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{Special Issue: Neutrosophy and Plithogeny: Fundamentals and Applications}

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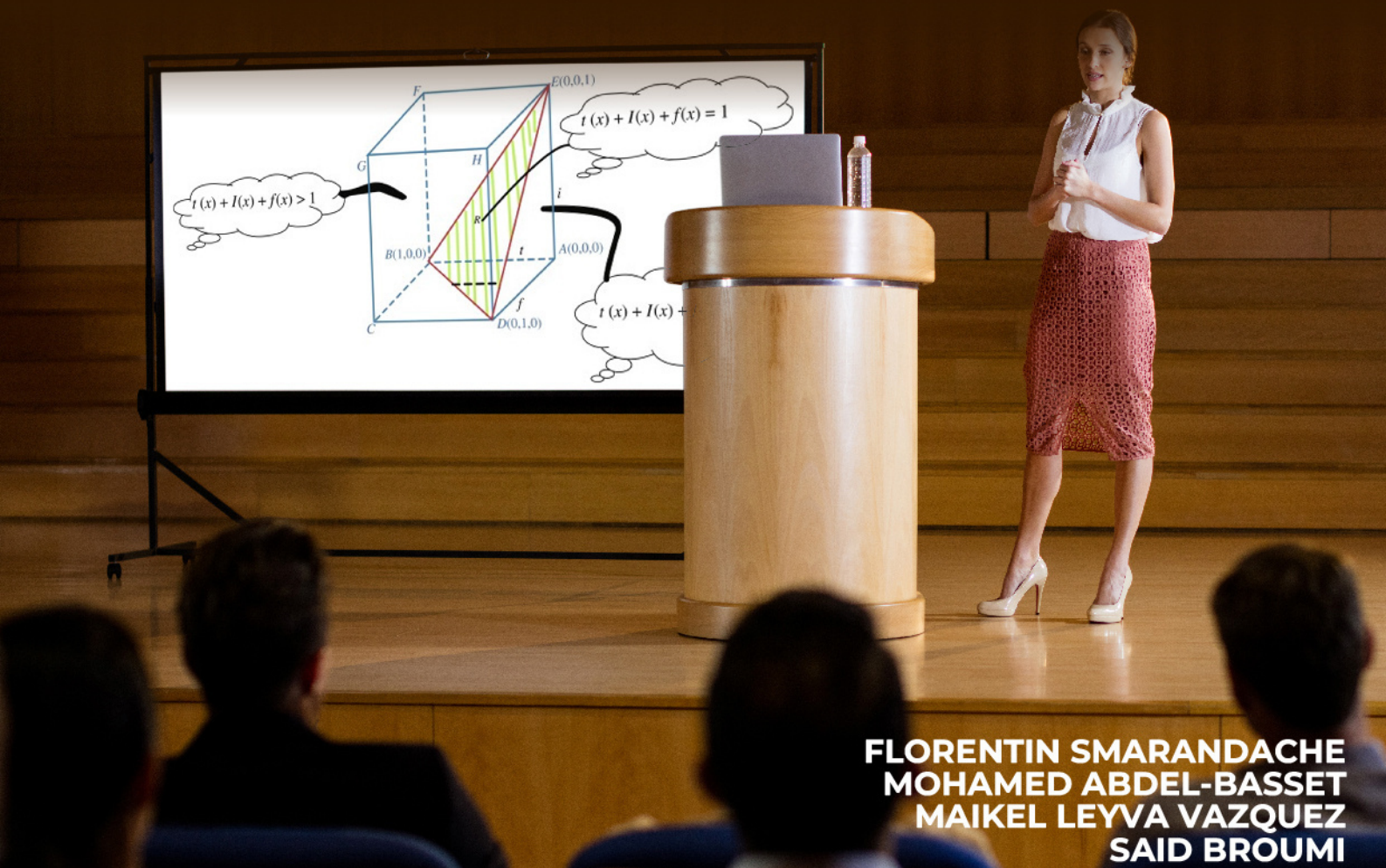

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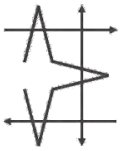

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Information for Authors and Suscribers

"Neutrosophic Sets and Systems" has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results. Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their inter actions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only). According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and Neutrosophic Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of]-0, 1+[.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.
Neutrosophic Statistics is a generalization of the classical statistics.
What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, igno- rance, imprecision, etc.

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Preface: Neutrosophy and Plithogeny: Fundamentals and Applications

We are pleased to present this special issue dedicated to the Neutrosophic approaches in research, on the occasion of the international and multidisciplinary conference held at the Universidad César Vallejo in Lima, Peru, on July 8 and 9. This event marks a significant milestone, as it is the first time that the Andean region and Latin America host scholars and researchers dedicated to studying various theoretical and applicative issues in the expansive and diverse field of Neutrosophic approaches.

Since its conception, Neutrosophic theory has proven to be an interdisciplinary and innovative field, notably growing with the introduction of several generalizations of Neutrosophic Sets, such as Plithogenic Sets, Hypersoft Sets, IndetermSoft Sets, SuperHyperSoft Sets, and MultiAlism. These advanced conceptualizations have further expanded the versatility and application range of Neutrosophic theory, allowing its adoption in an ever-increasing spectrum of disciplines.

The conference, with its international and multidisciplinary character, has brought together experts and scholars from various fields, providing a unique platform for the discussion and exchange of ideas on the multiple applications of Neutrosophic approaches. The topics covered in this event include, but are not limited to:

- **Neutrosophic Approaches in various disciplines**, including:
  - Computer Science
  - Biological Sciences
  - Social Sciences
  - Engineering
  - Art
  - Philosophy
- **Neutrosophic Sets and their generalizations**, such as:
  - Plithogenic Sets
  - Hypersoft Sets
  - IndetermSoft Sets
  - TreeSoft Sets
- **MultiAlism System of Thought**

This special issue also addresses how scientific production in Neutrosophy focuses on social issues specific to Latin American philosophy. In the regional context of Latin America, it is possible to state that Neutrosophic tools and knowledge are used for the identification, analysis, and resolution of social problems, offering unique approaches or distinctive contributions to the field of Neutrosophy, influenced by its cultural and philosophical context.

Neutrosophic science in Latin America shows a clear pattern of how scientific production addresses social problems, standing out for its innovative approaches that reflect the cultural and philosophical particularities of the region. This approach has allowed Neutrosophy not only to advance in theoretical terms but also to provide practical and contextually relevant solutions to social challenges.

This special issue compiles works presented at the conference, reflecting the richness and diversity of current research in this field. We hope that these articles not only contribute to the advancement of knowledge in Neutrosophic theory but also inspire new research and applications in multiple disciplines.

We thank all participants and collaborators for their valuable contributions and hope that this special issue serves as an important reference for future studies in this fascinating and constantly evolving field of research.

Conference Organizers:

1. Prof. Dr. Florentin Smarandache, PhD, Postdoc, Mathematics Department, University of New Mexico, Gallup, NM 87301, USA
2. Dr. Maikel Leyva Vazquez, Universidad de Guayaquil, Guayas, Ecuador
3. Dr. Jesús Estupiñán Ricardo, Instituto Superior Tecnológico de Investigación Científica e Innovación, Ecuador
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University of New Mexico

Application of NCMs and MultiAlism in Indigenous Art Analysis

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Abstract. This study utilizes Neutrosophic Cognitive Maps (NCMs) to investigate cultural interactions in Tigua, an indigenous village in Ecuador known for its rich artistic heritage. The approach evaluates five crucial factors: ancestral traditions (A), external influences (B), cultural synthesis (C), artistic creativity (D), and education and cultural transmission (E). The activation of cultural synthesis (node C) is crucial for merging traditions with new influences, leading to the development of cultural and artistic breakthroughs. By integrating MultiAlism notions into the discourse, the comprehension of Tigua artworks is heightened since it establishes a correlation between indigenous and postcolonial art. This synthesis not only alters and strengthens historic traditions but also improves educational programs and fosters the incorporation of external influences. The findings highlight the importance of education in protecting and promoting culture, highlighting that making changes to customs in a creative way is essential for maintaining cultural vitality in a global society. In the end, this strategy offers a distinct and comprehensive blueprint for other towns that want to protect their cultural heritage during a changing and developing environment.

Keywords: MultiAlism, art conservation, cultural adaptation, neutrosophic cognitive maps.

1 Introduction

The indigenous village of Tigua, situated in the rural parish of Zumbahua in the Pujilí canton of the Co-topaxi region, has successfully maintained its traditions and customs throughout the years. By engaging in this practice, it has preserved its cultural identity by means of creative manifestations as Tigua paintings [1]. These artworks, which integrate traditional methods with indigenous cultural motifs [2], exemplify a distinctive kind of Ecuadorian folk art that has been created from 1970 till now.

The Toaquiza family, led by Julio Toaquiza and his children, pioneered this pictorial style that has garnered the interest of historians and enthusiasts due to its abundant iconography and significant cultural, artistic, symbolic, and anthropological significance. Nevertheless, the production of these works has been shaped by three primary elements. Globalization, as facilitated by technology means, brings about new production practices and foreign cultural references, thereby impacting the traditional knowledge and comprehension of the Andean communities.

Immigration is the second aspect that results in the relocation of inhabitants, leading to the assimilation of foreign customs by modern generations. This assimilation causes a loss of identity in Andean cities [3]. Finally, the third component is education, which, despite its function in acquiring knowledge, frequently neglects to include cultural art in its curriculum.

These factors demonstrate how the art of Tigua has been conditioned over time. For the indigenous people of Tigua, maintaining ancestral techniques in their creative processes is crucial, as these techniques foster creativity and the inclusion of neighboring communities [4]. Added to this is the diversity of cultural art, which is seen as a significant resource for knowledge and dialogue. Furthermore, it helps to preserve the diversity of native art from any territory or locality, serving as a source of creative inspiration alongside innovation.

Therefore, this study aims to analyze and understand the interrelationships between different aspects of Tigua's culture, in order to identify strategies that promote the conservation and cultural innovation in a balanced manner. To this end, the following specific objectives are proposed:

- Analyze the cultural environment of Tigua through artistic expressions.

- Analyze the interrelationships between the components of indigenous art using the neutrosophic conceptual map.
- Propose strategies that enhance the culture and art of Tigua.

2 Materials and Methods

2.1 Qualitative Analysis of the Cultural Environment. Erwin Panofsky Method.

The study of Tigua paintings employs a qualitative methodology, specifically relying on Erwin Panofsky's method for iconographic analysis [5] [6]. The study commenced by conducting interviews with three prominent artists from the community, employing a standardized questionnaire to examine the diversity in the materials utilized in the production of their artworks. A total of 100 paintings were considered, but only thirty were chosen. The paintings that were not in the naive style or did not have Andean themes were excluded. The paintings were sourced from two primary galleries, namely the Primer Pintor Gallery and the Wilson Toaquiza Gallery, to ensure that the chosen artworks retained their ancestral attributes (Table 1).

Table 1: Coded record of the painting First Dream. Source: own elaboration.

Code	**Theme:** First dream	**Processes**	**Materials**
012		His initial sketches were made on pieces of paper, followed by placing the cured leather in a laurel wood frame and making undetailed pencil sketches. Subsequently, he painted from top to bottom using various types of brushes, giving detailed characteristics to the painting.	-Sheep leather -Wooden frame -Hair of their children -Aniline -Taino Plant -Petromax - Light
Dimension:	Unknown	**Technique**	Naive
Location:	Primer Pintor Gallery of Tigua	**Author**	Julio Toaquiza
Date	1973	**Cost:**	6200

By applying Erwin Panofsky's method, the cultural environment analysis of the Tigua paintings has facilitated a methodical and comprehensive examination, uncovering profound levels of significance and artistic skill. The analysis was organized into three levels: pre-iconographic, iconographic, and iconological, with each level providing a more profound comprehension of the visual aspects and their cultural and artistic importance.

At the pre-iconographic level, the paintings were analyzed to identify their pure forms and basic qualities. The compositions exhibit a diverse array of colors and textures that accurately depict the lives and natural surroundings of the Tigua population. The depictions of plant and animal life are particularly intricate, accurately portraying the variety of species and the interplay between humans and their surroundings.

Iconographic Level: This level entails the examination of traditional themes and narratives depicted in the artworks. Tigua paintings often depict ordinary occurrences within their society, combining elements of their traditional culture with their modern interactions. Recurring patterns were observed, including the use of geometric and organic shapes, which aid in structuring the compositions and guiding the viewer's comprehension towards the core concepts of unity and cultural continuity.

An in-depth analysis was conducted at the iconological level to interpret the intrinsic meanings of the works. This level showcased how Tigua artists effectively employed art to communicate fundamental aspects of their worldview, spirituality, and philosophy. Garments, tools, and creatures are displayed not merely for their practical or ornamental value, but also as symbols that have important cultural and spiritual meanings.

2.2 Neutrosophy

Definition 1. Let X be a universe of discourse[7]. A Neutrosophic Set (NS) is characterized by three membership functions[8], $u_A(x), r_A(x), v_A(x): X \rightarrow$]$^-0.1^+$[,that satisfy the condition $-0 \leq inf u_A(x) + inf r_A(x) + inf v_A(x)\ sup\ u_A(x) + sup\ r_A(x) + \sup v_A(x) \leq 3^+$ for all $x \in X$. $u_A(x), r_A(x)$ and $v_A(x)$ denote the true, indeterminate, and false membership functions of x in A, respectively, and their images are standard or non-standard subsets of $]-0,1+[$.

Definition 2. Let X be a universe of discourse. A Single Value Neutrosophic Set (SVNS) A over X is an object of the form [9]:

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\} \quad (1)$$

Where $u_A, r_A, v_A : X \to [0,1]$, satisfy condition $0 \leq u_A(x), r_A(x), v_A(x) \leq 3$ for all $x \in X$. $u_A(x), r_A(x)$ and $v_A(x)$ denote the true, indeterminate, and false membership functions of x in A, respectively. For convenience, a Single Valued Neutrosophic Number (SVNN) will be expressed as A = (a, b, c), where a, b, c [0,1] and satisfies $0 \leq a + b + c \leq 3$.

2.3 Neutrosophic Cognitive Maps (NCMs)

Starting from the previous elements, in this particular work the use of Neutrosophic Cognitive Maps (NCMs) is proposed considering the advantages that this technique offers compared to other soft-computing techniques [10], in terms of interpretability, scalability, aggregation of knowledge, dynamism and its ability to represent feed-back and indeterminacy relationships [11].

NCMs were introduced in 2003. NCMs are an integration of the Fuzzy Cognitive Maps (FCM) introduced by Kosko in 1986 and the Neutrosophic Sets (NSs) introduced by Smarandache in 1995. This technique overcomes the inability of traditional FCMs to represent indeterminacy. The inclusion of indeterminacy establishes that neutrality and ignorance are also forms of uncertainty. Exposes that FCMs constitute a technique that has received increasing attention due to its possibilities for representing causality. The following is a set of definitions necessary for working with NCMs. Firstly, let us formally expose the original definition of neutrosophic logic as it is shown in [12].

Definition 1 [13]: $Let\ N = ((T, I, F) : T, I, F \in [0,1])$

Let C be a *neutrosophic set of evaluations*. v: $P \to$N is a mapping of a group of propositional formulas into N, i.e., each sentence $p \in P$ is associated with a value in N, as it is exposed in Equation 2, meaning that p is T% true, I% indeterminate and F% false.

$$v\,(p) = (T, I, F) \quad (2)$$

Hence, neutrosophic logic is a generalization of fuzzy logic, based on the concept of neutrosophy.

Definition 2 [14]: Let K be a ring of real numbers. The ring generated by $K \cup I$ is called a *neutrosophic ring* if it involves the indeterminacy factor in it, where I satisfy $I2 = I, I + I = 2I$ and in general, $I + I + \ldots + I = nI, if\ k \in K, then\ k.I = kI, 0I = 0$. The neutrosophic ring is denoted by $K(I)$, which is generated by $K \cup I, i.e., K(I) = < K \cup I >, where\ < K \cup I >$ denotes the ring generated by K and I.

Definition 3: A *neutrosophic matrix* is a matrix $A = [a_{ij}]ij$ i =1, 2, …, m and j = 1, 2, …, n; $m, n \geq 1$, such that each $a_{ij} \in K(I)$, where K(I) is a neutrosophic ring.

Let us observe that an element of the matrix can have the form $a + bI$, where "a" and "b" are real numbers, whereas I is the indeterminacy factor. The usual operations of neutrosophic matrices can be extended from the classical matrix operations.

For example,

$$\begin{pmatrix} -1 & \mathrm{I} & 5\mathrm{I} \\ \mathrm{I} & 4 & 7 \end{pmatrix} \begin{pmatrix} \mathrm{I} & 9\mathrm{I} & 6 \\ 0 & \mathrm{I} & 0 \\ -4 & 7 & 5 \end{pmatrix} = \begin{pmatrix} -21\mathrm{I} & 27\mathrm{I} & -6 + 25\mathrm{I} \\ -28 + \mathrm{I} & 49 + 13\mathrm{I} & 35 + 6\mathrm{I} \end{pmatrix}$$

Additionally, a *neutrosophic graph* is a graph that has at least one indeterminate edge or one indeterminate node. The *neutrosophic adjacency matrix* is an extension of the adjacency matrix in classical graph theory. $a_{ij} = 0$ means nodes i and j are not connected, $a_{ij} = 1$ means that these nodes are connected and $a_{ij} = I$, which means the connection is indeterminate (unknown if there is a connection or not). Fuzzy set theory does not use such notions.

On the other hand, if indeterminacy is introduced in a cognitive map, then this cognitive map is called a *neutrosophic cognitive map*, which is especially useful in the representation of causal knowledge. It is formally defined in Definition 4.

Definition 4 [15,16]. A *Neutrosophic Cognitive Map* (NCM) is a neutrosophic directed graph with concepts like policies, and events, among others, as nodes and causalities or indeterminacies as edges. It represents the causal relationship between concepts.

In this case

The measures described below are used in the proposed model, they are based on the absolute values of the adjacency matrix [17]:

- Outdegree (v_i) is the sum of the row elements in the neutrosophic adjacency matrix. It reflects the strength of the outgoing relationships (c_{ij}) of the variable.

$$od(v_i) = \sum_{i=1}^{n} c_{ij} \tag{3}$$

- Indegree (v_i) is the sum of the column elements. It reflects the strength of relations (c_{ij}) outgoing from the variable.

$$id(v_i) = \sum_{i=1}^{n} c_{ji} \tag{4}$$

- Total centrality (total degree $td\ (v_i)$), is the sum of the indegree and the outdegree of the variable.

$$td(v_i) = od(v_i) + id(v_i) \tag{5}$$

The static analysis is applied using the adjacency matrix, taking into consideration the absolute value of the weights. Static analysis in Neutrosophic Cognitive Maps (NCMs), initially contains the neutrosophic number of the form ($a + bI$, where I = indetermination). It requires a process of deneutrosophication, where $I \in [0, 1]$ and is replaced by their maximum and minimum values.

Finally, a sum of the values is calculated using (5), which is useful to obtain a single value. This value contributes to the identification of the characteristics to be attended, according to the factors obtained, for our case study.

$$\lambda([a_1, a_2]) = \frac{a_1 + a_2}{2} \tag{6}$$

$$A \succ B \Leftrightarrow \frac{a_1 + a_2}{2} > \frac{b_1 + b_2}{2} \tag{7}$$

In this paper nodes conections are representes with Single Valued Neutrosophic Number (SVNN) A deneutrosification process is used in this case in line with [18]

Let $AN = \{x, (TA(x), IA(x), FA(x)) : x \in X\}$ be an NS. Its equivalent fuzzy membership set is defined as $AF = \{(x, \mu A(x)) : x \in X\}$, where $\mu A(x) = s\left((TA(x), IA(x), FA(x)), (1,0,0)\right)$. So, using the equation of similarity:

$$\mu A(x) = 1 - \tfrac{1}{2}[(1 - T_A(x)) + \max\{I_A(x), F_A(x)\}] \tag{8}$$

As the range of the similarity measure function is the unit interval [0,1], μA(x)∈[0,1] for all x∈X. Hence, the membership function of the derived fuzzy set belongs to [0,1] and thus it satisfies the property of a membership function of a fuzzy set (FS).

The density function of the random variable X is defined as :

$f(x) = \frac{\mu A(x)}{\Delta}\Delta$

where $\Delta = \int_{-\infty}^{\infty} \mu A(x)\, dx$ (9)

2.4 Philosophy and Art Integration via MultiAlism

MultiAlism proposes a paradigm that allows for the coexistence and interaction of other philosophical elements, including monism, dualism, pluralism, and more complex varieties. MultiAlism, created by Florentin Smarandache, promotes the investigation and combination of various dimensions. This makes it especially appropriate for environments that have a wide range of philosophical traditions, such as Latin America [19].

Leyva and Smarandache examine Latin American philosophy via the perspective of MultiAlism, which can be incorporated into indigenous art to provide a more profound and nuanced comprehension of the cultural and spiritual customs of the region. This study examines the incorporation of native philosophies and postcolonial modernist theories into the MultiAlism framework, emphasizing the possibility of dialogue and combination between these many traditions in Latin America.

Philosophical Aspects [18]:

- Monism posits the existence of a singular fundamental reality or principle, denoted as $< A > = \infty$, where $< A >$ represents an abstract concept or substance, and ∞ represents the entirety of the world, reality, or everything.

- Dualism is a philosophical concept that acknowledges the existence of two fundamental and frequently opposing forces or principles. In this context, $< A >$ represents an abstract concept, a material substance, or any other entity, whereas $< antiA >$ represents its opposite or negation. The symbol ∞ represents the encompassing notion of the world, reality, or everything.

- Pluralism encompasses a multitude of coexisting principles or realities. In this context, $< pluriA >$ represents a quantity more than two (perhaps three) 'ideas', and ∞ symbolizes 'world', 'reality', or 'all'.

- .MultiAlism surpasses these classifications by promoting a framework in which these and other possible philosophical categories might dynamically interact, i.e., $< (multi)A > + < (multi)neutA > + < (multi)antiA > = \infty$.

- Pluralism, by going beyond these categorizations, encourages a framework in which different philosophical notions can interact in a dynamic manner. The equation $< (multi)A > + < (multi)neutA > + < (multi)antiA > = \infty$ represents this relationship.

MultiAlism offers a significant framework for comprehending and valuing the intricate interplay of diverse cultural, spiritual, and philosophical components found in Latin American indigenous art.

3 Results

3.1 Analysis of the cultural environment of Tigua

The study examines the progression of techniques and substances employed in Tigua paintings, emphasizing traditional customs and contemporary modifications. Since 1973, the paintings have consistently had a Naive style and have been executed on sheepskin canvases. Originally, aniline was utilized, but it did not adhere well to the sleek leather surface. Therefore, it was combined with the extract from the Tañe stem to obtain a successful blending of colors. Enamel blended with thinner, which created a satin finish, was discontinued because of its detrimental health effects, despite its cosmetic advantages. Therefore, acrylic and oil paints were ultimately selected in order to reduce the potential hazards to the respiratory system.

Interviews indicate that Julio Toaquiza initially painted on wooden drums, but later incorporated suggestions from visitor Olga Fisch, who recommended the use of wooden frames. Traditional painting materials, including chicken feathers and human hair, were utilized until 2007, at which point they were substituted with contemporary brushes.

The works explore themes such as religion, culture, traditions, and the daily lives of Tigua, showcasing a consistent thematic thread from its inception to the present day. At the pre-iconographic level, the paintings depict customary landscapes featuring mountains, volcanoes, and everyday tasks like spinning and herding. Iconographic analysis utilized Gestalt rules to deconstruct the visual arrangement, discerning the grouping of people, animals, and natural components to convey unity and resemblance (refer to Figures 1 to 4).

Figure 1: Iconographic analysis of human beings, clothing. Source: Taken from Wilson Toaquiza Gallery

.

Figure 2: Iconographic analysis of animals, and elements of nature. Source: Taken from Wilson Toaquiza Gallery.

Figure 3: Iconographic analysis of infrastructure, and work tools. Source: Taken from Julio Toaquiza Gallery.

Figure 4: Iconographic analysis of festive characters. Source: Taken from Julio Toaquiza Gallery.

The study examines the symbolic depictions found in Tigua paintings, with a specific focus on animals, infrastructure, work instruments, and clothing. This piece of work emphasizes the correlation between nature, sustainability, and modernism, while also underscoring the significance of colors and patterns in communicating cultural meanings. The paintings function as a dynamic testament to the community.

3.2 NCMs for Indigenous Art

To enhance the culture of Tigua, it is necessary to visualize the relationships between the components of indigenous art through neutrosophic conceptual maps. This approach highlights the complexity and cultural dynamics inherent to their study and practice. It is essential to recognize that interactions between tradition and external influences are not binary and that indeterminacy in these interactions can lead to enriching and respectful forms of cultural expression. Consequently, the following neutrosophic conceptual map for indigenous art is presented:

1. Central Node: Indigenous Art
   - Description: This node acts as the core of the analysis, focused on how indigenous art serves as a dynamic manifestation of cultural identity and community creativity.
2. Neutrosophic Nodes and States:
   A. Ancestral Traditions

Neutrosophic State: They represent the essence and unalterable foundations of the community, but their interpretation and application can vary, showing how traditions adapt without losing their fundamental meaning.

B. External Influences
Neutrosophic State: While external influence may be seen as a dilution of cultural authenticity, it can also act as a catalyst for adaptation and change. It provides a new context for traditional expression without completely denying its value.

C. Cultural Synthesis
Neutrosophic State: This node highlights the emergence of cultural forms that are neither completely traditional nor entirely modern, but an amalgam that reflects both continuity and transformation.

D. Artistic Innovation.
Neutrosophic State: Focuses on how innovation within the indigenous artistic community creates new forms of art that may not clearly conform to traditional or modern categories but flow between both.

E. Education and Cultural Transmission.
Neutrosophic State: Education and cultural transmission in indigenous communities often face the challenge of balancing the preservation of ancestral knowledge with the incorporation of new ideas and techniques, which can result in practices that are both conservative and progressive.

3. Measurement Scales and Linguistic Terminology: Strong: from (1,0,0) to (0.8,0.2,0); moderate from (0.7,0.3,0) to (0.5,0.3,0.2); weak (0.3,0.4,0.3) to (0.2,0.6,0.2).

The analysis of the NCMs underscores the complexity and dynamics of the interactions between tradition and modernity in the context of indigenous art (see Tables 2 and 3). It accepts that the interaction between traditional and modern elements is not binary and recognizes that areas of indeterminacy and contradiction can be sources of cultural and artistic enrichment.

Table 2: NCM adjacency matrix.

Node	A	B	C	D	E
A	(1, 0, 0)	(0.2, 0.5, 0.3)	(0.8, 0.2, 0)	(0.7, 0.3, 0)	(0.9, 0.1, 0)
B	(0.3, 0.4, 0.3)	(1, 0, 0)	(0.5, 0.3, 0.2)	(0.6, 0.4, 0)	(0.2, 0.6, 0.2)
C	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(1, 0, 0)	(0.9, 0.1, 0)	(0.8, 0.2, 0)
D	(0.5, 0.3, 0.2)	(0.7, 0.2, 0.1)	(0.8, 0.1, 0.1)	(1, 0, 0)	(0.7, 0.2, 0.1)
E	(0.9, 0.1, 0)	(0.3, 0.4, 0.3)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(1, 0, 0)

Table3: Deneutrosified adjacency matrix.

Node	A	B	C	D	E	$od(v_i)$
A	1	0.35	0.2	0.65	0.95	3.15
B	0.45	1	0.5	0.7	0.5	3.15
C	0.75	0.65	1	0.95	0.9	4.25
D	0.6	0.75	0.85	1	0.75	3.95
E	0.95	0.45	0.85	0.75	1	4
$id(v_i)$	3.75	3.2	3.4	4.05	4.1	

Table4: Centrality analysis

Nodes	$od(v_i)$	$id(v_i)$	$td(v_i)$
Ancestral traditions	3.15	3.75	6.90
External influences	3.15	3.2	6.35
Cultural synthesis	4.25	3.4	7.65
Artistic innovation	3.95	4.05	8.00
Education and cultural transmission	4	4.1	8.10

Therefore, the following order of importance of the nodes is defined: E≻D≻C≻B≻A. These results help us understand how the elements of art and indigenous culture intertwine to form a dynamic cultural ecosystem. The

nodes and their relationships, as well as the effect of activating the most influential node, are described below:

I. Node A (Ancestral Traditions)
- Relationships: Connects bidirectionally with all other nodes (B, C, D, E).
- Meaning: Represents the customs and practices transmitted that form the base of indigenous culture.

II. Node B (External Influences)
- Relationships: Has a bidirectional connection with A and an indeterminate connection with C, representing the uncertainty about how external influences directly affect cultural synthesis.
- Meaning: Symbolizes the external elements and forces that can influence or modify traditions.

III. Node C (Cultural Synthesis)
- Relationships: Connects bidirectionally with all nodes, highlighting its central role in integrating traditions and innovations.
- Meaning: Embodies the fusion of ancestral traditions with modern or external influences, generating new cultural expressions.

IV. Node D (Artistic Innovation)
- Relationships: Connects bidirectionally with A, C, and E, showing how innovation is fueled by education, cultural synthesis, and traditions.
- Meaning: Represents the new forms of expression and creativity that arise within the culture.

V. Node E (Education and Cultural Transmission)
- Relationships: Connects bidirectionally with all nodes except B, indicating that it plays a crucial role in teaching and preserving both traditions and innovations.
- Meaning: Refers to how cultural practices are taught and kept alive through generations.

In this context, the strongest node is Node C (Cultural Synthesis), due to its central position and bidirectional connections with all nodes. If this node is activated, it implies a strong integration and adaptation of traditions and external influences, resulting in new cultural forms that may include artistic innovations and adapted educational methods. The activation of this node has a cascading effect on the others:

- Increase in Artistic Innovation (D): The new cultural synthesis can inspire innovative artistic forms.
- Reinforcement of Ancestral Traditions (A): Traditions adapt and become more resilient by being reinterpreted in new contexts.
- Stimulation of Education and Cultural Transmission (E): Education incorporates these new cultural elements, ensuring transmission to future generations.
- Interaction with External Influences (B): External influences are assimilated more effectively within the culture, allowing a dynamic interaction between the old and the new.

In summary, the activation of Node C facilitates a dynamic and healthy exchange among all aspects of the culture. It also allows for constant evolution and adaptation in the face of new influences and internal and external challenges.

4 Discussion

An analysis of Tigua paintings, employing Erwin Panofsky's methodology and iconographic documentation, uncovers a noteworthy artistic progression within the community. The study emphasizes the complexity of their visual arrangement and varied range of colors, showcasing the concepts of Gestalt. The iconological study examines the symbolism of the paintings, emphasizing the relationship between the society and its natural surroundings and indigenous wildlife, offering a significant record of their existence and encounters.

To maximize the potential of the findings described and promote the conservation and dissemination of the art and culture of Tigua, the following action plan is proposed, which includes specific strategies and activities (see Table 5). This plan is designed to strengthen artistic innovation, preserve ancestral traditions, and encourage education and cultural transmission.

Table 5: Action plan to promote the culture and art of Tigua. Source: own elaboration.

Strategy	Key activities	Indicators of success
Conservation of ancestral art.	- Cataloging and digitization of existing works. - Workshops on conservation techniques for local artists.	- Number of digitized works. - Participation in workshops.
Artistic innovation and development.	- Creation of an art laboratory to experiment with new techniques and materials. - Annual exhibitions to showcase innovative works.	- New techniques developed. - Assistance and sales at exhibitions.
Education and cultural	- Educational programs in schools that include the history	- Number of participating

Strategy	Key activities	Indicators of success
transmission.	and art techniques of Tigua.	schools.
	- Scholarships for young artists to study both locally and internationally.	- Number of scholarships awarded.
Integration of external influences.	- Invitations to international artists for workshops and collaborations.	- Participating international artists.
	- Webinars and conferences on the global influence of traditional art.	- Attendance at online events.
Cultural synthesis.	- Collaborative projects that blend traditional and modern elements.	- Fulfilled projects.
	- Documentaries and publications that narrate the evolution of Tigua art.	- Viewings and distribution of documentaries.

The objective of this plan is to not only safeguard the cultural heritage of Tigua, but also to modify and rejuvenate it in response to contemporary difficulties and worldwide prospects. By doing so, it guarantees the continuous flourishing of Tigua's cultural diversity while also allowing it to flexibly change over time. Each strategy in the previous list is specifically meant to tackle various facets of cultural and artistic advancement in Tigua, encompassing the preservation of ancient techniques as well as the adaptation and creation of novel forms of artistic expression. It efficiently incorporates external influences and encourages education and cultural dissemination through inventive platforms and international collaborations. This systematic method guarantees that every component of cultural development is given thorough attention and sufficient resources, so aiding the accomplishment of the established goals.

By incorporating the ideas of MultiAlism [9] into the debate, the understanding of Tigua artworks is enhanced since it establishes a connection between indigenous art and postcolonial art. This approach places cultural diversity as a top priority and examines power structures, fostering a deeper comprehension and appreciation for the great indigenous heritage of Latin America [20]. The proposed action plan for preserving and promoting Tigua art involves MultAlism, which aims to conserve indigenous techniques while also advocating for creative and inclusive policies. These strategies are formulated to address current issues and guarantee the ongoing development and worldwide importance of Tigua's cultural heritage.

4 Conclusion

Neutrosophic Cognitive Maps (NCMs) have been successfully utilized to detect and analyze the intricate cultural dynamics of Tigua. Using NCMs, the evaluation of the connections among various cultural elements, such as traditional customs, creative advancements, and education, has facilitated the identification of specific focal points. Notably, node C (cultural synthesis) plays a crucial role in driving significant transformations in the cultural ecology. This technique has enabled the visualization and prediction of how actions in one area can impact others, so promoting more efficient strategic planning that considers the interconnection of cultural aspects.

Education and efficient cultural transmission are essential for the preservation and continuous advancement of Tigua's culture. By incorporating the art history, techniques, and cultural symbolism of Tigua, educational programs guarantee the preservation of ancestral knowledge for future generations. Furthermore, by cultivating a profound comprehension and admiration for indigenous art among young individuals, they not only sustain the culture but also promote the generation of novel artistic manifestations that preserve the tradition. This transmission is not one-way; it is enhanced by the integration of novel ideas and perspectives that young people can contribute, so assuring an ongoing evolution that honors the past while also focusing on the future.

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Analysis Using Treesoft Set of the Strategic Development Plan for Extreme Poverty Municipalities

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Abstract. The analysis of the Strategic Development Plan for extreme poverty municipalities in the Province of Huancayo, Peru in 2019 using Treesoft Set and neutrosophic sets represents an innovative and rigorous approach to evaluate development strategies in challenging socioeconomic contexts. In this study, advanced data analysis and uncertainty management tools are used to thoroughly examine the actions proposed in the strategic plan and their potential impact on the most vulnerable communities. The combination of Treesoft Set, a comprehensive platform for data analysis, and neutrosophic sets, which allow the modeling of imprecision and ambiguity in collected data and opinions, offers a robust methodology to address the complexity inherent in planning and executing strategy strategies. development in municipalities with high levels of poverty.This analysis seeks to identify both the strengths and possible areas of improvement of the Strategic Development Plan, providing an objective and well-founded basis for decision-making in public policies aimed at reducing poverty and promoting sustainable development. By exploring the multiple dimensions of socioeconomic development through the application of Treesoft Set and neutrosophic sets, this study aims to contribute to the design of more effective interventions adapted to the specific needs of extreme poverty municipalities in the Province of Huancayo. This methodological approach has the potential to generate significant insights that can inform and improve local development strategies, thus promoting a positive and lasting impact on the living conditions of the most vulnerable populations.

Keywords: Treesoft Set, Neutrosophic Set; Strategic Development Plan

1 Introduction

This investigation begins, after becoming aware of the report provided by the National Center for Strategic Planning, hereinafter CEPLAN, on the province of Huancayo, capital of the Junín Region. , in its 2017 Report where extreme poverty districts are cataloged Pariahuanca , Chacapampa , Santo Domingo de Acobamba , Cullhuas , Huasicancha , Carhuacallanga , Chongos Alto, Pucara (heroic city), Colca and Chicche , although the city of Huancayo is classified as one of the leading cities in economic growth in the central region of the country [1].

Given the lack of professionals with government experience to provide their services ad honorem to these local governments, there is a conviction of the need to propose a Strategic Development Plan model, that is, to achieve better management in the Extreme Poverty Municipalities of the Province of Huancayo [2].

Similarly, in Peru, the decentralization process is based on economic and administrative autonomy in the management of district municipalities, and the Public Budget Law establishes guidelines for the formulation of short and long-term plans, which cannot be applied in local governments living in extreme poverty. Therefore, this article describes the strategic lines for the development of these municipalities. The same will be evaluated and updated every four (04) years, by the representative actors of the municipality, to eradicate extreme poverty in its population [3].

However, the application of these standards has not been possible, due to the municipal management of local governments of extreme poverty [4]. That is why this thesis research work is proposed as an instrument that provides strategic lines for the development of these municipalities. The same will be evaluated and updated every four (04) years, by the representative actors of the community recognized by the District Municipality of extreme poverty. It must monitor

them as well as make continuous improvements and also articulate efforts to take advantage of the predominant natural resources in each of them.

In this paper, we utilized the MCDM [5] methodology to examine the strategic development plan for extreme poverty municipalities in the province of Huancayo, Peru. To facilitate this evaluation, we employed Treesoft Set and neutrosophic sets as analytical tools. By applying the MCDM methodology, we evaluated a variety of criteria and used the VIKOR[6] method to rank the available alternatives. The integration of MCDM methodology with neutrosophic sets addressed the uncertainty inherent in the evaluation process, combining them with the Treesoft Set. The participation of three decision-makers and experts was instrumental in classifying the criteria and options, opting to use BNN over the opinions expressed by the experts. Utilizing the VIKOR method, we created three decision matrices, each providing defined values that were combined to form a final matrix. This study focused on the evaluation of 12 criteria and 10 alternatives, aiming to ensure a greater allocation of economic resources from the MEF and improve municipal management and tax collection strategies.

2. Related Work

An interval valued neutrosophic set (IVNS) is defines as

Definition 1[6]:

Let Y be a universe of discourse with a generic element in Y denoted by y. We can define the neutrosophic variable y as $y = (T, I, F)$ where T, I and F refer to the degrees of truth, indeterminacy and falsity.

$$0 \leq \sup \ (\mathrm{T}(y)) + \sup(I(y)) + \sup(F(y)) \leq 3 \quad (1)$$

We can define the IVNS as:

$$y = [\mathrm{T}^L\ T^U], [I^L, I^U], [F^L, F^U] \quad (2)$$

Definition 2

We can define bipolar neutrosophic sets (BNS) [7] :

$$A = \{< x, T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x) >\} \quad (3)$$

$T^+, I^+, F^+: X \rightarrow [0,1]$and $T^-, I^-, F^-: X \rightarrow [0,1]$

The positive membership degree represents the truth, indeterminate, and false membership of an element with respect to a bipolar neutrosophic set. Conversely, the negative membership degree signifies the truth, indeterminate, and false membership of the same element related to some implicit counter property corresponding to the bipolar neutrosophic set [8, 9].

Let U universe of discourse and H a non-empty subset of, with $P(\mathrm{H})$ *be a powerset of* H[10, 11] .

Let TSR a set of attributes of the problem (criteria),

$$TSR = \{\mathrm{T}SR_1, TSR_2, \ldots, TSR_n\}, n \geq 1 \quad (4)$$

Where $TSR_1, TSR_2, \ldots, TSR_n$ are criteria from the first level of the tree.

Each attribute TSR_1, $1 \leq i \leq n$, *is formed by subattributes* :

$TSR_1 = \{ TSR_{1,1}, TSR_{1,2} \ldots, \}$

$TSR_2 = \{ TSR_{2.1}, TSR_{2.2} \ldots, \}$

$TSR_n = \{ \mathrm{T}SR_{n,1}, TSR_{n,2} \ldots, \}$

Where $TSR_{i,j}$ They are sub attributes.

Tree TreeSoft Set [12] can be as:

$$F: P\,(Tree\,(TSR)) \rightarrow P\,(H) \tag{5}$$

$$Tree\,(SR) = \{ TSR_i \mid i_1 = 1,2,3,\ldots \} \cup \{ TSR_i \mid i_1, i_2 = 1,2,3, \ldots \} \cup \{ TSR_i \mid i_1, i_2, i_3 = 1,2,3, \ldots \} \cup \ldots \cup \{ TSR_i \mid i_1, i_2, \ldots i_m = 1,2,3, \ldots \} \tag{6}$$

3. Material and Methods

The methodology employed using the neutrosophic TreeSoft Set with the VIKOR method used in this paper is presented below (Figure 1).

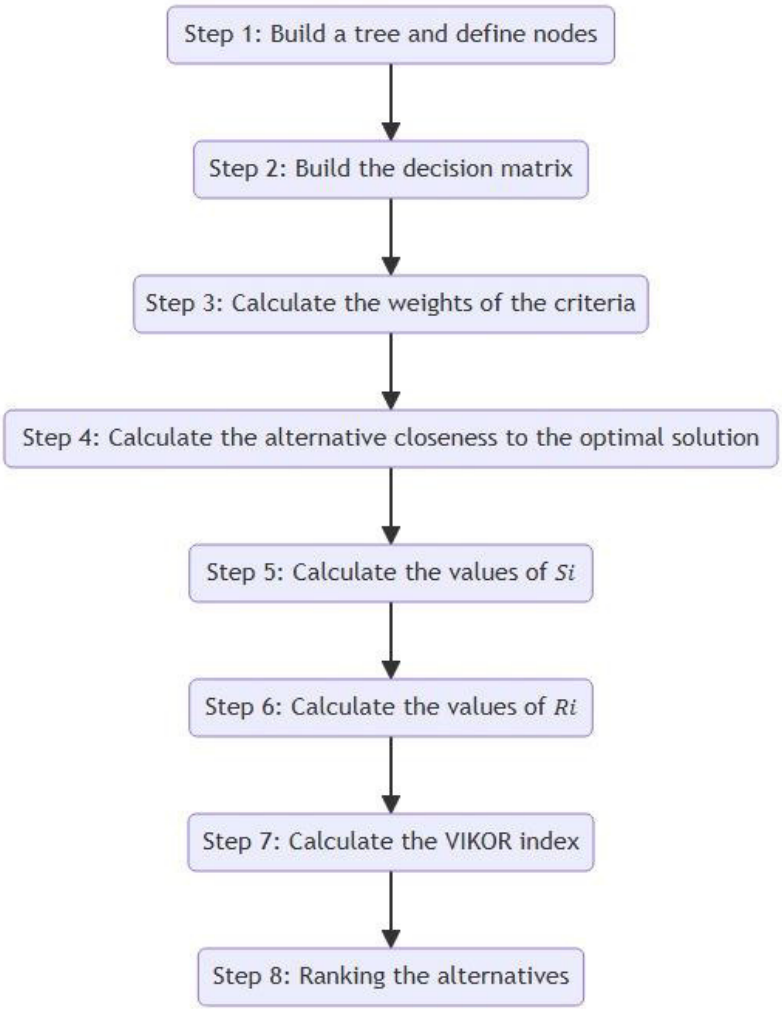

Figure 1. TreeSoft Set with the VIKOR method

Step 1. Build a tree and define nodes [11,12]

The tree has more than one level, at the first level, the main criteria are entered as $SWM_1, SWM_2 \ldots, SWM_n$

At the second level, the sub-criteria are introduced as $SWM_{1.1}, SWM_{1.2}, \ldots$. And $SWM_{2.1}, SWM_{2.2}, \ldots$.

Step 2. Build the decision matrix

The decision matrix is constructed using information from decision makers and experts between criteria and alternatives.

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}; \; i=1,2,\ldots,m; j=1,2,\ldots,n \tag{7}$$

Step 3. Calculate the weights of the criteria.

