{Ts}(\check{l}\star\check{m}),\left(\mathbb{k}_A^{T_r}\right)_\varrho^{Ts}(\check{m})\right\}\\
\left(\mathbb{k}_A^{I_d}\right)_\zeta^{Ts}(\check{l}) &\geq \wedge\left\{\left(\mathbb{k}_A^{I_d}\right)_\zeta^{Ts}((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\left(\mathbb{k}_A^{I_d}\right)_\zeta^{Ts}(\check{m})\right\}\\
&put\ \check{n}=0\\
\left(\mathbb{k}_A^{I_d}\right)_\zeta^{Ts}(\check{l}) &\geq \wedge\left\{\left(\mathbb{k}_A^{I_d}\right)_\zeta^{Ts}(0\star\check{l})\star(0\star\check{m}),\left(\mathbb{k}_A^{I_d}\right)_\zeta^{Ts}(\check{m})\right\}\\
\left(\mathbb{k}_A^{I_d}\right)_\zeta^{Ts}(\check{l}) &\geq \wedge\left\{\left(\mathbb{k}_A^{I_d}\right)_\zeta^{Ts}(\check{l}\star\check{m}),\left(\mathbb{k}_A^{I_d}\right)_\zeta^{Ts}(\check{m})\right\}\\
\left(\mathbb{k}_A^{F_s}\right)_\nu^{Ts}(\check{l}) &\leq \vee\left\{\left(\mathbb{k}_A^{F_s}\right)_\nu^{Ts}((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\left(\mathbb{k}_A^{F_s}\right)_\nu^{Ts}(\check{m})\right\}\\
&put\ \check{n}=0\\
\left(\mathbb{k}_A^{F_s}\right)_\nu^{Ts}(\check{l}) &\leq \vee\left\{\left(\mathbb{k}_A^{F_s}\right)_\nu^{Ts}(0\star\check{l})\star(0\star\check{m}),\left(\mathbb{k}_A^{F_s}\right)_\nu^{Ts}(\check{m})\right\}\\
\left(\mathbb{k}_A^{F_s}\right)_\nu^{Ts}(\check{l}) &\leq \vee\left\{\left(\mathbb{k}_A^{F_s}\right)_\nu^{Ts}(\check{l}\star\check{m}),\left(\mathbb{k}_A^{F_s}\right)_\nu^{Ts}(\check{m})\right\}
\end{aligned}
$$

□

Theorem 4.3. *Let the NT* $\left(A^{T_r,I_d,F_s}\right)_{\varrho,\zeta,\nu}^{Ts}$ *of* A *is a NINK-I of* $\daleth$, *for* $\varrho,\zeta,\nu\in[0,1]$. *If* $\check{l}\leq\check{m}$, *then* $(\mathbb{k}_A^{T_r})_\varrho^{Ts}(\check{l})\geq(\mathbb{k}_A^{T_r})_\varrho^{Ts}(\check{m}),(\mathbb{k}_A^{I_d})_\zeta^{Ts}(\check{l})\geq(\mathbb{k}_A^{I_d})_\zeta^{Ts}(\check{m})$ *and* $(\mathbb{k}_A^{F_s})_\nu^{Ts}(\check{l})\leq(\mathbb{k}_A^{F_s})_\nu^{Ts}(\check{m})$.

Proof. Let $\check{l},\check{m},\check{n}\in\daleth$ we have,

$$
\begin{aligned}
(\mathbb{k}_A^{T_r})_\varrho^{Ts}(\check{l}) &\geq \wedge\left\{(\mathbb{k}_A^{T_r})_\varrho^{Ts}((\check{n}\star\check{l})\star(\check{n}\star\check{m})),(\mathbb{k}_A^{T_r})_\varrho^{Ts}(\check{m})\right\}\\
&= \wedge\left\{(\mathbb{k}_A^{T_r})_\varrho^{Ts}(\check{l}\star\check{m}),(\mathbb{k}_A^{T_r})_\varrho^{Ts}(\check{m})\right\}\\
&= \wedge\left\{(\mathbb{k}_A^{T_r})_\varrho^{Ts}(0),(\mathbb{k}_A^{T_r})_\varrho^{Ts}(\check{m})\right\}\\
&= (\mathbb{k}_A^{T_r})_\varrho^{Ts}(\check{m})\\
(\mathbb{k}_A^{I_d})_\zeta^{Ts}(\check{l}) &\geq \wedge\left\{(\mathbb{k}_{Ł}^{I})_\zeta^{Ts}((\check{n}\star\check{l})\star(\check{n}\star\check{m})),(\mathbb{k}_A^{I_d})_\zeta^{Ts}(\check{m})\right\}\\
&= \wedge\left\{(\mathbb{k}_A^{I_d})_\zeta^{Ts}(\check{l}\star\check{m}),(\mathbb{k}_A^{I_d})_\zeta^{Ts}(\check{m})\right\}\\
&= \wedge\left\{(\mathbb{k}_A^{I_d})_\zeta^{Ts}(0),(\mathbb{k}_A^{I_d})_\zeta^{Ts}(\check{m})\right\}\\
&= (\mathbb{k}_A^{I_d})_\zeta^{Ts}(\check{m})
\end{aligned}
$$

$$
(\mathbb{k}_A^{F_s})_\nu^{Ts}(\check{l}) \leq \vee\left\{(\mathbb{k}_A^{F_s})_\nu^{Ts}((\check{n}\star\check{l})\star(\check{n}\star\check{m})),(\mathbb{k}_A^{F_s})_\nu^{Ts}(\check{m})\right\}
$$

$$\begin{aligned} &= \vee\left\{(\Bbbk_A^{F_s})_\nu^{Ts}(\check{l}\star\check{m}), (\Bbbk_A^{F_s})_\nu^{Ts}(\check{m})\right\} \\ &= \vee\left\{(\Bbbk_A^{F_s})_\nu^{Ts}(0), (\Bbbk_A^{F_s})_\nu^{Ts}(\check{m})\right\} \\ &= (\Bbbk_A^{F_s})_\nu^{Ts}(\check{m}) \end{aligned}$$

□

Theorem 4.4. *Let A is a NINK-I of ℶ, then the NT $\left(A^{T_r,I_d,F_s}\right)_{\varrho,\zeta,\nu}^{Ts}$ of A is a NINK-S of ℶ.*

Proof.

$$\begin{aligned} (\Bbbk_A^{T_r})_\varrho^{Ts}(\check{l}\star\check{m}) &= \Bbbk_A^{T_r}(\check{l}\star\check{m}) + \varrho \\ &= \wedge\left\{\Bbbk_A^{T_r}((\check{n}\star(\check{l}\star\check{m})\star(\check{n}\star\check{m})), \Bbbk_A^{T_r}(\check{m})\right\} + \varrho \quad (Def.4.1\ in\ (2)) \\ &= \wedge\left\{\Bbbk_A^{T_r}((\check{l}\star\check{m})\star(\check{n}\star\check{m})), \Bbbk_A^{T_r}(\check{m})\right\} + \varrho \\ &= \wedge\left\{\Bbbk_A^{T_r}(0), \Bbbk_A^{T_r}(\check{m})\right\} + \varrho \\ &\geq \wedge\left\{\Bbbk_A^{T_r}(\check{l}), \Bbbk_A^{T_r}(\check{m})\right\} + \varrho \\ (\Bbbk_A^{T_r})_\varrho^{Ts}(\check{l}\star\check{m}) &\geq \wedge\left\{(\Bbbk_A^{T_r})_\varrho^{Ts}(\check{l}), (\Bbbk_A^{T_r})_\varrho^{Ts}(\check{m})\right\} + \varrho \\ (\Bbbk_A^{I_d})_\zeta^{Ts}(\check{l}\star\check{m}) &= \Bbbk_A^{I_d}(\check{l}\star\check{m}) + \zeta \\ &= \wedge\left\{\Bbbk_A^{I_d}((\check{n}\star(\check{l}\star\check{m})\star(\check{n}\star\check{m})), \Bbbk_A^{I_d}(\check{m})\right\} + \zeta \quad (Def.4.1\ in\ (3)) \\ &= \wedge\left\{\Bbbk_A^{I_d}((\check{l}\star\check{m})\star(\check{n}\star\check{m})), \Bbbk_A^{I_d}(\check{m})\right\} + \varrho \\ &= \wedge\left\{\Bbbk_A^{I_d}(0), \Bbbk_A^{I_d}(\check{m})\right\} + \zeta \\ &\geq \wedge\left\{\Bbbk_A^{I_d}(\check{l}), \Bbbk_A^{I_d}(\check{m})\right\} + \zeta \\ &\geq \wedge\left\{(\Bbbk_A^{I_d})_\zeta^{Ts}(\check{l}), (\Bbbk_A^{I_d})_\zeta^{Ts}(\check{m})\right\} + \zeta \\ (\Bbbk_A^{F_s})_\nu^{Ts}(\check{l}\star\check{m}) &= \Bbbk_A^{F_s}(\check{l}\star\check{m}) + \nu \\ &= ma\check{l}\left\{\Bbbk_A^{F_s}((\check{n}\star(\check{l}\star\check{m})\star(\check{n}\star\check{m})), \Bbbk_A^{F_s}(\check{m})\right\} - \nu \\ &= \vee\left\{\Bbbk_A^{F_s}((\check{l}\star\check{m})\star(\check{n}\star\check{m})), \Bbbk_A^{F_s}(\check{m})\right\} - \nu \quad (Def.4.1\ in\ (4)) \\ &= \vee\left\{\Bbbk_A^{F_s}(0), \Bbbk_A^{F_s}(\check{m})\right\} - \nu \\ &\leq \vee\left\{\Bbbk_A^{F_s}(\check{l}), \Bbbk_A^{F_s}(\check{m})\right\} - \nu \\ &\leq \vee\left\{(\Bbbk_A^{F_s})_\nu^{Ts}(\check{l}), (\Bbbk_A^{F_s})_\nu^{Ts}(\check{m})\right\} + \nu. \end{aligned}$$

□

Theorem 4.5. *Every neutrosphic Translation $(\mathbb{k}_A^{T_r})^{Ts}_{\varrho,\zeta,\nu}$ of A is a NINK-ideal of* ℷ*, if A is a NTINK-I of* ℷ*,for all $\varrho,\zeta,\nu \in [0,T]$.*

Proof. Assume that A is a NINK-I of ℷ and let $\varrho,\zeta,\nu \in [0,T]$.

$$\begin{aligned}
(\mathbb{k}_A^{T_r})^{Ts}_{\varrho}(0) &= (\mathbb{k}_A^{T_r})(\check{l}) + \varrho \\
&\geq \wedge\left\{(\mathbb{k}_A^{T_r})((\check{n}\star\check{l})\star(\check{n}\star\check{m})), (\mathbb{k}_A^{T_r})(\check{n})\right\} + \varrho \\
&= \wedge\left\{(\mathbb{k}_A^{T_r})((\check{n}\star\check{l})\star(\check{n}\star\check{m})) + \varrho, (\mathbb{k}_A^{T_r})(\check{m}) + \varrho\right\} \\
(\mathbb{k}_A^{T_r})^{Ts}_{\varrho}(0) &= \wedge\left\{(\mathbb{k}_A^{T_r})^{Ts}_{\varrho}((\check{n}\star\check{l})\star(\check{n}\star\check{m})), (\mathbb{k}_A^{T_r})^{Ts}_{\varrho}(\check{m})\right\}. \\
(\mathbb{k}_A^{I_d})^{Ts}_{\varrho}(0) &= (\mathbb{k}_A^{I_d})(\check{l}) + \zeta \\
&\geq \wedge\left\{(\mathbb{k}_A^{I_d})((\check{n}\star\check{l})\star(\check{n}\star\check{m})), (\mathbb{k}_A^{I_d})(\check{m})\right\} + \zeta \\
&= \wedge\left\{(\mathbb{k}_A^{I_d})((\check{n}\star\check{l})\star(\check{n}\star\check{m})) + \zeta, (\mathbb{k}_A^{I_d})(\check{m}) + \zeta\right\} \\
(\mathbb{k}_A^{I_d})^{Ts}_{\zeta}(0) &= \wedge\left\{(\mathbb{k}_A^{I_d})^{Ts}_{\zeta}((\check{n}\star\check{l})\star(\check{n}\star\check{m})), (\mathbb{k}_A^{I_d})^{Ts}_{\zeta}(\check{m})\right\}. \\
(\mathbb{k}_A^{F_s})^{Ts}_{\nu}(0) &= (\mathbb{k}_A^{F_s})(\check{l}) - \nu \\
&\leq \vee\left\{(\mathbb{k}_A^{F_s})((\check{n}\star\check{l})\star(\check{n}\star\check{m})), (\mathbb{k}_A^{F_s})(\check{m})\right\} - \nu \\
&= \vee\left\{(\mathbb{k}_A^{F_s})((\check{n}\star\check{l})\star(\check{l}\star\check{m})) + \nu, (\mathbb{k}_A^{F_s})(\check{m}) - \nu\right\} \\
(\mathbb{k}_A^{F_s})^{Ts}_{\mathbb{k}}(0) &= \vee\left\{(\mathbb{k}_A^{F_s})^{Ts}_{\nu}((\check{n}\star\check{l})\star(\check{n}\star\check{m})), (\mathbb{k}_A^{F_s})^{Ts}_{\nu}(\check{m})\right\}.
\end{aligned}$$

□

Theorem 4.6. *Let A be a neutrosophic subset of* ℷ *such that neutrosophic ϱ,ζ,ν-multiplication of A is a neutrosophic INK-ideal* ℷ *for some $\varrho,\zeta,\nu \in [0,1]$, then A is a neutrosophic INK-ideal of* ℷ*.*

Proof. Assume that $(\mathbb{k}_A^{T_r})^{M}_{\varrho,\zeta,\nu}$ is a NINK-ideal of ℷ,for some $\varrho,\zeta,\nu \in [0,1]$.
Let $\varrho,\zeta,\nu \in [0,1]$. Then

$$\begin{aligned}
\varrho\cdot(\mathbb{k}_A^{T_r})(0) &= (\mathbb{k}_A^{T_r})^{M}_{\varrho}(0) \\
&\geq (\mathbb{k}_A^{T_r})^{M}_{\varrho}(0) \\
&= \varrho\cdot(\mathbb{k}_A^{T_r})(\check{l})
\end{aligned}$$

$$\begin{aligned}
(\mathbb{k}_A^{T_r})(0) &\geq (\mathbb{k}_A^{T_r})(\check{l}) \\
\varrho\cdot(\mathbb{k}_A^{T_r})(\check{l}) &= (\mathbb{k}_A^{T_r})_\varrho^M(\check{l}) \\
&\geq \wedge\left\{\left(\mathbb{k}_A^{T_r}\right)_\varrho^M((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\left(\mathbb{k}_{\text{Ł}}^{T}\right)_\varrho^M(\check{m})\right\} \\
&\geq \wedge\left\{\varrho\cdot\left(\mathbb{k}_A^{T_r}\right)((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\varrho\cdot\left(\mathbb{k}_A^{T_r}\right)(\check{m})\right\} \\
\varrho\cdot(\mathbb{k}_A^{T_r})(\check{l}) &= \varrho\cdot\wedge\left\{\left(\mathbb{k}_A^{T_r}\right)((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\left(\mathbb{k}_A^{T_r}\right)(\check{m})\right\} \\
(\mathbb{k}_A^{T_r})(\check{l}) &= \wedge\left\{\left(\mathbb{k}_A^{T_r}\right)((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\left(\mathbb{k}_A^{T_r}\right)(\check{m})\right\} \\
\zeta\cdot(\mathbb{k}_A^{I_d})(0) &= (\mathbb{k}_A^{I_d})_\zeta^M(0) \\
&\geq (\mathbb{k}_A^{I_d})_\zeta^M(0) \\
&= \zeta\cdot(\mathbb{k}_A^{I_d})(\check{l}) \\
(\mathbb{k}_A^{I_d})(0) &\geq (\mathbb{k}_A^{I_d})(\check{l}) \\
\zeta\cdot(\mathbb{k}_A^{I_d})(\check{l}) &= (\mathbb{k}_A^{I_d})_\zeta^M(\check{l}) \\
&\geq \wedge\left\{\left(\mathbb{k}_A^{I_d}\right)_\zeta^M((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\left(\mathbb{k}_A^{I_d}\right)_\zeta^M(\check{m})\right\} \\
&\geq \wedge\left\{\zeta\cdot\left(\mathbb{k}_A^{I_d}\right)((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\zeta\cdot\left(\mathbb{k}_A^{I_d}\right)(\check{m})\right\} \\
\zeta\cdot(\mathbb{k}_A^{I_d})(\check{l}) &= \zeta\cdot\wedge\left\{\left(\mathbb{k}_A^{I_d}\right)((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\left(\mathbb{k}_A^{I_d}\right)(\check{m})\right\} \\
(\mathbb{k}_A^{I_d})(\check{l}) &= \wedge\left\{\left(\mathbb{k}_A^{I_d}\right)((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\left(\mathbb{k}_A^{I_d}\right)(\check{m})\right\} \\
\nu\cdot(\mathbb{k}_A^{F_s})(0) &= (\mathbb{k}_A^{F_s})_\nu^M(0) \\
&\leq (\mathbb{k}_A^{F_s})_\varrho^M(0) \\
&= \nu\cdot(\mathbb{k}_A^{F_s})(\check{l}) \\
(\mathbb{k}_A^{F_s})(0) &\leq (\mathbb{k}_A^{F_s})(\check{l}) \\
\nu\cdot(\mathbb{k}_A^{F_s})(\check{l}) &= (\mathbb{k}_A^{F_s})_\nu^M(\check{l}) \\
&\leq \vee\left\{\left(\mathbb{k}_A^{F_s}\right)_\nu^M((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\left(\mathbb{k}_A^{F_s}\right)_\nu^M(\check{m})\right\} \\
&\leq \vee\left\{\nu\cdot\left(\mathbb{k}_A^{F_s}\right)((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\nu\cdot\left(\mathbb{k}_A^{F_s}\right)(\check{m})\right\} \\
\nu\cdot(\mathbb{k}_A^{F_s})(\check{l}) &= \nu\cdot\vee\left\{\left(\mathbb{k}_A^{F_s}\right)((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\left(\mathbb{k}_A^{F_s}\right)(\check{m})\right\} \\
(\mathbb{k}_A^{F_s})(\check{l}) &= \vee\left\{\left(\mathbb{k}_A^{F_s}\right)((\check{n}\star\check{l})\star(\check{n}\star\check{m})),\left(\mathbb{k}_A^{F_s}\right)(\check{m})\right\}
\end{aligned}$$

Hence $(\mathbb{k}_A^{T_r})_\varrho^M, (\mathbb{k}_A^{I_d})_\zeta^M$ and $(\mathbb{k}_A^{F_s})_\nu^M$ is a multilication of NINK-ideal of ℷ. □

Theorem 4.7. *If the NT $\left(A^{T_r,I_d,F_s}\right)_{\varrho,\zeta,\nu}^M$ of A is a NINK-ideal of ℷ, $(\varrho,\zeta,\nu)\in[0,1]$, then A is a NINK-S of ℷ.*

Proof. Let us assume that $\left(A^{T_r,I_d,F_s}\right)^M_{\varrho,\zeta,\nu}$ of A is a NINK-ideal of $\beth$. Then

$$\begin{aligned}
\varrho\cdot(\Bbbk_A^{T_r})(\check{l}\star\check{m}) &= (\Bbbk_A^{T_r})^M_\varrho(\check{l}\star\check{m})\\
&\geq \wedge\left\{\left(\Bbbk_A^{T_r}\right)^M_\varrho((\check{n}\star(\check{l}\star\check{m}))\star(\check{n}\star\check{m})),\left(\Bbbk_A^{T_r}\right)^M_\varrho(\check{m})\right\}\\
&\geq \wedge\left\{\left(\Bbbk_A^{T_r}\right)^M_\varrho(((\check{l}\star\check{m})\star\check{m})),\left(\Bbbk_A^{T_r}\right)^M_\varrho(\check{m})\right\} \quad (Def\ 4.1.\ (2))\\
&\geq \wedge\left\{\left(\Bbbk_A^{T_r}\right)^M_\varrho(0),\left(\Bbbk_A^{T_r}\right)^M_\varrho(\check{m})\right\}\\
&= \wedge\left\{\left(\Bbbk_A^{T_r}\right)^M_\varrho(\check{l}),\left(\Bbbk_A^{T_r}\right)^M_\varrho(\check{m})\right\}\\
&= \wedge\left\{\left(\varrho\cdot\Bbbk_A^{T_r}\right)(\check{l}),\varrho\cdot\left(\Bbbk_A^{T_r}\right)(\check{m})\right\}\\
\varrho\cdot(\Bbbk_A^{T_r})(\check{l}\star\check{m}) &= \varrho\cdot\wedge\left\{\left((\Bbbk_A^{T_r}\right)(\check{l}),\left(\Bbbk_A^{T_r}\right)(\check{m})\right\}\\
(\Bbbk_A^{T_r})(\check{l}\star\check{m}) &= \wedge\left\{\left((\Bbbk_A^{T_r}\right)(\check{l}),\left(\Bbbk_A^{T_r}\right)(\check{m})\right\}\\
\zeta\cdot(\Bbbk_A^{I_d})(\check{l}\star\check{m}) &= (\Bbbk_A^{I_d})^M_\zeta(\check{l}\star\check{m})\\
&\geq \wedge\left\{\left(\Bbbk_A^{I_d}\right)^M_\zeta((\check{n}\star(\check{l}\star\check{m}))\star(\check{n}\star\check{m})),\left(\Bbbk_A^{I_d}\right)^M_\zeta(\check{m})\right\}\\
&\geq \wedge\left\{\left(\Bbbk_A^{I_d}\right)^M_\zeta(((\check{l}\star\check{m})\star\check{m})),\left(\Bbbk_A^{I_d}\right)^M_\zeta(\check{m})\right\} \quad (Def\ 4.1.\ (3))\\
&\geq \wedge\left\{\left(\Bbbk_A^{I_d}\right)^M_\zeta(0),\left(\Bbbk_A^{I_d}\right)^M_\zeta(\check{m})\right\}\\
&= \wedge\left\{\left(\Bbbk_A^{I_d}\right)^M_\zeta(\check{l}),\left(\Bbbk_A^{I_d}\right)^M_\zeta(\check{m})\right\}\\
&= \wedge\left\{\left(\zeta\cdot\Bbbk_A^{I_d}\right)(\check{l}),\zeta\cdot\left(\Bbbk_A^{I_d}\right)(\check{m})\right\}\\
\zeta\cdot(\Bbbk_A^{I_d})(\check{l}\star\check{m}) &= \zeta\cdot\wedge\left\{\left((\Bbbk_A^{I_d}\right)(\check{l}),\left(\Bbbk_A^{I_d}\right)(\check{m})\right\}\\
(\Bbbk_A^{I_d})(\check{l}\star\check{m}) &= \wedge\left\{\left((\Bbbk_A^{I_d}\right)(\check{l}),\left(\Bbbk_A^{I_d}\right)(\check{m})\right\}\\
\nu\cdot(\Bbbk_A^{F_s})(\check{l}\star\check{m}) &= (\Bbbk_A^{F_s})^M_\nu(\check{l}\star\check{m})\\
&\leq \vee\left\{\left(\Bbbk_A^{F_s}\right)^M_\nu((\check{n}\star(\check{l}\star\check{m}))\star(\check{n}\star\check{m})),\left(\Bbbk_A^{F_s}\right)^M_\nu(\check{m})\right\}\\
&\leq \vee\left\{\left(\Bbbk_A^{F_s}\right)^M_\nu(((\check{l}\star\check{m})\star\check{m})),\left(\Bbbk_A^{F_s}\right)^M_\nu(\check{m})\right\} \quad (Def\ 4.1.\ (4))\\
&\leq \vee\left\{\left(\Bbbk_A^{F_s}\right)^M_\nu(0),\left(\Bbbk_A^{F_s}\right)^M_\nu(\check{m})\right\}\\
&= \vee\left\{\left(\Bbbk_A^{F_s}\right)^M_\nu(\check{l}),\left(\Bbbk_A^{F_s}\right)^M_\nu(\check{m})\right\}\\
&= \vee\left\{\left(\nu\cdot\Bbbk_A^{F_s}\right)(\check{l}),\nu\cdot\left(\Bbbk_A^{F_s}\right)(\check{m})\right\}\\
\nu\cdot(\Bbbk_A^{F_s})(\check{l}\star\check{m}) &= \nu\cdot\vee\left\{\left((\Bbbk_A^{F_s}\right)(\check{l}),\left(\Bbbk_A^{F_s}\right)(\check{m})\right\}
\end{aligned}$$

$$(\Bbbk_A^{F_s})(\check{l} \star \check{m}) = \vee\left\{\left((\Bbbk_A^{F_s}\right)(\check{l}), \left(\Bbbk_{\S}^{F}\right)(\check{m})\right\}$$

□

Theorem 4.8. *Intersection and union of any two neutrosophic translations of a NINK-ideal of ℶ is also a NINK-ideal of ℶ*

Proof. Let $\left(A^{T_r,I_d,F_s}\right)_{\varrho_1,\zeta_1,\nu_1}$ and $\left(A^{T,I,F}\right)_{\varrho_2,\zeta_2,\nu_2}$ be two neutrosophic translations of a NINK-ideal of ℶ where $(\varrho_1, \zeta_1, \nu_1, \nu_2, \varrho_2, \zeta_2) \in [0,1]$. Assume that $\varrho_1 \leq \varrho_2, \zeta_1 \leq \zeta_2$ and $\nu_1 \geq \nu_2$. Then $\left(A^{T_r,Id,Fs}\right)_{\varrho_1,\zeta_1,\nu_1}$ and $\left(A^{T,I,F}\right)_{\varrho_2,\zeta_2,\nu_2}$ are NINK-ideal of ℶ.

$$\begin{aligned}
((\Bbbk_A^{T_r})_{\varrho_1} \cap (\Bbbk_A^{T})_{\varrho_2})(\check{l}) &= \wedge\{(\Bbbk_A^{T_r})_{\varrho_1}(\check{l}), (\Bbbk_A^{T})_{\varrho_2}(\check{l})\} \\
&= \wedge\{(\Bbbk_A(\check{l}) + \varrho_1, \Bbbk_A(\check{l}) + \varrho_2\} \\
&= \Bbbk_A(\check{l}) + \varrho_1 \\
((\Bbbk_A^{T_r})_{\varrho_1} \cap (\Bbbk_A^{T})_{\varrho_2})(\check{l}) &= \Bbbk_A^{T_r}(\check{l}). \\
((\Bbbk_A^{I_d})_{\zeta_1} \cap (\Bbbk_A^{I})_{\zeta_2})(\check{l}) &= \wedge\{(\Bbbk_A^{I_d})_{\zeta_1}(\check{l}), (\Bbbk_A^{I})_{\zeta_2}(\check{l})\} \\
&= \wedge\{(\Bbbk_A(\check{l}) + \zeta_1, \Bbbk_A(\check{l}) + \zeta_2\} \\
&= \Bbbk_A(\check{l}) + \zeta_1 \\
((\Bbbk_A^{I_d})_{\zeta_1} \cap (\Bbbk_A^{I})_{\zeta_2})(\check{l}) &= \Bbbk_A^{I_d}(\check{l}). \\
((\Bbbk_A^{F_s})_{\nu_1} \cap (\Bbbk_A^{T})_{\nu_2})(\check{l}) &= \vee\{(\Bbbk_A^{F_s})_{\nu_1}(\check{l}), (\Bbbk_A^{F})_{\nu_2}(\check{l})\} \\
&= \vee\{(\Bbbk_A(\check{l}) + \nu_1, \Bbbk_A(\check{l}) + \nu_2\} \\
&= \Bbbk_A(\check{l}) + \nu_1 \\
((\Bbbk_A^{F_s})_{\nu_1} \cap (\Bbbk_A^{F})_{\nu_2})(\check{l}) &= \Bbbk_A^{F_s}(\check{l}).
\end{aligned}$$

and

$$\begin{aligned}
((\Bbbk_A^{T_r})_{\varrho_1} \cup (\Bbbk_A^{T})_{\varrho_2})(\check{l}) &= \wedge\{(\Bbbk_A^{T_r})_{\varrho_1}(\check{l}), (\Bbbk_A^{T})_{\varrho_2}(\check{l})\} \\
&= \wedge\{(\Bbbk_A(\check{l}) + \varrho_1, \Bbbk_A(\check{l}) + \varrho_2\} \\
&= \Bbbk_A(\check{l}) + \varrho_1 \\
((\Bbbk_A^{T_r})_{\varrho_1} \cup (\Bbbk_A^{T})_{\varrho_2})(\check{l}) &= \Bbbk_A^{T_r}(\check{l}). \\
((\Bbbk_A^{I_d})_{\zeta_1} \cup (\Bbbk_A^{I})_{\zeta_2})(\check{l}) &= \wedge\{(\Bbbk_A^{I_d})_{\zeta_1}(\check{l}), (\Bbbk_A^{I})_{\zeta_2}(\check{l})\} \\
&= \wedge\{(\Bbbk_A(\check{l}) + \zeta_1, \Bbbk_A(\check{l}) + \zeta_2\} \\
&= \Bbbk_A(\check{l}) + \zeta_1 \\
((\Bbbk_A^{I_d})_{\zeta_1} \cup (\Bbbk_A^{I})_{\zeta_2})(\check{l}) &= \Bbbk_A^{I_d}(\check{l}).
\end{aligned}$$

$$\begin{aligned}((\Bbbk_A^{F_s})_{\nu_1} \cup (\Bbbk_A^{T})_{\nu_2})(\check{l}) &= \vee\{(\Bbbk_A^{F_s})_{\nu_1}(\check{l}), (\Bbbk_A^{F})_{\nu_2}(\check{l})\} \\ &= \vee\{(\Bbbk_A(\check{l}) + \nu_1, \Bbbk_A(\check{l}) + \nu_2\} \\ &= \Bbbk_A(\check{l}) + \nu_1 \\ ((\Bbbk_A^{F_s})_{\nu_1} \cup (\Bbbk_A^{F})_{\nu_2})(\check{l}) &= \Bbbk_A^{F_s}(\check{l}).\end{aligned}$$

Hence $\left(A^{T_r,I_d,F_s}\right)_{\varrho_1,\zeta_1,\nu_1}$ and $\left(A^{T,I,F}\right)_{\varrho_2,\zeta_2,\nu_2}$ are NINK-ideal of $\daleth$. □

5. Conclusion

This paper introduces and explores the translation of NINK-ideals within the framework of INK-algebras, shedding light on some of their valuable properties. The investigation establishes connections between neutrosophic fuzzy translations and the N-Ms inherent in these INK-ideals. This research lays the groundwork for further exploration into the theory of INK-algebras. Looking ahead, our future studies on the neutrosophic structure of INK-algebras may include the following topics:

(1) Translation of neutrosophic soft a-ideals in INK-algebras
(2) Product of translation of neutrosophic soft d-ideals in INK-algebras
(3) Exploring the translation dynamics of neutrosophic soft INK-ideals in INK-algebras. We believe that these areas of investigation will contribute to the ongoing development of this field.

6. Acknowledgment

I would like to acknowledge the initial support received from Jadara University under grant number Jadara-SR-Full2023. This support played a vital role in facilitating this research.

References

1. A. A. A. Agboola and B. Davvaz, Introduction to Neutrosophic BCI/BCKAlgebras,Information.International Journal of Mathematics and Mathematical Sciences, 2015,130. https://doi.org/10.1155/2015/370267.
2. Atanassov, K.T, Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, 1986, 20, 87-96.
3. Bordbar, Hashem; Xiao Long Xin; Rajab Ali Borzooei; and Young Bae Jun, Positive implicative ideals of BCK-algebras based on neutrosophic sets and falling shadows. Neutrosophic Sets and Systems, 2022, 48(1).
4. F. Smarandache, Neutrosophic set-A generalization of the intuitionistic fuzzy set. Internat. J. Pure Appl. Math. 2005, 24, 287297.
5. F. Smarandache, Neutrosophy, Neutrosophic Probability, Set, and Logic. Pro Quest Information and Learning, Ann Arbor, Michigan, USA. 1998.
6. F Smarandache, New Types of Soft Sets HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set: An Improved Version. Neutrosophic Systems With Applications,2023, 8, 3541. https://doi.org/10.61356/j.nswa.2023.41

7. Jun, Young Bae and Roh, Eun Hwan. MBJ-neutrosophic ideals of BCK/BCI-algebras, Open Mathematics, 1999, 17(1), 588-601.
8. Lee, K. J., Y. B. Jun, M. I. Doh, Fuzzy translations and fuzzy multiplications of BCK/BCI-algebras. Commun. Korean Math.Soc., 2009, 24, 35336.
9. L. A. Zadeh,Fuzzy sets. Information and Control., 1965,338-353.
10. M. Kaviyarasu., K. Indhira., V.M. Chandrasekaran, Fuzzy p-ideal in INK-Algebra. Journal of Xian University of Architecture and Technology, 2000, 12, 4746-4752.
11. M. Kaviyarasu., K. Indhira., V.M. Chandrasekaran, Neutrosophic set in INK-Algebra, Adv. Math., Sci. J., 2000, 9, 43454352.
12. M. Kaviyarasu., K. Indhira., V.M. Chandrasekaran, Direct product of Neutrosophic INKAlgebras. Infinite Study., 2020, 38.
13. M Kaviyarasu and K Indhira 2017 IOP Conf. Ser.: Mater. Sci. Eng. 263 042142.DOI 10.1088/1757-899X/263/4/042142.
14. Manokaran, Surya, and Muralikrishna Prakasam. On MBJNeutrosophic B-Subalgebra. Neutrosophic Sets and Systems, 2019, 28, 216-227.
15. Mohsin Khalid , Neha Andaleeb Khalid , Rakib Iqbal, MBJ-neutrosophic T-ideal on B-algebra. International Journal of Neutrosophic Science, 2020, 1(1), 29-39.
16. M. Mohseni Takallo., R.A. Borzooei., Young Bae Jun.,MBJ-neutrosophic structures and its applications in BCK/BCI-algebras. Neutrosophic Sets and Systems, 2018, 23.
17. M.Kaviyarasu , Rajeshwari M., Direct Product of Neutrosophic h-ideal in INK-Algebra. International Journal of Neutrosophic Science, 2023, 20(1), 119-127. Doi : https://doi.org/10.54216/IJNS.200110.
18. Ranulfo Paiva Barbosa , & Smarandache, F. Pura Vida Neutrosophic Algebra. Neutrosophic Systems With Applications, 2023, 9, 101106. https://doi.org/10.61356/j.nswa.2023.68
19. Song, S.-Z.; Ozturk, M.A.; Jun, Y.-B. Commutative Ideals of BCI-Algebras Using MBJ-Neutrosophic Structures. Mathematics, 2021, 9, 2047.
20. Senapati, T., Fuzzy Translations of Fuzzy H-ideals in BCK/BCI-Algebras. Journal of the Indonesian Mathematical Society, 2015, 21(1), 4558.
21. Senapati,T.,M. Bhowmik,M.Pal, B. Davvaz. Atanassovs intuitionistic fuzzy translations of intuitionistic fuzzy subalgebras and ideals in BCK/BCI-algebras. Eurasian Mathematical Journal, 2015, 6 (1), 96114.
22. Smarandache, F.,Foundation of the SuperHyperSoft Set and the Fuzzy Extension Super HyperS oft Set: A New Vision. Neutrosophic Systems With Applications,2023, 11, 4851. https://doi.org/10.61356/j.nswa.2023.95
23. WadeiF. Al-Omeri, Saeid Jafari and Smarandache, F. (2020). Neutrosophic Fixed Point Theorems and Cone Metric Spaces. Neutrosophic Sets and Systems, 2020, 31, 250-265.
24. Wadei Faris Al-Omeri., On Mixed b-Fuzzy Topological Spaces. International Journal of Fuzzy Logic and Intelligent Systems, 20(3), 242-246.
25. Wadei Faris Al-Omeri., On almost e-I-continuous functions. Demonstratio Mathematica, 2021, 54(1), 168-177.
26. Wadei F. Al-Omeri , O. H. Khalil and A. Ghareeb, Degree of (L, M)-fuzzy semi-precontinuous and (L, M)-fuzzy semi preirresolute functions. Demonstr. Math., 51(1), 182197.
27. Wadei F. Al-Omeri.et.al., On Ra operator in ideal topological spaces. Creat. Math. Inform, 2016, 25, 2016,1-10.
28. Y.B. Jun., J. Meng, Fuzzy p-ideals in BCI-algebras. Mathematica Japonica, 1994, 40, 271282.
29. Y.B.Jun, Translations of fuzzy ideals in BCK/BCI algebras. Hacettepe Journal of Mathematics and Statistics, 2021, 40, 349358.
30. Iseki, K. On BCI-algebras. Math. Semin. Notes 1980.

31. Imai, Y.; Iseki, K. On axiom systems of propositional calculi XIV. Proc. Jpn. Acad. 1966, 42, 1922.
32. Iseki, K.; Tanaka, S. An introduction to the theory of BCK algebras. Math. Jpn. 1978, 23, 126.

Received: Jan 9, 2024. Accepted: Mar 31, 2024

Neutrosophic Sets and Systems, Vol. 66, 2024

University of New Mexico

Child Labor, Informality, and Poverty: Leveraging Logistic Regression, Indeterminate Likert Scales, and Similarity Measures for Insightful Analysis in Ecuador

Arnaldo Vergara-Romero[1] , Guido Macas-Acosta[2] , Fidel Márquez-Sánchez[3] , Orlando Arencibia-Montero[4]

[1]Universidad Ecotec, Km 13.5 vía Samborondón, Samborondón-Ecuador., E-mail: avergarar@ecotec.edu.ec

[2]Universidad Ecotec, Km 13.5 vía Samborondón, Samborondón-Ecuador., E-mail: gmacas@ecotec.edu.ec

[3]Universidad Espritu Santo Km 2.5 vía Samborondón, Samborondón-Ecuador, fmarquez@uees.edu.ec,

[4]VSB-Technical University of Ostrava 17. listopadu 2172/15. 708 00 Ostrava-Poruba. Czech Republic. E-mail: orlando.arencibia@vsb.cz

* Correspondence: gmacas@ecotec.edu.ec

Abstract.

The paper presents a comprehensive analysis of child labor in Ecuador, employing advanced statistical tools like logistic regression, neutrosophic Likert scales, and similarity measures to deepen the understanding of this social issue. The integration of these methodologies allows for a nuanced assessment of the various socio-economic factors contributing to child labor. By capturing the uncertainty in human responses, the research highlights the complex interplay between poverty, household income, education levels, and labor types on the incidence of child labor. Key findings suggest that rural location, the age of the child, and the informal nature of the head of the household's work are the most significant predictors of child labor. Notably, parental education appears to have a less direct influence. Despite various efforts, including government monetary transfers through programs like the BDH, child labor persists, indicating the need for more targeted interventions.The paper proposes future research to extend these models to a broader demographic and geographic data set, emphasizing the potential for these methods to be applied to a variety of social issues. The development of computational tools to automate neutrosophic analysis could greatly benefit large-scale studies, potentially aiding policymakers in designing more effective interventions for vulnerable populations.

Keywords: Child Labor, Logistic Regression, Neutrosophic Scales, Indeterminacy

1. Introduction.

Child labor continues unabated globally, exacerbated by increasing poverty, informality, inequality, and insufficient governmental support, particularly in developing nations. According to the International Labor Organization (ILO), over 200 million children are involved in labor, depending on definitions and data sources [1]. In the poorest countries, more than one in five children is involved in child labor [2]. Ecuador presents a concerning scenario where one in ten children and adolescents are working, with 85% of these residing in rural areas. This study aims to explore how the distribution of labor activities among minors in Ecuador varies over time in response to household income constraints and minimal state intervention to ensure education access and revitalize the economy in areas vulnerable to child labor.

Numerous studies have identified poverty as a critical factor influencing child labor dynamics, potentially worsened by ineffective state policies [3]. Research also delves into the types of work minors engage in, the impact on their education and health, and the associated externalities [4]. Some argue that government economic

compensations, despite being intended to alleviate poverty and hence reduce child labor, have been insufficient [5][6]. In Ecuador, the Bono de Desarrollo Humano (BDH), a monetary transfer to impoverished and unemployed heads of households or single mothers with minor children, ranges from $50 to $260 monthly, which has not substantially resolved the issue [7]. Conditional Transfer Programs (CTPs) across Latin America aim to mitigate poverty and child labor, showing improvements in income, food consumption, and access to education and health, thus reducing child labor and empowering mothers [8]. Edmonds & Schady [9] found that such transfers in Ecuador significantly reduce child labor, including paid employment, particularly among children who were students.
Factors driving children into labor are predominantly linked to poverty, including family income levels, credit restrictions, and lack of access to microcredit, among others Basu [10]. Furthermore, minimal state assistance and economic crises exacerbate family members' reliance on "family savings," intensifying unpaid domestic work, primarily performed by women and girls. However, non-poverty-related factors, such as parental perspectives, education level, household dynamics, proximity to schools, and educational costs, play a significant role [11].