Criteria weights are calculated using the average method.

$$\sum_{j=1}^{n} w_j = 1 \tag{8}$$

Step 4. Calculate the alternative closeness to the optimal solution.

$$U\,i = \left\{ \sum_{j=1}^{n} \left[\frac{w_j (r_j{}^* - r_{ij})}{(r_j{}^* - r_j{}^-)} \right]^p \right\}^{\frac{1}{p}} \tag{9}$$

Where r_j is the best and r_j^- correspond to the best and worst alternative of the system, respectively

Step 5. Calculate the values of S_i

$$S_i = \sum_{j=1}^{n} w_j \frac{(r_j^* - r_{ij}^-)}{(r_j^* - r_j^-)} \; \text{i=1,2,…,m, j=1,2…,n} \tag{10}$$

Step 6. Calculate the values of R_i

$$R_i = \max_j \left[w_j \frac{(r_j^* - r_{ij}^-)}{(r_j^* - r_j^-)} \right] \; \text{i=1,2,…,m, j=1,2…,n} \tag{11}$$

Step 7. Calculate the VIKOR index[14].

$$Q_i = t * \left[\frac{S_i - S^*}{S^- - S^*} \right] + (1 + \text{t}) * \left[\frac{R_i - R^*}{R^- - R^*} \right] \; \text{i=1,2,…,m, j=1,2…,n} \tag{12}$$

$$S^* = \min_i S_i \,, S^- \max_i S_i \,, R^* = \min_i R_i \,, R^- \max_i R_i$$

Where $t = 0.5$

Step 8. Classify the alternatives.

Alternatives are classified as descending values of Q_i

4. Case Study

This section presents the outcomes of the TreeSoft analysis combined with the BNS and VIKOR methods. The study utilized 8 criteria and 8 alternatives, as illustrated. Three experts evaluated these criteria and alternatives using bipolar neutrosophic numbers (BNNs).

1. **Local Economic Development Programs:** Implement programs that promote economic diversification and local entrepreneurship, including business training, access to microcredit, and support for small businesses.
2. **Promotion of Education and Training:** Develop educational and job training programs that strengthen the skills of the population to access better employment and entrepreneurship opportunities.
3. **Access to Health Services:** Establish health centers and health care programs that improve access to basic and specialized health services for the vulnerable population.
4. **Affordable Housing Development:** Implement social housing projects that provide adequate and affordable housing solutions for low-income families.
5. **Basic Infrastructure Improvement:** Prioritize infrastructure projects that improve access to basic services such as drinking water, sanitation, electricity, roads and public transportation.

6. **Environmental Preservation and Sustainable Development:** Promote environmental conservation practices and sustainable management of natural resources, including reforestation programs, sustainable agriculture and waste management.
7. **Strengthening Community Participation:** Encourage active community participation in decision-making and project execution, through the creation of local committees and inclusive dialogue spaces.
8. **Promotion of Sustainable Tourism:** Develop strategies to take advantage of local tourism potential in a sustainable manner, promoting ecotourism and the valorization of local culture as a source of income.

Evaluation criteria:

1. **Impact on poverty reduction:** Evaluate to what extent the strategic plan has contributed to the effective reduction of poverty in the municipality, considering indicators such as the decrease in the multidimensional poverty index and the improvement in the living conditions of the population. vulnerable.
2. **Inclusion and community participation** : Analyze whether the plan has promoted the active participation of the local community in the identification of needs, design of strategies and decision making, ensuring the inclusion of marginalized or excluded groups.
3. **Sustainability and resilience:** Evaluate the sustainability of the actions proposed in the strategic plan, considering aspects such as the efficient use of natural resources, the promotion of environmentally sustainable practices and the community's capacity to face future challenges.
4. **Access to basic services** : Verify whether the plan has improved the population's access to essential services such as health, education, drinking water, basic sanitation and adequate housing, especially in areas where there are significant deficits.
5. **Local economic development:** Analyze the impact of the plan on the generation of employment, the promotion of sustainable economic activities, the diversification of the local economy and the strengthening of the business capabilities of the population.
6. **Governance and transparency:** Evaluate the effectiveness of the governance mechanisms implemented for the execution of the plan, including transparency in the allocation of resources, accountability and the participation of key actors.
7. **Institutional capacities:** Measure the strengthening of local institutional capacities to plan, implement, monitor and evaluate development programs and projects, including intersectoral coordination and effective resource management.
8. **Monitoring and evaluation:** Analyze the existence of adequate monitoring and evaluation systems that allow measuring progress and results achieved based on specific indicators, facilitating informed decision-making and continuous adaptation of the plan.

This section shows the results of TreeSoft with the BNS and VIKOR methods. This study used 8 criteria as shown and 8 alternatives. Three experts used bipolar neutrosophic numbers (BNN) to evaluate the criteria and alternatives.

Step 1. Build a tree and define the nodes.

The tree has more than one level, at the first level, the main criteria are entered as

$SWM_1, SWM_2 \ldots, SWM_n$

At the second level, the sub-criteria are introduced as $SWM_{1.1}, SWM_{1.2}, \ldots$. And $SWM_{2.1}, SWM_{2.2}, \ldots$.

Step 2. Construct the decision matrix using Eq. (8). Table 1 shows the decision matrix.

Table 1. The decision matrix.

	$SWM1$	$SWM2$	$SWM3$	$SWM4$	$SWM5$	$SWM6$	$SWM7$	$SWM8$
ALT 1	[0,9O, O,1O, O,1O, −O,4O, −O,7O, −O,9O]	[O,9O, O,1O, O,1O, −O,4O, −O,7O, −O,9O]	[O,7O, O,6O, O,6O, −O,4O, −O,7O, −O,7O]	[O,9O, O,1O, O,1O, −O,4O, −O,7O, −O,9O]	[O,9O, O,1O, O,1O, −O,4O, −O,7O, −O,9O]	[O,7O, O,6O, O,6O, −O,4O, −O,7O, −O,7O]	[O,9O, O,1O, O,1O, −O,4O, −O,7O, −O,9O]	[O,9O, O,1O, O,1O, −O,4O, −O,7O, −O,9O]
ALT 2	[O,7O, O,6O, O,6O , −O,4O, −O,7O, −O,7O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, −O,9O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, −O,9O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, −O,9O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, −O,9O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, −O,9O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, −O,9O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, −O,9O]
ALT 3	[O,4O, O,2O, O,5O , −O,6O, −O,2O, −O,1O]	[O,1O, O,7O, O,5O , −O,9O, −O,2O, −O,1O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, −O,9O]	[O,1O, O,7O, O,5O , −O,9O, −O,2O, −O,1O]	[O,1O, O,7O, O,5O , −O,9O, −O,2O, −O,1O]	[O,7O, O,6O, O,6O, −O,4O, −O,7O, −O,7O]	[O,1O, O,7O, O,5O , −O,9O, −O,2O, −O,1O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, −O,9O]
ALT 4	[O,4O, O,2O, O,5O , −O,6O, −O,2O, −O,1O]	[O,7O, O,6O, O,6O , −O,4O, −O,7O, −O,7O]	[O,1O, O,7O, O,5O , −O,9O, −O,2O, −O,1O]	[O,7O, O,6O, O,6O , −O,4O, −O,7O, −O,7O]	[O,7O, O,6O, O,6O , −O,4O, −O,7O, −O,7O]	[O,4O, O,2O, O,5O, −O,6O, −O,2O, −O,1O]	[O,7O, O,6O, O,6O , −O,4O, −O,7O, −O,7O]	[O,7O, O,6O, O,6O , −O,4O, −O,7O, −O,7O]
ALT 5	[O,1O, O,7O, O,5O , −O,9O, −O,2O, −O,1O]	[O,6O, O,6O, O,6O , −O,6O, −O,6O, −O,6O]	[O,7O, O,6O, O,6O , −O,4O, −O,7O, −O,7O]	[O,6O, O,6O, O,6O , −O,6O, −O,6O, −O,6O]	[O,6O, O,6O, O,6O , −O,6O, −O,6O, −O,6O]	[O,4O, O,2O, O,5O , O,6O, −O,2O, −O,1O]	[O,6O, O,6O, O,6O , −O,6O, −O,6O, −O,6O]	[O,4O, O,2O, O,5O , −O,6O, −O,2O, −O,1O]
ALT 6	[O,9O, O,1O, O,1O, −O,4O, −O,7O, −O,9O]	[O,4O, O,2O, O,5O , −O,6O, −O,2O, −O,1O]	[O,6O, O,6O, O,6O , −O,6O, −O,6O, −O,6O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, −O,9O]	[O,4O, O,2O, O,5O , −O,6O, −O,2O, −O,1O]	[O,1O, O,7O, O,5O, −O,9O, −O,2O, −O ,1O]	[O,4O, O,2O, O,5O , −O,6O, −O,2O, −O,1O]	[O,4O, O,2O, O,5O , −O,6O, −O,2O, −O,1O]
ALT 7	[O,9O, O,1O, O,1O −O,4O, −O,7O, −O,9O]	[O,1O, O,7O, O,5O, −O,9O, −O,2O, −O,1O]	[O,4O, O,2O, O,5O, −O,6O, −O,2O, −O,1O]	[O,7O, O,6O, O,6O, −O,4O, −O,7O, −O,7O]	[O,4O, O,2O, O,5O , −O,6O, −O,2O, −O,1O]	[O,4O, O,2O, O,5O O,6O, −O,2O, −O,1O]	[O,4O, O,2O, O,5O , −O,6O, −O,2O, −O,1O]	[O,1O, O,7O, O,5O, −O,9O, −O,2O, −O,1O]

	$SWM1$	$SWM2$	$SWM3$	$SWM4$	$SWM5$	$SWM6$	$SWM7$	$SWM8$
ALT 8	[O,1O, O,7O, O,5O, −O,9O, −O,2O, − O,1O]	[O,9O, O,1O, O,1O, −O,4O, −O,7O, − O,9O]	[O,1O, O,7O, O, , −O,9O, −O,2O, − O,1O]	[O,4O, O,2O, O,5O, −O,6O, −O,2O, − O,1O]	[O,1O, O,7O, O,5O, −O,9O, −O,2O, − O,1O]	[O,9O, O,1O, O,1O, O,4O, −O,7O, − O,9O]	[O,1O, O,7O, O,5O, −O,9O, −O,2O, − O,1O]	[O,7O, O,6O, O,6O, −O,4O, −O,7O, − O,7O]
	$SWM1$	$SWM2$	$SWM3$	$SWM4$	$SWM5$	$SWM6$	$SWM7$	$SWM8$
ALT 1	[0.40, 0.20, 0.50 , −0.50, −0.20, − 0.10]	[0.40, 0.20, 0.50 , −0.50, −0.20, − 0.10]	[0.70, 0.50, 0.50 , −0.40, −0.70, − 0.70]	[0.40, 0.20, 0.50 , −0.50, −0.20, − 0.10]	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]	[0.40, 0.20, 0.50 , 0.50, −0.20, − 0.10]	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]	[0.40, 0.20, 0.50 , −0.50, −0.20, − 0.10]
ALT 2	[0.70, 0.50, 0.50 , −0.40, −0.70, − 0.70]	[0.70, 0.50, 0.50 , −0.40, −0.70, − 0.70]	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]	[0.70, 0.50, 0.50 , −0.40, −0.70, − 0.70]	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]	[0.70, 0.50, 0.50 , 0.40, −0.70, − 0.70]	[0.90, 0.10, 0.10, −0.40, −0.70, − 0.90]	[0.70, 0.50, 0.50 , −0.40, −0.70, − 0.70]
ALT 3	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]	[0.10, 0.70, 0.50 , −0.90, −0.20, − 0.10]	[0.90, 0.10, 0.10 , 0.40, −0.70, − 0.90]	[0.10, 0.70, 0.50 , −0.90, −0.20, − 0.10]	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]
ALT 4	[0.10, 0.70, 0.50 , −0.90, −0.20, − 0.10]	[0.10, 0.70, 0.50 , −0.90, −0.20, − 0.10]	[0.10, 0.70, 0.50 , −0.90, −0.20, − 0.10]	[0.10, 0.70, 0.50 , −0.90, −0.20, − 0.10]	[0.70, 0.50, 0.50 , −0.40, −0.70, − 0.70]	[0.10, 0.70, 0.50 , 0.90, −0.20, − 0.10]	[0.70, 0.50, 0.50 , −0.40, −0.70, − 0.70]	[0.10, 0.70, 0.50 , −0.90, −0.20, − 0.10]
ALT 5	[0.10, 0.70, 0.50 , −0.90, −0.20, − 0.10]	[0.50, 0.50, 0.50 , −0.50, −0.50, − 0.50]	[0.70, 0.50, 0.50 , −0.40, −0.70, − 0.70]	[0.50, 0.50, 0.50 , −0.50, −0.50, − 0.50]	[0.50, 0.50, 0.50 , −0.50, −0.50, − 0.50]	[0.40, 0.20, 0.50 , 0.50, −0.20, − 0.10]	[0.50, 0.50, 0.50 , −0.50, −0.50, − 0.50]	[0.40, 0.20, 0.50 , −0.50, −0.20, − 0.10]

	$SWM1$	$SWM2$	$SWM3$	$SWM4$	$SWM5$	$SWM6$	$SWM7$	$SWM8$
ALT 6	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]	[0.40, 0.20, 0.50 , −0.50, −0.20, − 0.10]	[0.50, 0.50, 0.50 , −0.50, −0.50, − 0.50]	[0.40, 0.20, 0.50 , −0.50, −0.20, − 0.10]	[0.40, 0.20, 0.50 , −0.50, −0.20, − 0.10]	[0.10, 0.70, 0.50 , .90, −0.20, − 0.10]	[0.40, 0.20, 0.50 , −0.50, −0.20, − 0.10]	[0.40, 0.20, 0.50 , −0.50, −0.20, − 0.10]
ALT 7	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]	[0.70, 0.50, 0.50 , −0.40, −0.70, − 0.70]	[0.40, 0.20, 0.50 , −0.50, −0.20, − 0.10]	[0.70, 0.50, 0.50 , −0.40, −0.70, − 0.70]	[0.70, 0.50, 0.50 , −0.40, −0.70, − 0.70]	[0.40, 0.20, 0.50 , 0.50, −0.20, − 0.10]	[0.70, 0.50, 0.50 , −0.40, −0.70, − 0.70]	[0.10, 0.70, 0.50 , −0.90, −0.20, − 0.10]
ALT 8	[0.10, 0.70, 0.50 , −0.90, −0.20, − 0.10]	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]	[0.10, 0.70, 0.50 , −0.90, −0.20, − 0.10]	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]	[0.90, 0.10, 0.10 , 0.40, −0.70, − 0.90]	[0.90, 0.10, 0.10 , −0.40, −0.70, − 0.90]	[0.70, 0.50, 0.50 , −0.40, −0.70, − 0.70]
	$SWM1$	$SWM2$	$SWM3$	$SWM4$	$SWM5$	$SWM6$	$SWM7$	$SWM8$
ALT 1	[O,4O, O,2O, O,5O , −O,6O, −O,2O, − O,1O]	[O,4O, O,2O, O,5O , −O,6O, −O,2O, − O,1O]	[O,7O, O,6O, O,6O , −O,4O, −O,7O, − O,7O]	[O,4O, O,2O, O,5O , −O,6O, −O,2O, − O,1O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, − O,9O]	[O,4O, O,2O, O,5O , O,6O, −O,2O, − O,1O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, − O,9O]	[O,4O, O,2O, O,5O , −O,6O, −O,2O, − O,1O]
ALT 2	[O,7O, O,6O, O,6O , −O,4O, −O,7O, − O,7O]	[O,7O, O,6O, O,6O , −O,4O, −O,7O, − O,7O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, − O,9O]	[O,7O, O,6O, O,6O , −O,4O, −O,7O, − O,7O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, − O,9O]	[O,7O, O,6O, O,6O , O,4O, −O,7O, − O,7O]	[O,9O, O,1O, O,1O, −O,4O, −O,7O, − O,9O]	[O,7O, O,6O, O,6O , −O,4O, −O,7O, − O,7O]
ALT 3	[O,9O, O,1O, O,1O , −O,4O, −O,7O, − O,9O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, − O,9O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, − O,9O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, − O,9O]	[O,1O, O,7O, O,5O , −O,9O, −O,2O, − O,1O]	[O,9O, O,1O, O,1O , O,4O, −O,7O, − O,9O]	[O,1O, O,7O, O,5O , −O,9O, −O,2O, − O,1O]	[O,9O, O,1O, O,1O , −O,4O, −O,7O, − O,9O]
ALT 4	[O,1O, O,7O, O,5O	[O,1O, O,7O, O,5O	[O,1O, O,7O, O,5O	[O,1O, O,7O, O,5O	[O,7O, O,6O, O,6O	[O,1O, O,7O, O,5O	[O,7O, O,6O, O,6O	[O,1O, O,7O, O,5O

	$SWM1$	$SWM2$	$SWM3$	$SWM4$	$SWM5$	$SWM6$	$SWM7$	$SWM8$
	, −0,90, −0,20, −0,10]	, −0,90, −0,20, −0,10]	, −0,90, −0,20, −0,10]	, −0,90, −0,20, −0,10]	, −0,40, −0,70, −0,70]	, 0,90, −0,20, −0,10]	, −0,40, −0,70, −0,70]	, −0,90, −0,20, −0,10]
ALT 5	[0,10, 0,70, 0,50 , −0,90, −0,20, −0,10]	[0,60, 0,60, 0,60 , −0,60, −0,60, −0,60]	[0,70, 0,60, 0,60 , −0,40, −0,70, −0,70]	[0,60, 0,60, 0,60 , −0,60, −0,60, −0,60]	[0,60, 0,60, 0,60 , −0,60, −0,60, −0,60]	[0,40, 0,20, 0,50 , 0,60, −0,20, −0,10]	[0,60, 0,60, 0,60 , −0,60, −0,60, −0,60]	[0,40, 0,20, 0,50 , −0,60, −0,20, −0,10]
ALT 6	[0,90, 0,10, 0,10 , −0,40, −0,70, −0,90]	[0,40, 0,20, 0,50 , −0,60, −0,20, −0,10]	[0,60, 0,60, 0,60 , −0,60, −0,60, −0,60]	[0,40, 0,20, 0,50 , −0,60, −0,20, −0,10]	[0,40, 0,20, 0,50 , −0,60, −0,20, −0,10]	[0,10, 0,70, 0,50 , ,90, −0,20, −0,10]	[0,40, 0,20, 0,50 , −0,60, −0,20, −0,10]	[0,40, 0,20, 0,50 , −0,60, −0,20, −0,10]
ALT 7	[0,90, 0,10, 0,10 , −0,40, −0,70, −0,90]	[0,70, 0,60, 0,60 , −0,40, −0,70, −0,70]	[0,40, 0,20, 0,50 , −0,60, −0,20, −0,10]	[0,70, 0,60, 0,60 , −0,40, −0,70, −0,70]	[0,70, 0,60, 0,60 , −0,40, −0,70, −0,70]	[0,40, 0,20, 0,50 , 0,60, −0,20, −0,10]	[0,70, 0,60, 0,60 , −0,40, −0,70, −0,70]	[0,10, 0,70, 0,50 , −0,90, −0,20, −0,10]
ALT 8	[0,10, 0,70, 0,50 , −0,90, −0,20, −0,10]	[0,90, 0,10, 0,10 , −0,40, −0,70, −0,90]	[0,10, 0,70, 0,50 , −0,90, −0,20, −0,10]	[0,90, 0,10, 0,10 , −0,40, −0,70, −0,90]	[0,90, 0,10, 0,10 , −0,40, −0,70, −0,90]	[0,90, 0,10, 0,10 , 0,40, −0,70, −0,90]	[0,90, 0,10, 0,10 , −0,40, −0,70, −0,90]	[0,70, 0,60, 0,60 , −0,40, −0,70, −0,70]

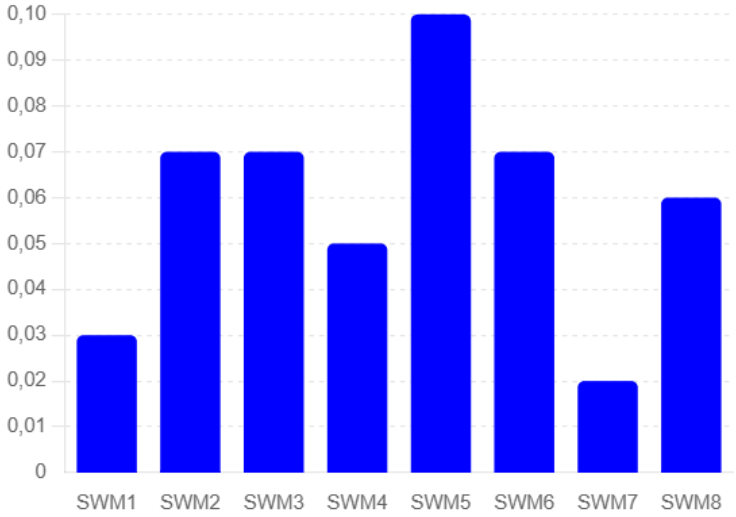

Figure 2. The weights of the criteria.

Step 3. Calculate the criteria weights as shown in Figure 3.

Step 4. Calculate the alternative closeness to the optimal solution using Eq. (fifteen).

Step 5. Calculate the values of S_i using the equation. (18).

Step 6. Calculate the values of R_i using the equation. (19).

Step 7. Calculate the VIKOR index using Eqs . (20 and 21)

Step 8. Rank the alternatives as shown in Figure 4.

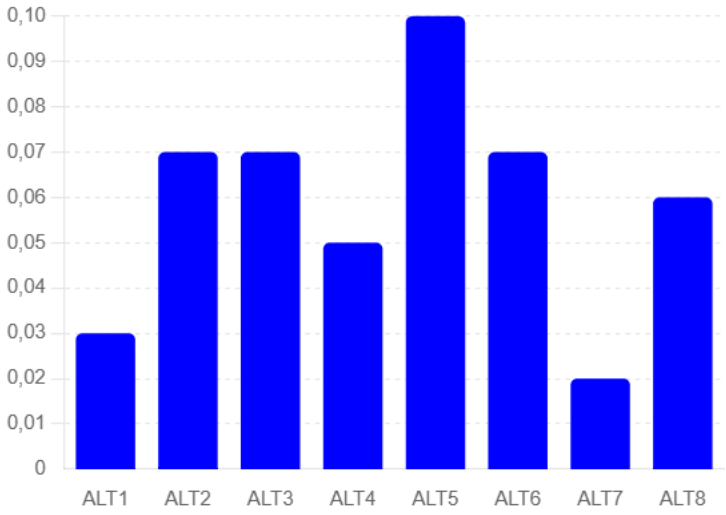

Figure 3. The VIKOR index values.

Alternative 5 is the best: **Basic Infrastructure Improvement:** Prioritize infrastructure projects that improve access to basic services such as drinking water, sanitation, electricity, roads and public transportation.
Alternative 7 is the worst: **Strengthening Community Participation:** Encourage active community participation in decision-making and project execution, through the creation of local committees and inclusive dialogue spaces.

3. Conclusions

In this analysis, the MCDM methodology was used to examine the strategic development plan for extreme poverty municipalities in the province of Huancayo, Peru. To carry out this evaluation, Treesoft Set was used together with neutrosophic sets as analysis tools. The MCDM methodology was applied to evaluate a variety of criteria and the VIKOR method was used to rank the various alternatives available. The MCDM methodology was integrated with neutrosophic sets to address the uncertainty inherent in the evaluation process, combining them with the TreeSoft Set. Three decision makers and experts participated in the classification of criteria and options, and it was decided to use BNN instead of the opinions expressed by the experts. Using the VIKOR method, three decision matrices were created that provided defined values in each, which were combined to form a final matrix. This study was based on the evaluation of 12 criteria and 10 alternatives.

The district municipalities of extreme poverty in the province of Huancayo already have a Strategic Development Plan, the implementation of which will be essential to improve municipal management and reduce extreme poverty. When implementing a Strategic Development Plan, it is essential to fully consider the guidelines to improve management control in these municipalities. The objectives outlined in the Strategic Development Plan must be adequately defined by the extreme poverty municipalities of Huancayo to ensure a greater allocation of economic resources from the MEF. It is

necessary for these municipalities to establish easy-to-implement strategies that can be integrated into the Strategic Development Plan and contribute positively to improving the collection of taxes and tributes.

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Plithogenic Analysis of the Psychosocial Perceptions of Call Center Operators in Quito

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Abstract. This study focused on evaluating the perceptions of a group of call center operators in Quito regarding their work environment using Plithogenic statistics to address the complexity and ambiguity of the responses. The sample included 114 workers from three companies, assessed using the Battery of the Ministry of Social Protection of Colombia and a work stress questionnaire. The results revealed high job demands, perceived as stressful by employees, although satisfaction with the rewards received was shown. However, significant uncertainty was identified regarding control over work, leadership, and social relationships, suggesting critical areas for improvement. The use of Plithogenic logic allowed for an in-depth analysis of these phenomena, providing valuable information for future organizational interventions. It is recommended to implement stress management programs and improve communication and leadership to strengthen role clarity and increase job satisfaction and efficiency.

Keywords: call center operators, Plithogenic statistics, work perceptions, stress management, organizational leadership

1 Introduction

In the context of a global economy dominated by capitalist imperatives, organizations face increased pressures to comply with various essential requirements for market sustainability. These changes have profoundly modified work methodologies in recent years. Within this framework, competitiveness and productivity emerge as central axes in the formulation of corporate strategies across all economic sectors, imposing high-performance standards on employees [1].

Consequently, the intensification of work demands implies that workers must develop advanced skills and competencies to respond to global market dynamics. This requirement may increase pressure on employees to stay updated and competitive, which could potentially affect the balance between work and personal life, as well as job satisfaction. Furthermore, employees are expected to participate in multiple projects, collaborate with teams from various cultures, and take on additional tasks that were not previously common in their respective regions or countries. These demands can increase the workload and generate additional stress, particularly if adequate resources and support are not available [2].

In the telemarketing sector, competition, both nationally and internationally, drives employers or managers to exert considerable pressure on operators. This situation is exacerbated by the ability to outsource services to regions where labor is less costly and technological demands are high, putting additional pressure on Ecuadorian organizations with high operational costs [3]. Thus, the performance of operators may be intrinsically linked to the quality of leadership within the company.

Additionally, the work environment for telemarketing operators is a source of high tension, exacerbated by frequent interactions with emotionally affected clients and limited assertive communication skills. Verbal aggressions are recurrent, restricting the expression of operators and subjecting them to the pressure of meeting specific objectives, whose non-achievement can result in sanctions or dismissals, increasing their work stress [4]. Therefore, call center operators face a significant emotional burden when managing interactions with upset clients, which can deteriorate their psychosocial well-being.

The leadership style adopted by the company plays a crucial role in the work experience of call center operators, considered vital for the success of any economic, political, and organizational system [5]. Promoting leadership practices that foster virtuous and eudaimonic behaviors can be decisive in promoting both organizational and occupational well-being at individual and collective levels.

In this sense, prioritizing the management of psychosocial risk factors is critical. Exposure to psychosocial risks can induce high levels of tension in workers. According to [6], when this tension is maintained for prolonged periods with high intensity and combined with insufficient relaxation periods, it can lead to a state of chronic stress

that deteriorates the health and general well-being of employees. This phenomenon also has direct repercussions on business performance, manifested through an increase in absenteeism rates due to illness, disabilities, attitudes of indiscipline, and a decrease in work performance attributable to psychological stress [7].

Regarding the dimension of psychosocial factors, both positive and negative, these are closely linked to the work environment and the specific functions performed. An illustrative example of this is Call Centers, which emerged in the United States during the 1960s as a strategy of the Ford Company to implement telemarketing campaigns [8]. Originally, these services focused on providing information as a complementary service to the main product. Throughout the 1990s, this sector experienced significant growth both in Europe and Latin America, expanding its functions to debt management and emergency services, driven by technological advances.

The workload for call center operators is considerable, as their evaluation depends on the number of calls managed, the time invested, and the effectiveness of communication. This work configuration imposes on operators the obligation to meet call quotas and ensure their effectiveness, which often leads to pressure on customers [9]. This cycle can reverse the desired effectiveness into negativity, exacerbating stress levels among workers.

Psychosocial factors are defined as elements that can be internal or external to an organization, and that encompass both working conditions and the personal characteristics of employees. These factors establish a dynamic interaction based on individual perceptions and experiences, exerting a significant influence on the health and work performance of workers [10]. In this framework, this study aims to assess the perception of a group of call center operators in the city of Quito regarding the psychosocial factors associated with their work environment.

To achieve this end, the use of contributions made by plithogenic logic is proposed, a tool that allows analyzing the evolution of variables in related groups, adapting it to investigate the correlations between working conditions and their effects on employee well-being. Plithogenic statistics, developed from traditional multivariate statistics, provide a theoretical and methodological approach to managing uncertainty and variability in perceptions [11]. By incorporating uncertainty and diversity of viewpoints into statistical analysis, this methodology facilitates a more detailed investigation of complex phenomena, such as the perception of objectivity.

This approach could reveal significant patterns that help better understand psychosocial dynamics in intensive work environments like call centers. From a scientific perspective, the use of this methodology in intensive work environments like call centers not only enriches scientific research in the area of work psychology and organizations but also provides a solid framework for improving human resource management in practical and effective terms. This approach underscores the importance of considering the diversity of experiences and perceptions within a workforce, and how these can be essential for the strategic and operational development of the company.

2 Preliminaries

2.1 Notions on Plithogenic Logic

F. Smarandache's theory of Plithogeny articulates the formation, development, progression, and enhancement of novel entities arising from the dynamic integration of pre-existing entities, which may be antagonistic, neutral, or synergistic. This concept advocates for the amalgamation and integration of theoretical constructs and insights across multiple disciplines, thereby endorsing a transdisciplinary fusion of knowledge that encompasses the soft sciences, hard sciences, arts, and theoretical aspects of literature. This approach facilitates a comprehensive understanding by bridging disparate fields of study. [12]

Within this framework, a plithogenic Set is identified as a non-trivial set P, situated within a given domain $U(P \subseteq U)$ characterized by one or more distinguishing attributes $A_1, A_2, \dots, A_m, m \geq 1$. Each attribute within this set can assume values from a wide spectrum S of potential states. This spectrum can present itself in diverse forms—it may be finite or infinite, discrete or continuous, and its range may be either open or closed. [13]

This characterization highlights the inherent versatility and dynamism of plithogenic sets, reflecting the multifaceted and intricate nature of the knowledge and phenomena they aim to represent. By integrating a diverse range of attributes and their corresponding values within a cohesive framework, the plithogenic methodology enables a more detailed and holistic examination of entities. This approach promotes an interdisciplinary discourse and investigation, facilitating the transcendence of traditional demarcations among distinct domains of study.

Each element $x \in P$, it is characterized by the entire range of potential values for the attributes contained within the set $V = \{v_1, v_2, \cdots, v_n\}$. An attribute's value has a degree of belonging $d(x, v)$ for an element x in set P based on a specific criterion. This degree of belonging can manifest as fuzzy, intuitionistic fuzzy, or neutrosophic, among other types. [14]

This signifies that for every element x in the set P, there exists a function $d: PxV \rightarrow \wp([0,1]^z)$, as shown in equation (1), where $d(x, v) \subseteq [0,1]^z$ and $\wp([0,1]^z)$ represents the power set of $[0,1]^z$. Here, z indicates the degree of appurtenance, with $z = 1$ corresponding to the fuzzy degree, $z = 2$ to the intuitionistic fuzzy degree, and $z = 3$ to the neutrosophic degree of appurtenance.

$$\forall x \in P, d: PxV \rightarrow \wp([0,1]^z) \quad (1)$$

In this advanced exposition of plithogenic sets, a nuanced mechanism is introduced for evaluating the degree

of contradiction between different attribute values within such sets. If we denote V as the value set with its cardinality being greater than or equal to 1, we define a specialized function $c: V \times V \rightarrow [0,1]^2$. This function, termed the attribute value contradiction degree function, is designed to quantify the level of contradiction between any pair of attribute values v_a, v_b. The operation of this function is guided by several key axioms: [15]

$c(v_a, v_a) = 0$, which asserts that there is no contradiction in an attribute value when compared with itself, encapsulating the principle of non-contradiction.

$c(v_a, v_b) = c(v_b, v_a)$, which underscores the symmetry in the degree of contradiction between any two distinct attribute values, suggesting that the contradiction is mutual and unaffected by the order of comparison.

The notation c is specifically chosen to highlight that this function operates within the realm of fuzzy logic, implying a continuum of contradiction degrees rather than binary or discrete states. Additionally, variations of this function, such as $c_{IF}: V \times V \rightarrow [0,1]^2$, are conceptualized to accommodate the framework of neutrosophic logic, thereby acknowledging and quantifying varying levels of certainty or contradiction inherent in the attribute values.

Definition 1. In the context of a plithogenic set defined as (P, A, V, d, c), a Plithogenic Neutrosophic Aggregation Operator is described by Equation 2:

$$(a_1, a_2, a_3)AND_p(b_1, b_2, b_3) = ((1-c)(a_1 \wedge_F b_1) + c(a_1 \vee_F b_1), \frac{1}{2}(a_2 \wedge_F b_2 + a_2 \vee_F b_2), (1-c)(a_3 \vee_F b_3) + c(a_3 \wedge_F b_3)) \quad (2)$$

Here, $c \in [0,1]$, where $\wedge_F$ denotes a t-norm and $\vee_F$ denotes a t-conorm, as discussed in [16]. The operation acts as a Plithogenic Neutrosophic Intersection when c=0 and transforms into a Plithogenic Neutrosophic Union when c =1. This aggregation approach is noted for its superior accuracy compared to standard n-norms and n-conorms used between neutrosophic sets.

Additionally, a plithogenic neutrosophic set can be simplified into a crisp value using the formula outlined in Equation 3.

$$S(T, I, F) = \frac{1}{3}(2 + T - I - F) \quad (3)$$

In the framework of a plithogenic set, defined as (P, a, V, d, c), the structure consists of the principal set P, the attribute set A, the value set V, a membership function m, and the contradiction degree function d, which is conceptually aligned with the harmony coefficient c. This contradiction function is instrumental in assessing and quantifying the level of contradiction among the attributes, especially in regard to a primary attribute, should such an attribute be deemed critical in comparison to others. This framework thus serves as a potent instrument for analyzing and elucidating the intricate interactions among attributes within a plithogenic set. It provides valuable insights into the dynamics of contradiction and coherence, enhancing our understanding of the complex relationships within the set. [17]

In contrast, (U, a, V, d, c) is designated as Plithogenic Probability, wherein E represents the event space. Plithogenic probability is conceptualized as the probability of an event's occurrence considering the influence of all associated random variables, where each variable may conform to various probabilistic models including classical, T, I, F-neutrosophic, I-neutrosophic, T, F-intuitionistic fuzzy, T, N, F-picture fuzzy, T, N, F-spherical fuzzy, or other fuzzy extensions in their distribution functions. This approach significantly broadens the classical framework of multivariate probability by incorporating a diverse array of distribution functions that accommodate uncertainty and imprecision inherent in the data, providing a more comprehensive understanding of probabilistic events across different contexts and scenarios. [18]

Plithogenic statistics expands upon traditional multivariate statistics by integrating the principles of plithogenic probability. This integration not only broadens the conventional analytical framework but also enhances its descriptive power by allowing for a more nuanced decomposition of probabilities into distinct components: truth, indeterminacy, and falsehood. Such a multifaceted approach provides a deeper, more comprehensive analysis of probabilistic data, acknowledging and addressing the complexities often inherent in real-world scenarios. This methodology enables researchers and practitioners to capture and quantify the various degrees of uncertainty and vagueness that traditional statistical methods might overlook, offering a richer, more detailed understanding of the underlying dynamics of the data. Specifically, it breaks down probabilities into detailed segments represented as $T_1, T_2, \ldots, T_p$ for truths; $I_1, I_2, \ldots, I_q$ for indeterminacies; and $F_1, F_2, \ldots, F_r$ for falsehoods. This granular approach ensures that at least one set among the truths, indeterminacies, or falsehoods is comprised of more than one element, indicating the multifaceted nature of probabilities within the plithogenic framework.

This sophisticated subdivision allows plithogenic statistics to capture the complexity of real-world phenomena more accurately than classical methods. By acknowledging and quantifying the degrees of truth, the potential for indeterminacy, and the possibility of falsehood in any given situation, plithogenic statistics provide a richer, more dimensional view of statistical analysis. This methodological advancement enables researchers and analysts to account for the inherent uncertainties and ambiguities in data, offering a more refined and nuanced understanding of statistical outcomes. [19]

2.2 Method

This study adopts a descriptive cross-sectional quantitative approach to assess the perception of psychosocial factors in the work environment of a group of call center operators in Quito. The research focuses on the application of plithogenic statistics to analyze the perceptions and responses of workers to various work conditions. This approach allows for a detailed interpretation of the collected data, covering the complexity and heterogeneity of individual perceptions through a robust and adaptive statistical model.

The selection of the target population was meticulously planned to include call center operators who work under indefinite and full-time contract conditions, in three pre-selected companies located in Quito. This specific criterion was chosen to ensure homogeneity in employment conditions, providing a stable base for assessing psychosocial impacts without the confounding variables that might be introduced by variable duration contracts or different work schedules.

For participant selection, simple random sampling was implemented. This method was used to choose 114 workers from the payrolls of the selected companies, thus ensuring representativeness and randomness in the sample selection. This approach ensures that each member of the population has the same probability of being chosen, which is fundamental for the statistical validity and generalization of the study results.

The main instrument used in the research was the Psychosocial Risk Battery from the Ministry of Social Protection of Colombia, a validated and widely recognized tool for its effectiveness in measuring levels of stress and other psychosocial factors in work environments. This tool classifies work conditions into three main domains: intra-labor, extra-labor, and individual. Each of these domains is broken down into various dimensions that address specific aspects of the work environment:

1. Labor Demands: This domain evaluates the inherent demands of the job, including aspects such as quantitative demands, mental load, emotional load, responsibilities, physical and environmental effort, duration of the workday, and role consistency.
2. Control over Work: This domain investigates the extent to which the worker can influence or make decisions about their work, covering role clarity, training received, participation and management of change, development opportunities, and the use of skills and knowledge.
3. Leadership and Social Relations: Focuses on the quality of social and leadership interactions at work, considering leadership characteristics, social relationships, feedback on performance, and relations with colleagues.
4. Rewards: Assesses the perception of the rewards received, both in terms of recognition and compensation, as well as rewards derived from a sense of belonging to the organization.

Additionally, a stress assessment questionnaire was administered to identify symptoms of stress, classified into physiological, social behavioral, intellectual, work, and psychoemotional categories. This questionnaire complements the Battery by providing a direct analysis of stress reactions, allowing for a more comprehensive assessment of the psychosocial well-being of workers.

The combination of these instruments offers a comprehensive and detailed analysis of work conditions and their impact on the well-being of call center operators, thereby allowing the identification of critical factors that could be targeted for future interventions aimed at improving the quality of work life and organizational productivity.

Data collection was conducted using linguistic scales. These scales are associated with plithogenic numbers for their analysis, as represented in Table 1. This step facilitates a more precise and nuanced evaluation of the responses, using plithogenic numbers that represent degrees of affirmation and negation within a neutrosophic framework.

Table 1: Linguistic scale associated with plithogenic numbers for the evaluation of respondents

Linguistic scale	(T, I, F)
Strongly disagree	(0.1; 0.75; 0.85)
Disagree	(0.4; 0.7; 0.5)
Undefined	(0.5; 0.4; 0.6)
Agree	(0.65; 0.3; 0.45)
Strongly Agree	(0.95; 0.05; 0.05)

The analysis was conducted using plithogenic statistical techniques, which allow for the aggregation and complex analysis of neutrosophic data. This method is particularly useful for decomposing and understanding the variabilities and contradictions in employees' perceptions. Correlations and emerging patterns among the different domains and dimensions were evaluated to identify the main psychosocial factors affecting the well-being and job satisfaction of call center operators.

The study was conducted respecting the ethical principles of confidentiality, voluntariness, and informed consent. All participants were informed about the objectives of the research and gave their consent before participating. It was ensured that all information collected was treated confidentially and used exclusively for research purposes.

The adoption of plithogenic statistics in this study provides a valuable tool for exploring and understanding the complexity of psychosocial perceptions in the work environment of call center operators in Quito, opening new avenues for future research and for the development of evidence-based interventions that improve working conditions in this sector.

This methodology not only highlights the richness of the data obtained but also allows for handling uncertainty and the diversity of opinions in a structured manner, facilitating the interpretation of the complex psychosocial factors that influence the work environment of call center operators. The plithogenic approach, therefore, presents itself as a valuable tool in the context of data analysis where responses can be intrinsically nebulous or where individual perceptions may significantly diverge.

3 Results

The use of selected instruments facilitated data collection, capturing both individual and collective perceptions related to job demands, control over work, leadership, social relations, and rewards, as well as symptoms related to stress. Each response was meticulously recorded and organized to ensure the integrity and coherence of the information gathered. Data processing was carried out using statistical techniques utilizing the facilities provided by plithogenic numbers. This technique allowed for the decomposition of responses into neutrosophic components of truth, falsity, and indeterminacy, offering a deeper and more nuanced view of the perceptions of the work environment.

Figure 1 shows a summary of the responses obtained regarding each of the domains analyzed, as well as the level of perception expressed by the respondents.

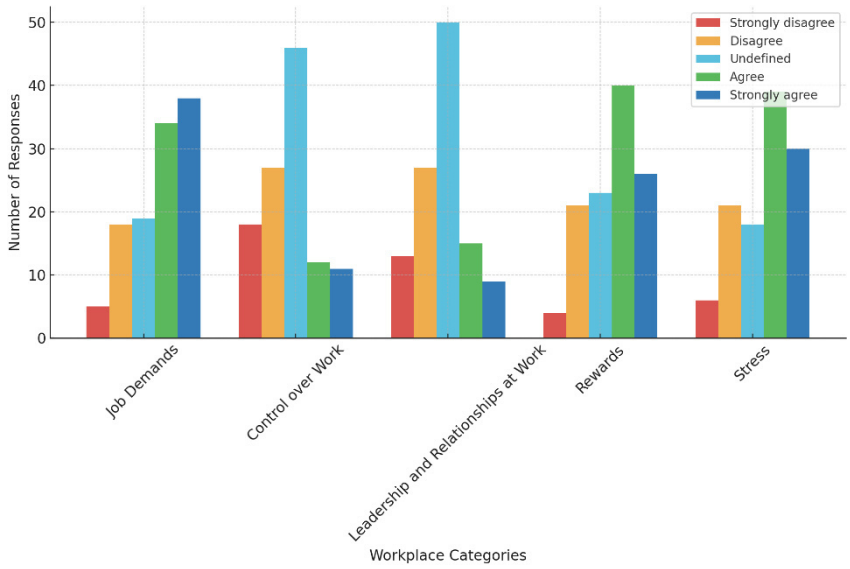

Figure1: Summary of questionnaire responses by domain analyzed

The evaluation of the "Labor Demands" dimension shows a pattern, particularly manifested in the responses grouped under the "Agree" and "Strongly Agree" categories. This phenomenon indicates that most call center operators perceive a significant level of job demands, including aspects such as mental and emotional load, assigned responsibilities, and the physical and environmental effort required in their daily work.

These results suggest that call center operators perceive the influence of an intensive work environment with high psychosocial demands. These results may indicate work stress that is not fully recognized or articulated by the workers. This is particularly relevant in the case of mental and emotional loads, which are known for their cumulative effects and, if not properly managed, can lead to long-term negative consequences for mental and physical health.

On the other hand, regarding the "Control over Work" domain, the majority of responses categorized as "Undefined" illustrate a lack of clarity and consensus among respondents about their ability to influence and make decisions in their job roles. This ambiguity in responses suggests a diffuse and possibly incoherent perception of autonomy and individual management capacity in the workplace, as well as the clarity of roles and responsibilities

assigned. High variability in responses indicating uncertainty or negative answers could reflect significant differences in individual employee experiences.