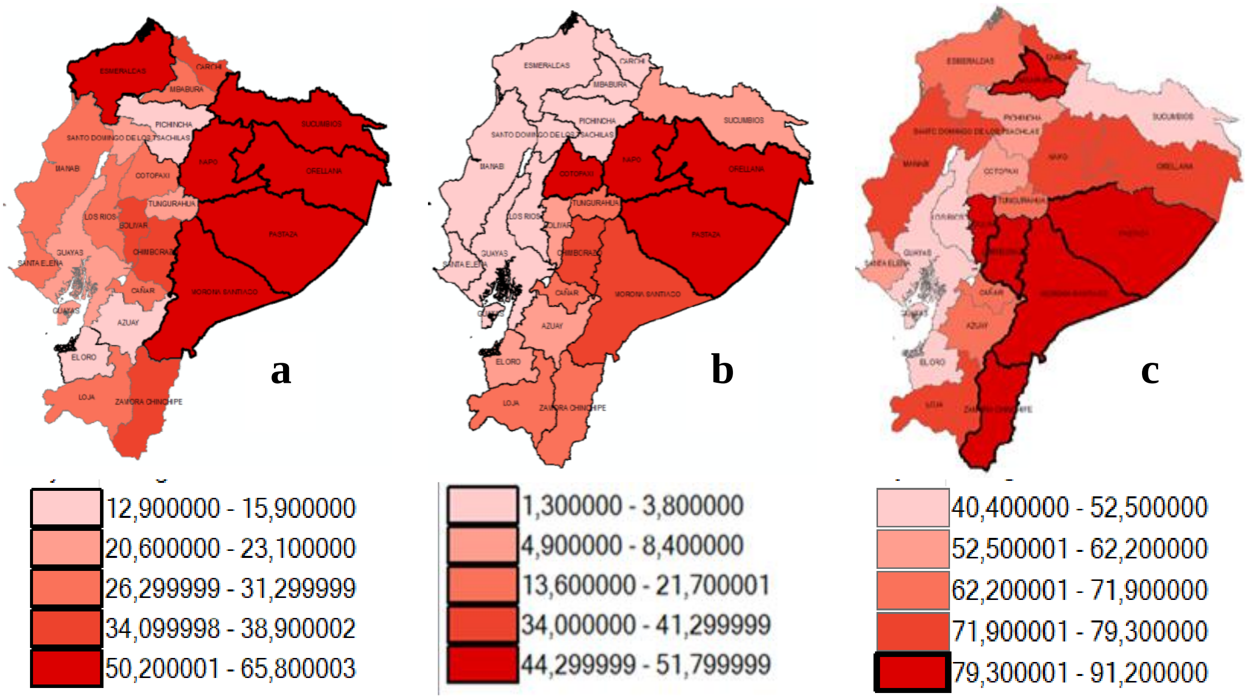


Figure 1. Percentage of Population by Province of Ecuador under poverty (a), with child labor (b) and informal work (c). [12].

In the paper of child labor in Ecuador, logistic regression [13], neutrosophic Likert scales [14] , and similarity measures[15] are employed as advanced statistical and analytical tools to identify and quantify the factors influencing these social issues. Logistic regression is used to model the probability of binary events occurring, such as whether a child participates in labor activities or not, based on a set of predictor variables, such as family income level, access to education, and housing conditions. On the other hand, neutrosophic Likert scales capture the uncertainty and indeterminacy in respondents' perceptions and opinions on the causes and consequences of child labor and poverty, facilitating a more nuanced assessment of attitudes and experiences. Similarity measures are employed to determine the relationship between variables that influence child labor. By analyzing how closely related these variables are, researchers can identify common patterns and factors contributing to child labor.
This method allows for a nuanced understanding of the complex interplay between socioeconomic conditions, family characteristics, and educational access, among others, in influencing the likelihood of children entering the workforce. Through the use of similarity measures, it becomes possible to cluster similar variables and discern which variables most significantly impact child labor, aiding in the development of targeted interventions and policies to address this issue effectively. By combining these approaches, researchers can gain deeper insights into the complex dynamics underlying child labor and poverty in Ecuador, which is crucial for designing more effective public policies specifically targeted to the needs of vulnerable populations.

2. Preliminaries

2.1 Logistic Regesion Model

The logit model represents the relationship between the likelihood of an event occurring and its independent factors. The following serves as the basis for deriving the logistic function used for modelling the probability that a specific event will occur, which in this case is whether or not the child or adolescent works.

The following is an example of how to write a logit function [16]:

$$logit(l) = log\left(\frac{P}{(1-P)} = Z = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_n X_n + \mu_i\right) \quad (1)$$

In this case, P represents the likelihood that an event will occur and the likelihood that the event will occur. At the same time, Z may be considered a linear combination of the independent variables and their coefficients. Equation 2 mentioned above may be solved further to arrive at the following function, which can then be used to estimate the probability of the event[17].

$$log\left(\frac{P_i}{(1-P_i)}\right) = logit(P) \quad (2)$$

The logit model's capacity to represent the relationship between the likelihood of an event and its independent factors provides a robust framework for statistical analysis in various fields, including the study of child labor. However, while logistic regression can offer insights into the probabilities associated with certain outcomes, it does not inherently determine causality. This is where the concept of neutrosophy and the use of indeterminate Likert scales[18] become particularly valuable.

2.2 Indeterminate Likert Scales and Similarity Measures

The foundational concepts of neutrosophic set [19] and its application through indeterminate Likert scales and similarity measures form a pivotal framework for addressing uncertainty and indeterminacy in various fields. Neutrosophic Likert scales extend traditional Likert scales by incorporating degrees of indeterminacy, allowing respondents to express not just agreement or disagreement but also uncertainty regarding statements or questions. This nuanced approach to survey responses captures a broader spectrum of human perception and opinion, making it invaluable for research that deals with subjective information. Concurrently, neutrosophic similarity measures evaluate the degree of similarity between entities represented by neutrosophic sets, facilitating the comparison and clustering of data with inherent uncertainties.

These concepts are crucial for the advanced analysis of complex systems where traditional binary logic falls short, enabling researchers to delve deeper into the intricacies of human cognition and decision-making processes.

Definition 1 ([19]). The *Single-Valued Neutrosophic Set* (SVNS) N over U is $A = \{< x; T_A(x), I_A(x), F_A(x) >: x \in U\}$, where $T_A: U \to [0,1]$, $I_A: U \to [0,1]$, and $F_A: U \to [0,1]$, $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2 ([20]). The *refined neutrosophic logic* is defined such that: a truth T is divided into several types of truths: $T_1, T_2, \ldots, T_p$, I into various indeterminacies: $I_1, I_2, \ldots, I_r$ and F into various falsities: $F_1, F_2, \ldots, F_s$, where all $p, r, s \geq 1$ are integers, and $p + r + s = n$.

Definition 3 ([19]). A *triple refined indeterminate neutrosophic set* (TRINS) A in X is characterized by positive $P_A(x)$, indeterminacy $I_A(x)$, negative $N_A(x)$, positive indeterminacy $I_{P_A}(x)$ and negative indeterminacy $I_{N_A}(x)$ membership functions. Each of them has a weight $w_m \in [0,1]$ associated with it. For each $x \in X$, there are $P_A(x), I_{P_A}(x), I_A(x), I_{N_A}(x), N_A(x) \in [0,1]$,
$w_P^m(P_A(x)), w_{I_P}^m(I_{P_A}(x)), w_I^m(I_A(x)), w_{I_N}^m(I_{N_A}(x)), w_N^m(N_A(x)) \in [0,1]$ and $0 \leq P_A(x) + I_{P_A}(x) + I_A(x) + I_{N_A}(x)(x) + N_A(x) \leq 5$. Therefore, a TRINS A can be represented by $A = \{\langle x; P_A(x), I_{P_A}(x), I_A(x), I_{N_A}(x), N_A(x)\rangle | x \in X\}$.

Let A and B be two TRINS in a finite universe of discourse, $X = \{x_1, x_2, \ldots, x_n\}$, which are denoted by:

$A = \{\langle x; P_A(x), I_{P_A}(x), I_A(x), I_{N_A}(x), N_A(x)\rangle | x \in X\}$ and $B = \{\langle x; P_B(x), I_{P_B}(x), I_B(x), I_{N_B}(x), N_B(x)\rangle | x \in X\}$,

Where $P_A(x_i), I_{P_A}(x_i), I_A(x_i), I_{N_A}(x_i), N_A(x_i), P_B(x_i), I_{P_B}(x_i), I_B(x_i), I_{N_B}(x_i), N_B(x_i) \in [0,1]$, for every $x_i \in X$. Let w_i(i = 1, 2,...,n) be the weight of an element x_i (i = 1,2,...,n), with $w_i \geq 0$ (i = 1,2,...,n) and $\sum_{i=1}^{n} w_i = 1$.

The weighted distance for generalized TRINS is defined as ([19, 21]):

$$d_\lambda(A,B) = \left\{\frac{1}{5}\sum_{i=1}^{n} w_i \left[|P_A(x_i) - P_B(x_i)|^\lambda + \left|I_{P_A}(x_i) - I_{P_B}(x_i)\right|^\lambda + |I_A(x_i) - I_B(x_i)|^\lambda + \left|I_{N_A}(x_i) - I_{N_B}(x_i)\right|^\lambda + |N_A(x_i) - N_B(x_i)|^\lambda\right]\right\}^{1/\lambda} \quad (3)$$

Where $\lambda > 0$.

Definition 9: ([22,23]) Let $A, B \in \mathcal{N}(X)$ where $X = \{x_1, x_2, \cdots, x_n\}$, then a measure of similarity between sets A and B is determined by calculating the distance between them, which reflects their convergence or divergence within a neutrosophic framework calculated by :

$$S^1(A,B) = \frac{1}{1+d(A,B)} \quad (4)$$

Such that $d(A,B)$ is a distance function between the two single-valued neutrosophic sets.

Let us recall that the distance function satisfies the following axioms $\forall A, B, C \in \mathcal{N}(X)$:

(1) $d(A,B) \geq 0$ and $d(A,B) = 0$ if and only if $A = B$,

(2) $d(A,B) = d(B,A)$,

(3) $d(A,C) \leq d(A,B) + d(B,C)$.

The Indeterminate Likert Scale is formed by the following five elements:
– Negative membership,
– Indeterminacy leaning towards negative membership,
– Indeterminate membership,
– Indeterminacy leaning towards positive membership,
– Positive membership.
These values substitute the classical Likert scale with values (Figure 2):
–Strongly disagree,
– Disagree,
–Neutral,
– Agree,
–Strongly agree.

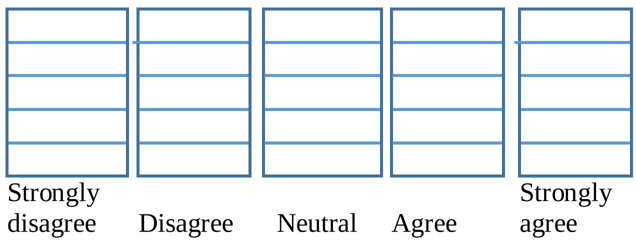


Figure 2. Visual representation of Indeterminate Likert Scale

Respondents are asked to give their opinion on a scale of 0-5 about their agreement in each of the possible degrees, which are “Strongly disagree”, “Disagree”, “Neutral”, “Agree”, “Strongly agree”, for this end, they were provided with a visual scale like the one shown in Figure 2.

3. Material and Methods

The information in this research comes from the National Survey of Employment, Unemployment, and Underemployment (ENEMDU), carried out by the Ecuadorian Institute of Statistics and Census (INEC) in December 2022. The incidence of child labor is calculated with the questions that determine the occupancy condition (questions 20, 21, and 22) in the reference week. The initial data was processed and filtered for the most efficient and consistent estimation. The database was configured with a two-stage, stratified, and cluster sampling design to obtain population estimators. Then, the children and adolescents between the ages of 5 and 17 who were or were not working during the survey were selected. To this is added additional information on the characteristics of the heads of the household, such as income, years of education, area of residence, gender, and age, among others. The employment income used in this investigation will equal or less than $3,500. 95% of entrepreneurs register payments of fewer than 1,200 dollars, which is why the barrier of 3,500 was located, as is also done by the Central Bank of Ecuador in its poverty and income reports [24]. Thus, there was a sample size of 64,847 observations.

This research used a logistic regression estimate considered as a tool to analyze data where there is a categorical response variable with two levels, in this case, if the minor works or not, also if he is poor or not poor, whether or not the head of household studied, among others.

The results of applying the econometric model (Logit) are exposed concerning the factors that influence child labor in 2022, and it is essential to mention that the target population is children aged 5 to 17 years. Age, regardless of their sex, area, ethnic self-identification, or any other factors inferring the model. The independent variables used for the model are the following:

Table 1. Description of the independent and dependent variables.

Variables	Description	Type	Scale
Age Range	1 (5 to 11 years) 2 (12 to 14 years) 3 (15 to 17 years)	Quantitative	Ordinal
Head of Household Income	Income	Quantitative	Ordinal
Jetrainfo	0 (Formal) 1 (Informal)	Qualitative	Nominal
Educajeho	Years	Quantitative	Ordinal
Zone	0 (Urban) 1 (Rural)	Qualitative	Nominal
Gender	0 (Women) 1 (Man)	Qualitative	Nominal
Pobhogti	0 (Not poor) 1 (Poor)	Qualitative	Nominal

With the complex variables, the Logit model is carried out, as established by the maximum likelihood methodology, as expressed below:

$$L_i = \ln\left(\frac{P_i}{1-P_i}\right) = \beta_0 + \beta_1(rangodad) + \beta_2(ingrejeti) + \beta_3(jetrainfo) + \beta_4(educajeho) + \beta_5(pobhogti) + \beta_6(zona) + \beta_7(sexo) + \mu_i \quad (5)$$

Where:

$L_i = \ln\left(\frac{P_i}{1-P_i}\right)$ The logarithmic probability that a child and adolescent is in a situation of child labor in the country in 2022.

$X_1 \ldots X_7$: Social, economic, and demographic factors.

β_0: Logit model constant.

$\beta_1 \ldots \beta_7$: Model earrings.

μ_i: Stochastic error.

It is essential to mention that these variables used for the model were transformed using the statistical package Stata 17. In addition, this model is not intended to look for a causal relationship but rather to see the degree and direction of association of a set of dimensions versus the probability of choosing among the options presented.

The influence of various variables on child labor was assessed by consulting a panel of five experts using indeterminate Likert scales. To quantify the level of agreement or disagreement, responses were aggregated as follows: the degree of "Strongly agree" was derived from the sum of truth by $\sum_{i=1}^{5} T_A(x_i)$, the degree of “Agree” by $\sum_{i=1}^{5} I_{T_A}(x_i)$, the degree of “Neutral” by $\sum_{i=1}^{5} I_A(x_i)$, the degree of “Disagree” by $\sum_{i=1}^{5} I_{F_A}(x_i)$, and the degree of “Strongly disagree” by $\sum_{i=1}^{5} F_A(x_i)$.

Each of these sums was then averaged by dividing by 5 and scaled to percentages.

The questions posed to experts were structured as follows:

Q1: To what extent do you agree that the age range influences child labor?

Q2: How much do you believe the income of the head of the household influences the incidence of child labor?

Q3: How strongly do you agree that the type of work (formal vs. informal) affects children's participation in child labor?

Q4: To what degree do you think the level of education influences the likelihood of a child being involved in child labor?
Q5: Do you agree that the area of residence impacts the incidence of child labor?
Q6: Do you consider that the gender of the child significantly influences child labor?
Q7: To what extent do you believe poverty impacts child labor?

The distance between TRINS (Triple Refined Indeterminate Neutrosophic Sets) was calculated using the Euclidean distance formula, providing a quantitative measure of the closeness or similarity(4) between the sets of responses for each question. This methodological approach allows for a nuanced analysis of expert opinions on factors affecting child labor, incorporating the complexity and uncertainty inherent in social phenomena[19].

4. Results

The factor that most affects child labor is the location of the family in rural areas(Table 2), followed by the minor's age and then by the everyday work of the head of the household. It can be noted that parental education does not influence child labor and that the average years of study for people who live in rural areas is eight while those who live in urban areas are ten years. The results imply that most parents do not finish high school.

In short, it can be described that if the little one lives in a rural area, the probability of working increases by 10.5%, and each time he approaches the age of 15, his options to work gain by 4%. Further, it can also be noted that if the father works informally, the probability that minors work is 2.2%. In this case, there is no significant difference if the head of the household is male or female, as seen in the table of logit model estimation.

Table 2. Logit Model estimation.

	Parameter	Standard Error	t	p-value	Odd Ratio	Marginals Effects dy/dx	p-value
Constant	**-5,41**	0,144	-37,711	**0,000**	**0,00**		
trinfjeti	**0,51**	0,039	13,161	**0,000**	**1,67**	**2,2%**	0,000***
edujehoti	**-0,02**	0,004	-3,852	**0,000**	**0,98**	**0,0%**	0,000***
pobreti	**0,21**	0,038	5,406	**0,010**	**1,23**	**1,0%**	0,000***
sexti	**0,35**	0,031	11,380	**0,000**	**1,43**	**1,5%**	0,000***
ruralti	**1,72**	0,035	49,486	**0,000**	**5,57**	**10,5%**	0,000***
lingjehogati	**-0,04**	0,020	-1,826	**0,068**	**0,96**	**-0,2%**	0,0678*
ranedad	**0,98**	0,019	50,719	**0,000**	**2,65**	**4,1%**	0,000***
Statistical evaluation							
R-squared	**20,4%**						
Information Criteria							
AIC	29743						
BIC	29816						

However, the situation may increase in areas with higher levels of informality, and the population is concentrated in rural areas, such as in various provinces of the Oriente and the Sierra. Moreover, unpaid economic activities focus on farms, plots, and even family businesses, where child labor increases among the poorest households with children over 14 years of age and decreases among the least imperfect.

On the other hand, for unpaid domestic workers, where due domestic services are also included, more than 200,000 children are working, and more intensely, those between 12 and 14 years of age. The result can be a reason for reduced academic performance or delays in school or secondary school processes, especially in rural areas, which, on average, have two fewer years of schooling than urban areas.

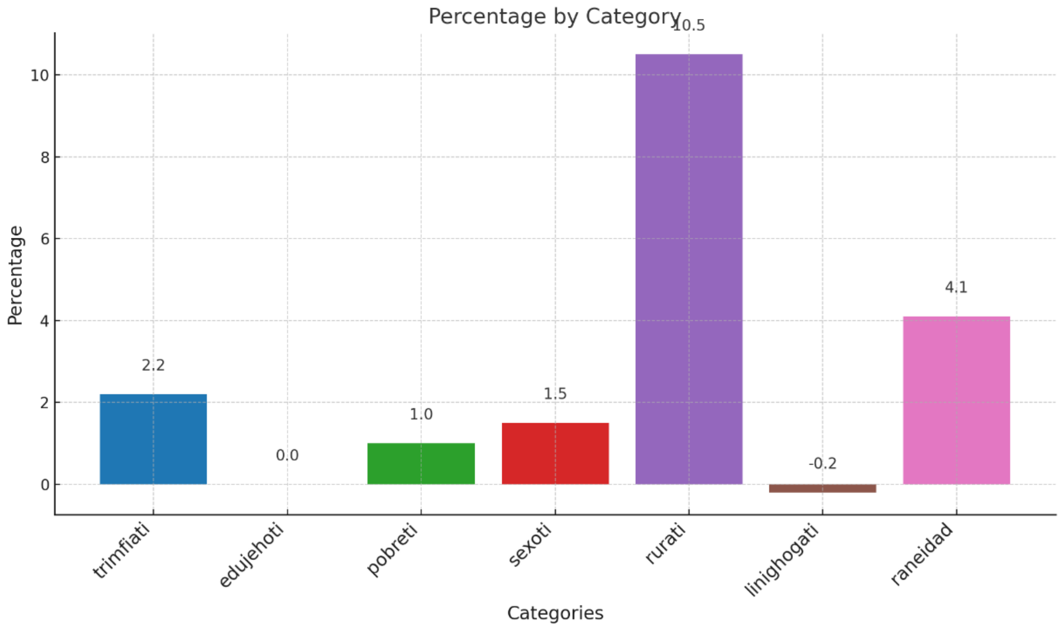


Figure 3. Logistic Regression Outcomes: Impact of Socioeconomic Variables on Child Labor

Experts were requested to evaluate the impact of each variable on child labor through the use of Likert-type scales, and Figure 3 showcases the gathered responses.

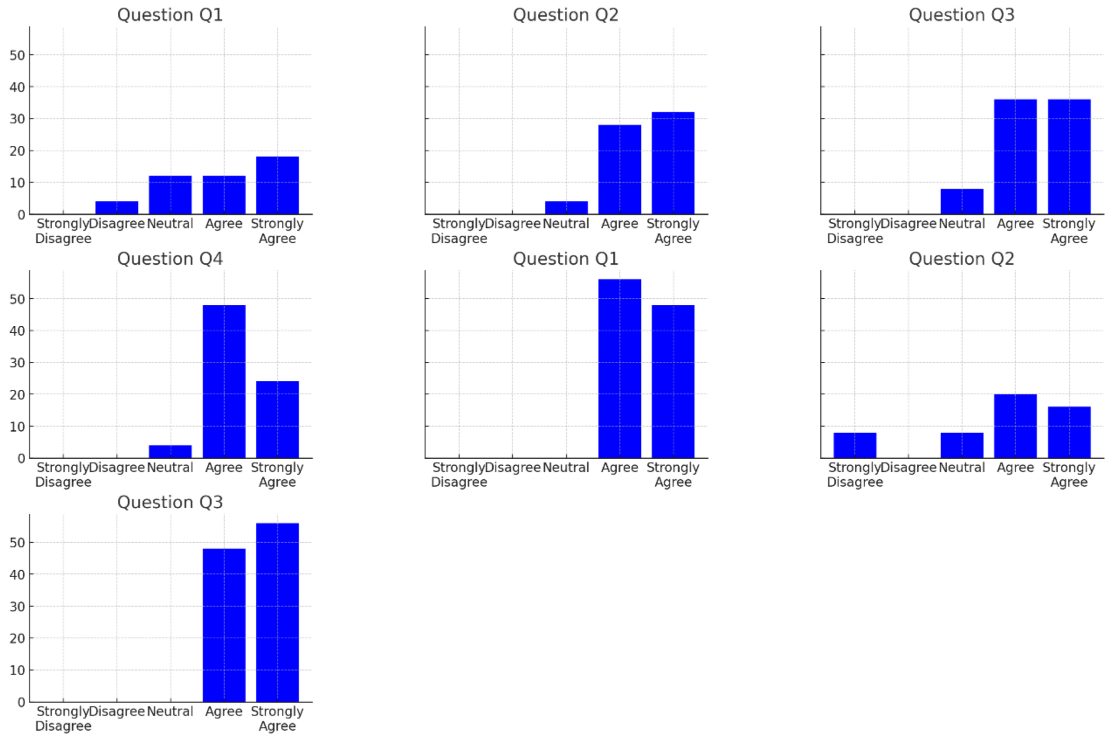


Figura 3. Neutrosophic Likert Scale Analysis

The results indicate varied perceptions on factors influencing child labor:

- Age Range: There’s a general agreement that age range influences child labor, with the majority falling in the 'Agree' to 'Strongly Agree' categories.
- Household Income: The belief that the head of household's income affects child labor incidence is strong, as reflected by the higher number of respondents in the 'Agree' and 'Strongly Agree' responses.

- Type of Work: There's a strong consensus that the type of work, whether formal or informal, has a significant effect on child labor, with most responses again in the 'Agree' and 'Strongly Agree' categories.
- Level of Education: The level of education is deemed quite influential on child labor involvement, with a significant lean towards 'Agree'.
- Area of Residence: Respondents overwhelmingly agree that the area of residence impacts the incidence of child labor, as evidenced by the high counts in 'Agree' and 'Strongly Agree'.
- Gender Influence: Opinions are more varied regarding gender's influence on child labor. While there are still more responses in the 'Agree' category than any other, there's a notable count in 'Strongly Disagree' and 'Neutral', indicating less consensus.
- Influence of Poverty: Poverty is perceived as having a very strong influence on child labor, with the majority of respondents placing this in the 'Agree' and 'Strongly Agree' categories.

Overall, it appears that socio-economic factors such as household income and poverty, as well as education levels, are considered to be significant influencers of child labor. Gender and age range are also seen as important but have a more varied consensus among the respondents.

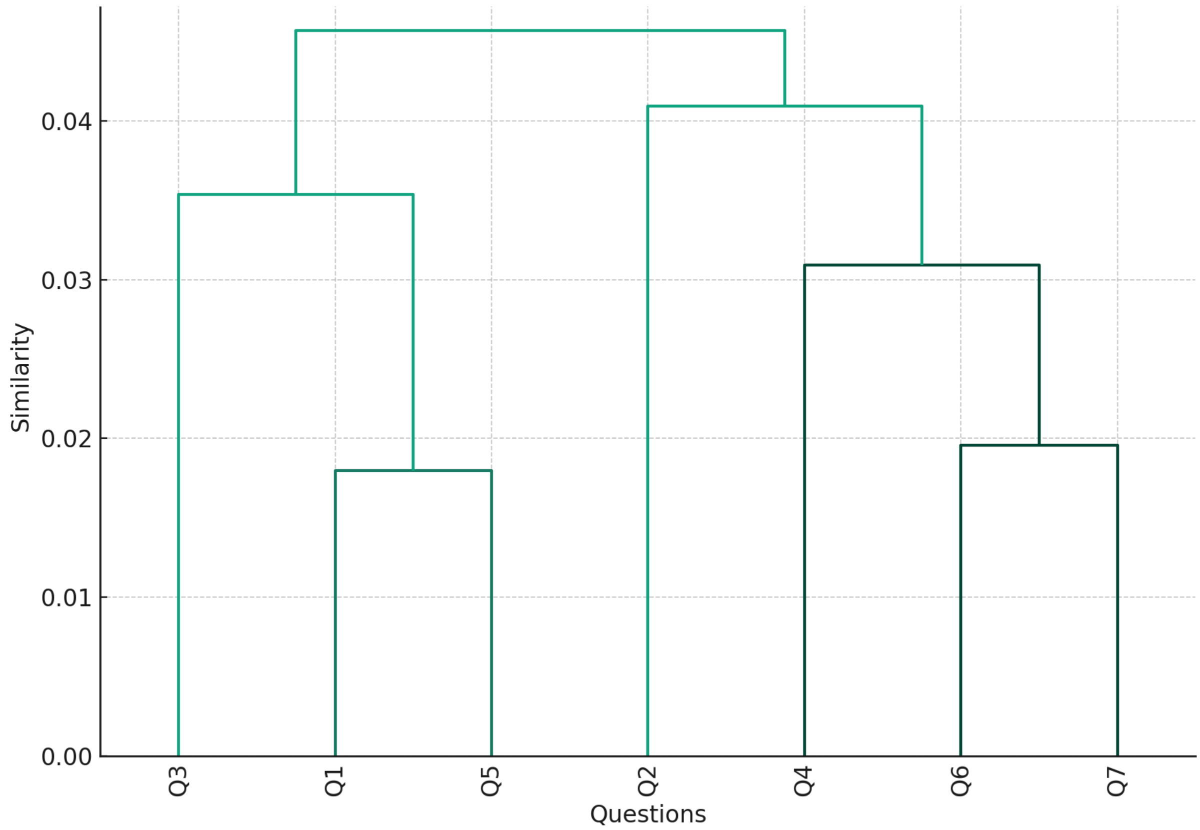


Figure 4. Dendrogram of Responses to Questions

Based on the dendrogram and the questions provided, here is an interpretation of the clustering:

Questions Q1 (age range's influence) and Q5 (area of residence's impact) seem to be the most similar in terms of responses. This suggests that respondents may perceive the influence of age and the area of residence on child labor as having similar levels of impact.

Questions Q2 (household income) and Q7 (poverty's influence) are clustered together at the highest level, indicating that the respondents see a strong relationship between household income and poverty when it comes to influencing child labor. This is intuitive, as these two factors are economically related.

Questions Q3 (type of work) and Q4 (level of education) are also grouped closely, suggesting that the nature of employment (formal vs. informal) and the level of education are perceived to similarly affect child labor.

Question Q6 (child's gender influence) stands alone and joins the other clusters at a higher threshold, which implies that the perceptions of gender influence on child labor are distinct from the other factors.

From these clustering, the dendrogram indicates that economic factors (household income and poverty) are viewed as closely linked in the context of child labor. Similarly, factors related to the work environment and education are also related. Meanwhile, gender stands out as a factor with a different pattern of influence according to the survey responses. This kind of analysis can be instrumental in targeting specific areas for intervention by identifying which factors are considered similar in their influence on child labor.

5. Conclusion

This paper underscores the critical importance of integrating logistic regression with the analysis of neutrosophic linguistic scales and similarity assessments. By examining the interrelationships between various socio-economic factors influencing child labor through this multifaceted approach, we achieve a more nuanced understanding that transcends traditional binary logistic regression models. The use of neutrosophic scales allows for capturing the uncertainty and indeterminacy inherent in human responses, while similarity assessments enable us to identify patterns and clusters in the data that might otherwise be overlooked. This integration facilitates a deeper exploration into the complex dynamics at play, offering a more robust and comprehensive framework for understanding the subtleties of human opinion and its impact on child labor.

For future work, the paper suggests expanding the current model to include a broader set of variables and a larger dataset, potentially drawn from diverse geographical regions to validate the universality of the findings. Additionally, the development of advanced computational tools to automate the neutrosophic analysis could significantly streamline the process, making it more accessible for larger-scale studies. Further research could also explore the application of this integrated approach to other social issues, examining whether the improved insights provided by the combination of logistic regression and neutrosophic linguistic scales hold consistent across different domains. Such studies could provide invaluable information for policymakers and social scientists working towards the mitigation of complex social problems.

6. References

[1]. UNICEF. Trabajo infantil: se elevan a 160 millones al alza por primera vez en dos decenios. Available online: https://www.unicef.org/es/comunicados-prensa/trabajo-infantil-elevan-160-millones-al-alza-primera-vez-dos-decenios (accessed on 20 January 2024).

[2]. BUREAU OF INTERNATIONAL LABOR AFFAIRS (ILAB, B.O.). Child Labor and Forced Labor Reports: Ecuador. Available online: https://www.dol.gov/agencies/ilab/resources/reports/child-labor/ecuador (accessed on 2 February 2024)

[3]. Basu, K.; Genicot, G.; Stiglitz, J. Household labor supply, unemployment, and minimum wage legislation. Policy and Research Working Papers 1999. Available online: https://ssrn.com/abstract=620651 (accessed on 13 January 2024).

[4]. Edmonds, E. Child Labor. Handbook of Development Economics 2007, 4, 3607–3709

[5]. Edmonds, E.; Schady, N. Poverty Alleviation and Child Labor. Economic Policy 2012, 4(4), 100-124. Available online: https://doi.org/10.1257/pol.4.4.100 (accessed on 6 January 2024).

[6]. Jara, H. X.; Montesdeoca, L.; Tasseva, I. The role of automatic stabilizers and emergency tax–benefit policies during the COVID-19 pandemic: Evidence from Ecuador. The European Journal of Development Research 2022, 34(6), 2787-2809. Available online: https://doi.org/10.1057/s41287-021-00490-1 (accessed on 2 April 2024).

[7]. Ministerio de Inclusión Económica y Social (MIES). (2023). Available online: https://www.inclusion.gob.ec/que-tipos-de-bonos-y-pensiones-entrega-el-mies/?preview=true (accessed on 2 April 2024).

[8]. Cecchini, S.; Atuesta, B. Programas de transferencias condicionadas en América Latina y el Caribe: Tendencias de cobertura e inversión. Santiago: Naciones Unidas, 2017. Available online: https://repositorio.cepal.org/handle/11362/41811 (accessed on 2 April 2024)

[9]. Edmonds, E.; Schady, N. Poverty Alleviation and Child Labor. Economic Policy 2012, 4(4), 100-124.

[10]. Basu, K.; Tzannatos, Z. The Global Child Labor Problem: What Do We Know and What Do We Do? World Bank Economic Review 2003, 17(2), 147-173.

[11]. Sikandar; Panezai, S.; Saqib, S.; Muhammad, S.; Bilal; Khan, I. Factors Determining Child Labor: Empirical Evidence from Khyber Pakhtunkhwa, Pakistan. Changing Societies & Personalities 2022, 6(1), 123–143.

[12]. INEC, Instituto Nacional de Estadística y Censos. (2022). Boletín Técnico - ENEMDU. Available online: https://www.ecuadorencifras.gob.ec/sistema-estadisticas-laborales-empresariales/ (accessed on 16 January 2024).

[13]. Rahman, M. A. Household characteristics and poverty: A logistic regression analysis. The Journal of Developing Areas 2013, pages 303-317.

[14]. Kandasamy, I.; Kandasamy, W. V.; Obbineni, J. M.; Smarandache, F. Indeterminate Likert scale: feedback based on neutrosophy, its distance measures and clustering algorithm. Soft Computing 2020, 24, 7459-7468.

[15]. Broumi, S.; Smarandache, F. Several Similarity Measures of Neutrosophic Sets. Neutrosophic Sets & Systems 2013, (1).

[16]. Das, A. Logistic regression. In Encyclopedia of Quality of Life and Well-Being Research; Springer International Publishing: Cham, 2021; pp. 1-2.

[17]. Lohmann, A.; Groenwold, R.H.; van Smeden, M. Comparison of likelihood penalization and variance decomposition approaches for clinical prediction models: A simulation study. Biometrical J. 2024, 66(1), 2200108.
[18]. Vásquez-Ramírez, M.R.; Moscoso-Paucarchuco, K.M.; Beraún-Espíritu, M.M.; Yupanqui-Villanueva, H.R.; Vivanco-Nuñez, O.A.; Yupanqui-Villanueva, W.F.; Gutiérrez-Gómez, E. Determination of the degree of relationship between Activity Cost and Financial Management in beef cattle production in a region of Peru, based on Indeterminate Likert Scale and Neutrosophic Similarity. Neutrosophic Sets Syst. 2024, 64, 84-90.
[19]. Kandasamy, I., and Smarandache, F. (2016) Triple refined indeterminate neutrosophic sets for personality classification. Paper presented at the 2016 IEEE Symposium Series on Computational Intelligence (SSCI).
[20]. Smarandache, F. n-Valued Refined Neutrosophic Logic and Its Applications to Physics. Prog. Phys. 2013, 4, 143.
[21]. Leonor, M. M., Easud, G. S., and Fernando, P. P. (2022) Indeterminate Likert Scale in Social Sciences Research. International Journal of Neutrosophic Science, 19, 289-297.
[22]. Chatterjee, R., Majumdar, P. and Samanta, S. K. (2019). Similarity Measures in Neutrosophic Sets-I in Fuzzy Multi-criteria Decision-Making Using Neutrosophic Sets, Switzerland, Springer Nature, pp. 249-294.
[23]. Chatterjee, R., Majumdar, P. and Samanta, S. K. (2019). Similarity Measures in Neutrosophic Sets-II in Fuzzy Multi-criteria Decision-Making Using Neutrosophic Sets, Switzerland, Springer Nature, pp. 295-326.
[24]. Banco Central del Ecuador. Reporte de pobreza, ingreso y desigualdad. Available online: https://contenido.bce.fin.ec/documentos/Estadisticas/SectorReal/Previsiones/IndCoyuntura/Empleo/indice_pobreza.htm (accessed on 18 January 2024).

Received: Jan 8, 2024. Accepted: Mar 29, 2024

Neutrosophic Sets and Systems, Vol. 66, 2024
University of New Mexico

The Problem of Planning Neutrosophic Hydropower Systems (Converting Some Nonlinear Neutrosophic Models into Linear Neutrosophic Models)

Maissam Jdid
Faculty of Science, Damascus University, Damascus, Syria
maissam.jdid66@damascusuniversity.edu.sy

Abstract:
The core of operations research activity focuses on creating and using models. These models may be linear models, non-linear models, dynamic models, and others. Linear models are considered one of the most important and most widely used operations research models due to the availability of appropriate algorithms through which we can obtain the optimal solution, which prompts us to benefit from the nature of the topic under study and the information available to us about the variables in it to transform it into linear models. In classical logic, many nonlinear programming problems have been processed and transformed into linear programming problems. In this research, we present a study of the issue of planning hydroelectric systems, where the general policy for operating this system specifies two prices for selling the produced electricity, which makes this issue a non-linear programming issue. We will turn it into a linear programming problem using linear programming concepts, and then we will use Boolean concepts. Neutrosophic studies of linear programming and non-linear programming are presented to provide a neutrosophic formulation of the issue of planning hydroelectric systems, through which we obtain a neutrosophic linear model whose optimal solution fits all the conditions that the system's operating environment may experience during the operating period. During the two operating periods.

key words:
Operations research; linear models; nonlinear models; neutrosophic logic; neutrosophic linear models; neutrosophic nonlinear models; converting neutrosophic nonlinear models into linear neutrosophic models; the issue of planning neutrosophic hydroelectric systems; converting nonlinear neutrosophic models into linear neutrosophic models.

1. Introduction:

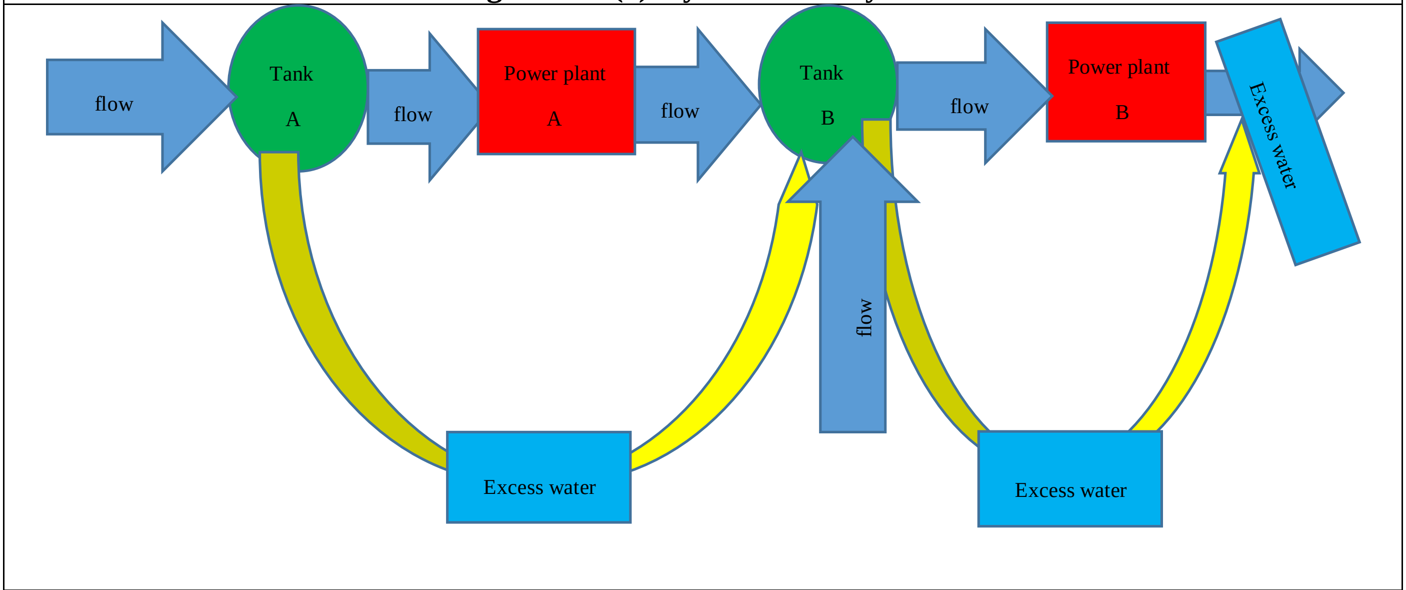
Mathematical programming problems are generally concerned with allocating scarce resources of labor, machinery, and capital and using them in the best possible way, such that costs are reduced to their minimum or profits are maximized, by choosing the optimal solution from a set of possible solutions, and in doing so it depends on transforming the problem under study into a model. Mathematical and appropriate techniques are used for this type of model to reach the optimal solution. Linear programming problems are among the most widely used problems in most fields, and the reason for this is the availability of many appropriate techniques to find the optimal solution. Therefore, we find that students and researchers in the field of operations research always seek to Converting realistic problems into linear models if the data of the problem allow it. In this research, we present a study of one of the important issues, which is the issue of generating electrical power through the flow of water to power plants, where we described a hydroelectric system that operates in two periods and produces an amount of electricity that is sold at two different prices. This depends on the quantity produced, and because of this difference in price for the quantities produced, we find that the mathematical model that we will obtain is a non-linear model, based on the information contained in reference [1]. In this research, we will present the issue using classical logic in a detailed manner through which we explain how to address some Nonlinear programming issues using linear programming concepts, and in order to obtain accurate results that take into account all conditions and factors that can affect the amount of water used in the process of generating electrical power, we will use the concepts of neutrosophic logic, the logic whose studies and research, whose concepts have been used in most fields, have provided more results. Accuracy of the results that we were obtaining using the concepts of classical logic see [2-12], using previous studies that we presented using the concepts of neutrosophic logic for the topics of linear programming and nonlinear programming [13-24], and the result of this study will be to transform a neutrosophic nonlinear programming problem into A neutrosophic linear programming problem. The studies presented in the papers [17-20] can be used to find the optimal solution.

2. Discussion:

Based on the study mentioned in Reference [1], we present the following example through which we present the classic formulation of the issue of planning hydroelectric energy systems:

The administration controls the operation of a system consisting of two water tanks, each equipped with two electrical power plants, as shown in the following figure:

Figure No. (1) Hydroelectric system



This system works by sending a quantity of water to power plants to generate electrical power. The experts provided the following information:

- **Information about the power plants during the two periods:**

1- A volume of water of 1 Kilo-Acre-Foot (*KAF*), can generate 400 Megawatt-hour (*MWh*) of electricity in power plant *A* and 200 *MWh* in power plant *B*.

2- The maximum power that can be generated in power plant *A* is 60000 *MWh*, and in power plant *B* the maximum power that can be generated is 35000 *MWh*.

3- An amount of electricity amounting to 50000 *MWh*, can be sold at a price of 20$ *MWh*, while the excess quantity is sold at a price of 14$ *MWh*.

- **Information about the tanks during the two periods:**

1- The maximum capacity of reservoir *A* is 2000 *KAF* and the maximum capacity of reservoir *B* is 1500 *KAF*.