Similarly, the "Leadership and Social Relations" domain reveals an interesting pattern with a significant amount of responses classified as "Undefined". This trend suggests notable uncertainty or diversity of opinions among employees about the quality of social interactions and leadership within their work environment. The presence of such indeterminate responses reflects the existence of varied and possibly contradictory perceptions about these critical aspects of organizational climate. Differences in perception can arise from variations in employees' direct experiences with their superiors, as well as from the effectiveness with which interpersonal relations are managed and conflicts are resolved within teams.

Meanwhile, the results show positive responses and general satisfaction with the rewards received, encompassing both recognition and compensation. This pattern of responses suggests a positive feeling of appreciation and belonging within the organization, crucial elements for employee motivation and engagement. Despite this, samples were collected that indicate a significant inclination towards "Strongly Agree" in the perception of work stress. This result is particularly relevant, as it indicates that a considerable segment of the workforce experiences high levels of stress, a factor that can have profound implications for both the individual well-being of employees and the overall productivity of the organization. The marked tendency to strongly agree on perceiving stress highlights the presence of significant stressors in the work environment. Work stress not only affects the physical and mental health of employees but can also lead to a decrease in efficiency, increased absenteeism, and high turnover rates, which in turn negatively impacts the operability and costs for the company.

In the analysis of the collected data, the plithogenic AND_p aggregation operator formula was implemented, specifically designed to handle and synthesize information from contexts where variables may present inherent ambiguities or potential contradictions between analyzed elements. The formulation used incorporates minimum (min) and maximum (max) operations to perform conjunction $\wedge_F$ and disjunction $\vee_F$ operations, respectively, with a value of c=0.5. This value was chosen to balance the importance between conjunction and disjunction, reflecting a neutral position where neither operation is prioritized over the other, as there is no clear justification to favor one at the expense of the other. Moreover, by using equations (2) and (3), it was possible to determine the behavior of the parameters of interest for each particular dimension and in general.

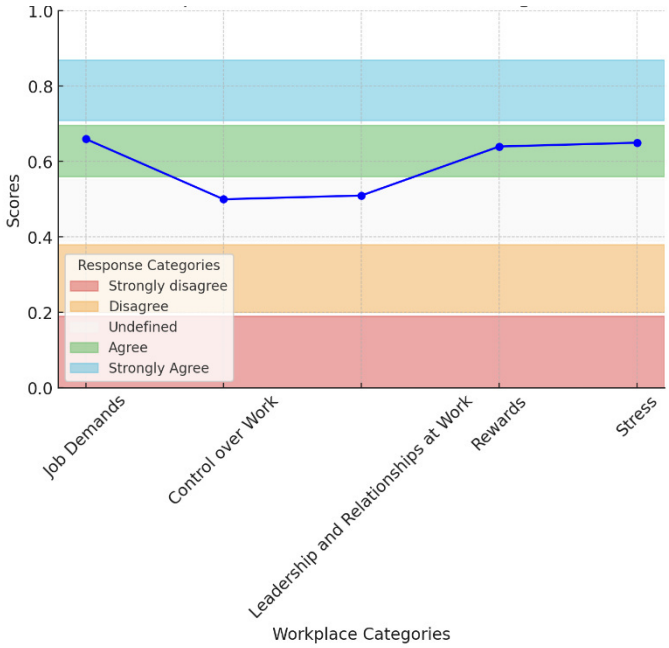

Figure 2: Overall plithogenic scores for each dimension evaluated

As observed in Figure 2, in general, employees indicate that high job demands, both physical and mental, are a prominent feature of their work environment. These demands are significantly associated with the perception of stress among workers. In this regard, it can be inferred that the intensity and volume of work, along with the required emotional and mental load, contribute to elevated levels of stress. This finding is consistent with existing literature that links high job demands with occupational stress, which can have adverse effects on the physical and mental health of employees.

Despite the challenges mentioned, there is general satisfaction with the rewards obtained. This aspect of the

study reflects a positive perception of the compensation and recognition policies implemented by the participating companies. Satisfaction with rewards can be a mitigating factor of the negative impact of work stress, as it provides employees with a sense of appreciation and fairness, essential for maintaining motivation and organizational commitment.

However, there is a generalized indeterminacy or uncertainty regarding control over work and in positions of leadership and labor relations. This result suggests that there is a lack of clarity or consistency in how employees perceive their ability to influence decisions and work processes, as well as the quality of interactions and leadership in the workplace. The ambiguity in these domains is problematic, as the perception of lack of control and poor labor relations are known factors that contribute to job dissatisfaction and a decrease in well-being at work.

4 Discussion

The implementation of plithogenic logic and the use of plithogenic numbers in the conducted study constitutes a methodological approach of great relevance, providing an analytical framework capable of addressing the complexity and multifaceted nature of human data, especially in the field of social and behavioral sciences. On one hand, the use of plithogenic logic allows for the manipulation and analysis of information that is inherently uncertain, imprecise, and/or contradictory. This capability is particularly relevant in studies where variables, such as human perceptions and emotional responses, do not lend themselves to strict or binary categorizations. Plithogenic numbers, which express degrees of truth, falsity, and indeterminacy, facilitate a more faithful representation of the subjective reality of respondents, allowing for a deeper understanding of their experiences and opinions.

The use of this methodology in the study provided several key advantages. First, it enabled capturing the complexity of employees' perceptions in a way that traditional statistical methods might have overly simplified. Additionally, the ability to break down responses into neutrosophic components helped to identify specific areas of consensus and dissent among employees, thereby facilitating more targeted and effective interventions.

From a decision-making perspective, the information generated through plithogenic logic allowed managers and policymakers to have a rich and detailed database on which they could base their strategies for improving the work climate. The identification of areas with high indeterminacy points to the need for greater clarity in company policies or in communicating expectations to employees.

Finally, the adoption of plithogenic numbers and plithogenic logic in this study not only enriches the analysis within the current project but also contributes to the methodological literature, proposing a path for future research facing similar challenges in measuring complex and subjective phenomena. This approach offers a valuable alternative to conventional methods, suggesting new possibilities for exploring and understanding the depths of human experience in various organizational and social contexts.

5 Conclusion

This study comprehensively explored the perceptions of relevant psychosocial factors among a group of call center operators in the city of Quito. Using advanced statistical methods and tools, such as plithogenic logic and plithogenic numbers, it was possible to capture and analyze the complexity and ambiguity inherent in human perceptions, providing valuable information for organizational management and the development of labor policies. Validated instruments were used to assess psychosocial factors and the perception of work-related stress. The use of elements of plithogenic logic and plithogenic numbers allowed for a detailed representation and analysis of employee responses, addressing the uncertainty and contradictions in their perceptions.

The results of the study revealed that call center operators face high job demands that significantly contribute to the perception of stress. This underscores the importance of developing strategies to manage the workload and improve employee well-being. Despite the demands and stress, employees expressed high satisfaction with the rewards received, indicating that compensation and recognition policies are effective and valued by workers. The indeterminacy in perceptions about control over work and the quality of leadership and social relationships suggests that these are areas that require additional attention and improvements to ensure a clearer and more cohesive work environment. The use of plithogenic statistics in the study was crucial for unraveling the complexity and ambiguity of employees' perceptions of their work environment, allowing for a detailed and nuanced analysis that goes beyond conventional statistical techniques. It is suggested to implement comprehensive stress management programs and review leadership policies to improve clarity in responsibilities and increase employee control and satisfaction at work.

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Composite Sustainability Indicator for the Conservation of Ashigua Páramo: Integration of Neutrosophic Logic and the PROMETHEE TODIM Method

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Abstract. The project was conducted in the Ashigua páramo, Mulaló, Latacunga, Cotopaxi province. Its objective was to develop a composite environmental sustainability indicator for the conservation of the páramo. Neutrosophic logic and the PROMETHEE TODIM method were used to select and evaluate sub-indicators in a context of uncertain and complex environmental data. Neutrosophic logic facilitated the management of ambiguities and contradictions in the data. The findings highlighted the relevance of sub-indicators linked to carbon storage and ecological productivity. Indicators of regulatory compliance and sustainable management were evaluated, emphasizing the importance of adjusting environmental policies to regulations and social expectations. This study introduced a novel methodological approach to environmental assessment, providing valuable insights for policy development and effective management of natural resources and ecosystems.

Keywords: composite indicator, neutrosophic indicator, neutrosophic TODIM, PROMETHEE, environmental sustainability, Ecuador.

1 Introduction

In Ecuador, páramo ecosystems cover approximately 1,260,000 hectares, which represents 5% of the national territory. These ecosystems, characterized by their semi-humid and cold nature, extend almost continuously along the Andes Mountain Range, exceeding the current or potential limits of the forest [1]. Fourteen of the country's thirty-five protected areas include páramos, in addition to other conserved areas such as protective forests and private reserves. Páramos provide vital ecosystem services, such as hydrological regulation and the capture of atmospheric carbon, which are essential for both local communities and society in general, contributing to the control of global warming and the sustained provision of high-quality water. [2]

However, these ecosystems face significant threats due to the expansion of the agricultural frontier, grazing, climate change, unregulated tourism, and extreme sports, which compromise the integrity of the sensitive soils in these areas [3]. Despite the potential productivity of the páramos and their inclusion in various protection categories, inadequate management, especially in the province of Cotopaxi, can lead to irreversible environmental deterioration due to the fragility of their soils.

The páramos of the Cotopaxi province in Ecuador comprise approximately 105,000 hectares, representing 8% of the national páramos. Within this region, the parish of Mulaló stands out for its rich biodiversity and cultural heritage, hosting Andean ecosystems, forests, lakes, rivers, and cliffs. Nevertheless, it is observed that conservation strategies in these páramos are insufficient.

In the field of environmental policies, the incorporation of environmental variables is crucial. This process involves the creation and application of environmental sustainability indicators, developed by various entities and community organizations. These indicators are fundamental for assessing the interaction between human activities and the environment, providing clear and understandable data on the current state and environmental trends. [4]

Environmental sustainability indicators not only reflect the impact of productive practices on ecosystems but also calculate the environmental responsibility and sustainability of individuals, organizations, and communities [5]. This tool is indispensable for discerning how human activities affect the planet and for guiding policies that promote a balance between human development and environmental conservation.

Sustainable Development Indicators (SDIs) constitute an integral system of metrics that facilitate the assessment of progress towards sustainable development in various geographies. These indicators function as essential practical tools in the design and evaluation of public policies, improving informed decision-making and fostering citizen participation, essential for guiding countries toward more sustainable practices. [6]

Furthermore, SDIs allow quantifying corporate commitment to the environment and society. It is widely recognized in environmental sciences that humanity consumes the planet's resources at an unsustainable pace, increasing global environmental damage. If adequate control measures are not implemented, this deterioration continues to escalate. Therefore, it is critical to analyze and measure human activities to manage and potentially reduce their environmental impact. [7]

Globally, some environmental indicators are designed to be universally applied, while others are specifically developed to measure the quality of ecosystems in particular locations. In the context of Ecuador, for example, the creation of indicators is underway through the Unified National Information System (SUIN), led by the Ministry of the Environment. Currently, although national environmental indicators are limited, they represent a crucial step towards more robust environmental management and greater ecological responsibility.

Environmental indicators are fundamental tools for articulating sustainability goals, being essential both at the sectoral and comprehensive levels. Their value lies in their ability to be formulated in unique and specific social, administrative, and territorial contexts [8]. These indicators, when carefully selected and related to variables to be evaluated, provide crucial information for an optimal interpretation of sustainability, desired by local managers. Additionally, they can be considered as scientifically configured variables that condense a social interest in the environment, thus facilitating their inclusion in the decision-making process. [9]

This research project is vital to ensure the protection and conservation of fragile ecosystems, such as the páramos, contributing to the well-being of the community. The development of a system of specific indicators will allow for the proper management of these ecosystems, especially in the Ashigua páramo of the Mulaló parish, whose inhabitants will be the main beneficiaries. Through this system, it will be possible to assess and act on the collected data, improving the conservation of these vital areas.

The limited environmental awareness of the inhabitants about the use of natural resources can lead to excessive exploitation, compromising the capacity of ecosystems to meet human needs. In this context, generating environmental development indicators is key to promoting sustainable development and effectively addressing local environmental issues. Additionally, soil compaction, a result of agricultural activities and human settlements, reduces vegetation cover and accelerates soil erosion, threatening the integrity of the páramos and their ability to sustain life.

The insufficient environmental awareness and the profound lack of knowledge among the population about the significance of ecosystems, along with a deficient understanding of the environmental regulations that govern the management of these, contribute significantly to their deterioration. Factors such as the expansion of the agricultural and livestock frontier, the burning of grasslands, and deforestation.

2. Preliminaries

2.1 Neutrosophic Theory

The implementation of Single-Valued Neutrosophic Sets (SVNS) constitutes a significant advancement in the domains of set theory and logic, providing a robust framework for the precise representation of ambiguity and uncertainty. These SVNS are essential for detailing the truth, indeterminacy, and falsity associated with elements within a set. This capability makes them valuable tools for multiple fields, including decision-making in contexts of uncertainty, artificial intelligence, and information management.

Within the framework of SVNS, let's consider X as a space that contains points or objects, with generic elements in X denoted by x. A Single-Valued Neutrosophic Set A in X is defined by three characteristic functions: the truth-membership function $T_A(x)$, the indeterminacy-membership function $I_A(x)$, and the falsity-membership function $F_A(x)$. Therefore, an SVNS A can be formally expressed as $A = \{x, T_{A(x)}, I_{A(x)}, F_{A(x)x} \in X\}$, where $T_{A(x)}, I_{A(x)}, F_{A(x)} \in [0,1]$ for each point x in X. In this way, the sum of $T_{A(x)}, I_{A(x)}$, and $F_{A(x)}$ meets the condition $0 \leq T_{A(x)} + I_{A(x)} + F_{A(x)} \leq 3$. [15]

This formalism allows each element x in the space X to be evaluated under these three metrics, thereby facilitating a more nuanced and detailed understanding of its state in terms of truth, falseness, and indeterminacy. This approach not only enriches traditional set theory with an additional dimension of analysis but also optimizes decision-making and analysis processes in complex and dynamic environments.

The modeling of membership functions in the range [0,1] in SVNS provides greater flexibility and precision in the analysis of contexts where uncertainty is a predominant factor. This interval ensures that the total sum of the truth-membership, indeterminacy-membership, and falsity-membership functions does not exceed the value of 3, thus maintaining structural coherence within the theoretical framework of SVNS. This methodology provides robust support for handling ambiguity and uncertainty in various fields of application.

Decision-making in many contexts is facilitated by the use of linguistic variables, which are terms commonly used in human language to express judgments and preferences. These variables facilitate the articulation and understanding of complex evaluations in a format that is accessible and intuitive for participants in the decision-

making process. For example, terms such as "very important," "important," "somewhat important," and "not important" are frequently used to reflect the relative valuation of different criteria. See Table 1.

The integration of these linguistic variables into the SVNS scheme allows for a more faithful representation of how individuals perceive and prioritize aspects in specific situations. By converting linguistic evaluations into SVNS, a valuable tool is obtained for modeling and effectively analyzing the uncertainty and ambiguity inherent in decision processes, particularly in scenarios where available information is vague or incomplete. This approach provides a bridge between natural human communication and formal decision analysis systems, thus optimizing information management in critical decisions.

Table 1. Linguistic variable and Single-Valued Neutrosophic Numbers (SVNNs). Source:[16]

Integer	Linguistic variable	SVNNs
0	Not important	(0.10;0.90;0.90
1	Low important	(0.35;0.75;0.80)
2	Medium important	(0.50;0.5;0.50)
3	High important	(0.75;0.25;0.20)
4	Very high important	(0.9;0.1;0.1))

In accordance with [16], if $E_k = (T_k, I_k, F_k)$ is a neutrosophic number defined for the rating of the k-th decision-maker, then the weight of the k-th decision maker can be expressed as:

$$\psi_k = \frac{1-\sqrt{[(1-T_k(x))^2+(I_k(x))^2+(\mathrm{F}(x))^2]/3}}{\sum_{k=1}^{p}\sqrt{[(1-T_k(x))^2+(I_k(x))^2+(\mathrm{F}(x))^2]/3}} \quad (1)$$

This equation allows calculating the weight of each decision-maker in the context of a group decision, considering the multiple perspectives and evaluations provided by various individuals. This approach can enrich the decision-making process and lead to more robust and equitable solutions.

In the group decision-making process, all evaluations from individual decision-makers must be aggregated into an aggregated neutrosophic decision matrix using the Single-Valued Neutrosophic Weighted Average (SVNWA) aggregation operator, as proposed in reference [17]. The use of SVNWA facilitates the combination of individual neutrosophic evaluations into a single matrix that represents the group decision more completely and accurately. The evaluations from all decision-makers can be compiled into a single decision matrix that reflects the consensus or weighting of individual evaluations based on the weights assigned to each decision-maker.

In such a case, with $D_k = \left(d_{ij}(k)\right)_{mxn}$ being the single-valued neutrosophic decision matrix of the k-th decision-maker and $\psi = (\psi_1, \psi_2, \dots, \psi_p)^T$ the weight vector of the decision-makers, such that each $\psi_k \in [0,1]$, the weighted decision matrix can be obtained considering that [17]:

$$d_{ij} = \langle 1 - \prod_{k=1}^{p}\left(1 - T_{ij}^{(p)}\right)^{\psi_k}, \prod_{k=1}^{p}\left(I_{ij}^{(p)}\right)^{\psi_k}, \prod_{k=1}^{p}\left(F_{ij}^{(p)}\right)^{\psi_k}\rangle \quad (2)$$

On the other hand, if A and B are assumed to be two single-valued neutrosophic numbers, the normalized Hamming distance between them is defined as:

$$d(A,B)\frac{|TA-TB|+|IA-IB|+|FA-FB|}{3} \quad (3)$$

The normalized Hamming distance between two Single-Valued Neutrosophic Numbers, A and B, measures the discrepancy or difference between them based on their components of truth, falseness, and indeterminacy. It is an important indicator for evaluating how similar or different two SVNNs are in terms of their neutrosophic characteristics. The smaller the normalized Hamming distance, the greater the similarity between A and B, and vice versa. Meanwhile, the complement of an SVNN $A = (T_A, I_A, F_A)$ can be defined as:

$$A^C = (F_A, 1 - I_A, T_A) \quad (4)$$

The complement of an SVNN reflects the complementary degrees of truth, indeterminacy, and falseness of A. This concept is useful for analyzing and comparing inverse or opposing properties in neutrosophic analysis, allowing for a deeper understanding of the dynamics involved in contexts where uncertainty and ambiguity play a central role.

2.2 Combined TODIM/PROMETHEE approach

The methodological approach that combines TODIM (a methodology based on the Dominance of Interactive and Multicriteria Criteria) with PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) constitutes an advanced technique for multi-criteria decision-making in contexts of uncertainty. This method

relies on Single-Valued Neutrosophic Numbers (SVNNs) to integrate both the certainties and uncertainties of attributes, providing a robust tool for evaluating alternatives considering their strengths and weaknesses.

In practice, this approach involves defining alternatives and attributes as follows:

Alternatives: They are represented as $A = (A_1, \dots, A_m)$, where each A_i is a specific alternative within the set of available options.

Attributes: They are denoted as $G = (G_1, G_2, \dots, G_n)$, each of which describes a relevant criterion in the evaluation of the alternatives.

To proceed with the evaluation, it is essential to assign weights to each attribute, represented as $W = (w_1, w_2, \dots, w_n)$, where $0 \leq w_j \leq 1$ and the total sum of the weights is 1. This ensures that the attributes are appropriately weighted in the overall evaluation.

The attribute values for each alternative are organized in a matrix $A = (a_{ij})$ with dimensions m×n, where a_{ij} represents the value of the attribute G_j for alternative A_i. Each element a_{ij} is a Single-Valued Neutrosophic Number, represented as T_{ij}, I_{ij}, and F_{ij}, where:

- T_{ij} indicates the degree of membership or truth of attribute G_j for the alternative A_i.
- I_{ij} represents the degree of indeterminacy, reflecting the uncertainty or ambiguity of the value.
- F_{ij} denotes the degree of non-membership or falseness, indicating the extent to which the attribute is not characteristic of the alternative.

Let's consider the alternatives as $A = (A_1, \dots, A_m)$ and the attributes as $G = (G_1, G_2, \dots, G_n)$. We assign weights to the attributes as $W = (w_1, w_2, \dots, w_n)$, where the sum of all weights equals 1, i.e., $\sum_{j=1}^{n} w_j = 1$. Let us denote a_{ij}, where i = 1, 2, ..., m and j = 1, 2, ..., n, as the value of the attribute G_j for the alternative A_i. We create a matrix $A = (a_{ij})$ with dimensions m×n, which is a matrix of Single-Valued Neutrosophic Numbers (SVNNs), represented as $\langle (T_{ij}, I_{ij}, F_{ij}) \rangle_{mxn}$, where T_{ij}, I_{ij}, and F_{ij} are the degrees of membership, degrees of indeterminacy, and degrees of non-membership.

The implementation of this method allows for a detailed and nuanced evaluation of the available options, facilitating decision-making in complex environments where criteria can be numerous and information may be incomplete or uncertain. This approach not only helps capture the multiple and often contradictory dimensions of decision problems but also promotes a more informed and balanced choice among the proposed alternatives.

The procedure for applying the TODIM-PROMETHEE method using Single-Valued Neutrosophic Numbers (SVNN) for decision-making can be broken down into several structured steps, as described below:

Step 1: Identification of Treatment Techniques: In this first step, the relevant treatment techniques or alternatives that will be evaluated are identified. This is a crucial step where the options among which a decision will be made are established.

Step 2: Assignment of Weights to Decision Makers: Each decision maker is assigned a weight that reflects their experience and knowledge about the problem in question. These weights are expressed using linguistic variables that are then converted into Single-Valued Neutrosophic Numbers (SVNN) using a specific equation, allowing for precise quantification of each expert's influence on the final decision.

Step 3: Conversion of Linguistic Evaluations into SVNN: Based on the evaluations provided by the experts, which initially may be clear linguistic expressions, individual neutrosophic matrices are constructed for each decision maker. This step transforms qualitative evaluations into quantitative ones, suitable for more detailed mathematical analysis.

Step 4: Creation of the Initial Relation Matrix: A matrix $A = (a_{ij})_{mxn}$ is formed where each element a_{ij} represents the value of the attribute G_j for the alternative A_i. This matrix is expressed in neutrosophic form as $\langle (T_{ij}, I_{ij}, F_{ij}) \rangle_{mxn}$, where T_{ij}, I_{ij}, and F_{ij} represent the degrees of membership, indeterminacy of membership, and non-membership, respectively.

Step 5: Standardization of Decision Information: the standardization process normalizes the matrix $A = (a_{ij})_{mxn}$ to transform it into a matrix $B = (b_{ij})_{mxn}$. If the attribute is a cost factor, it is transformed using its complementary set to reflect the preference for lower cost; for efficiency factors, this transformation is not necessary.

Step 6: Construction of the Preference Function: A preference function $P_j(B_i, B_r)$ is developed to evaluate the alternative B_i in relation to B_r under attribute G_j. This function is based on a specific equation that measures the preference of one alternative over another, considering the degrees of membership, indeterminacy, and non-membership in the neutrosophic context. The procedure described in equation (5) is followed.

$$P_j(B_i, B_r) = \begin{cases} 0, d \leq p \\ \frac{d-p}{q-p}, p < d < q \\ 1, d \geq q \end{cases} \quad (5)$$

Step 7: Calculation of the Relative Weight of the Attributes

This step involves calculating the relative weight G_j of one attribute G_j with respect to another G_r. Mathematically, this is expressed as:

$$w_{jr} = \frac{w_j}{w_r} = (j, r = 1,2, \dots, n) \quad (6)$$

This calculation helps to determine the relative importance of each attribute in comparison to others, providing a basis for more detailed comparative analyses in the subsequent steps.

Step 8: Definition of the Priority Index: The priority index $\pi(B_i, B_r)$ evaluates the scheme B_i in relation to B_r using the following formula:

$$\pi(B_i, B_r) = \frac{\sum_{j=1}^{n} w_{jr} P_j(B_i, B_r)}{\sum_{j=1}^{n} w_{jr}} \quad (7)$$

This index synthesizes preferences under all attributes, weighting each preference by the relative weight of the corresponding attribute.

Step 9: Calculation of Inflow, Outflow, and Net Flow: The inflow $\Phi^+(B_i)$, outflow $\Phi^-(B_i)$, and net flow $\Phi(B_i)$ of each alternative are calculated to determine how the alternatives compare to each other within the group. The formulas are the following:

$$\Phi^+(\mathrm{B}_i) = \frac{\sum_{r=1}^{m} \pi(B_i, B_r) - \min_{1 \le i \le m} \{\sum_{r=1}^{m} \pi(B_i, B_r)\}}{\max_{1 \le i \le m} \{\sum_{r=1}^{m} \pi(B_i, B_r)\} - \min_{1 \le i \le m} \{\sum_{r=1}^{m} \pi(B_i, B_r)\}} \quad (8)$$

$$\Phi^-(\mathrm{B}_i) = \frac{\sum_{r=1}^{m} \pi(B_r, B_i) - \min_{1 \le i \le m} \{\sum_{r=1}^{m} \pi(B_r, B_i)\}}{\max_{1 \le i \le m} \{\sum_{r=1}^{m} \pi(B_r, B_i)\} - \min_{1 \le i \le m} \{\sum_{r=1}^{m} \pi(B_r, B_i)\}} \quad (9)$$

$$\Phi(\mathrm{B}_i) = \Phi^+(\mathrm{B}_i) - \Phi^-(\mathrm{B}_i) \quad (10)$$

Step 10: Classification of Alternatives: Finally, all alternatives are ranked according to the value of $\Phi(B_i)$. The alternatives are ordered from the highest to lowest value of Φ, facilitating the identification of the best option in the decision-making context. The alternative with the highest $\Phi(B_i)$ is considered optimal, as it indicates a favorable balance between the inflows and outflows in terms of aggregated preferences.

This detailed process ensures that all alternatives are evaluated and compared fairly and thoroughly, considering both individual preference metrics and the relative weights of the attributes, and culminating in an informed selection based on objective and nuanced criteria.

3 Method

For the community development of the Ashigua páramo and the implementation of conservation strategies, the Community Self-Management Process (PAC) methodology was applied. This methodology is part of an ethnic revitalization process in Ecuador, where indigenous communities are emerging as key actors in social planning and the management of participatory development initiatives. The PAC methodology is based on the rich history of autonomous planning of pre-colonial Andean peoples and adapts to current cultural structures, which preserve clear methods and principles for organizing and executing their activities.

This study is categorized as applied research that adopts quantitative and qualitative methods to develop a composite indicator aimed at the environmental sustainability of the páramo. The methodological approach is based on the integration of various environmental sub-indicators, using advanced decision-making techniques grounded in neutrosophic logic, which is essential for managing the indeterminacy and uncertainty characteristic of environmental studies.

Initially, a thorough review of the literature is conducted to identify potential sub-indicators used in páramo conservation. The selected sub-indicators are those that meet pre-established criteria of scientific relevance and practical applicability. In addition, rounds of questionnaires are conducted among selected experts for the study, who provide valuable considerations for the study. A combined strategy of the TODIM and PROMETHEE methodologies in their neutrosophic variant is adopted. This combination allows a meticulous evaluation of the sub-indicators according to their relevance and importance, based on 5 specific evaluation criteria:

1. Sensitivity: The sub-indicator's ability to detect significant changes both in the natural environment and in anthropogenic pressures on natural resources.
2. Ease of Measurement: Consideration of the practicality and cost associated with data collection, including the availability of technology and the need for specialized training.
3. Integrability: Evaluation of the sub-indicator's ability to integrate with others and form a coherent and robust composite indicator.

4. Predictability: The sub-indicator's potential to anticipate future changes in the environment, based on historical and current trends.
5. Acceptability: The degree of acceptance of the sub-indicator by the scientific community and relevant stakeholders.

The neutrosophic methodology of the TODIM and PROMETHEE methods proposed by [18] is used to integrate the evaluations of experts, who assign values to each sub-indicator according to the mentioned criteria. Each criterion is evaluated using a scale of neutrosophic numbers, which allows for capturing the opinions of the experts in terms of certainty, uncertainty, and indeterminacy. For the study, criteria 1 and 3 are considered to have the highest weight of importance with a value of 0.3, while criteria 2 and 4 share an importance of 0.15, and criterion 5 has a value of importance or weight of 0.1. The results are synthesized to form a composite environmental sustainability indicator. Ethical considerations related to data collection and management are taken into account, ensuring that all personal information of participants is treated confidentially and in accordance with current regulations.

This methodological approach provides a rigorous framework for developing an indicator that not only reflects the conditions of the páramo but also integrates the complexity and multidimensionality of the factors influencing its sustainability. By employing neutrosophic logic, this study positions itself at the forefront of research that incorporates uncertainty and indeterminacy as central elements in environmental evaluation.

4 Results

The literature review conducted and consultations with experts facilitate the identification of a set of sub-indicators whose relevance is suitable for inclusion in a composite environmental sustainability indicator that is being developed. This methodological process results in the initial proposal of the sub-indicators detailed in Table 2. These sub-indicators have been selected based on their ability to capture critical aspects of environmental sustainability, reflecting essential dimensions that are fundamental for a comprehensive assessment of environmental impact and effective conservation of natural resources.

Table 2: Indicators considered for the preparation of the composite indicator

FACTOR	INDICATORS
Water	Water Flow
	Water Quality
	Transport of Contaminants in Water
Biodiversity	Ecological Footprint
	Deforestation
	Biocapacity
	Biodiversity Quantification
	Invasive Species
Soil	Advancement of the Agricultural Frontier
	Soil Erosion
	Degree of Erosion by Visitors in Protected Areas
	Level of Vegetation Cover
	Biomass Calculation
	Soil Carbon Concentration
Air	Burning of Grasslands
	Air Pollution
Legal aspects	Compliance with Environmental Legislation
Protection	Good Practices for Páramo Management
	Environmental Preservation

In the current study, it is established that the five experts involved have equivalent influence and relevance in the decision-making process, thus ensuring an impartial and consensual evaluation method. The decision-makers proceed with the analysis of the selected indicators, examining in detail each of the criteria previously determined for their evaluation. To implement this procedure, a transformation of the individual decision matrices of each expert is performed using equation (2), to derive matrix A. This matrix, whose details are specified in Table 3, synthesizes the individual evaluations made by the experts regarding the available options and the criteria set for their evaluation.

Table 3: Normalized decision matrix

	Criterion 1	**Criterion 2**	**Criterion 3**	**Criterion 4**	**Criterion 5**
A1	(0.83428;0.16572;0.15849)	(0.83428;0.16572;0.15849)	(0.83428;0.16572;0.15849)	(0.8343;0.1657;0.1585)	(0.8343;0.1657;0.1585)
A2	(0.6059;0.41731;0.40953)	(0.80963;0.19037;0.19037)	(0.87989;0.12011;0.11487)	(0.8799;0.1201;0.1149)	(0.8799;0.1201;0.1149)
A3	(0.87989;0.12011;0.11487)	(0.82671;0.17329;0.15157)	(0.87989;0.12011;0.11487)	(0.8799;0.1201;0.1149)	(0.8799;0.1201;0.1149)
A4	(0.80963;0.19037;0.19037)	(0.62107;0.37893;0.34657)	(0.67012;0.32988;0.28854)	(0.7254;0.2746;0.2512)	(0.7254;0.2746;0.2512)
A5	(0.82671;0.17329;0.15157)	(0.83428;0.16572;0.15849)	(0.62107;0.37893;0.34657)	(0.8343;0.1657;0.1585)	(0.6211;0.3789;0.3466)
A6	(0.83428;0.16572;0.15849)	(0.87989;0.12011;0.11487)	(0.83428;0.16572;0.15849)	(0.862;0.138;0.138)	(0.862;0.138;0.138)
A7	(0.83428;0.16572;0.15849)	(0.83428;0.16572;0.15849)	(0.83428;0.16572;0.15849)	(0.8343;0.1657;0.1585)	(0.8343;0.1657;0.1585)
A8	(0.75;0.25;0.2)	(0.82671;0.17329;0.15157)	(0.83428;0.16572;0.15849)	(0.8343;0.1657;0.1585)	(0.8343;0.1657;0.1585)
A9	(0.72536;0.27464;0.25119)	(0.83428;0.16572;0.15849)	(0.83428;0.16572;0.15849)	(0.8343;0.1657;0.1585)	(0.8343;0.1657;0.1585)
A10	(0.6059;0.41731;0.40953)	(0.83428;0.16572;0.15849)	(0.87989;0.12011;0.11487)	(0.8799;0.1201;0.1149)	(0.8799;0.1201;0.1149)
A11	(0.80095;0.19905;0.18206)	(0.75;0.25;0.2)	(0.80095;0.19905;0.18206)	(0.8009;0.1991;0.1821)	(0.7254;0.2746;0.2512)
A12	(0.80963;0.19037;0.19037)	(0.87989;0.12011;0.11487)	(0.80963;0.19037;0.19037)	(0.8096;0.1904;0.1904)	(0.6211;0.3789;0.3466)
A13	(0.67012;0.32988;0.28854)	(0.80963;0.19037;0.19037)	(0.67012;0.32988;0.28854)	(0.6701;0.3299;0.2885)	(0.8096;0.1904;0.1904)
A14	(0.62107;0.37893;0.34657)	(0.82671;0.17329;0.15157)	(0.62107;0.37893;0.34657)	(0.6211;0.3789;0.3466)	(0.7254;0.2746;0.2512)
A15	(0.83428;0.16572;0.15849)	(0.87989;0.12011;0.11487)	(0.87989;0.12011;0.11487)	(0.8799;0.1201;0.1149)	(0.8799;0.1201;0.1149)
A16	(0.71283;0.28717;0.24022)	(0.83428;0.16572;0.15849)	(0.71283;0.28717;0.24022)	(0.7128;0.2872;0.2402)	(0.8096;0.1904;0.1904)
A17	(0.62107;0.37893;0.34657)	(0.75;0.25;0.2)	(0.83428;0.16572;0.15849)	(0.8343;0.1657;0.1585)	(0.8096;0.1904;0.1904)
A18	(0.62107;0.37893;0.34657)	(0.83428;0.16572;0.15849)	(0.83428;0.16572;0.15849)	(0.8343;0.1657;0.1585)	(0.7254;0.2746;0.2512)
A19	(0.83428;0.16572;0.15849)	(0.44935;0.22865;0.52661)	(0.83428;0.16572;0.15849)	(0.8343;0.1657;0.1585)	(0.6211;0.3789;0.3466)

Subsequently, the development of matrices that represent the degrees of preference $P_j(B_i, B_r)$ with respect to the attribute G_j is addressed. These degrees are calculated using a linear function, defined in Equation 5, where the parameters q = 1 and p = 0 are set. This approach is adopted under the premise that it allows for a direct and

simplified interpretation of preferences, thus facilitating the comparison between alternatives under specific criteria.

To determine the comprehensive priority index of the evaluated alternatives, Equation 7 is used. The information provided supplies the inflow, outflow, and net flows for each of the alternatives, as specified in Table 4. These flows are crucial for elucidating the sub-indicators of greater preference according to the parameters established in the study. The articulation of these flows allows for a quantitative evaluation of how each alternative compares in terms of receiving preferences (inflows), yielding to other alternatives (outflows), and their net balance, which is decisive for identifying the most prominent sub-indicators within the defined criteria framework. This analysis is instrumental for a deep understanding of preference dynamics and for grounding strategic decisions based on empirical evidence.

Table 4: Inflows, outflows, and net flows for each indicator evaluated

Sub Indicators	Φ+	Φ-	Φ
Water Flow	0.1	0.569	-0.506
Water Quality	0.6	0.428	0.162
Transport of Contaminants in Water	0	1,000	-1,000
Ecological Footprint	0.6	0.220	0.421
Deforestation	0.4	0.393	0.052
Biocapacity	0	0.687	-0.644
Biodiversity Quantification	0.1	0.569	-0.506
Invasive Species	0.1	0.472	-0.349
Advancement of the Agricultural Frontier	0.2	0.369	-0.137
Soil Erosion	0.6	0.450	0.112
Degree of Erosion by Visitors in Protected Areas	0.2	0.404	-0.186
Level of Vegetation Cover	0.2	0.466	-0.239
Biomass Calculation	0.7	0.078	0.634
Soil Carbon Concentration	1	0.000	1,000
Burning of Grasslands	0	0.887	-0.878
Air Pollution	0.4	0.228	0.216
Compliance with Environmental Legislation	0.5	0.216	0.257
Good Practices for Páramo Management	0.4	0.233	0.213
Environmental Preservation	0.1	0.517	-0.375

The analyzed data revealed that "Soil Carbon Concentration" obtained the highest preference among the indicators analyzed, reflecting unanimity in its favorability, given that no negative preference flows were recorded. This result highlights the critical importance of this sub-indicator in environmental evaluation, possibly due to its direct relevance in carbon capture and climate change mitigation. This indicator is crucial not only for its role in carbon capture but also for its part in the global carbon cycle, which directly affects climate regulation and environmental sustainability.

On the other hand, "Biomass Calculation" reached a value with the highest positive preference flow after Soil Carbon. This suggests that the quantification of biomass is highly valued by experts, possibly for its role in understanding ecosystem productivity and the overall health of the ecosystem. Interestingly, indicators such as "Ecological Footprint," "Compliance with Environmental Legislation," and "Good Practices for Páramo Management" presented moderately positive values, reflecting a balanced acceptance and recognizing their usefulness in a comprehensive environmental evaluation framework.

The results of the evaluation of these indicators highlight a favorable trend towards those intrinsically linked with the carbon storage capacity and the conservation of the ecological functionality of ecosystems. This indicates a conscious and strategic approach towards the selection of sub-indicators that not only monitor ecological health but also strengthen actions against global environmental challenges like climate change. It is inferred that, by prioritizing these sub-indicators, environmental management policies, and practices could be improved, steering them towards effective conservation and restoration of ecosystems.

In general, the proposed composite indicator is composed as follows:

$$\boldsymbol{Index} = \sum \boldsymbol{i} = \boldsymbol{1} \boldsymbol{W_i} * \boldsymbol{V_i}$$

- W = Weight of the considered indicator.
- V = Indicator value.
- n = Number of indicators used.
- $\sum W_i = 100$

Indicators	Calculation formula	Definition of variables
Water Quality	$\boldsymbol{ICA_{OBJ}} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \boldsymbol{Pi}.\,(\boldsymbol{ICA})\boldsymbol{i}$	Number of parameters selected. $(ICA)_i$: Environmental quality index for the parameter. Pi: Weight attributed to parameter i.
Ecological Footprint	$\boldsymbol{F = D/Y}$	EF = Ecological Footprint. D = Annual demand for a product. Y = Annual yield of the same product.
Deforestation	$\frac{\boldsymbol{Deforested\ areas\ (h2)}}{\boldsymbol{Conservation\ areas\ (h2)}}$	Conserved Areas (m2): páramo conservation area in the last year. Deforested Areas (m2): geographic area that has undergone deforestation, whether caused by humans or not, in the last year.
Soil Erosion	$\frac{\boldsymbol{Area\ used\ for\ grazing\ (ha)}}{\boldsymbol{Geographical\ area\ per\ hectare\ (ha)}}$	Area used for grazing (ha): Increase in livestock farming in the páramos in recent years. Geographical Areas (ha): Loss from soil erosion and loss of carbon retention.
Biomass Calculation	$\boldsymbol{Biomass}(\frac{\boldsymbol{ugC}}{\boldsymbol{ml}}) = \boldsymbol{N} * \boldsymbol{By} * \boldsymbol{F}$	N: The number of microorganisms enumerated per ml of sample. Bv: The biovolume expressed as µm^3 per microorganism. F: Conversion factor, µg of Carbon per μm^3.
Soil Carbon Concentration	$\boldsymbol{CT} = \boldsymbol{C}_{bushy\ comp} + \boldsymbol{C}_{herbaceous\ comp} + \boldsymbol{C}_{necromass\ comp\ (a+h)} + \boldsymbol{C}_{soil+(a+h)}$	C_{TOTAL}: Total Carbon Content of the Páramo ecosystem. $C_{comp\ bushes}$: C in the bush compartment (biomass). $C_{comp.\ herbs}$: C in the herbaceous compartment (biomass). $C_{comp.\ necromass\ (a+h)}$: C in the necromass of the shrubby and herbaceous páramos. $C_{comp.\ soil\ (a+h)}$: C in the soil compartment in the shrubby and herbaceous páramos.
Air Pollution	$\frac{\boldsymbol{NOx\ emissions(kg)}}{\boldsymbol{n\ hab\ or\ visitors}}$	NOx emissions in the páramo area under study. Population in the year of measurement.

Indicators	Calculation formula	Definition of variables
		Ratio of NOx emissions to the population in the year of calculation.
Compliance with Environmental Legislation	$\mathbf{I} = (\mathbf{NAC} / \mathbf{NTAE}) * \mathbf{100}$	NAC = Number of environmental regulations complied with. NTAE = Total number of environmental regulations required.
Good Practices for Páramo Management	Promote and require the performance and documentation of monitoring, training in environmental issues, inspections, and compliance with environmental legislation in the páramo and among the inhabitants of the surrounding rural areas who benefit from its nearby ecosystem services.	

5 Conclusion

This study facilitated the development of an environmental sustainability indicator based on the rigorous selection and evaluation of specific environmental sub-indicators. To achieve this objective, advanced methodologies were applied, including neutrosophic logic and a combination of multi-criteria evaluation methods. These tools enabled the integration and analysis of multiple dimensions of environmental data, characterized by their complexity and ambiguity.

The application of neutrosophic logic was crucial for managing the inherent uncertainty in environmental data, allowing for a more accurate evaluation of the indicators. This approach provided a solid foundation for incorporating multiple perspectives and managing contradictions in the available information, resulting in a more inclusive and representative decision-making process. On the other hand, the use of TODIM and PROMETHEE methods in their neutrosophic variant facilitated the objective comparison and prioritization of sub-indicators, based on criteria specifically selected to reflect the most relevant aspects of sustainability and environmental impact.

The prioritization of indicators directly related to carbon storage and ecosystem productivity was confirmed, reflecting their critical relevance in mitigating climate change and conserving biodiversity, according to experts. Additionally, the utility of integrating regulatory compliance measures and sustainable management practices related to compliance with environmental legislation and good practices for páramo management was highlighted. These sub-indicators are essential to ensure that environmental management policies and practices are not only effective but also aligned with legal requirements and social expectations. The combination of analytical tools and advanced logical approaches proved effective in addressing the complexity of environmental systems, suggesting their continued and expanded application in future environmental monitoring and evaluation efforts.

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Neutrosophic Hypothesis to Validate the Efficacy and Safety of Propolis in the Treatment of External Bacterial Otitis in Canines

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Abstract. Propolis, known for its antibacterial, antifungal, antioxidant, antiviral, immunomodulatory, and anti-parasitic properties, has been the subject of this study to evaluate its efficacy and safety in the treatment of bacterial external otitis in domestic canines. Through questionnaires administered to veterinary experts and the use of neutrosophic logic for hypothesis testing, perceptions based on clinical experiences and scientific literature were analyzed. The results indicate strong agreement that propolis is effective and safe, surpassing some aspects of conventional antibiotic treatments and demonstrating high tolerability without serious side effects. This acceptance underscores its potential as a natural alternative in veterinary treatments, particularly valuable in cases of antimicrobial resistance or allergies to traditional medications. This study provides significant evidence supporting the integration of propolis into veterinary practices, suggesting the need for further research and possible revisions of current clinical practices for managing bacterial conditions in animals. The use of neutrosophic hypotheses facilitated a more detailed and deeply informed interpretation of the level of expert agreement on the properties of propolis in the condition analyzed.

Keywords: propolis, external bacterial otitis, domestic canines, neutrosophic hypothesis.