2- The level of reservoir *A* at the beginning of the first period is 1900 *KAF* and the level of reservoir *B* is 850 *KAF*.

3- The minimum level allowed in tank *A* is 1200 *KAF* and in tank *B* is 800 *KAF*.

4- The flow to tank A during the first period is 200 KAF and to tank B is 40 KAF.
5- The flow to tank A during the second period is 130 KAF and to tank B is 15 KAF.

we should be noted here:

- Power plant A is supplied from tank A and power plant B from tank B.
- When the tanks are completely full, some of the water is drained through the drainage channels so that flooding does not occur.
- For tank B, water is supplied from the following sources:
  - The aforementioned outflow.
  - From power plant A, the water that is supplied to the power plant after its use goes out to tank B.
  - Water drained from tank A so that flooding does not occur.

What is required is to build a mathematical model of the functioning of this system so that it achieves the maximum profit from the process of selling electricity. We know that to formulate a mathematical programming program, we follow the following three steps:

First step: We identify the unknowns in the problem (decision variables) and express them in algebraic symbols.

Second step: We define all constraints and express them with equations or inequalities that are mathematical functions of the unknown variables.

Step Three: We define the objective function and represent it as a linear function of the unknown variables. It should be made as large or as small as possible.

According to the text of the issue, in order to obtain the required mathematical model, the issue must be studied according to the data for each period and the necessary relationships determined. Here we find that a study must be presented for the first and second steps, specific to each period separately.

The study for the first period:

based on the previous information, we organize the following tables:

Information about the power plants of the first period:

Table No. (1) Information about power plants A and B during the first period

power plants / Information	power plant A	power plant B
Water supplied to the power plant during the first period	unknown	unknown
Maximum generating capacity during the first period	60000	35000
The power it can generate for a volume of water of 1 KAF	400	200

Information about tanks during the first period:

Table No. (2) Information about reservoirs A and B during the first period

Tanks / Information	Tank A	Tank B
Maximum capacity	2000	1500
Minimum permissible level	1200	800
The ratio at the beginning of the first period	1900	850
Excess water drained from the tank to prevent flooding	unknown	unknown
Flow during the first period	200	40

Reservoir level at the end of the first period	unknown	unknown

The first step regarding this issue during the first period of work, we find that the unknowns

1- The amount of power produced is divided into two parts: a quantity that is sold for 20$ *MWh*, and a quantity that is sold for 14$ *MWh*.
2- The amount of water that must be supplied to power plant A will be denote x_1.
3- The amount of water supplied to power plant B will be denoted x_2.
4- The amount of water that must be drained from tank A so that flooding does not occur. We will denote it x_3.
5- The amount of water that must be drained from tank B so that flooding does not occur. We will denote it x_4.
6- Tank A level at the end of the first period. We will denote it x_5.
7- Tank B level at the end of the first period. We will denote it x_6.

The second step regarding this issue during the first period of work we find the following restrictions:

1- We assume that the amount of power sold during the first period at a price of 20$ *MWh*, is X_1 and the amount of power sold at a price of 14$ *MWh*, is X_2. The amount of power produced during the first period is:

$$X = X_1 + X_2$$

Electrical power constraint: Since every 1 *KAF* generates electrical power of 400 *MWh*, in power plant A when supplying this plant with the quantity x_1*KAF*, the power produced during the first period of this plant will be $400\,x_1$ .The same situation applies to power plant B, the amount of power produced is $200x_2$, and therefore the amount of power produced during the first period of the two plants is:

$$400x_1 + 200x_2$$

The amount of power produced during the first period must equal the amount of power sold during this period, meaning we get the following restriction:

$$400x_1 + 200x_2 = X_1 + X_2 \quad (1)$$

2- Constraint on the amount of water that must be supplied to Power Plant A: Since the maximum power that this plant can produce during the first period is 60000 *MWh*, and every 1 *KAF* of water in this plant produces 400 *MWh*, we get the following constraint:

$$\frac{60000}{400} = 150 \Rightarrow$$

$$x_1 \leq 150 \quad (2)$$

Restricting the amount of water that must be supplied to power plant B: Since the maximum power that this plant can produce during the first period is 35000 *MWh*, and every 1 *KAF* of water in this plant produces 200 *MWh*, we get the following restriction:

$$\frac{35000}{200} = 175 \Rightarrow$$

$$x_2 \leq 175 \quad (3)$$

3- Maintaining the amount of water in tank A:
The level of tank A at the beginning of the first period, plus the amount of flow into this tank, must equal the amount of water supplied to power plant A from tank A + the amount of water that must be drained so that a flood does not occur + the tank level at the end of the first period, i.e.:

$$1900 + 200 = x_1 + x_3 + x_5 \Rightarrow$$

$$x_1 + x_3 + x_5 = 2100 \quad (4)$$

Since the minimum level allowed in this tank is 1200 and the maximum capacity of this tank is 2000, the level of tank A at the end of the first period must be limited between the values 1200 and 2000 and thus we obtain the following double restriction:

$$1200 \leq x_5 \leq 2000 \quad (5)$$

4- Maintaining the amount of water in tank B:

The water supplied to power plant B+ the water drained from tank B so that there is no flooding + the level of tank B at the end of the first period must equal the amount of water supplied to tank B from power plant A (it is the same amount of water that was supplied to power plant A from the tank A) + the amount of water that must be drained from tank A so that a flood does not occur + the level of tank B at the beginning of the first period + the amount of flow into tank B in the first period, i.e.:

$$x_2 + x_4 + x_6 = x_1 + x_3 + 850 + 40 \Longrightarrow$$
$$x_2 + x_4 + x_6 - x_1 - x_3 = 890 \quad (6)$$

Since the minimum level allowed in this tank is 800 and the maximum capacity of this tank is 1500, the level of tank Bat the end of the first period must be limited between the values 800 and 1500, and thus we obtain the following double restriction:

$$800 \leq x_6 \leq 1500 \quad (7)$$

- **The study for the second period:**

Information about the power plants of the second period:

Table No. (1) Information about power plants A and B during the second period		
power plants / Information	power plant A	power plant B
Water supplied to the power plant during the second period	Unknown	Unknown
Maximum generating capacity during the second period	60000	35000
The power it can generate for a volume of water of 1 KAF	400	200

Information about tanks during the first period:

Table No. (2) Information about reservoirs A and B during the second period		
Tanks / Information	Tank A	Tank B
Maximum capacity	2000	1500
Minimum permissible level	1200	800
The ratio at the beginning of the second period is the same as the ratio at the end of the first period	Unknown	Unknown
Excess water drained from the tank to prevent flooding	Unknown	Unknown
Flow during the second period	130	15
Reservoir level at the end of the second period	Unknown	Unknown

The first step regarding this issue during the second period of work, we find that the unknowns are:

1- The amount of power produced is divided into two parts: a quantity that is sold for 20$ *MWh*, and a quantity that is sold for 14$ *MWh.*
2- The amount of water that must be supplied to power plant A will be denoted y_1.
3- The amount of water supplied to power plant B will be denoted y_2.
4- The amount of water that must be drained from tank A so that flooding does not occur. We will denote it y_3.
5- The amount of water that must be drained from tank B so that flooding does not occur. We will denote it y_4.
6- Tank A level at the end of the second period. We will denote it y_5.
7- Tank B level at the end of the second period. We will denote it y_6.

The second step regarding this issue during the second period of work we find the following restrictions:

1- We assume that the amount of power sold during the first period at a price of 20$ per megawatt-hour is Y_1 and the amount of power sold at a price of 14$ per megawatt-hour is Y_2. The amount of power produced during the first period is:

$$Y = Y_1 + Y_2$$

Electrical power constraint: Since every 1 *KAF* generates electrical power of 400 *MWh,* in power plant A, when supplying this plant with the quantity y_1 *KAF*, the power produced during the second period of this plant will be $400\,y_1$. The same situation is for power plant B, the amount of power produced is $200y_2$, and therefore the amount of power produced during the second period of the two plants is:

$$400y_1 + 200y_2$$

The amount of power produced during the second period must equal the amount of power sold during this period, meaning we get the restriction:

$$400y_1 + 200y_2 = Y_1 + Y_2 \quad (8)$$

2- Constraint on the amount of water that must be supplied to power Plant A: Since the maximum power that this plant can produce during the second period is 60000 *MWh,* and every 1 *KAF* of water in this plant produces 400 *MWh,* we get the following constraint:

$$\frac{60000}{400} = 150 \implies$$
$$y_1 \leq 150 \quad (9)$$

Restricting the amount of water that must be supplied to power plant B: Since the maximum power that this plant can produce during the second period is 35000 *MWh,* and every 1 *KAF* of water in this plant produces 200 *MWh,* we get the following restriction:

$$\frac{35000}{200} = 175 \implies$$
$$y_2 \leq 175 \quad (10)$$

3- Maintaining the amount of water in tank A:

The level of tank A at the beginning of the second period, plus the amount flowing into this tank, must equal the amount of water supplied to it Power factor A of tank A + the amount of water that must be drained so that there is no flooding + the tank level at the end the first period, i.e.:

$$x_5 + 130 = y_1 + y_3 + y_5 \implies$$
$$y_1 + y_3 + y_5 - x_5 = 130 \quad (11)$$

Since the minimum level allowed in this tank is 1200 and the maximum capacity of this tank is 2000, the level of tank A at the end of the first period must be limited between the values 1200 and 2000 and thus we obtain the following double restriction:

$$1200 \leq y_5 \leq 2000 \quad (12)$$

4- Maintaining the amount of water in tank B:

The water supplied to power plant B + the water drained from tank B so that there is no flooding + the level of tank B at the end of the second period must equal the amount of water supplied to tank B from power plant A (it is the same amount of water that was supplied to power plant A from the tank A) + the amount of water that must be drained from tank A so that a flood does not occur + the level of tank B at the beginning of the second period + the amount of flow into tank B in the first period, i.e.:

$$y_2 + y_4 + y_6 = x_1 + y_3 + x_6 + 15 \Rightarrow$$
$$y_2 + y_4 + y_6 - x_1 - y_3 - x_6 = 15 \quad (13)$$

Since the minimum level allowed in this tank is 800 and the maximum capacity of this tank is 1500, the level tank B at the end of the second period must be confined between the values 800 and 1500, and thus we obtain the double entry.

$$800 \leq y_6 \leq 1500 \quad (14)$$

The third step: In the problem, determine the objective function relation:

From the data of the issue, we found that the department responsible for the workflow set two prices for selling the electrical power produced during the two periods, according to the quantity sold, and then they are:

The amount of power produced during the first period is given by the following relation:

$$X = X_1 + X_2$$

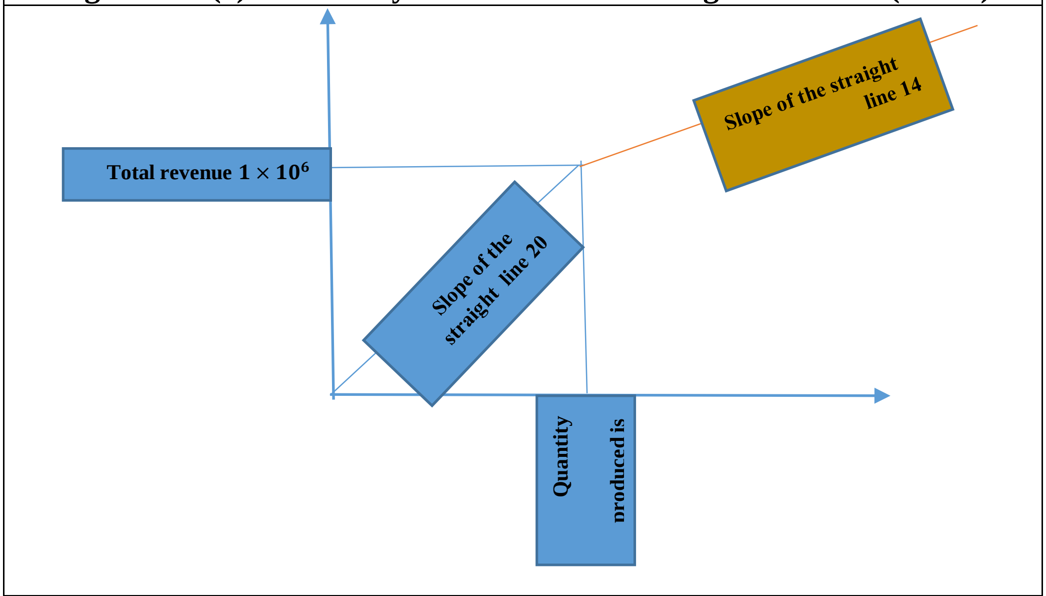
Where X_1 is the amount of power sold during this period at a price of 20$ *MWh*, and X_2 is the amount of power sold during this period is at a price of 14$ *MWh*, the amount of power produced during the second period is given by the following relation:

$$Y = Y_1 + Y_2$$

Where Y_1 is the amount of power sold during this period at a price of 20$ *MWh*, and Y_2 is the amount of power sold during this period is at a price of 14$ *MWh*, in the following figure we show the maximum revenue and the quantities sold of power corresponding to the price of 20$ *MWh*, and the smallest revenue that we can obtain from power and the quantities sold of power corresponding to the price of 14$ *MWh*.

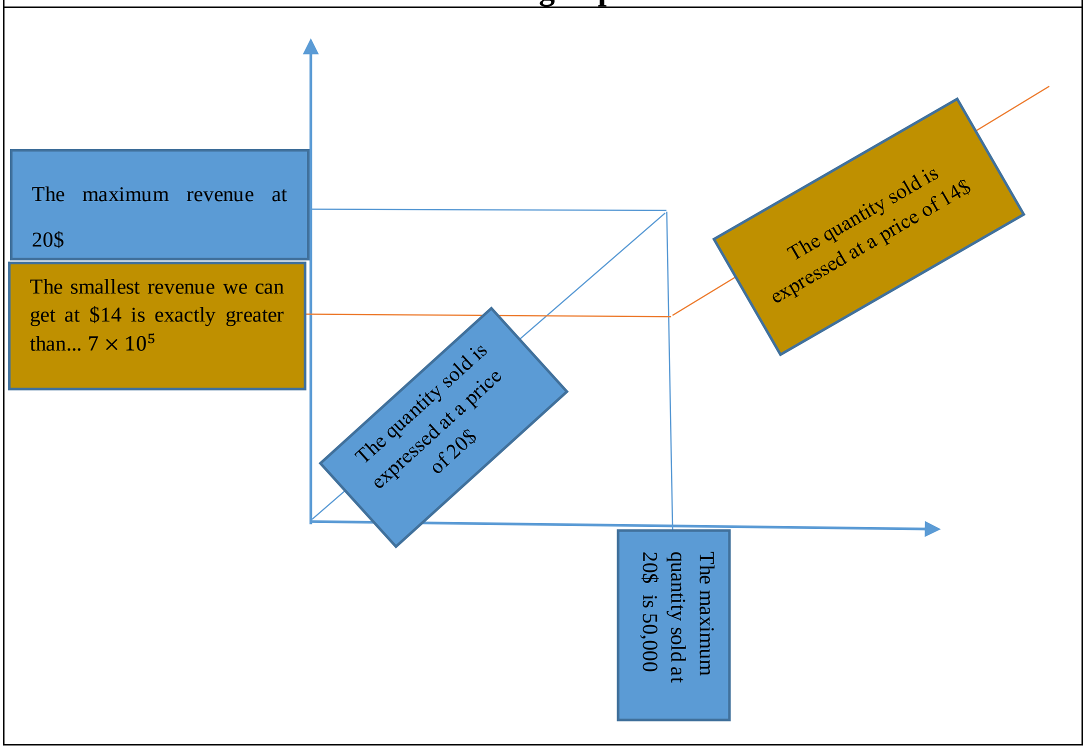
The objective function can be represented as follows:

Figure No. (2): Electricity sales estimated in megawatt hours (MWh)



From the previous figure, we notice that the objective function is a discrete linear function. It is linear in the fields $[0,50000]$ and $]50000, \infty[$. Therefore, the quantity of electricity sold can be divided into two parts: the part sold at a price of 20$/ MWh, and the part sold at a price: 14$/ MWh, and the following figure shows the quantities produced and the maximum revenue for power sold at a price of 20$ and the quantities produced and the smallest revenue we can get from power sold at a price of 14$.

Figure No. (3) Maximum revenue, minimum revenue, and quantities sold according to price



From the above, we can represent the objective function with a linear function as follows:

$$Z = 20(X_1 + Y_1) + 14(X_2 + Y_2) \quad (15)$$

Mathematical model:

Find:

$$Z = 20(X_1 + Y_1) + 14(X_2 + Y_2) \longrightarrow Max$$

Within the conditions:

$$400x_1 + 200x_2 = X_1 + X_2$$
$$x_1 \leq 150$$
$$x_2 \leq 175$$
$$x_1 + x_3 + x_5 = 2100$$
$$x_5 \leq 2000$$
$$x_5 \geq 1200$$
$$x_2 + x_4 + x_6 - x_1 - x_3 = 890$$
$$x_6 \leq 1500$$
$$x_6 \geq 800$$
$$400y_1 + 200y_2 = Y_1 + Y_2$$
$$y_1 \leq 150$$
$$y_2 \leq 175$$
$$y_1 + y_3 + y_5 - x_5 = 130$$
$$y_5 \leq 2000$$
$$y_5 \geq 1200$$
$$y_2 + y_4 + y_6 - x_1 - y_3 - x_6 = 15$$
$$y_6 \leq 1500$$
$$y_6 \geq 800$$
$$x_j \geq 0 , y_j \geq 0 ; j = 1,2,\dots,6 \ and (X_1, X_2, Y_1, Y_2) \geq 0$$

It is a linear model in which the direct Simplex algorithm and its modifications can be used to find the optimal solution through which we achieve the maximum profit, but this solution will be a classic value, a specific value, appropriate to the data that was used, and any change in the conditions of the work environment will affect this data, and therefore the solution we obtain will be inappropriate. It may cause the organization to suffer significant losses, so we suggest that this issue be studied using the concepts of neutrosophic logic by taking data that is subject to change in neutrosophic values, as in the following generalized study:

- **The general neutrosophical formulation of the issue of planning hydroelectric power systems:**

In this section, we present a general neutrosophic formulation of the issue of planning hydroelectric systems, because, as we know, some data in the text of the issue are subject to change during the course of work in the system due to many natural and other factors. In order to obtain a more accurate and appropriate study for all circumstances, we will take these data as neutrosophic values.

Relying on the information contained in references [3,14,21], we present the following neutrosophic study:

Neutrosophic numbers [2]:

The neutrosophic number is given by the following formula: $a \pm bI$, where $a \ and \ b$ are real or complex coefficients, and I is the indeterminacy. It can be any domain, set, or any neighborhood of real values. Therefore, in order to obtain the neutrosophic mathematical model for the problem of planning hydroelectric systems, we will take the data that Affected by the factors and conditions surrounding the system's operating environment, neutrosophic numbers are any of the form Nb_i and Na_{ij}, indefinite values. Completely determined, they can be any neighborhood of the real numbers a_{ij} and b_i written in one of the forms:

$Na_{ij} = a_{ij} + \varepsilon_{ij}$ and $Nb_i = b_i + \mu_i$ where $\varepsilon_{ij} \in [\lambda_{1ij}, \lambda_{2ij}]$ or $\varepsilon_{ij} \in \{\lambda_{1ij}, \lambda_{2ij}\}$.

Neutrosophic Mathematical Model [21]:

In case of examples in which the goal and constraints are in the form of neutrosophic mathematical functions, then the neutrosophic mathematical model is written in the following form:

$$Nf = Nf(x_1, x_2, --, x_n) \rightarrow (Max) or (Min)$$

Subject to the following restrictions:

$$Ng_i(x_1, x_2, --, x_n) \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} Nb_i \quad ; i = 1,2, ---, m$$

$$x_1, x_2, --, x_n \geq 0$$

The general form of the neutrosophic linear model [14]:

The general neutrosophic form of the linear mathematical model is given in short form as follows:

$$Nf = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \rightarrow (Max) or (Min)$$

Within the restrictions:

$$\sum_{j=1}^{n} a_{ij} x_j \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} b_i \pm \delta_i \quad ; i = 1,2, \ldots, m$$

$$x_j \geq 0$$

Then we get the following neutrosophic formulation:

A department controls the operation of a system consisting of two water tanks, each equipped with a plant to generate electrical power. This system works by sending a quantity of water to power plants to generate electrical power. The experts provided the following information:

- **Information about the power plants during the two periods:**

1- A volume of water of 1 *KAF*, can generate p_A *MWh*, of electricity in power plant A and p_B *MWh*, in power plant B.

2- The maximum power that can be generated in power plant A is NP_A *MWh*, and in power plant B the maximum power that can be generated is NP_B *MWh*.

3- An amount of electricity amounting to K *MWh*, can be sold at a price of C_{N1}\$ *MWh*, while the excess quantity is sold at a price of C_{N2}\$ *MWh*.

- **Information about the tanks during the two periods:**

1- The maximum capacity of reservoir A is S_A *KAF* and the maximum capacity of reservoir B is S_B *KAF*.

2- The level of reservoir A at the beginning of the first period is NM_{A1} *KAF* and the level of reservoir B is NM_{B1} *KAF*.

3- The minimum level allowed in tank A is NL_A *KAF* and in tank B is NL_B *KAF*.

4- The flow to tank A during the first period is T_{AN1} *KAF* and to tank B is T_{BN1} *KAF*.

5- The flow to tank A during the second period is T_{AN2} *KAF* and to tank B is T_{BN2} *KAF*.

We should be noted here:

- Power plant A is supplied from tank A and power plant B from tank B.
- When the tanks are completely full, some of the water is drained through the drainage channels so that flooding does not occur.

- For tank B, water is supplied from the following sources:
  - The aforementioned outflow.
  - From power plant A, the water that is supplied to the power plant after its use goes out to tank B.
  - Water drained from tank A so that flooding does not occur.

What is required is to build a mathematical model of the functioning of this system so that it achieves the maximum profit from the process of selling electricity. We know that to formulate a mathematical programming program, we follow the following three steps:

First step: We identify the unknowns in the problem (decision variables) and express them in algebraic symbols.

Second step: We define all constraints and express them with equations or inequalities that are mathematical functions of the unknown variables.

Step Three: We define the objective function and represent it as a linear function of the unknown variables. It should be made as large or as small as possible.

According to the text of the issue, in order to obtain the required mathematical model, the issue must be studied according to the data for each period and the necessary relationships determined. Here we find that a study must be presented for the first and second steps, specific to each period separately.

- **The study for the first period:**

Based on the previous information, we organize the following tables:

Information about the power plants of the first period:

Table No. (1) Information about power plants A and B during the first period

power plants / Information	power plant A	power plant B
Water supplied to the power plant during the first period	Unknown	Unknown
Maximum generating capacity during the first period	NP_A	NP_B
The power it can generate for a volume of water of 1 KAF	p_A	p_B

Information about tanks during the first period:

Table No. (2) Information about reservoirs A and B during the first period

Tanks / Information	Tank A	Tank B
Maximum capacity	S_A	S_B
Minimum permissible level	NL_A	NL_B
The ratio at the beginning of the first period	NM_{A1}	NM_{B1}
Excess water drained from the tank to prevent flooding	Unknown	Unknown
Flow during the first period	T_{AN1}	T_{BN1}
Reservoir level at the end of the first period	Unknown	Unknown

The first step regarding this issue during the first period of work, we find that the unknowns:

1- The amount of power produced is divided into two parts: a quantity that is sold for $C_{N1}\$ MWh$, and a quantity that is sold for $C_{N2}\$ MWh$.
2- The amount of water that must be supplied to power plant A will be denoted x_1.
3- The amount of water supplied to power plant B will be denoted x_2.
4- The amount of water that must be drained from tank A so that flooding does not occur. We will denote it x_3.
5- The amount of water that must be drained from tank B so that flooding does not occur. We will denote it x_4.
6- Tank A level at the end of the first period. We will denote it x_5.
7- Tank B level at the end of the first period. We will denote it x_6.

The second step regarding this issue during the first period of work we find the following restrictions:

1- We assume that the amount of power sold during the first period at a price of $C_{N1}\$$ *MWh*, is X_1 and the amount of power sold at a price of $C_{N2}\$$ *MWh*, is X_2. The amount of power produced during the first period is:

$$X = X_1 + X_2$$

Electrical power constraint: Since every 1 *KAF* generates electrical power of 400 *MWh*, in power plant A when supplying this plant with the quantity x_1*KAF*, the power produced during the first period of this plant will be $p_{A1}\,x_1$ .The same situation applies to power plant B, the amount of power produced is $p_B x_2$, and therefore the amount of power produced during the first period of the two plants is:

$$p_A x_1 + p_B x_2$$

The amount of power produced during the first period must equal the amount of power sold during this period, meaning we get the following restriction:

$$p_A x_1 + p_B x_2 = X_1 + X_2 \quad (1)$$

2- Constraint on the amount of water that must be supplied to power plant A: Since the maximum power that this plant can produce during the first period is NP_A *MWh*, and every 1 *KAF* of water in this plant produces p_A *MWh*, we get the following constraint:

$$\frac{NP_A}{p_A} \Rightarrow x_1 \le \frac{NP_A}{p_A} \quad (2)$$

Restricting the amount of water that must be supplied to power plant B: Since the maximum power that this plant can produce during the first period is NP_B megawatt-hours, and every 1 *KAF* of water in this plant produces p_B *MWh*, we get the following restriction:

$$\frac{NP_B}{p_B} \Rightarrow x_2 \le \frac{NP_B}{p_B} \quad (3)$$

3- Maintaining the amount of water in tank A:

The level of tank A at the beginning of the first period, plus the amount of flow into this tank, must equal the amount of water supplied to power plant A from tank A + the amount of water that must be drained so that a flood does not occur + the tank level at the end of the first period, i.e.:

$$NM_{A1} + T_{AN1} = x_1 + x_3 + x_5 \Rightarrow$$
$$x_1 + x_3 + x_5 = NM_{A1} + T_{AN1} \quad (4)$$

Since the minimum level allowed in this tank is S_A and the maximum capacity of this tank is NL_A, the level of tank A at the end of the first period must be limited between the values NL_A and S_A thus we obtain the following double restriction:

$$NL_A \le x_5 \le S_A \quad (5)$$

4- Maintaining the amount of water in tank B:

The water supplied to power plant B+ the water drained from tank B so that there is no flooding + the level of tank B at the end of the first period must equal the amount of water supplied to tank B from power plant A (it is the same amount of water that was supplied to power plant A from the tank A) + the amount of water that must be drained from tank A so that a flood does not occur + the level of tank B at the beginning of the first period + the amount of flow into tank B in the first period, i.e.:

$$x_2 + x_4 + x_6 = x_1 + x_3 + NM_{B1} + T_{BN1} \Longrightarrow$$

$$x_2 + x_4 + x_6 - x_1 - x_3 = NM_{B1} + T_{BN1} \quad (6)$$

Since the minimum level allowed in this tank is NL_B and the maximum capacity of this tank is S_B, the level of tank Bat the end of the first period must be limited between the values NL_B and S_B, thus we obtain the following double restriction:

$$NL_B \leq x_6 \leq S_B \quad (7)$$

- **The study for the second period:**

Information about the power plants of the second period:

Table No. (1) Information about power plants A and B during the second period

Information \ power plants	power plant A	power plant B
Water supplied to the power plant during the second period	Unknown	Unknown
Maximum generating capacity during the second period	NP_A	NP_B
The power it can generate for a volume of water of 1 KAF	p_A	p_B

Information about tanks during the first period:

Table No. (2) Information about reservoirs A and B during the second period

Information \ Tanks	Tank A	Tank B
Maximum capacity	S_A	S_B
Minimum permissible level	NL_A	NL_B
The ratio at the beginning of the second period is the same as the ratio at the end of the first period	Unknown	Unknown
Excess water drained from the tank to prevent flooding	Unknown	Unknown
Flow during the second period	T_{AN2}	T_{BN2}
Reservoir level at the end of the second period	Unknown	Unknown

The first step regarding this issue during the second period of work, we find that the unknowns are:

1- The amount of power produced is divided into two parts: a quantity that is sold for C_{N1}\$ MWh, and a quantity that is sold for C_{N2}\$ MWh.
2- The amount of water that must be supplied to power plant A will be denoted y_1.
3- The amount of water supplied to power plant B will be denoted y_2.
4- The amount of water that must be drained from tank A so that flooding does not occur. We will denote it y_3.

5- The amount of water that must be drained from tank B so that flooding does not occur. We will denote it y_4.
6- Tank A level at the end of the second period. We will denote it y_5.
7- Tank B level at the end of the second period. We will denote it y_6.

The second step regarding this issue during the second period of work we find the following restrictions:

1- We assume that the amount of power sold during the first period at a price of C_{N1}\$ MWh, is Y_1 and the amount of power sold at a price of C_{N2}\$ MWh, is Y_2. The amount of power produced during the first period is:

$$Y = Y_1 + Y_2$$

Electrical power constraint: Since every 1 *KAF* generates electrical power of p_A *MWh*, in power plant A, when supplying this plant with the quantity y_1 *KAF*, the power produced during the second period of this plant will be $p_A\, y_1$. The same situation is for power plant B, the amount of power produced is $p_B y_2$, and therefore the amount of power produced during the second period of the two plants is:

$$p_A y_1 + p_B y_2$$

The amount of power produced during the second period must equal the amount of power sold during this period, meaning we get the restriction:

$$p_A y_1 + p_B y_2 = Y_1 + Y_2 \quad (8)$$

2- Constraint on the amount of water that must be supplied to power plant A: Since the maximum power that this plant can produce during the second period is NP_A *MWh*, and every 1 *KAF* of water in this plant produces p_A *MWh*, we get the following constraint:

$$\frac{NP_A}{p_A} \Rightarrow y_1 \leq \frac{NP_A}{p_A} \quad (9)$$

Restricting the amount of water that must be supplied to power plant B: Since the maximum power that this plant can produce during the second period is NP_B *MWh*, and every 1 *KAF* of water in this plant produces p_B *MWh*, we get the following restriction:

$$\frac{NP_B}{p_B} \Rightarrow y_2 \leq \frac{NP_B}{p_B} \quad (10)$$

3- Maintaining the amount of water in tank A:

The level of tank A at the beginning of the second period, plus the amount flowing into this tank, must equal the amount of water supplied to it Power factor A of tank A + the amount of water that must be drained so that there is no flooding + the tank level at the end the first period, i.e.:

$$x_5 + T_{A2} = y_1 + y_3 + y_5 \Rightarrow$$
$$y_1 + y_3 + y_5 - x_5 = T_{A2} \quad (11)$$

Since the minimum level allowed in this tank is NL_A and the maximum capacity of this tank is S_A, the level of tank A at the end of the first period must be limited between the values NL_A and S_A thus we obtain the following double restriction:

$$NL_A \leq y_5 \leq S_A \quad (12)$$

4- Maintaining the amount of water in tank B:

The water supplied to power plant B + the water drained from tank B so that there is no flooding + the level of tank B at the end of the second period must equal the amount of water supplied to tank B from power plant A (it is the same amount of water that was supplied to power plant A from the tank A) + the amount of water that must be drained from tank A so that a flood does not occur + the level of tank B

at the beginning of the second period + the amount of flow into tank B in the first period, i.e.:

$$y_2 + y_4 + y_6 = x_1 + y_3 + x_6 + T_{BN2} \Longrightarrow$$
$$y_2 + y_4 + y_6 - x_1 - y_3 - x_6 = T_{BN2} \quad (13)$$

Since the minimum level allowed in this tank is NL_B and the maximum capacity of this tank is S_B, the level tank B at the end of the second period must be confined between the values NL_B and S_B, and thus we obtain the double entry.

$$NL_B \le y_6 \le S_B \quad (14)$$

The third step: In the problem, determine the objective function relation:

From the data of the issue, we found that the department responsible for the workflow set two prices for selling the electrical power produced during the two periods, according to the quantity sold, and then they are:

The amount of power produced during the first period is given by the following relation:

$$X = X_1 + X_2$$

Where X_1 is the amount of power sold during this period at a price of $C_{N1}\$\ MWh$, and X_2 is the amount of power sold during this period is at a price of $C_{N2}\$\ MWh$, the amount of power produced during the second period is given by the following relation:

$$Y = Y_1 + Y_2$$

Where Y_1 is the amount of power sold during this period at a price of $C_{N1}\$\ MWh$, and Y_2 is the amount of power sold during this period is at a price of $C_{N2}\$\ MWh$, in the following figure we show the maximum revenue and the quantities sold of power corresponding to the price of $C_{N1}\$\ MWh$, and the smallest revenue that we can obtain from power and the quantities sold of power corresponding to the price of $C_{N2}\$\ MWh$.

From the previous figure, we notice that the objective function is a discrete linear function. It is linear in the fields $[0,50000]$ and $]50000, \infty[$. Therefore, the quantity of electricity sold can be divided into two parts: the part sold at a price of $C_{N1}\$/MWh$, and the part sold at a price: $C_{N2}\$/MWh$, and the following figure shows the quantities produced and the maximum revenue for power sold at a price of $C_{N1}\$$ and the quantities produced and the smallest revenue we can get from power sold at a price of $C_{N2}\$\ MWh$.

From the above, we can represent the objective function with a linear function as follows:

$$Z = C_{N1}(X_1 + Y_1) + C_{N2}(X_2 + Y_2) \quad (15)$$

Mathematical model:

Find:

$$Z = C_{N1}(X_1 + Y_1) + C_{N2}(X_2 + Y_2) \longrightarrow Max$$

Within the conditions:

$$p_A x_1 + p_B x_2 = X_1 + X_2$$
$$x_1 \le \frac{NP_A}{p_A}$$
$$x_2 \le \frac{NP_B}{p_B}$$
$$x_1 + x_3 + x_5 = NM_{A1} + T_{AN1}$$
$$y_5 \ge NL_A$$
$$y_5 \le S_A$$
$$x_2 + x_4 + x_6 - x_1 - x_3 = NM_{B1} + T_{BN1}$$

$$x_6 \leq S_B$$
$$x_6 \geq NL_B$$
$$p_A y_1 + p_B y_2 = Y_1 + Y_2$$
$$y_1 \leq \frac{NP_A}{p_A}$$
$$y_2 \leq \frac{NP_B}{p_B}$$
$$y_1 + y_3 + y_5 - x_5 = T_{AN2}$$
$$y_5 \leq S_A$$
$$y_5 \geq NL_A$$
$$y_2 + y_4 + y_6 - x_1 - y_3 - x_6 = T_{BN2}$$
$$y_6 \leq S_B$$
$$y_6 \geq NL_B$$
$$x_j \geq 0 \,, y_j \geq 0 \,; j = 1,2,\dots,6 \ \ and(X_1, X_2, Y_1, Y_2) \geq 0$$

- **Example text using neutrosophic values:**

  A department controls the operation of a system consisting of two water tanks, each equipped with a plant to generate electrical energy. This system works by sending a quantity of water to power plants to generate electrical energy. The experts provided the following information:

Information about the power plants during the two periods:

1- A volume of water of 1 *KAF*, can generate 400 *MWh*, of electricity in power plant A and 200 *MWh*, in power plant B.
2- The maximum power that can be generated in power plant A is $[50000,70000]$ *MWh*, and in power plant B the maximum power that can be generated is $[30000,40000]$ *MWh*.
3- An amount of electricity amounting to 50000 *MWh*, can be sold at a price of $\{15,20,25\}$\$ *MWh*, while the excess quantity is sold at a price of $\{10,14,15\}$\$ *MWh*.

Information about the tanks during the two periods:

1- The maximum capacity of reservoir A is 2000 *KAF* and the maximum capacity of reservoir B is 1500 *KAF*.
2- The level of reservoir A at the beginning of the first period is $[1700,1900]$ *KAF* and the level of reservoir B is $[650,850]$ *KAF*.
3- The minimum level allowed in tank A is $[1000,1200]$ *KAF* and in tank B is $[600,800]$ *KAF*.
4- The flow to tank A during the first period is $[150,250]$ *KAF* and to tank B is $[30,50]$ *KAF*.
5- The flow to tank A during the second period is $[80,180]$ *KAF* and to tank B is $[10,20]$ *KAF*.

It should be noted here:

- Power plant A is supplied from tank A and power plant B from tank B.
- When the tanks are completely full, some of the water is drained through the drainage channels so that flooding does not occur.
- For tank B, water is supplied from the following sources:
  - The aforementioned outflow.
  - From power plant A, the water that is supplied to the power plant after its use goes out to tank B.
  - Water drained from tank A so that flooding does not occur.

What is required is to build a mathematical model of the functioning of this system so that it achieves the maximum profit from the process of selling electricity, we know that to formulate a mathematical programming program, we follow the following three steps:

First step: We identify the unknowns in the problem (decision variables) and express them in algebraic symbols.

Second step: We define all constraints and express them with equations or inequalities that are mathematical functions of the unknown variables.

Step Three: We define the objective function and represent it as a linear function of the unknown variables. It should be made as large or as small as possible.

According to the text of the issue:

In order to obtain the required mathematical model, the issue must be studied according to the data for each period and the necessary relationships determined.

Here we find that a study must be presented for the first and second steps, specific to each period separately.

- **The study for the first period:**

Based on the previous information, we organize the following tables:

Information about the power plants of the first period:

Table No. (1) Information about power plants A and B during the first period		
power plants / Information	power plant A	power plant B
Water supplied to the power plant during the first period	Unknown	Unknown
Maximum generating capacity during the first period	[50000,70000]	[30000,40000]
The power it can generate for a volume of water of 1 KAF	400	200

Information about tanks during the first period:

Table No. (2) Information about reservoirs A and B during the first period		
Tanks / Information	Tank A	Tank B
Maximum capacity	2000	1500
Minimum permissible level	[1000,1200]	[600,800]
The ratio at the beginning of the first period	[1700,1900]	[650,850]
Excess water drained from the tank to prevent flooding	Unknown	Unknown
Flow during the first period	[150,250]	[30,50]
Reservoir level at the end of the first period	Unknown	Unknown

The first step regarding this issue during the first period of work, we find that the unknowns:

1- The amount of power produced is divided into two parts: a quantity that is sold for $\{15,20,25\}\$$ MWh, and a quantity that is sold for $\{10,14,15\}\$$ MWh.
2- The amount of water that must be supplied to power plant A will be denoted x_1.
3- The amount of water supplied to power plant B will be denoted x_2.

4- The amount of water that must be drained from tank A so that flooding does not occur. We will denote it x_3.
5- The amount of water that must be drained from tank B so that flooding does not occur. We will denote it x_4.
6- Tank A level at the end of the first period. We will denote it x_5.
7- Tank B level at the end of the first period. We will denote it x_6.

The second step regarding this issue during the first period of work we find the following restrictions:

1- We assume that the amount of power sold during the first period at a price of $\{15,20,25\}\$$ *MWh*, is X_1 and the amount of power sold at a price of $\{10,14,15\}\$$ *MWh*, is X_2. The amount of power produced during the first period is:

$$X = X_1 + X_2$$

Electrical power constraint: Since every 1 *KAF* generates electrical power of 400 *MWh*, in power plant A when supplying this plant with the quantity x_1*KAF*, the power produced during the first period of this plant will be $400\,x_1$ .The same situation applies to power plant B, the amount of power produced is $200x_2$, and therefore the amount of power produced during the first period of the two plants is:

$$400x_1 + 200x_2$$

The amount of power produced during the first period must equal the amount of power sold during this period, meaning we get the following restriction:

$$400x_1 + 200x_2 = X_1 + X_2 \quad (1)$$

2- Constraint on the amount of water that must be supplied to Power Plant A: Since the maximum power that this plant can produce during the first period is $[50000,70000]$ *MWh*, and every 1 *KAF* of water in this plant produces 400 *MWh*, we get the following constraint:

$$\frac{[50000,70000]}{400} \Rightarrow x_1 \leq [125,175] \quad (2)$$

Restricting the amount of water that must be supplied to power plant B: Since the maximum power that this plant can produce during the first period is $[30000,40000]$ *MWh*, and every 1 *KAF* of water in this plant produces 200 *MWh*, we get the following restriction:

$$\frac{[30000,40000]}{200} \Rightarrow x_2 \leq [150,200] \quad (3)$$

3- Maintaining the amount of water in tank A**:**

The level of tank A at the beginning of the first period, plus the amount of flow into this tank, must equal the amount of water supplied to power plant A from tank A + the amount of water that must be drained so that a flood does not occur + the tank level at the end of the first period, i.e.:

$$[50000,70000] + [150,250] = x_1 + x_3 + x_5 \Rightarrow$$
$$x_1 + x_3 + x_5 = [50150,70250] \quad (4)$$

Since the minimum level allowed in this tank is $[1000,1200]$ and the maximum capacity of this tank is 2000, the level of tank A at the end of the first period must be limited between the values $[1000,1200]$ and 2000 and thus we obtain the following double restriction:

$$[1000,1200] \leq x_5 \leq 2000 \quad (5)$$

4- Maintaining the amount of water in tank B:

The water supplied to power plant B+ the water drained from tank B so that there is no flooding + the level of tank B at the end of the first period must equal the amount of water supplied to tank B from power plant A (it is the same amount of water that was supplied to power plant A from the tank A) + the amount of water that must be drained from tank A so that a flood does not occur + the level of tank B at the beginning of the first period + the amount of flow into tank B in the first period, i.e.:

$$x_2 + x_4 + x_6 = x_1 + x_3 + [650,850] + [30,50] \Longrightarrow$$
$$x_2 + x_4 + x_6 - x_1 - x_3 = [680,900] \quad (6)$$

Since the minimum level allowed in this tank is $[600,800]$ and the maximum capacity of this tank is 1500, the level of tank Bat the end of the first period must be limited between the values $[600,800]$ and 1500, and thus we obtain the following double restriction:

$$[600,800] \le x_6 \le 1500 \quad (7)$$

- **The study for the second period:**

Information about the power plants of the second period:

Table No. (1) Information about power plants A and B during the second period		
power plants / Information	power plant A	power plant B
Water supplied to the power plant during the second period	Unknown	Unknown
Maximum generating capacity during the second period	[50000,70000]	[30000,40000]
The power it can generate for a volume of water of 1 KAF	400	200

Information about tanks during the first period:

Table No. (2) Information about reservoirs A and B during the second period		
Tanks / Information	Tank A	Tank B
Maximum capacity	2000	1500
Minimum permissible level	[1000,1200]	[600,800]
The ratio at the beginning of the second period is the same as the ratio at the end of the first period	Unknown	Unknown
Excess water drained from the tank to prevent flooding	Unknown	Unknown
Flow during the second period	[80,180]	[10,20]
Reservoir level at the end of the second period	Unknown	Unknown

The first step regarding this issue during the second period of work, we find that the unknowns:

1- The amount of power produced is divided into two parts: a quantity that is sold for $\{15,20,25\}\$ MWh$, and a quantity that is sold for $\{10,14,15\}\$ MWh$.
2- The amount of water that must be supplied to power plant A will be denoted y_1.
3- The amount of water supplied to power plant B will be denoted y_2.
4- The amount of water that must be drained from tank A so that flooding does not occur. We will denote it y_3.