1 Introduction

The indiscriminate use of antibiotics in both human medicine and veterinary medicine has induced side effects ranging from mild to severe, significantly contributing to the emergence of bacterial strains with multiple resistances to these medications [1]. Microbial adaptability has led to the frequent need to modify initial therapeutic regimens, due to the progressive reduction of the effectiveness of antibiotic drugs [2]. In this context, the use of propolis, an apiary derivative of a resinous nature and complex composition, has been proposed. This natural product presents variations in its coloration, ranging from ochre and red to light brown and green, and in its texture, which varies from friable and firm to gummy and elastic. [3]

Propolis has been extensively studied for its antibacterial properties, and it has also been found to have antifungal, antioxidant, antiviral, immunomodulatory, and anti-parasitic capabilities [4]. Recent research has linked these beneficial properties to the presence of terpenoids, flavonoids, and anthraquinones in its composition. [5]

External otitis is characterized by an inflammation of the skin lining the external auditory canal and, sometimes, the ear pinna. When this condition is not effectively resolved, it can evolve into a chronic form and, in unfortunate situations, compromise the tympanic membrane, extending the infection to the middle ear [6]. This pathology results from a confluence of etiological factors that include ectoparasites, allergic conditions, dermatological problems, endocrine disorders, and the presence of foreign bodies. The persistence of the inflammatory process in the outer ear for a period longer than six months is defined as chronic external otitis. [7]

Both Gram-positive and Gram-negative bacteria play a crucial role as etiological agents in ear infectious processes. These microorganisms, which include species such as *Staphylococcus intermedius* and *Pseudomonas aeruginosa,* are commonly identified in chronic otic infections and can coexist as part of the normal microbiota of the auditory canal. Despite their usual presence in this environment, under certain conditions, such as a significant increase in their population density or a decrease in local defenses, they can become opportunistic pathogens and cause diseases. [8]

In particular, *Staphylococcus intermedius* is frequently associated with infections in canines, showing a notable adaptation to this host [9]. On the other hand, *Pseudomonas aeruginosa* is known for its resistance to multiple antibiotics and its ability to colonize moist environments, making it a formidable pathogen in cases of external otitis, especially in chronic and recurrent forms.

Besides these main pathogens, other bacteria such as *Proteus mirabilis, Escherichia coli, Corynebacterium*

spp., Enterococcus spp., and Streptococcus spp. also contribute to the etiology of otitis, although their involvement is relatively less frequent. These microorganisms can be part of the commensal or transient flora of the auditory canal, but under conducive conditions, such as alterations in the normal microflora or changes in host immunity, they can cause infections.

Propolis is a waxy substance produced by bees from resinous exudates collected from various plants. This resin is used by bees not only for the construction and repair of their hives but also as a defense mechanism to seal openings and protect the hive from external elements. Currently, scientific interest in propolis has significantly increased, especially in fields like biology and medicine, where its potential as a dietary supplement and its applicability in the pharmaceutical industry are explored. [10]

Historically, the use of propolis dates back to ancient civilizations such as the Greeks, Romans, and Egyptians, who used it to treat medical conditions, including bacterial infections. This traditional use has persisted to the present day within alternative medicine, where propolis continues to be valued for its therapeutic properties, particularly its antibacterial activity.

The relevance of propolis in contemporary medicine is based on its rich biochemical composition, which includes compounds such as flavonoids, terpenoids, and phenolics, known for their antioxidant, anti-inflammatory, antibacterial, and antifungal properties. Recent studies have expanded our understanding of how these compounds can contribute to managing infectious and chronic diseases, providing a scientific basis for their integration into more modern and effective treatments. Given this context, ongoing research is essential to scientifically validate the use of propolis in clinical contexts, establish precise dosages, and explore its mechanisms of action at the cellular and molecular levels. This could facilitate its acceptance and regulation as a component in medicines and nutritional supplements, thereby broadening its potential use in public health and therapeutics.

Informed decision-making in veterinary medicine, especially in the treatment of conditions like bacterial external otitis in domestic canines, requires a comprehensive and nuanced approach due to the complexity and variability of the clinical factors involved. In this context, the neutrosophic logic proposed by Florentin Smarandache offers a promising theoretical framework by allowing the integration of elements of truth, falsehood, and indeterminacy in the analysis of information and uncertain situations that often arise in clinical practice. [11][12]

External Bacterial otitis in canines is a multifactorial condition influenced by various factors such as genetics, environment, and the presence of multiple pathogens resistant to conventional treatments. The variability in treatment response among individuals further complicates decision-making. Neutrosophic logic, by addressing the uncertainties and contradictions inherent in such cases, facilitates a more holistic and realistic approach to evaluating therapeutic options.

The theory of neutrosophy has given rise to numerous scientific disciplines, including neutrosophic logic [13], neutrosophic sets [14], neutrosophic probability, and neutrosophic statistics [15], which have found varied applications in fields such as engineering, computer science, and medical research [16][17]. In this study, a neutrosophic hypothesis is employed, which differs from traditional statistical hypotheses in that the variables describing the population's characteristics are of a neutrosophic nature. That is, they have indeterminate values, some unknown or an inexact number of terms if the variable is discrete, or at least one of the compared characteristics of the population is neutrosophic, characterized by being indeterminate, unknown, or vague.

Using neutrosophic logic, veterinarians can consider not only the expected outcomes (truth) and the possible adverse effects or failures of the treatment (falsehood) but also states of indetermination [18], such as the unpredictable reactions of the patient to treatment or the unknown interaction between concurrent medications. This allows for a more comprehensive assessment of risks and benefits, resulting in more informed and case-adapted decision-making.

Neutrosophic statistics extend this approach to data analysis, incorporating the indetermination and uncertainty of the real world into statistical interpretation [19], [20]. In researching and treating various conditions, neutrosophic statistics can be used to analyze the efficacy of different therapeutic interventions, considering not only typical responses to treatment but also inexplicable anomalies and variations. [21]

In this sense, the proposed study aims to collect and analyze the perceptions of veterinary experts on the antimicrobial and healing qualities of propolis in managing bacterial external otitis in domestic canines. This approach seeks to improve the precision and personalization of veterinary treatments, optimizing clinical outcomes through a more thorough and representative analysis of the variables involved.

2 Method

2.1 Methodology

This study is categorized as a quantitative exploratory study with the primary objective of evaluating the application of neutrosophic logic and statistics in determining the level of agreement among veterinary experts regarding the antimicrobial and healing utilities of propolis, as well as its efficacy and safety in the treatment of bacterial external otitis in domestic canines compared to conventional treatments. The adopted design is cross-

sectional, where data is collected and analyzed at a single point in time through a structured survey.

For data collection, a combination of simple random sampling and purposive sampling was used. Simple random sampling ensures that every member of the population of veterinarians experienced in treatments for canines has an equal probability of being selected. This approach was complemented by purposive sampling to focus on those professionals with limited access. Thus ensuring a broad and representative coverage of experienced opinions.

A survey based on the Likert format was designed, specifically created to measure the concordance among veterinary experts on various statements about multiple aspects of the use of propolis, including its efficacy, safety, and antimicrobial and healing properties in bacterial external otitis, as well as some comparisons with conventional treatments. Respondents were asked to express their degree of agreement or disagreement on a scale from "Strongly agree" to "Strongly disagree," according to the items shown in Table 1.

Table 1. Linguistic variable and Single-Valued Neutrosophic Numbers (SVNNs) Note: Source:[22]

Integer	Linguistic variable	SVNNs
0	Strongly disagree	(0, 1, 1)
1	Disagree	(0.20, 0.85, 0.80)
2	Partially Disagree	(0.40, 0.65, 0.60)
3	Neither agree or disagree	(0.50;0.5;0.50)
4	Partially agree	(0.60, 0.35, 0.40)
5	Agree	(0.8, 0.15, 0.20)
6	Strongly agree	(1, 0, 0)

The data collected through the survey were processed using neutrosophic statistics. This involved transforming the linguistic responses into a set of single-valued neutrosophic numbers, thereby allowing for the management of indetermination in the responses. This facilitated the conducting of neutrosophic hypothesis tests to assess the concordance of opinions and provide a solid basis for informed decisions. This method is particularly useful in studies where perceptions and expert evaluations play a crucial role and where responses may not be absolutely positive or negative.

Informed consent was obtained from all participants involved in the survey, ensuring the confidentiality and anonymity of their responses. All relevant ethical guidelines for research with human subjects were adhered to, guaranteeing an ethical and professional process in the collection and analysis of data. This methodological approach provides a robust framework for deeply exploring expert perceptions and applying advanced theoretical frameworks such as neutrosophic logic and statistics in the field of veterinary and precision medicine.

2.2 Single Valued Neutrosophic Sets

The implementation of single-valued neutrosophic sets (SVNS) marks a significant development within the realms of set theory and logic, offering a robust framework for accurately representing ambiguity and uncertainty. These SVNS are fundamental for precisely describing the truth, indeterminacy, and falsity of elements within a set. This capability makes them valuable tools in various sectors, including decision-making in uncertain environments, artificial intelligence, and information management, facilitating the analysis and handling of complex data in these disciplines.

Within the framework of SVNS, let us consider X as a space containing points or objects, with generic elements in X denoted by x. A single-valued neutrosophic set A in X is defined through three characteristic functions: the truth-membership function $T_A(x)$, the indeterminacy-membership function $I_A(x)$, and the falsity-membership function $F_A(x)$. Thus, an SVNS A can be formally expressed as $A = \{x, T_{A(x)}, I_{A(x)}, F_{A(x)x} \in X\}$, where $T_{A(x)}, I_{A(x)}$, and $F_{A(x)} \in [0,1]$ for each point x in X. In this way, the sum of $T_{A(x)}, I_{A(x)}$, and $F_{A(x)}$ meets the condition $0 \leq T_{A(x)} + I_{A(x)} + F_{A(x)} \leq 3$. [23]

This formalism allows each element x in the space X to be evaluated under these three metrics, thus facilitating a more nuanced and detailed understanding of its state in terms of truth, falsity, and indeterminacy. This approach not only enriches traditional set theory with an additional dimension of analysis but also optimizes decision-making and analysis processes in complex and dynamic environments.

Modeling membership functions within the range $[0,1]$ in SVNS provides increased flexibility and precision for analysis in contexts where uncertainty is a predominant element. This range ensures that the total sum of the membership functions related to truth, indeterminacy, and falsity does not exceed the maximum value of 3, thus

maintaining structural coherence within the theoretical framework of SVNS. Such methodology offers solid support for managing ambiguity and uncertainty across a wide range of application fields, facilitating a more systematic and detailed approach in situations that challenge conventional analytical methods.

A single-valued neutrosophic number (SVN number) facilitates the incorporation of linguistic variables into the analysis, allowing for a more nuanced interpretation of qualitative data and ambivalent contexts. To transform these SVN numbers into precise and clear values, a scoring function is used. This scoring function is crucial for quantifying the linguistic responses provided in surveys, thereby allowing for an accurate numerical evaluation of the opinions or perceptions expressed by respondents in each statement. This method ensures that subjective data are converted into quantifiable information, which is essential for subsequent statistical analysis and evidence-based decision-making.

$$p(x) = 2 + T(x) + I(x) + F(x) \quad (1)$$

In the context of this research, the scoring function can be mathematically expressed in the following way for each statement in the survey:

$$p(x)_s = 2 + T(x)_s + I(x)_s + F(x)_s \quad (2)$$

Where x represents the number of respondents, s is the number of statements, and $p(x)_s$ is the scoring function value of respondent x for statement s. The scoring function reflects each respondent's assessment of a specific statement based on their level of agreement or disagreement. The sum of the scores for each statement across all respondents is used to calculate the average scoring function for that specific statement.

To interpret the obtained values and assign degrees of agreement, the total possible range of the average scoring function (ranging from 0 to 3) is divided into 7 intervals. Each interval represents a specific degree of agreement or disagreement, detailed in Table 2 of the study. This segmentation allows a more granular interpretation of how respondents perceive and value each statement presented in the survey.

This method provides a solid foundation for analyzing the responses in statistical terms and enables a quantitative evaluation of opinions, which is crucial for making informed and evidence-based decisions in the context of the research.

Table 2. Intervals of the average score by agreement degree.

Linguistic variable	Interval
Strongly disagree	$0 < \bar{p}(s) \leq 0.43$
Disagree	$0.43 < \bar{p}(s) \leq 0.86$
Partially Disagree	$0.86 < \bar{p}(s) \leq 1.29$
Neither agree nor disagree	$1.29 < \bar{p}(s) \leq 1.71$
Partially agree	$1.71 < \bar{p}(s) \leq 2.14$
Agree	$2.14 < \bar{p}(s) \leq 2{,}57$
Strongly agree	$2.57 < \bar{p}(s) \leq 3$

This methodology facilitated the application of a neutral hypothesis test for the average scoring function assigned to each statement in the survey. A neutrosophic hypothesis is a statement about the neutrosophic values of one or several characteristics of a population. The distinction between classical (statistical) hypothesis and neutrosophic hypothesis is that in neutrosophic statistics, the variables describing the characteristics of the population are neutrosophic (i.e., they have some indeterminate values, several unknown values, or an imprecise number of terms if the variable is discrete), or for the compared values at least one of the population's characteristics is neutrosophic (i.e., it has an indeterminate, unclear, or vague value). Similar to classical statistics, a neutrosophic null hypothesis, denoted by NH_0, is the statement initially assumed to be true. The neutrosophic alternative hypothesis, denoted by NH_a, is the other hypothesis. When testing NH_0 versus NH_a, there are two possible outcomes: reject NH_0 (if the sample evidence strongly suggests NH_a is false), or do not reject NH_0 (if the sample does not support strong evidence against it).

As in classical statistics, this study employs the classic standard normal distribution of a random variable z, characterized by having a mean value μ=0 and a standard deviation σ=1. Applying this distribution allows for statistical inferences under conditions of normality, providing a solid basis for the comparison and analysis of data.

In the neutrosophic context, if we consider the null hypothesis about variable x, it could be established in the following way to adapt to the neutrosophic approach:

$$NH_0: \mu_x \in [a, b]$$

where [a, b] is an interval that contains the hypothetical mean value μ_x of variable x. This interval represents a range of possible values for μ_x, reflecting the nature of indeterminacy and uncertainty that characterize neutrosophic statistics. Here, a and b are values that bound the interval within which the true mean value of the variable under study is presumed to lie, allowing a margin of flexibility essential in analyses where absolute precision cannot be guaranteed due to the variability and ambiguity inherent in respondents' answers or the nature of the phenomenon under study. Therefore, the neutrosophic statistical test is:

$$z = \frac{\hat{x} - [a,b]}{s/\sqrt{n}} \quad (3)$$

A Neutrosophic P-value, similar to classical statistics, is defined as the lowest level of significance at which a null hypothesis can be rejected. However, unlike traditional statistics where the P-value is a precise number, the neutrosophic P-value is presented as a set, and in many applications, it is expressed as an interval. This approach allows for a more flexible and adaptive interpretation of data, reflecting the indeterminacy and ambiguity that often characterize real research contexts.

In this specific study, this neutrosophic statistical methodology was used to examine the degree of agreement among experts. The goal was to determine if there was at least partial agreement, using a significance level α_N that ranges between 0.90% and 0.95%. This range reflects the neutral character of the hypothesis testing carried out, highlighting the usefulness of neutrosophic statistics in managing uncertainty and making informed decisions in contexts where data may be incomplete, ambiguous, or contradictory.

By employing neutrosophic tests, it was possible to effectively assess the degree of consensus or dissent among veterinary experts regarding claims about the antimicrobial, healing, efficacy, and safety of propolis in treating bacterial external otitis in canines. This statistical approach allowed for the incorporation and management of the uncertainty and ambiguity inherent in the subjective responses of participants, providing a deeper and more nuanced analysis than with traditional statistical methods.

3 Results

For this study, a group of 62 experts in the specific field of study was selected to participate. After administering the survey to these experts, preliminary statistical data were obtained that provided an initial view of the perceptions and opinions regarding the subject under study. The results obtained are organized and presented in a frequency table, which is essential for initial interpretation and to formulate more detailed analyses later. See Table 3.

Table 3: Frequency table for the responses obtained.

Linguistic variable	Propolis is effective in reducing symptoms	Propolis has significant antimicrobial properties that justify its use in veterinary treatments.	The use of propolis in the treatment of external otitis in canines is safer than traditional antibiotic treatments	Propolis accelerates the healing process in cases of bacterial external otitis in canines, compared to other conventional treatments	The application of propolis in treatments for canines with external otitis is well tolerated by animals, without severe side effects.	I would recommend the use of propolis as an alternative to antibiotics for the treatment of bacterial external otitis in canines.
SD	0	0	0	0	0	0
D	0	0	0	1	0	0
PD	4	6	6	1	0	0
NAD	15	13	3	5	9	15
PA	3	5	23	25	13	8
A	24	25	22	20	16	16
SA	16	13	8	10	24	23

In the table presented, it's evident that a substantial majority of respondents adopt a favorable stance regarding the statements about the properties of propolis in mitigating various aspects of bacterial external otitis in canines. Specifically, categories that reflect some level of disagreement—such as "Disagree" (D), "Partially Disagree" (PD), and "Neither Agree nor Disagree" (NAD)—record the lowest frequencies during the initial phase of the study.

This pattern suggests a generally positive perception of the use of propolis among veterinary experts, indicating

recognition of its potential benefits in treating medical conditions in canines. The scarcity of responses in the disagreement categories might be indicative of an emerging consensus on the efficacy of propolis.

With the data obtained, it is possible to apply Equation 2 to calculate the average scoring functions corresponding to each survey question. This process involves a mathematical aggregation of individual responses to obtain a central measure that reflects the general trend of the data per question. Once calculated, these average scores are categorized according to the intervals specified in Table 2, thus facilitating their systematic interpretation and comparison. See Table 4.

Table 4: Average score function for each questionnaire statement

	Propolis is effective in reducing symptoms	Propolis has significant antimicrobial properties that justify its use in veterinary treatments.	The use of propolis in the treatment of external otitis in canines is safer than traditional antibiotic treatments	Propolis accelerates the healing process in cases of bacterial external otitis in canines, compared to other conventional treatments	The application of propolis in treatments for canines with external otitis is well tolerated by animals, without severe side effects.	I would recommend the use of propolis as an alternative to antibiotics for the treatment of bacterial external otitis in canines.
Average score function	2,249	2,192	2,127	2,169	2,399	2,347

Subsequently, these categorized data are graphed to enhance the understanding and visualization of the information gathered. Graphical representation allows for a more intuitive and direct interpretation of the trends and patterns within the data set, providing a visual perspective that can reveal insights not immediately apparent through numerical analysis alone.

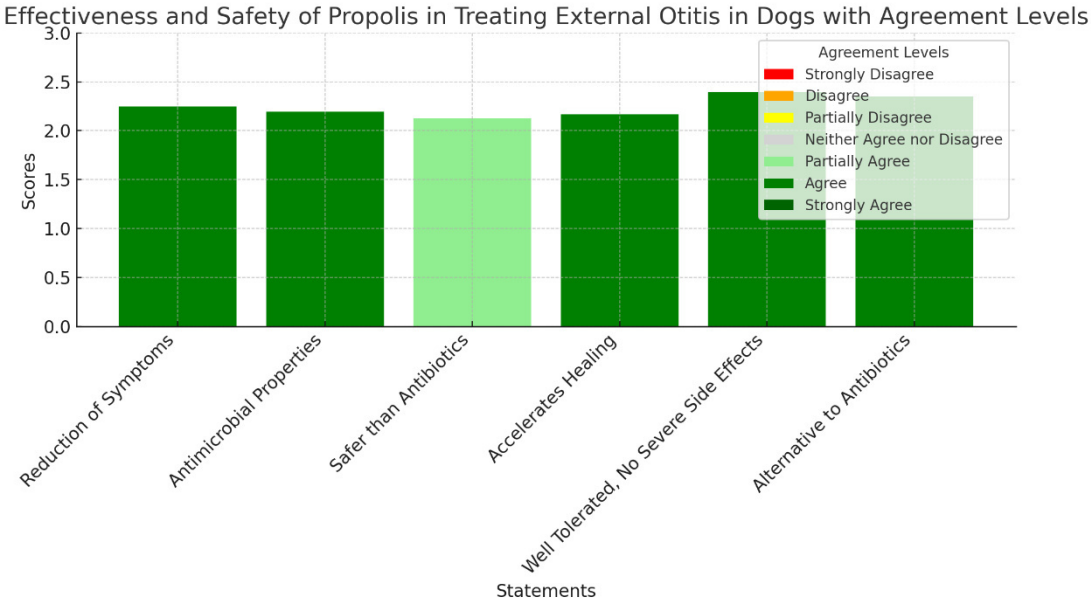

Figure 1: Effectiveness and safety of propolis according to the calculated average scoring functions

In the study conducted the average scoring function values for each statement on the questionnaire were consistently within the acceptance intervals. This indicates that, overall, the consulted experts showed favorable agreement with the proposed assertions about the properties and benefits of propolis in treating external otitis in domestic canines.

Particularly, the third statement, which claimed that "The use of propolis in the treatment of external otitis in canines is safer than traditional antibiotic treatments," scored slightly lower on average compared to the other statements on the questionnaire. While this claim was still generally accepted, the lower score suggests that there

are some reservations or a lesser degree of agreement among the experts regarding the safety of propolis compared to conventional antibiotics.

On the other hand, the rest of the statements received average scores higher than 2.14, which confirms strong concordance among the experts regarding the efficacy and tolerability of propolis. Notably, the statement that "The application of propolis in treatments for canines with external otitis is well tolerated by the animals, without severe side effects," achieved the highest average score of 2.4. This result indicates significant recognition of the tolerability and perceived safety of propolis, critical aspects in the clinical management of veterinary treatments.

The collection and analysis of these data provide valuable evidence supporting the use of propolis as a safe and effective alternative to traditional treatments for external otitis in domestic canines.

To demonstrate the claims made, as well as the overall level of agreement with respect to the study topic, the following neutral hypothesis was proposed:

$NH_0: \mu \in [0, 1.71]$

$NH_1: \mu > 1.71$

In this study, it was estimated that the sample mean corresponds to the minimum and maximum values obtained from the statements of the applied questionnaire. That is, the mean was calculated based on the range of the highest and lowest scores reported for each statement. In this case, the calculated standard deviation was 1.11. This standard deviation value indicates moderate variability in the experts' responses, reflecting differences in the perception and evaluation of propolis among the respondents:

$$z = \frac{[2.127, 2.399] - [0, 1.71]}{1.11/\sqrt{62}}$$

$$z = \frac{[0.417, 2.299]}{0.141}$$

$$z = [2.98, 2.02]$$

In the statistical analysis of the data collected from the questionnaire, a specific criterion was employed for decision-making about the null hypothesis NH_0, based on comparing the critical value with the observed test statistic. According to this criterion, NH_0 should be rejected if the minimum of the critical value exceeds the maximum of the value $\{Z_{1-\alpha N}\}$. In this case, with a significance level given as $\alpha_N = [0.05, 0.1]$, the associated critical values, calculated from standard normal distribution tables or statistical software, are in the interval [1.28, 1.64]. This means that if the test statistic calculated from the data exceeds the highest value in this range, the null hypothesis can be confidently rejected.

For the data obtained in this study, the value of the test statistic was 2.02, which is greater than the critical interval [1.28, 1.64]. Therefore, based on this analysis, NH_0 can be rejected. This implies that, statistically, there is sufficient evidence to assert that the results obtained are significantly different from what the null hypothesis had predicted, and these differences are statistically significant within the established confidence level. This result implies that the claims about the properties and effectiveness of propolis in the treatment of external otitis in canines, as valued by the experts, have statistical backing to be considered valid in clinical practices, thus supporting the integration or increased use of propolis in veterinary treatments.

4 Conclusion

This study focused on collecting and analyzing the perceptions of veterinary experts on the antimicrobial and healing qualities of propolis in managing bacterial otitis externa in domestic canines, supported by evidence derived from both clinical experience and existing scientific literature. The design and implementation of a questionnaire based on the Likert scale, which used linguistic scales, allowed for the systematic capture and evaluation of veterinarians' opinions regarding various statements about the use of propolis. The adoption of neutrosophic statistical techniques to address the uncertainty and ambiguity in respondents' answers enabled the calculation of average scores and the conduct of hypothesis tests aimed at determining the statistical relevance of the findings.

The results revealed that a large majority of experts agree that propolis is effective and safe for the treatment of external otitis in canines, surpassing certain aspects of traditional antibiotic treatments. Additionally, its good tolerability in these treatments was highlighted, demonstrating its acceptance by the animals without inducing serious side effects, an aspect of special importance in clinical settings where treatment safety is a priority.

The study provides encouraging evidence on the viability of propolis in veterinary medicine, supporting its efficacy and safety through quantitative data and expert evaluations. The use of neutrosophic hypotheses has been fundamental in effectively managing the uncertainties and indeterminacies present in expert responses, facilitating a more detailed and deeply informed interpretation of propolis properties. These findings establish a solid foundation for future research and could prompt a revision of current clinical practices in treating bacterial conditions in animals.

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Evaluation of the challenges and opportunities of Artificial Intelligence applied in current Latin American Education with the help of Neutrosophic SWOT and Neutrosophic Cognitive Maps

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Abstract. Artificial intelligence (AI) has revolutionized many areas, including education, by improving teaching methods and addressing challenges such as equity of access, data privacy, quality of algorithms, and adaptation of traditional education systems. This paper aims to analyze and understand the challenges and opportunities of artificial intelligence in Latin American education today. For the proposed evaluation we use the criteria of five experts on the subject. Specifically, we use Neutrosophic SWOT and static analysis of Neutrosophic Cognitive Maps techniques. The use of tools in the field of Neutrosophy allows us to incorporate indeterminacy into the calculations carried out, which allows greater accuracy in the results. Indeterminacy is an intrinsic component of decision-making.

Keywords: Artificial Intelligence, Education, Neutrosophic SWOT, Neutrosophic Cognitive Map, Static Analysis, Neutrosophic Number, Single-Valued Triangular Neutrosophic Number, Neutrosophic Graph.

1 Introduction

Artificial Intelligence (AI) impacts society, education is no exception. In recent decades, the accelerated advancement in AI technologies has allowed the development of tools and systems that promise to revolutionize learning and teaching methods. However, integrating AI into education presents challenges that must be addressed to increase its benefits and reduce risks.

The main challenge of AI is to guarantee equitable access to these technologies in education. Socioeconomic differences between students can be amplified if AI tools are not equitably available, which could exacerbate pre-existing educational gaps. Data must be private and secure because the educational AI system often requires collecting and analyzing extensive personal data from students.

The objective of this article is to evaluate the use of AI in higher education in any Latin American country, determining its benefits and deficiencies for both, students and teachers. The algorithms used in training can make unfair or discriminatory decisions, affecting and generating distrust between educators and students, which is another great challenge.

Adapting traditional educational systems to new AI technologies requires a profound restructuring. Educators need ongoing training to effectively use these tools, and curricula must be updated to include essential digital competencies. This adaptation process must also consider technology and the preservation of human interactions fundamental to learning equitably.

Addressing challenges proactively and thoughtfully mitigates the risks associated with AI in education. The potential to generate more inclusive, personalized, and effective learning environments must also be harnessed.

To carry out the measurement we propose to use a series of decision-making tools adapted to the framework of neutrosophy. According to F. Smarandache, Neutrosophy is the branch of philosophy dedicated to the study of neutralities, which includes the neutral, unknown, incoherent, inconsistent, contradictory, and paradoxical, among

others [1]. That is why with the incorporation of neutrosophic knowledge we can include knowledge of which there is no partial or total certainty because it is indeterminate and therefore the results of the studies carried out are more reliable to reality.

One of these tools is the Neutrosophic SWOT (Strengths, Weaknesses, Opportunities, and Threats) method [2-4]. The application of this technique allows us to quantify the Challenges and Opportunities of applying AI in education in the current era. The second step is the determination of the causal relationship between the identified concepts, in this way, neutrosophic graphs will be designed [5].

A neutrosophic graph is a graph where at least one of the edges is considered indeterminate. That is, it cannot be determined in the case of two vertices if there is an edge that joins them and this is represented graphically with the help of dashed lines.

Finally, these relationships are represented in Neutrosophic Cognitive Maps (NCM) [6-11]. These are neutrosophic graphs whose edges have associated numerical weight values and where indeterminate edges are symbolically represented by I to indicate indeterminacy. A static analysis of the NCM is used for the calculations. The indices associated with this type of Cognitive Map will allow us to determine and measure the importance of each of the factors or concepts determined to be important in this study.

The structure of this paper is a Preliminaries section where the basic notions of Neutrosophy, Neutrosophic SWOT, and Neutrosophic Cognitive Maps theories are explained. The Results section contains the results obtained from the evaluations given by the experts. At the end, the conclusions are given.

2 Preliminaries

2.1 SWOT analysis

SWOT analysis is a methodology for studying the situation of a company or a project, analyzing its internal characteristics (Weaknesses and Strengths) and its external situation (Threats and Opportunities) in a [2-4].

This analysis is carried out in four stages:

- External analysis,
- Internal analysis,
- Preparation of the SWOT matrix,
- Determination of the strategy to use.

The organization depends on the environment around it for its existence. This framework has opportunities and threats for the organization. These are the elements that are assessed in the external analysis. The organization excels in some areas but has room for growth in others. The outcome of these matters relies on the management of the organization, these are the strengths and weaknesses. The organization's growth can be affected positively or negatively by these four factors.

When utilized, opportunities can be beneficial aspects of the environment, aiding in the advancement and prosperity of an organization or project. Threats come from outside and can harm the organization or project. These problems can be a danger to the organization or the project. We need to come up with plans to solve them. Weaknesses within an organization or project are areas that require improvement and are within the control of its fundamental components. Meanwhile, strengths are good components that are the opposite of weaknesses. We should use them and make them even better.

During the SWOT analysis, we look at what our company is good at (strengths) and what we need to work on (weaknesses). This encompasses our assets, employees, merchandise, and public perception. This is why looking inside the organization helps us figure out how much and how good the resources and processes are, and whether the organization can control them.

The four parts of the analysis are put in a chart and looked at by professionals. These results are combined using the percentages of their ratings. During the assessment, we consider the strengths and areas for growth to determine the most effective strategies for supporting the organization or project.

These positive actions can be held back by joining weaknesses with threats. We should always remember this combination because it prevents the organization from moving excessively fast. The organization should consider both the positive and negative potential outcomes and devise a strategic plan for the future. Thus, potential is measured using Strengths + Opportunities, Risk = Strength + Threats, Challenges = Weaknesses + Opportunities, and Limitations = Weaknesses + Threats.

Especially in the Neutrosophic SWOT, Neutrosophy-based assessment tools are used.

2.2 Neutrosophic Cognitive Maps

Definition 1: ([5]) Let X be a universe of discourse. A *Neutrosophic Set* (NS) is characterized by three membership functions, $u_A(x), r_A(x), v_A(x) : X \rightarrow \,]^-0, 1^+[$, which satisfy the condition $^-0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^+$ for all $x \in X$. $u_A(x), r_A(x)$, and $v_A(x)$ denote the

membership functions of truthfulness, indeterminacy, and falseness of x in A, respectively, and their images are standard or non-standard subsets of $]^{-}0,1^{+}[$.

NS is useful only as a philosophical approach, so *a Single-Valued Neutrosophic Set* is defined to guarantee the applicability of neutrosophy, see Definition 2.

Definition 2: ([5]) Let X be a universe of discourse. A *Single-Valued Neutrosophic Set* (SVNS) A on X is an object of the form:

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\} \quad (1)$$

Where $u_A, r_A, v_A : X \rightarrow [0,1]$, satisfy the condition $0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3$ for all $x \in X$. $u_A(x), r_A(x)$, and $v_A(x)$ denote the membership functions of truthfulness, indeterminacy, and falseness of x in A, respectively. For convenience, a *Single-Valued Neutrosophic Number* (SVNN) will be expressed as $A = (a, b, c)$, where $a, b, c \in [0,1]$ and satisfies $0 \leq a + b + c \leq 3$.

Neutrosophic Logic (NL) extends fuzzy logic. A proposition P is characterized by three components; see [5, 12-13]:

$$NL(P) = (T,I,F) \quad (2)$$

Where component T is the degree of truthfulness, F is the degree of falsity and I is the degree of indeterminacy. T, I, and F belong to the interval [0, 1], and they are independent of each other.

A *neutrosophic number* is formed by the algebraic structure a+bI, where I = indeterminacy. In the following, we formally describe some important concepts.

Definition 3: ([14]) Let R be a ring. The *neutrosophic ring* $\langle R \cup I\rangle$ is also a ring, generated by R and I under the operation of R, where I is a neutrosophic element that satisfies the property $I^2 = I$. Given an integer n, then, n+I and nI are neutrosophic elements of $\langle R \cup I\rangle$, and in addition $0 \cdot I = 0$. Also, I^{-1}, the inverse of I is not defined.

E.g., a neutrosophic ring is $\langle \mathbb{Q} \cup I\rangle$ generated by $\mathbb{Q}$, which is the set of rationals.

Some operations using I are $I + I + \ldots + I = nI$.

Definition 4: ([14]) A *neutrosophic number* N is also defined as a number as follows:

$$N = d + I \quad (3)$$

Where d is the *determined part* and I is the *indeterminate part* of N.

Let $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ be two neutrosophic numbers, then some operations between them are:

1. $N_1 + N_2 = a_1 + a_1 + (b_1 + b_2)I$ (Addition),
2. $N_1 - N_2 = a_1 - a_1 + (b_1 - b_2)I$ (Difference),
3. $N_1 \times N_2 = a_1 a_2 + (a_1 b_2 + b_1 a_2 + b_1 b_2)I$ (Product),
4. $\frac{N_1}{N_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$ (Division).

A *neutrosophic matrix* is a matrix whose components are elements of $\langle R \cup I\rangle$.

Thus, it is possible to generalize the operations between vectors and matrices on R to the ring $\langle R \cup I\rangle$.

A *neutrosophic graph* is a graph with at least one neutrosophic edge linking two nodes, that is to say, there is an edge with an indeterminacy on its two nodes connection, [5], see Figure 1.

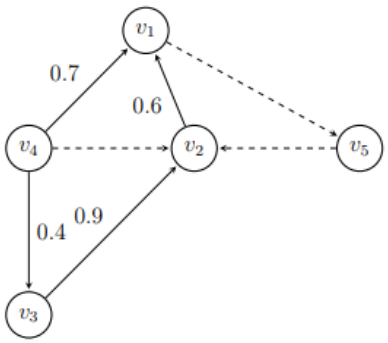

Figure 1: Example of neutrosophic graph. Source [5].

The des-neutrosophication process was introduced by Salmeron and Smarandache [15], which converts a neutrosophic number into one numerical value. This process provides a range of numbers for centrality using as a base the maximum and minimum values of $I = [a_1, a_2] \subseteq [0,1]$, based on Equation 4:

$$\lambda([a_1, a_2]) = \frac{a_1 + a_2}{2} \quad (4)$$

Each node constitutes a causal concept. This characteristic makes the representation flexible to visualize human knowledge. The adjacency matrix is obtained from the values assigned to the arcs.

The values obtained by the group of experts involved in the process are aggregated, conforming to the general knowledge of the relationships between the criteria. Activity results in the NCM [5]. From the assessment of the causal relationships, the static analysis is performed. The knowledge stored in the adjacency matrix is taken as a reference. For the development of the present method, we work with the *indegree (id)* of output as shown by Equation 5, [5].

$$id_i = \sum_{j=1}^{n} |I_{ij}| \tag{5}$$

The *outdegree* (*od*) is calculated by Equation 6, and the total degree (*td*) by Equation 7:

$$od_i = \sum_{j=1}^{n} |I_{ji}| \tag{6}$$

$$td_i = id_i + od_i \tag{7}$$

Definition 5 ([16, 17]): The Single-Valued Triangular Neutrosophic Number, $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set on $\mathbb{R}$, whose truth, indeterminacy, and falsity membership functions are defined as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), a_1 \le x \le a_2 \\ \alpha_{\tilde{a}}, x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), a_2 < x \le a_3 \\ 0, \text{otherwise} \end{cases} \tag{8}$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \le x \le a_2 \\ \beta_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\beta_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \le a_3 \\ 1, \text{otherwise} \end{cases} \tag{9}$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\gamma_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \le x \le a_2 \\ \gamma_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\gamma_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \le a_3 \\ 1, \text{otherwise} \end{cases} \tag{10}$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0,1]$,$a_1,\ a_2, a_3 \in \mathbb{R}$ and $a_1 \le a_2 \le a_3$.

Definition 6 ([16, 17]): Given $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$ two single-valued triangular neutrosophic numbers and λ is any non-null number in the real line. Then, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
2. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
3. Inversion: $\tilde{a}^{-1} = \langle ({a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3 \neq 0$.
4. Multiplication by a scalar number:

$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda < 0 \end{cases}$$

5. Division of two triangular neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

6. Multiplication of two triangular neutrosophic numbers:

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle,\ a_3 > 0 \text{ and } b_3 > 0 \\ \langle (a_1 b_3, a_2 b_2, a_3 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle (a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

Where, ∧ is a t-norm and ∨ is a t-conorm.

3 Results

The factors that influence the use of AI in Latin American classrooms were determined. These are the following:

1. AI develops new skills in students,
2. The use of these technologies must be balanced with other pedagogical practices that are traditional and effective,
3. There is no adequate literacy, teacher training, and government regulations for the use of AI in classrooms,
4. There is no ethical and multidisciplinary approach to teaching with AI,
5. There are no periodic updates to the technology nor it is done for personal and environmental benefit.
6. International collaboration is not encouraged nor there is an investment in AI talent,
7. There are flaws in the privacy and security of personal data,
8. AI is truly capable of transforming the teaching and learning process in education, through sophisticated studies, personalized tutoring, chatbots, and other tools,
9. AI in education causes benefits of adaptive courses on student performance and motivation,
10. AI improves educational quality and reduces teachers' workload,
11. AI is used in assessments to improve performance prediction, the use of educational robots, and the transparency of assessments,
12. Technology meets the objective of optimizing teaching, personalizing learning, conducting effective evaluations, and improving educational administration,
13. AI generates new knowledge,
14. There is inequality in capturing the benefits of these tools, due to the social and economic inequality of students,
15. To implement these technologies, high-quality data is needed and there are difficulties in customizing the algorithms,
16. There is the problem of interpretation and bias in the results that AI gives,
17. Teachers do not have computer tools, cutting-edge technology, and training in educational institutions, which makes it difficult to use AI as a teaching strategy in classrooms.
18. Educators must be continuously trained in the use of these tools.
19. Human interaction can be neglected when these technologies are used continuously.

The previous aspects were classified into Strengths, Weaknesses, Opportunities, and Threats as indicated below according to the number with which we have listed them previously:

Strengths: 1, 2;
Weaknesses: 3, 4, 5, 6, 7;
Opportunities: 8, 9, 10, 11, 12, 13;
Threats: 14, 15, 16, 17, 18, 19.

Five experts were selected to carry out the evaluations according to the scale that appears in Table 1.

Table 1. Linguistic terms for evaluations and their associated SVTNNs. See [18].

Linguistic Terms	SVTNN
Very low (VL)	$\langle(0,0,1);0.00,1.00,1.00\rangle$
Low (L)	$\langle(0,1,3);0.17,0.85,0.83\rangle$
Medium Low (MDL)	$\langle(1,3,5);0.33,0.75,0.67\rangle$
Medium (M)	$\langle(3,5,7);0.50,0.50,0.50\rangle$
Medium High (MDH)	$\langle(5,7,9);0.67,0.25,0.33\rangle$
High (H)	$\langle(7,9,10);0.83,0.15,0.17\rangle$
Very High (VH)	$\langle(9,10,10);0.00,1.00,1.00\rangle$

To reach a conjoint evaluation of all the experts, they were asked to agree on an evaluation for each of the possible combinations between an internal factor and an external one. This implied the need for deliberation among the specialists, who in the end arrived at the requested results.

Table 2. SWOT neutrosophic matrix obtained from the criteria of the five specialists.

		Strengths		**Weaknesses**				
		S_1	**S_2**	**W_1**	**W_2**	**W_3**	**W_4**	**W_5**
Opportunities	**O_1**	VH	MDH	M	MDH	MDH	MDH	MDH
	O_2	VH	M	M	MDH	MDH	MDH	MDH
	O_3	H	M	MDH	L	H	M	M
	O_4	VH	M	M	H	H	MDH	MDL
	O_5	VH	M	M	H	MDH	M	MDL
	O_6	H	M	M	L	M	MDH	MDH
Threats	**T_1**	M	MDH	VL	L	MDL	L	L
	T_2	MDL	MDL	L	H	MDL	L	L
	T_3	L	VH	L	L	L	M	L
	T_4	L	VH	L	H	L	L	L
	T_5	MDH	MDH	L	MDL	MDL	L	L
	T_6	L	VH	H	VL	L	MDH	M

The data in Table 2 are processed for each quadrant by finding the average of the results of the quadrant using the algebra between SVTNNs, the results are summarized in Table 3.

Table 3. Aggregation of the results of Table 2 for each quadrant using the average of the SVTNNs.

	Strengths	**Weaknesses**
Opportunities	$\langle(5.8333, 7.5, 8.6667); 0.67, 0.25, 0.33\rangle$	$\langle(4.0667, 6.0000, 7.8667); 0.17, 0.85, 0.83\rangle$
Threats	$\langle(3.9167, 5.4167, 6.9167); 0.17, 0.85, 0.83\rangle$	$\langle(1.2, 2.4667, 4.3); 0.00, 1.00, 1.00\rangle$

To have a crisp score value between 0 and 10 for each of the quadrants, we have the following accuracy function equation:

$$A(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \quad (11)$$

So, we have:
For the SW quadrant: 7.5625,
ST Quadrant: 4.3672,
WO Quadrant: 4.8196,
WT: 1.9917.

From here it can be concluded that the potential = SO is 7.5625 on a scale of 10.
However, risk and challenge are rated at 4.3672 and 4.8196, respectively, which is less than average.
While the evaluation of the limitations is low, equal to 1.9917. This means that there are major limitations.
Now we will study the problem from the perspective of the dynamics between the variables, using the Neutrosophic Cognitive Maps through a static analysis.
We will represent this using the adjacency matrix that appears in Tables 4-5.

Table 4. Neutrosophic Cognitive Map Matrix according to experts.

Variable	1	2	3	4	5	6	7	8	9	10
1	0	I	I	0	0	0.1	0.1	0.9	0.9	0.7
2	I	0	I	0.9	0	0	0	0.6	0	0.5
3	I	I	0	0.8	I	1	0	0.1	0	0.9
4	0	0.8	0.7	0	0	0.7	0	0.7	0.5	0.9
5	0	0	I	0	0	I	1	0.1	0.1	0.2
6	0.1	0	1	0.6	I	0	0.6	0.8	0.6	0.7
7	0.1	0	0	0	1	0.7	0	0.6	0.4	0.6
8	0.9	0.7	0.2	0.8	0.1	0.7	0.5	0	1	1
9	0.9	0	0	0.5	0.1	0.7	0.4	1	0	1
10	0.7	0.5	0.8	0.9	0.2	0.7	0.7	1	1	0
11	0.9	0.6	0.5	0.9	0.8	0.4	0.9	1	1	1
12	0.9	0.5	0.4	0.6	0.4	0.4	0.6	1	1	1
13	0.9	0.7	0.6	0.6	0.6	0.8	0	0.8	1	1
14	0.9	0.5	0.7	0	1	0.6	0	0.8	0	0
15	0.4	0.6	0	0.2	0.6	0.7	1	0.6	0.7	0.8
16	0.2	0	0.6	0	0.8	0.7	1	0.9	0.7	0.9
17	0.3	0.3	1	0	0.6	1	1	0.9	0.8	0.8
18	0.4	0	1	1	0.8	1	1	0.9	0.9	0.8
19	0	0	0.7	1	0	0	I	0	0	I

Table 5. Neutrosophic Cognitive Map Matrix according to experts (Continued).

Variable	11	12	13	14	15	16	17	18	19
1	0.9	0.9	0.9	0.9	0.5	0.1	0.4	0.4	0
2	0.6	0.5	0.6	0.7	0.5	0.7	0	0.3	0
3	0.6	0.3	0.7	0.6	0	0.7	1	1	0.8
4	0.9	0.7	0.6	0	0.3	0	0	1	1
5	0.8	0.4	0.6	1	0.6	0.8	0.6	0.8	0
6	0.4	0.3	0.9	0.7	0.8	0.8	1	1	0
7	0.9	0.6	0	0	1	1	1	1	I
8	1	1	0.9	0.7	0.6	0.9	0.9	0.9	0
9	1	1	1	0	0.7	0.8	0.9	0.9	0
10	1	1	1	0	0.9	0.9	0.7	0.7	I
11	0	1	1	I	1	0.8	0.8	0.9	0.9
12	1	0	1	0.9	0.8	1	0.6	0.9	0
13	1	1	0	0.8	0.8	1	0.7	0.8	0
14	I	0.9	0.8	0	0	1	0	I	0
15	1	0.9	0.8	0	0	0.8	1	1	0
16	0.7	1	1	1	0.8	0	0	0	0
17	0.8	0.6	0.7	0	1	0	0	1	0
18	0.9	0.8	0.7	I	1	0	1	0	0
19	0.9	0	0	0	0	0	0	0	0

The results presented in the two previous tables correspond to the consensus of the five experts in evaluating the strength of the relationship between two variables or concepts on a scale between 0 and 1. Furthermore, it is possible to associate a weight symbolically represented as I when the relationship between the concepts is not known or there is a disagreement between the evaluations given by the experts.

To help the specialists, they were asked to evaluate on a scale from 0 to 10 or I, and then the numerical values obtained were divided by 10.

Table 6 summarizes the results of the static analysis of the variables.

Table 6. Indegree, Outdegree, Total Degree, and Total Degree des-neutrosophied were calculated from the studied variables.

Variable	Indegree	Outdegree	Total Degree	Total Degree des-neutrosophied
1	7.7 + 2I	7.6 + 2I	15.3 + 4I	17.3
2	5.9 + 2I	5.2 + 2I	11.1 + 4I	13.1
3	8.5 + 3I	8.2 + 3I	16.7 + 6I	19.7
4	8.8	8.8	17.6	17.6
5	7 + 2I	7 + 2I	14 + 4I	16
6	10.3 + I	10.2 + I	20.5 + 2I	21.5
7	8.9 + I	8.8 + I	17.7 + 2I	18.7
8	12.8	12.7	25.5	25.5
9	10.9	10.6	21.5	21.5
10	12.7 + I	12.8 + I	25.5 + 2I	26.5
11	14.4 + I	14.4 + I	28.8 + 2I	29.8
12	13	12.9	25.9	25.9
13	13.1	13.2	26.3	26.3
14	7.2 + 2I	7.3 + 2I	14.5 + 4I	16.5
15	11.1	11.3	22.4	22.4
16	10.3	11.3	21.6	21.6
17	10.8	10.6	21.4	21.4
18	12.2 + I	12.6 + I	24.8 + 2I	25.8
19	2.6 + 2I	2.7 + 2I	5.3 + 4I	7.3

Table 6 shows that the order of influence of the variables studied can be summarized as follows according to the Total Degree des-neutrosophied:

$v_{19} \prec v_2 \prec v_5 \prec v_{14} \prec v_1 \prec v_4 \prec v_7 \prec v_3 \prec v_{17} \prec v_6 \approx v_9 \prec v_{16} \prec v_{15} \prec v_8 \prec v_{18} \prec v_{12} \prec v_{13} \prec v_{10} \prec v_{11}$.

4 Conclusion

The use of Artificial Intelligence in daily life is a fact that cannot be denied. This has advantages and disadvantages. However, the advantages can be profited in its application in teaching. The disadvantages are a challenge that must be dealt with. This article studies the pros and cons of using these technologies in a Latin American country today. To do this, we use neutrosophic tools that allow us to include indeterminacy within the study, and this makes the study more complete. Specifically, we used the evaluation of five experts for the use of two techniques, the Neutrosophic SWOT and the static analysis of the Neutrosophic Cognitive Maps. Nineteen variables were determined to be important. It was concluded that the potential is promising. However, the risks and challenges are both medium and the limitations that exist are high. On the other hand, the most important variables to take into account according to the Neutrosophic Cognitive Map are the potential that includes the improvement of

the quality of education, the generation of new knowledge, and the support it provides to teachers for teaching. Therefore, it is estimated that there is a positive balance in the potential offered by its application concerning the limitations, risks, and challenges.

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Using Plithogenic Statistics to Determine Optimal Guinea Pig Manure Dosing in Prickly Pear Cultivation

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Abstract: This study applied plithogenic statistics to evaluate how different doses of guinea pig manure affect the cultivation of Opuntia ficus-indica (commonly known as prickly pear) and soil quality. It was determined that 7.5 tons/ha, referred to as treatment T3, is the optimal dose, as it maximizes the growth and production of the plant, improves plant height, and increases the production of buttons and berries. T3 also optimized organic matter and adjusted the pH to ideal levels, maintaining a nutritional balance that ensures the availability of nitrogen, phosphorus, and potassium. This balance enhances agricultural production without sacrificing soil sustainability. In conclusion, the research showed that T3 provides the best performance, improving soil quality without the risks associated with higher doses. This underscores the need to carefully calibrate organic fertilizers to maximize efficiency and minimize negative environmental impacts.

Keywords: Plithogenic statistics, sustainable development, agricultural production, prickly pear cultivation.

1 Introduction

Opuntia ficus-indica, commonly known as prickly pear [1], is a plant species of great relevance for rural economies and arid areas [2]. It is valued for its ability to adapt to adverse conditions such as low rainfall soils and droughts, where other species might not survive. This adaptability makes it a viable option for production in areas with limited water and soil resources [3].

In the context of organic farming, which emerges as a sustainable alternative to conventional agriculture, organic fertilizers play a crucial role [4]. These fertilizers enhance the fertility of the soil and improve its physical, chemical, and biological characteristics, thereby increasing crop yields and quality. Soil, a non-renewable natural resource on a human timescale, is essential for the sustainability of terrestrial ecosystems.

Manure, as a principal source of organic fertilizer, is vital for providing nutrients to plants and enhancing soil properties. In shallow soils, it is recommended to use about 5 tons/ha of cattle manure, or 8 kg per plant, applied at the onset of the rainy season and distributed around the plant. General manure application levels vary from 10 to 20 tons per hectare, with 100 to 200 kg of nitrogen and 80 to 100 kg of phosphorus.

Guinea pig manure, rich in nitrogen, phosphorus, and potassium, is noted for significantly improving the physical, chemical, and biological characteristics of the soil, resulting in healthy and high-yielding crops. This study focused on evaluating different doses of guinea pig manure, referred to as cuyasa, in the cultivation of prickly pear. The aim is to determine the optimal dose for crop adaptation in the eroded soils of the Salache campus and the differences in soil properties with their application. It was hypothesized that at least one dose of guinea pig manure would have a positive impact on the adaptation of prickly pear [5, 6].

2 Materials and methods

2.1 Plithogenic statistics.

Plithogenic statistics are used to address and model complex data where the interactions between multiple factors and their contradictions are crucial for understanding the phenomena being studied [7]. In the case of the effect of guinea pig manure on the cultivation of prickly pear, plithogenic statistics allow for a detailed analysis of how different doses of guinea pig manure affect multiple aspects of the soil and plant simultaneously. To do this, the dynamics of Plithogeny must first be defined [8].

Plithogeny is the dynamics of various types of opposites, and/or their neutrals, and/or non-opposites and their organic fusion. Plithogeny is a generalization of dialectics (dynamics of one type of opposites: <A> and <antiA>), neutrosophy (dynamics of one type of opposites and their neutrals: <A>, <antiA>, and <neutA>), as Plithogeny studies the dynamics of many types of opposites and their neutrals and non-opposites (<A>, <antiA>, <neutA>, <B>, <antiB>, <neutB>, etc.), and many non-opposites (<C>, <D>, etc.) altogether. As an application and specific

case derived from Plithogeny, a plithogenic set is an extension of the classical set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set [9], and has multiple scientific applications [10].

Thus, it is called a plithogenic set (P, a, V, d, c):

- Where "P" is a set, "a" is an attribute (generally multidimensional), "V" is the range of values of the attribute, "d" is the degree of membership of the attribute value of each element x in the set P according to some given criteria ($x \in P$), and "d" signifies "d_F", " d_{IF} ", or "d_N", when it is a degree of fuzzy membership, an intuitionistic fuzzy membership, or a neutrosophic membership, respectively, of an element x to the plithogenic set P;
- "c" means "c_F", "c_{IF}", or "c_N", when it is a function of the degree of contradiction of a fuzzy attribute value, an intuitionistic fuzzy attribute value, or a neutrosophic attribute value, respectively. [11].

The functions are defined according to the applications that experts need to address. $d(\cdot,\cdot)$ and $c(\cdot,\cdot)$ are then used, adopting the following notation: $x(d(x, V))$, where $d(x, V) = \{d(x, v) \text{ for all } v \in V\}, \forall x \in P$.

So, plithogenic statistical analysis allows for addressing the complexity of the perceptions of the analyzed sample. This requires a linguistic evaluation system adapted to the plithogenic model to accurately capture the opinions of experts. Consequently, the dataset is evaluated, which is formed wholly or partly by data with some degree of indeterminacy and contradiction. For this, the plithogenic statistical method is used, which allows interpreting and organizing plithogenic data (data that can be found in the generalization of dialectics) to reveal the underlying patterns [12].

For the plithogenic statistical modeling in this study, reference is made to a random variable P, which represents the lower and upper level respectively that the studied plithogenic variable can reach, within an indeterminate and contradictory interval. Thus, it follows the plithogenic mean of the variable ($\overline{P}$) when formulating:

$$\overline{P} = \frac{1}{n_P} \sum_{i=1}^{n_P} P_i \tag{1}$$

Where n_P is a plithogenic random sample from the studied population. Once the plithogenic mean is defined, the next step is to calculate the variance of the plithogenic sample. To do this, it is necessary to convert a plithogenic number to a scalar number according to the methodology analyzed in the study materials. Subsequently, the following equation is defined to calculate S_P^2:

$$S_P^2 = \frac{\sum_{i=1}^{n_P} (P_i - \bar{P}_i)^2}{n_P} \tag{2}$$

Subsequently, the calculation proceeds for the plithogenic coefficient (CV_P) which measures the consistency of the variable. The smaller the value of CV_P, the more consistent the performance of the analyzed element compared to others studied. For this, the following equation is proposed:

$$CV_P = \sqrt{S_P^2} \times 100 \tag{3}$$

2.2 Plithogenic scales for measuring the elements that most influence the development of the prickly pear.

To model plithogenic statistics, it is necessary to define scales for each element to be evaluated, as well as the behavior of the prickly pear crop over 8 months. First, a plithogenic scale is defined for each analyzed variable [13,14], and then, in the development of plithogenic statistics, these terms are applied to the behavior of the prickly pear crop (see Tables 1 to 8).

Table 1: Plithogenic scales for plant height

Code	Plithogenic linguistic expression	Plithogenic number (T, I, F, C)	Variable range	Description
H0	Very Low	(0.10, 0.85, 0.05, 0.20)	0-10cm	Insufficient growth for the month.
H1	Low	(0.20, 0.75, 0.05, 0.15)	10-20cm	Low growth, improvable.
H2	Moderate	(0.40, 0.55, 0.05, 0.10)	20-30cm	Moderate growth, in process.
H3	High	(0.65, 0.30, 0.05, 0.05)	30-40cm	Good growth, almost ideal.
H4	Very high	(0.85, 0.10, 0.05, 0.02)	>40cm	Excellent growth, ideal for the month.

Table 2: Plithogenic scales for the number of buds.

Code	Plithogenic linguistic expression	Plithogenic number (T, I, F, C)	Variable range	Description
B0	Very Low	(0.05, 0.90, 0.05, 0.20)	0-0.5	Insufficient bud development.
B1	Low	(0.15, 0.80, 0.05, 0.15)	0.5-1.0	Few buds, under development.
B2	Moderate	(0.30, 0.65, 0.05, 0.10)	1.0-1.5	Moderate bud development.
B3	High	(0.55, 0.40, 0.05, 0.05)	1.5-2.0	Good number of buds, almost ideal.
B4	Very high	(0.75, 0.20, 0.05, 0.02)	>2.0	Excellent number of buds, ideal.

Table 3: Plithogenic scales for berry production.

Code	Plithogenic linguistic expression	Plithogenic number (T, I, F, C)	Variable range	Description
Y0	Very Low	(0.05, 0.90, 0.05, 0.20)	0-0.5	Insufficient bud development.
Y1	Low	(0.15, 0.80, 0.05, 0.15)	0.5-1.0	Few buds, under development.
Y2	Moderate	(0.30, 0.65, 0.05, 0.10)	1.0-1.5	Moderate bud development.
Y3	High	(0.55, 0.40, 0.05, 0.05)	1.5-2.0	Good number of buds, almost ideal.
Y4	Very high	(0.75, 0.20, 0.05, 0.02)	>2.0	Excellent number of buds, ideal.

Table 4: Plithogenic scales for soil pH.

Code	Plithogenic linguistic expression	Plithogenic number (T, I, F, C)	Variable range	Description
P0	Very high	(0.05, 0.10, 0.85, 0.20)	>9.0	pH too alkaline, adverse for prickly pear.
P1	High	(0.20, 0.15, 0.65, 0.15)	8.5-9.0	High pH, needs correction.
P2	Moderate	(0.40, 0.20, 0.40, 0.10)	8.0-8.5	Moderate, acceptable pH.
P3	Optimum	(0.70, 0.05, 0.25, 0.05)	7.5-8.0	Optimal pH for prickly pear.
P4	Very Optimal	(0.90, 0.05, 0.05, 0.02)	<7.5	Ideal pH, perfect for prickly pear.

Table 5: Plithogenic scales for organic matter.

Code	Plithogenic linguistic expression	Plithogenic number (T, I, F, C)	Variable range	Description
M0	Very Low	(0.10, 0.85, 0.05, 0.25)	<0.8	Very little organic matter, deficient.
M1	Low	(0.25, 0.70, 0.05, 0.20)	0.8-1.2	Low organic matter, improvable.
M2	Moderate	(0.45, 0.50, 0.05, 0.15)	1.2-1.6	Moderate organic matter, good.
M3	High	(0.70, 0.25, 0.05, 0.10)	1.6-2.0	High organic matter, very good.
M4	Very high	(0.85, 0.10, 0.05, 0.05)	>2.0	Excellent organic matter, ideal.

Table 6: Plithogenic scales for nitrogen.

Code	Plithogenic linguistic expression	Plithogenic number (T, I, F, C)	Variable range	Description
N0	Very Low	(0.10, 0.85, 0.05, 0.25)	<40	Severe nitrogen deficiency.
N1	Low	(0.25, 0.70, 0.05, 0.20)	40-50	Low nitrogen, improve.
N2	Moderate	(0.45, 0.50, 0.05, 0.15)	50-60	Moderate nitrogen, acceptable.
N3	High	(0.70, 0.25, 0.05, 0.10)	60-70	High nitrogen, good.
N4	Very high	(0.85, 0.10, 0.05, 0.05)	>70	Excellent level of nitrogen, ideal.

Table 7: Plithogenic scales for phosphorus.

Code	Plithogenic linguistic expression	Plithogenic number (T, I, F, C)	Variable range	Description
F0	Very Low	(0.10, 0.85, 0.05, 0.25)	<40	Very low phosphorus, adverse for Prickly Pear.
F1	Low	(0.25, 0.70, 0.05, 0.20)	40-50	Low phosphorus, improve.
F2	Moderate	(0.45, 0.50, 0.05, 0.15)	50-60	Moderate phosphorus, acceptable.
F3	High	(0.70, 0.25, 0.05, 0.10)	60-70	High phosphorus, good.
F4	Very high	(0.85, 0.10, 0.05, 0.05)	>70	Excellent level of nitrogen, ideal.

Table 8: Plithogenic scales for potassium.

Code	Plithogenic linguistic expression	Plithogenic number (T, I, F, C)	Variable range	Description
K0	Very Low	(0.10, 0.85, 0.05, 0.25)	<2.0	Very low potassium, deficient.
K1	Low	(0.25, 0.70, 0.05, 0.20)	2.0-2.5	Low potassium, improve.
K2	Moderate	(0.45, 0.50, 0.05, 0.15)	2.5-3.0	Moderate potassium, acceptable.
K3	High	(0.70, 0.25, 0.05, 0.10)	3.0-3.5	High potassium, good.
K4	Very high	(0.85, 0.10, 0.05, 0.05)	>3.5	Excellent level of potassium, ideal.

This plithogenic approach provides a comprehensive and detailed view of how different elements interact and contributes to improving soil properties in prickly pear crops treated with guinea pig manure. These scales help to evaluate and determine more informed and efficient agronomic decisions. Thus, it optimizes the selection and application of treatments for the sustainable management of agricultural resources.

3 Results

3.1 Case study.

The research was conducted in Salache, Eloy Alfaro parish, Latacunga canton, Cotopaxi province, where sandy loam and alkaline soils predominate, which restrict the cultivation of various species of agricultural and economic interest. This area is located at coordinates X: 764249, Y: 9889461, at an altitude of 2734 meters, with an average annual temperature of 14°C. The materials used included prickly pear seedlings and guinea pig manure. The methodological approach was inductive experimental, based on field observations and practices. Soil sampling was carried out at the beginning and the end of the first prickly pear cultivation cycle, with bimonthly applications of guinea pig manure.

The study included five guinea pig manure dosage treatments: T0 (0 tons/ha), T1 (2.5 tons/ha), T2 (5 tons/ha), T3 (7.5 tons/ha), and T4 (10 tons/ha). The bank terraces used had average dimensions of 25 meters in length by 2.5 meters in width, while the experimental units were 4 meters long by 2.5 meters wide, with a 1-meter separation between units to facilitate the development of the experiment.

The statistical design adopted was the completely randomized block design (CRBD), with five treatments and four replications, resulting in a total of 20 experimental units. This setup effectively evaluated the influence of different doses of guinea pig manure on the development of prickly pear cultivation under the specific conditions of Salache.

The research conducted on the Salache Campus, Latacunga, demonstrated significant changes in soil chemical properties after applying various doses of guinea pig manure in prickly pear cultivation. Soil chemical analyses showed a decrease in pH and an increase in organic matter, indicating an improvement in soil quality for agriculture in this area with sandy loam and alkaline soils (see Table 9).

Table 9: Summary of initial and final soil chemical analysis with prickly pear cultivation.

Treatment	Initial pH	Final pH	Initial MO (%)	Final MO (%)	Initial N (ppm)	Final N (ppm)	Initial P (ppm)	Final P (ppm)	Initial K (meq/100gr)	Final K (meq/100gr)
Witness (T0)	10.18	10.00	0.60	1.00	52	17	48	58	3.92	1.8
T1 (2.5 Ton/ha)	10.18	9.93	0.60	1.30	52	27	48	45	3.92	2.7
T2 (5 Ton/ha)	10.18	9.61	0.60	1.20	52	26	48	93	3.92	3.0
T3 (7.5 Ton/ha)	10.18	9.93	0.60	1.30	52	20	48	74	3.92	2.8
T4 (10 Ton/ha)	10.18	9.80	0.60	1.40	52	23	48	100	3.92	3.5

Observations from the initial and final soil chemical analysis with prickly pear cultivation:

- Soil pH: All treatments showed a decrease in pH, with T2 (5 Ton/ha) being the most effective in reducing soil alkalinity.
- Organic Matter Percentage (% OM): Increased in all treatments, with the greatest increase in T4 (10 Ton/ha).
- Nitrogen (N): There was a decrease in all treatments, with the smallest decrease in T1 (2.5 Ton/ha).
- Phosphorus (P): Significantly increased in treatments T2, T3, and T4, indicating an enrichment of this essential nutrient.
- Potassium (K): Decreased in all treatments, though all maintained levels were considered high.

3.2 Plithogenic statistics.

3.2.1 Analysis and Interpretation.

The implementation of plithogenic statistics allows for a deeper and more holistic analysis of the data collected in the research on the effect of guinea pig manure on prickly pear cultivation. This approach is particularly useful for managing the indeterminacy and contradiction inherent in agricultural processes and in measuring the effectiveness of treatments under varied soil conditions, as evidenced in Tables 10 and 11.

Table 10: Evaluation of the behavior of the variables that affect the cultivation of Prickly Pear

Month	Plant Height	Number of Buttons	Berry Production	Soil pH	Organic material	Nitrogen	Match	Potassium
1	H0 (Very Low)	B0 (Very Low)	Y0 (Very Low)	P0 (Very High)	M0 (Very Low)	N1 (Low)	F1 (Low)	K4 (Very High)
2	H1 (Low)	B0 (Very Low)	Y0 (Very Low)	P1 (High)	M1 (Low)	N1 (Low)	F1 (Low)	K3 (High)
3	H1 (Low)	B1 (Low)	Y0 (Very Low)	P2 (Moderate)	M1 (Low)	N2 (Moderate)	F2 (Moderate)	K3 (High)
4	H2 (Moderate)	B1 (Low)	Y1 (Low)	P2 (Moderate)	M2 (Moderate)	N2 (Moderate)	F2 (Moderate)	K2 (Moderate)
5	H2 (Moderate)	B2 (Moderate)	Y1 (Low)	P3 (Optimal)	M2 (Moderate)	N2 (Moderate)	F2 (Moderate)	K2 (Moderate)
6	H3 (High)	B3 (High)	Y2 (Moderate)	P3 (Optimal)	M3 (High)	N3 (High)	F3 (High)	K1 (Low)
7	H3 (High)	B3 (High)	Y2 (Moderate)	P3 (Optimal)	M3 (High)	N3 (High)	F3 (High)	K1 (Low)
8	H4 (Very High)	B4 (Very High)	Y3 (High)	P4 (Very Optimal)	M4 (Very High)	N4 (Very High)	F4 (Very High)	K0 (Very Low)

Table 11: Obtaining $\bar{x}_P$, S_P, and CV_P in the growth of the prickly pear crop

Variable	$\bar{x}_P$	SP	CVP
Plant Height	0.500	0.0600	0.2449
Number of buds	0.450	0.0775	0.2784
Berry Production	0.325	0.0444	0.2107
Soil pH	0.450	0.0575	0.2398
Organic material	0.500	0.0600	0.2449
Nitrogen	0.550	0.0375	0.1936
Phosphorus	0.550	0.0375	0.1936
Potassium	0.500	0.0600	0.2449

To interpret the statistical results obtained for the prickly pear crop, it is important to understand how $\bar{x}_P$, S_P and CV_P reflect the behavior of each analyzed variable. Here is the interpretation for each of these metrics:

Plant Height

- Average (0.5): This value indicates that, on average, the height of prickly pear plants over the observed 8 months is moderate, according to our plithogenic scale. This means that, in general, the plants have reached an intermediate height (between Low and High in linguistic terms), suggesting steady and stable growth during the study period.
- Variance (0.060): The variance indicates the dispersion of the plant heights around their mean. A value of 0.060 shows that the heights are relatively close to the average, indicating no extreme variability in plant growth over the different months. The plants have maintained uniform growth with little fluctuation in height.
- Standard Deviation (0.244): The standard deviation, being the square root of the variance, provides a measure of dispersion in the same units as the mean. A value of 0.244 means that most plant heights tend to be within 0.244 units of the average on our scale, underscoring a consistency in growth.

Number of Buds

- Average (0.45): This average indicates that, generally, the number of buds on prickly pear plants is moderate, close to the middle value between Low and Moderate. This suggests that while the plants grow in height, they also produce a reasonable number of buds, though not in maximum quantities.
- Variance (0.0775): This variance shows that there is moderate dispersion in the number of buds over the months. This indicates that in some months, the number of buds has varied more significantly than in others, possibly reflecting different phases of floral development or environmental impacts.
- Standard Deviation (0.278): A standard deviation of 0.278 shows that bud count values are mostly within this distance from the average, indicating moderate variability in bud production month-to-month.

Berry Production

- Average (0.325): This average indicates that berry production is generally low, close to the midpoint between Very Low and Moderate. This suggests that although there is berry production, it does not reach an optimal or highly productive level during the observed period.
- Variance (0.0444): The variance here shows a dispersion similar to the number of buds, indicating that berry production also has month-to-month variability but without significant extremes.
- Standard Deviation (0.210): The standard deviation indicates that most data on berry production are within 0.210 units of the average, showing variability but within a controlled range.

Soil pH

- Average (0.45): An average of 0.45 indicates a moderately high soil pH on the plithogenic scale. This suggests that the pH tends to be at a level that may be suitable or slightly adverse for some plants but has been stable.
- Variance (0.0575): The variance in soil pH is moderate, indicating that there have been fluctuations in pH over time, but these have not been extremely wide.
- Standard Deviation (0.239): This indicates that most soil pH values are within 0.239 units of the average, showing some consistency but with the capacity to vary within a moderate range.

Organic Matter

- Average (0.6): This value indicates that, on average, the organic matter content in the soil is between moderate and high. This is beneficial as a high amount of organic matter is crucial for fertile and healthy soil that supports plant growth.
- Variance (0.0686): With a variance similar to plant height, organic matter shows that its amount in the soil has been quite uniform, without significant variations that might indicate problems of

degradation or excessive enrichment.

- Standard Deviation (0.244): This reflects that most measurements of organic matter are close to the average, indicating stability in soil quality in terms of organic content.

These results provide a comprehensive and detailed view of the performance and conditions of the prickly pear crop, highlighting the importance of maintaining nutritional balances and stable growth conditions to optimize production.

3.2.2. Comparative analysis regarding the treatments applied.

Plant Height: The height of the prickly pear plant is a fundamental indicator of its growth and development. Observing the calculated averages:

- T0 (no fertilizer): Plants show minimal growth, reflected in a lower average height value (0.1 on the plithogenic scale). This indicates that without fertilizer, plants struggle to reach optimal development.
- T1 (2.5 tons/ha): With a low dose of guinea pig manure, there is a slight increase in height (0.3), showing that fertilization begins to have a positive effect.
- T2 (5 tons/ha): Increasing the fertilizer dose to 5 tons/ha significantly improves plant growth (0.5), indicating a favorable response to the increase in nutrients.
- T3 (7.5 tons/ha): This treatment shows an even greater height (0.7), suggesting that this dose might be close to optimal for maximum vertical development of the prickly pear.
- T4 (10 tons/ha): Here, the greatest growth in height is observed (0.9), which could indicate that the plant is reaching its maximum growth potential given the nutrient increase.

Number of Buds and Berry Production: These parameters are crucial for assessing the reproductive and productive capacity of the crop.

- T0 (no fertilizer): The production of buds and berries is minimal (0.1), reflecting the low fertility and productivity without the use of fertilizer.
- T1 (2.5 tons/ha): There is a slight improvement in the production of buds and berries (0.3), showing that a little fertilizer helps, but is not enough for ideal production.
- T2 (5 tons/ha): With this dose, there is a moderate improvement (0.5) in both parameters, indicating that increasing the fertilizer improves fruiting and flowering.
- T3 (7.5 tons/ha): A good increase in the production of buds (0.7) and berries (0.5) is registered, suggesting that this could be a more suitable dose to promote greater production.
- T4 (10 tons/ha): This treatment shows the maximum production of buds (0.9) and high production of berries (0.7), indicating that increasing the dose of fertilizer enhances the reproductive capacity of the crop.

Soil Quality: pH, Organic Matter, Nitrogen, Phosphorus, and Potassium: Analyzing these variables is essential to understanding the quality of the land and its capacity to support the cultivation of prickly pear.

- Soil pH:
  Higher in T0: A high pH (0.9) in the treatment without fertilizer indicates less favorable conditions for prickly pear, which prefers a lower pH.
  Optimization with fertilizer: The application of guinea pig manure reduces the pH to more optimal levels (0.1-0.3 in T3 and T4), which is beneficial for the crop.
- Organic Matter:
  Increase with fertilizer: Organic matter is lower without fertilizer (0.1) and increases significantly with the application of guinea pig manure, reaching the highest point in T4 (0.9). This improves the structure and fertility of the soil.
- Nitrogen:
  Deficiency without fertilizer: Very low in T0 (0.1), indicating a deficiency.
  Improvement with fertilizer: Increases with the dose of guinea pig manure, being higher in T4 (0.9), which favors the overall growth and development of the crop.
- Phosphorus:
  Low in T0: Reflects an initial deficiency (0.1).
  Increases with fertilizer: Improves with doses of guinea pig manure, especially in T4 (0.9), essential for flowering and root development.
- Potassium:
  Adequate levels with fertilizer: Low in T0 (0.1) and increases up to T4 (0.9), which is vital for photosynthesis, water balance, and other metabolic processes of the plant.

3.2.3. Comparative plithogenic decision.

The use of plithogenic statistics allows for the integration of uncertainty and inherent variability in agricultural and environmental processes in the study of prickly pear cultivation and soil recovery at the Salache Campus. This

advanced methodology has facilitated a deeper understanding of how different doses of guinea pig manure affect both the adaptation of the prickly pear and the soil quality, by evaluating variables such as plant height, number of buds, berry production, and key soil parameters such as pH, organic matter, nitrogen, phosphorus, and potassium.

The results revealed that the T3 treatment (7.5 tons/ha) is the most effective in promoting an ideal balance between the growth and development of the prickly pear crop and the improvement of soil health. Specifically, this treatment not only optimized plant height by maximizing the production of buds and berries but also balanced pH levels. Additionally, it increased organic matter and ensured the adequate availability of nitrogen, phosphorus, and potassium. The application of 7.5 tons/ha of guinea pig manure showed significant improvements in soil conditions, evidenced by an increase in organic matter and an adjustment in pH to levels more conducive to the prickly pear. Thus, it has demonstrated how the interaction of physical and chemical factors can be effectively modeled under the plithogenic framework.

In contrast, the T4 treatment (10 tons/ha), although it promoted the highest plant height and the maximum concentration of organic matter, was revealed to be potentially excessive, suggesting that more fertilizer does not always translate into better agronomic or environmental results. Therefore, T3 not only aligns with the agronomic needs for optimal prickly pear production but also supports the long-term sustainability of the soil. Together, with the application of plithogenic statistics, it highlights the importance of precisely dosing agricultural inputs to maximize both crop yield and the health of the ecosystem in which it develops.

4 Conclusion

The treatment with 7.5 tons per hectare (T3) of guinea pig manure emerged as the most balanced, effectively optimizing the growth and production of prickly pear. This dose not only maximized plant height and the production of buds and berries but also promoted a favorable soil environment. This balance is crucial, as it allows for the improvement of agricultural production without compromising the health and sustainability of the soil. Additionally, the applied plithogenic statistics showed that T3 facilitates a robust adaptation of the prickly pear, by improving organic matter, adjusting pH to more suitable levels, and ensuring the availability of essential nutrients such as nitrogen, phosphorus, and potassium.

The application of guinea pig manure has proven to be instrumental in enriching soil quality, particularly with the T3 treatment. This treatment significantly raised organic matter and maintained an optimal nutritional balance, indicative of effective soil recovery. The adequate reduction of pH, especially in originally alkaline soils, and the increase in key nutrients underscore how precise management of fertilizer can transform soil properties to create more conducive conditions for cultivation. This improvement in soil quality is essential not only for the current crop cycle but also for the long-term health of the land, ensuring productivity and sustainability.

The research has reaffirmed the importance of carefully calibrating the application of organic fertilizers like guinea pig manure to obtain the maximum agricultural benefits without incurring unsustainable practices. While higher doses (such as T4 with 10 tons/ha) offered certain benefits in terms of growth and organic matter, T3 (7.5 tons/ha) provided optimal performance without the potential risks of excess, which can include nutritional imbalances or long-term environmental damage. Consequently, it is vital to guide sustainable agricultural practices, where the goal is to maximize the efficiency of inputs and minimize negative impacts on the environment.

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Neutrosophic Clustering Analysis with Data from Cysticercus Tenuicollis DNA Samples

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Abstract. Parasitic diseases caused by *Cysticercus tenuicollis* significantly impact the livestock industry. The quantification of such impacts presents challenges due to the handling of data obtained through modern molecular techniques. This investigation explores the application of neutrosophic cluster analysis to the study of *Cysticercus tenuicollis* DNA. Neutrosophic clustering and the use of linguistic terms help interpret analyses, employing a five-step neutrosophic clustering algorithm: selection of the clustering algorithm, definition of distance metrics, data preparation, algorithm execution, and examination of results. Samples from 20 sheep exhibiting liver cysts were collected in Cotopaxi Province, Ecuador. The analysis resulted in a neutrosophic cluster where one group displayed low truth and falsity values but a moderate level of indeterminacy. The stability of the clusters was evaluated using the Adjusted Rand Index (ARI) with 100 bootstrapping repetitions, yielding a moderate value of approximately 0.52, which implies that the detected cluster structures are robust and representative of the underlying patterns in the data. The clusters show a clear differentiation based on the initial DNA concentration. Samples with high DNA concentrations tend to group together, suggesting that the quality and quantity of DNA may be critical indicators of the samples' condition and possibly the presence and state of the parasite.

Keywords: Neutrosophic Clustering Analysis, DNA, clustering algorithm, Cysticercus tenuicollis, molecular techniques.

1 Introduction

The study of parasitic diseases, such as those caused by *Cysticercus tenuicollis*, a significant parasite in the livestock sector, is crucial for the development of effective control and eradication strategies [1]. This parasite, whose prevalence in animals like sheep can lead to substantial economic losses, has been the subject of numerous studies aiming to better understand its biology and transmission. Recently, molecular identification has enabled significant advancements in this area, providing more accurate methods for studying and diagnosing the parasite [2].

However, analyzing the vast amount of genetic data obtained through modern molecular techniques presents challenges, especially in terms of managing uncertainty and interpreting genetic variability [3]. Neutrosophic cluster analysis emerges as a powerful tool, facilitating the grouping of complex DNA data from Cysticercus tenuicollis into clusters that reflect significant similarities and differences, even under conditions of uncertainty [4]. This approach enhances understanding of the parasite's genetic structure and variability, supporting the development of more targeted and efficient intervention measures [5].

This is a globally occurring disease with a low incidence, but when it does occur, it can cause losses due to death ranging from 3% to 25%. The disease was first reported in California, USA in 1916 and has since been described in other parts of the world. Thus, this disease has a global distribution, concentrating in humid areas and lowlands with predominantly alkaline soils and waters [5].

This article explores the application of neutrosophic cluster analysis in the study of Cysticercus tenuicollis DNA, highlighting how this method can overcome the limitations of traditional analytical approaches and significantly contribute to the field of veterinary parasitology [6]. By integrating neutrosophic set theory into cluster analysis, a new dimension is opened in the study of pathogens, enhancing our ability to interpret complex data and make informed decisions in the fight against parasitic diseases.

2 Background of Cysticercus tenuicollis in sheep

Currently, sheep are highly valued mammals in our region, especially in the Central Sierra of Ecuador. Their geographic distribution is broad because these animals exist worldwide, and their adaptability allows them to be constantly managed in rustic conditions across various climates and ecologies; therefore, this species has been produced and exploited in extensive pasture areas [7]. Moreover, sheep have very distinct and specific feeding habits; unlike most species, they refuse to eat or drink from dirty feeders or troughs. They require clean food and water, free of feces or any other contaminants [8].

Animal behavior is the ultimate expression of adaptability or adjusting internal conditions to the external environment, considering that the animal's body responds as a whole to a stimulus. Thus, every manifestation of behavior is directed toward satisfying one of the three basic needs of life: feeding, defense, and reproduction. For sheep, nutritional requirements are stronger than defensive ones, and defensive needs are stronger than reproductive ones [9].

In this sense, most sheep are found infested by external parasites and, more so, by internal parasites, which have secondary life cycles that occur across different pastures. Without a proper parasite control program or a deworming schedule, sheep can suffer severe parasitism, leading to weight gain deficiencies, especially in lambs and adult animals intended for reproduction, resulting in complex cases of anemia, diarrhea, and in the most neglected cases, death at any age [9].

Cysticercosis in wild and domestic animals is caused by the larval stages (metacestodes) of tapeworms (cestodes), with these adult phases located in the intestines of dogs and wild canids, and even findings in cats [10]. This type of cysticercosis particularly causes economic losses due to the seizure of meats and residues infected with these types of parasites, as well as specific organs such as livers and even lungs [11].

Based on various research studies and investigations on Cysticercus tenuicollis, it is determined that a parasite is any living being that spends all or part of its life at the expense of another living being, called a host (intermediate or definitive), from which the parasite lives, causing harm or not, and on which it has an obligatory dependence. Similarly, a parasite is an animal or plant organism that lives only at the expense of another organism, either on it or inside it [12]. In this regard, there are currently two types of parasites; internal parasites that maintain a part of their cycle in the bodies of animals and eventually humans, causing disorders [13], and external parasites that live outside an animal organism, such as fleas, ticks, lice, among others, which are easier to detect with some exceptions, like the agents producing scabies, but which can be effectively treated with a simple clinical diagnosis [13].

Indeed, the etiological agent causing hepatoperitoneal cysticercosis in sheep, which typically affects other mammalian animals including humans, is Cysticercus tenuicollis, the metacestode of Taenia Hydatigena [10]. According to recent studies, the type of parasites Taenia Hydatigena and in its larval stages such as Cysticercus tenuicollis and Toxocara canis can sometimes be transmitted to humans and can be widely distributed in the cells of the parenchyma, the basal membrane of the tegument, and the apical surface of the epithelial cells, which surround and cover the ceca; this type of cysticercus in large quantities can lead to severe parasitic infections that may induce the development of specific cancer [10].

In the case of Cysticercus tenuicollis, it has also been found in the pudú, a native deer of South America commonly found in countries like Chile and Argentina. This parasite has been observed in the bronchi of this deer, in both males and females, young and adults; these cases are among the first records of deer acting as hosts, due to the establishment of European deer populations, which presumably causes the spread of Cysticercus tenuicollis by sharing various pasture sites that practically cause a high additional impact, both in exotic mammals and in farm mammals [7].

In the case of cattle, diagnosis, as in sheep, is normally carried out during post-mortem inspection, where the cysticerci (vesicles) found present whitish or slightly turbid vesicles, where a single scolex is clearly recognized. In certain cases, these vesicles may present a reddish hue, with the scolex difficult to identify due to its coloring, and in some Cysticercus tenuicollis, rounded formations are found, already encapsulated or calcified in specific organs such as the liver and lungs [9].

2.1 Description of clinical symptoms

Due to the acute to subacute progression of this disease, it is very difficult to observe clinical symptoms, especially under field conditions. What was observed upon arrival at the facility was that the animals exhibited bloody diarrhea, colored urine, a retracted abdomen, and a staggering gait. Animals showing clinical symptoms would fall and exhibit pedaling motions, opisthotonos, nystagmus, bloody foam from the nose, and vocalizations. Rectal temperature remained within normal parameters. The oral, ocular, and vulvar mucosa exhibited an anemic coloration with a jaundiced tint. Subsequently, it was observed that these symptoms led to the facility manager requesting the euthanasia of the animal for subsequent necropsy.

3 Neutrosophic analysis of the data

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, created by Professor Florentin Smarandache. Its incorporation ensures that the inherent uncertainty in decision-making is considered, including indeterminacies where experts will issue their judgments evaluating linguistic rather than numerical terms, which constitutes the most natural form of measurement in humans [14]. Neutrosophic logic and sets, on the other hand, represent a generalization of Zadeh's fuzzy logic and sets, particularly of Atanassov's intuitionistic logic, with multiple applications in the field of decision-making and machine learning [14].

The veracity of a proposition in neutrosophic analysis is as follows [15]: Let $N = \{(T, I, F): T, I, F \subseteq [0,1]\}n$, a neutrosophic evaluation is a mapping of a group of proposition formulas to N, and therefore p:

$$v(p) = (T, I, F) \quad (1)$$

All this facilitates practical application in real-world problems [16], where the analysis of Single-Value Neutrosophic Sets (SVNS) also provides linguistic terms to achieve a greater interpretation of them [14]. In this context, X represents the universe as an SVNS. Thus, X can be determined in the following way [14]:

$$A = \{\langle x, u_a(x), r_a(x), v_a(x)\rangle : x \in X\} \quad (2)$$

Where $u_a(x): X \to [0,1], r_a(x): X \to [0,1]$ y $v_a(x): X \to [0,1]$

With

$$0 \leq u_a(x), r_a(x), v_a(x) \leq 3, \forall\, x \in X \quad (3)$$

The interval $u_a(x), r_a(x)$ and $v_a(x)$ denotes the neutrosophic true, indeterminate, and false values from x in A, respectively [15]. For convenience, an SVNN is expressed as A = (a, b, c), where a, b, c are in the range [0, 1] and $0 \leq a + b + c \leq 3$ [17].

Neutrosophic numbers and linguistic terms in the form (T, I, F) are shown in Table 1.

Table 1: Linguistic terms according to the method and single-valued neutrosophic numbers.

Initial Concentration		DNA Volume		Water Volume	
[0.35, 0.65, 0.05]	(Medium, Medium, Minimum)	0.40, 0.60, 0.05]	(Medium, Medium, Minimum)	[0.60, 0.40, 0.05]	(Medium, Medium, Minimum)
[0.60, 0.40, 0.05]	(Medium, Medium, Minimum)	[0.13, 0.87, 0.05]	(Low, High, Minimum)	[0.87, 0.13, 0.05]	(High, Low, Minimum)
[1.00, 0.00, 0.05]	(High, Low, Minimum)	[0.00, 1.00, 0.05]	(Low, High, Minimum)	[1.00, 0.00, 0.05]	(High, Low, Minimum)
[0.61, 0.39, 0.05]	(Medium, Medium, Minimum)	[0.12, 0.88, 0.05]	(Low, High, Minimum)	[0.87, 0.13, 0.05]	(High, Low, Minimum)
[0.35, 0.65, 0.05]	(Medium, Medium, Minimum)	[0.33, 0.67, 0.05]	(Low, High, Minimum)	[0.67, 0.33, 0.05]	(High, Low, Minimum)

These terms help interpret the degrees of truth, falsehood, and indeterminacy in more accessible terms such as "Low", "Medium", and "High" for T and F, and "Minimal", "Moderate", and "High" for I. This structure enriches the analysis by allowing not only the viewing of numerical data but also a quick understanding of their meaning in the context of neutrosophic analysis.

3.1 Neutrosophic Cluster Analysis

Cluster Analysis[18, 19] is the generic name for a wide variety of procedures that can be used to create a classification. More specifically, a clustering method is a multivariate statistical procedure that starts with a dataset

containing information about a sample of entities and attempts to reorganize them into relatively homogeneous groups, which we will call clusters [20].

In Cluster Analysis, little or no information is known about the structure of the categories, which differentiates it from multivariate assignment and discrimination methods. What is available is a collection of observations, with the operational objective in this case being to discover the structure of the categories into which the observations fit. More specifically, the goal is to organize the observations into groups such that the degree of natural association is high among members of the same group and low among members of different groups. Although little or nothing is known about the structure of the categories a priori, there are often some notions about desirable and undesirable characteristics when establishing a particular classification scheme. Operationally, the analyst is sufficiently informed about the problem such that they can distinguish between good and bad category structures when they encounter them [21].

To carry out the neutrosophic cluster analysis with the DNA sample data of Cysticercus tenuicollis in sheep from Ecuador, the following steps were taken:

- **Step 1.** Selecting the Clustering Algorithm

Given that we are working with neutrosophic data, a clustering method that can adapt or be modified to handle this type of data is necessary. The K-means algorithm [22]is commonly used, but it must be adapted to work with the T, I, and F components.

- **Step 2**. Definition of the Distance Metric

Euclidean distance is not suitable for neutrosophic numbers due to their three-dimensional nature and the interpretation of T, I, and F. It will be required to define a metric that can effectively measure the distance between neutrosophic data points considering all components.

- **Step 3.** Data Preparation

The original measurements of each sample were converted into neutrosophic values as we have already calculated (T, I, F). We will use these values for clustering.

- **Step 4**. Execution of the Algorithm

The selected algorithm is implemented in Python, using the defined distance metric, and we apply the clustering to the prepared data.

- **Step 5.** Analysis of the Results

Evaluate the formed clusters to understand how the samples are grouped according to their neutrosophic characteristics and to interpret the results biologically.