5- The amount of water that must be drained from tank B so that flooding does not occur. We will denote it y_4.
6- Tank A level at the end of the second period. We will denote it y_5.
7- Tank B level at the end of the second period. We will denote it y_6.

The second step regarding this issue during the second period of work we find the following restrictions:

1- We assume that the amount of power sold during the first period at a price of $\{15,20,25\}\$$ MWh, is Y_1 and the amount of power sold at a price of $\{10,14,15\}\$MWh$, is Y_2. The amount of power produced during the first period is:

$$Y = Y_1 + Y_2$$

Electrical power constraint: Since every 1 KAF, generates electrical power of 400 MWh, in power plant A, when supplying this plant with the quantity y_1 KAF, the power produced during the second period of this plant will be $400\ y_1$. The same situation is for power plant B, the amount of power produced is $200y_2$, and therefore the amount of power produced during the second period of the two plants is:

$$400y_1 + 200y_2$$

The amount of power produced during the second period must equal the amount of power sold during this period, meaning we get the restriction:

$$400y_1 + 200y_2 = Y_1 + Y_2 \quad (8)$$

2- Constraint on the amount of water that must be supplied to power Plant A: Since the maximum power that this plant can produce during the second period is $[50000,70000]MWh$, and every 1 KAF of water in this plant produces 400 MWh, we get the following constraint:

$$\frac{[50000,70000]}{400} \Rightarrow y_1 \leq\ [125,175]\ (9)$$

Restricting the amount of water that must be supplied to power plant B: Since the maximum power that this plant can produce during the second period is $[30000,40000]$ MWh, and every 1 KAF of water in this plant produces 200 MWh, we get the following restriction:

$$\frac{[30000,40000]}{200} \Rightarrow y_2 \leq [150,200] \quad (10)$$

3- Maintaining the amount of water in tank A:

The level of tank A at the beginning of the second period, plus the amount flowing into this tank, must equal the amount of water supplied to it Power factor A of tank A + the amount of water that must be drained so that there is no flooding + the tank level at the end the first period, i.e.:

$$x_5 + [150,250] = y_1 + y_3 + y_5 \ \Rightarrow$$
$$y_1 + y_3 +\ y_5 - x_5 = [150,250] \quad (11)$$

Since the minimum level allowed in this tank is $[1000,1200]$ and the maximum capacity of this tank is 2000, the level of tank A at the end of the first period must be limited between the values $[1000,1200]$ and 2000 and thus we obtain the following double restriction:

$$[1000,1200] \leq y_5 \leq 2000 \quad (12)$$

4- Maintaining the amount of water in tank B:

The water supplied to power plant B + the water drained from tank B so that there is no flooding + the level of tank B at the end of the second period must equal the amount of water supplied to tank B from power plant A (it is the same amount of water that was supplied to power plant A from the tank A) + the amount of water that must be drained from tank A so that a flood does not occur + the level of tank B at the beginning of the second period + the amount of flow into tank B in the first period, i.e.:

$$y_2 + y_4 + y_6 = x_1 + y_3 + x_6 + [30,50] \Longrightarrow$$
$$y_2 + y_4 + y_6 - x_1 - y_3 - x_6 = [30,50] \quad (13)$$

Since the minimum level allowed in this tank is $[600,800]$ and the maximum capacity of this tank is 1500, the level tank B at the end of the second period must be confined between the values$[600,800]$and 1500, and thus we obtain the double entry.

$$[600,800] \leq y_6 \leq 1500 \quad (14)$$

The third step: In the problem, determine the objective function relation:

From the data of the issue, we found that the department responsible for the workflow set two prices for selling the electrical power produced during the two periods, according to the quantity sold, and then they are:

The amount of power produced during the first period is given by the following relation:

$$X = X_1 + X_2$$

Where X_1 is the amount of power sold during this period at a price of $\{15,20,25\}$\$ *MWh*, and X_2 is the amount of power sold during this period is at a price of $\{10,14,15\}$\$ *MWh*, the amount of power produced during the second period is given by the following relation:

$$Y = Y_1 + Y_2$$

Where Y_1 is the amount of power sold during this period at a price of $\{15,20,25\}$\$ *MWh*, and Y_2 is the amount of power sold during this period is at a price of $\{10,14,15\}$\$ *MWh*, in the following figure we show the maximum revenue and the quantities sold of power corresponding to the price of $\{15,20,25\}$\$ *MWh*, and the smallest revenue that we can obtain from power and the quantities sold of power corresponding to the price of $\{10,14,15\}$\$ *MWh*.

From the previous figure, we notice that the objective function is a discrete linear function. It is linear in the fields $[0,50000]$ and $]50000,\infty[$. Therefore, the quantity of electricity sold can be divided into two parts: the part sold at a price of $\{15,20,25\}$\$/ *MWh*, and the part sold at a price: $\{10,14,15\}$\$/ *MWh*, and the following figure shows the quantities produced and the maximum revenue for power sold at a price of $\{15,20,25\}$\$ and the quantities produced and the smallest revenue we can get from power sold at a price of $\{10,14,15\}$\$.

From the above, we can represent the objective function with a linear function as follows:

$$Z = \{15,20,25\}(X_1 + Y_1) + \{10,14,15\}(X_2 + Y_2) \quad (15)$$

Mathematical model:

Find:

$$Z = \{15,20,25\}(X_1 + Y_1) + \{10,14,15\}(X_2 + Y_2) \longrightarrow Max$$

Within the conditions:

$$400x_1 + 200x_2 = X_1 + X_2$$
$$x_1 \leq [125,175]$$
$$x_2 \leq [150,200]$$
$$x_1 + x_3 + x_5 = [50150,70250]$$
$$y_5 \leq 2000$$

$$y_5 \geq [1000,1200]$$
$$x_2 + x_4 + x_6 - x_1 - x_3 = [680,900]$$
$$x_6 \leq 1500$$
$$x_6 \geq [600,800]$$
$$200y_1 + 400y_2 = Y_1 + Y_2$$
$$y_1 \leq [125,175]$$
$$y_2 \leq [150,200]$$
$$y_1 + y_3 + y_5 - x_5 = [150,250]$$
$$y_5 \leq 2000$$
$$y_5 \geq [1000,1200]$$
$$y_2 + y_4 + y_6 - x_1 - y_3 - x_6 = [30,50]$$
$$y_6 \leq 1500$$
$$y_6 \geq [600,800]$$
$$x_j \geq 0\,, y_j \geq 0\,; j = 1,2,\dots,6 \;\; and(X_1, X_2, Y_1, Y_2) \geq 0$$

It is a linear neutrosophic model. The direct simplex neutrosophic algorithm and its modifications can be used to find the optimal solution through which we achieve the maximum profit, and it takes into account all the conditions that the system's working environment may experience, through the indeterminacy present in the neutrosophic values that were taken for data that is subject to change due to factors and conditions that can occur. To go through the work environment.

Conclusion and results:

In this research, we presented a study of the issue of planning hydroelectric systems. From the information contained in reference [1], we reformulated this issue in an expanded way using classical values. Since some of the data in this issue are affected by natural or other factors, we found that the solution we can get from While solving the linear model, there may be an inaccurate solution that does not fit with the conditions that the system's operating environment may experience. Therefore, we presented a complex formula for the problem of planning hydroelectric systems using the concepts of neutrosophic logic, and we obtained a neutrosophic linear model. Special neutrosophic algorithms can be used to solve the linear models that were presented. In previous research to obtain the optimal neutrosophic solution suitable for all conditions

References:

1- Bukajh J.S -. Mualla, W… and others – Operations Research Book translated into Arabic – The Arab Center for Arabization, Translation, Authoring and Publishing -Damascus -1998.

2- Florentin Smarandache , Maissam Jdid, On Overview of Neutrosophic and Plithogenic Theories and Applications, Prospects for Applied Mathematics and Data Analysis, Volume 2 , Issue 1, PP: 19-26 , 2023 ,Doi :https://doi.org/10.54216/PAMDA.020102

3- Smarandache, F., Introduction to Neutrosophic statistics, Sitech & Education Publishing, 2014.

4- Maissam Jdid , Florentin Smarandache, The Box and Muller Technique for Generating Neutrosophic Random Variables Follow a Normal Distribution, International Journal of Neutrosophic Science, Volume 23 , Issue 4, PP: 83-87 , 2024, Doi :https://doi.org/10.54216/IJNS.230406

5- Maissam Jdid, Florentin Smarandache, Double Indeterminacy - Neutrosophic study of an Approximation Techniques Used to Find Random Variables Prospects for Applied Mathematics and Data Analysis, Volume 3 , Issue 1, PP: 21-28 , 2023,Doi :https://doi.org/10.54216/PAMDA.030102

6- Mohammad Abobala, M. Bisher Ziena, R. Iqbal Doewes, Zahraa Hussein, The Representation of Refined Neutrosophic Matrices By Refined Neutrosophic Linear Functions, Doi:https://doi.org/10.54216/IJNS.190131

7- Adeeb Alhebri, Safeguarding Financial Integrity with Interval-Valued Neutrosophic Analytic Hierarchy Process for Sustainable Accounting Systems, Doi:https://doi.org/10.54216/IJNS.230426
8- Şule Bayazit Bedirhanoğlu, & Mahmut Atlas. (2024). Production Planning with The Neutrosophic Fuzzy Multi-Objective Optimization Technique. Neutrosophic Sets and Systems, 65, 119-150. https://fs.unm.edu/nss8/index.php/111/article/view/4301
9- S. Loganayaki, N. Rajeswari, K. Kirupa, & S. Broumi. (2024). Optimization of triangular neutrosophic based economic order quantity model under preservation technology and power demand with shortages. Neutrosophic Sets and Systems, 65, 11-31. https://fs.unm.edu/nss8/index.php/111/article/view/4292
10- Maissam Jdid, Florentin Smarandache Converting Some Zero-One Neutrosophic Nonlinear Programming Problems into Zero-One Neutrosophic Linear Programming Problems, Neutrosophic Optimization and Intelligent systems, Vol. 1 (2024) 39-45, DOI: https://doi.org/10.61356/j.nois.2024.17489 , https://sciencesforce.com/index.php/nois/article/view/74/67
11- Maissam Jdid, & Florentin Smarandache. (2024). Neutrosophic Vision of the Expected Opportunity Loss Criterion (NEOL) Decision Making Under Risk. Neutrosophic Sets and Systems, 65, 110-118. https://fs.unm.edu/nss8/index.php/111/article/view/4300
12- Manas Karak, Pramodh Bharati, Animesh Mahata, Subrata Paul, Santosh Biswas, Supriya Mukherjee, Said Broumi, Mahendra Rong, & Banamali Roy. (2024). Ranking of Neutrosophic number based on values and ambiguities and its application to linear programming problem. Neutrosophic Sets and Systems, 63, 203-218. https://fs.unm.edu/nss8/index.php/111/article/view/3886
13- Maissam Jdid , Florentin Smarandache, A Study of Systems of Neutrosophic Linear Equations, International Journal of Neutrosophic Science, Volume 23 , Issue 2, PP: 16-25 , 2024, Doi :https://doi.org/10.54216/IJNS.230202
14- Maissam Jdid, Huda E. Khalid, Mysterious Neutrosophic Linear Models, International Journal of Neutrosophic Science, Volume 18 , Issue 2, PP: 243-253 , 2022,Doi :https://doi.org/10.54216/IJNS.180207
15- Maissam Jdid, Some Important Theories about Duality and the Economic Interpretation of Neutrosophic Linear Models and Their Dual Models, Prospects for Applied Mathematics and Data Analysis, Volume 2 , Issue 2, PP: 27-33 , 2023 Doi:https://doi.org/10.54216/PAMDA.020203
16- Maissam Jdid, Florentin Smarandache, Neutrosophic Treatment of Duality Linear Models and the Binary Simplex Algorithm, Prospects for Applied Mathematics and Data Analysis, Volume 2 , Issue 2, PP: 08-21 , 2023,Doi :https://doi.org/10.54216/PAMDA.020202
17- Maissam Jdid, & Florentin Smarandache. (2023). The Graphical Method for Finding the Optimal Solution for Neutrosophic linear Models and Taking Advantage of Non-Negativity Constraints to Find the Optimal Solution for Some Neutrosophic linear Models in Which the Number of Unknowns is More than Three. Neutrosophic Sets and Systems, 58, 68-80. http://fs.unm.edu/nss8/index.php/111/article/view/3532
18- Maissam Jdid, A. A. Salama, Huda E. Khalid, Neutrosophic Handling of the Simplex Direct Algorithm to Define the Optimal Solution in Linear Programming, International Journal of Neutrosophic Science, Volume 18 , Issue 1, PP: 30-41 , 2022,Doi :https://doi.org/10.54216/IJNS.180104
19- Maissam Jdid, Florentin Smarandache, Neutrosophic Treatment of the Modified Simplex Algorithm to find the Optimal Solution for Linear Models, International Journal of Neutrosophic Science, Volume 23 , Issue 1, PP: 117-124 , 2024 Doi :https://doi.org/10.54216/IJNS.230110
20- Jdid, M., & Smarandache, F. (2024). Finding a Basic Feasible Solution for Neutrosophic Linear Programming Models: Case Studies, Analysis, and Improvements. Neutrosophic Systems with Applications, 14, 30–37. https://doi.org/10.61356/j.nswa.2024.130

21- Maissam Jdid, Neutrosophic Nonlinear Models, Prospects for Applied Mathematics and Data Analysis, Volume 2 , Issue 1, PP: 42-46 , 2023, Doi :https://doi.org/10.54216/PAMDA.020104
22- Maissam Jdid and Florentin Smarandache, Graphical Method for Solving Neutrosophical Nonlinear Programming Models, / Int.J.Data.Sci. & Big Data Anal. 3(2) (2023) 66-72 https://dx.doi.org/10.51483/IJDSBDA.3.2.2023.66-72
23- Maissam Jdid, & Florentin Smarandache. (2023). Lagrange Multipliers and Neutrosophic Nonlinear Programming Problems Constrained by Equality Constraints. Neutrosophic Systems with Applications, 6, 25–31. https://doi.org/10.61356/j.nswa.2023.35

Received: Jan 10, 2024. Accepted: Mar 30, 2024

NSS

Neutrosophic Sets and Systems, Vol. 66, 2024

University of New Mexico

Exploring Neutrosophic Linear Programming in Advanced Fuzzy Contexts

Shubham Kumar Tripathi[1], Arindam Dey[1], Said Broumi[1] and Ranjan, Kumar[1,*]

[1]VIT-AP University, Inavolu, Beside AP Secretariat, Amaravati AP, India

[2]Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Morocco.

*Correspondence: ranjank.nit52@gmail.com

Abstract. A neutrosophic set is a mathematical framework that extends fuzzy and intuitionistic sets to handle indeterminate or contradictory information using three components: truth, falsity, and indeterminacy - membership degrees which deal with handling indeterminate, imprecise, and uncertain data, while Extended Fuzzy Theory extends the standard fuzzy set theory to manage more intricate membership degrees. The primary objective of this review is to thoroughly investigate and summarize the existing literature on trapezoidal neutrosophic environments, particularly focusing on aspects such as the NFMOLP Problems in SVTpN environments. Ultimately, this comprehensive review article aims to enhance the understanding of the potential of these integrated methodologies for effectively presenting decision-making amidst complex and uncertain conditions.

Keywords: Fuzzy Linear Programming problems; Linear Programming problems; Uncertainty principle; Membership function; Single valued trapezoidal neutrosophic numbers; NFMOLP- problems

---

—-

1. **Introduction**

Linear programming (LP) problem is a powerful mathematical technique in optimization employed to address problems where the objective is to maximize or minimize a linear function while adhering to a set of linear constraints. In linear programming, the decision maker first identifies the decision variables, objective function, and constraints. Various methods find the feasible solution of LPP using the various methods simplex method [1], dual simplex [2], graphical method [3] and so on which satisfy the constraints and optimize the objective value.

The area of linear programming is very broad. The applications of LPP contain various areas such as Production planning [4], transport services [5], water management [6] and so on. Over the years researchers have worked in these areas such as using a modified dual simplex method Patil et al. [7] developed an android application for cattle feed and Zhou et al. [8] introduced the SMSS optimization algorithm for a data-clustering problem, demonstrating its potential and effectiveness with eleven benchmark datasets. Chen et al. [9] presented a parameter identification approach for photovoltaic models using a hybrid adaptive Nelder-Mead simplex algorithm based on eagle strategy. Furthermore, Liero et al. [10] developed a comprehensive theory for the novel class of Optimal Entropy-Transport problems, focusing on nonnegative and finite Radon measures in general topological spaces. In another study, Tang et al. [11] proposed a novel deep learning-based algorithm-based partial channel assignment algorithm to intelligently allocate channels to each link in the SDN-IoT network.

Our intention is that this comprehensive literature review will be very useful for students or researchers to understand both Fuzzy and trapezoidal neutrosophic environments through a single review paper. After reviewing the introduction, we found that there is a significant gap in the study of FLPP. Therefore, we have presented some important methodologies and applications in a unified manner, considering different aspects of uncertainty. As a result, we are compelled to explore the application of the Trapezoidal neutrosophic environments to LPP and provide an updated analysis of applications and methodologies to discourse these gaps. The summary of the trapezoidal neutrosophic environments will support researchers in promoting and discovering improvements and progress in the field of Linear Programming Problems (LPP). Through this comprehensive review paper, we attempt to support students and researchers in developing a profound understanding of Neutrosophic, LR Fuzzy, Intuitionistic, and Hesitant fuzzy models. Ultimately, our work aims to contribute to fostering progress in this area. .

The paper is organized as follows. Section 2 discuss difficulties in the classical linear programming model and Section 3 define the key concepts and definitions of fuzzy theory. In Section 4 introduce the FLPP then after in section 5 discuss some important definitions related to Fuzzy extended theory and Section 6 short literature review on LPP under the neutrosophic principle. Finally in Sction 7 we conclude the review.

1.1. *List of Abbreviations used throughout this paper.*

- **FLP:** stands for ”Fuzzy Linear Programming”
- **ITpNNs:** stands for ”Interval trapezoidal neutrosophic numbers”
- **IFS:** stands for ”Intuitionistic fuzzy Set”
- **IFNs:** stands for ”Intuitionistic fuzzy numbers”

- **TpFn:** stands for ”Trapezoidal Fuzzy Number”
- **SVTpNNs:** stands for ”Single-valued trapezoidal neutrosophic numbers”
- **GTpFn:** stands for ”General Trapezoidal Fuzzy Number”
- **WTpFn:** stands for ”Weighted Trapezoidal Fuzzy Number”
- **TpIFN:** stands for ”Trapezoidal Intuitionistic Fuzzy Number ”
- **MADM:** stands for ”Multiple Attribute Decision Making”
- **NTpLWAA:** stands for ”Neutrosophic trapezoid linguistic weighted arithmetic averaging aggregation”
- **NTpLWGA:** stands for ”Neutrosophic trapezoid linguistic weighted geometric aggregation.”
- **R-NWFS:** stands for ”Robust no-wait flow shop scheduling”
- **NFMOLP:** stands for ”Neutrosophic Fuzzy Multi-Objective Linear Programming”
- **SMSS:** stands for ”Simplex method-based social spider”
- **MOLP:** stands for “Multi-Objective Linear Programming”

2. Discussion Difficulties in The Classical Linear Programming Model

Classical Linear Programming (LP) have several demerits that can be addressed using fuzzy logic. Firstly, classical LP assumes precise and deterministic input data, which can lead to unrealistic and rigid solutions in the face of uncertainties and imprecisions in real-world problems. Fuzzy logic, on the other hand, allows for the representation of vagueness and uncertainty in the input data, enabling a more robust and flexible approach to problem-solving. Secondly, classical LP may struggle to handle subjective and qualitative factors adequately, while fuzzy logic can incorporate linguistic variables and expert opinions through fuzzy set theory, providing a more human-like decision-making framework. Additionally, classical LP may not handle imprecise or ambiguous constraints effectively, leading to suboptimal solutions, whereas fuzzy logic can model and manage fuzzy constraints more naturally, resulting in more meaningful and context-sensitive outcomes. In summary, fuzzy logic addresses the demerits of classical LP by embracing uncertainty, imprecision, and subjectivity, making it a valuable tool for handling complex real-world problems. In specific decision-making situations where multiple factors come into play, some of which may be difficult to quantify precisely, fuzzy logic presents a highly flexible and subtle approach when contrasted with an LP (Linear Programming) model. The versatility of fuzzy logic lies in its capability to encompass a wider array of factors and integrate subjective or qualitative inputs. As a result, fuzzy logic has demonstrated its superiority over Linear Programming models. The pioneering work introducing fuzzy logic was accomplished by Lotfi A. Zadeh [12] in 1965.

3. Fuzzy Theory: Key Concepts and Definitions

Fuzzy set:

Zadeh [12] introduced the thought of fuzzy logic in 1965 to address systems with ill-defined, vague, or incomplete information. If U is a collection of elements denoted by u, then a fuzzy set $\widetilde{F}$ in U is a set of ordered pairs: $\widetilde{F} = \left\{\left(u, \mu_{\widetilde{F}}(u)\right) : u \in U\right\}$ Where $\mu_{\widetilde{F}}(u) : U \rightarrow [0,1]$is called grade of membership or the membership function or degree of compatibility or degree of truth of u in $\widetilde{F}$.

Notably, there are different types of fuzzy membership functions, including Triangular and Trapezoidal fuzzy functions, which are defined as below: -

Triangular fuzzy Numbers [13]:

A fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$, defined on the universal set as a set of real numbers is said to be a Triangular fuzzy number if its grade of membership function, $\mu_{\widetilde{M}}(n)$is follow the below condition:

(1) $\mu_{\widetilde{M}}(n)$ a strictly increasing and continuous function between the intervals $[\ a_1, a_2]$.

(2) $\mu_{\widetilde{M}}(n)$ a strictly decreasing and continuous function between the intervals $[\ a_2, a_3]$.

(3) $\mu_{\widetilde{M}}(n)$, a continuous function under the interval $[\ 0,1]$.

And given by

$$\mu_{\widetilde{M}}(n) = \left\{\begin{array}{cc} \frac{n-a_1}{a_2-a_{\ 1}} & \text{for } a_1 \leq n \leq a_2 \\ \frac{a_3-n}{a_3-a_2} & \text{for } a_2 \leq n \leq a_3 \\ 0 & \text{othewise} \end{array}\right\}$$

Trapezoidal Fuzzy Number (TpFN) [14]:

A TpFN can be written as $\widetilde{Tp_f} = \langle m_1, m_2, m_3, m_4 \rangle$ whose membership function $\mu_{\widetilde{Tp_f}}$ as follows:

$$\mu_{\widetilde{Tp_f}}(t) = \begin{cases} \frac{t-m_1}{m_2-m_1}, & m_1 \leq t \leq m_2; \\ 1, & m_2 \leq t \leq m_3; \\ \frac{m_4-t}{tf_4-tf_3}, & m_3 \leq t \leq m_4; \\ 0, & Otherwise \end{cases}$$

$$where \quad m_1, m_2, m_3, m_4 \in \mathbb{R}$$

Weighted Trapezoidal Fuzzy Number [15]:

Let set $\tilde{S}_p = \left(\tilde{s}_1^p, \tilde{s}_2^p, \tilde{s}_3^p, \tilde{s}_4^p : \omega_l^p, \omega_r^p\right)$ (where $\tilde{s}_1^p, \tilde{s}_2^p, \tilde{s}_3^p$and $\tilde{s}_4^p$ real number with $\tilde{s}_1^p \leq \tilde{s}_2^p \leq \tilde{s}_3^p \leq \tilde{s}_4^p$, $\omega_r^p, \omega_l^p$the right height and the left height of $\tilde{S}_p$) is GTpFn and the grade of membership function is define as :

$$\mu_{\tilde{S}_p}(t) = \left\{\begin{array}{cc} 0 & \text{for t}\tilde{s}_1^p or \text{ t}\tilde{s}_4^p \\ \omega_l^p \cdot \frac{t-\tilde{s}_1^p}{\tilde{s}_2^p-\tilde{s}_1^p} & \text{for } \tilde{s}_1^p \leq t \leq \tilde{s}_2^p \\ \omega_l^p + \left(\omega_l^p - \omega_l^p\right) \frac{t-\tilde{s}_2^p}{\tilde{s}_3^p-\tilde{s}_2^p} & \text{for } \tilde{s}_2^p \leq t \leq \tilde{s}_3^p \\ \omega_r^p \cdot \frac{t-\tilde{s}_4^p}{\tilde{s}_3^p-\tilde{s}_4^p} & \text{for } \tilde{s}_3^p \leq t \leq \tilde{s}_4^p \end{array}\right\}$$

Where $0 \prec \omega_l^p \leq 1$ and $0 \prec \omega_r^p \leq 1$; If $\omega_l^p = \omega_r^p$ then $\tilde{S}_p$become $\tilde{S}_p = (\tilde{s}_1^p, \tilde{s}_2^p, \tilde{s}_3^p, \tilde{s}_4^p : \omega)$ also known as WTpFn.

LR flat Trapezoidal Fuzzy Number [16]:

Let $\tilde{\varphi} = (\tilde{\varphi}_1, \tilde{\varphi}_2, \tilde{\varphi}_l, \tilde{\varphi}_r)_{LR}$ is said L-R flat TpFn iff the grade of membership function $\mu_{\tilde{\varphi}}(t)$ is defined by-

$$\mu_{\tilde{\varphi}}(t) = \left\{ \begin{array}{ll} L\left(\frac{\tilde{\varphi}_1 - t}{\tilde{\varphi}_l}\right) & \text{For } t \leq \tilde{\varphi}_1 \text{ and } \tilde{\varphi}_l > 0 \\ R\left(\frac{t - \tilde{\varphi}_2}{\tilde{\varphi}_r}\right) & \text{For } t \geq \tilde{\varphi}_2 \text{ and } \tilde{\varphi}_r > 0 \\ 1 & \text{For } \tilde{\varphi}_1 \leq t \leq \tilde{\varphi}_2 \end{array} \right\}$$

Hence, Fuzzy Logic presents a broader range of considerations and holds vagueness and uncertainty during the optimization solution, rendering it a more advantageous alternative to LP Models in certain scenarios. As demonstrated in Section 2, conventional LP struggles to adequately address uncertainty, leading to decreased accuracy in identifying optimal solutions. To overcome this limitation, Zimmermann proposed fuzzy Linear Programming Problems, which we will thoroughly examine in Section 4.

4. **Introduction of FLPP**

Fuzzy linear programming (FLP) is a prevailing mathematical framework that extends classical linear programming by including uncertainties and imprecisions through fuzzy logic. The FLP model, first introduced by Zimmermann [17], provides a powerful approach to tackle Linear Programming (LP) problems by integrating fuzzy linear constraints and keeping track of the evolution of related research and advancements. The goal of FLPP is to attain an optimal solution for a given objective function while adhering to a fuzzy set of constraints. In real-life situations, uncertainties are universal, making classical linear programming insufficient for accurately representing complex problems but FLP addresses this limitation by utilizing fuzzy sets and fuzzy numbers to model imprecise data, enabling decision-makers to optimize systems and resources under uncertain conditions. Its significance lies in various practical domains, such as supply chain management [18], financial planning [19], construction projects [20], environmental management [21], and so on. This paper explores various real-world applications to showcase the practical relevance and efficacy of FLPP in handling imprecision and uncertainty. By embracing the inherent fuzziness of real-world problems, FLPP equips decision-makers with a powerful tool to address intricate optimization challenges effectively.

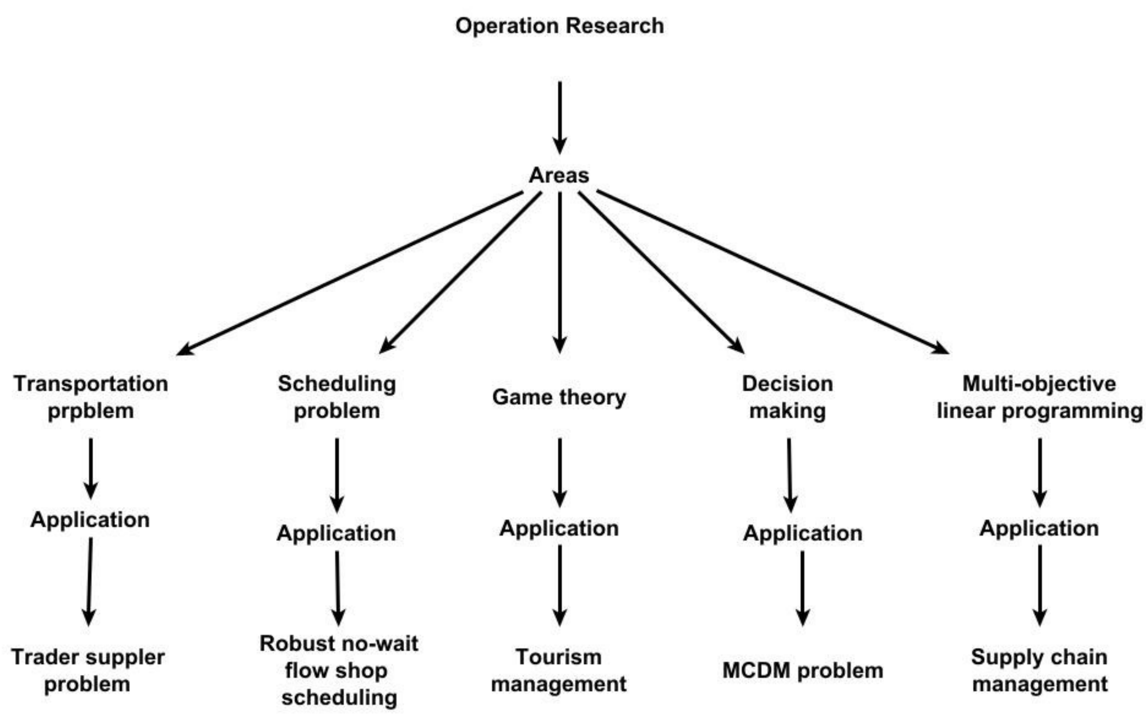
Table 1. Researchers from diverse domains have extensively investigated the real-life applications of FLPP, revealing its profound impact.

Authors	**Year**	**Environment**	**Application**	**Significance**
Ebrahimnejad [22]	2016	Transportation prpblem	Trader suppler problem	The author proposed a novel approach to address the transportation problem by incorporating interval-valued trapezoidal fuzzy numbers to represent transportation costs, supplies, and demands and solve it.
Sun et al. [23]	2021	Scheduling problem	To solve the robust no-wait flow shop scheduling	This article aims to investigate the R-NWFS problem with interval-valued fuzzy processing times, with the objective of minimizing the make span while adhering to an upper bound on the total completion time.
Bhaumik and Roy [24]	2021	Game theory	Tourism management	The primary objective of this article is to devise and examine a matrix game with multiple objectives. The focus lies in resolving the problem within a single-valued neutrosophic environment, utilizing a linguistic approach.
Deli and Karaaslan [25]	2021	Decision making	MCDM problem	Using generalized hesitant trapezoidal fuzzy numbers, which express membership degrees through multiple possible trapezoidal fuzzy numbers, proves to be a more appropriate and effective approach for solving real-life MCDM problems compared to real life problem.

Continued on next page

Table 1 – Continued the Literature survey

Authors	Year	Environment	Application	Significance
Hassanpour et al. [26]	2023	Multi-objective linear programming	Supply chain management	This paper aims to address a challenging problem of intuitionistic fuzzy multi-objective linear programming, where objective functions and constraints involve intuitionistic fuzzy parameters and using linear ranking function to transform the intuitionistic fuzzy parameters into a crisp problem.



Different subfield of operation research and applications

Throughout our continuous search, we have widely scrutinized the distinct attributes of fuzzy concept. However, it is essential to acknowledge the existence of particular challenges associated with this approach, which necessitate a comprehensive discussion in the subsequent section.

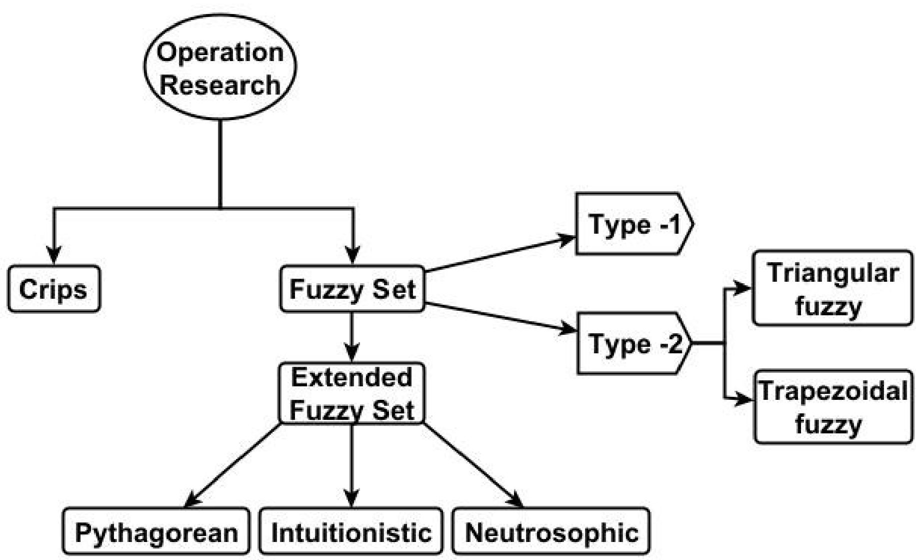
4.1. *Some Challenges of The Fuzzy Linear Programming:*

FLP play important role in decision making problem but Fuzzy Linear Programming (FLP) poses several challenges due to the inherent uncertainty and imprecision associated with fuzzy logic. Additionally, solving problems in fuzzy logic can be computationally intensive, especially when dealing with non-linear functions. As the number of variables and constraints increases, finding optimal solutions becomes more difficult, which can cause scalability issues. It is evident from the above literature that fuzzy theory alone is not sufficient to address uncertainty effectively. Therefore, researchers have developed several theories to overcome these challenges. These theories include Intuitionistic theory (1983) by Atanassov, neutrosophic theory (1990) by Samarandche, Pythagorean theory (2013) by Yager, so on.

5. **Some Important Definitions, Introduction Related to Fuzzy Extended Theory**

The Extended Fuzzy Principle starts by providing an extensive overview of the fundamental concepts and principles underlying fuzzy logic. The key growths in the extended fuzzy set include the following types:

- Intuitionistic
- Neutrosophic
- Pythagorean and so on.



Different types of fuzzy extended principle

Intuitionistic Fuzzy:

Intuitionistic fuzzy are part of extension Fuzzy sets that offer more information about the degree of ambiguity through non- grade of membership. Intuitionistic fuzzy first introduced by Atanassov [27] in 1986 to deal with uncertainty using non- grade of membership and grade of membership functions. The key definition and concept of intuitionistic is discuss below:

Intuitionistic fuzzy Set [27]

A set$\tilde{\theta}_{IF}$ on H is defined as $\tilde{\theta}_{IF} = \{(h, [(\rho(h), \upsilon(h))]) : h \in H\}$ Where $\rho_{IF}(h) : H \to [0,1]$ is membership functions and $\upsilon_{IF}(h) : H \to [0,1]$ is non-membership function. And $\rho_{IF}(h), \upsilon_{IF}(h)$ satisfies the following relation:

$$0 \leq \rho_{IF}(h) + \upsilon_{IF}(h) \leq 1$$

Intuitionistic Fuzzy Number [28]:

An intuitionistic fuzzy number is represented as,$\tilde{M}_k$with the grade of membership function $\sigma_{\tilde{M}_k}(t)$and non- grade of membership function $\eta_{\tilde{M}_k}(t)$. The following properties hold:

(1) For all, the non- grade of membership function is concave, meaning that$t_1, t_2 \in t, \eta_{\tilde{M}_k}(\lambda_k t_1 + (1-\lambda_k)t_2) \leq \max\{\eta_{\tilde{M}_k}(t_1), \eta_{\tilde{M}_k}(t_2)\}$, where $\lambda_k \in [0,1]$.
(2) For all, the membership function is convex, meaning that $t_1, t_2 \in t, \sigma_{\tilde{M}_k}(\lambda_k t_1 + (1-\lambda_k)t_2) \geq \min\{\sigma_{\tilde{M}_k}(t_1), \sigma_{\tilde{M}_k}(t_2)\}$, where $\lambda_k \in [0,1]$.
(3) The number is considered normal as there exists a $t_0 \in t$value such that $\sigma_{\tilde{M}_k}(t_0) = 1$ and $\eta_{\tilde{M}_k}(t_0) = 0$.
(4) It is classified as an intuitionistic fuzzy subset of the real line.

In summary, an intuitionistic fuzzy shows concavity for its non- grade of membership function, convexity for its membership function, satisfies the normal property, and can be categorized as an intuitionistic fuzzy subset within the real set.

Trapezoidal Intuitionistic Fuzzy Number (TpIFN) [29]:

Trapezoidal Intuitionistic fuzzy number denoted as $\widetilde{ITF}$, which possesses a grade of membership $\tau_{\widetilde{ITS}}(\psi)$, and a non- grade of membership $\gamma_{\widetilde{ITF}}(\psi)$ functions follow as:

$$\tau_{\widetilde{ITS}}(\psi) = \begin{cases} \frac{(\psi-q)}{(n-q)}\tau_{\widetilde{ITF}}, & \text{q}\leq\psi\leq r; \\ \tau_{\widetilde{ITF}}, & \text{r}\leq\psi\leq s; \\ \frac{(t-\psi)}{(t-s)}\tau_{\widetilde{ITF}}, & \text{s}<\psi\leq t; \\ 0 & \text{Otherwise.} \end{cases}$$

$$\gamma_{\widetilde{ITF}}(\psi) = \begin{cases} \frac{(r-\psi)+\nu_{\widetilde{ITF}}(\psi-q_1)}{(r-m_1)}\gamma_{\widetilde{ITF}}, & \text{q}\leq\psi\leq r; \\ \gamma_{\widetilde{ITF}}, & \text{r}\leq\psi\leq s; \\ \frac{(\psi-s)+\nu_{\widetilde{ITF}}(t_1-\psi)}{(t_1-o)}\gamma_{\widetilde{ITF}}, & \text{s}<\psi\leq t; \\ 0, & \text{Otherwise.} \end{cases}$$

Where $0 \leq \tau_{\widetilde{ITF}}(\alpha) \leq 1; 0 \leq \gamma_{\widetilde{ITF}}(\alpha) \leq 1$; and $\tau_{\widetilde{ITF}} + \gamma_{\widetilde{ITF}} \leq 1; q, r, s, t \in \mathbb{R}$. Then $\widetilde{ITF} = \left\langle \left([q, r, s, t] ; \tau_{\widetilde{ITF}} \right), \left([q_1, r, s, t_1]; \gamma_{\widetilde{ITF}} \right) \right\rangle$ is called as an intuitionistic trapezoidal fuzzy number.

Neutrosophic:

A neutrosophic set is a mathematical framework that extends fuzzy and intuitionistic sets to handle indeterminate or contradictory information using three components: falsity - membership, indeterminacy-membership, and truth-membership degrees which handling indeterminate, imprecise, and uncertain data. Neutrosophic theory introduced by Florentin Smarandache in the 1990s [30], is a remarkable fusion of classical and fuzzy logic. In neutrosophic logic, are considered as false (F), true (T), or indeterminate (I). The key definition and concept of neutrosophic is discuss below:

Neutrosophic Set [13]:

A set $\widetilde{neuK}$ in the universal set U, is said to be Neutrosophic Set if $\widetilde{neuK} = \left\{ \left(u, \left[t_{\widetilde{neuK}}(u), i_{\widetilde{neuK}}(u), f_{\widetilde{neuK}}(u) \right] \right) : u \in U \right\}$ Where $f_{\widetilde{neuK}}(u) : U \rightarrow [0, 1]$,$i_{\widetilde{neuK}}(u) : U \rightarrow [0, 1]$, and $t_{\widetilde{neuK}}(u) : U \rightarrow [0, 1]$ are the Falsity, indeterminacy, and truth membership function respectively and $f_{\widetilde{neuK}}(u), i_{\widetilde{neuK}}(u), t_{\widetilde{neuK}}(u)$ satisfy the following relation.

$$0 \leq sup\left\{ t_{\widetilde{neuK}}(u) \right\} + sup\left\{ i_{\widetilde{neuK}}(u) \right\} + sup\left\{ f_{\widetilde{neuK}}(u) \right\} \leq 3$$

.

Single valued trapezoidal neutrosophic number (SVTpNN) [31]:

A set $\widetilde{neuT_p}$ is said to be SVTpNN if the set $\widetilde{neuT_p}$ is defined as: $\widetilde{neuT_p} = \left\{ \left((p_1, p_2, p_3, p_4) ; \left[\phi_{T_p}, \varphi_{T_p}, \gamma_{T_p} \right] \right) : p_1, p_2, p_3, p_4 \in \mathbb{R} \right\}$ where truth-membership, indeterminacy-membership, and a falsity-membership are defined as respectively:

$$t_{\widetilde{neuT_p}}(t) = \left\{ \begin{array}{ll} \frac{(t-p_1)\phi_{T_p}}{p_2-p_1} & p_1 \leq t \leq p_2 \\ \phi_{T_p} & p_2 \leq t \leq p_3 \\ \frac{(p_4-t)\phi_{T_p}}{p_4-p_3} & p_3 \leq t \leq p_4 \\ 0, & \text{Otherwice} \end{array} \right\}$$

$$i_{\widetilde{neuT_p}}(t) = \left\{ \begin{array}{ll} \frac{\left(p_2-t+(t-p_1)\varphi_{T_p}\right)}{p_2-p_1} & p_1 \leq t \leq p_2 \\ \phi_{T_p} & p_2 \leq t \leq p_3 \\ \frac{\left(t-p_3+(p_4-t)\varphi_{T_p}\right)}{p_4-p_3} & p_3 \leq t \leq p_4 \\ 1, & \text{Otherwice} \end{array} \right\}$$

$$f_{\widetilde{neuT_p}}(t) = \left\{ \begin{array}{ll} \frac{\left(p_2-t+(t-p_1)\gamma_{T_p}\right)}{p_2-p_1} & p_1 \leq t \leq p_2 \\ \gamma_{T_p} & p_2 \leq t \leq p_3 \\ \frac{\left(t-p_3+(p_4-t)\gamma_{T_p}\right)}{p_4-p_3} & p_3 \leq t \leq p_4 \\ 1, & \text{Otherwice} \end{array} \right\}$$

Where, $0 \leq \phi_{T_p}, \varphi_{T_p}, \gamma_{T_p} \leq 1$ and $0 \leq \phi_{T_p} + \varphi_{T_p} + \gamma_{T_p} \leq 3$

6. Different Environment of the Neutrosophic Linear Programming Problem in different real-life problems

Table 2. Researchers from various domains have widely investigated the real-life applications of NFLPP, revealing its deep impact.

Authors	Year	Environment	Application	Significance
Alrefaei et al. [32]	2014	Supply Chain Management	Trapezoidal Neutrosophic Numbers	This paper proposes a flexible approach to fuzzy linear programming, which is employed to address the supply chain management challenges faced by a steel manufacturing company.
Broumi et al. [33]	2016	Decision making	Trapezoidal neutrosophic numbers.	This paper introduces a novel MADM method utilizing the Single valued NTpLWAA operator and the Single valued NTpLWGA operator.
Biswas et al. [34]	2018	Decision making	ITpNNs	The author proposes the development of a MADM strategy utilizing interval trapezoidal neutrosophic numbers (ITpNNs) as the distance measure.
Hamiden Khalifa [35]	2020	Assignment problem	Single-valued TpNN	The objective of this article is to optimize the multi-objective assignment problem using neutrosophic numbers.
Badr et al. [36]	2021	LPP	Trapezoidal neutrosophic numbers	Badr et al. proposed a new Exterior Point Simplex Algorithm to Solve the Neutrosophic Linear Programming Problems.

6.1. *The LPP under the Neutrosophic Principle*

Kamal et al. [37]

The author introduced a Multi-Objective Transportation Problem in which the objective functions are characterized as Type-2 TpFn and demand are expressed as multi-choice and supply are expressed as probabilistic random variables in constraints. Additionally, the study considers the " rate of decrement in profit and rate of increment in Transportation Cost (TC) on transporting the products from destinations to sources due to" factors affecting such as late deliveries, product damage, weather conditions, and other problems, which add to the overall cost. Given the presence of these uncertainties, obtaining a direct optimum solution becomes challenging. Hence, the first step is to convert these uncertainties into a crisp equivalent form. The aim of this article to transform type 2 TpFn into the classical equivalents by using two-phase defuzzification technique. Multi-choice variables are converted into equivalent values using the Stochastic Programming (SP) approach and probabilistic random variables are converted into equivalent values using the binary variables approach. The supply parameters are assumed to follow various types of probabilistic distributions, such as Weibull, Extreme value, Cauchy, and Pareto, while demand parameters are assumed as Normal distribution.

Hosseinzadeh and Tayyebi [38]

This paper presents an effective optimization and modeling framework for Fuzzy Multi-Objective neutrosophic Linear Programming Problem. The proposed framework employs SVTpNNs in Neural Networks to represent the coefficients of the right-hand side parameters, constraints, and objective functions. The main objective is to transform NFMOLP problem into an equivalent crisp MOLP Problem using a ranking function of SVTpNNs. To avoid decision deadlock situations within a hierarchical structure, the proposed method defines appropriate membership functions for each objective function based on the best and worst values of the objectives. This approach aims to select an optimal compromise solution that maximizes the degree of acceptance and minimizes the degree of rejection to some extent, considering all objectives simultaneously. The concept of fuzzy decision sets is employed to achieve this goal effectively. The practical applicability of the proposed method is demonstrated through the resolution of a transportation problem, highlighting its usefulness in real-world scenarios. Industries and business organizations dealing with imprecise and contradictory information can benefit from this approach to handle complex decision-making tasks efficiently.

Fathya and Ammar [39]

In this research, Fathya and Ammar [39] propose an innovative collaborative strategy for addressing multi-objective neutrosophic multi-level linear programming (MNMLP) problems, utilizing the harmonic mean technique. The coefficients of the objective function and the coefficients of constraints are neutrosophic numbers of the level decision makers. To transform the MNMLP problem effectively, we employ the interval programming technique, which splits it into two classical MMLP problems. One of these problems features all coefficients of neutrosophic numbers as upper approximations, while the other uses as lower approximations. Subsequently, they employ the harmonic mean method to amalgamate the various objectives from each classical problem converted into a single objective. By solving the crisp linear single-objective programming problem, they obtain a chosen solution for MNMLP problems. Fathya and Ammar (Fathy & Ammar, 2023) illustrate the practicality of our research through an application that addresses the optimization of the cost for a multi-objective of the transportation problem with a neutrosophic environment.

7. Conclusion:

In conclusion, this review article highlights the integration of Neutrosophic Theory and trapezoidal neutrosophic environments in the situation of LP problems. A neutrosophic theory is a mathematical framework that extends fuzzy and intuitionistic sets to handle uncertain, imprecise, and indeterminate information using three components: truth, falsity, and indeterminacy. On the other hand, Extended Fuzzy Theory extends the traditional fuzzy theory to manage more intricate grade of membership degrees. The primary objective of this review is to summarize and analyze the prevailing literature on trapezoidal neutrosophic environments, particularly focusing on aspects such as the NFMOLP Problems in SVTpN environments. This comprehensive review article aims to enhance the understanding of the potential of these integrated methodologies for effectively presenting decision-making problems amidst complex and uncertain conditions.

References

1. JC Nash. The (dantzig) simplex method for linear programming. *Computing in Science & Engineering*, 2(1):29–31, 2000.
2. Achim Koberstein. The dual simplex method, techniques for a fast and stable implementation. *Unpublished doctoral thesis, Universität Paderborn, Paderborn, Germany*, 2005.
3. James Edmund Reeb, Scott A Leavengood, et al. Using the graphical method to solve linear programs. 1998.
4. Reay-Chen Wang and Tien-Fu Liang. Applying possibilistic linear programming to aggregate production planning. *International Journal of Production Economics*, 98(3):328–341, 2005.
5. Carlos Monardes-Concha, Carlos Serrano-Julio, and Christian Hoffmann. Linear programming based decision support system for grapes transport planning in capel. *International Transactions in Operational Research*, 30(4):1874–1900, 2023.
6. Yan Han, Yue-Fei Huang, Guang-Qian Wang, and Imran Maqsood. A multi-objective linear programming model with interval parameters for water resources allocation in dalian city. *Water Resources Management*, 25:449–463, 2011.
7. Patil V;Gupta R;Duraisamy R;Jain RV. Cattle feed formulation using modified dual simplex method—an android application. In *Soft Computing for Problem Solving*, volume 1, pages 137–154. Springer, 2021.
8. M Zhou Y;Zhou, Y; Luo Q; Abdel-Basset. A simplex method-based social spider optimization algorithm for clustering analysis. *Engineering Applications of Artificial Intelligence*, 64:67–82, 2017.
9. L;Lin P;Wu Y; Cheng-S Chen, Z;Wu. Parameters identification of photovoltaic models using hybrid adaptive nelder-mead simplex algorithm based on eagle strategy. *Applied Energy*, 182:47–57, 2016.
10. G Liero, M; Mielke A; Savare. Optimal entropy-transport problems and a new hellinger–kantorovich distance between positive measures. *Inventiones Mathematicae*, 211(3):969–1117, 2018.

11. ZM;Mao B;Kato N Tang, F;Fadlullah. An intelligent traffic load prediction-based adaptive channel assignment algorithm in sdn-iot: A deep learning approach. *IEEE Internet of Things Journal*, 5(6):5141–5154, 2018.
12. Lotfi A Zadeh. Fuzzy sets. *Information and Control*, 8(3):338–353, 1965.
13. Singh N;Chakraborty A; Biswas SB;Majumdar M. Impact of social media in banking sector under triangular neutrosophic arena using mcgdm technique. *Neutrosophic Sets and Systems*, 35:153–176, 2020.
14. Wang JQ;Zhang Z. Multi-criteria decision-making method with incomplete certain information based on intuitionistic fuzzy number. *Control and Decision*, 24:226–230, 2009.
15. Ronald R Yager. A procedure for ordering fuzzy subsets of the unit interval. *Information Sciences*, 24(2):143–161, 1981.
16. DJ Dubois. *Fuzzy sets and systems: theory and applications*, volume 144. Academic press, 1980.
17. HJ Zimmermann. Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*, 1(1):45–55, 1978.
18. Lisa M Ellram. Supply-chain management: the industrial organisation perspective. *International Journal of Physical Distribution & Logistics Management*, 21(1):13–22, 1991.
19. Sheldon D Balbirer and David Shaw. An application of linear programming to bank financial planning. *Interfaces*, 11(5):77–83, 1981.
20. Yasser MR Aboelmagd. Linear programming applications in construction sites. *Alexandria Engineering Journal*, 57(4):4177–4187, 2018.
21. YR Fan and GH Huang. A robust two-step method for solving interval linear programming problems within an environmental management context. *Journal of Environmental Informatics*, 19(1), 2012.
22. Ebrahimnejad A; Verdegay JL;. A survey on models and methods for solving fuzzy linear programming problems. *Fuzzy Logic in Its 50th Year New Developments, Directions and Challenges*, 41(3):327–368, 2016.
23. A;Ge D; Zheng X;Gao-F Sun, H;Jiang. A chance constrained programming approach for no-wait flow shop scheduling problem under the interval-valued fuzzy processing time. *Processes*, 9(5):789, 2021.
24. Ankan Bhaumik, Sankar Kumar Roy, and Gerhard Wilhelm Weber. Multi-objective linguistic-neutrosophic matrix game and its applications to tourism management. *Journal of Dynamics & Games*, 8(2), 2021.
25. F Deli I; Karaaslan. Generalized trapezoidal hesitant fuzzy numbers and their applications to multi criteria decision-making problems. *Soft Computing*, 25(2):1017–1032, 2021.
26. Hassanpour H;Hosseinzadeh E; Moodi M;. Solving intuitionistic fuzzy multi-objective linear programming problem and its application in supply chain management. *Applications of Mathematics*, 68:269–287, 2023.
27. KT; Stoeva S Atanassov. Intuitionistic fuzzy sets. *Fuzzy sets and Systems*, 20(1):87–96, 1986.
28. Sharma MK;Dhiman N; Mishra VN. Mediative fuzzy logic mathematical model a contradictory management prediction in covid-19 pandemic. *Applied Soft Computing*, 105:107–285, 2021.
29. Wang J. Multi-criteria decision-making approach with incomplete certain information based on ternary ahp. *Journal of Systems Engineering and Electronics*, 17:109–114, 2006.
30. Florentin Smarandache. A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic, 1999.
31. Y Deli, I;Subas. A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems. *International Journal of Machine Learning and Cybernetics*, 8(4):1309–1322, 2017.
32. Alrefaei M;Alawneh A;Hassan M. Fuzzy linear programming for supply chain management in steel industry. *Applied Mathematical Sciences*, 8(103):5105 – 5114, 2014.
33. A;Talea M; Smarandache-F; Vladareanu L Broumi, S; Bakali. Computation of shortest path problem in a network with sv-trapezoidal neutrosophic numbers. In *International Conference on Advanced Mechatronic Systems (ICAMechS)*, pages 417–422. IEEE, 2016.

34. Pranab Biswas, Surapati Pramanik, and Bibhas Chandra Giri. Distance measure based madm strategy with interval trapezoidal neutrosophic numbers. *Neutrosophic Sets and Systems*, 19:40–46, 2018.
35. H Khalifa. *An approach to the optimization of multi-objective assignment problems with neutrosophic numbers*. Infinite Study, 2020.
36. Badr E;Nada S; Ali S; Elrokh A. Solving neutrosophic linear programming problems using exterior point simplex algorithm. *Neutrosophic Sets and Systems*, 45:21, 2021.
37. A; Haq A; Yusufi FNK; Ali I Kamal, M; Alarjani. Multi-objective transportation problem under type-2 trapezoidal fuzzy numbers with parameters estimation and goodness of fit. *Transport*, 36(4):317–338, 2021.
38. Hosseinzadeh E;Tayyebi J. A compromise solution for the neutrosophic multi-objective linear programming problem and its application in transportation problem. *Journal of Applied Research on Industrial Engineering*, 10:1–10, 2023.
39. E Fathy, E; Ammar. On neutrosophic multi-level multi-objective linear programming problem with application in transportation problem. *Journal of Intelligent and Fuzzy Systems*, (Preprint):1–17, 2023.

Received: Jan 10, 2024. Accepted: Mar 29, 2024

Logarithmic Similarity Measure of Neutrosophic Z-Number Sets for Undergraduate Teaching Quality Evaluation

Wen-Ping Chen, Yi-Ming Fang, Wen-Hua Cui *

Department of Electrical Engineering and Automation, Shaoxing University, Shaoxing 312000, Zhejiang, P.R. China
cwp@usx.edu.cn, ymfang@usx.edu.cn, wenhuacui@usx.edu.cn

* Correspondence: wenhuacui@usx.edu.cn

Abstract: In practical decision-making problems, the different reliability levels of decision-makers in their evaluation information are usually ignored by most existing decision-making methods. Then, a neutrosophic Z-number (NZN) is a powerful model for simultaneously describing the restraint and associated reliability of evaluation information in view of truth, indeterminacy, and falsity Z-numbers. This paper first proposed a logarithmic similarity measure (LSM) of NZN sets, which is more flexible and can change its measure result by adjusting the exponential parameters of the restraint and reliability. Then, a multi-attribute decision-making approach is developed based on the presented LSM of NZN sets, and its applicability and flexibility are further illustrated by a case study of undergraduate teaching quality assessment.

Keywords: teaching quality evaluation; neutrosophic Z-number; logarithmic similarity measure; confidence degree

1. Introduction

Teaching quality assessment is an effective way to accelerate the development of universities and to ensure the quality of university education. Resulting from the ambiguity and uncertainty of human thinking, various fuzzy theories have already been widely used for the teaching quality assessment, such as the spherical fuzzy set (FS) [1], the q-rung orthopair FS [2], the triangular Pythagorean FS [3], the fuzzy rough set [4], the interval-valued (IV) Pythagorean FS [5], the IV dual-hesitant FS [6], the IV intuitionistic FS [7], the IV hesitant fuzzy linguistic sets [8], and the Plithogenic cubic vague sets [9]. Especially in recent years, the neutrosophic decision-making (DM) approaches [10-11] were introduced in incomplete, uncertainty and inconsistent environments. For example, researchers successively proposed various DM approaches using the neutrosophic reducible weighted Maclaurin symmetric mean [12], the hyperbolic sine similarity measure for neutrosophic multivalued sets [13], the tangent trigonometric single-valued neutrosophic number (SvNN) aggregation operators [14], the TOPSIS method of simplified neutrosophic indeterminate sets [15], the grey relation analysis (GRA) method of 2-tuple linguistic neutrosophic numbers [16], the improved GRA method of probabilistic simplified neutrosophic sets [17], the combined compromise solution method of double-valued neutrosophic sets(NSs) [18], the GRA method of interval-valued NSs [19], and so on. However, the above methods only emphasize the assessment data, but ignore the fact that the evaluators usually imply the measures/levels of reliability related to their assessment data.

Fortunately, to fully describe human judgments, Zadeh [20] first defined the notion of Z-numbers, where a pair of fuzzy numbers is used to represent the restraint and reliability of evaluation information. Subsequently, Z-numbers had been further studied in both theory and applications, such as the arithmetic for discrete [21] or continuous Z-numbers [22], the approximate reasoning [23] according to Z-number-valued if-then rules, and the DM approaches for supplier selection [24], medicine selection [25], and environmental assessment [26]. Furthermore, to express indeterminate and inconsistent information with reliability measures, a neutrosophic Z-number (NZN) [27] was proposed, which uses three order pairs of fuzzy numbers to demonstrate the restraint and reliability of evaluation information in terms of truth, indeterminacy, and falsity. Weighted aggregation operators such as Dombi [28], Aczel-Alsina [29], and similarity measures on the basis of the generalized distance [30], and the correlation coefficient based on mean, variance, and covariance [31] had been proposed for multi-attribute DM (MADM) in NZN setting. As the extended forms of NZN, the trapezoidal NZN [32], and the linguistic NZN [33] had also been proposed for DM problems.

However, the logarithmic similarity measure (LSM) [34] is an effective tool for MADM. Then, NZN can ensure the levels of reliability of true, false and indeterminate values. Therefore, this paper proposes a generalized LSM of NZN sets (NZNLSM), in which the weights of the restraint and reliability of evaluation information can be adjusted according to the decision preferences of the evaluators. Specifically, as the weight of the reliability measure increases, the outcome of the decision will be more important by the reliability level of the evaluator. Furthermore, based on the proposed LSM of NZNSs, this paper developed a generalized DM method for performing MADM problems, and demonstrates the effectiveness and flexibility of the approach using the undergraduate teaching quality evaluation (UTQE) as an example.

In the rest of the paper, the basic notion of a NZN set and the definition of LSM of NZNSs are introduced in Section 2. Then, the MADM approach applying LSM of NZNSs is presented in Section 3. Furthermore, an example of UTQE is illustrated in Section 4, and both the comparison analysis and the sensitivity analysis are carried out in more detail. Lastly, the conclusion and the relative future research plan are shown in Section 5.

2. Neutrosophic Z-number sets

Du et al [27] put forward the neutrosophic Z-number set (NZNS) and its basic operational laws from the definition of the neutrosophic set [10-11] and Z-number [20].

Definition 1 [27]**.** A NZNS in a universe set S can be expressed as

$$X = \left\{ \left\langle s, \left(T_D(s), T_R(s)\right), \left(I_D(s), I_R(s)\right), \left(V_D(s), V_R(s)\right) \right\rangle \middle| s \in S \right\}. \tag{1}$$

where ($T_D(s)$, $T_R(s)$), ($I_D(s)$, $I_R(s)$), and ($V_D(s)$, $V_R(s)$) are the sequence pairs of truth, indeterminacy and falsity fuzzy values, and each pair is given by a Z-number that consists of the evaluation values such as $T_D(s_k)$, $I_D(s_k)$, $V_D(s_k)$ and the reliability measures such as $T_R(s_k)$, $I_R(s_k)$ and $V_R(s_k)$ corresponding to the evaluation values. Here, the element x = <s, ($T_D(s)$, $T_R(s)$), ($I_D(s)$, $I_R(s)$), ($V_D(s)$, $V_R(s)$)> in X is a NZN, with the conditions $T_D(s) + I_D(s) + V_D(s) \in [0, 3]$ and $T_R(s) + I_R(s) + V_R(s) \in [0, 3]$. For convenience, the element <s, ($T_D(s)$, $T_R(s)$), ($I_D(s)$, $I_R(s)$), ($V_D(s)$, $V_R(s)$) > in X can be simply written as x = <(T_D, T_R), (I_D, I_R), (V_D, V_R)>, which is called NZN.

For two NZNs denoted by x_a = <(T_{Da}, T_{Ra}), (I_{Da}, I_{Ra}), (V_{Da}, V_{Ra})> and x_b = <(T_{Db}, T_{Rb}), (I_{Db}, I_{Rb}), (V_{Db}, V_{Rb})>, there are the following relations:

(1) $x_a \oplus x_b = \left\langle \left(T_{Da} + T_{Db} - T_{Da}T_{Db}, T_{Ra} + T_{Rb} - T_{Ra}T_{Rb}\right), \left(I_{Da}I_{Db}, I_{Ra}I_{Rb}\right), \left(V_{Da}V_{Db}, V_{Ra}V_{Rb}\right) \right\rangle$;

(2) $x_a \otimes x_b = \left\langle \left(T_{Da}T_{Db}, T_{Ra}T_{Rb}\right), \left(I_{Da} + I_{Db} - I_{Da}I_{Db}, I_{Ra} + I_{Rb} - I_{Ra}I_{Rb}\right), \left(V_{Da} + V_{Db} - V_{Da}V_{Db}, V_{Ra} + V_{Rb} - V_{Ra}V_{Rb}\right) \right\rangle$;

(3) $\lambda x_a = \left\langle \left(1 - \left(1 - V_{Da}\right)^{\lambda}, 1 - \left(1 - V_{Ra}\right)^{\lambda}\right), \left(I_{Da}^{\lambda}, I_{Ra}^{\lambda}\right), \left(V_{Da}^{\lambda}, V_{Ra}^{\lambda}\right) \right\rangle$;

(4) $x_a^{\lambda} = \left\langle \left(T_{Da}^{\lambda}, T_{Ra}^{\lambda}\right), \left(1 - \left(1 - I_{Da}\right)^{\lambda}, 1 - \left(1 - I_{Ra}\right)^{\lambda}\right), \left(1 - \left(1 - V_{Da}\right)^{\lambda}, 1 - \left(1 - V_{Ra}\right)^{\lambda}\right) \right\rangle$;

(5) $x_a \supseteq x_b \Leftrightarrow T_{Da} \ge T_{Db}, T_{Ra} \ge T_{Rb}, I_{Da} \le I_{Db}, I_{Ra} \le I_{Rb}, V_{Da} \le V_{Db}$, and $V_{Ra} \le V_{Rb}$;

(6) $x_a = x_b \Leftrightarrow x_a \supseteq x_b$ and $x_a \subseteq x_b$;

(7) $x_a \cup x_b = \left\langle \left(T_{Da} \vee T_{Db}, T_{Ra} \vee T_{Rb}\right), \left(I_{Da} \wedge I_{Db}, I_{Ra} \wedge I_{Rb}\right), \left(V_{Da} \wedge V_{Db}, V_{Ra} \wedge V_{Rb}\right) \right\rangle$;

(8) $x_a \cap x_b = \left\langle \left(T_{Da} \wedge T_{Db}, T_{Ra} \wedge T_{Rb}\right), \left(I_{Da} \vee I_{Db}, I_{Ra} \vee I_{Rb}\right), \left(V_{Da} \vee V_{Db}, V_{Ra} \vee V_{Rb}\right) \right\rangle$;

(9) $\left(x_a\right)^C = \left\langle \left(V_{Da}, V_{Ra}\right), \left(1-I_{Da}, 1-I_{Ra}\right), \left(T_{Da}, T_{Ra}\right) \right\rangle$.

3. Logarithmic Similarity Measure of NZNSs

This section presents LSM of NZNSs as an extension of the LSM of dynamic neutrosophic cubic sets [34].

Assume that there are two NZNs x_a = <(T_{Da}, T_{Ra}), (I_{Da}, I_{Ra}), (V_{Da}, V_{Ra})> and x_b= <(T_{Db}, T_{Rb}), (I_{Db}, I_{Rb}), (V_{Db}, V_{Rb})>. We can define a function $\zeta(x_a, x_b)$ as

$$\zeta\left(x_a, x_b\right) = \frac{\left|T_{Da}^{\lambda} T_{Da}^{(1-\lambda)} - \left(T_{Db}\right)^{\lambda} \left(T_{Rb}\right)^{(1-\lambda)}\right| + \left|I_{Da}^{\lambda} I_{Da}^{(1-\lambda)} - \left(I_{Db}\right)^{\lambda} \left(I_{Rb}\right)^{(1-\lambda)}\right| + \left|V_{Da}^{\lambda} V_{Da}^{(1-\lambda)} - \left(V_{Db}\right)^{\lambda} \left(V_{Rb}\right)^{(1-\lambda)}\right|}{3}, \tag{2}$$

where the exponential parameter $\lambda \in (0, 1]$.

Let $X(s)$ = {<s_κ, ($T_{Dx}(s_\kappa)$, $T_{Rx}(s_\kappa)$), ($I_{Dx}(s_\kappa)$, $I_{Rx}(s_\kappa)$), ($V_{Dx}(s_\kappa)$, $V_{Rx}(s_\kappa)$)>|$s_\kappa \in S$} and $Y(s)$ = {<s_κ, ($T_{Dy}(s_\kappa)$, $T_{Ry}(s_\kappa)$), ($I_{Dy}(s_\kappa)$, $I_{Ry}(s_\kappa)$), ($V_{Dy}(s_\kappa)$, $V_{Ry}(s_\kappa)$)>|$s_\kappa \in S$} be two NZNSs in $S = \{s_1, s_2, \ldots, s_\eta\}$ whose weights are given by a vector $\tau = \{\tau(s_1), \tau(s_2), \ldots, \tau(s_\eta)\}$ with $\sum_{\kappa=1}^{\eta} \tau\left(s_\kappa\right) = 1$. Therefore, the weighted LSM between $X(s)$ and $Y(s)$ can be calculated by

$$L_w\left(X(s), Y(s)\right) = \sum_{\kappa=1}^{\eta} \tau\left(s_\kappa\right) \log_\varepsilon \left\{\varepsilon - (\varepsilon-1) \times \zeta\left(X\left(s_\kappa\right), Y\left(s_\kappa\right)\right)\right\}$$

$$= \sum_{\kappa=1}^{\eta} \tau\left(s_\kappa\right) \left\{ \log_\varepsilon \left[\varepsilon - (\varepsilon-1) \times \frac{\left|\left(T_{Dx}(s_\kappa)\right)^{\lambda} \left(T_{Rx}(s_\kappa)\right)^{1-\lambda} - \left(T_{Dy}(s_\kappa)\right)^{\lambda} \left(T_{Ry}(s_\kappa)\right)^{1-\lambda}\right| + \left|\left(I_{Dx}(s_\kappa)\right)^{\lambda} \left(I_{Rx}(s_\kappa)\right)^{1-\lambda} - \left(I_{Dy}(s_\kappa)\right)^{\lambda} \left(I_{Ry}(s_\kappa)\right)^{1-\lambda}\right| + \left|\left(V_{Dx}(s_\kappa)\right)^{\lambda} \left(V_{Rx}(s_\kappa)\right)^{1-\lambda} - \left(V_{Dy}(s_\kappa)\right)^{\lambda} \left(V_{Ry}(s_\kappa)\right)^{1-\lambda}\right|}{3} \right] \right\} \text{ for } \varepsilon \geq 2, \lambda \in (0, 1], \tag{3}$$

where ε is an integer, and $s_\kappa \in S$ (κ = 1, 2, …, η). Especially when the parameter λ = 1, the LSM of NZNSs will be reduced to that of SvNSs.

Example 1. Let $X(s) = \left[\left\langle(0.8,0.7),(0.2,0.6),(0.1,0.7)\right\rangle \ \left\langle(0.7,0.6),(0.1,0.7),(0.3,0.8)\right\rangle \ \left\langle(0.7,0.6),(0.2,0.8),(0.2,0.8)\right\rangle\right]$ and $Y(s) = \left[\left\langle(0.7,0.7),(0.2,0.7),(0.2,0.9)\right\rangle \ \left\langle(0.6,0.7),(0.2,0.7),(0.1,0.9)\right\rangle \ \left\langle(0.7,0.8),(0.1,0.7),(0.2,0.6)\right\rangle\right]$ be two NZNSs in $S = \{s_1, s_2, s_3\}$ whose weights are given by the vector $\tau = \{0.3, 0.4, 0.3\}$. If ε = 2 and λ = 0.5 are chosen, the weighted LSM between $X(s)$ and $Y(s)$ is $L_w(X(s),Y(s))$ = 0.3×0.28265 + 0.4*0.37045 + 0.3*0.27861 = 0.9317 according to Eq.(3). While when λ = 1, $L_w(X(s),Y(s))$ = 0.3×0.28533 + 0.4*0.36019 + 0.3*0.29273 = 0.9382.

Theorem 1. Let $X(s)$ = {<s_κ, ($T_{Dx}(s_\kappa)$, $T_{Rx}(s_\kappa)$), ($I_{Dx}(s_\kappa)$, $I_{Rx}(s_\kappa)$), ($V_{Dx}(s_\kappa)$, $V_{Rx}(s_\kappa)$)>|$s_\kappa \in S$} and $Y(s)$ = {<s_κ, ($T_{Dy}(s_\kappa)$, $T_{Ry}(s_\kappa)$), ($I_{Dy}(s_\kappa)$, $I_{Ry}(s_\kappa)$), ($V_{Dy}(s_\kappa)$, $V_{Ry}(s_\kappa)$)>|$s_\kappa \in S$ } be two NZNSs in $S = \{s_1, s_2, \ldots, s_\eta\}$ whose weights are given by a vector $\tau = \{\tau(s_1), \tau(s_2), \ldots, \tau(s_\eta)\}$ with $\sum_{\kappa=1}^{\eta} \tau\left(s_\kappa\right) = 1$. The weighted LSM of NZNSs denoted by $L_w(X(s), Y(s))$ contains the properties as follows:

(R1) $0 \leq L_w(X(s), Y(s)) \leq 1$;

(R2) $L_w(X(s), Y(s)) = 1$ if and only if $X(s) = Y(s)$;
(R3) $L_w(X(s), Y(s)) = L_w(Y(s), X(s))$;
(R4) If $M(s)$ is a NZNS in S, and $X(s) \subseteq Y(s) \subseteq M(s)$, then $L_w(X(s), M(s)) \le L_w(X(s), Y(s))$ and $L_w(X(s), M(s)) \le L_w(Y(s), M(s))$.

Proof. (R1) Since all variable values of $X(s)$ and $Y(s)$ are in the range of 0 to 1, $\zeta(X(s_\kappa), Y(s_\kappa)) \in [0,1]$ can be easily determined from Eq. (2). Then, when $\kappa = 1, 2, \ldots, \eta$, one arrives at $\varepsilon-(\varepsilon-1)\times\zeta(X(s_\kappa), Y(s_\kappa)) \in [1, \varepsilon]$, whose logarithm to the base ε is in [0,1]. Therefore, $L_w(X(s), Y(s)) \in [0,1]$ can be derived by Eq. (3) when the condition $\sum_{\kappa=1}^{\eta}\tau(s_\kappa)=1$ is satisfied.

(R2) If $X(s) = Y(s)$, then for $\kappa = 1, 2, \ldots, \eta$, there is $T_{Dx}(s_\kappa) = T_{Dy}(s_\kappa)$, $T_{Rx}(s_\kappa) = T_{Ry}(s_\kappa)$, $I_{Dx}(s_\kappa) = I_{Dy}(s_\kappa)$, $I_{Rx}(s_\kappa) = I_{Ry}(s_\kappa)$, $V_{Dx}(s_\kappa) = V_{Dy}(s_\kappa)$, and $V_{Rx}(s_\kappa) = V_{Ry}(s_\kappa)$. Thus, $\zeta(X(s_\kappa), Y(s_\kappa)) = 0$ can be further inferred from Eq. (2), and that the logarithm of $\varepsilon-(\varepsilon-1)\times\zeta(X(s_\kappa), Y(s_\kappa))$ to the base ε equals 1 by Eq. (3). Therefore, $L_w(X(s), Y(s)) = 1$ can be gotten for the condition $\sum_{\kappa=1}^{\eta}\tau(s_\kappa)=1$.

Conversely, if $L_w(X(s), Y(s)) = 1$, by Eq. (3) and the condition $\sum_{\kappa=1}^{\eta}\tau(s_\kappa)=1$, there exists the logarithm of $\varepsilon-(\varepsilon-1)\times\zeta(X(s_\kappa), Y(s_\kappa))$ to the base ε equals 1 for $\kappa = 1, 2, \ldots, \eta$. Then, $\zeta(X(s_\kappa), Y(s_\kappa))$ must be zero for $\kappa = 1, 2, \ldots, \eta$. Thus, by Eq. (2) there is $T_{Dx}(s_\kappa) = T_{Dy}(s_\kappa)$, $T_{Rx}(s_\kappa) = T_{Ry}(s_\kappa)$, $I_{Dx}(s_\kappa) = I_{Dy}(s_\kappa)$, $I_{Rx}(s_\kappa) = I_{Ry}(s_\kappa)$, $V_{Dx}(s_\kappa) = V_{Dy}(s_\kappa)$, and $V_{Rx}(s_\kappa) = V_{Ry}(s_\kappa)$ for $\kappa = 1, 2, \ldots, \eta$, that is $X(s) = Y(s)$.

(R3) By Eq. (3), $L_w(X(s), Y(s)) = L_w(Y(s), X(s))$ can be straightforwardly obtained.

(R4) The condition $X(s) \subseteq Y(s) \subseteq P(s)$ implies that $T_{Dx}(s_\kappa) \le T_{Dy}(s_\kappa) \le T_{Dp}(s_\kappa)$, $T_{Rx}(s_\kappa) \le T_{Ry}(s_\kappa) \le T_{Rp}(s_\kappa)$, $I_{Dx}(s_\kappa) \ge I_{Dy}(s_\kappa) \ge I_{Dp}(s_\kappa)$, $I_{Rx}(s_\kappa) \ge I_{Ry}(s_\kappa) \ge I_{Rp}(s_\kappa)$, $V_{Dx}(s_\kappa) \ge V_{Dy}(s_\kappa) \ge V_{Dp}(s_\kappa)$, and $V_{Rx}(s_\kappa) \ge V_{Ry}(s_\kappa) \ge V_{Rp}(s_\kappa)$ for $\kappa = 1, 2, \ldots, \eta$. Since the power function with a positive base increases monotonically in the range of positive exponents, for $\lambda \in (0,1]$, there is $(T_{Dx}(s_\kappa))^\lambda \le (T_{Dy}(s_\kappa))^\lambda \le (T_{Dp}(s_\kappa))^\lambda$, $(T_{Rx}(s_\kappa))^{(1-\lambda)} \le (T_{Ry}(s_\kappa))^{(1-\lambda)} \le (T_{Rp}(s_\kappa))^{(1-\lambda)}$, $(I_{Dx}(s_\kappa))^\lambda \ge (I_{Dy}(s_\kappa))^\lambda \ge (I_{Dp}(s_\kappa))^\lambda$, $(I_{Rx}(s_\kappa))^{(1-\lambda)} \ge (I_{Ry}(s_\kappa))^{(1-\lambda)} \ge (I_{Rp}(s_\kappa))^{(1-\lambda)}$, $(V_{Dx}(s_\kappa))^\lambda \ge (V_{Dy}(s_\kappa))^\lambda \ge (V_{Dp}(s_\kappa))^\lambda$, $(V_{Rx}(s_\kappa))^{(1-\lambda)} \ge (V_{Ry}(s_\kappa))^{(1-\lambda)} \ge (V_{Rp}(s_\kappa))^{(1-\lambda)}$. Hence, we can obtain the follows:

$$\left|(T_{Dx}(s_\kappa))^\lambda(T_{Rx}(s_\kappa))^{(1-\lambda)}-(T_{Dy}(s_\kappa))^\lambda(T_{Ry}(s_\kappa))^{(1-\lambda)}\right| \le \left|(T_{Dx}(s_\kappa))^\lambda(T_{Rx}(s_\kappa))^{(1-\lambda)}-(T_{Dp}(s_\kappa))^\lambda(T_{Rp}(s_\kappa))^{(1-\lambda)}\right|,$$
$$\left|(T_{Dy}(s_\kappa))^\lambda(T_{Ry}(s_\kappa))^{(1-\lambda)}-(T_{Dp}(s_\kappa))^\lambda(T_{Rp}(s_\kappa))^{(1-\lambda)}\right| \le \left|(T_{Dx}(s_\kappa))^\lambda(T_{Rx}(s_\kappa))^{(1-\lambda)}-(T_{Dp}(s_\kappa))^\lambda(T_{Rp}(s_\kappa))^{(1-\lambda)}\right|,$$
$$\left|(I_{Dx}(s_\kappa))^\lambda(I_{Rx}(s_\kappa))^{(1-\lambda)}-(I_{Dy}(s_\kappa))^\lambda(I_{Ry}(s_\kappa))^{(1-\lambda)}\right| \le \left|(I_{Dx}(s_\kappa))^\lambda(I_{Rx}(s_\kappa))^{(1-\lambda)}-(I_{Dp}(s_\kappa))^\lambda(I_{Rp}(s_\kappa))^{(1-\lambda)}\right|,$$
$$\left|(I_{Dy}(s_\kappa))^\lambda(I_{Ry}(s_\kappa))^{(1-\lambda)}-(I_{Dp}(s_\kappa))^\lambda(I_{Rp}(s_\kappa))^{(1-\lambda)}\right| \le \left|(I_{Dx}(s_\kappa))^\lambda(I_{Rx}(s_\kappa))^{(1-\lambda)}-(I_{Dp}(s_\kappa))^\lambda(I_{Rp}(s_\kappa))^{(1-\lambda)}\right|,$$
$$\left|(V_{Dx}(s_\kappa))^\lambda(V_{Rx}(s_\kappa))^{(1-\lambda)}-(V_{Dy}(s_\kappa))^\lambda(V_{Ry}(s_\kappa))^{(1-\lambda)}\right| \le \left|(V_{Dx}(s_\kappa))^\lambda(V_{Rx}(s_\kappa))^{(1-\lambda)}-(V_{Dp}(s_\kappa))^\lambda(V_{Rp}(s_\kappa))^{(1-\lambda)}\right|,$$
$$\left|(V_{Dy}(s_\kappa))^\lambda(V_{Ry}(s_\kappa))^{(1-\lambda)}-(V_{Dp}(s_\kappa))^\lambda(V_{Rp}(s_\kappa))^{(1-\lambda)}\right| \le \left|(V_{Dx}(s_\kappa))^\lambda(V_{Rx}(s_\kappa))^{(1-\lambda)}-(V_{Dp}(s_\kappa))^\lambda(V_{Rp}(s_\kappa))^{(1-\lambda)}\right|.$$

According to the above conclusion and Eq. (2), it is not difficult to get $\zeta(X(s), Y(s)) \le \zeta(X(s), P(s))$ and $\zeta(Y(s), P(s)) \le \zeta(X(s), P(s))$. And $L_w(X(s), P(s)) \le L_w(X(s), Y(s))$ and $L_w(X(s), P(s)) \le L_w(Y(s), P(s))$ can be further determined for the reason that the logarithm to the base ε increases with the function ζ decreasing.

Thus, the above properties are completely proved. □

3. MADM Approach Applying the Proposed LSM in NZN Setting

To solve the DM problems in NZN setting, this section developed a MADM approach based on the presented NZNLSM. Assume that the decision makers need to evaluate α alternatives represented by $U = \{U_1, U_2, \ldots, U_\alpha\}$ on the η attributes represented by $S = \{s_1, s_2, \ldots, s_\eta\}$ to obtain the best option, and the attribute importance is given by the weight vector $\tau = \{\tau_1, \tau_2, \ldots, \tau_\eta\}$, where τ_κ is the weight of the attribute s_κ for $\kappa = 1, 2, \ldots, \eta$. Then, the evaluation value of the option U_ι for the attribute s_κ can be expressed as the NZN $x_{\iota\kappa} = \langle(T_{D\iota\kappa}, T_{R\iota\kappa}), (I_{D\iota\kappa}, I_{R\iota\kappa}), (V_{D\iota\kappa}, V_{R\iota\kappa})\rangle$, where $T_{D\iota\kappa}, I_{D\iota\kappa}, V_{D\iota\kappa} \in [0, 1]$ and $T_{R\iota\kappa}, I_{R\iota\kappa}, V_{R\iota\kappa} \in [0, 1]$. Therefore, for all attributes s_κ ($\kappa = 1, 2, \ldots, \eta$), the evaluated values of all options U_ι ($\iota = 1, 2, \ldots, \alpha$) can be constructed by the NZN matrix $E = (x_{\iota\kappa})_{\alpha\times\eta}$.

Then, the decision process of using the weighted LSM of NZNSs can be given as follows.