3.2 Collection and processing of Cysticercus tenuicollis DNA samples in sheep

Twenty cyst samples (ranging from 2 to 5 cm in size) were collected from sheep livers slaughtered at the Metropolitan Public Company of Rastro Quito (EMRAQ-EP) and in the communities of the Cantons Pujilí and Saquisilí in the Province of Cotopaxi, Ecuador. Molecular analysis of the samples was conducted at the Research Laboratory of the Universidad de las Américas (UDLA), Quito-Ecuador. DNA was extracted from a portion of tissue from each cysticercus using the Phenol-Chloroform method [23]:

- Dissect 2 mm of tissue sample and transfer it to a new 1.5 mL tube.
- Add 500 µL of Extraction Buffer and macerate with a pipette tip or pestle for 5 minutes inside a cold block.
- Vortex for 1 minute and add 5 µL of Proteinase K (20 mg/mL).
- Incubate the samples at 56°C overnight with shaking at 300 rpm.
- Remove the samples from the heating block, and let them rest for 5 minutes at room temperature.
- Add 750 µL of Phenol/Chloroform/Isoamyl and vortex until a milky emulsion forms.
- Centrifuge for 10 minutes at 4°C at maximum speed. Transfer all the aqueous phases to a new 1.5 mL microtube.
- To this phase, add 500 µL of Chloroform: Isoamyl, and homogenize by vortexing.
- Centrifuge for 10 minutes at 4°C at maximum speed. Transfer all the aqueous phases to a new 1.5 mL microtube.
- To the new microtube, add 400 µL of 100% Isopropanol and 40 µL of 3M NaCl.
- Mix well and let it sit at room temperature for two hours.
- Centrifuge for 30 minutes at 4°C at 15,000 rpm.
- Carefully discard the supernatant without disturbing the pellet.
- Add 1000 uL of ice-cold 70% Ethanol and centrifuge for 15 min at 15,000 rpm.
- Discard the supernatant and allow it to dry.
- Resuspend in 40 µL of Milli Q Water or TE and incubate the sample at 37°C for two hours.
- Quantify the DNA concentration.

The quantification of DNA concentrations was performed using a spectrophotometer-nanodrop, after which it was diluted to a final concentration of 5 ng/µL as shown in Table 2. The samples were stored in a freezer at -20°C until use.

Table 2: Dilutions of Cysticercus tenuicollis DNA samples.

CYSTICERCUS DILUTIONS			
FINAL VOLUME (µL)			20
FINAL CONCENTRATION (ng/µL)			5
SAMPLE ID	**INITIAL C.**	**V. DNA**	**V. WATER**
C001	73.3	1.3	18.64
C002	212.8	0.47	19.53
C003	305.4	0.33	19.67
C004	215.5	0.46	19.54
C005	150	0.67	19.33
C006	275.5	0.36	19.64
C007	126.2	0.79	19.21
C008	273.5	0.37	19.63
C009	236.5	0.42	19.58
C010	109.3	0.91	19.09
C011	136.3	0.73	19.27
C012	73.1	1.37	18.63
C013	124.8	0.8	19.2
C014	155.2	0.64	19.36
C015	107	0.93	19.07
C016	76.8	1.3	18.7
C017	101.6	0.98	19.02
C018	95.9	1.04	18.96
C019	109.3	0.91	19.09
C020	126.4	0.79	19.21

3.3 Amplification of DNA fragments

In the study, a total of 454 bp of subunit 1 of the NADH dehydrogenase (nad1) from the mitochondrial DNA of T. hydatigena isolated from sheep were sequenced [11]. PCR amplification of the mitochondrial nad1 gene was performed using the following pair of primers specific for cestode parasites, described by [24]:

5′-CARTTTCGTAAGGGBCCWAAWAAGGT-3′ forward
5′-CCAATTTCYTGAAGTTAACAGCATCA-3′ reverse

The PCR reactions were carried out in a final volume of 15 µl for each sample as shown in Table 3.

Table 3: Components for the nad1 gene amplification PCR. Source: own elaboration.

COMPONENT	C. Initial	C. End	Vol. 1 Rx (µL)	Vol. 60 Rx (µL)
MQ water			8.8	528
Buffer	10X	1X	1.5	90
MgCl2	50mM	1.5mM	0.45	27
dNTP Mix	10mM	0.2	0.3	18
First Fw	20 µM	0.5µM	0.375	22.5
First RV	20 µM	0.5µM	0.375	22.5
Taq Polymerase*	10 U/uL	2 U/uL	0.2	12
DNA	5ng/µL	1ng/µL	3	
Final Volume			15	

The PCR reactions were performed with a final volume of 15 µl for each sample. The fragments were generated by PCR using thermocyclers, under the following conditions: initial denaturation temperature of 94°C for 2 min (1 cycle), followed by 35 cycles of denaturation at 94°C for 1 min, primer annealing at 58°C for 30 seconds, extension at 72°C for 1 min, and a final extension of the products at 72°C for 5 min, ending with a 10 min rest at 4°C.

The PCR products were visualized on 2% agarose gels to identify possible failures. The gels were stained with SYBR Safe, analyzed under ultraviolet light, and photographed. In each case, a negative control was run to check for contamination. The amplified product was sequenced using the BigDye v3.1 sequencing kit (Applied Biosystems). The sequencing reactions were purified using an enzymatic purification protocol based on gel filtration resin. The Sanger sequencing matrix (BigDye - 3.1) was analyzed by capillary electrophoresis. The sequences were aligned and edited with the MEGA X software, using the reference sequence (MN175584.1), available on GenBank. The determination of the identified polymorphisms (number of haplotypes, number of mutations, as well as haplotype diversity and nucleotide diversity), was performed using DnaSP version 6.

Phylogenetic trees generated among sequences of the Nad1 gene (454 bp) from Cysticercus tenuicollis and the comparison with other sequences deposited in GenBank with accession numbers: MN175584.1 for Taenia hydatigena from Nigeria and FJ440841.1 for Taenia regis from Uganda, were constructed using the UPGMA (unweighted pair group method using arithmetic averages) and NJ (neighbor-joining) methods [12]. The reliability of the obtained trees was determined with a bootstrap test (500 replicates), shown next to the branches [11].

4 Results

Figure 1 displays the neutrosophic cluster obtained from the analysis of DNA data from Cysticercus tenuicollis in sheep from Ecuador. The charts reveal groupings based on the neutrosophic characteristics of the samples, aiding in identifying relationships and differences among data groups. The variability among the clusters in each chart could indicate different behaviors or characteristics inherent to the samples, useful for more detailed analysis or specific decisions based on the identified groups.

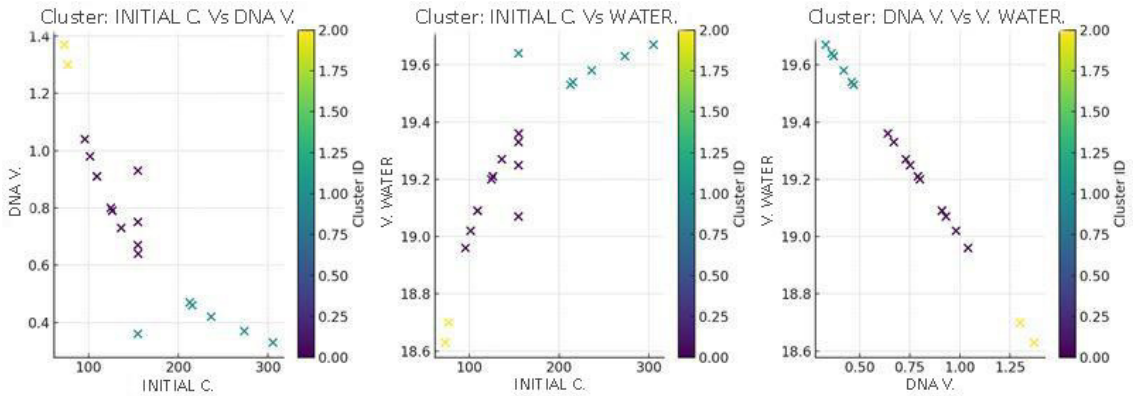

Figure 1: Neutrosophic clusters obtained from the analysis of DNA data of Cysticercus tenuicollis in sheep from Ecuador. Source: own elaboration.

1. INITIAL C. vs V. ADN: This chart shows how the samples are distributed in terms of ' INITIAL C. ' and 'V. ADN'. The clusters are clearly differentiated, indicating distinct patterns in these two parameters.

2. INITIAL C. vs V. WATER: Similar to the first chart, this one displays the distribution of samples for ' INITIAL C. ' and 'V. WATER'. A separation of clusters is observed, suggesting differences in how these two variables interact.

3. V. ADN vs V. WATER: This chart compares 'V. ADN' with 'V. WATER', also shows a clear distinction between clusters.

To assess the stability of the neutrosophic clusters, a resampling or bootstrapping approach is applied. This method involves repeating the clustering process multiple times with different subsets of data and observing how consistently the samples are grouped into the same clusters in each iteration. The stability of the clusters can indicate how robust and reliable the detected grouping patterns are.

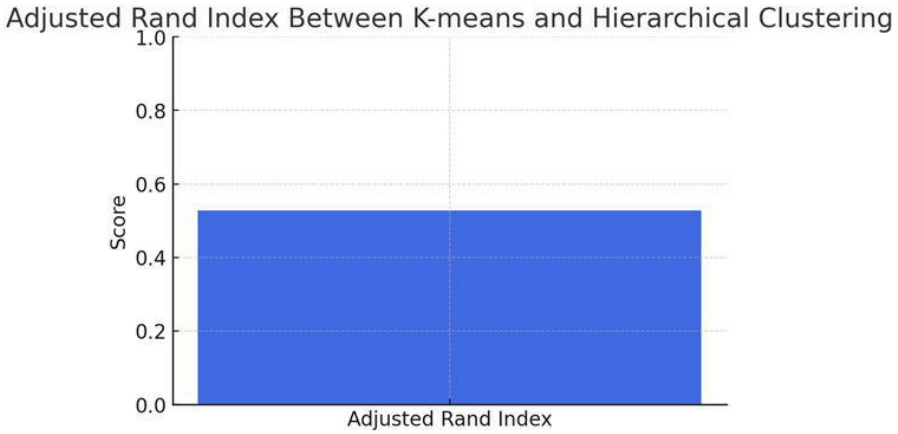

Figure 2: Stability analysis of the neutrosophic cluster obtained from DNA of Cysticercus tenuicollis in sheep from Ecuador. Source: Own elaboration.

The evaluation of cluster stability, using the Adjusted Rand Index (ARI) over 100 bootstrapping iterations, resulted in an average value of approximately 0.52 as shown in Fig. 2. This suggests that, although there is some consistency between the clusters generated by both methods, there are also significant differences. This may be due to the nature of the algorithms and how they handle the structure of the data in the clusters generated from the neutrosophic DNA samples of Cysticercus tenuicollis in sheep from Ecuador [19]. A high ARI suggests that samples tend to be grouped consistently into the same clusters across different subsets of data, which supports the reliability of the clustering analysis performed. This implies that the detected cluster structures are robust and representative of the underlying patterns in the data.

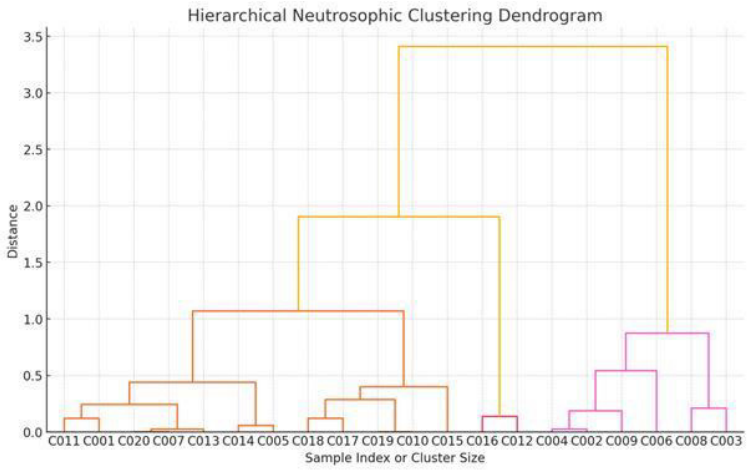

Figure 3: Hierarchical clustering analysis with the neutrosophic data from DNA samples of Cysticercus tenuicollis isolated in sheep from Ecuador. Source: Own elaboration.

Figure 3 shows the hierarchical clustering analysis of the neutrosophic data from DNA samples of Cysticercus tenuicollis isolated in sheep from Ecuador. This graph demonstrates how the samples are grouped based on their neutrosophic characteristics (T, I, F). The height in the dendrogram where two groups join reflects the clustering distance between them, which is a measure of their dissimilarity. Different 'cuts' in the dendrogram can be observed to decide the most appropriate number of clusters. For example, a cut at a height of about 10 could indicate the formation of two or three main groups, depending on how you prefer to interpret the separation between the groups.

A total of 2 polymorphic sites were detected in the Nad1 gene of the mitochondrial DNA, all being parsimony informative. Three different haplotypes (h) were observed, with haplotype 1 being predominant, represented by 85% of the sequences.

Hap_1: 17 sequences [Nad-Cox1-Fw-CT001 Nad-Cox1-Fw-CT002 Nad-Cox1-Fw-CT003 Nad-Cox1-Fw-CT004 Nad-Cox1-Fw-CT005 Nad-Cox1-Fw-CT007 Nad-Cox1-Fw-CT008 Nad-Cox1-Fw-CT009 Nad-Cox1-Fw-CT011 Nad-Cox1-Fw-CT012 Nad-Cox1-Fw-CT013 Nad-Cox1-Fw-CT014 Nad-Cox1-Fw-CT015 Nad-Cox1-Fw-CT016 Nad-Cox1-Fw-CT017 Nad-Cox1-Fw-CT018 Nad-Cox1-Fw-CT019]

Hap_2: 2 sequences [Nad-Cox1-Fw-CT006 Nad-Cox1-Fw-CT020]

Hap_3: 1 sequence [Nad-Cox1-Fw-CT010]

The effectiveness of molecular tools in identifying and characterizing different species and genotypes of tapeworms has been proposed by authors such as [3-6], with results based on the existence of interspecific variations for molecular characters [8].

Table 4 shows values of (0.279) in reference to haplotype diversity and (0.00101) for nucleotide diversity when evaluating the 454 bp fragment of the Nad1 gene from the mitochondrial DNA of Cysticercus tenuicollis.

Table 4: Variability parameters in 641 bp of the Nad1-Cox1 gene, from Cysticercus tenuicollis sequences.

Characteristic	Total
Number of sequences	20
Number of mutations	2
Haplotypes	3
Haplotype Diversity	0.279
Standard deviation	0.123
Significance	0.00101
Nucleotide Diversity	0.00046

These results coincide with those reported for Taenia hydatigena by [7] who conducted a molecular description, phylogeny, and genetic variation of Taenia hydatigena from sheep and goats in Nigeria, based on the mitochondrial Nad1 gene, and are very close to those described by others who developed the molecular identification of Taenia hydatigena from sheep in Khartoum, Sudan [9].

5 Discussion

Choosing a good protocol for DNA extraction is crucial for DNA amplification by PCR. The most commonly used technique is organic solvents like phenol/chloroform; however, this method has drawbacks in terms of time and toxicity, making it not entirely feasible since extraction protocols should be simple, quick, and reproducible while maintaining the safety and integrity of the sample. For this reason, the DNA extraction protocol based on organic solvent phenol/chloroform was used.

All 20 samples were analyzed using the selected protocol, achieving a 100% yield, meaning 20 samples were identified using specific primers for Cysticercus Tenuicollis; thus, the chosen DNA extraction protocol was ideal. However, a study by [16] determined that this classic protocol had a low yield, which could be attributed to possible oxidation of the used phenol; it also involves many steps, making it lengthy and laborious, potentially leading to DNA loss during the various steps of the technique, including the use of polluting and toxic solvents. Nevertheless, the integrity of the DNA was maintained in optimal conditions, showing a single large band as in the processed samples of Cysticercus Tenuicollis

This research generates new topics for further study to identify the gender of Cysticercus Tenuicollis that takes sheep as hosts and the possible damage they can cause to them. The impact of this project affects animal health since contracting gastrointestinal parasites decreases the vitality and well-being of canines, besides becoming a

zoonotic factor related to public health issues since sheep may ingest their eggs expelled in the feces of infected canines The environmental impact occurs when the parasite's eggs are expelled through feces and spread in the environment, causing reinfection in livestock (such as sheep, cattle, and pigs) and humans, thus continuing the biological cycle of the parasite. As a disease that affects the animal and deteriorates its health, it also has an economic impact as the diagnosis and treatment of canines reduce the owner's finances, and in most cases, the animals are not treated The technical impact is evidenced in the performance of molecular diagnosis of the parasite to generate specific data on the identification of the genus Cysticercus Tenuicollis in slaughtered sheep. Finally, the social impact must be considered in informing pet owners about the results found in the laboratory diagnosis and molecular identification and subsequently informing them about the symptoms and lesions that cause the deterioration of the animal's health and the potential contagion through the spread of zoonosis.

6 Conclusion

The clusters formed indicate a clear differentiation based on the initial DNA concentration. Samples with high DNA concentration tend to cluster together, suggesting that the quality and quantity of DNA can be critical indicators of the sample's condition and possibly the presence and state of the parasite.

The neutrosophic values (T, I, F) provide deeper insights into the uncertainty and quality of the samples:

- High Truth (T): Samples with high DNA concentrations tend to have a high T value, indicating better sample quality for future analysis.
- Low Falsehood (F): Samples with less water volume (less dilution) exhibit lower F values, which could be interpreted as less contamination or interference in the sample.
- Indetermination (I): Samples with medium DNA volume values show variability in I, reflecting the uncertainty in the precision of DNA measurement.

The use of neutrosophic numbers has allowed for the evaluation of multiple aspects of the samples simultaneously, highlighting the utility of this approach in contexts where uncertainty and variability are significant. This methodology could be adapted to other types of biological or medical analysis where data interpretation is complex. The evaluation of cluster stability through bootstrapping techniques showed that the clusters are consistently reproducible, indicating that the groupings found are robust and reliable.

Clustering samples based on neutrosophic values and their DNA concentration can have important implications for epidemiological studies of Cysticercus tenuicollis in sheep, helping to identify potential infection foci or differences in parasitic load.

Recommendations for future analyses should explore other factors such as the geographic location of the sheep, age, and other clinical data to see how these influence the clusters. Validation of these findings with alternative analysis methods and in a larger dataset is necessary to generalize the results. Practical applications of using these findings include improving sampling and diagnostic strategies for cysticercosis in sheep, thus optimizing control and prevention efforts.

This neutrosophic analysis provides a solid foundation for better understanding the variability and quality of DNA samples, offering a valuable tool for scientific research and practical applications in veterinary and public health.

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Neutrosophic Analysis in the Evaluation of the Use of Beauveria Bassiana against the Potato White Grub

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Abstract. Beauveria bassiana is an entomopathogenic fungus that has been shown to have the potential to control the potato white grub (Premnotrypes vorax), a pest that significantly affects potato production, especially in Ecuador. Therefore, the present study has focused on evaluating the effectiveness of Beauveria bassiana in the control of the potato white grub under laboratory conditions. To this end, neutrosophic statistics have been used to analyze larval mortality, by applying concentrations of $\mathbf{10^4}$, $\mathbf{10^6}$ and $\mathbf{10^8}$ conidia of Beauveria bassiana by spraying and immersion. Among the results, it was determined that treatment at a concentration of 10^8 conidia per spray has proven to be the best option, with a neutrosophic coding from very effective to extremely effective in mortality in larvae. In conclusion, neutrosophic analysis has shown that Beauveria bassiana is effective in controlling white grub, especially with high concentrations and spraying.

Keywords: Sustainable agriculture, treatment efficacy, entomopathogen, neutrosophic statistics.

1 Introduction

In this study, the effectiveness of *Beauveria bassiana*, an entomopathogenic fungus isolated from rabbit manure, was evaluated for the control of the white potato grub (*Premnotrypes vorax*) under laboratory conditions [1]. *Beauveria bassiana* is known for its ability to act as a natural biological insecticide in soils, controlling a wide variety of pests. This fungus thrives in cool, moist environments with little sun exposure. It infects pests with muscardine disease, characterized by a whitish-yellowish cotton-like covering [2].

The use of *Beauveria bassiana* is especially relevant since it does not harm crops, soil, and is harmless to humans and animals [3]. This represents a sustainable alternative to the use of agrochemicals for pest control [4], helping to avoid environmental pollution and reduce the economic costs associated with purchasing pesticides.

Therefore, it constitutes a solution for the protection of potato crops, considered among the most important globally, with an estimated production of 341 million tons on an area of 20 million hectares. The main producing countries include China, with a production of between 66 and 71 million tons, followed by Russia, India, Poland, the United States, Ukraine, and Germany.

Among the most impactful afflictions that affect this crop is the white potato grub (Premnotrypes vorax), which is a pest that significantly affects the quality of the tubers. Therefore, the use of entomopathogenic fungi such as Beauveria bassiana has gained popularity as an efficient alternative to chemical insecticides [5] [6], which are harmful to human health and ecosystems [7] [8] [9].

Recent studies indicate that farmers report the white grub as the main pest of potato cultivation [10], affecting it consistently every year. They point out that the price of potatoes can decrease by up to 50% if the damage ranges between 10% and 30%. Whereas, higher damage even prevents the marketing of the product. As a control measure, farmers often defoliate the crop before it reaches its normal maturity, which can reduce yield between 10% and 40%.

Therefore, this study has focused on evaluating the efficacy of *Beauveria bassiana* in controlling the white potato grub under laboratory conditions. For this purpose, neutrosophic statistics were used to analyze larval mortality, applying concentrations of 10^4, 10^6, and 10^8 conidia of Beauveria bassiana by spraying and immersion in a sample of 35 experimental units.

For the development of this research, it is necessary to apply a neutrosophic analysis due to the level of indeterminacy in the study. Thus, indeterminacies are included as part of the statistical processing in evaluating larval mortality in controlling the white grub.

2 Materials and methods

2.1 Neutrosophic Statistics

Before analyzing the neutrosophic statistics of the method, it is necessary to define the neutrosophic set being analyzed [11]. The neutrosophic set is defined by the following elements: true ϑ, indeterminate η, and false δ of x in Q, respectively, and their images constitute standard or non-standard subsets within the range (0,1). For X from the universe of discourse, the single-valued neutrosophic set Q over X is defined as an object in the representation $l = \{\langle x, \vartheta_A(x), \eta_A(x), \delta_A(x)\rangle : x \epsilon X\}$.

Neutrosophic probabilities and statistics are a generalization of classical and imprecise probabilities and statistics [12]. The neutrosophic probability of an event E is the probability that the event E occurs, the probability that the event E does not occur, and the probability of indeterminacy (not knowing whether the event E occurs or not). In classical probability, the highest value (nsup) is less than or equal to 1, while in neutrosophic probability, nsup is less than or equal to 3. The function that models the neutrosophic probability of a random variable X is called the neutrosophic distribution:

Where $\vartheta_A(x), \eta_A(x), \delta_A(x)$ meet the following condition: $0 \leq \vartheta_A(x), \eta_A(x), \delta_A(x) \leq 3$ for all $x \in X$. Thus, to define each neutrosophic number, it is expressed in the form h, i, j for the modeling of the neutrosophic methodology to be used. Therefore, the following functions are defined:

$h = \vartheta_A(x)$ for true membership functions, where $\in \{0,1\}$.

$i = \eta_A(x)$ for indeterminate membership functions, where $\in \{0,1\}$.

$j = \delta_A(x)$ for false membership functions, where $\in \{0,1\}$.

Therefore, the neutrosophic number defined for the study is determined as $L = (h, i, j)$, where h, i, j $\in \{0,1\}$ and satisfies the following condition $0 \leq h + i + j \leq 3$. So, it is defined as a B-score function of a neutrosophic number according to the proposal of Smarandache or Basset.

In the given text, where h represents the probability that the value x occurs, j(x) represents the probability that the value x does not occur, and I(x) represents the indeterminate or unknown probability of x. Neutrosophic statistics is the analysis of neutrosophic events and deals with neutrosophic numbers, neutrosophic probability distribution, neutrosophic estimation, neutrosophic regression, etc.

Single-Valued Neutrosophic Sets (SVNS) emerged with the idea of applying neutrosophic sets for practical purposes [13]. Some operations among Single-Valued Neutrosophic Numbers (SVNN) are expressed below:

Given $L_1 = (h_1, i_1, j_1)$ and $L_2 = (h_2, i_2, j_2)$, two Single-Valued Neutrosophic Numbers (SVNN), the sum of L_1 and L_2 is defined as follows:

$$L_1 + L_2 = (h_1 + h_2 - h_1 h_2, i_1 i_2, j_1 j_2) \tag{1}$$

Given $L_1 = (h_1, i_1, j_1)$ and $L_2 = (h_2, i_2, j_2)$, two Single-Valued Neutrosophic Numbers (SVNN), the multiplication of L_1 and L_2 is defined as follows:

$$L_1 \cdot L_2 = (h_1 h_2, i_1 + i_2 - i_1 i_2, j_1 + j_2 - j_1 j_2) \tag{2}$$

The product of a positive scalar with an SVNN, $L = (h, i, j)$c) is defined by:

$$\mathrm{L} = (1 - (1 - h), \mathrm{i}, \mathrm{j}) \tag{3}$$

It refers to a dataset, which is wholly or partly composed of data with some degree of indeterminacy and the methods for analyzing them. Neutrosophic statistical methods allow for the interpretation and organization of neutrosophic data (data that can be ambiguous, vague, imprecise, incomplete, or even unknown) to reveal underlying patterns.

In conclusion, neutrosophic logic [14], neutrosophic sets, and neutrosophic probabilities and statistics have broad applications in various research fields and constitute a novel reference for study in full development. Neutrosophic descriptive statistics encompass all the techniques for summarizing and describing the characteristics of neutrosophic numerical data.

The study of neutrosophic statistics refers to a neutrosophic random variable, where L represents the corresponding lower and upper levels that the variable under study can reach within an indeterminate interval. Thus, it follows the neutrosophic mean of the variable ($\bar{\mathrm{L}}$) when formulating:

$$\bar{\mathrm{L}} = \frac{1}{n_N} \sum_{i=1}^{n_N} L_i \tag{4}$$

Where n_N is a neutrosophic random sample from the studied population. Once the neutrosophic mean is defined, the next step is the calculation of the variance of the neutrosophic sample. For this, the following equation is defined:

$$S_N^2 = \frac{\sum_{i=1}^{n_N}(L_i - \bar{L}_i)^2}{n_N} \quad (5)$$

Subsequently, the calculation of the Neutrosophic Coefficient of Variation (NCV)[15] is carried out, which measures the consistency of the variable. The smaller the value of the NCV, the more consistent the performance of the factor compared to other factors. For this purpose, the following equation is proposed:

$$CV_N = \sqrt{S_N^2} \times 100 \quad (6)$$

To measure the variable, the Neutrosophic Argumentation Coefficient is used. This evaluates the criteria through Linguistic Terms with Single-Valued Neutrosophic Numbers (SVNN) of consensus of justification of the expert opinion (see Table 1).

Table 1: Linguistic terms that represent the weight of larval mortality.

Coding	Linguistic Term	SVNN (T, I, F)	Description
EE	Extremely Effective	(1,0,0)	Indicates 100% efficacy in daily larval mortality, without indeterminacies or uncertainty.
VE	Very Effective	(0.9,0.05,0.1)	Almost complete efficacy with slight indeterminacies about daily consistency.
E	Effective	(0.8,0.15,0.2)	High efficacy with some uncertainties about mortality results.
ME	Moderately Effective	(0.7,0.25,0.3)	Good overall efficacy but with notable daily fluctuations.
SE	Somewhat Effective	(0.6,0.35,0.4)	Medium efficacy with considerable daily uncertainty.
N	Neutral	(0.5,0.45,0.5)	Balance between efficacy and inefficacy with high indeterminacy.
SHI	Somewhat High Ineffective	(0.4,0.55,0.65)	Somewhat low efficiency with significant indeterminacy and a tendency towards inefficacy.
SI	Somewhat Ineffective	(0.3,0.65,0.7)	Low efficacy with a predominance of negative results.
I	Ineffective	(0.2,0.75,0.8)	Mostly ineffective with few incidents of effective mortality.
VI	Very Ineffective	(0.1,0.85,0.9)	Almost completely ineffective with sporadic larval deaths.
EI	Extremely Ineffective	(0,0.95,1)	Total inefficacy in inducing mortality, without indeterminacies.

2.2 Sample Selection.

For the study, the sample size of respondents is determined using Equation 7, which takes the probabilities as 50% or 0.05, according to the following results:

- Maximum allowable error margin = 10.0%
- Population size = 55 experimental units.
- Size for a confidence level of 95%: 35
- Size for a confidence level of 97%: 38
- Size for a confidence level of 99%: 42

Additionally, the following formula was used for statistical processing to calculate the sample size.

$$n = \frac{ZNpq}{E^2(N-1) + Z^2pq} \quad (7)$$

Where:

- n: Sample size
- Z: Value from the normal distribution corresponding to the assigned confidence level

- E: Desired sampling error

- N: Population size

3 Results

3.1 Data collection

In this research, a Randomized Complete Block Design (RCBD) was used with 6 treatments and 1 control, covering a total of 35 experimental units. This design allowed for the evaluation of the efficacy of Beauveria bassiana, isolated from rabbit manure, for controlling the white grub of the potato (Premnotrypes vorax), by measuring variables such as the number of live and dead larvae, and the condition of the tubers (healthy and infected).

Study factors:

- Factor A (Dose concentration): A1 (10^4), A2 (10^6), and A3 (10^8).
- Factor B (Types of application): B1 (immersion) and B2 (aspersion).

Operationalization of variables: The study variables are defined where the independent variable is the dosage and type of application of Beauveria bassiana, and the dependent variables are effectiveness (% of pest invasion) and yield (percentage of healthy and infected tubers).

Specific management of the experiment: The study was conducted on the Salache campus of the Technical University of Cotopaxi, in the Agronomy laboratory. Meanwhile, the collection of rabbit manure was carried out in the American neighborhood, Cayambe, Pichincha.

Method of application of treatments (see Table 2):

- Aspersion: 10 ml of the Beauveria bassiana bioformulate was used per treatment using a small pump.
- Immersion: 15 worms were submerged in 10 ml of the bioformulate per treatment.

Variables evaluated:

- Mortality: the initial number of worms was counted, with daily readings, to obtain the percentage of mortality. To determine the levels of inconsistencies and indeterminacies in the study, work is carried out with 95% confidence in the analyzed variable (see Table 3).

Table 2: Treatment applied. Source: own elaboration.

Treatment	Conidia Concentration (per ml)	Application Method	Treatment description
T1	10^4	Aspersion	Use of low concentration applied through aspersion, aiming to cover wider areas in a less concentrated manner.
T2	10^4	Immersion	Application by immersion with low concentration to allow more prolonged and direct contact.
T3	10^6	Aspersion	Increase of concentration to evaluate a more potent effect through aspersion.
T4	10^6	Immersion	Medium concentration applied by immersion, seeking efficiency in use and balance in effectiveness.
T5	10^8	Immersion	Use of the highest concentration in immersion to maximize direct efficacy against larvae.
T6	10^8	Aspersion	Application of the maximum concentration through aspersion to test effectiveness under conditions of wide dispersion.

Table 3: Variable characteristics. Source: own elaboration.

Variable	Coding	Sample	Scale $[0\ ;\ 1], \forall\ F_n$
Larval mortality in the control of the white grub	MLCGB	[25;35]	Complete efficacy: There is no indeterminacy; the treatment with the highest concentration has proven to be completely effective in larval mortality, achieving the highest levels of mortality observed **(1,0,0)**. Mostly effective: The treatment is generally effective with some minor indeterminacies, suggesting that although it is effective, there might be minor variations in efficacy among replicates or similar conditions **(0.75,0.20,0.05)**. Moderately effective: There is considerable indeterminacy. The treatment is partially effective, showing inconsistent efficacy that might require adjustments or combination with other methods **(0.50,0.50,0)**.

Variable	Coding	Sample	Scale $[0\ ;\ 1], \forall\ F_n$
			Mostly ineffective: Most of the time, the treatment fails to effectively control larval mortality, indicating the need for review or substantial improvement of the treatment approach **(0.30,0.65,0.05)**.
			Completely ineffective: The treatment has no impact on larval mortality, indicating a total lack of effectiveness and the urgent need for a complete review or change of strategy **(0,0,1)**.

Development of the method: For the neutrosophic statistical modeling, observations and evaluations were carried out on each of the experimental units to define the level of MLCGB. Statistical analysis would determine the best treatment to apply based on the impact on the mortality of live larvae and the condition of healthy tubers. For the evaluation of the sample, linguistic terms are applied to each experimental unit according to the treatment applied over 25 days (the scale used according to the linguistic term from Table 1). For modeling, neutrosophic statistics are used to include various complex criteria at the time of evaluation (see Table 4).

Table 4: Neutrosophic frequency of daily evaluations in the monitoring of MLCGB. Source: own elaboration.

Day	Treatment 1 (T1)	Treatment 2 (T2)	Treatment 3 (T3)	Treatment 4 (T4)	Treatment 5 (T5)	Treatment 6 (T6)
1	(0.2,0.75,0.8)	(0.3,0.65,0.7)	(0.6,0.35,0.4)	(0.7,0.25,0.3)	(0.8,0.15,0.2)	(0.9,0.05,0.1)
2	(0.3,0.65,0.7)	(0.4,0.55,0.65)	(0.7,0.25,0.3)	(0.6,0.35,0.4)	(0.7,0.25,0.3)	(0.8,0.15,0.2)
3	(0.1,0.85,0.9)	(0.3,0.65,0.7)	(0.8,0.15,0.2)	(0.7,0.25,0.3)	(0.9,0.05,0.1)	(1,0,0)
4	(0.3,0.65,0.7)	(0.5,0.45,0.5)	(0.6,0.35,0.4)	(0.8,0.15,0.2)	(0.8,0.15,0.2)	(0.9,0.05,0.1)
5	(0.5,0.45,0.5)	(0.4,0.55,0.65)	(0.7,0.25,0.3)	(0.6,0.35,0.4)	(0.9,0.05,0.1)	(0.8,0.15,0.2)
6	(0.4,0.55,0.65)	(0.3,0.65,0.7)	(0.8,0.15,0.2)	(0.7,0.25,0.3)	(1,0,0)	(0.9,0.05,0.1)
7	(0.5,0.45,0.5)	(0.3,0.65,0.7)	(0.6,0.35,0.4)	(0.7,0.25,0.3)	(0.8,0.15,0.2)	(0.9,0.05,0.1)
8	(0.3,0.65,0.7)	(0.4,0.55,0.65)	(0.7,0.25,0.3)	(0.6,0.35,0.4)	(0.7,0.25,0.3)	(1,0,0)
9	(0.1,0.85,0.9)	(0.3,0.65,0.7)	(0.8,0.15,0.2)	(0.7,0.25,0.3)	(0.8,0.15,0.2)	(0.9,0.05,0.1)
10	(0.3,0.65,0.7)	(0.5,0.45,0.5)	(0.6,0.35,0.4)	(0.8,0.15,0.2)	(0.7,0.25,0.3)	(0.9,0.05,0.1)
11	(0.4,0.55,0.65)	(0.3,0.65,0.7)	(0.2,0.75,0.8)	(0.6,0.35,0.4)	(0.8,0.15,0.2)	(0.9,0.05,0.1)
12	(0.2,0.75,0.8)	(0.4,0.55,0.65)	(0.8,0.15,0.2)	(0.7,0.25,0.3)	(0.9,0.05,0.1)	(0.9,0.05,0.1)
13	(0.3,0.65,0.7)	(0.5,0.45,0.5)	(0.6,0.35,0.4)	(0.8,0.15,0.2)	(0.9,0.05,0.1)	(0.9,0.05,0.1)
14	(0.4,0.55,0.65)	(0.3,0.65,0.7)	(0.7,0.25,0.3)	(0.6,0.35,0.4)	(0.8,0.15,0.2)	(0.9,0.05,0.1)
15	(0.3,0.65,0.7)	(0.4,0.55,0.65)	(0.6,0.35,0.4)	(0.7,0.25,0.3)	(0.9,0.05,0.1)	(0.9,0.05,0.1)
16	(0.2,0.75,0.8)	(0.3,0.65,0.7)	(0.2,0.75,0.8)	(0.6,0.35,0.4)	(0.7,0.25,0.3)	(0.9,0.05,0.1)
17	(0.4,0.55,0.65)	(0.5,0.45,0.5)	(0.6,0.35,0.4)	(0.8,0.15,0.2)	(0.9,0.05,0.1)	(0.9,0.05,0.1)
18	(0.3,0.65,0.7)	(0.4,0.55,0.65)	(0.7,0.25,0.3)	(0.6,0.35,0.4)	(0.8,0.15,0.2)	(1,0,0)
19	(0.1,0.85,0.9)	(0.3,0.65,0.7)	(0.8,0.15,0.2)	(0.7,0.25,0.3)	(0.9,0.05,0.1)	(0.9,0.05,0.1)
20	(0.3,0.65,0.7)	(0.5,0.45,0.5)	(0.6,0.35,0.4)	(0.8,0.15,0.2)	(0.9,0.05,0.1)	(0.9,0.05,0.1)
21	(0.4,0.55,0.65)	(0.3,0.65,0.7)	(0.7,0.25,0.3)	(0.6,0.35,0.4)	(0.8,0.15,0.2)	(0.9,0.05,0.1)
22	(0.5,0.45,0.5)	(0.4,0.55,0.65)	(0.2,0.75,0.8)	(0.7,0.25,0.3)	(0.9,0.05,0.1)	(0.9,0.05,0.1)
23	(0.3,0.65,0.7)	(0.5,0.45,0.5)	(0.6,0.35,0.4)	(0.8,0.15,0.2)	(0.9,0.05,0.1)	(0.9,0.05,0.1)
24	(0.4,0.55,0.65)	(0.3,0.65,0.7)	(0.7,0.25,0.3)	(0.6,0.35,0.4)	(0.8,0.15,0.2)	(0.9,0.05,0.1)
25	(0.3,0.65,0.7)	(0.4,0.55,0.65)	(0.6,0.35,0.4)	(0.7,0.25,0.3)	(0.9,0.05,0.1)	(1,0,0)
$\overline{x}$	**[(0.3,0.65,0.7); (0.4,0.55,0.65)]**	**[(0.3,0.65,0.7); (0.4,0.55,0.65)]**	**[(0.6,0.35,0.4); (0.7,0.25,0.3)]**	**[(0.6,0.35,0.4); (0.7,0.25,0.3)]**	**[(0.8,0.15,0.2); (0.9,0.05,0.1)]**	**[(0.9,0.05,0.1); (1,0,0)]**

Neutrosophic statistical analysis: The preliminary results of the treatments applied to the experimental units define varied and complex evaluations within the neutrosophic linguistic scales on average. The average analysis of evaluations regarding the effectiveness of the treatments in the experimental units shows that:

- For *treatment T1 and T2*: The average evaluations are between SI and SHI, indicating that the efficacy of this treatment with the assigned concentration tends towards ineffectiveness. The same applies to these two treatments that use the same concentration of 10^4, but with the difference that one applies the method of aspersion and the other of immersion.
- For *treatment T3 and T4*: The evaluations of these two alternatives on average cover a state of effectiveness from ME to SE. Thus, these alternatives represent a tendency to be effective but with notable daily fluctuations, where it does not completely eliminate the pest from the tubers. Therefore, the variation for both methods of application of the treatments T3 and T4 is found in the same neutrosophic area and at the

same concentration of 10^6.

- For *treatment T5*: This treatment performed by immersion at a concentration of 10^8 is observed that the average evaluations are between E and VE. Therefore, this treatment indicates that it is an effective neutrosophic area to combat the pest with positive results and with slight indeterminacies about daily consistency.
- For *treatment T6*: It is considered according to the neutrosophic average as the alternative with the best results for combating the pest at a concentration of 10^8. By applying this treatment through the aspersion method, effectiveness from VE to EE was achieved. This indicates that, although it does not achieve 100% mortality of the pest, it is positioned close to this neutrosophic point of greater impact. Nevertheless, the application of T6 achieves higher tuber yields.

Once the results are analyzed, it is observed that the treatments are located in four neutrosophic areas, two of which are accepted and positive impact on the control of the white grub. For this, the following areas where the daily results obtained converge the most are defined:

- Neutrosophic Area AN1: With the classification [SI; SHI] and a tendency towards ineffectiveness. It also has a high probability of negative results and significant indeterminacy. In this area, treatments T1 and T2 are found and it is located in a neutral zone in the analyzed neutrosophic set.

- Neutrosophic Area AN2: With classification [ME; SE], with a tendency towards the limit of effectiveness. With low daily fluctuations in the results of the mortality of the pest. In this area, treatments T3 and T4 are located at the extreme of the neutral zone in the analyzed neutrosophic set.

- Neutrosophic Area AN3: With classification [E; VE]. It is characterized by being an area of high effectiveness and slight uncertainties about daily consistency. In it, treatment T5 is located, being part of the neutrosophic acceptance zone.

- Neutrosophic Area AN4: With classification [VE; EE]. It is characterized by being an almost completely effective area, where the desired alternatives to the identified problem are found. In it, treatment T6 is located and forms part of the neutrosophic acceptance zone.

Comparative analysis: To determine the best treatment that complies in terms of consistency and reliability, the variance and standard deviation are calculated. The results are shown in Figure 1, where it can be observed that treatments T1, T2, and T3 have the least effectiveness and high variability. While treatments T4 significantly improve in effectiveness and consistency. Furthermore, treatment T5 has high effectiveness and good consistency, but treatment T6 stands out more, possessing the highest effectiveness and the best consistency (with a lower variance of 0.0012 and standard deviation of 0.0346).

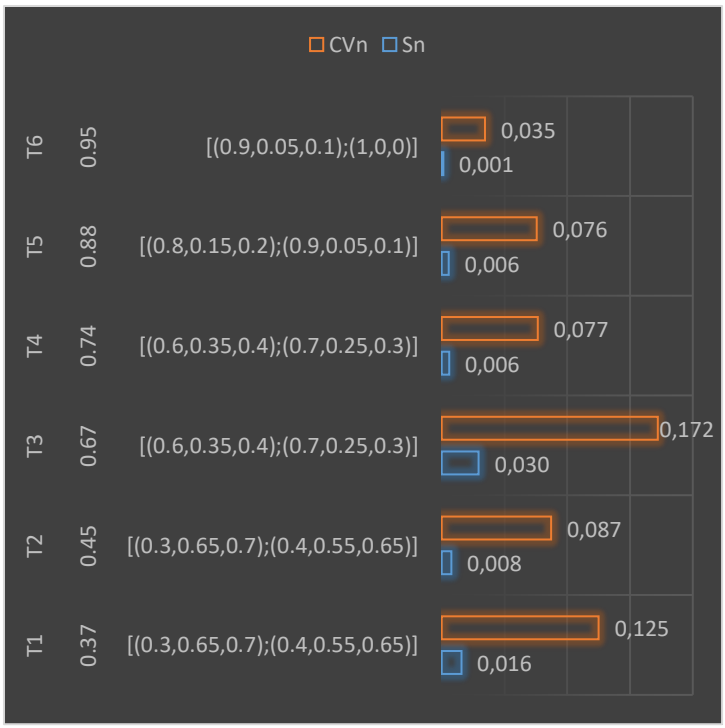

Figure 1: Calculation of the $\bar{L}, S_N$ and CV_N of the MLCGB variable. Source: own elaboration.

The concentration of 10^8 conidia of Beauveria bassiana (T6), especially through aspersion, is highly effective in controlling the white grub, especially with high concentrations and aspersion. The application of neutrosophic statistics allowed for precisely identifying the optimal conditions for its use. This emphasizes the need to adjust application practices to maximize efficacy and promote sustainable agriculture less dependent on chemical insecticides. Integrating these findings with agronomic practices could significantly improve integrated pest management. This treatment should be considered the preferred option for practical application, subject to additional considerations such as cost, applicability in the field, and environmental sustainability. Therefore, solutions should be proposed to enhance the application in other regions of Ecuador, such as promoting acceptance by potato growers.