Step 1. Since the ideal one of all evaluated NZNs on the attribute $s_\kappa(\kappa = 1, 2, \ldots, \eta)$ can be obtained by

$$\begin{aligned} x_\kappa^* &= \left\langle s_\kappa, \left\langle \left(T_{D\kappa}^*, T_{R\kappa}^*\right), \left(I_{D\kappa}^*, I_{R\kappa}^*\right), \left(V_{D\kappa}^*, V_{R\kappa}^*\right) \right\rangle \right\rangle \\ &= \left\langle s_\kappa, \left\langle \left(\max_\iota\left(T_{D\iota\kappa}\right), \max_\iota\left(T_{R\iota\kappa}\right)\right), \left(\min_\iota\left(I_{D\iota\kappa}\right), \min_\iota\left(I_{R\iota\kappa}\right)\right), \left(\min_\iota\left(V_{D\iota\kappa}\right), \min_\iota\left(V_{R\iota\kappa}\right)\right) \right\rangle \right\rangle, \end{aligned} \tag{4}$$

where $\iota = 1, 2, \ldots, \alpha$. The ideal NZNS considering all attributes can be derived from the formula

$$X^* = \left\{ \left\langle s_\kappa, \left\langle \left(T_{D\kappa}^*, T_{R\kappa}^*\right), \left(I_{D\kappa}^*, I_{R\kappa}^*\right), \left(V_{D\kappa}^*, V_{R\kappa}^*\right) \right\rangle \right\rangle \middle| s_\kappa \in S, \kappa = 1, 2, \ldots, \eta \right\}. \tag{5}$$

Step 2. By Eq. (3), the weighted LSM between X_ι and X^* is calculated as

$$L_w\left(X_\iota, X^*\right) = \sum_{\kappa=1}^{\eta} \tau_\kappa \log_\varepsilon \left[\varepsilon - (\varepsilon - 1) \times \frac{\begin{array}{c} \left| T_{D\iota\kappa}^{\lambda} T_{R\iota\kappa}^{(1-\lambda)} - \left(T_{D\kappa}^*\right)^{\lambda} \left(T_{R\kappa}^*\right)^{(1-\lambda)} \right| + \left| I_{D\iota\kappa}^{\lambda} I_{R\iota\kappa}^{(1-\lambda)} - \left(I_{D\kappa}^*\right)^{\lambda} \left(I_{R\kappa}^*\right)^{(1-\lambda)} \right| \\ + \left| V_{D\iota\kappa}^{\lambda} V_{R\iota\kappa}^{(1-\lambda)} - \left(V_{D\kappa}^*\right)^{\lambda} \left(V_{R\kappa}^*\right)^{(1-\lambda)} \right| \end{array}}{3} \right]. \tag{6}$$

Step 3. According to the largest value of $L_w(X_\iota, X^*)$ for $\iota = 1, 2, \ldots, \alpha$, the optimal alternative can be determined.

Step 4. End.

4. Example and Analysis

The applicability and effectiveness of the presented DM approach are demonstrated through a case of UTQE in this section. Then, its robustness and sensitivity are further analyzed by comparing with the existing methods [27-30] in NZN setting.

4.1 Example of UTQE

Suppose that there are four universities represented by $U = \{U_1, U_2, U_3, U_4\}$ to participate in UTQE. Experts are required to assess them in terms of five aspects denoted by $S = \{s_1, s_2, s_3, s_4, s_5\}$, in which s_1 is the adaptability degree; s_2 is the achievement degree; s_3 is the guarantee degree; s_4 is the satisfied degree; and s_5 is the effective degree, with the corresponding weight vector given by $\tau = \{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5\} = \{0.2, 0.3, 0.15, 0.15, 0.2\}$. Then, all decision information of the UTQE can be finally established as the following NZN matrix:

$$E = \left(x_{\iota\kappa}\right)_{4\times5} = \begin{bmatrix} \langle(0.7,0.8),(0.1,0.7),(0.2,0.8)\rangle & \langle(0.7,0.8),(0.1,0.7),(0.2,0.8)\rangle & \langle(0.7,0.7),(0.2,0.7),(0.2,0.9)\rangle & \langle(0.8,0.6),(0.2,0.8),(0.3,0.8)\rangle & \langle(0.6,0.8),(0.2,0.9),(0.2,0.8)\rangle \\ \langle(0.8,0.7),(0.2,0.6),(0.1,0.7)\rangle & \langle(0.6,0.7),(0.2,0.7),(0.1,0.9)\rangle & \langle(0.6,0.8),(0.3,0.8),(0.3,0.8)\rangle & \langle(0.8,0.7),(0.2,0.7),(0.2,0.6)\rangle & \langle(0.7,0.8),(0.2,0.8),(0.1,0.8)\rangle \\ \langle(0.9,0.8),(0.1,0.8),(0.1,0.7)\rangle & \langle(0.7,0.6),(0.1,0.7),(0.3,0.8)\rangle & \langle(0.7,0.7),(0.4,0.7),(0.6,0.7)\rangle & \langle(0.9,0.8),(0.1,0.7),(0.1,0.8)\rangle & \langle(0.8,0.7),(0.1,0.7),(0.1,0.7)\rangle \\ \langle(0.7,0.8),(0.1,0.7),(0.2,0.6)\rangle & \langle(0.6,0.7),(0.1,0.8),(0.2,0.7)\rangle & \langle(0.7,0.6),(0.2,0.8),(0.2,0.8)\rangle & \langle(0.7,0.8),(0.2,0.6),(0.3,0.7)\rangle & \langle(0.5,0.6),(0.2,0.8),(0.1,0.8)\rangle \end{bmatrix}.$$

Thus, the novel MADM approach based upon the NZNLSM can be applied to the UTQE problem. First, according to Eqs. (4) and (5), the ideal option can be calculated from the decision matrix X as

X^* = {<(0.9,0.8), (0.1,0.6), (0.1,0.6)>, <(0.7,0.8), (0.1,0.7), (0.1,0.7)>, <(0.7,0.8), (0.2,0.7), (0.2,0.7)>, <(0.9,0.8), (0.1,0.6), (0.1,0.6)>, <(0.8,0.8), (0.1,0.7), (0.1,0.7)>}.

Then, using Eq. (6), we calculate the weighted LSM between the alternative X_ι (ι = 1, 2, 3, 4) and the ideal option X^*. To verify whether the alternative ranking changes with the parameter value of λ, the LSM values between X_ι (ι = 1, 2, 3, 4) and X^* are calculated with λ changing from 0.1 to 1 for ε = 2, which are shown in Table 1.

Table 1. The MADM results applying the proposed approach for ε = 2 with $\lambda \in (0, 1]$

	$L_w(X_\iota, X^*)$ for ι = 1, 2, 3, 4	Ranking order	The optimal one
λ=0.1	0.9278, 0.9408, 0.9445, 0.9379	U_3> U_2> U_4> U_1	U_3
λ=0.2	0.9277, 0.9390, 0.9458, 0.9350	U_3> U_2> U_4> U_1	U_3
λ=0.3	0.9285, 0.9381, 0.9471, 0.9330	U_3> U_2> U_4> U_1	U_3
λ=0.4	0.9300, 0.9379, 0.9487, 0.9318	U_3> U_2> U_4> U_1	U_3
λ=0.5	0.9320, 0.9382, 0.9506, 0.9311	U_3> U_2> U_1> U_4	U_3
λ=0.6	0.9341, 0.9388, 0.9526, 0.9308	U_3> U_2> U_1> U_4	U_3
λ=0.7	0.9364, 0.9398, 0.9548, 0.9308	U_3> U_2> U_1> U_4	U_3
λ=0.8	0.9387, 0.9409, 0.9572, 0.9311	U_3> U_2> U_1> U_4	U_3
λ=0.9	0.9409, 0.9422, 0.9598, 0.9314	U_3> U_2> U_1> U_4	U_3
λ=1.0	0.9431, 0.9436, 0.9625, 0.9318	U_3> U_2> U_1> U_4	U_3

From Table 1, the use of different values of λ affects the ranking results of the alternatives. In this case, although the best alternative keeps the same as U_3, the ranking of the alternatives is $U_3 > U_2 > U_4 > U_1$ for $\lambda < 0.5$, while it changes into $U_3 > U_2 > U_1 > U_4$ for $\lambda \geq 0.5$. This is because the exponent of the restraint in Eq. (6) is λ and the exponent of reliability is $1-\lambda$. Therefore, the ranking order is more sensitive to the reliability measure when λ is smaller than 0.5, while it seems to be more sensitive to the restraint when λ is bigger than 0.5, which illustrates the flexibility of the proposed LSM-based MADM. In practice, the value of λ can be made based on the decision maker's preferences and DM requirements.

4.2 Comparative Analysis

A comparison of the presented MADM approach with the published MADM approaches was carried out in NZN setting. Table 2 lists the DM results using the published MADM methods based on the operators including the NZN weighted arithmetic average (NZNWAA) and the NZN weighted geometric average (NZNWGA) [27], the NZN Dombi weighted arithmetic average (NZNDWAA) and the NZN Dombi weighted geometric average (NZNDWGA) [28], the NZN Aczel-Alsina weighted arithmetic average (NZNAAWAA) and the NZN Aczel-Alsina weighted geometric average (NZNAAWGA) [29], or based on the similarity measures including the NZN generalized distance-based similarity measure (NZNDSM) [30], the NZN cosine similarity measure (NZNCSM) [30], and the NZN cotangent similarity measure (NZNCTSM) [30].

Table 2. The MADM results applying published approaches in NZN setting

DM Method	Parameter value	Score value	Ranking order	The optimal one
NZNWAA [27]		0.7521,0.7561,0.7859,0.7390	U_3>U_2>U_1>U_4	U_3
NZNWGA [27]		0.7420,0.7421,0.7478,0.7255	U_3>U_2>U_1>U_4	U_3
NZNDWAA [28]	q = 1	0.7582,0.7642,0.8051,0.7484	U_3>U_2>U_1>U_4	U_3
NZNDWGA [28]	q = 1	0.7381,0.7373,0.7343,0.7213	U_1>U_2>U_3>U_4	U_1
NZNAAWAA [29]		0.7570,0.7633,0.7991,0.7466	U_3>U_2>U_1>U_4	U_3
NZNAAWGA [29]		0.7329,0.7298,0.7094,0.7149	U_1>U_2>U_4>U_3	U_1
NZNDSM [30]	ρ = 1	0.9142, 0.9233, 0.9367, 0.9150	U_3>U_2>U_4>U_1	U_3
NZNCSM [30]	ρ = 1	0.9909, 0.9928, 0.9951, 0.9911	U_3>U_2>U_4>U_1	U_3
NZNCTSM [30]	ρ = 1	0.8735, 0.8863, 0.9052, 0.8747	U_3>U_2>U_4>U_1	U_3

Compared to the outcomes of the proposed MADM approaches in Table 1, the nine published approaches yield four different sorting orders and two different best alternatives. Then, the methods based on the operators of NZNWAA, NZNWGA, NZNDWAA, and NZNAAWAA yield a uniform ranking result of $U_3 > U_2 > U_1 > U_4$, which is as similar as that of the proposed one for $\lambda \geq 0.5$ as shown in Table 1. The existing MADM methods applying the similarity measures of the NZNDSM, the NZNCSM, and the NZNCTSM for ρ = 1 yield the ranking result of $U_3 > U_2 > U_1 > U_4$, which is as the same as that of the proposed one with $\lambda < 0.5$.

In a word, the LSM of NZNSs-based MADM method then demonstrates more possible ranking results than the existing methods, and is more flexible as the parameter value of λ changes. Furthermore, LSM of NZNSs with $\lambda = 1$ can deal with DM problems in SvNN setting, which cannot be handled by the existing MADM methods of NZNs. And LSM of NZNSs with $\lambda < 1$ can handle the MADM problem in NZN setting, which cannot be solved by the existing MADM methods of the SvNN. In short, the MADM methods based on LSM of NZNSs is very useful and flexible.

4.3 Sensitivity Analysis

4.3.1 Sensitivity Analysis to Logarithmic Base

To verify whether the ranking order is affected by the value of the logarithmic base ε, the LSM values between X_ι (ι = 1, 2, 3, 4) and X^* with ε in the range of 2 to 100 for λ = 0.1, 0.4, 0.6, 0.9 was calculated by Eq. (6), as shown in Fig.1.

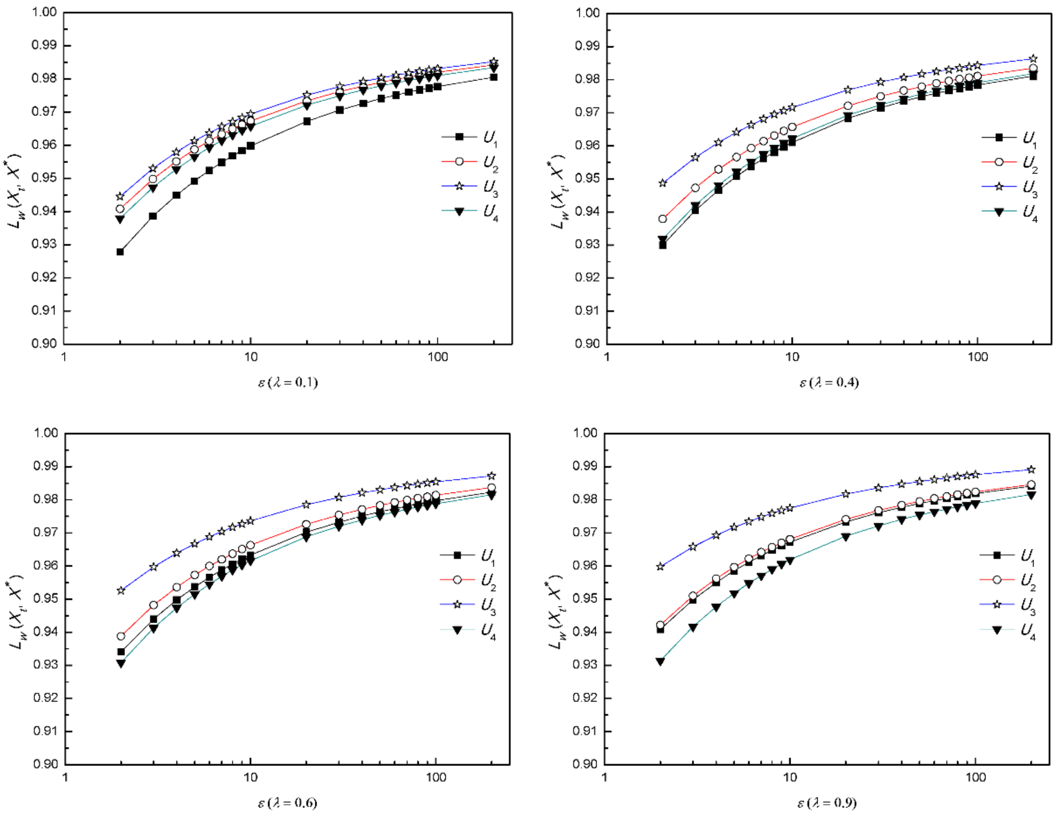


Figure 1. Values of LSM of NZNSs for all alternatives with ε ∈ [2, 200] for λ = 0.1, 0.4, 0.6, 0.9

It is clear from Fig.1 that for the same value of λ, the sorting results are consistent over the entire range of ε. In details, when λ = 0.1 or 0.4, the sort result keeps as $U_3 > U_2 > U_4 > U_1$, while when λ = 0.6 or 0.9, it is as consistent as $U_3 > U_2 > U_1 > U_4$. Therefore, the proposed LSM of NZNSs is a generalized

LSM of NZNSs, in which the decision maker can choose a LSM of NZNSs with any valid ε value, since ε has little effect on the final ranking result.

4.3.2 Sensitivity Analysis to Reliability Measure

The sensitivity of the presented MADM approach to the level of reliability is demonstrated by using four different cases, which are all adapted from the matrix E, changing only the parameter values of U_1. As shown in Table 3, all T_Rs for U_1 in Case 1, I_Rs for U_1 in Case 2, and V_Rs for U_1 in Case 3 are increased by 0.1 compared to the original U_1 data in matrix E represented by Case 0. Table 4 shows the DM results for different cases by using the proposed method and existing methods.

Table 3. U_1 data for different cases

		s_1	s_2	s_3	s_4	s_5
Case 0	U_1	<(0.7,0.8),(0.1,0.7),(0.2,0.8)>	<(0.7,0.8),(0.1,0.7),(0.2,0.8)>	<(0.7,0.7),(0.2,0.7),(0.2, 0.9)>	<(0.8,0.6),(0.2,0.8),(0.3,0.8)>	<(0.6,0.8),(0.2,0.9),(0.2,0.8)>
Case 1	U_1	<(0.7,0.9),(0.1,0.7),(0.2,0.8)>	<(0.7,0.9),(0.1,0.7),(0.2,0.8)>	<(0.7,0.8),(0.2,0.7),(0.2, 0.9)>	<(0.8,0.7),(0.2,0.8),(0.3,0.8)>	<(0.6,0.9),(0.2,0.9),(0.2,0.8)>
Case 2	U_1	<(0.7,0.8),(0.1,0.8),(0.2,0.8)>	<(0.7,0.8),(0.1,0.8),(0.2,0.8)>	<(0.7,0.7),(0.2,0.8),(0.2, 0.9)>	<(0.8,0.6),(0.2,0.9),(0.3,0.8)>	<(0.6,0.8),(0.2,1.0),(0.2,0.8)>
Case 3	U_1	<(0.7,0.8),(0.1,0.7), (0.2,0.9)>	<(0.7,0.8),(0.1,0.7),(0.2,0.9)>	<(0.7,0.7),(0.2,0.7),(0.2,1.0)>	<(0.8,0.6),(0.2,0.8),(0.3,0.9)>	<(0.6,0.8),(0.2,0.9),(0.2,0.9)>

Table 4. Ranking orders of different MADM methods

DM method	Parameter value	Ranking order for case 0	Ranking order for case 1	Ranking order for case 2	Ranking order for case 3
The proposed NZNLSM	λ=0.1	$U_3>U_2>U_4>U_1$	$U_1>U_3>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
	λ=0.2	$U_3>U_2>U_4>U_1$	$U_1>U_3>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
	λ=0.3	$U_3>U_2>U_4>U_1$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
	λ=0.4	$U_3>U_2>U_4>U_1$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
	λ=0.5	$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
	λ=0.6	$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
	λ=0.7	$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
	λ=0.8	$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
	λ=0.9	$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
	λ=1.0	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
NZNWAA [27]		$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
NZNWGA [27]		$U_3>U_2>U_1>U_4$	$U_1>U_3>U_2>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
NZNDWAA [28]	q=1	$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
NZNDWGA [28]	q=1	$U_1>U_2>U_3>U_4$	$U_1>U_2>U_3>U_4$	$U_3>U_2>U_1>U_4$	$U_2>U_3>U_1>U_4$
NZNAAWAA [29]		$U_3>U_2>U_1>U_4$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_1>U_4$	$U_3>U_2>U_1>U_4$
NZNAAWGA [29]		$U_1>U_2>U_4>U_3$	$U_1>U_2>U_4>U_3$	$U_2>U_1>U_4>U_3$	$U_2>U_1>U_4>U_3$
NZNDSM [30]	ρ=1	$U_3>U_2>U_4>U_1$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
NZNCSM [30]	ρ=1	$U_3>U_2>U_4>U_1$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$
NZNCTSM [30]	ρ=1	$U_3>U_2>U_4>U_1$	$U_3>U_1>U_2>U_4$	$U_3>U_2>U_4>U_1$	$U_3>U_2>U_4>U_1$

From Table 4, compared with the results for Case 0, the sorting position of U_1 in Case 1 is shifted forward, and backward in both Case 2 and Case 3. This is because the proposed LSM value of U_1 increases with the increase of T_R and decreases with the increase of I_R or V_R. Moreover, the smaller the value of the parameter λ, the more obvious the shift of the position of U_1. For example, in Case 1, when λ is less than or equal to 0.2, U_1 is shifted forward from the fourth position to the first position in the sequence, whereas when λ = 0.3 or 0.4, U_1 moves forward to the second position; when λ is in the range of 0.5 to 0.9, U_1 only moves to the third position in the sequence; and when λ = 1, the position of U_1 in the sequence even remains unchanged. The reason can be deduced from Eq. (6) that the exponent of the confidence level is $1 - \lambda$. Therefore, the smaller the parameter value of λ, the more sensitive the proposed LSM is to the reliability level. And when λ = 1, the proposed LSM is reduced to LSM of SvNSs without considering the reliability level, and the related MADM method can process the SvNN information.

In addition, the DM results of the proposed approach at each parameter value of λ almost cover most of the DM results of the published ones. In details, the ranking results with λ= 0.3 or 0.4 are identical to the similarity measures of NZNDSM, NZNCSM and NZNCTSM [30], while the ranking results at λ = 0.7, 0.8 and 0.9 are the same as those of the NZNWAA [27], NZNDWAA [28] and NZNAAWAA [29] operators.

In summary, a smaller value of λ can be used if the decision result is desired to be more sensitive to the reliability measure. On the contrary, a larger value of λ should be chosen if the decision outcome requires more consideration of the restraint. In order to balance the restraint and reliability, an intermediate value of λ, such as 0.5, can be used.

5. Conclusions

In this study, a generalized LSM of NZNSs was presented and its properties were investigated. After that, a MADM approach was put forward based on the LSM of NZNSs to deal with DM problems in NZN setting. At last, the application of the presented MADM method was illustrated by a UTQE example, and the effectiveness and flexibility of the DM method was further verified by comparative analysis and sensitivity analysis. The UTQE results showed that the LSM of NZNSs-based MADM approach demonstrates more possible ranking results by determining the index parameter of the restraint and reliability in terms of the evaluator's preferences. The proposed NZNLSM is more of a generalized LSM because the logarithmic base ε hardly affects the ranking results. However, the logarithmic base ε of the proposed NZNLSM is finite under the condition of $\varepsilon \geq 2$. Therefore, future research can focus on more reliable similarity measures or aggregation operators of NZNSs to overcome the above limitations and develop more reliable DM methods of NZNSs for applications in UTQE, medical diagnosis, and other areas.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Wang, R.; Rong, X. Integrated MAGDM framework based on spherical fuzzy sets for teaching quality evaluation of ideological and political courses in universities. *Journal of Intelligent & Fuzzy Systems* **2023**, 45, 2799–2811.
2. Gong, J. W.; Li, Q.; Yin, L.; Liu H.C. Undergraduate teaching audit and evaluation using an extended MABAC method under q-rung orthopair fuzzy environment. *International Journal of Intelligent Systems* **2020**, 35, 1912–1933.
3. Dou, W. W. MADM framework based on the triangular Pythagorean fuzzy sets and applications to college public English teaching quality evaluation. *Journal of Intelligent & Fuzzy Systems* **2023**, 45, 4395–4414.
4. Zhou, Y.; Hang, L. Evaluation method of online education quality based on fuzzy rough set. *International journal of continuing engineering education and life-long Learning* **2023**, 33, 403–417.
5. Zhang, Y. Y. Approaches to multiple attribute group decision making under interval-valued Pythagorean fuzzy sets and applications to environmental design majors teaching quality evaluation. *International Journal of Knowledge-Based and Intelligent Engineering Systems* **2023**, 27, 289–301.
6. Ali, J.; Bashir, Z.; Rashid, T. Weighted interval-valued dual-hesitant fuzzy sets and its application in teaching quality assessment. *Soft Computing* **2021**, 25, 3503–3530.

7. Han, Y. G.; Yan, R.; Gou, C. An integrated multiple attribute decision making methodology for quality evaluation of innovation and entrepreneurship education with interval-valued intuitionistic fuzzy information. *Journal of Intelligent & Fuzzy Systems* **2023**, 44, 2231–2249.
8. Mao, L. X.; Lan, J.; Li, Z. F.; Shi, H. Undergraduate teaching audit and evaluation using an extended Oreste method with interval-valued hesitant fuzzy linguistic sets. *Systems* **2023**, 11, 2079–8954.
9. Anitha, S.; Shalini, A. F. Similarity measure of Plithogenic cubic vague sets: examples and possibilities. Neutrosophic Systems With Applications **2023**, 11, 39–47.
10. Smarandache, F. Neutrosophy: neutrosophic probability, set, and logic; American Research Press: Rehoboth, USA, **1998**.
11. Ye, J. A multi-criteria decision making method using aggregation operators for simplified neutrosophic sets. *Journal of Intelligent & Fuzzy Systems* **2014**, *26*, 2459–2466.
12. Ramya, G.; Francina Shalini, A. Trigonometric similarity measures of Pythagorean neutrosophic hypersoft sets. *Neutrosophic Systems With Applications* **2023**, *9*, 91–100.
13. Zhao, M. L.; Ye, J. MCGDM approach using the weighted hyperbolic sine similarity measure of neutrosophic (indeterminate fuzzy) multivalued sets for the teaching quality assessment of teachers. *Neutrosophic Sets and Systems* **2022**, *48*, 1–8.
14. Zhao, M. L.; Ye, J. MADM technique using tangent trigonometric SVNN aggregation operators for the teaching quality assessment of teachers. *Neutrosophic Sets and Systems* **2022**, *50*, 651–662.
15. Lu, X. P.; Zhang, T.; Fang, Y. M.; Ye, J. TOPSIS method-based decision-making model of simplified neutrosophic indeterminate sets for teaching quality evaluation. *Neutrosophic Sets and Systems* **2023**, *53*, 97–106.
16. Song, T. Method for 2-tuple linguistic neutrosophic number MAGDM and applications to physical education teaching quality evaluation in colleges and universities. *Journal of Intelligent & Fuzzy Systems* **2023**, 44, 4233–4244.
17. Gao, K. L.; Yu, F. Q. An improved GRA algorithm for teaching quality evaluation of college physical education based on the probabilistic simplified neutrosophic sets. *Journal of Intelligent & Fuzzy Systems* **2023**, 45, 5199–5210.
18. Wei, L. An integrated decision-making framework for blended teaching quality evaluation in college english courses based on the double-valued neutrosophic sets. *Journal of Intelligent & Fuzzy Systems* **2023**,45, 3259–3266.
19. Geng, X. N. Combining grey relation analysis with MCGDM to evaluate the higher education management quality with interval-valued neutrosophic sets. *Journal of Intelligent & Fuzzy Systems* **2023**, 45, 3791–3805.
20. Zadeh, L. A. A note on z-numbers. *Inf Sci* **2011**, *181*, 2923–2932.
21. Aliev, R. A.; Alizadeh, A. V.; Huseynov, O. H. The arithmetic of discrete Z-numbers. *Information Sciences* **2015**, *290*, 134–155.
22. Aliev, R. A.; Huseynov, O. H.; Zeinalova, L. M. The arithmetic of continuous Z-numbers. *Information Sciences* **2016**, *373*, 441–460.
23. Aliev, R. A.; Pedrycz, W.; Huseynov, O. H.; Eyupoglu, S. Z. Approximate reasoning on a basis of Z-number-valued if–then rules. *IEEE Transactions on Fuzzy Systems* **2017**, *25*, 1589–1600.
24. Aboutorab, H.; Saberi, M.; Asadabadi, M. R.; Hussain, O.; Chang, E. ZBWM: the Z-number extension of best worst method and its application for supplier development. *Expert Systems with Applications* **2018**, *107*, 115–125.
25. Ren, Z.; Liao, H.; Liu, Y. Generalized Z-numbers with hesitant fuzzy linguistic information and its application to medicine selection for the patients with mild symptoms of the COVID-19. *Computers & Industrial Engineering* **2020**, *145*, 106517.
26. Kang, B.; Zhang, P.; Gao, Z.; Chhipi-Shrestha, G.; Hewage, K.; Sadiq, R. Environmental assessment under uncertainty using Dempster–Shafer theory and Z-numbers. *Journal of Ambient Intelligence and Humanized Computing* **2020**, *11*, 2041–2060.
27. Du, S.; Ye, J.; Yong, R.; Zhang, F. Some aggregation operators of neutrosophic Z-numbers and their multicriteria decision making method. *Complex and Intelligent Systems* **2021**, *7*, 429–438.
28. Ye, J.; Du, S.; Yong, R. Dombi weighted aggregation operators of neutrosophic Z-numbers for multiple attribute decision making in equipment supplier selection. *Intelligent Decision Technologies* **2022**, *16*, 9–21.

29. Ye, J.; Du, S.; Yong, R. Aczel–Alsina weighted aggregation operators of neutrosophic Z-numbers and their multiple attribute decision-making method. *International Journal of Intelligent Systems* **2022**, *24*, 2397–2410.
30. Ye, J. Similarity measures based on the generalized distance of neutrosophic Z-number sets and their multi-attribute decision making method. *Soft Computing* **2021**, *25*, 13975–13985.
31. Karabacak, M. Correlation coefficient for neutrosophic Z-numbers and its applications in decision making. *Journal of Intelligent & Fuzzy Systems* **2023**, 45(1), 215–228.
32. Yong, R.; Ye, J.; Du, S. Multicriteria decision-making method and application in the setting of trapezoidal neutrosophic Z-numbers. *Journal of Mathematics* **2021**, 2021, Article ID 6664330, 13 pages.
33. Ye, J.; Yong, R.; Du, S. Multiattribute group decision-making method in terms of linguistic neutrosophic Z-number weighted aggregation operators. *Journal of Mathematics* **2022**, 2022, 9509823, 12 pages.
34. Cui, W. H.; Ye, J. Logarithmic similarity measure of dynamic neutrosophic cubic sets and its application in medical diagnosis. *Computers in Industry* **2019**, 111, 198–206.

Received: Jan 13, 2024. Accepted: Mar 30, 2024

Analyzing the Income-Education Nexus in Ecuador: A Neutrosophic Statistical Approach

Guido Macas-Acosta[1], Juan-Antonio Jimber-del-Río [2], Fidel Márquez-Sánchez [3]* and Arnaldo Vergara-Romero[4]

[1] Universidad ECOTEC; Guayaquil, Ecuador, gmacas@ecotec.edu.ec
[2] Universidad de Córdoba (España), WEARE-Water, Environmental, and Agricultural Resources Economics Research Group.; España, jjimber@uco.es
[3] Universidad de Especialidades Espíritu Santo, Ecuador; fmarquez@uees.edu.ec
[4] Universidad ECOTEC; Guayaquil, Ecuador, avergarar@ecotec.edu.ec

* Correspondence: fmarquez@uees.edu.ec

Abstract: This study examines the relationship between education and entrepreneur income in Ecuador, a prototypical small economy grappling with employment challenges. Utilizing quantile regression and neutrosophic statistical methods, the research uncovers varied impacts of education on earnings across different income levels. The findings reveal a decrease in indeterminacy as individuals ascend income quintiles, indicating that higher education levels yield more predictable and substantial economic returns at the upper end of the income spectrum. These insights are vital for policymakers to develop targeted educational interventions. The study's methodological novelty lies in its adoption of neutrosophic statistics, which embraces the indeterminacy intrinsic to economic data, offering a refined lens for understanding the income-education interplay. Future research directions include longitudinal studies to trace the temporal effects of education and cross-comparative analyses across economies. The potential integration of machine learning with neutrosophic statistics promises enhanced predictive models, contributing to data-driven economic policy formulation.

Keywords: Neutrosophic Statistics, Quantile Regression, Education, Entrepreneur Income, Economic Policy

---

1. Introduction

In the dynamic landscape of global economies, small economies face the pressing challenge of generating sufficient employment to sustain their populations. This scarcity of traditional job opportunities compels individuals, irrespective of age or gender, to seek alternative income sources to meet their basic survival needs. Amidst this backdrop, entrepreneurship and self-employment have emerged as significant avenues for employment, accounting for 33.3% of job occupation worldwide, trailing behind paid employment (53.4%), which includes positions in both the private and public sectors as reported by the International Labor Organization [1]. In Ecuador, a country representative of these small economies, understanding the dynamics that influence entrepreneur income is critical for economic development and policy formulation [2].

This study delves into the intricacies of the income-education nexus within Ecuador, employing a neutrosophic statistical approach [3] to uncover the principal determinants of entrepreneur income.

By leveraging quantile regression, we aim to dissect the varying impacts of independent variables across different distribution points, providing insights into the heterogeneity of effects. This methodological choice is particularly pertinent in the presence of heteroscedasticity [4], a condition where error variances are not constant, thereby allowing for a nuanced analysis that traditional regression models might overlook.

Furthermore, the application of neutrosophic statistics introduces a novel perspective to our analysis. This theoretical framework operates on the premise that models and parameters should be defined over intervals rather than fixed numerical values, offering a unique advantage in terms of flexibility and adaptability [5]. By embracing the indeterminacy inherent in economic data [6] and modeling, we aspire to achieve a more accurate depiction of the income-education relationship in Ecuador, thereby contributing to a deeper understanding of economic phenomena in small economies.

The paper continues with a description of the methodologies employed in the investigation. Subsequently, the outcomes of the analysis are presented showing the impact of socioeconomic determinants such as educational attainment, marital status, gender, and geographical location on the income distribution among entrepreneurs. Following the presentation of results, the discussion segment interprets the findings. Finally, conclusions consolidate the principal discoveries and propose future areas of investigation.

2. Some notions on Neutrosophic Statistics

This section introduces foundational aspects of neutrosophic statistics, an extension of classical statistics that utilizes set values instead of precise numerical values. It encompasses Neutrosophic Descriptive Statistics, which summarize and delineate the characteristics of neutrosophic numerical data, and Neutrosophic Inferential Statistics, allowing for extrapolation from a neutrosophic sample to its broader population [7,8].

Neutrosophic Data embodies indeterminacy, with discrete neutrosophic data represented by distinct points and continuous neutrosophic data by one or multiple intervals. This data can be quantitative, such as an uncertain number within an interval, or qualitative, like color identification with uncertainty. Observations can be univariate, focusing on a single neutrosophic attribute, or multivariate, encompassing multiple attributes [9].

A Neutrosophic Statistical Number [10] consists of a determinate part (d) and an indeterminate part (I), expressed as N = d + I. The concept of Neutrosophic Frequency Distribution arises from categorizing frequencies and relative frequencies amidst indeterminacy, often stemming from imprecise or unknown data, thus affecting the precision of relative frequencies. Similarly, Neutrosophic Survey Results and Populations incorporate indeterminacy, impacting the certainty of membership within a population.

Sampling methods, like simple random neutrosophic sampling and stratified random neutrosophic sampling, adapt to this framework by accounting for indeterminacy in sample selection. Unlike interval statistics, which accumulate uncertainty from one operation to the next, neutrosophic statistics reduce or even eliminate uncertainty. The paper also delves operations between neutrosophic numbers, with operations defined as follows [11]:

Addition $(N_1 + N_2) = a_1 + a_2 + (b_1 + b_2)I$
Subtraction $(N_1 - N_2) = a_1 - a_2 + (b_1 - b_2)I$
Multiplication $(N_1 \times N_2) = a_1a_2 + (a_1b_2 + b_1a_2 + b_1b_2)I$
Division $\frac{N_1}{N_2} = \frac{a_1+b_1I}{a_2+b_2I} = \frac{a_1}{a_2} + \frac{a_2b_1-a_1b_2}{a_2(a_2+b_2)}I$

In machine learning, intervals play a crucial role in gauging the level of uncertainty tied to estimates and predictive analyses. Prediction intervals [12] set probabilistic boundaries, both upper and lower, for the anticipated outcome based on a predetermined confidence level, such as 95%. This suggests that, typically, 95 of every 100 future observations should fall within these boundaries. On the other hand, confidence intervals [13] aim to capture the uncertainty of an estimate to demonstrate a machine learning algorithm's effectiveness on unfamiliar data. Differing from prediction intervals that delineate the range for a single data point, confidence intervals define the expected range for a population metric, like the average.

2. Materials and Methods

The dataset in this research comes from the National Survey of Employment, Unemployment, and Underemployment (ENEMDU), carried out by the Ecuadorian Institute of Statistics and Census (INEC) in December 2021. The initial data was processed and filtered to obtain the most efficient and consistent estimates. The database was configured with a two-stage, stratified, and cluster sampling design to obtain population estimators. Then, individuals between the ages of 15 and 80 are selected, and their job occupation is classified as self-employed workers, which is considered a business or enterprise.

This research will focus on quantile regression [14] to measure this relationship. It involves fitting separate line segments, restricted by a quartile or quintile, that account for nonlinearity between the predictor and the outcome.

The next step is quantile regression to predict the earnings (lingemp) logarithm in the different quartiles. It will produce a model for various quintiles or quartiles ranging from 5 to 95. The following expression expresses the model:

$$lingem_i = \delta eduemp_i + \vartheta edaemp_i + \gamma exp_i + \theta sex_i + \varphi urbano_i + u_i \quad (1)$$

lingem, the logarithm of income of entrepreneurs.
eduemp, years of entrepreneurship education.
edaemp, age of entrepreneurs.
exp, work experience in entrepreneurship.
sexo, gender of entrepreneurs.
urbano, urban location.
u, is the stochastic disturbance term or random error.

With quantile regression, it is possible to observe if there are differences in the effects of the independent variables depending on the point of the distribution that is analyzed. Furthermore, this type of regression is helpful in the presence of heteroscedasticity. It occurs when the variance of the errors is not constant.

A quantile regression is estimated that establishes the relationship between the variation in income and a series of explanatory variables for seven quintiles of the endogenous variable with the following specifications:

$$lingem_{i1} = \beta_{0.05} + \delta_{0.05} eduemp_{i1} + \alpha_{0.05} edaemp_{i1} + \gamma_{0.05} exp_{i1} + \theta_{0.05} sex_{i1} + \varphi_{0.05} urbano_{i1} + u_{i1} \quad (2)$$

$$lingem_{i2} = \beta_{0.10} + \delta_{0.10} eduemp_{i2} + \alpha_{0.10} edaemp_{i2} + \gamma_{0.10} exp_{i2} + \theta_{0.10} sex_{i2} + \varphi_{0.10} urbano_{i2} + u_{i2} \quad (3)$$

$$lingem_{i3} = \beta_{0.25} + \delta_{0.25} eduemp_{i3} + \alpha_{0.25} edaemp_{i3} + \gamma_{0.25} exp_{i3} + \theta_{0.25} sex_{i3} + \varphi_{0.25} urbano_{i3} + u_{i3} \quad (4)$$

$$lingem_{i4} = \beta_{0.50} + \delta_{0.50} eduemp_{i4} + \alpha_{0.50} edaemp_{i4} + \gamma_{0.50} exp_{i4} + \theta_{0.50} sex_{i4} + \varphi_{0.50} urbano_{i4} + u_{i4} \quad (5)$$

$$lingem_{i5} = \beta_{0.75} + \delta_{0.75} eduemp_{i5} + \alpha_{0.75} edaemp_{i5} + \gamma_{0.75} exp_{i5} + \theta_{0.75} sex_{i5} + \varphi_{0.75} urbano_{i5} + u_{i5} \quad (6)$$

$$lingem_{i6} = \beta_{0.90} + \delta_{0.90} eduemp_{i6} + \alpha_{0.90} edaemp_{i6} + \gamma_{0.90} exp_{i6} + \theta_{0.90} sex_{i6} + \varphi_{0.90} urbano_{i6} + u_{i6} \quad (7)$$

$$lingem_{i7} = \beta_{0.95} + \delta_{0.95} eduemp_{i7} + \alpha_{0.95} edaemp_{i7} + \gamma_{0.95} exp_{i7} + \theta_{0.95} sex_{i7} + \varphi_{0.95} urbano_{i7} + u_{i7} \quad (8)$$

In neutrosophic statistics, the confidence interval can be interpreted as an indicator of indeterminacy. In this context, Equation 9 defines the indeterminacy of an interval Im as the difference between the upper limit [15]:

$$\gamma(Im) = a2 - a1 \quad (9)$$

where Im = [a1, a2] be an interval and a2, a2 are lower and upper bounds of the confidence interval, respectively

The measure γ(Im) quantifies how much we do not know about the parameter we are estimating. The greater this value, the more indeterminacy there is. Graphically, a wider confidence band (a larger shaded area) would indicate more uncertainty in the estimation of the coefficients at that specific quantile [16].

3. Results

Quantile regression was developed (Table 1) to delve into how the returns on various factors such as education, marital status, sex, and location differ across the income distribution of entrepreneurs.

Table 11. Result of the Models with Logarithm of the Dependent Variable.

Variables	OLS	QR_05	QR_10	QR_25	QR_50	QR_75	QR_90	QR_95
eduemp	0.056***	0.049***	0.0477***	0.052***	0.054***	0.056***	0.062***	0.067***
	(0.001)	(0.003)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
edaemp	0.079***	0.102***	0.0951***	0.091***	0.077***	0.064***	0.058***	0.065***
	(0.002)	(0.005)	(0.004)	(0.003)	(0.002)	(0.002)	(0.002)	(0.003)
edaemp2	-0.001***	-0.001***	-0.001***	-0.001***	-0.001***	-0.001***	-0.001***	-0.001***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
exp	0.003***	0.006***	0.006***	0.005***	0.003***	0.002***	0.003***	0.003***
	(0.000)	(0.001)	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)	(0.001)
sex	0.547***	0.854***	0.848***	0.730***	0.513***	0.352***	0.265***	0.278***
	(0.009)	(0.021)	(0.016)	(0.013)	(0.010)	(0.009)	(0.012)	(0.017)
married	0.115***	0.059***	0.078***	0.103***	0.147***	0.151***	0.166***	0.180***
	(0.009)	(0.025)	(0.021)	(0.013)	(0.010)	(0.009)	(0.012)	(0.017)
urban	0.300***	0.460***	0.423***	0.378***	0.315***	0.222***	0.165***	0.163***
	(0.010)	(0.027)	(0.022)	(0.014)	(0.012)	(0.011)	(0.014)	(0.18)
_cons	2.295***	0.023***	0.632***	1.393***	2.447***	3.374***	3.938***	3.996***
	(0.043)	(0.1113)	(0.087)	(0.061)	(0.050)	(0.047)	(0.057)	(0.081)
Observations	55.857	55.857	55.857	55.857	55.857	55.857	55.857	55.857
R-squared	0.1982	0.1124	0.1172	0.1233	0.1106	0.0963	0.0927	0.0965

The main findings are summarized below:

- Education: The study found that the benefit of an additional year of education on earnings varies across the income distribution but remains significant across all quintiles. Notably, higher returns on education are evident at the higher end of the income spectrum. In the 5th quintile, an additional year of education is associated with an increase in income by $1.84, while in the 95th quintile, the increase is $52.45.

- Marital Status: Being married or in a common-law union positively influences earnings across the distribution. The effect is more pronounced at the upper end, where partnered entrepreneurs earn 12.1% more than their single peers in the 95th quintile.
- Gender: The coefficient analysis reveals that male entrepreneurs consistently out-earn their female counterparts across all educational quintiles, with a substantial 80% higher earnings at the lower conditional quintiles.
- Urban Location: Entrepreneurs located in urban areas have a clear income advantage over those in rural areas, with earnings between 10% to 45% higher, showcasing the importance of geographical location in economic success.
- Age: Age shows a positive correlation with earnings, but its impact diminishes in the higher income quintiles, indicating that younger entrepreneurs tend to earn more in the lower quintiles of the distribution.
- Experience: While experience contributes to income, its impact is relatively minor, not exceeding 1%.
- Coefficients and Confidence Intervals: The study also highlights that the constants and slopes of these relationships significantly differ across quintiles, with a steeper slope observed from the 75th quintile upwards. This suggests an increasing return on these variables, especially education, in higher income brackets.

The quantile regression plot (Figure 1) depicts the impact of years of entrepreneurship education, labeled as "eduemp," on the dependent variable across different points of its distribution. The Y-axis measures the size of the "eduemp" coefficient, indicating the strength and direction of its association with the outcome variable at various quantiles, displayed on the X-axis, ranging from the lowest (0) to the highest (1).

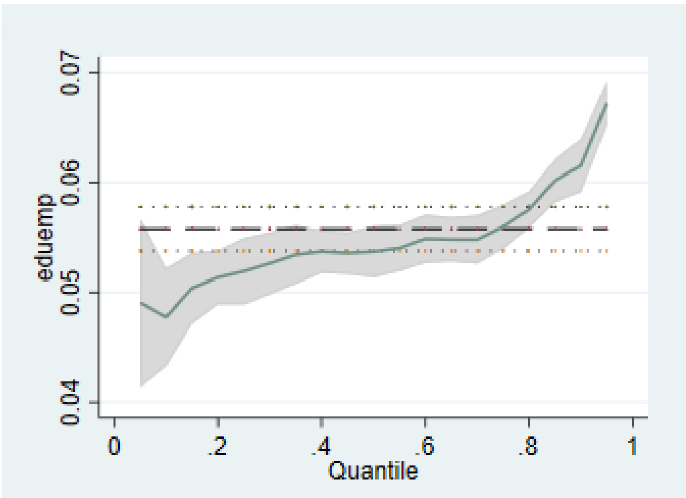


Figure 1. Quantile Regression of Entrepreneurship Education Years ('eduemp') on Outcome Variable

Indeterminacy is calculated (Figure 2) and normalized to observe the relative variability and trends more clearly in the data across various quintiles.

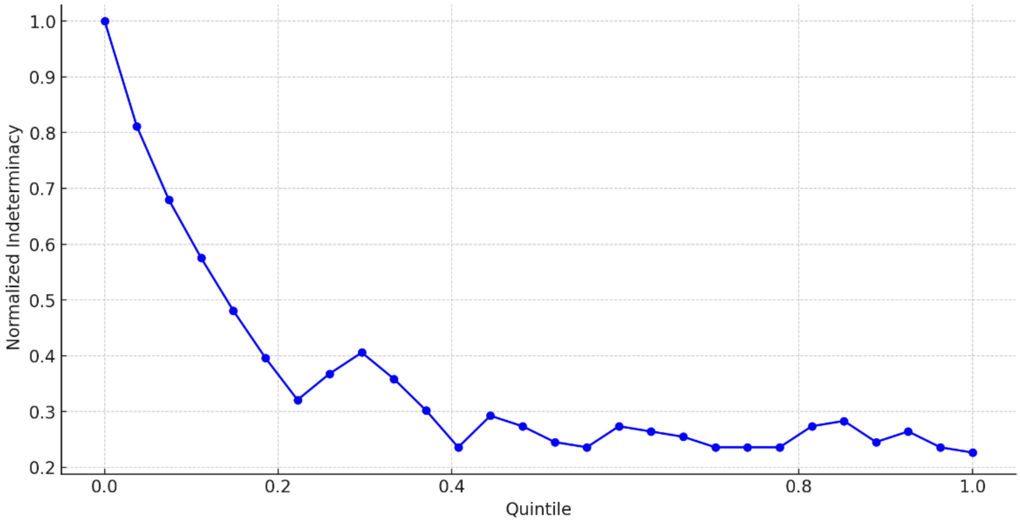


Figure 2. Normalized Indeterminacy Across Quintiles

The pattern depicted in the figure reveals a decline in indeterminacy, hinting at a progressively uniform influence of educational attainment on earnings as one ascends the income quintiles. This visual evidence underscores a pivotal consideration: the impact of education on income is not uniform across the spectrum. Consequently, this necessitates a nuanced approach in policy-making that acknowledges and addresses the variable effects of educational investment across different income segments.

4. Discussion

A few key observations can be drawn and related to the findings on education's impact on earnings and the calculation of indeterminacy [17, 18, 19]:

1. Decreasing Indeterminacy with Income: The graph shows a steep initial decline in indeterminacy, which then stabilizes as it progresses through the quintiles. This could suggest that at lower income levels, the impact of additional education on earnings is more variable. As we move up the income distribution, the effect of education becomes more predictable, with higher quintiles showing less indeterminacy and hence a more consistent return on investment in education.
2. Higher Returns at Higher Income Levels: The research indicated that additional education has higher returns at the upper end of the income spectrum, which corresponds to the lower indeterminacy at higher quintiles seen in the graph. This relationship suggests a positive correlation between the certainty of returns on education and the income level, reinforcing the finding that additional education is a more significant predictor of income increases for those already at the higher end of the income distribution.
3. Implications for Neutrosophic Statistics: In the framework of neutrosophic statistics, the varying levels of indeterminacy across quintiles can be seen as an opportunity to explore the elements of truth, falsity, and indeterminacy in the data. Neutrosophic statistics could be used to model these elements more explicitly, potentially offering richer insights into the nuanced way education impacts earnings.

For instance, where indeterminacy is high, the neutrosophic approach could help to uncover the underlying factors contributing to this uncertainty, such as differing quality of education, the varying economic value of different fields of study, or the impact of network effects and social capital that might not be captured in a traditional statistical model[20]. Conversely, at higher quintiles, where there is less indeterminacy, neutrosophic statistics could help validate the robustness of the observed trends, confirming the strong positive impact of additional education on earnings for individuals in these segments.

Incorporating neutrosophic statistics into this analysis could thus enhance the understanding of educational returns [21] across income levels, enabling more targeted and effective educational policies and interventions that are responsive to the observed heterogeneity in the data [22].

5. Conclusions

The research presented in this paper reveals a pattern of decreasing indeterminacy with ascending income quintiles, indicating that the influence of educational attainment on earnings stabilizes at higher income levels. This suggests that the economic returns on additional education are more predictable and potentially greater for individuals in the upper levels of the income distribution. These findings are instrumental for policymakers, as they highlight the necessity of crafting nuanced educational policies that consider the heterogeneous effects of education across the income spectrum. By focusing on the areas where educational investment yields substantial benefits, policymakers can better allocate resources to enhance economic outcomes.

Looking ahead, the application of neutrosophic statistics in this study introduces an innovative methodology that more accurately reflects the uncertainties present in economic data. Prospective research could build on this foundation by applying a neutrosophic framework to analyze other socio-economic factors, thereby unraveling the intricate factors influencing income. Future initiatives could include longitudinal analyses to track the long-term effects of education on earnings or cross-country comparisons to understand these dynamics in various economic contexts. Moreover, integrating machine learning with neutrosophic statistical methods could lead to the development of more sophisticated and flexible predictive models, thus equipping policymakers with advanced tools for fostering economic growth and development.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

[1]. ILO, International Labour Organization. Social and Employment Perspectives in the World. Geneva: International Labour Organization, 2022. Accessed on January 17, 2024. Available at: https://www.ilo.org/wcmsp5/groups/public/---dgreports/---dcomm/---publ/documents/publication/wcms_848464.pdf.

[2]. Pérez-Campdesuñer, R., García-Vidal, G., Sánchez-Rodríguez, A., Martínez-Vivar, R., de Miguel-Guzmán, M., & Guilarte-Barinaga, E. Influence of the socio-economic environment on the entrepreneurs behavior. Cases of Cuba and Ecuador. International Journal of Engineering Business Management 2021, 13, 1847979021994509.

[3]. Smarandache, F. Plithogenic Probability & Statistics are generalizations of MultiVariate Probability & Statistics. Neutrosophic Sets and Systems 2021, 43(1), 20.

[4]. Guan, H.; Guan, S.; Zhao, A. Forecasting model based on neutrosophic logical relationship and Jaccard similarity. Symmetry 2017, 9(9), 191.

[5]. Kandemir, H. Ş.; Aral, N. D.; Karakaş, M.; Et, M. Neutrosophic Statistical Analysis of Temperatures of Cities in the Southeastern Anatolia Region of Turkey. Neutrosophic Systems with Applications 2024, 14, 50-59.

[6]. Moscoso-Paucarchuco, K.; Raiser Vásquez-Ramírez, M.; Yupanqui-Villanueva, H. R.; Yupanqui-Villanueva, W. F.; Vivanco-Nuñez, O. A.; Beraún-Espíritu, M. M.; Fernández-Jaime, R. J.; Gutiérrez-Gómez, E. Study of the relationship among economic variables in cattle production in a region of Peru,

based on Plithogenic Statistics. Neutrosophic Sets and Systems 2024, 65, 1-10. DOI: 10.5281/zenodo.10729952.

[7]. Patro, S. K.; Smarandache, F. The Neutrosophic Statistical Distribution - More Problems, More Solutions. Neutrosophic Sets and Systems 2016, 12, 73-79. doi:10.5281/zenodo.571153

[8]. Reales Chacón, L.; Campos Moposita, A.; Marquina Amón, D.; Garcia Camacho, V.; Bonilla Ayala, J.; Caiza Lema, J.; Aguilar Zapata, Y.; Cantuña Vallejo, P. Study of the impact of cardiovascular exercises and their functional responses on diabetic peripheral neuropathy using neutrosophic statistics. Neutrosophic Sets and Systems 2024, 64, 77-83. DOI: 10.5281/zenodo.10699498

[9]. Smarandache, F. Neutrosophic statistics is an extension of interval statistics, while plithogenic statistics is the most general form of statistics (third version). Bulletin of Pure & Applied Sciences-Mathematics and Statistics 2022, 41(2), 172-183.

[10]. Hassan, M. K.; Aslam, M. Birnbaum Saunders distribution for imprecise data: statistical properties, estimation methods, and real life applications. Scientific Reports 2024, 14(1), 6955.

[11]. Smarandache, F. Neutrosophic Statistics is an extension of Interval Statistics, while Plithogenic Statistics is the most general form of statistics (second version). International Journal of Neutrosophic Science (IJNS) 2022, 19(1).

[12]. Nourani, V.; Paknezhad, N. J.; Tanaka, H. Review of Prediction Interval Estimation Methods in Artificial Neural Network (ANN)-based Hydro-climatic Modeling. Sustainability 2021, 13(4), 1633.

[13]. Bayle, P.; Bayle, A.; Janson, L.; Mackey, L. Establishing Confidence Intervals for Test Error Through Cross-validation. Advances in Neural Information Processing Systems 2020, 33, 16339-16350.

[14]. Koenker, R. Quantile Regression: 40 Years On. Annual Review of Economics 2017, 9, 155-176.

[15]. Huera Castro, D. E.; Dávila Castillo, M. R.; Escobar Gonzáles, E. J.; Hidalgo Ruiz, M. R. Applying Neutrosophic Statistics to Analyze Tax Evasion Causes. Neutrosophic Sets and Systems 2021, 44, 427-435. DOI: 10.5281/zenodo.5163700.

[16]. Shah, F.; Aslam, M.; Khan, Z. New Control Chart Based on Neutrosophic Maxwell Distribution with Decision Making Applications. Neutrosophic Sets and Systems 2023, 53(1), 18.

[17]. Hernández, N. B.; Vázquez, M. Y. L.; Caballero, E. G.; Cruzaty, L. E. V.; Chávez, W. O.; Smarandache, F. A New Method to Assess Entrepreneurship Competence in University Students Using Plithogenic Numbers and SWOT Analysis. International Journal of Fuzzy Logic and Intelligent Systems 2021, 21(3), 280-292.

[18]. Nagarajan, D.; Broumi, S.; Smarandache, F.; Kavikumar, J. Analysis of Neutrosophic Multiple Regression. Neutrosophic Sets and Systems 2021, 43, 44-54.

[19]. Pupo, I. P.; Pérez, P. Y. P.; Vacacela, R. G.; Bello, R.; Santos, O.; Vázquez, M. Y. L. Extensions to Linguistic Summaries Indicators Based on Neutrosophic Theory, Applications in Project Management Decisions. Neutrosophic Sets & Systems 2018, 22.

[20]. Caballero, E. G.; Smarandache, F.; Leyva Vázquez, M. On Neutrosophic Offuninorms. Symmetry 2019, 11(9), 1136.

[21]. Barrientos Gutiérrez, P.; Flores-Ledesma, K. N.; Guanilo Gómez, S. L.; López Bulnes, J. L.; Jave Nakayo, J. L.; Cabrera Carranza, C. F.; Neri Ayala, A. C.; Ramos Yovera, S. E. Plithogenic Iadov Model to Study University Teaching Practices in the Complexity of the Educational Process of Comprehensive Training by Competencies. Neutrosophic Sets and Systems 2023, 62, 78-85. DOI: 10.5281/zenodo.10436629.

[22]. Sneyers, E., & De Witte, K. (2018). Interventions in Higher Education and Their Effect on Student Success: A Meta-Analysis. Educational Review, 70(2), 208-228.

Received: Jan 15, 2024. Accepted: Mar 30, 2024

Neutrosophic Sets and Systems, Vol. 66, 2024
University of New Mexico

An Intelligent Neutrosophic Type-II Model for Selecting Optimal Internet of Things (IoT) Service Provider: Analysis and Application

Ibrahim Alrashdi
Department of Computer Science, College of Computer and Information Sciences, Jouf University, Sakaka 2014, Saudi Arabia
Email: irrashdi@ju.edu.sa

Abstract: The Internet of Things, or IoT, is a rapidly expanding technology, and one of its most important application areas is sustainable transportation. Choosing the right IoT service provider is a complex process that is considered a multi-criteria decision issue. The multi-criteria decision-making (MCDM) methodology deals with massive criteria. This study proposed an MCDM methodology for selecting the best IoT service provider under different sustainable criteria. The Multi-Objective Optimization Ratio Analysis (MOORA) method is an MCDM methodology used to rank the alternatives. The MOORA method is integrated with a type 2 neutrosophic set to deal with uncertain information. The proposed MCDM methodology applied to six criteria and 13 alternatives. The results show that alternative 3 is the best and alternative 5 is the worst. The sensitivity analysis was conducted to show the stability of the rank. The sensitivity analysis was performed under seven cases; the results show that the rank of alternatives was stable under different cases. The comparative study was conducted to show the effectiveness of the proposed methodology.

Keywords: Internet of Things; Soft Computing; MCDM; Neutrosophic Logic; IoT Services; Artificial Intelligence.

1. Introduction

The Internet of Things, or IoT, is a network of different things with sensors installed. These items generate and send data to the cloud for processing, which allows for the inference of essential information for making decisions. Following cloud computing and services computing, it is the next emerging paradigm since the user may more efficiently use the diverse capabilities of the Internet of Things based on service-oriented computing. Many large corporations, like Microsoft, IBM, Cisco, Samsung, and Nokia, are investing in technology advancement to establish IoT services and make them viable for their clients. The increasing number of devices with sensors, the widespread availability of high-speed Internet, the decreased cost of device connections to the Internet, etc., are the main factors contributing to the market of IoT[1].

As the Internet of Things grows and expands, many new IoT service providers are jumping on board to provide a wide range of these services. These providers assert that their case studies, which include several noteworthy customer references, provide stronger support. A service's quality might differ depending on the application. A service's ability to satisfy customer needs effectively determines its success. Defining and identifying their needs to choose an appropriate service can be difficult for users. Users can recognize their needs and evaluate the characteristics of various services to select the best option from those offered with an efficient tool or set of recommendations. An IoT customer may wind up paying more than necessary in addition to vendor lock-in if quality of service (QoS) criteria connected to IoT are thoroughly described in one location. This is something that can be beneficial[2].

Understanding QoS metrics in IoT requires exploring the different components involved in IoT application development. To illustrate this, let's consider one of the most popular IoT applications—the smart home. In a smart home, several sensor-equipped household equipment run without human input. These devices make wise decisions, such as adjusting the air conditioner's temperature based on the weather, controlling the lights based on the number of people, and using motion detection for security. This not only enhances the home's functionality but also gives the owner peace of mind[3], [4].

The following are this application's four primary components: The first element consists of a network of different sensors, such as temperature sensors, that are either independently operating or connected to the appliances to perceive data from the surroundings. The information transmission is the second element. Sensors may be linked to a local hub that securely transmits and stores the data after it's been gathered, analyzed, and stored. If necessary, it can also send an alarm to the owner's family members. The third element is an interface allowing users to control the household's gadgets and get notifications. The last part is service management, which handles the service providers' billing, upgrading, and other activities. From the preceding, it is clear that the three main components of nearly all Internet of objects applications are computation, communication, and objects. To comprehend IoT services, one must have access to QoS data about every component. Because IoT infrastructure requires a significant amount of hardware and software, experts predict it is impractical for one business to provide an end-to-end IoT solution. However, significant participants may work together to create quicker, more effective, and more affordable solutions by enhancing one another's technological and commercial expertise[5], [6].

When a service comprises several sub-services, users or organizations require sub-service specialists to help them make the best possible decisions about which services to choose. IoT support is, therefore, only valid when the best and most practical IoT services are chosen through the integration and assessment of computing, communication, and object services. For these three services, the computing service provider (CSP), communication service provider (CSP), and Internet of Things service provider (IoTSP) are the accountable providers, in that order[7], [8].

The opinions of many decision-makers (DMs) or experts must be merged to determine the best choice in this kind of selection setting. Multi-criteria group decision-making difficulties are another name for these kinds of dilemmas. The goal of designing a group decision-making model is to give DM preferences a solid mathematical foundation so they can pick the best option out of those offered while considering the significance of different KPIs. The multi-criteria decision-making (MCDM) methodology is used to deal with various criteria. MCDM methodology is applied in various applications such as assessment of health sustainability [9], sustainability energy [10], supplier selection supply chain management [11], construction [12], cloud platform selection for smart framing [13], in bioenergy systems [14], in agriculture [15], in financial decision making [16], selecting optimal charcoal company [17].

DM has to deal with a variety of unknowns. Zadeh [18]created fuzzy set theory, which is widely used to address uncertainty. However, because it is hard to give a precise membership value, fuzzy set theory does not always account for all uncertainties. Zadeh [19]addressed this restriction by introducing the idea of type-2 fuzzy sets (T2FS). The fuzzy set was applied in various real-life applications such as solar hydrogen production [20], improvement projects [21], and blockchain technology strategies assessment [22]. However, in many real-world applications, the uncertainty is predicated on both indeterminacy grades and membership and non-membership degrees. Smarandache [23], [24]created the idea of neutrosophic sets (NS) to remedy this. Neutrosophic sets were applied in various applications such as the evaluation of solar power plants [25] and construction projects [26].

Brauers invented the Multi-Objective Optimisation Ratio Analysis (MOORA) method, which is regarded as an objective (non-subjective) approach. In addition, ranking is done using both desired and unwanted criteria at the same time to choose a higher or better option from the group of alternatives. There are a lot of uses for this method[27], [28]. The following characteristics of the MOORA approach are:

- *It is a component of compensating techniques.*
- *Features stand alone;*
- *The process transforms the qualitative characteristics into quantitative ones.*

2. Preliminaries

This section introduces some concepts of neutrosophic type 2 and their operations[29].

Definition 1.

Let x be the limited universe of discourse and F[0,1] be the set of all triangular neutrosophic numbers on F[0,1]. A type 2 neutrosophic number set (T2NNS) A in X is represented by $A = \{x, T_A(x), I_A(x), F_A(x) | \, x \in X\}$, where $T_A(x): X \rightarrow F[0,1], I_A(x): X \rightarrow F[0,1], and\ F_A(x): X \rightarrow F[0,1]$. A T2NNS

$T_A(x) = \left(T_{T_A}(x), T_{I_A}(x), T_{F_A}(x)\right), I_A(x) = \left(I_{T_A}(x), I_{I_A}(x), I_{F_A}(x)\right), and\ F_A(x) =$ $\left(F_{T_A}(x), F_{I_A}(x), F_{F_A}(x)\right)$ refers to the truth, indeterminacy, and falsified membership degrees.

$0 \leq T_A(x)^3 + I_A(x)^3 + F_A(x)^3 \leq 3$

$A = \left(\left(T_{T_A}(x), T_{I_A}(x), T_{F_A}(x)\right), \left(I_{T_A}(x), I_{I_A}(x), I_{F_A}(x)\right), \left(F_{T_A}(x), F_{I_A}(x), F_{F_A}(x)\right)\right)$ as a T2NNS.

Definition 2.

$$let\ A_1 = \begin{pmatrix} \left(T_{T_{A_1}}(x), T_{I_{A_1}}(x), T_{F_{A_1}}(x)\right), \\ \left(I_{T_{A_1}}(x), I_{I_{A_1}}(x), I_{F_{A_1}}(x)\right), \\ \left(F_{T_{A_1}}(x), F_{I_{A_1}}(x), F_{F_{A_1}}(x)\right) \end{pmatrix} \text{ and } A_2 = \begin{pmatrix} \left(T_{T_{A_2}}(x), T_{I_{A_2}}(x), T_{F_{A_2}}(x)\right), \\ \left(I_{T_{A_2}}(x), I_{I_{A_2}}(x), I_{F_{A_2}}(x)\right), \\ \left(F_{T_{A_2}}(x), F_{I_{A_2}}(x), F_{F_{A_2}}(x)\right) \end{pmatrix} \quad (1)$$

The sum of two T2NNS is:

$$A_1 \oplus A_2 = \begin{pmatrix} \begin{pmatrix} \left(T_{T_{A_1}}(x) + T_{T_{A_2}}(x) - T_{T_{A_1}}(x) T_{T_{A_2}}(x)\right) \\ \left(T_{I_{A_1}}(x) + T_{I_{A_2}}(x) - T_{I_{A_1}}(x) T_{I_{A_2}}(x)\right), \\ T_{F_{A_1}}(x) + T_{F_{A_2}}(x) - T_{F_{A_1}}(x) T_{F_{A_2}}(x) \end{pmatrix}, \\ \left(I_{T_{A_1}}(x) I_{T_{A_2}}(x), I_{I_{A_1}}(x)\, I_{I_{A_2}}(x), I_{F_{A_1}}(x) I_{F_{A_2}}(x)\right), \\ \left(F_{T_{A_1}}(x) F_{T_{A_2}}(x), F_{I_{A_1}}(x) F_{I_{A_2}}(x), F_{F_{A_1}}(x) F_{F_{A_2}}(x)\right) \end{pmatrix} \quad (2)$$

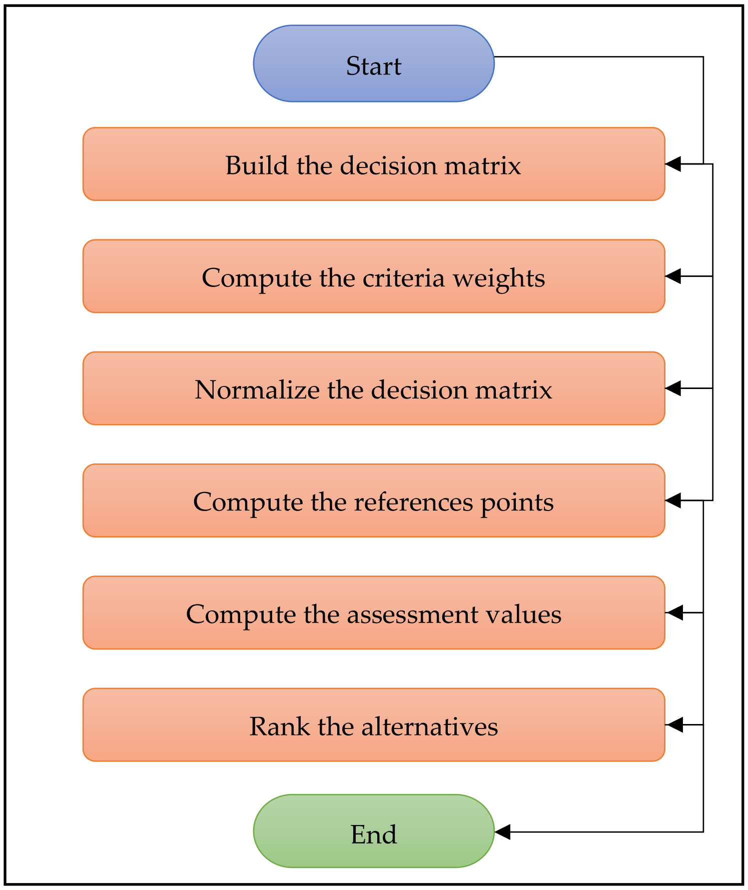


Figure 1. The steps of the proposed method.

Definition 3.

The multiplication of two T2NNS is:

$$A_1 \otimes A_2 = \begin{pmatrix} \left(T_{T_{A_1}}(x)T_{T_{A_2}}(x), T_{I_{A_1}}(x)\, T_{I_{A_2}}(x), T_{F_{A_1}}(x)T_{F_{A_2}}(x)\right), \\ \begin{pmatrix} \left(I_{T_{A_1}}(x) + I_{T_{A_2}}(x) - I_{T_{A_1}}(x)I_{T_{A_2}}(x)\right) \\ \left(I_{I_{A_1}}(x) + I_{I_{A_2}}(x) - I_{I_{A_1}}(x)I_{I_{A_2}}(x)\right), \\ I_{F_{A_1}}(x) + I_{F_{A_2}}(x) - I_{F_{A_1}}(x)I_{F_{A_2}}(x) \end{pmatrix}, \\ \begin{pmatrix} \left(F_{T_{A_1}}(x) + F_{T_{A_2}}(x) - F_{T_{A_1}}(x)F_{T_{A_2}}(x)\right) \\ \left(F_{I_{A_1}}(x) + F_{I_{A_2}}(x) - F_{I_{A_1}}(x)F_{I_{A_2}}(x)\right), \\ F_{F_{A_1}}(x) + F_{F_{A_2}}(x) - F_{F_{A_1}}(x)F_{F_{A_2}}(x) \end{pmatrix} \end{pmatrix} \quad (3)$$

3. Materials and Methods

This section presents the MCDM methodology to compute the weights of criteria and rank the alternatives under T2NSs to deal with vague information. Figure 1 shows the proposed MCDM methodology under

T2NSs. This study uses the MOORA method to rank the alternatives. The following are the steps of the proposed MCDM methodology:

Step 1. Build the decision matrix between criteria and alternatives.

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}_{m\times n} ; i = 1,2, \dots , m, \quad j = 1,2, \dots , n \tag{4}$$

Experts used T2NNS to evaluate the criteria and alternatives. Then we obtain the crisp values, then we combine these matrices into one matrix.

Step 2. Compute the criteria weights.

$$\sum_{j=1}^{n} w_j = 1 \tag{5}$$

Step 3. Normalize the decision matrix.

$$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{6}$$

Step 4. Compute the reference points.

Step 5. Compute the assessment values.

$$u_i = \sum_{j=1}^{g} x_{ij}^* w_j - \sum_{j=g+1}^{n} x_{ij}^* w_j \tag{7}$$

Where g refers to the number of positive criteria and n-g refers to the number of negative criteria.

Step 6. Rank the alternatives in descending order based on the value of u_i.

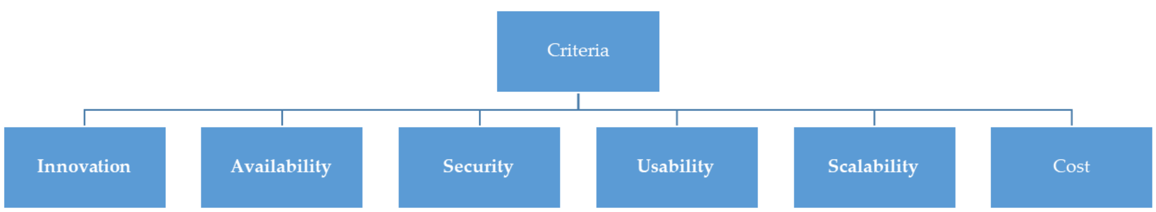


Figure 2. A group of criteria.

4. Results

This section presents the results of the proposed methodology for selecting the best IoT service provider. Three experts have experience in the industry and academic degrees to evaluate the criteria and alternatives. They used linguistic terms to evaluate the criteria and alternatives to their opinions. Then we used the T2NNS to evaluate the criteria and alternatives. This study used 6 criteria and 13 alternatives as shown in Figure 2.

Step 1. Eq. (4) was used to build the decision matrices between criteria and alternatives as shown in Tables A1-A3. We used the score function to obtain the crisp values[30]. Then we used the aggregated method to combine these matrices.

Step 2. We compute the weights of the criteria as shown in Figure 3. We show that criterion 6 has the highest weight and criterion 2 has the lowest weight.

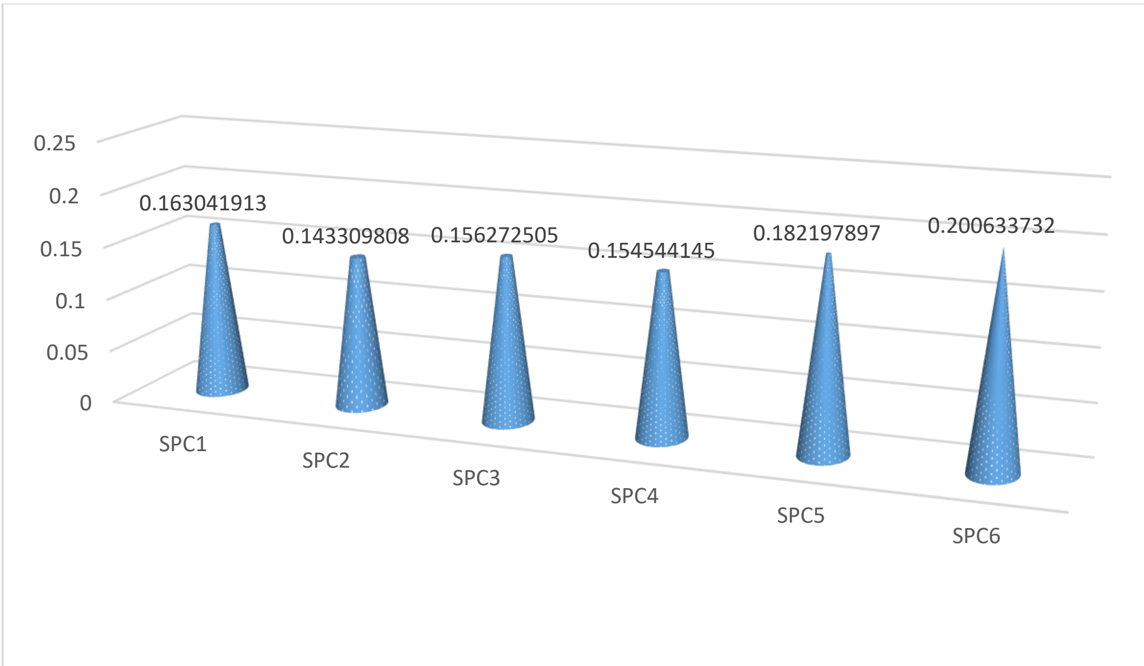


Figure 3. The criteria weights.

Table 1. Normalization decision matrix.

	SPC_1	SPC_2	SPC_3	SPC_4	SPC_5	SPC_6
SPA_1	0.270784	0.198809	0.214399	0.242543	0.308194	0.239285
SPA_2	0.305979	0.260247	0.243459	0.198733	0.261438	0.300217
SPA_3	0.236308	0.227802	0.242813	0.240004	0.263413	0.344647
SPA_4	0.275094	0.260247	0.243459	0.284449	0.322681	0.285619
SPA_5	0.173819	0.425231	0.3978	0.297147	0.275925	0.208819
SPA_6	0.339737	0.304426	0.217627	0.351751	0.221926	0.166928
SPA_7	0.390016	0.211925	0.227314	0.299687	0.142902	0.213262
SPA_8	0.300951	0.216067	0.302224	0.333973	0.21995	0.311007
SPA_9	0.317471	0.259556	0.316432	0.266036	0.279218	0.371305
SPA_{10}	0.344047	0.276124	0.349366	0.208892	0.35034	0.404944
SPA_{11}	0.221943	0.310639	0.125281	0.13778	0.290413	0.213897
SPA_{12}	0.172383	0.325826	0.227314	0.333973	0.346389	0.182796
SPA_{13}	0.122823	0.248511	0.371969	0.323815	0.252877	0.23992

Step 3. Eq. (6) used to normalize the decision matrix as shown in Table 1.

Step 4. Compute the reference points. Then compute the weighted normalized decision matrix as shown in Table 2.

Table 2. Weighted normalization decision matrix.

	SPC_1	SPC_2	SPC_3	SPC_4	SPC_5	SPC_6
SPA_1	0.044149	0.028491	0.033505	0.037484	0.056152	0.048009
SPA_2	0.049887	0.037296	0.038046	0.030713	0.047633	0.060234
SPA_3	0.038528	0.032646	0.037945	0.037091	0.047993	0.069148
SPA_4	0.044852	0.037296	0.038046	0.04396	0.058792	0.057305
SPA_5	0.02834	0.06094	0.062165	0.045922	0.050273	0.041896
SPA_6	0.055391	0.043627	0.034009	0.054361	0.040434	0.033491
SPA_7	0.063589	0.030371	0.035523	0.046315	0.026036	0.042788
SPA_8	0.049068	0.030964	0.047229	0.051614	0.040074	0.062399
SPA_9	0.051761	0.037197	0.04945	0.041114	0.050873	0.074496
SPA_{10}	0.056094	0.039571	0.054596	0.032283	0.063831	0.081246
SPA_{11}	0.036186	0.044518	0.019578	0.021293	0.052913	0.042915
SPA_{12}	0.028106	0.046694	0.035523	0.051614	0.063111	0.036675
SPA_{13}	0.020025	0.035614	0.058128	0.050044	0.046074	0.048136

Step 5. Eq. (7) is used to compute the assessment values as shown in Figure 4.

Step 6. Rank the alternatives in descending order based on the value of u_i as shown in Figure 5. Alternative 3 is the best and alternative 5 is the worst.

Figure 4. The assessment values.

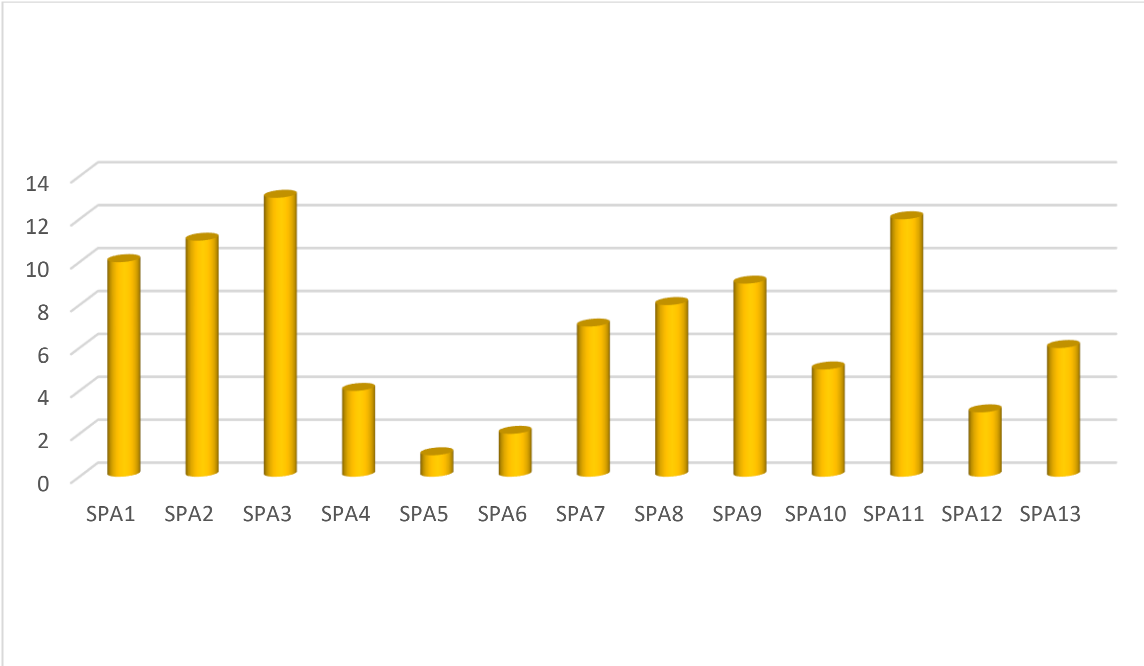


Figure 5. The rank of alternatives.

5. Sensitivity analysis

This section presents the sensitivity analysis to ensure the stability of the rank. We changed the weights of the criteria, then we applied the MOORA method. The weights of the criteria are changed under different seven cases as shown in Figure 6. In the first case, the equal weights are determined. In the second case, the first criterion weighs 0.2, and other criteria have the same weight.

We applied the MOORA method under different seven cases. We show the rank of alternatives is stable under different cases. All cases have alternative 5 as the worst and alternative 3 as the best except cases 4 and 5. Cases 4 and 5 have alternative 11 is the best and alternative 5 is the worst. Figure 7 shows the rank of alternatives.

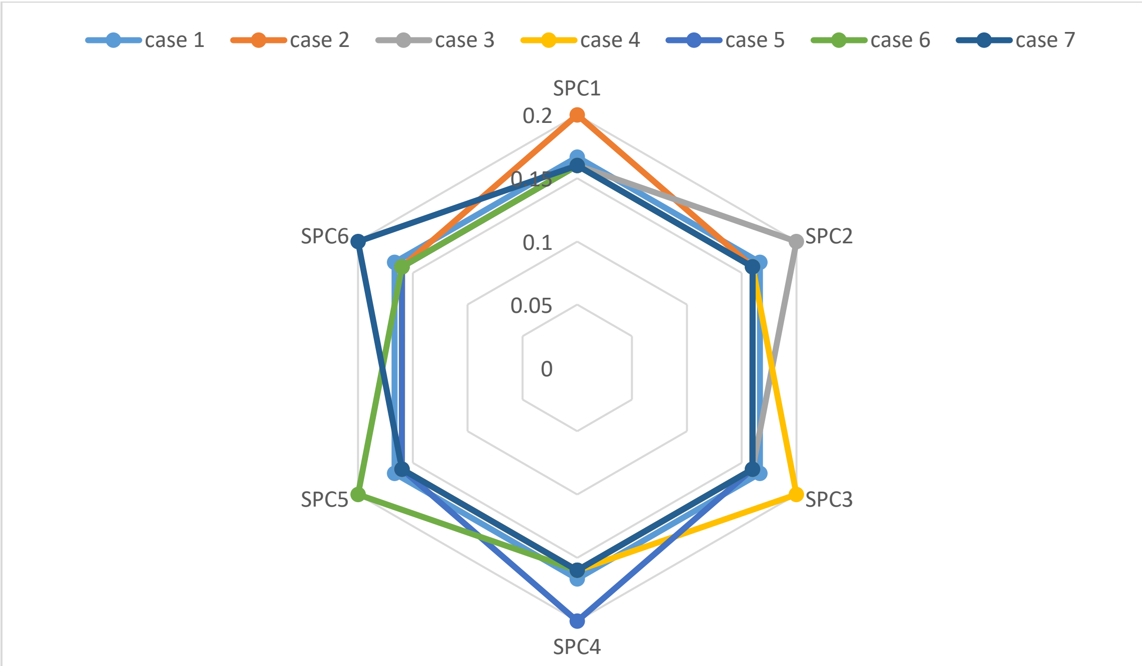


Figure 6. Criteria weights under sensitivity analysis.

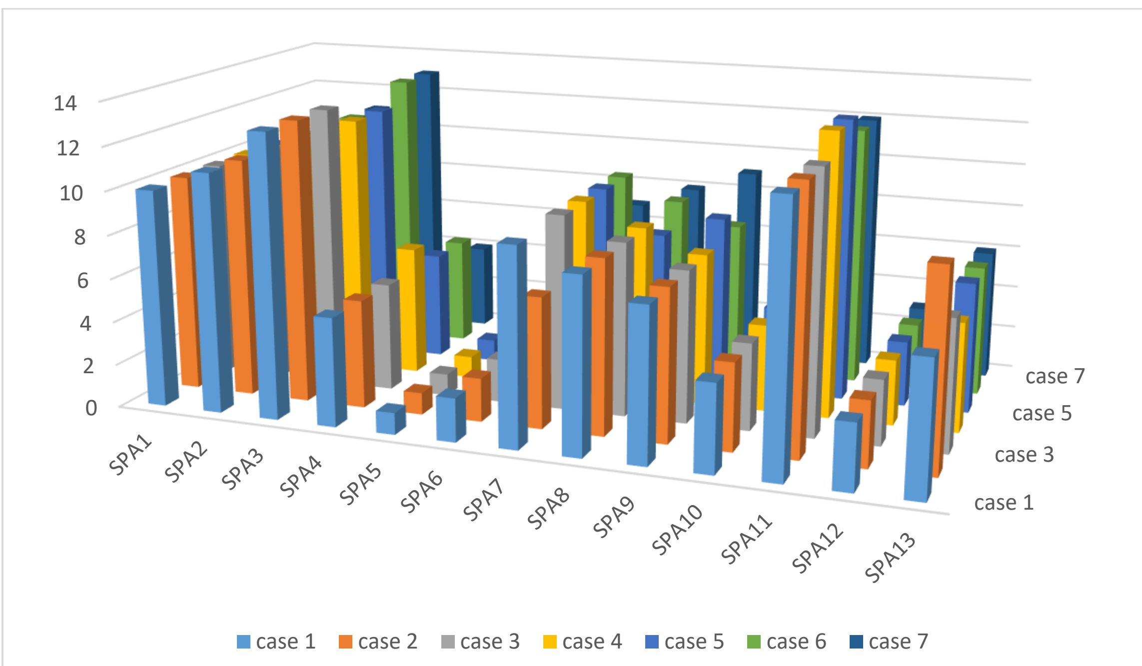


Figure 7. The rank of alternatives under different cases.