3.2 Solutions to enhance the effectiveness of T6 Treatment.

To enhance the T6 treatment of Beauveria bassiana for the control of the white grub of the potato (Premnotrypes vorax) in Ecuador, two research projects are proposed as follows:

Project 1: Optimization of Beauveria bassiana Application by Aspersion for White Grub Control in Potato Crops

Scope: Develop improved aspersion application methodologies of Beauveria bassiana that increase the effectiveness in controlling the white grub in potato crops in Ecuador.

Estimated duration of the project: 2 years.

General objective: Increase the effectiveness of the T6 treatment through the optimization of aspersion techniques and dosing of Beauveria bassiana.

Specific objectives:

- Determine the optimal dosage of conidia for aspersion application.
- Evaluate the effectiveness of different types of aspersion equipment under field conditions.
- Develop application protocols that minimize the degradation of the biological agent by environmental factors.

Stages:

- Experimental design and preliminary data collection.
- Field trials under different climatic conditions and soil types.
- Data analysis and formulation of recommendations.

Resources:

- Biotechnology and agronomy laboratories.
- Experimental fields.
- Aspersion equipment.
- Funding by governmental and academic entities.

Expected impact: Reduction in the use of chemical pesticides and an increase in pest-free potato production.

Qualified personnel: Researchers in biotechnology, agronomy, and entomology.

Approval levels: Requires ethical and environmental approval.

Funding: Seeking funding through governmental research funds and international collaborations.

Results: Development of a best practices manual for the application of Beauveria bassiana in agriculture.

Benefits: Improvement of sustainability in agriculture and cost reduction for farmers.

Project 2: Long-term Environmental and Agronomic Impact Assessment of Beauveria bassiana on Potato Crops

Scope: Study the long-term effects of the continued use of Beauveria bassiana on local biodiversity and soil health in potato-growing areas.

Estimated duration of the project: 3 years.

General objective: Evaluate the environmental and agronomic impacts of prolonged use of Beauveria bassiana to ensure its viability as a sustainable solution.

Specific objectives:

- Monitor the biodiversity of insects and soil microorganisms in areas treated with Beauveria bassiana.
- Analyze the impact on soil health and the quality of potato crops.
- Develop strategies to mitigate any observed negative impacts.

Stages:

- Selection of study sites and methodological design.
- Implementation of treatments and continuous monitoring.
- Analysis of results and development of management guidelines.

Resources:

- Environmental monitoring equipment.
- Laboratories for soil and biodiversity analysis.
- Technical and scientific staff.

Expected impact: Validation of Beauveria bassiana as an ecologically efficient and long-term treatment.
Qualified personnel: Specialists in ecology, microbiology, and crop protection.
Approval levels: Regulatory approvals for long-term field studies.
Funding: Sustainability research funds and academic grants.
Results: Publication of studies in scientific journals and presentations at international conferences.
Benefits: Promote integrated pest management that is effective and ecological, thereby reducing dependence on synthetic pesticides and improving ecosystem health.

These projects involve active collaboration from Ecuadorian universities, providing academic and scientific resources. Additionally, they offer opportunities for students to conduct applied research and theses related to sustainable agriculture and biological pest control.

4 Conclusion

The use of neutrosophic statistics in this study has enabled a richer and more nuanced interpretation of the results. The research demonstrated that Beauveria bassiana, isolated from rabbit manure, is highly effective in controlling the white grub of the potato, especially when applied in concentrations of 10^8 conidia and by aspersion. This form of treatment showed significant efficacy, with very effective mortality of the white grub for an evaluation between (0.9,0.05,0.1) and (1,0,0) compared to the control group, where no significant mortality was observed. This result underscores the potential of Beauveria bassiana as a viable and effective biocontrol agent that can be integrated into pest management programs to reduce the use of chemical insecticides and promote more sustainable agriculture.

To maximize the efficacy of Beauveria bassiana and its sustainable use in agriculture, it is essential to adopt a comprehensive approach. This includes adjusting the conidial concentration and preferring aspersion for effective coverage, to actively control environmental conditions such as humidity and temperature to maintain the effectiveness of the biocontrol agent. Additionally, combining Beauveria bassiana with other pest management methods and agronomic practices strengthens the control of threats and minimizes dependence on single methods. Training farmers and technicians in best practices and the interpretation of results through neutrosophic statistics is crucial for improving decision-making and fostering resilient and productive agriculture. This deepened understanding helps to formulate more precise and adaptive strategies for the use of Beauveria bassiana, ensuring a more effective and efficient application.

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Model of Tax Culture Impact on the Financial Sustainability of Small and Medium Enterprises in Ecuador Based on Neutrosophic HyperSoft Sets

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Abstract. The tax culture of a country consists of several factors to take into account. Tax evasion is a widespread problem in the world today. Ecuadorian Small and Medium Enterprises (SMEs) as a whole have a great impact on the country's economy and have a weight to consider. This is why it is important to measure the tax behavior of Ecuadorian SMEs. This paper aims to propose a neutrosophic model to measure the tax culture of SMEs in Ecuador. To do this, we base on a model designed from the Neutrosophic theory and HyperSoft Sets. Hypersoft Sets extend Soft Sets from a single parameter to multiple parameters. On the other hand, when it is hybridized with the Single-Valued Neutrosophic Sets theory allows the inclusion of indeterminacy in this model. The article illustrates the model with an example.

Keywords: Tax Culture, Financial Sustainability, Small and Medium Enterprises (SMEs), Single-Valued Neutrosophic Set, Hypersoft Set, Neutrosophic Hypersoft Set, Decision-Making.

1 Introduction

The study of the impact of tax culture on the financial sustainability of small and medium-sized companies in Ecuador is crucial for several reasons. Firstly, SMEs represent a significant part of the Ecuadorian economy and their financial stability is essential for economic growth and job creation. In addition, the tax culture influences the investment decisions, competitiveness, and compliance capacity of these companies. Understanding how tax culture affects SMEs will identify barriers and opportunities to improve their economic performance, and promote a more sustainable and prosperous business environment in Ecuador.

Defining tax culture implies having several definitions. It refers to the attitudes, beliefs, and social norms of a population regarding the payment of taxes. It includes the perception of the fairness of the tax system, trust in tax authorities and the willingness to voluntarily comply with tax obligations. A strong tax culture can promote voluntary tax compliance, while a weak culture can lead to high levels of tax evasion.

Tax culture can also be understood as the set of values, norms and attitudes shared by the members of a society with respect to taxes and their role in public life. This includes the perception of taxes as a fair and necessary contribution to social well-being, as well as confidence in the government's ability to use fiscal resources efficiently and equitably.

To evaluate the tax culture of medium and small companies in Ecuador, it is necessary to take into account a group of indicators. By evaluating these dimensions, we can obtain a complete vision of the level of tax culture in Ecuadorian SMEs and its impact on their financial sustainability. This can serve as a basis for identifying areas of improvement and designing strategies to strengthen the tax culture and optimize the financial situation of SMEs in Ecuador.

On the other hand, there are diverse and dissimilar components within this single concept. In addition, to measure the behavior of the tax culture, the legal, and psychological part of the taxpayers, social, economic, and financial, among many others, must be taken into account. The combination of all these factors makes the study of this concept complex. From the epistemological point of view, its study must encompass the components of uncertainty and indeterminacy that characterize it.

The purpose of this article is to design a model to measure how tax culture impacts the financial sustainability of Ecuadorian SMEs. To achieve this, we apply Neutrosophic Hypersoft sets [1-4]. Neutrosophic Sets were first

defined by Professor F. Smarandache to generalize the theories of Fuzzy Sets, Intuitionistic Fuzzy Sets, Interval-Valued Fuzzy Sets, among other similar theories of uncertainty [5]. Smarandache's contribution is in the explicit incorporation of a membership function of indeterminacy that does not necessarily have a restrictive condition with respect to the membership and non-membership functions. The types of Neutrosophic Sets that have practical importance are the Single-Valued Neutrosophic Sets and Interval-Valued Neutrosophic Sets.

On the other hand, Professor Molodtsov defined Soft Sets that generalize fuzzy sets, where a soft set is a parameterized family of subsets of an initial set [6, 7]. This theory has applications in both pure mathematics such as topology and applied mathematics such as decision-making. There were already hybrid theories where soft sets were combined with fuzzy sets, among others [8-13].

Smarandache also defined Hypersoft Sets that extend the approximation function from a single attribute to multiple attributes [14-16]. The Neutrosophic Hypersoft Sets assign to each element of the Hypersoft Set a set of neutrosophic truth values to each element for each attribute [1-4, 7]. This is the tool we selected to model this problem.

The paper consists of the following structure, a Preliminaries section where the basic foundations of Neutrosophic theory and Hypersoft Sets are summarized. The next section contains the details of the proposed model and an illustrative example. The paper ends with the conclusions.

2 Preliminaries

This section contains the fundamental definitions of Neutrosophic Hypersoft Sets.

Definition 1: ([5, 18]) Let X be a universe of discourse. A *Neutrosophic Set* (NS) is characterized by three membership functions, $u_A(x), r_A(x), v_A(x): X \to \,]^-0, 1^+[$, which satisfy the condition $^-0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^+$ for all $x \in X$. $u_A(x), r_A(x)$ and $v_A(x)$ are the membership functions of truthfulness, indeterminacy and falseness of x in A, respectively, and their images are standard or non-standard subsets of $]^-0, 1^+[$.

Definition 2: ([5, 18]) Let X be a universe of discourse. A *Single-Valued Neutrosophic Set* (SVNS) A on X is a set of the form:

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\} \quad (1)$$

Where $u_A, r_A, v_A: X \to [0,1]$, satisfy the condition $0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3$ for all $x \in X$. $u_A(x), r_A(x)$and $v_A(x)$ denote the membership functions of truthfulness, indeterminate, and falseness of x in A, respectively. For convenience, a *Single-Valued Neutrosophic Number* (SVNN) will be expressed as $A = (a, b, c)$, where $a, b, c \in [0,1]$ and satisfy $0 \leq a + b + c \leq 3$.

Definition 3: ([18]) Given U is the initial universe set and E is the set of parameters. A pair (F, E) is called a *soft set* (over U) if and only if F is a mapping of E into the set of all subsets of U.

That is to say, having a set E of parameters and fixing a parameter $\varepsilon \in E$, then $F(\varepsilon) \in \mathcal{P}(U)$, where $\mathcal{P}(U)$ denotes the power set of U and $F(\varepsilon)$ is considered the set of ε-elements of the Soft Set (F, E) or the set of ε-approximate elements of the Soft Set.

It is not difficult to realize that fuzzy sets are soft sets, this is a consequence of the α-levels definition of a membership function μ_A where we have the following:

$F(\alpha) = \{x \in U \mid \mu_A(x) \geq \alpha\}$, $\alpha \in [0, 1]$. Thus, if we know the family F, then we can reconstruct the function μ_A by using the following formula:

$$\mu_A(x) = \sup_{\substack{\alpha \in [0,1] \\ x \in F(\alpha)}} \alpha$$

Thus, a fuzzy set is a $(F, [0, 1])$ soft set.

Given a binary operation * for subsets of the set U, where (F, A) and (G, B) are soft sets over U. Then, the operation * for soft sets is defined as follows:

$(F, A) * (G, B) = (J, A \times B)$, where $J(\alpha, \beta) = F(\alpha) * G(\beta)$; $\alpha \in A$, $\beta \in B$, and $A \times B$ is the Cartesian product of the sets A and B.

Definition 4: ([18]) Given U the initial universe set and $\mathcal{P}(U)$ is the power set of U, and for $n \geq 1$ there are n distinct attributes $a_1, a_2, \ldots, a_n$ whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(F, A_1 \times A_2 \times \ldots \times A_n)$ where $F: A_1 \times A_2 \times \ldots \times A_n \to \mathcal{P}(U)$ is called a *Hypersoft Set* over U.

Definition 5: ([18]) Given X is the initial universe set and $\mathcal{P}(X)$ is the power set of X, and for $n \geq 1$ there are n distinct attributes $a_1, a_2, \ldots, a_n$ whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then, the pair (ψ, Λ) is a *Neutrosophic HyperSoft Set* (NHSS) over X if there exists a relation $A_1 \times A_2 \times \ldots \times A_n = \Lambda$. ψ is a mapping from $A_1 \times A_2 \times \ldots \times A_n$ to $\mathcal{P}(X)$ and $\psi_\Lambda(A_1 \times A_2 \times \ldots \times A_n) = \{\langle x, u_\Lambda(x), r_\Lambda(x), v_\Lambda(x)\rangle : x \in X\}$, where $u_\Lambda, r_\Lambda, v_\Lambda : X \to [0,1]$, satisfy the condition $0 \leq u_\Lambda(x) + r_\Lambda(x) + v_\Lambda(x) \leq 3$ for all $x \in X$. $u_\Lambda(x), r_\Lambda(x)$, and $v_\Lambda(x)$ denote the membership functions of truthfulness, indeterminacy, and falseness of x in Λ, respectively.

Definition 6: ([18]) Given two Neutrosophic Hypersoft sets ψ_{Λ_1} and ψ_{Λ_2} over U, ψ_{Λ_1} is called a *Neutrosophic Hypersoft Subset* of ψ_{Λ_2} if $T(\psi_{\Lambda_1}) \leq T(\psi_{\Lambda_2})$, $I(\psi_{\Lambda_1}) \geq I(\psi_{\Lambda_2})$ and $F(\psi_{\Lambda_1}) \geq F(\psi_{\Lambda_2})$.

Definition 7: ([18]) Given two Neutrosophic Hypersoft sets ψ_{Λ_1} and ψ_{Λ_2} over U, ψ_{Λ_1} and ψ_{Λ_2} are *equal* if and only if $T(\psi_{\Lambda_1}) = T(\psi_{\Lambda_2})$, $I(\psi_{\Lambda_1}) = I(\psi_{\Lambda_2})$, and $F(\psi_{\Lambda_1}) = F(\psi_{\Lambda_2})$.

Definition 8: ([18]) Given a Neutrosophic Hypersoft set ψ_Λ, its *complement* $(\psi_\Lambda)^c$ is defined by $(\psi_\Lambda)^c = \{\langle u, T((\psi_\Lambda)^c) = F(\psi_\Lambda), I((\psi_\Lambda)^c) = I(\psi_\Lambda), F((\psi_\Lambda)^c) = T(\psi_\Lambda)\rangle : u \in U\}$.

Definition 9: ([18]) Given two Neutrosophic Hypersoft sets ψ_{Λ_1} and ψ_{Λ_2} over the common universe U. The *union* of them which is denoted by $\psi_{\Lambda_1} \cup \psi_{\Lambda_2}$ is the Neutrosophic Hypersoft Set defined as follows:

$$T(\psi_{\Lambda_1} \cup \psi_{\Lambda_2}) = \begin{cases} T(\psi_{\Lambda_1}) \text{ if } u \in \Lambda_1 \setminus \Lambda_2, \\ T(\psi_{\Lambda_2}) \text{ if } u \in \Lambda_2 \setminus \Lambda_1, \\ \max(\psi_{\Lambda_1}, \psi_{\Lambda_2}) \text{ if } u \in \Lambda_1 \cap \Lambda_2, \end{cases}$$

$$I(\psi_{\Lambda_1} \cup \psi_{\Lambda_2}) = \begin{cases} I(\psi_{\Lambda_1}) \text{ if } u \in \Lambda_1 \setminus \Lambda_2, \\ I(\psi_{\Lambda_2}) \text{ if } u \in \Lambda_2 \setminus \Lambda_1, \\ \min(\psi_{\Lambda_1}, \psi_{\Lambda_2}) \text{ if } u \in \Lambda_1 \cap \Lambda_2, \end{cases}$$

$$F(\psi_{\Lambda_1} \cup \psi_{\Lambda_2}) = \begin{cases} F(\psi_{\Lambda_1}) \text{ if } u \in \Lambda_1 \setminus \Lambda_2, \\ F(\psi_{\Lambda_2}) \text{ if } u \in \Lambda_2 \setminus \Lambda_1, \\ \min(\psi_{\Lambda_1}, \psi_{\Lambda_2}) \text{ if } u \in \Lambda_1 \cap \Lambda_2, \end{cases}$$

Definition 10: ([18]) Given two Neutrosophic Hypersoft sets ψ_{Λ_1} and ψ_{Λ_2} over the common universe U. The *intersection* of them which is denoted by $\psi_{\Lambda_1} \cap \psi_{\Lambda_2}$ is the Neutrosophic Hypersoft Set defined as follows:

$$T(\psi_{\Lambda_1} \cap \psi_{\Lambda_2}) = \begin{cases} T(\psi_{\Lambda_1}) \text{ if } u \in \Lambda_1 \setminus \Lambda_2, \\ T(\psi_{\Lambda_2}) \text{ if } u \in \Lambda_2 \setminus \Lambda_1, \\ \min(\psi_{\Lambda_1}, \psi_{\Lambda_2}) \text{ if } u \in \Lambda_1 \cap \Lambda_2, \end{cases}$$

$$I(\psi_{\Lambda_1} \cap \psi_{\Lambda_2}) = \begin{cases} I(\psi_{\Lambda_1}) \text{ if } u \in \Lambda_1 \setminus \Lambda_2, \\ I(\psi_{\Lambda_2}) \text{ if } u \in \Lambda_2 \setminus \Lambda_1, \\ \max(\psi_{\Lambda_1}, \psi_{\Lambda_2}) \text{ if } u \in \Lambda_1 \cap \Lambda_2, \end{cases}$$

$$F(\psi_{\Lambda_1} \cap \psi_{\Lambda_2}) = \begin{cases} F(\psi_{\Lambda_1}) \text{ if } u \in \Lambda_1 \setminus \Lambda_2, \\ F(\psi_{\Lambda_2}) \text{ if } u \in \Lambda_2 \setminus \Lambda_1, \\ \max(\psi_{\Lambda_1}, \psi_{\Lambda_2}) \text{ if } u \in \Lambda_1 \cap \Lambda_2, \end{cases}$$

Definition 11: ([18]) Given two Neutrosophic Hypersoft sets ψ_{Λ_1} and ψ_{Λ_2} over the common universe U. The *AND-Operation* between them $\psi_{\Lambda_1} \wedge \psi_{\Lambda_2}$ is the Neutrosophic Hypersoft Set defined as follows:

$$T(\psi_{\Lambda_1} \wedge \psi_{\Lambda_2}) = \min(T(\psi_{\Lambda_1}), T(\psi_{\Lambda_2})),$$

$$I(\psi_{\Lambda_1} \wedge \psi_{\Lambda_2}) = \max(I(\psi_{\Lambda_1}), I(\psi_{\Lambda_2})),$$

$$F(\psi_{\Lambda_1} \wedge \psi_{\Lambda_2}) = \max(F(\psi_{\Lambda_1}), F(\psi_{\Lambda_2})).$$

Definition 12: ([18, 19]) Given two Neutrosophic Hypersoft sets ψ_{Λ_1} and ψ_{Λ_2} over the common universe U. The *OR-Operation* between them $\psi_{\Lambda_1} \wedge \psi_{\Lambda_2}$ is the Neutrosophic Hypersoft Set defined as follows:

$$T(\psi_{\Lambda_1} \wedge \psi_{\Lambda_2}) = \max(T(\psi_{\Lambda_1}), T(\psi_{\Lambda_2})),$$

$$I(\psi_{\Lambda_1} \wedge \psi_{\Lambda_2}) = \min(I(\psi_{\Lambda_1}), I(\psi_{\Lambda_2})),$$

$$F(\psi_{\Lambda_1} \wedge \psi_{\Lambda_2}) = \min(F(\psi_{\Lambda_1}), F(\psi_{\Lambda_2})).$$

3 The proposed model

In this section, we present the details of the proposed model. We must have a set of experts denoted by $E = \{e_1, e_2, \ldots, e_n\}$, where $n \geq 1$.

The points to measure the tax culture of an SME that are important for economic sustainability are the following:

1- Tax Knowledge:

a) Understanding of tax laws and regulations applicable to SMEs in Ecuador.
b) Knowledge of different taxes (VAT, income tax, municipal taxes, etc.) and their corresponding tax obligations.
c) Familiarity with procedures for compliance with tax obligations, such as filing and paying taxes.

2- Tax Compliance:

a) Degree of compliance with tax obligations by SMEs, including timely filing of returns and payment of taxes.
b) Non-existence of tax evasion or avoidance practices within SMEs.
c) Use of tax incentives or tax benefits available to SMEs and their impact on tax compliance.

3- Attitudes and Perceptions towards Taxes:

a) SME owners, managers, and employees towards the tax system.
b) Perceptions about the equity of the tax system and distributive justice.
c) Level of trust in tax authorities and their ability to effectively manage fiscal resources.

4- Tax management:

a) Existence of departments or personnel dedicated to tax management within SMEs.
b) Use of software or technological tools to facilitate tax management and compliance with tax obligations.
c) Strategies used to legally and ethically minimize the tax burden, such as tax planning and tax credit optimization.

5- Economic and Financial Impact:

a) Costs associated with compliance with tax obligations and their impact on the profitability and liquidity of SMEs.
b) Effects of changes in tax policy on the financial viability of SMEs.
c) Relationship between the tax burden and the capacity for investment, growth, and employment generation in SMEs.

Note that each dimension to be measured was listed using Arabic numerals and each of them has certain important points that constitute it, and for this we use letters. In summary, we have five dimensions, each of them made up of three aspects to measure.

Let us denote the dimensions with the set $D = \{d_1, d_2, d_3, d_4, d_5\}$, each of the subcriteria to be measured by the i[th] dimension is denoted by d_{ia}, d_{ib}, d_{ic}.

There is a measurement scale with linguistic values that is recommended to experts to be used in decision-making. See Table 1.

Table 1: Scale of linguistic terms associated with neutrosophic values.

Linguistic expressions	**(T, I, F)**
Very Bad (VB)	(0.10, 0.75, 0.85)
Bad (B)	(0.25, 0.60, 0.80)
Medium Bad (MB)	(0.40, 0.70, 0.50)
Medium (M)	(0.50, 0.40, 0.60)

Linguistic expressions	**(T, I, F)**
Medium Good (MG)	$(0.65, 0.30, 0.45)$
Good	$(0.80, 0.10, 0.30)$
Very Good (VG)	$(0.95, 0.05, 0.05)$

The steps to follow for the evaluation are given below:

Steps to follow to evaluate the tax culture of an SME

1. Input: E $= \{e_1, e_2, \ldots, e_n\}$ is the set of experts, where $n \geq 1$.
D $= \{d_1, d_2, d_3, d_4, d_5\}$: Dimensions to measure,
d_{ia}, d_{ib}, d_{ic}: Each of the three aspects to measure the i[th] dimension. The dimensions constitute the parameters to measure, where the elements of the set are $d_{ia} \times d_{ib} \times d_{ic}$ which means all the conditions of the i[th] dimension are met, or the element $\sim(d_{ia} \times d_{ib} \times d_{ic})$ which means at least one of these conditions is not met.
Given an SME, or a group of SMEs in a city, region, etc. in Ecuador, which we wish to study regarding their tax culture. This set is denoted by U $= \{P_1, P_2, \ldots, P_m\}$, $m \geq 1$.
Each expert in E evaluates each of the subcriteria to be measured for each of the SMEs.

$a_{i_k jl}$ is the evaluation according to the scale shown in Table 1, by the j[th] expert concerning the subcriterion i_k of the i[th] dimension for the l[th] SME. j $\in \{1, 2, \ldots, n\}$, i $\in \{1, 2, \ldots, 5\}$, k $\in \{a, b, c\}$ and l $\in \{1, 2, \ldots, m\}$.

2. Given i, k, and l fixed, we have a single triple of numerical values $\bar{a}_{i_k l}$ that is the median of the values $a_{i_k jl}$ for all experts. These values will be taken as the evaluation to be processed.

3. It is established to include in the image ψ_Λ those values greater than or equal to M in Table 1.

4. Now fixing the i[th] dimension, two-parameter options are obtained: "Satisfied" and "Not Satisfied" for each dimension.

When $\min(\bar{a}_{i_a l}, \bar{a}_{i_b l}, \bar{a}_{i_c l}) \succcurlyeq M$ then the dimension is " Satisfied " with a Single-Valued Neutrosophic Number equals to the minimum of the neutrosophic valuations of the three values.

Otherwise, that is, when $\min(\bar{a}_{i_a l}, \bar{a}_{i_b l}, \bar{a}_{i_c l}) \prec M$, then the dimension is considered to be "Not Satisfied" with a Single-Valued Neutrosophic Number equals to the minimum of the neutrosophic valuations of the three values.

5. Thus, there is a Cartesian set made up of five sets Λ: $D_1 \times D_2 \times D_3 \times D_4 \times D_5$ to measure the tax culture of SMEs. There is also the function:

ψ_Λ: $D_1 \times D_2 \times D_3 \times D_4 \times D_5 \rightarrow \mathcal{P}(\mathrm{U})$.

6. With these results, queries can be made using an AND-Operation (Definition 11) or an OR- Operation (Definition 12).

Let us illustrate this model using an example.

Example 1. Suppose that three experts evaluate three Ecuadorian SMEs according to the 15 elements that constitute the subcriteria using the scale that appears in Table 1.

Tables 2-4 collect these results:

Table 2: Evaluation by Expert 1 on the 15 sub-criteria for the three SMEs.

Subcriterion /SME	**P_1**	**P_2**	**P_3**
d_{1a}	M	MB	MB
d_{1b}	G	M	MB
d_{1c}	M	M.B	M
d_{2a}	MG	B	MB
d_{2b}	MG	B	B
d_{2c}	MG	MG	M
d_{3a}	M	B	MB

Subcriterion /SME	P_1	P_2	P_3
d_{3b}	M	MG	VB
d_{3c}	M	M	VB
d_{4a}	MG	MG	MB
d_{4b}	G	M	VB
d_{4c}	G	M	VB
d_{5a}	M	B	MB
d_{5b}	MG	MB	B
d_{5c}	MG	MB	B

Table 3: Evaluation by Expert 2 on the 15 sub-criteria for the three SMEs.

Subcriterion /SME	P_1	P_2	P_3
d_{1a}	VG.	MG	MB.
d_{1b}	M	MB	M
d_{1c}	MG	MG	VB
d_{2a}	M	MG	M
d_{2b}	MG	B	VB
d_{2c}	M	MB	VB
d_{3a}	M	B	M
d_{3b}	VG.	M	MB
d_{3c}	G	MB	B
d_{4a}	MG	M	M
d_{4b}	VG.	B	VB
d_{4c}	G	MB	MB
d_{5a}	VG.	B	M
d_{5b}	M	MG	B
d_{5c}	M	B	M

Table 4: Evaluation by Expert 3 on the 15 sub-criteria for the three SMEs.

Subcriterion /SME	P_1	P_2	P_3
d_{1a}	VG	MG	B
d_{1b}	VG	MB	MB
d_{1c}	G	MB	M
d_{2a}	VG.	B	VB
d_{2b}	MG	B	M
d_{2c}	MG	M	MB
d_{3a}	VG	M	B
d_{3b}	M	MB	MB
d_{3c}	G	M	VB
d_{4a}	VG	B	VB
d_{4b}	VG	MB	B
d_{4c}	VG	B	M
d_{5a}	VG	MB	B
d_{5b}	VG	MB	M
d_{5c}	MG	B	B

The results of calculating the medians for the evaluations of all the experts from Tables 2-4 are shown in Table 5.

Table 5: Aggregated evaluation of all experts on the 15 sub-criteria for the three SMEs.

Subcriterion /SME	P_1	P_2	P_3
d_{1a}	VG.	MG	MB
d_{1b}	G	MB.	MB
d_{1c}	MG	MB	M

Subcriterion /SME	P_1	P_2	P_3
d_{2a}	MG	B	MB
d_{2b}	MG	B	B
d_{2c}	MG	M	MB
d_{3a}	M	B	MB
d_{3b}	M	M	MB
d_{3c}	G	M	VB
d_{4a}	MG	M	MB
d_{4b}	VG	MB	VB
d_{4c}	G	MB	MB
d_{5a}	VG	B	MB
d_{5b}	MG	MB	B
d_{5c}	MG	B	B

With the data in Table 5, it can be reduced to the "Satisfied" or "Not Satisfied" members of each dimension as shown in Table 6.

Table 6: Evaluation of each dimension for the three SMEs according to the parameters "Satisfied" and "Not Satisfied".

Dimension/Parameters	Satisfied	Not Satisfied
D_1	P_1 (MG)	P_2 (MB), P_3 (MB)
D_2	P_1 (MG)	P_2 (B), P_3 (B)
D_3	P_1 (M)	P_2 (B), P_3 (VB)
D_4	P_1 (MG)	P_2 (MB), P_3 (VB)
D_5	P_1 (MG)	P_2 (B), P_3 (B)

Therefore, from Table 6, if we want to consult the status of SMEs then:

$\psi_A(Satisfied, Satisfied, Satisfied, Satisfied, Satisfied) =$
$\{\langle P_1, ((0.65, 0.30, 0.45), (0.65, 0.30, 0.45), (0.50, 0.40, 0.60)), (0.65, 0.30, 0.45), (0.65, 0.30, 0.45)\rangle\}$.

That is, the SME denoted by P_1 is the only one that satisfies all the conditions imposed to be considered as complying with the tax culture. Note the degrees of truthfulness, indeterminacy, and falseness indicated by each dimension.

4 Conclusion

Ecuadorean SMEs have an important economic and social weight for the proper economic functioning of this Andean country. This is why a good tax culture guarantees the economic sustainability of the country, the community, and therefore the SME itself. This article proposes a neutrosophic model to measure the correct tax functioning of SMEs. For this, we use the theory of Neutrosophic Hypersoft Sets. The advantages of this model are the ease with which it can be automated, it is capable of dealing with uncertainty and indeterminacy, in addition to the fact that experts base their evaluation on an easily understandable linguistic scale. We illustrate the use of the model with an example that shows all the advantages that we have highlighted.

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Neutrosophic Statistical Analysis of the Efficacy of Biostimulants in Amaranth Varieties

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Abstract. This study evaluated the effects of biostimulants on two varieties of amaranth, Valentina (Amaranthus tricolor) and Nezhenka (Amaranthus hybridus), in terms of germination, growth, and biomass production. Using neutrosophic statistics allowed for the measurement of indeterminacies and the analysis of variables such as days to germination, stem length, number of leaves, and biomass production. The results showed that seaweed extract was the most effective biostimulant, accelerating germination, increasing stem length and the number of leaves, and maximizing biomass production. Meanwhile, Nezhenka stood out for its superior biomass production, especially with seaweed extract, while Valentina was notable for its rapid germination. These findings suggest that the selection of biostimulants and varieties should be strategic to optimize agricultural yield.

Keywords: Germination, seaweed extract, cultivation, neutrosophic Statistics

1 Introduction

The genus Amaranthus includes between 60 and 90 species, with approximately 55 of them distributed globally. Specifically, Russia hosts about 16 species in the wild. In Ecuador, the Alegría variety is cultivated by the National Institute of Agricultural Research (INIAP). Historical and genetic studies have demonstrated that the origin of Amaranthus is in America [1], with A. cruentus, A. caudatus, and A. hipochondriacus as the main species domesticated for grain [2]. Another notable point about Amaranthus is that they descend from the wild species A. powelli, A. quitensis, and A. hybridus respectively. It is suggested that A. quitensis might be synonymous with A. hybridus, which would be the ancestor of these cultivated species.

The species suitable for foliage production include A. dubius, A. blitum, A. hybridus, A. quitensis, and A. tricolor [3]. In terms of selection and hybridization, modern varieties of amaranth include A. cruentus, A. caudatus, A. hipochondriacus, A. mantegazzianus, and A. paniculatus for seed production, while A. tricolor, A. caudatus, A. spinosus, and A. cruentus are used both in horticulture and in decorative and medicinal applications [4].

Historically, amaranth has been a fundamental crop in America for over 4000 years, promoted by the Mayas in Mexico and Guatemala and the Incas in Ecuador, Peru, and Bolivia. White seed varieties were preferred over dark ones, aiding their domestication. Currently, grain amaranth production is most notable in the inter-Andean valleys of Peru, Bolivia, and northern Argentina, although it is marginal in the Colombian and Ecuadorian highlands.

In India, China, and some African countries, amaranth is mainly cultivated for biomass production as a vegetable. Nutritionally [5], 150-200 grams of amaranth leaves are equivalent to 1 kg of tomatoes or cucumbers, and 100 grams of leaves contain 13.1 g of dry matter, 3.5 g of protein, 0.5 g of fat, 256 mg of calcium, 67 mg of phosphorus, 3.9 mg of iron, 411 mg of sodium, and 80 mg of vitamin C [6]. Additionally, 18 sterols have been identified in the leaves of amaranth [7], some with medical applications for treating atherosclerosis [8].

The use of biostimulants in this study is motivated by the high dependence on agrochemicals in Ecuador, where between 60% and 70% of cultivated lands are treated with these products. A biostimulant is a substance or microorganism that enhances the efficiency of plants in nutrient absorption and assimilation, increases stress tolerance, and improves agronomic characteristics, irrespective of its nutritional content.

This study underscores the agronomic and nutritional relevance of amaranth, as well as the need to optimize its production through the conscious use of biostimulants to reduce dependence on agrochemicals. Therefore, the research aims to evaluate the effect of different biostimulators on the growth and development of the amaranth

varieties Valentina and Nezhenka, to determine which of these products offers the greatest improvement in terms of days to germination, stem length, number of leaves, and biomass production. For the modeling and development of the study, the use of neutrosophic statistics is necessary to include, analyze, and assess the existing indeterminacies.

2 Materials and Methods

2.1 Neutrosophic Statistics

Neutrosophic probabilities and statistics are an extension of classical methods that incorporate a third component of indeterminacy [9], allowing for the analysis of events with imprecise, incomplete, or unknown information. This approach utilizes neutrosophic numbers and neutrosophic random variables to describe uncertainty [10] and is based on the definition of neutrosophic probabilities that sum up to a maximum of three, instead of one [11]. Neutrosophic descriptive statistics, such as neutrosophic mean, variance, and coefficient of variation, are used to summarize and analyze data with indeterminate or complex characteristics (according to the methodology in referenced studies [12] [13] [14] [15]).

Neutrosophic inferential statistics involve techniques that enable the extension of neutrosophic sampling to a population that the sample was taken from.

Neutrosophic data are data that possess a degree of indeterminacy. Similar to classical statistics, it can be categorized as:

- Discrete neutrosophic data refers to values that are individual points, such as 7+i_1, where i_1 is a number between 0 and 1, 2, 38+i_2, where i_2 is a number between 10 and 12. - Continuous neutrosophic data refers to values that form one or more intervals, such as [0.05, 0.1] or [0.9, 1.0] (meaning it is uncertain which specific value within the interval is being referred to).

Additional categorization:

- Neutrosophic data can be classified into two types: quantitative (numerical) and qualitative (categorical). - Quantitative neutrosophic data refers to numerical values that are uncertain, such as a number within the range of 3 to 8, or a set of numbers like 50, 53, 58, or 61, where the exact value is unknown.

- Qualitative neutrosophic data refers to categorical values that are uncertain, such as colors like blue or red, or a set of colors like white, black, green, or yellow, where the exact color is unknown.

In addition, we can categorize the data into two types: neutrosophic data univariate, which refers to data that consists of observations on a single neutrosophic attribute, and multivariate neutrosophic data, which refers to data that consists of observations on two or more attributes. Specifically, we refer to the bivariate neutrosophic data and the trivariate neutrosophic data in certain instances [16, 17].

3 Results

3.1 Study Environment

In the study conducted in Cunchibamba, Tungurahua, a 2x4 factorial experimental design with four replications and a total of 32 experimental units was used to evaluate the effect of different biostimulants on two varieties of amaranth, A. tricolor (Valentina) and A. hybridus (Nezhenka). Each experimental unit measured 3m x 1m, totaling 3 m², with 20 plants per unit and 10 plants per evaluated plot. The evaluated factors included:

- Factor A: Amaranthus Varieties
- V1: Valentina (A. tricolor)
- V2: Nezhenka (A. hybridus)
- Factor B: Growth Biostimulants (see Table 1)

For each variety, agronomic parameters such as days to emergence, stem length, number of leaves per plant, and biomass production were measured. Stem length was measured every 15 days from the base of the stem to the beginning of the inflorescence, using a tape measure. Additionally, the number of leaves was assessed every 30 days from the base of the stem to the base of the panicle throughout the vegetative cycle.

Table 1: Description of biostimulant factors.

Code	Biostimulant	Description	Main application
B0	Without biostimulant	Control, no application of additional biostimulants.	Used to establish a baseline in the research, by comparing the effects of other treatments.
B1	seaweed extract	Contains micronutrients, vitamins, and natural hormones that promote growth and health.	Enhances germination and vegetative growth, and increases resistance to abiotic stress.

Code	Biostimulant	Description	Main application
B2	basfoliar algae	Prepared from seaweed, rich in essential nutrients such as potassium, iron, and zinc.	Stimulates root and leaf growth, improves nutrient absorption and enhances biomass quality.
B3	Active basfoliar	Combination of microelements and growth promoters designed for rapid absorption.	Suitable for critical stages of development, increases biomass production, and improves photosynthetic efficiency.

Yield in kg/ha was determined by collecting leaf samples from 1m² per experimental unit. Fresh samples were weighed and then dried on aluminum trays and manila envelopes inside an incubator. The drying process began at 30°C for 48 hours, increasing to 60°C for an additional 24 hours, after which the dry weight was recorded in grams and converted to kg/ha.

Each of these biostimulants offers different advantages and mechanisms of action, which are explored in the study to determine their impact on the growth and development of the amaranth varieties Nezhenka and Valentina. Therefore, it allows researchers and agricultural practitioners to better understand how each type of biostimulant should be used to optimize cultivation practices and achieve desirable agronomic results.

3.2 Germination parameter

Once the study environment has been analyzed, the neutrosophic ranges of the germination variable are defined. Subsequently, the Valentina and Nezhenka samples are evaluated and analyzed against this parameter, reflecting the previous results and observations. Below, the ranges and germination results of the Amaranths species are presented (see Tables 2 and 3).

Table 2: Neutrosophic ranges for the germination variable.

Code	Neutrosophic expression	Neutrosophic number	Variable range	Description
G1	Very fast	(0.95, 0.05, 0.00)	≤ 4 days	Germination in less than 5 days
G2	Fast	(0.75, 0.20, 0.05)	5-7 days	fast germination
G3	Moderate	(0.50, 0.30, 0.20)	8-10 days	Moderate germination
G4	Slow	(0.25, 0.50, 0.25)	11-14 days	slow germination
G5	Very slow	(0.05, 0.70, 0.25)	≥ 15 days	Very slow germination

Table 3: Germination results of Amaranths species.

Variety	Days until germination	Neutrosophic evaluation	Triple neutrosophic number	Description
Valentina	4	G1 (Very Fast)	(0.95, 0.05, 0.00)	Germination < 5 days
Valentina	6	G2 (Fast)	(0.75, 0.20, 0.05)	Germination 5-7 days
Nezhenka	9	G3 (Moderate)	(0.50, 0.30, 0.20)	Germination 8-10 days
Nezhenka	12	G4 (Slow)	(0.25, 0.50, 0.25)	Germination 11-14 days

Neutrosophic statistical analysis:

I. Valentina:
   - It exhibits "Very Fast" germination in just 4 days, aligning with the G1 range. This reflects its adaptability and efficiency under optimal conditions, with early seedling emergence.
   - After 6 days, its germination is classified as "Rapid" (G2), indicating that even slightly outside the ideal conditions, Valentina maintains a good emergence rate.

II. Nezhenka:
   - At 9 days, it falls into the "Moderate" range (G3), showing slower germination compared to Valentina, but still within an acceptable framework for healthy development.
   - For 12 days, its germination is described as "Slow" (G4), which underlines its longer cycle nature and slower adaptation compared to Valentina.

Based on the neutrosophic data and the description of the previous results, it underscores how Valentina and Nezhenka respond to germination conditions. Valentina demonstrates greater speed and effectiveness in seedling emergence, while Nezhenka shows a more gradual process. This reflects the importance of selecting the right variety according to the specific conditions and goals of the crop.

3.3 Stem Length Parameter

Secondly, the neutrosophic ranges of the stem length variable are defined, to evaluate and analyze the samples of Valentina and Nezhenka and their development in relation to this parameter. Below, the ranges and the results

of stem length in the Amaranths varieties Valentina and Nezhenka are presented (see Tables 4 and 5).

Table 4: Neutrosophic ranges for variable stem length.

Code	Neutrosophic expression	Neutrosophic number	Variable range	Description
L1	Very Low	(0.10, 0.85, 0.05)	0-10cm	Insufficient growth
L2	Low	(0.30, 0.60, 0.10)	10-20cm	Low but improvable growth
L3	Moderate	(0.50, 0.45, 0.05)	20-30cm	Moderate growth
L4	High	(0.75, 0.20, 0.05)	30-40cm	Good growth almost ideal
L5	Very high	(0.90, 0.05, 0.05)	>40cm	Ideal excellent growth

Table 5: Results of variable stem length in Amaranths varieties.

Variety	Stem length (cm)	Neutrosophic evaluation	triple neutrosophic number	Description
Valentina	eleven	L2 (Low)	(0.30, 0.60, 0.10)	Low growth
Valentina	18	L3 (Moderate)	(0.50, 0.45, 0.05)	Moderate growth
Nezhenka	44	L5 (Very High)	(0.90, 0.05, 0.05)	Excellent growth
Nezhenka	30	L4 (High)	(0.75, 0.20, 0.05)	good growth

Neutrosophic statistical analysis:

I. Valentina:
- At 11 cm, Valentina falls into the "Low" range (L2), indicating insufficient growth, but with potential for improvement.
- At 18 cm, the plant shows "Moderate Growth" (L3), reflecting a substantial improvement under more favorable conditions, especially with the use of algae extract.

II. Nezhenka:
- At 44 cm, Nezhenka shows "Excellent Growth" (L5), indicating optimal adaptation and maximum use of resources under the seaweed extract treatment.
- At 30 cm, it is classified as "High" (L4), showing robust and productive growth, suitable for biomass production.

The neutrosophic data illustrate how Valentina and Nezhenka respond in terms of stem length under different treatments. While Valentina shows moderate growth, Nezhenka excels with very high growth, especially under the use of seaweed extract. These results underline the importance of selecting the right variety and treatment to optimize stem growth and the overall productivity of the plants. This neutrosophic approach provides a rich and detailed perspective for understanding and improving agronomic strategies based on the specific characteristics and responses of each amaranth variety.

Analysis of stem length for combinations of varieties with biostimulants: This neutrosophic analysis helps to visualize and quantify how biostimulants differentially affect amaranth varieties. It emphasizes the importance of properly selecting both the variety and the treatment. Additionally, agronomic results must be optimized based on the combination of the selected variety and biostimulant (see Table 6).

Table 6: Behavior of stem length for Amaranths varieties Valentina and Nezhenka under different treatments with biostimulators

Variety + Biostimulator	Stem Length (cm)	Neutrosophic Evaluation	Triple Neutrosophic Number	Description	Variety + Biostimulator
Nezhenka + Seaweed Extract (V2B1)	54.30	L5 (Very High)	(0.90, 0.05, 0.05)	Excellent growth	Nezhenka + Seaweed Extract (V2B1)
Nezhenka + Basfoliar Algae (V2B2)	50.44	L5 (Very High)	(0.90, 0.05, 0.05)	Excellent growth	Nezhenka + Basfoliar Algae (V2B2)
Nezhenka + Without Biostimulator (V2B0)	49.61	L5 (Very High)	(0.90, 0.05, 0.05)	Excellent growth	Nezhenka + Without Biostimulator (V2B0)
Nezhenka + Active Basfoliar (V2B3)	44.97	L5 (Very High)	(0.90, 0.05, 0.05)	Excellent growth	Nezhenka + Active Basfoliar (V2B3)
Valentina + Seaweed Extract (V1B1)	13.53	L2 (Low)	(0.30, 0.60, 0.10)	Low growth	Valentina + Seaweed Extract (V1B1)
Valentina + Basfoliar Algae (V1B2)	12.72	L2 (Low)	(0.30, 0.60, 0.10)	Low growth	Valentina + Basfoliar Algae (V1B2)
Valentina + Without Biostimulator (V1B0)	11.88	L2 (Low)	(0.30, 0.60, 0.10)	Low growth	Valentina + Without Biostimulator (V1B0)

Variety + Biostimulator	Stem Length (cm)	Neutrosophic Evaluation	Triple Neutrosophic Number	Description	Variety + Biostimulator
Valentina + Active Basfoliar (V1B3)	11.36	L2 (Low)	(0.30, 0.60, 0.10)	Low growth	Valentina + Active Basfoliar (V1B3)

Neutrosophic statistical analysis:

I. Nezhenka:
- With Algae Extract (V2B1), Nezhenka achieves "Excellent Growth" with 54.30 cm, showing the maximum efficiency of this biostimulator.
- With Active Basfoliar (V2B3), although the length is the smallest for Nezhenka in these treatments, it is classified as "Excellent Growth" with 44.97 cm, indicating that even the least effective of the treatments evaluated maintains high growth.