6. Comparative analysis

We compared the proposed methodology with other MCDM methods such as TOPSIS, VIKOR, MABAC, and WASPAS to show the effectiveness of the proposed methodology. Figure 8 shows the rank of alternatives under comparison study. Alternative 3 is the best in all comparative studies and alternative 5 is the worst. So the proposed method is effective compared with other MCDM methods.

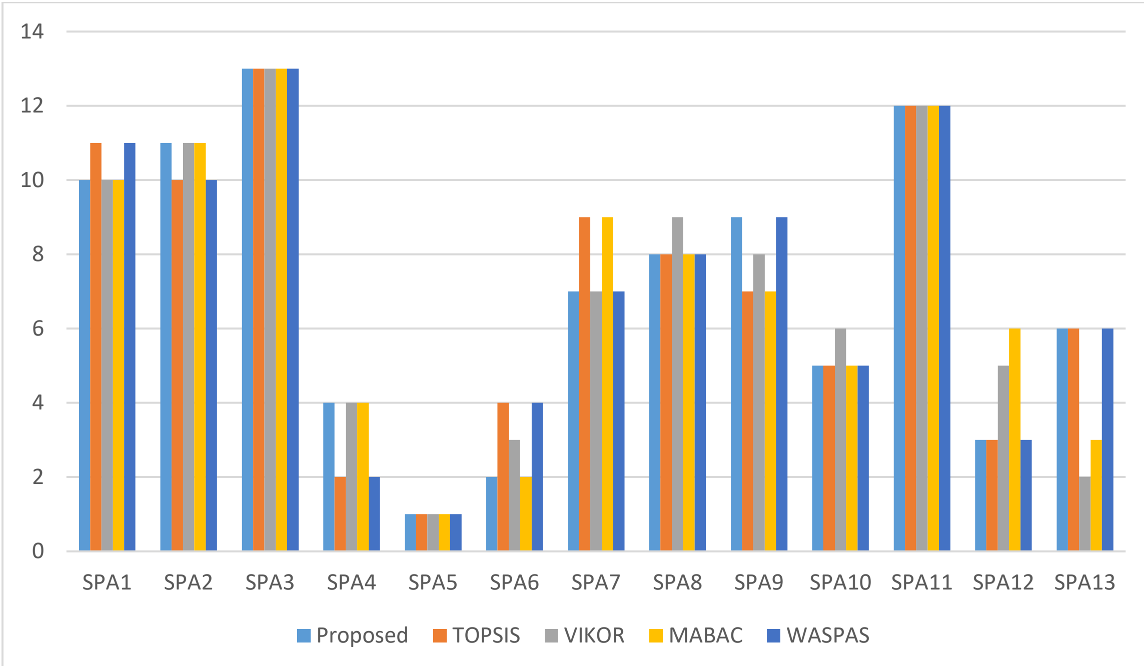


Figure 8. The rank of alternatives under comparative study.

We compute the correlation coefficient between the proposed methodology with other compared MCDM methods as shown in Table 3. We show the correlation between the proposed methodology and WASPAS is the highest and the correlation between the proposed methodology and the MABAC method is the lowest.

Table 3. The correlation between the proposed methodology and comparison MCDM methods.

	Proposed	TOPSIS	VIKOR	MABAC	WASPAS
Proposed	1	0.950549	0.934066	0.928571	0.972527
TOPSIS	0.950549	1	0.906593	0.923077	0.978022
VIKOR	0.934066	0.906593	1	0.972527	0.917582
MABAC	0.928571	0.923077	0.972527	1	0.901099
WASPAS	0.972527	0.978022	0.917582	0.901099	1

7. Organizational Impactions

This paper applied the MCDM methodology for selecting optimal IoT service providers. The MOORA method was used to rank the alternatives. The MOORA method was integrated with a neutrosophic set to deal with uncertain information. The proposed methodology can help managers select the best criteria for selecting IoT service providers. The proposed method can be applied in various organizations and firms. The proposed method is beneficial for selecting IoT service providers in intelligent cities.

8. Conclusions

This study proposed an MCDM methodology for ranking and selecting the best IoT service provider based on criteria and alternatives. The MOORA method was used to rank the other options. The neutrosophic set was integrated with MCDM methodology to deal with uncertain information. Three experts are invited to evaluate the criteria and alternatives. Then, we used the type 2 neutrosophic numbers to assess the requirements and alternatives. We applied the score function to obtain the crisp values. Then, we combine these matrices into one matrix. Then, we compute the weights of the criteria. The results show that criterion 6 has the highest and criterion 2 has the lowest. The MOORA method was applied based on the weights of the criteria. The results show that alternative 3 is the best and alternative 5 is the worst. The sensitivity analysis was conducted to show the stability rank of alternatives. There are seven cases created to change the weights of criteria. The sensitivity analysis results show that alternative 3 is the best and alternative 5 is the worst, so the rank of alternatives is stable in different cases. The proposed methodology was compared with four MCDM methods: TOPSIS, VIKOR, MABAC, and WASPAS. The results show that the proposed methodology is effective compared with other MCDM methods. The proposed methodology can be applied to different decision-making methodologies in future research. Other MCDM methods can be used for this problem.

References

[1] A. Zannou, A. Boulaalam, and E. H. Nfaoui, "System service provider–customer for IoT (SSPC-IoT)," in *Embedded Systems and Artificial Intelligence: Proceedings of ESAI 2019, Fez, Morocco*, Springer, 2020, pp. 731–739.

[2] F. Li, M. Vögler, M. Claeßens, and S. Dustdar, "Efficient and scalable IoT service delivery on cloud," in *2013 IEEE sixth international conference on cloud computing*, IEEE, 2013, pp. 740–747.

[3] G. Baranwal, M. Singh, and D. P. Vidyarthi, "A framework for IoT service selection," *The Journal of Supercomputing*, vol. 76, no. 4, pp. 2777–2814, 2020.

[4] S. P. Gochhayat *et al.*, "LISA: Lightweight context-aware IoT service architecture," *Journal of cleaner production*, vol. 212, pp. 1345–1356, 2019.

[5] J. Wang, Q. Zhu, and Y. Ma, "An agent-based hybrid service delivery for coordinating internet of things and 3rd party service providers," *Journal of Network and Computer Applications*, vol. 36, no. 6, pp. 1684–1695, 2013.

[6] J. Kim and J.-W. Lee, "OpenIoT: An open service framework for the Internet of Things," in *2014 IEEE world forum on internet of things (WF-IoT)*, IEEE, 2014, pp. 89–93.

[7] R. Krishankumar and F. Ecer, "Selection of IoT service provider for sustainable transport using q-rung orthopair fuzzy CRADIS and unknown weights," *Applied Soft Computing*, vol. 132, p. 109870, 2023.

[8] I. Farris, L. Militano, M. Nitti, L. Atzori, and A. Iera, "MIFaaS: A mobile-IoT-federation-as-a-service model for dynamic cooperation of IoT cloud providers," *Future Generation Computer Systems*, vol. 70, pp. 126–137, 2017.

[9] A. A. El-Douh, S. Lu, A. Abdelhafeez, and A. S. Aziz, "Assessment the Health Sustainability using Neutrosophic MCDM Methodology: Case Study COVID-19," *Sustainable Machine Intelligence Journal*, vol. 3, p. 1, 2023.

[10] J.-J. Wang, Y.-Y. Jing, C.-F. Zhang, and J.-H. Zhao, "Review on multi-criteria decision analysis aid in sustainable energy decision-making," *Renewable and sustainable energy reviews*, vol. 13, no. 9, pp. 2263–2278, 2009.

[11] S. I. A. Aal, "A Multi-Criteria Decision Making Model for Sustainable and Resilient Supplier

Selection and Management," *Neutrosophic Systems with Applications*, vol. 15, pp. 33–45, 2024.
[12] D. Jato-Espino, E. Castillo-Lopez, J. Rodriguez-Hernandez, and J. C. Canteras-Jordana, "A review of application of multi-criteria decision making methods in construction," *Automation in construction*, vol. 45, pp. 151–162, 2014.
[13] A. Elsayed and M. Abouhawwash, "An Effective Model for Selecting the Best Cloud Platform for Smart Farming in Smart Cities: A Case Study," *Optimization in Agriculture*, vol. 1, pp. 66–80, 2024.
[14] J. A. Scott, W. Ho, and P. K. Dey, "A review of multi-criteria decision-making methods for bioenergy systems," *Energy*, vol. 42, no. 1, pp. 146–156, 2012.
[15] A. Salam, M. Mohamed, and K. Venkatachalam, "An efficient Model for Selection of Unmanned Aerial Vehicles designed for Precision Agriculture," *Optimization in Agriculture*, vol. 1, pp. 22–39, 2024.
[16] C. Zopounidis and M. Doumpos, "Multi-criteria decision aid in financial decision making: methodologies and literature review," *Journal of Multi-Criteria Decision Analysis*, vol. 11, no. 4-5, pp. 167–186, 2002.
[17] S. I. A. Aal, J. Shreyas, and P. K. Udayaprasad, "Selecting Optimal Charcoal Company using Multi-Criteria Decision Making Methodology," *Multicriteria Algorithms with Applications*, vol. 3, pp. 15–22, 2024.
[18] L. A. Zadeh, "On fuzzy algorithms," in *fuzzy sets, fuzzy logic, and fuzzy systems: selected papers By Lotfi A Zadeh*, World Scientific, 1996, pp. 127–147.
[19] L. A. Zadeh, "Fuzzy logic," in *Granular, Fuzzy, and Soft Computing*, Springer, 2023, pp. 19–49.
[20] A. Alqazzaz, I. Alrashdi, and K. M. Sallam, "Assessment Criteria of Solar Hydrogen Production Plant Based on Fuzzy Multi-Criteria Decision-Making Methodology," *Information Sciences with Applications*, vol. 1, pp. 10–18, 2024.
[21] K. Rudnik, G. Bocewicz, A. Kucińska-Landwójtowicz, and I. D. Czabak-Górska, "Ordered fuzzy WASPAS method for selection of improvement projects," *Expert Systems with Applications*, vol. 169, p. 114471, 2021.
[22] A. Abdelhafeez, J. Shreyas, and P. K. Udayaprasad, "A Fuzzy TOPSIS Method for Assessment Blockchain Technology Strategies," *Information Sciences with Applications*, vol. 1, pp. 1–9, 2024.
[23] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, "Single valued neutrosophic sets," *Infinite study*, vol. 12, 2010.
[24] F. Smarandache, "A unifying field in Logics: Neutrosophic Logic.," in *Philosophy*, American Research Press, 1999, pp. 1–141.
[25] M. Abouhawwash and M. Jameel, "Evaluation Factors of Solar Power Plants to Reduce Cost Under Neutrosophic Multi-Criteria Decision-Making Model," *Sustainable Machine Intelligence Journal*, vol. 2, 2023.
[26] A. A. Mohammed, H. E. Khalid, R. W. Gadama, and T. P. Gadama, "Neutrosophic C-Mean Clustering Analysis of the Construction Projects Delay Causes," *Neutrosophic Optimization and Intelligent Systems*, vol. 2, pp. 28–42, 2024.
[27] S. Chakraborty, "Applications of the MOORA method for decision making in manufacturing environment," *The International Journal of Advanced Manufacturing Technology*, vol. 54, pp. 1155–1166, 2011.
[28] W. K. Brauers and E. K. Zavadskas, "The MOORA method and its application to privatization in a transition economy," *Control and cybernetics*, vol. 35, no. 2, pp. 445–469, 2006.
[29] M. Abdel-Basset, M. Saleh, A. Gamal, and F. Smarandache, "An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number," *Applied Soft Computing*, vol. 77, pp. 438–452, 2019, doi: https://doi.org/10.1016/j.asoc.2019.01.035.
[30] V. Simic, I. Gokasar, M. Deveci, and A. Karakurt, "An integrated CRITIC and MABAC based type-

2 neutrosophic model for public transportation pricing system selection," *Socio-Economic Planning Sciences*, vol. 80, p. 101157, 2022.

Appendix

Table A1. The first decision matrix.

	SPC_1	SPC_2	SPC_3	SPC_4	SPC_5	SPC_6
SPA_1	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.50,0.30,0.50),(0.50,0.35,0.45),(0.45,0.30,0.60))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))
SPA_2	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))
SPA_3	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.50,0.30,0.50),(0.50,0.35,0.45),(0.45,0.30,0.60))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))
SPA_4	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))
SPA_5	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))
SPA_6	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.50,0.30,0.50),(0.50,0.35,0.45),(0.45,0.30,0.60))
SPA_7	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))
SPA_8	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.50,0.30,0.50),(0.50,0.35,0.45),(0.45,0.30,0.60))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))
SPA_9	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.50,0.30,0.50),(0.50,0.35,0.45),(0.45,0.30,0.60))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))
SPA_{10}	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.50,0.30,0.50),(0.50,0.35,0.45),(0.45,0.30,0.60))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))
SPA_{11}	((0.50,0.30,0.50),(0.50,0.35,0.45),(0.45,0.30,0.60))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))
SPA_{12}	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))
SPA_{13}	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.50,0.30,0.50),(0.50,0.35,0.45),(0.45,0.30,0.60))

Table A2. The second decision matrix.

	SPC_1	SPC_2	SPC_3	SPC_4	SPC_5	SPC_6
SPA_1	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.50,0.30,0.50),(0.50,0.35,0.45),(0.45,0.30,0.60))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))
SPA_2	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))
SPA_3	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))
SPA_4	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))
SPA_5	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))
SPA_6	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))
SPA_7	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))
SPA_8	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))
SPA_9	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))
SPA_{10}	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))
SPA_{11}	((0.50,0.30,0.50),(0.50,0.35,0.45),(0.45,0.30,0.60))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))
SPA_{12}	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))
SPA_{13}	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.20,0.20,0.10),(0.65,0.80,0.85),(0.45,0.80,0.70))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.50,0.30,0.50),(0.50,0.35,0.45),(0.45,0.30,0.60))

Table A3. The third decision matrix.

	SPC_1	SPC_2	SPC_3	SPC_4	SPC_5	SPC_6
SPA_1	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.50,0.30,0.50),(0.50,0.35,0.45),(0.45,0.30,0.60))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))
SPA_2	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))
SPA_3	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))
SPA_4	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))
SPA_5	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))
SPA_6	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))
SPA_7	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))
SPA_8	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))
SPA_9	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.50,0.30,0.50),(0.50,0.35,0.45),(0.45,0.30,0.60))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))
SPA_{10}	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))
SPA_{11}	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))
SPA_{12}	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.95,0.90,0.95),(0.10,0.10,0.05),(0.05,0.05,0.05))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))
SPA_{13}	((0.20,0.20,0.10), (0.65,0.80,0.85), (0.45,0.80,0.70))	((0.35,0.5,0.10),(0.50,0.75,0.80),(0.50,0.75,0.65))	((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))	((0.60,0.45,0.50),(0.20,0.15,0.25),(0.10,0.25,0.15))	((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))	((0.50,0.30,0.50),(0.50,0.35,0.45),(0.45,0.30,0.60))

Received: Jan 16, 2024. **Accepted**: Mar 28, 2024

Neutrosophic Sets and Systems, Vol. 66, 2024
University of New Mexico

Analysis Of Neutrosophic Set, Julia Set In Aircraft Crash

M.N. Bharathi* And G. Jayalalitha **

*Research Scholar, Department of Mathematics, Vels Institute of Science, Technology, and Advanced Studies, Pallavaram, Chennai-117 Email: bharathimeenakshi70922@gmail.com
** Professor, Department of Mathematics, Vels Institute of Science, Technology, and Advanced Studies Pallavaram Chennai-117 Email: g.jayalalithamaths.sbs@velsuniv.ac.in

ABSTRACT

A neutrosophic fuzzy set that generalizes the classical set is represented by a closed interval *[0,1]*. Let us generalize the fuzzy set to the Neutrosophic set, which is defined as three membership functions between interval*]-0,1+ [*. This paper examines the bird collision problem within a more grounded Neutrosophic framework. The collision between the bird and the aircraft is represented in the Neutrosophic domain as a stationary point because it occurs at a distance too great for any other auditory signal to discern. Numerous methods exist for solving the airplane problem, including a Julia set in complex numbers. Bird collisions resulting from aircraft crashes add complication. An aviation signal is utilized to avoid an accident in a bird strike or bird collision.
Keywords: Neutrosophic set, Neutrosophic Fuzzy set, Fractals, Julia set, Bird collision.

1. INTRODUCTION

Lotfi.A. Zadeh introduced the Fuzzy set in 1965. Numerous real-world scenarios involving ambiguity and indeterminacy have found use for fuzzy sets and logic [19]. However, traditional fuzzy sets might not take member levels or membership values into account well, which might make it difficult to determine exact values in various situations. Interval-valued fuzzy sets were proposed to address the uncertainty of inclusion values; nonetheless, there are drawbacks to both conventional and interval-valued fuzzy sets [7]. In these situations, Atanassov's intuitionistic fuzzy sets provide some relief [2,6,10], but they might not be sufficient to handle conflicting and incomplete facts, which are common in systems of belief. Smarandache's neutrosophic sets provide a more thorough method of managing inaccurate and ambiguous data by extending conventional mathematical techniques [12].

Fuzzy systems (FS) are unable to solve the problem productively if the decision-maker's statement is confused regarding the correctness or improperness of the option [19]. Then, a few new hypotheses are needed to resolve the ambiguous issue. On the other hand, aside from being unexpected to the sets, the fuzzy method used in trade is vague and deficient [11]. Concurrently replicating the falsehood, indeterminacy, and truth values will produce a Neutrosophic set. Ye provided more straightforward Neutrosophic sets, which Peng et al. used to create their processes and operator groups [18]. Mathematician Benoit Mandelbrot is known to have used the term *fractal* for the first time in 1975. From the Latin word *fractus*, which means broken, Mandelbrot extended its application to patterns of geometry found in nature as well as the idea of conceptual fractional dimension [3,8,9]. Since there is only one genuine value, x, in neutrosophic statistics, this does not imply that x is also in $a + bI$ = N.

Rather, it means that $a + b\text{x} \in a + bI$. For this reason, it is necessary to present and study the appurtenance connection and equation [4,15,16].

Bird collisions claimed the lives of over 255 people in 1988; the first one happened in 1905 when Orville Wright hit a bird over an Ohio cornfield [1,14]. In the US, attacks by birds and other animals cost about 900 million dollars annually. Coyotes and deer are two examples of other wildlife attacks and aircraft [13]. The energy released during a collision is what causes serious financial and human repercussions and necessitates taking the possibility of a bird collision into account [17].

Recent work Utilizing a hybrid Eulerian-Lagrangian finite element formulation, the influence of an exceedingly deformable item on an inboard model has been effectively modeled. A prediction approach for aeronautical constructions must simulate harm in brass materials in composite and sandwich structures, operational parts in structure are typically constructed from items created by a range of resources [13]. The notion of a neutrosophic set is covered in Section 2 along with an example of how difficult it is to define the reasons for bird collisions. Aircraft accidents are caused by the complex number of bird strikes, and this shape which can be characterized as a Julia set may be discussed using a lemma and theorem.

2. METHODS

This section discusses the techniques used to look into bird collisions involving aircraft. It finds triangle-shaped collision patterns, which helps with hotspot identification and risk evaluation. By foreseeing collision hazards, this theory promotes the creation of predictive models, improving aircraft safety. All things considered, the Neutrosophic set theory advances our knowledge of bird-aircraft crashes and, by proactive steps and multidisciplinary collaboration, contributes to safer skies. The Neutrosophic set, which indicates that the birds make a triangle and strike the airplane with their bodies, is another name used to characterize these crashes. This idea applies only if there are several birds crashes taking place in the sky.

2.1 NEUTROSOPHIC SET

The phrase *neutrosophic set* refers to a set of elements that satisfy three conditions: functions T, I, and F: Y→ *[0,1]* [4]. For instance, because a cloud's boundaries are unclear and its constituent element (a drop of water) belongs to the Neutrosophic probability set, the cloud is considered a Neutrosophic set [2,11]. They therefore don't know where the clouds begin or stop, nor if some elements are part of the set or not [12,18]. Furthermore, a higher percentage of indeterminacy is needed for a more precise estimate and a more seamless organic process [10].

2.2 SIMULATED BIRD COLLISIONS

The bird collision event is included in the category of soft-body collisions from a numerical perspective. Because of stresses that significantly exceed the material's strength, this period of hitches is considered by the impacting material's liquid-like performance [13]. Numerical barriers that are present in the simulation of these occurrences are mostly caused by this behavior as well [14,17]. The most crucial of these challenges is to accurately model the weight transfer that occurs through soft-body influence.

In the direction of completing the assignment and ensuring dependable outcomes, three key areas of the problem must be adequately mathematically addressed the constituent model of the bird substantial, the significant deformations of the numerical substitute bird model, and the flop and damage reproductions of the wedged structure [1,13].

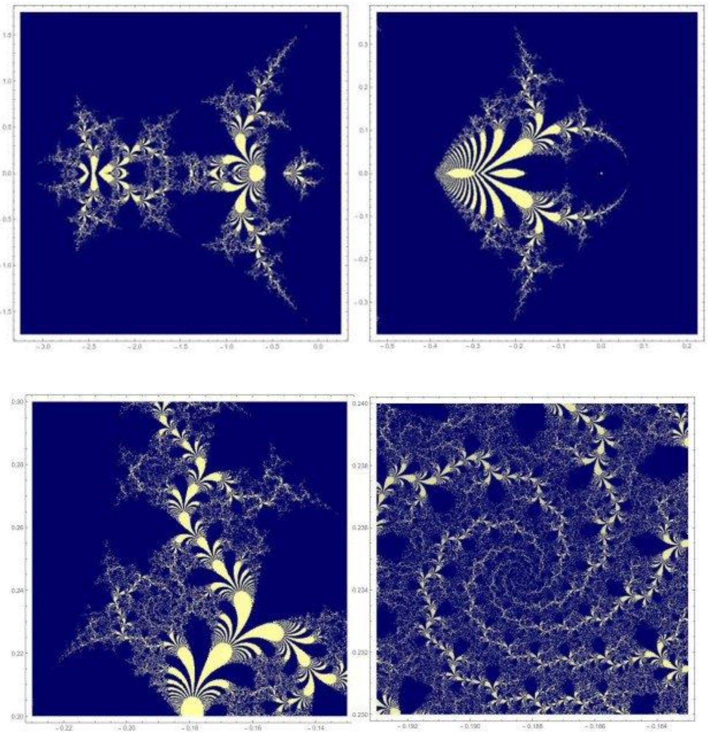


Figure 1 Birds Collision Complex

Examples

Several factors contribute to airplane crashes, including weather, aircraft mechanical issues, pilot experience, and training. The trio of Neutrosophic sets comprises.

i. There is little turbulence and a clear sky, according to the truth membership (T) weather prediction, indicating a minimal crash risk $T_{weather} = 0.2$. The weather forecast may have some indeterminacy (I) as unforeseen events could happen while the aircraft is in flight $I_{weather} = 0.6$. Although the prediction is usually correct, there is always a potential of inaccuracies or unanticipated weather phenomena Falsity membership (F) $F_{weather} = 0.1$.

ii. The likelihood of a mechanical breakdown is decreased when the aircraft truth membership (T) $T_{mechanical} = 0.2$.is subjected to routine maintenance inspections and found to be in excellent condition. Even with the Indeterminacy of membership $I_{mechanical} = 0.6$ maintenance checks, wear and tear or undiscovered problems might still present a danger to safety during the flight. Although Falsity membership (F) $F_{mechanical} = 0.2$ is uncommon, there is a remote chance that mechanical failures might still happen despite the tests.

iii. Due to their intensive training and wealth of expertise, Truth membership (T) pilots to help ensure safe operation $T_{pilot} = 0.3$. Unexpected events or exhaustion may have an impact on a pilot's performance during a flight, even with a membership (I) of indeterminacy $I_{pilot} = 0.5$. Although false membership (F) is rare, the pilot may have made mistakes or made a mistake in judgment $F_{pilot} = 0.2$.

2.3 NEUTROSOPHIC NUMBER OF APPURTENANCE

Let X be the universe and the Neutrosophic set P over X is defined as $P=\{(x, TP(x), IP(x), FP(X) :x$ $P = \{(x, TP(x), IP(x), FP(x): x \in X\}$ where the truth P, Indeterminacy P, and Falsity P are said to be real or non-standard subsets of the interval i.e, $0^- \leq T_{p^y}(x)+I_{p^y}(y) + F_{p^y}(y) \leq 3^+$[15].

Examples

Complexity of Neutrosophic number [16]

i. Let I_1= [0,1], $N_1 = 2 + 3i\, I_1$

$$N_1 = 2 + 3i\,[0,1] = 2 + [3i.0, 3i.1] = 2 + [0,3i] = 2 + 0, 2 + 3i = [2, 2 + 3i]$$

$$N_1 = [2, -3 \pm \frac{\sqrt{9} - 4(1)(0)}{2(2)} = \left[2, -\frac{3\sqrt{9}}{4}\right]$$

$$= \left[2, -3 \pm \frac{3}{4}\right] = [2, (0, -0.75)]$$

ii. Let $I_2 = \{0.025, 0.065, 0.582\}$ which has a three element or finite discrete set like

$$N_2 = \{2 + 3i\{0.025, 0.065, 0.582\} = 2 + \{-0.0474, -0.195, -1.746\}$$

$$N_2 = 1.526, 1.805, 0.254 \subset [2, -0.75]$$

iii. Let $I_3 = \{\frac{1}{n}, 1 \leq n \leq \infty, n\ is\ an\ integer\}$ which is an infinite discrete set

$$N_3 = 2 + 3iI_3 = \left\{2 + 3i\frac{1}{n}, 1 \leq n \leq \infty, n\ is\ an\ integer\right\}$$

$$N_3 = \left\{2 + \left(3\frac{i}{1}\right), 2 + \left(\frac{3i}{2}\right), 2 + \left(\frac{3i}{3}\right), \ldots 2 + 3.\frac{1}{n}, \ldots\right\} = \subset [2, -0.75]\ .$$

2.1 Theorem

Let the Neutrosophic sets be in complex numbers. Then $p \in P$ *and* $q \in Q$,

i. *Addition* $p + q \in P + Q$
ii. *Subtraction* $p - q \in P - Q$
iii. *Multiplication* $p \times q \in P \times Q$
iv. *Division* $\frac{p}{q} \in \frac{P}{Q}$
v. *Power* $p^q \in P^Q$

Proof

Let $*$ any operations above, then
$P * Q = \{x * y; where\ x \in P, y \in Q\}$, and the operation $*$ is said to be well-defined.
Then $x = p \in P$ and $y = q \in Q$ it seems to be of the form $p * q = P * Q$ [15].

2.4 JULIA SET

A complex fractal known as the Julia set is created when complex numbers are repeatedly used in a mathematical procedure. It displays geometric patterns that are complex and self-similar, frequently displaying chaotic and complicated behavior. These sets, which bear the name Gaston Julia after the French mathematician, are useful in dynamical systems, complex analysis, and computer graphics. They also provide light on the characteristics of complex numbers and how they behave in iterative systems [8].

If f^k k- stands for composition f ∘ ⋯ ∘ f the function f, $f^k(z)$ is iteration of k then f (f (· · · (f(z)) · · ·)) of z [14].

$$K(f) = \{z \in \mathbb{C} : f^k(z) \not\to \infty\} \qquad (1)$$

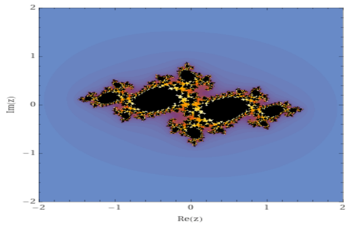


Figure 2 Julia set

2.1 Lemma

A polynomial $f(z) = a_n z^n + a_{n-1}z^{n-1} + \cdots + a_1 + a_0$ *with n and* $a_n \neq 0$, *there are some s such that if* $|z| \geq s$, *then* $|f(z)| \geq n|z|$. *In certain, if* $|f^m(z)| \geq s|$ *for around m* ⩾ *0, at that point* $f^{(k)}(z) \to \infty$ *ask* ⟶ ∞. *Hence, also* $f^{(k)}(z) \to \infty$ *or set* $\{f^{(k)}(z) : k = 0, 1, 2...\}$ *confined.*

Proof

Considering that f(z) is a complex function. A polynomial with the form $a_n z^n + a_{n-1}z^{n-1} + \cdots + a_1 + a_0$ and $\{n, n-1, n-2, ... 2,1,0\}$ is given a complex function f(z).

A Polynomial such as $a_n z^n + a_{n-1}z^{n-1} + \cdots + a_1 + a_0$ with $\{n, n-1, n-2, ... 2,1,0\}$ is desirable to explain that $f^{(k)}(z) \to \infty$.It can indicate is appropriately high to confirm that if $|z| \geq s$, then

$$\frac{1}{2}|a_n||z^n| \geq n|z| \text{ and } (|a_{n-1}||z^{n-1}| + \cdots |a_1||z| + |a_0|) \leq \frac{1}{2}|a_n||z^n| \quad (2)$$

Then, if absolute $|z| \geq s$,

$$|f(z)| \geq |a_n||z^n| - (|a_{n-1}||z^{n-1}| + \cdots |a_1||z| + |a_0|) \geq |a_n||z^n| \geq n|z| \quad (3)$$

Also, if $|f^m(z)| \geq s|$ for about m, then put on this inductively, then $|f^{m+k}(z)| \geq 2^m|f^k(z)|$, then $|f^m(z)| \to \infty$ [8].

2.2 Theorem

Let $f(z)$ be an analytic mapping in $G \subset \mathbb{C}$ such that $f(p) = p$ for about p in G. Then

(a) $|f'(p)| < 1$ then iff *p is an attracting fixed point,*

(b) $|f'(p)| > 1$ then iff *p is a repelling fixed point.*

Proof

Let $|f'(p)| = 0$, p is said to be *super attracting fixed point* and $|f'(p)| = 1$, p is said to be a *neutral fixed point*. It has intricate behavior.

It is possible to reveal a partially repellent and partially appealing nature. Sample of a neutral fixed point at $p = 0$ for the complex function $f(z) = \sin(z)$. P qualities in real-valued seeds, while $p = 0$ in fully imaginary-valued seeds has a repellent quality. For $z = bi$, $b \in \mathbb{R}, \sin(bi) = isinh(b)$. Then sin h(b) said to be an increasing function, for all $b \in \mathbb{R}^{+}$, $\{ sin^{m}(bi)\} \to \infty$ as $n \to \infty$ and for all $b \in \mathbb{R}^{-}$, $\{ sin^{m}(bi)\} \to -\infty$. Similarly, neutral fixed points are either an attracting or repelling fashion [8].

2.5 AIRPLANE CRASHES

The majority of large species with tremendous populations that are involved in bird collisions in the United States are geese and gulls. While feral Canada geese and Greylag geese populations are rising in rough areas of the European Union, raising the hazard of those enormous birds to aircraft, migratory snow geese and Canada populations have improved significantly in some areas of the US [1].

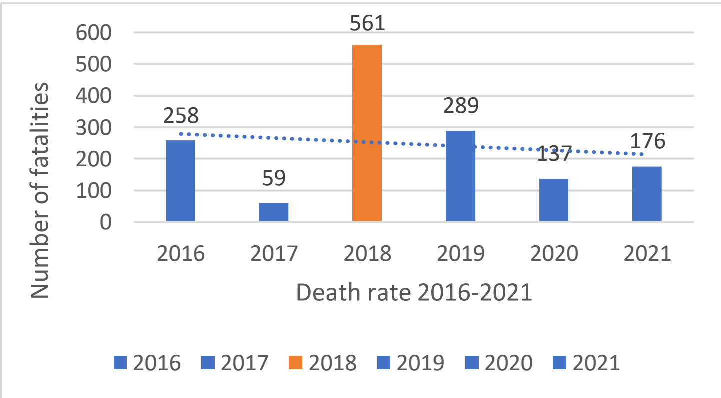


Figure 3 Number of worldwide air traffic fatalities from 2016 to 2021

In various regions globally, there is frequent interaction between sizable predatory birds like Milvus kites and Gyps vultures [9]. In the United States, the predominant species encountered include waterfowl (30%), gulls (22%), and raptors (20%), alongside pigeons and doves (7%) [13]. According to the Feathers Recognition Lab at the Smithsonian Institution, turkey vultures pose the highest risk, with Canada geese and white pelicans following closely behind in terms of potential hazards [14,17].

Figure 4 Complex Number of birds forms Julia set

Figure 4 shows geometric arrangements that are complex and chaotic, as complex numbers representing bird locations converge to form a Julia set. This depiction demonstrates the intricacy of the birds' movement patterns while capturing the dynamic interactions between them in their natural habitat. The Julia set highlights the complex dynamics found in avian populations and their influence on ecological systems by providing insights into the emergent behaviors and interactions among individual birds.

3. RESULTS

A bird strike occurs when an aircraft while it is in the air, generally on takeoff or landing. This bird crash might have been triggered by an aviation disaster in 2016 or 2021 and it may also be represented as a Neutrosophic set forms a complexity of neutrosophic numbers is explained in subsection 2.3 examples. A bird collision creates a Julia set (lemma 2.1) whose form is defined by the shape of the bird. The bird assaults the aircraft in opposing directions. This bird strike shape in Fig 2 is said to be the same shape as Julia set's fractal dimension in connectivity to another, more in-depth analysis of bird strike. It is discussed in the lemma, theorem, and section 2 about airplane crashes. Then Figure 4 depicts a triangle-shaped bird strike that appears to be sophisticated. It could symbolize the complex numbers of the Neutrosophic set and the Julia set's form.

Note:

- *[0,1]* denotes the membership values in the Fuzzy set.
- *]-0,1+ [* denotes that the membership functions of the Neutrosophic set.

4. CONCLUSION

This research uses the phrases Julia set and Neutrosophic set to distinguish between two concepts related to bird crashes in airplanes. The three categories indicate the interval*]-0,1+ [* remain stated in the Neutrosophic set. Because of their middle value, the Neutral term and the Fixed point of the neutrosophic indeterminacy are comparable. For instance, the middle value of the first and last membership functions, *[0,1],* is 0.5. However, there are some comparable events in bird crashes, and Julia set is an accumulation of the limit points of the entire reversed orbit. It is an array of a finite number of elements that, with repetition, converge to an infinite. Julia's set is a complex integer set that is elaborated with some examples above with bounded iterated values.

5. REFERENCES

1. Allan, J. R.; Bell, J. C.; Jackson, V. S. (1999). *An Assessment of the Worldwide Risk to Aircraft from Large Flocking Birds*, Bird Strike Committee Proceedings 1999 Bird Strike Committee-USA/Canada, Vancouver, BC.
2. Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3

3. Bray, K., Dwyer, J., Barnard, R. W., & Williams, G. B. (2020). Fixed Points, Symmetries, and Bounds for Basins of Attraction of Complex Trigonometric Functions. *International Journal of Mathematics and Mathematical Sciences*, *2020*, 1–10. https://doi.org/10.1155/2020/1853467
4. Chakraborty, A., Mondal, S. P., Ahmadian, A., Senu, N., Alam, S., & Salahshour, S. (2018). Different Forms of Triangular Neutrosophic Numbers, De-Neutrosophication Techniques, and their applications. *Symmetry*, *10*(8), 327. https://doi.org/10.3390/sym10080327
5. Dhouib, S. (2021). Neutrosophic Triangular Fuzzy Travelling Salesman Problem Based on Dhouib-Matrix-TSP1 Heuristic. *International Journal of Computer and Information Technology (2279-0764)*, *10*(5). https://doi.org/10.24203/ijcit.v10i5.154
6. F. Smarandache, Neutrosophic set, a generalization of the intuitionistic fuzzy sets, Inter. J. Pure Appl. Math. 24 (2005) 287–297.
7. I. Turksen, Interval-valued fuzzy sets based on normal forms, *Fuzzy Sets and Systems* 20 (1986) 191–210. https://doi.org/10.1016/0165-0114%2886%2990077-1
8. Kenneth Falconer, Fractal Geometry Mathematical Foundations and Applications Third Edition, *University of St Andrews, UK* 2014.
9. Mattila, P. (1975). Hausdorff dimension, orthogonal projections, and intersections with planes. *Annales Academiae Scientiarum Fennicae Series A I Mathematica*, *1*, 227–244. https://doi.org/10.5186/aasfm.1975.0110
10. Peng, J., Wang, J., Wu, X., Zhang, H., & Chen, X. (2015). The fuzzy cross-entropy for intuitionistic hesitant fuzzy sets and their application in multi-criteria decision-making. *International Journal of Systems Science*, *46*(13), 2335–2350. https://doi.org/10.1080/00207721.2014.993744.
11. Peng, J., Wang, J., Wang, J., Zhang, H., & Chen, X. (2016). Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. *International Journal of Systems Science*, *47*(10), 2342–2358. https://doi.org/10.1080/00207721.2014.994050
12. Smarandache, F. A Unifying Field in Logics Neutrosophy: Neutrosophic Probability, Set and Logic, 3rd ed.; American Research Press: Washington, DC, USA, 2003.
13. Smojver, I., & Ivančević, D. (2012). Advanced modeling of bird strike on high lift devices using hybrid Eulerian–Lagrangian formulation. *Aerospace Science and Technology*, *23*(1), 224–232. https://doi.org/10.1016/j.ast.2011.07.010
14. Smojver, I., & Ivancevic, D. (2010). Bird impact at aircraft structure – Damage analysis using Coupled Euler Lagrangian Approach. *IOP Conference Series: Materials Science and Engineering*, *10*, 012050. https://doi.org/10.1088/1757-899X/10/1/012050
15. Smarandache, F. (2024). Foundation of Appurtenance and Inclusion Equations for Constructing the Operations of Neutrosophic Numbers Needed in Neutrosophic Statistics. *Neutrosophic Systems With Applications*, *15*, 16-32. https://doi.org/10.61356/j.nswa.2024.1513856
16. Voskoglou, M. G., Smarandache, F., & Mohamed, M. (2024). q-Rung Neutrosophic Sets and Topological Spaces. *Neutrosophic Systems with Applications*, *15*, 58-66. https://doi.org/10.61356/j.nswa.2024.1515356
17. Thorpe, John (2003). "Fatalities and destroyed civil aircraft due to bird strikes, 1912–2002" (PDF). International Bird Strike Committee, IBSC 26 Warsaw. Archived from the original (PDF) on 2009-02-27. Retrieved 2009-01-17.
18. Ye, J. (2014). A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *Journal of Intelligent & Fuzzy Systems*, *26*(5), 2459–2466. https://doi.org/10.3233/IFS-130916
19. Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

Received: Jan 13, 2024. Accepted: Mar 30, 2024

Neutrosophic Sets and Systems (NSS) is an academic journal, published quarterly online and on paper, that has been created for publications of advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics etc. and their applications in any field.

All submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

ISSN (print): 2331-6055, ISSN (online): 2331-608X
Impact Factor: 1.739

NSS has been accepted by SCOPUS. Starting with Vol. 19, 2018, all NSS articles are indexed in Scopus.

NSS is also indexed by Google Scholar, Google Plus, Google Books, EBSCO, Cengage Thompson Gale (USA), Cengage Learning, ProQuest, Amazon Kindle, DOAJ (Sweden), University Grants Commission (UGC) - India, International Society for Research Activity (ISRA), Scientific Index Services (SIS), Academic Research Index (ResearchBib), Index Copernicus (European Union),CNKI (Tongfang Knowledge Network Technology Co., Beijing, China), etc.

Google Dictionary has translated the neologisms "neutrosophy" (1) and "neutrosophic" (2), coined in 1995 for the first time, into about 100 languages.

FOLDOC Dictionary of Computing (1, 2), Webster Dictionary (1, 2), Wordnik (1), Dictionary.com, The Free Dictionary (1),Wiktionary (2), YourDictionary (1, 2), OneLook Dictionary (1, 2), Dictionary / Thesaurus (1), Online Medical Dictionary (1, 2), and Encyclopedia (1, 2) have included these scientific neologisms.

DOI numbers are assigned to all published articles.

Registered by the Library of Congress, Washington DC, United States, https://lccn.loc.gov/2013203857.

Recently, NSS was also approved for Emerging Sources Citation Index (ESCI) available on the Web of Science platform, starting with Vol. 15, 2017.

Editors-in-Chief:

Prof. Dr. Florentin Smarandache, Postdoc, Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA, Email: smarand@unm.edu.
Dr. Mohamed Abdel-Basset, Faculty of Computers and Informatics, Zagazig University, Egypt, Email: mohamed.abdelbasset@fci.zu.edu.eg.
Dr. Said Broumi, Laboratory of Information Processing, Faculty of Science Ben M'Sik, University of Hassan II, Casablanca, Morocco, Email: s.broumi@flbenmsik.ma.

23316055

$39.95