II. Valentina:
- With Seaweed Extract (V1B1), Valentina shows low growth (13.53 cm), which is the best result for this variety but is still in a low range.
- With Active Basfoliar (V1B3), stem length is the lowest for Valentina in these treatments at 11.36 cm. So it keeps growth at a low level.

The results of the study clearly show the superiority of Nezhenka in terms of stem length under all treatments, with values consistently in the "Very High" range. In contrast, Valentina shows improvements with the biostimulants but remains in the "Low" range. This indicates a limitation in its ability to utilize these treatments to significantly enhance its vertical growth.

3.4 Number of sheets parameter

Thirdly, the neutrosophic ranges for the variable number of leaves are defined, to evaluate and analyze the samples of Valentina and Nezhenka. Below, the ranges and the results for the number of leaves in the varieties, under different treatments with biostimulants, are presented (see Tables 7 and 8).

Table 7: Neutrosophic ranges for the variable number of leaves.

Code	Neutrosophic expression	Neutrosophic number	Variable range	Description
H1	Very Low	(0.10, 0.85, 0.05)	≤ 5 sheets	Insufficient number of sheets
H2	Low	(0.30, 0.60, 0.10)	6-10 sheets	Few leaves, needs improvement
H3	Moderate	(0.50, 0.45, 0.05)	11-15 sheets	Moderate number of leaves
H4	High	(0.75, 0.20, 0.05)	16-20 sheets	Good number of leaves
H5	Very high	(0.90, 0.05, 0.05)	> 20 sheets	Excellent number of leaves

Table 8: Behavior of the number of leaves for the Amaranths varieties Valentina and Nezhenka under the different treatments with biostimulators.

Variety + Biostimulator	Number of sheets	Neutrosophic Evaluation	Neutrosophic Number	Description
Nezhenka + Seaweed Extract (V2B1)	40.9	H5 (Very High)	(0.90, 0.05, 0.05)	Excellent number of leaves
Nezhenka + Basfoliar Algae (V2B2)	38.98	H5 (Very High)	(0.90, 0.05, 0.05)	Excellent number of leaves
Nezhenka + Without Biostimulator (V2B0)	38.45	H5 (Very High)	(0.90, 0.05, 0.05)	Excellent number of leaves
Nezhenka + Basfoliar active (V2B3)	33.28	H5 (Very High)	(0.90, 0.05, 0.05)	Excellent number of leaves
Valentina + Seaweed Extract (V1B1)	15.93	H3 (Moderate)	(0.50, 0.45, 0.05)	Moderate number of leaves
Valentina + Basfoliar Algae (V1B2)	14.68	H3 (Moderate)	(0.50, 0.45, 0.05)	Moderate number of leaves
Valentina + Without Biostimulator (V1B0)	12.83	H3 (Moderate)	(0.50, 0.45, 0.05)	Moderate number of leaves
Valentina + Basfoliar active (V1B3)	12.53	H3 (Moderate)	(0.50, 0.45, 0.05)	Moderate number of leaves

Neutrosophic statistical analysis:

I. Nezhenka:
   - It exhibits an "Excellent number of leaves" in all treatments, greatly exceeding the threshold of 20 leaves, reflecting the ability to produce a large amount of foliage under various treatment conditions.

II. Valentina:
   - It shows a "Moderate number of leaves" in all treatments with biostimulators, ranging between 11 and 15 leaves. Although it does not reach the levels of Nezhenka, it maintains consistent leaf growth that reflects its adaptive capacity under different treatments.

From the analyzed data, it is determined that Valentina and Nezhenka respond in terms of leaf production under different treatments with biostimulants. While Nezhenka demonstrates a superior ability to generate an exceptional number of leaves, Valentina exhibits moderate performance, indicating significant differences in the biological response between the two varieties.

3.5 Biomass Performance Parameter

Lastly, the biomass yield parameter (Kg/ha) is analyzed, using the following neutrosophic ranges of the variable defined in table 9. Additionally, the results and observations of biomass yield in amaranth crops are analyzed (see Table 10).

Table 9: Neutrosophic ranges for the biomass yield variable.

Code	Neutrosophic expression	Neutrosophic number	Variable range	Description
B1	Very Low	(0.05, 0.90, 0.05)	≤ 50 kg/ha	Insufficient biomass production
B2	Low	(0.25, 0.70, 0.05)	51-100 kg/ha	Low production, requires improvements
B3	Moderate	(0.50, 0.45, 0.05)	101-150 kg/ha	Moderate production
B4	High	(0.75, 0.20, 0.05)	151-200 kg/ha	Good production, almost ideal
B5	Very high	(0.95, 0.05, 0.00)	>200 kg/ha	Exceptionally high biomass production

Table 10: Behavior of biomass yield under different treatments with biostimulators.

Biostimulator	Biomass Yield (kg/ha)	Neutrosophic Evaluation	Triple Neutrosophic Number	Description
Seaweed Extract (B1)	133.94	B3 (Moderate)	(0.50, 0.45, 0.05)	Moderate production
Basfoliar Algae (B2)	131.8	B3 (Moderate)	(0.50, 0.45, 0.05)	Moderate production
Without Biostimulator (B0)	107.43	B2 (Low)	(0.25, 0.70, 0.05)	Low production
Basfoliar Aktiv (B3)	106.74	B2 (Low)	(0.25, 0.70, 0.05)	Low production

Neutrosophic statistical analysis:

- ❖ Seaweed Extract and Basfoliar Seaweed show moderate yields, ranging from 101-150 kg/ha. This underlines their effectiveness as biostimulators that promote considerable biomass production.
- ❖ Without Biostimulator and Basfoliar Aktiv, on the other hand, present lower yields, in the range of 51-100 kg/ha, indicating lower effectiveness in promoting biomass growth.

The results reflect how different biostimulators affect biomass yield in amaranth crops. The Seaweed Extract and Basfoliar Seaweed show a moderate yield, which makes them more effective compared to treatments without a biostimulator or with Basfoliar Aktiv, as they present lower yields.

3.6 Comprehensive evaluation of amaranth varieties and applied biostimulators.

Neutrosophic Comparative Analysis: The following table provides a detailed and nuanced representation of the behavior of variables under different conditions. This is crucial for decision-making in agronomic management and the optimization of treatments in amaranth crops (see Table 11). For this purpose, neutrosophic categories for the mean, variance, and standard deviation are provided. These are assigned based on the interpretation of the neutrosophic results obtained, which better describe the variability and behavior of each variable under different treatments. These descriptions offer a richer and more contextualized perspective of the data, which is particularly useful in agricultural research where biological variability and indetermination are significant.

Table 11: Neutrosophic statistics of the variety and applied biostimulator. S

Variety + biostimulator	Variable	Neutrosophic mean	Neutrosophic variance	Neutrosophic standard deviation
Nezhenka + seaweed extract	Stem length	Very high	Moderate	Moderate
	Number of sheets	Very high	High	High
	Biomass yield	High	Moderate	Moderate
	Germination	Very fast	Low	Low
Nezhenka + basfoliar algae	stem length	High	Moderate	Moderate
	Number of sheets	Very high	High	High
	Biomass yield	High	Moderate	Moderate
	Germination	Fast	Low	Low
Nezhenka + without biostimulator	stem length	High	Moderate	Moderate
	Number of sheets	High	High	High
	Biomass yield	Moderate	Low	Low
	Germination	Moderate	Moderate	Moderate
Nezhenka + basfoliar Aktiv	stem length	Moderate	High	High
	Number of sheets	High	Very high	Very high
	Biomass yield	Moderate	Low	Low
	Germination	Slow	Moderate	Moderate
Valentina + seaweed extract	stem length	Low	Very low	Very low
	Number of sheets	Moderate	Moderate	Moderate
	Biomass yield	Low	Very low	Very low
	Germination	Moderate	High	High
Valentina + basfoliar algae	stem length	Very low	Very low	Very low
	Number of sheets	Moderate	Moderate	Moderate
	Biomass yield	Low	Very low	Very low
	Germination	Slow	High	High

Neutrosophic Statistical and Comparative Analysis on Amaranth Varieties and Biostimulants: Neutrosophic analysis offers a comprehensive approach to interpreting data where uncertainty and incomplete information play a crucial role. This allows for a deeper and more nuanced understanding than traditional statistical methods. The following integrated results are presented:

- Germination: Valentina showed superior germination (T: 0.85, I: 0.10, F: 0.05) compared to Nezhenka (T: 0.62, I: 0.20, F: 0.18), indicating rapid adaptability and robustness under controlled conditions. This is interpreted as Valentina's intrinsic ability to quickly adapt to new environments, which is essential in selecting crops for areas with climatic variability.
- Stem Length: Nezhenka exhibited significantly greater stem growth (T: 0.85, I: 0.10, F: 0.05) compared to Valentina, especially when seaweed extract was used. This indicates that Nezhenka responds well to certain biostimulants, showing significant agronomic potential in terms of vertical growth, which can be beneficial to maximize space use in controlled environments.
- Number of Leaves: Nezhenka also displayed a higher number of leaves under treatment with seaweed extract (T: 0.90, I: 0.05, F: 0.05), highlighting the ability to produce more foliar biomass, a desirable characteristic for forage production or leaf consumption.
- Biomass Yield: In terms of biomass, Nezhenka significantly outperformed Valentina with the use of seaweed extract, indicating that Nezhenka not only grows taller but also produces more total biomass. Thus, this variety constitutes a superior candidate for biomass production.
- This neutrosophic analysis reinforces the importance of selecting specific varieties and treatments according to their response to biostimulants and optimizing cultivation strategies to achieve the best possible agricultural outcomes.

4 Discussion

The results, along with previous studies conducted in both Ecuador and Russia, show consistency in Nezhenka's response to biostimulants. This suggests that Nezhenka's genetic characteristics may be well adapted to the use of biostimulants to optimize its growth. Meanwhile, although Valentina showed lower performance in several parameters, it presents more efficient germination, which is crucial for the initial crop establishment stage. However, the data also suggest that Valentina may be more sensitive to transplant conditions, a variable that must be carefully managed to maximize its potential. Lastly, the use of neutrosophic statistics not only allows for a richer and more complex evaluation of experimental data but also helps design more informed and adaptive agronomic strategies that take into account the variability and uncertainty of agricultural production.

5 Conclusion

The neutrosophic statistical analysis revealed that Nezhenka has demonstrated a notably positive response to the use of seaweed extract. This resulted in a significant increase in stem length and biomass production. This response suggests that Nezhenka could be especially beneficial in cultivation programs that incorporate biostimulants to enhance growth efficiency and productivity.

Valentina displayed a superior germination rate, particularly under controlled conditions, reflecting its ability to adapt quickly and establish itself effectively at the start of the crop cycle. This characteristic is critical for agricultural planning and can be a decisive factor in selecting varieties for areas with short growing seasons or unstable climate conditions.

The study highlighted Valentina's sensitivity to transplant conditions, which could adversely affect its development and yield. This underscores the importance of careful and well-planned agronomic management practices to ensure that all amaranth varieties reach their maximum potential. Thus, it emphasizes the need for strategies tailored to the specific characteristics of each variety.

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Cognitive Mapping in the Study of Agricultural Orality in the Apagua Community

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Abstract. This research explores the application of neutrosophy to understand and address the inherent complexity in the oral transmission of knowledge in family agriculture. It leverages its ability to handle contradictions and ambiguities, facilitating a richer analysis of agricultural practices and social interactions. Orality, as the primary form of communication in these communities, is essential for the transmission of agricultural and cultural knowledge. This research highlights how orality acts as a vehicle for the preservation of collective identity and the transmission of ancestral knowledge. The study used Fuzzy Cognitive Maps (FCM) to describe the causal relationships between different variables of interest. Fieldwork was conducted between October 2023 and January 2024, using semi-structured interviews to capture the oral narrative of the farmers in Apagua. The analysis of the FCM showed that the components of cultural preservation and knowledge transmission are the most influential. This study demonstrated that neutrosophy is an effective tool for analyzing the complex dynamics of oral knowledge transmission in family agriculture. The conclusions underline the interdependence between knowledge transmission, social relationships, adaptation to changes, and cultural preservation.

Keywords: Ambiguity, information transmission, neutrosophy, FCM.

1. Introduction

Orality in family agriculture often involves narratives and knowledge that are both contradictory and complementary. Communities may have different interpretations and practices that coexist, which can be difficult to analyze with traditional binary approaches. Neutrosophy allows for effectively addressing these contradictions, offering a framework that accepts and works with ambiguity.

The transmission of knowledge through orality is highly subjective and varied [1]. Neutrosophy, with its ability to handle indeterminacy, is particularly suited to analyze how different individuals or groups may interpret the same facts differently, allowing for a richer and deeper analysis of social and cultural interactions.

In family agricultural communities, knowledge is passed from generation to generation, often in ways that are both traditional and adaptive to new circumstances [2]. Neutrosophy helps to integrate these diverse perspectives without prioritizing one over another, facilitating a more inclusive and holistic approach.

In the implementation of policies and programs aimed at improving family agriculture and preserving oral culture, neutrosophy enables the evaluation of the effects of such policies not only in terms of success or failure but also considering neutral or indeterminate outcomes, which can lead to more precise and effective adjustments.

By explicitly recognizing areas of uncertainty and neutrality, neutrosophy facilitates dialogue between different parties or groups with opposing interests within the community. This is crucial in environments where decisions about agricultural practices and knowledge transmission need to consider the voices of multiple stakeholders [3].

Applying neutrosophy in the socio-communicational analysis of orality and family agriculture provides a robust framework for understanding and addressing the complexity inherent in these systems. It facilitates a deeper and more nuanced understanding of community dynamics, which is essential for formulating development strategies that are sustainable and culturally respectful.

The community of Apagua was established on January 11, 1939, and is located in the Canton of Pujilí in the province of Cotopaxi. It is situated at an altitude of 4200 meters above sea level. A notable aspect of Apagua, which makes it relevant to the current research, is that 70% of its economically active population is involved in agricultural activities such as planting and harvesting potatoes, melloco, and beans, among other products. The population of Apagua is approximately 1500 people, most of whom are between the ages of 20 and 65. Additionally, it is important to mention that the inhabitants of the community speak Kichwa.

The name Apagua is associated with the production of a variety of potatoes called 'Apagua', introduced and cultivated by Augustinian religious in this area. These religions were dedicated to cattle ranching and the cultivation of agricultural tubers, involving indigenous companions in the work of this estate.

It should be noted that communication is intrinsically linked with orality, as through words experiences directed at others are transmitted and expressed; language acts as a medium for the transmission of thoughts and the strengthening of socialization between thinking and speaking [4]. Accordingly, oral communication plays a key role in the exchange and preservation of the customs and traditions of indigenous cultures, thus establishing orality as a fundamental basis for the preservation of knowledge that shapes collective identity within cultural groups.

In the communicative sphere, indigenous peoples have transmitted their information through oral language, through which, by means of conversations, information is transferred from generation to generation, preserving important events for the peoples without the need to resort to writing. Thus, the way in which peasant conversations that refer to traditional practices can be scientifically analyzed becomes relevant [5].

Among the elements to be considered, it is possible to identify that orality is classified into two parts (Figure 1):

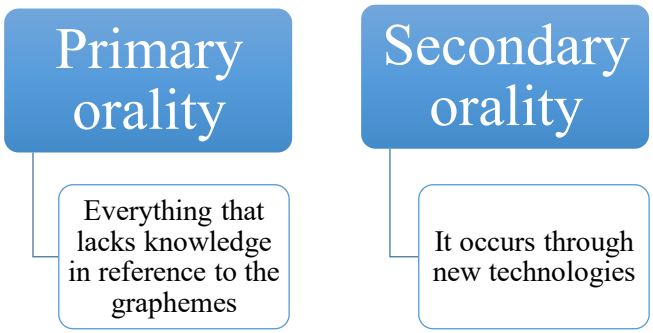

Figure 1: Classification of orality. Source: own elaboration

In this scenario and after reviewing the literature, it has been identified that various studies consider orality as the primary form of communication, as it allows the transmission of information in various ways. In the Latin American context, the process of orality has been analyzed using certain research parameters in relation to the recovery of speech as an effective strategy, not only for the reconstruction and preservation of ancestral traditions but also as a mechanism that facilitates dialogue about social and cultural trends [6].

In this same context, oral tradition plays a fundamental role in shaping a dynamic and creative culture. The exchange of stories creates a social space in which all elements that make up the identity of a group are shared and valued, avoiding exclusions or marginalization.

From the above, it can be seen that orality is linked to the essence of each people, as it allows preserving knowledge that is often also related to ancestry and the cultural heritage of the peoples, thus forming a cultural heritage that is maintained in families and transmitted from generation to generation. In this sense, the ancestral knowledge transmitted through orality gives meaning to the actions of social subjects and allows shaping a conception of present and future [7], [8].

In the case of Andean communities, stories, beliefs, knowledge, customs, and traditions that are loaded with meanings are involved. These narratives are part of the cultural heritage of the communities and, within the research conducted, there was an interest in identifying how knowledge related to agriculture is transmitted through orality.

The main objective of this study is to analyze how neutrosophy can be applied to understand and address the complexity of the oral transmission of knowledge in family agriculture in the community of Apagua. This approach seeks to capture the richness and variety of interpretations and agricultural practices that are transmitted from generation to generation, especially in contexts where contradictory and complementary narratives coexist.

2 Materials and methods

2.1 Fuzzy Cognitive Maps

Fuzzy Cognitive Maps (FCMs) extend over the range [-1, 1] to indicate the strength of causal relationships, see [9]–[11]. They describe the strength of the relationship using fuzzy values. FCMs allow for expressing causal relationships between variables, where each edge is associated with a weight in the set, where 0 indicates no causal relationship between the variables, -1 indicates an inverse causal relationship (if one variable increases, the other

decreases, and vice versa), and 1 indicates that there is a direct causal relationship (both variables increase or both decrease).

These three values do not capture the uncertainty that exists in these causal relationships, which is why Fuzzy Cognitive Maps emerge, where a gradation is introduced to the previous set of weights defined in the continuous interval [-1,1]. An FCM can be represented through a weighted directed graph. An adjacency matrix is constructed from the values assigned to the arcs generally in a numerical form [12].

In FCMs, there are three possible types of causal relationships between concepts:

- Positive causality ($W_{ij} > 0$): Indicates a positive causality between the concepts C_i and C_j, meaning an increase (decrease) in the value of C_i leads to an increase (decrease) in the value of C_j.
- Negative causality ($W_{ij} < 0$): Indicates a negative causality between the concepts C_i and C_j, meaning an increase (decrease) in the value of C_i leads to a decrease (increase) in the value of C_j.
- Non-existence of relationships ($W_{ij} = 0$): Indicates the absence of a causal relationship between C_i and C_j.

In this article, the calculation will proceed as follows:

1. Selection of relevant causals.
2. Elaboration of the adjacency matrix.
3. Static analysis: calculations are made for the absolute values of the adjacency matrix:
   - Outdegree, denoted by od(v_i), is the sum for each row of the absolute values of a variable from the fuzzy adjacency matrix. It measures the accumulated strength of the existing connections in the variable.
   - Indegree, denoted by id(v_i), which is the sum for each column of the absolute values of a variable from the fuzzy adjacency matrix. It measures the accumulated incoming strength of the variable.
   - The total centrality or degree of the variable is the sum of od(vi) with id(vi), as follows:

$$\text{td(vi)} = \text{od(vi)} + \text{id(vi)} \quad (1)$$

Finally, the variables are classified according to the following criterion, see [13]:

a) *Transmitter variables* are those with $od(v_i) > 0$ and $id(v_i) = 0$
b) *Receiver variables* are those with $od(v_i) = 0$ and $id(v_i) > 0$
c) *Ordinary variables* satisfy both $od(v_i) \neq 0$ and $id(v_i) \neq 0$

They are ranked in ascending order according to the degree of centrality.

When a group of individuals (k) participates, the adjacency matrix is formulated through an aggregation operator, such as the arithmetic mean. The simplest method consists of finding the arithmetic mean of each of the connections for each expert. For k experts, the final FCM adjacency matrix (E) is obtained as [14, 15]:

$$E = \frac{(E_1 + E_2 + \cdots + E_k)}{k} \quad (2)$$

2.2 Methodology

Fieldwork within the community of Apagua was conducted from October 2023 to January 2024. We operated under a qualitative approach, as we were interested in observing phenomena in their natural environment before analyzing them. We applied semi-structured interviews to consider oral narrative as an object of study within the research.

The criteria considered for the interview participants were as follows: male and female farmers aged between 24 and 70 years (Figure 2), engaged in agriculture and originating from Apagua.

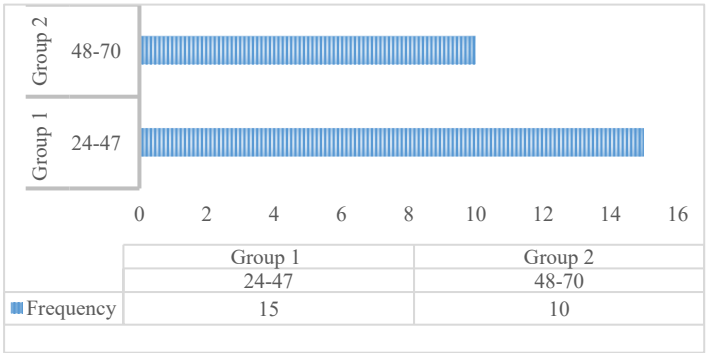

Figure2: Age groups. Source: own elaboration.

The topics covered in the interview script focused on four categories (knowledge transmission, social relationships, adaptation to changes, and cultural preservation). We highlight that during this process, the Pallamukuy Organization, affiliated with the Indigenous Peasant Movement of Cotopaxi (MICC), assisted us in establishing contact with community representatives, which allowed us to conduct interviews with various actors who have experiences and knowledge related to agriculture.

Once access to the community was obtained, interviews were conducted in the sectors of Corralpungo, Chilca, Milin, Redrován, Conucto, and Agua Centro. The interview script was structured through four categories and nine subcategories, which allowed for a deeper analysis of orality and family agriculture in the community of Apagua (Table 1).

Table 1: Categories and subcategories, interviews with farmers from the Apagua community, Pujilí canton, Cotopaxi province. Source: own elaboration.

Category	Subcategory
Knowledge Transmission	1a. Transmission of agricultural knowledge. 1b. Family stories in teaching agricultural techniques. 1c. Main sources of information on agricultural experiences and knowledge.
Social Relationships	2a. Interpersonal relationships between families. 2b. Verbal interactions and collaboration among families.
Adaptation to Changes	3a. Adaptation of new tools and machinery. 3b. Successful adaptation experiences.
Cultural Preservation	4a. Preservation of cultural identity. 4b. Stories that highlight the importance of maintaining traditional agricultural practices.

Once the interviews were conducted, open coding was developed to analyze the corresponding variables. This activity belongs to the analytical process where the researcher must identify basic concepts related to the text while discovering the same characteristics and dimensions through the narratives of the interviewees. After this process, the most important and representative fragments were selected and redundant data from each category were removed to avoid theoretical saturation. Subsequently, the responses obtained in the survey were combined with the results of the application of the fuzzy cognitive map for a more comprehensive assessment.

3 Results and Discussion

Once the corresponding analysis was conducted for each category, we proceeded to present the most relevant results in each of the areas previously outlined that structured the script of the research:

Category 1. Transmission of Knowledge.

The transmission of knowledge contributes to human personal and social development and establishes relationships with other individuals. Disseminating and sharing experiences through language is an activity that continues not only in indigenous communities but in any daily activity or geographical situation.

Testimonies identified that orality plays a very important role in ancestral communication, as it presents a path that allows the construction and transmission of agricultural knowledge through the voices of the actors themselves, such as our ancestors.

According to the results of the interviews, it is highlighted that agriculture in the Andean world is passed from generation to generation, as from an early age parents share activities linked to agriculture (land preparation, planting, and harvesting) with their progenitors.

For this reason, within the analyzed context, it is considered appropriate to explain and take children to agricultural activities together with their parents, as a result, they acquire knowledge that allows preserving wisdom and improving implemented techniques.

In agricultural activity, several processes are also required, such as the application of fertilizer in the soil after using the plow or after 'tolar'. Within this context, practices that have been used for a long time in the community within the planting process become important.

After the planting period, the harvest proceeds, of different products such as potatoes, beans, melloco, oca, mashwa, barley, carrot, beet, cabbage, and onion, these products are still maintained in the families of the Apagua community.

Through this dialogue, an important aspect that was highlighted was the collective minga, a tradition that consisted of unpaid work for a community or family. It is mentioned that "it has been like a kind of party for example the planting of potatoes after planting has been almost culturally like a wedding where guinea pigs, chickens, a lamb are killed."

Moreover, after planting they usually meet to share their products and at the same time engage in conversation with other individuals (neighbors), where they used to tell their stories, experiences, and anecdotes of their ancestors, there is much knowledge, much wisdom, through orality, grandparents share various stories about the proper process for planting and harvesting products in the chacras (family orchards).

Therefore, it can be seen that orality is very important in the transmission of knowledge, as it gives meaning to social activities through various interactions.

As already mentioned, orality has been from the beginning the first system of human expression, which gives us access to a cultural world full of knowledge, therefore, it is necessary to emphasize that history can also be told from the experience of the social actors of the community.

Another relevant aspect is that the community members of Apagua feel concerned to see that these stories and knowledge are being lost, since, not being written or disseminated in any physical or digital format, they run the risk of being forgotten, since currently within the community there is a minority of older adults who can still transmit their knowledge to new generations.

Therefore, orality is just as relevant as written forms of expression, in the case of orality the capacity to delve into both individual and collective memory takes on importance, which aids in the exchange of knowledge, this particularity can be enhanced by writing to safeguard knowledge.

Category 2. Social relations

While it is true that social relationships constitute a fundamental basis within the daily life of social groups, these interactions can involve various aspects. In the case of our research, for example, we identified that there is a direct link between orality, festivities, and agriculture.

Social relationships are differentiated by considering the level from which they are observed, such as interactions, organizations, and society. Therefore, the externalization of relationships as events suggests relationships are understood as interactions; however, in any case, structure and events are always co-present.

On the other hand, the family nucleus is the cultural support where the basic rules and norms of coexistence emerge, allowing the acquisition of skills for relationships and emotional control. In this sense, within the testimonies, and the result of interviews, it was found that most affirm that they still share these pieces of knowledge, stories, or tales from their grandparents to their children, grandchildren, and neighbors.

Another important aspect of social relationships is coexistence, solidarity, and reciprocity, which have been fundamental elements for agricultural work. The community of Apagua has traditions of Makita Mañachi (reciprocity) and Randy Randy (solidarity), these festivities are related to the community dynamics since previously to ask neighbors for help with planting and harvesting, they had to bring them some items like machica, potatoes, guinea pigs, and wedding gifts to formally ask for help. This activity serves as a symbol of showing respect towards the family and strengthening community bonds. These aspects directly influence the strengthening of the Andean people through community coexistence.

Therefore, it is important to highlight the relevance of orality within the family as it allows for articulating the collective dynamics with different field activities. Within this scope, it is worth noting the role of the women of the Pakarimuy organization who promote strengthening strategies for family agriculture. This organization aims to maintain its customs and traditions through orality; hence they have sought to spread the importance of the community's knowledge through media and digital platforms.

Therefore, social relationships go hand in hand with orality and communication, as they are the fundamental basis for all kinds of activities, organizations, and planning. Throughout our research, we identified how this is reflected in agricultural activities and that from generation to generation has allowed obtaining good production and at the same time maintaining a bond with the community.

Category 3. Adaptation to changes

In the context of family farming, each community has particular characteristics regarding soil preparation, seed selection, planting, and harvesting. This knowledge involves skills and experiences accumulated over generations, which allow for the improvement of agricultural practices. The way of life of people in the countryside has been crucial for production through the traditions of the taitas and mamas. This has led to the continued maintenance of ancestral knowledge and improvements with the implementation of machinery, fertilizers, and fertilizers.

It can be seen that the tractor is a very helpful tool for agriculture, but it is equally harmful to the soil since being heavy machinery, and exerting strong pressure on it tends to wear it out. It was mentioned that, in the past, there was a yoke, where they worked in mingas, performing the same activities as their grandparents. Also, the previous use of horses, donkeys, and mules, which were used to plow the soil, extract the grains, and take them to

the fairs, but this activity lasted several days and weeks. Currently, there are machines to shell corn and a threshing machine for barley. This has been an important contribution to agriculture because it speeds up the work of the farmers.

After analyzing several interviewees, it is evident that although innovation is a help for humans, the majority of the inhabitants of Apagua prefer to continue working in the traditional way, with manual labor, with the hoe and organic manure from animals, thus preserving the ancestral knowledge, since their ancestors always mention that the preservation of Pachamama is important for good living.

Category 4. Cultural preservation

Cultural preservation ensures that ancestral knowledge is kept for future generations, as through stories, tales, and myths, various traditions, customs, and festivals in honor of Pachamama have been generated. In this sense, all human beings experience community coexistence in terms of individual well-being and development (social, personal, economic) to improve quality of life.

In the Andean context, the diverse and ancient culture of the communities has been a reference for the exchange of knowledge through different festivals such as Inti Raymi (festival to the fertility of Mother Earth), Kulla Raymi (cult to the fertility of Mother Earth), Kapak Raymi (royal festival dedicated to germination) and Pawkar Raymi (festival to Pacha Mama).

Communities must continue maintaining these types of festivities, where they can interact not only with their relatives but also with people from other cities different from their culture. This helps to continue strengthening identity, and a sense of belonging, and at the same time interweaving customs and traditions.

To conclude, it is necessary to recognize that orality plays a very important role in the transmission of agricultural knowledge, as it facilitates communication, and promotes development, and social bonding. That is why, currently, in Apagua, family meetings have been chosen, including neighbors to maintain this agricultural knowledge to this day.

Following the line of the basic elements of the study, the experts analyzed the dimensions previously evaluated, from the point of view of the causal relationships between them. The development of the study is shown next in Figure 3, Tables 2 and 3.

Table 2: Adjacency matrix. Source: own elaboration

	Knowledge transmission	Adaptation to changes	Cultural preservation	Social relationships
Knowledge transmission		0.45	1	0.34
Adaptation to changes	-0.33		0.13	0.46
Cultural preservation	1	-0.39		0.7
Social relationships	0.19	0.3	0.28	

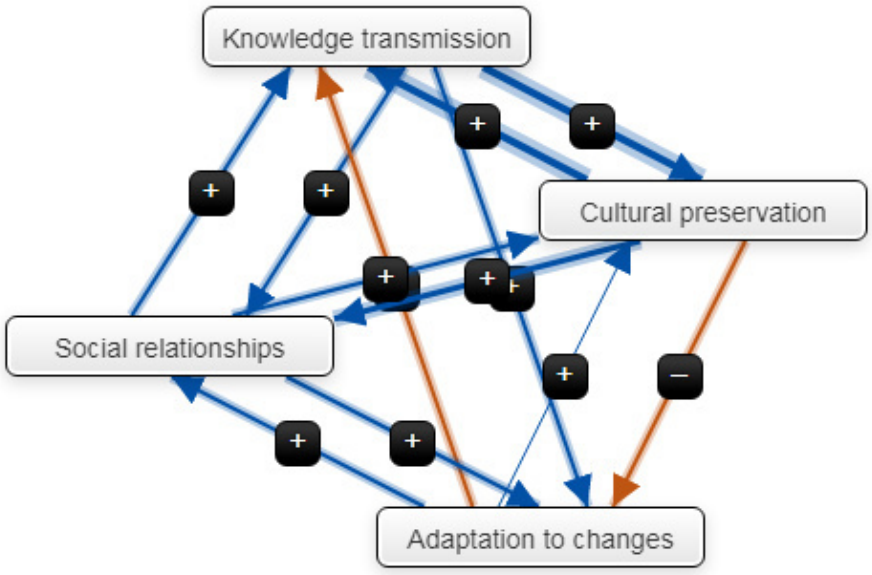

Figure 3: Fuzzy Cognitive Map. Source: own elaboration.

Table 3: FCM centrality analysis. Source: own elaboration.

Components	Indegree	Outdegree	Centrality	Type
Knowledge transmission	1.52	1.79	3.31	ordinary
Social relationships	1.5	0.77	2.27	ordinary
Cultural preservation	1.41	2.09	3.5	ordinary
Adaptation to changes	1.1400000000000001	0.9200000000000002	2.0600000000000005	ordinary

The interpretation of the centrality analysis reveals that Cultural Preservation and Knowledge Transmission are the most influential components, suggesting that interventions or policies focused on these aspects could have a broad impact on the network. Social Relationships, although relevant, seem to act more as receivers of influences than as transmitters, which can be significant when considering social intervention strategies. Adaptation to Changes requires more support or reinforcement to increase its capacity to influence the network, especially if the goal is to improve the resilience of a community or system.

Focusing on the interaction between both research efforts, the survey results were combined with the FCM to observe how they reflect and interconnect in terms of knowledge transmission, social relationships, adaptation to changes, and cultural preservation (Table 4).

Table 4: Fusion of methods. Source: own elaboration.

Category	Survey	FCM
Knowledge Transmission	Orality is highlighted as a crucial medium for the transmission of agricultural knowledge and ancestral practices in the Apagua community. The importance of generational interaction for the preservation of ancestral knowledge is emphasized.	This component is central in the matrix and shows a significant relationship with Cultural Preservation. Strengthening the oral transmission of knowledge is an effective way to preserve culture and improve social relationships in the community.
Social Relationships	Festivities and coexistence in agricultural activities strengthen social relationships and are essential for the transmission of knowledge. Orality facilitates cohesion and the strengthening of community ties.	It is influenced by other factors but has less external influence. Promoting activities that involve orality in a community context increases the influence of social relationships on other cultural and adaptive aspects.
Adaptation to Changes	There is a preference for maintaining traditional agricultural practices despite the introduction of modern technologies, reflecting resistance to change that affects the land. Technological adaptation is valued as long as it aligns with ancestral practices and respect for the land.	It has moderate influences on other components and vice versa. Interventions that seek to promote adaptation must consider ancestral knowledge and the needs of the community, integrating technologies that respect practices and the land.
Cultural Preservation	Culture and ancestral practices, such as festivals and planting according to the lunar calendar, are vital for community identity. The transmission of these knowledge and cultural practices is at risk due to a decline in oral transmission between generations.	This component has the highest degree of influence on other components and significant centrality. Reinforcing cultural preservation through formal and non-formal education can ensure the continuity of these ancestral practices and knowledge.

Combining these findings reveals the interdependence between knowledge transmission, social relationships, adaptation to changes, and cultural preservation. The most effective strategy for sustainable intervention would be one that not only addresses each of these components in isolation but seeks to strengthen them holistically, recognizing and leveraging their interconnections.

4 Conclusion

This study has demonstrated the richness and complexity of oral knowledge transmission in the community of Apagua, an essential nucleus in Andean family agriculture. The use of neutrosophy and Fuzzy Cognitive Maps (FCMs) has enabled a deeper and more nuanced analysis of the cultural and social dynamics that characterize this community.

The narratives and experiences shared orally between generations are fundamental for preserving ancestral techniques and knowledge. These practices are transmitted not only as practical instructions for agriculture but also as stories and anecdotes that reinforce cultural identity and social cohesion. Neutrosophy proved to be an effective framework for analyzing the complexity and contradictions inherent in these narratives, allowing for a more nuanced understanding of how knowledge is transmitted and evolves within the community.

Social relationships in Apagua are intrinsically linked to agriculture and orality. Festivities and collective agricultural activities, such as the minga, strengthen these ties, facilitating the exchange of knowledge and experiences. Orality not only serves to transmit technical information but also to maintain and reinforce networks of solidarity and reciprocity. The use of fuzzy cognitive maps revealed that, although social relationships are influenced by other cultural and knowledge factors, their capacity to influence other aspects is less direct. Enhancing these relationships through community activities could increase their effect on other cultural and adaptive aspects.

In Apagua, there is a clear preference for maintaining traditional agricultural practices despite the introduction of modern technologies. This resistance to change reflects a deep respect for the land and a valuation of ancestral practices that align with the care and preservation of Pachamama. Interventions to encourage adaptation must integrate these perceptions, using technologies that respect traditional practices and the land. Adaptation should not be imposed but should arise from dialogue and the integration of ancestral knowledge with appropriate innovations.

The preservation of culture and ancestral practices is crucial for the community identity of Apagua. The transmission of knowledge and cultural practices is at risk due to a decline in oral transmission between generations. Strengthening cultural preservation through formal and informal education can ensure the continuity of these practices and knowledge. The analysis showed that cultural preservation has the greatest degree of influence on other components, highlighting the importance of interventions focused on this aspect to have a broad impact on the community's network of knowledge and practices.

This study underscores the need for an integrated and respectful approach to intervention and development in Andean indigenous communities like Apagua. Neutrosophy and fuzzy cognitive maps have provided a rich and detailed perspective that allows for understanding and working with the complexity of orality and family agriculture, paving the way for more effective and culturally coherent policies and programs.

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The Rheological Analysis of Ancestral Beverages, Neutrosophic C-Means (NCM) Clustering Algorithm of Data

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Abstract. This study focuses on the rheological analysis of cassava and chonta chichas (Manihot esculenta Crantz and Bactris gasipaes) produced in the Province of Pastaza, aiming to preserve and disseminate the cultural and nutritional heritage of these traditional beverages. The research emphasizes the integration of ancestral knowledge into modern production practices, exploring sensory characteristics that could influence consumer preferences and production standardization. In Pastaza, it has been observed that traditional production methods, such as chewing cassava and chonta to initiate fermentation, expose the product to microbial risks, making it potentially unpalatable and unsafe for consumption. To address these issues, this work incorporates neutrosophic theory to manage the uncertainties in sensory evaluation and applies Neutrosophic C-means clustering to analyze the data obtained from both expert assessments and instrumental measurements. This approach aims to enhance our understanding of how variations in the production process affect the beverage's properties and to develop a standardized production protocol that ensures both the safety and quality of the chichas. Through empirical data and rheological studies, this research seeks to contribute significantly to the sustainable preservation and enhancement of traditional beverage production.

Keywords: Rheological Analysis, Traditional Beverages, Neutrosophic Theory, Neutrosophic C-means clustering

1. Introduction

A rheological study is an investigation that focuses on the behavior of matter in response to applied forces, specifically in relation to its flow, elasticity, and viscosity [1, 2]. Rheology is the branch of physics that studies the deformation and flow of matter, and a rheological study concentrates on measuring and analyzing how materials respond to mechanical forces, such as tension, compression, or shear. The purpose of a rheological study [3, 4] includes:

- Material Characterization: Allows understanding of the mechanical properties of materials, such as viscosity, elasticity, and plasticity, among others.
- Process Design: Assists in the design and control of industrial processes where the fluidity and behavior of materials are critical.
- Product Development: Fundamental in the formulation and improvement of products such as food, cosmetics, paints, and polymers, among others, to ensure their quality and performance.

The functional foods market is one of the fastest-growing segments in the food product development category, as an increasing number of consumers are concerned about health-related issues [5].

In Ecuador, there is limited research on this topic, making the integration of this analysis into the food industry novel, especially when applied to ancestral beverages such as chichas, which have significant historical recognition. Chicha is a traditional drink in various countries of Latin America, and its consistency and texture can vary depending on the region and recipe. The main problem at the beginning of the research is the cultural importance of chicha, as well as its nutritional and sensory value. Therefore, it can be said that the lack of information and valuation of the cultural importance of chicha de jora in the Sierra Norte is the motivation for this study, along with its various recipes and ways of preparation [6].

It can be said that studies conducted in this area of rheology do not provide regulations that set rheological parameters for fermented beverages that can be valued and referenced in research or production of yuca and chonta chichas. This is why entrepreneurs cannot validate the quality of the product endorsed by these types of studies. This lack of guidance or regulations is counterproductive because it does not provide a standardization that "guides" the producer to make yuca and chonta chichas in a way consistent with ancestral customs and allows the product to be endorsed for marketing.

For this reason, in this work, it was decided to emphasize the search for parameters based on sensory characteristics in order to rescue the ancestral knowledge of the drink, promote its preservation, and disseminate its cultural and nutritional wealth. It is intended to determine the rheological study of yuca (Manihot esculenta crantz) and chonta (Bactris gasipaes) chichas with enzymatic preparations made in the province of Pastaza. This work aims to better understand how variations in the production process affect the final properties of the product and to expand knowledge about the standardization of the production process of yuca and chonta chichas.

The study will be carried out in the Province of Pastaza where it was detected that the producers do not have raw materials nor a harmless process, making the process susceptible to microbial agents that can cause health damage, since it is traditionally done with the chewing of yuca and chonta masato which makes it unpalatable to the consumer. The study will be conducted involving neutrosophic theory in the processing of the magnitudes of the variables in the sensory evaluation for the rheological study before the experts. Given such inherent uncertainty in the use of experts, it was decided to apply neutrosophic sets for clustering using Neutrosophic C-means in the case of these evaluations and compare them with those obtained by the instruments and the proven properties, to issue criteria based on the contribution of the empirical data of the tests to be carried out in the rheological study. To illustrate the technological process, Table 1 shows a summary of the steps and significant details of this.

Table 1: Yuca and Chonta Masato Preparation Processes. Source: own elaboration based on direct observation.

Process	Description	Technical details
White Yuca Masato	Raw Material Preparation: Selection, washing, and cooking of cassava and sweet potato.	Weight: 5.4 kg of cassava, 333.3 g of sweet potato. Cooking: 80ºC for 40 min. Grinding: Homogeneous mass in a clean container.
Burnt Yuca Masato	Burning and Fermentation Process: Washing, burning, and controlled fermentation.	Burning: Directly at 89°C until soft. Fermentation: 4-5 days at room temperature, covered with bijao leaves and aluminum foil.
Wiwis Yuca Masato	Scraping and Cooking: Removal of the peel and cooking of cassava and sweet potato.	Cooking: 85°C for 40 min. Fermentation: 5 days at room temperature, covered with achira leaves.
Masato of Chonta	Cooking and Homogenization: Cooking of peach palm and sweet potato, followed by crushing.	Cooking: 90-94°C for 2.5 hours. Additions: 5% cooked sweet potato, 250 g of distilled water.
Enzymatic Hydrolysis	Preparation and Enzymation: Mixing masato with water and enzymes, temperature control.	Enzymes: α-amylase, β-amylase, amyloglucosidase (0.05%, 0.10%, 0.15%). Temperature: 55ºC initial, up to 95ºC for inactivation.
Rheological Analysis	Rheological Tests: Use of a rheometer to measure flow properties of the beverages.	Equipment: Anton Paar MCR 302 Rheometer. Conditions: 17.9 °C and 59.6% HR.
Physicochemical Analysis	Quality Evaluation: Measurement of acidity, Brix degrees, pH, alcoholic degrees, taste, color, and odor.	Comparison: Differences between chichas with and without enzymes.

2 Neutrosophic C-Means (NCM) to be applied to evaluations carried out by experts

A neutrosophic set is an extension of classical and fuzzy set theory that allows modeling uncertainty and indeterminacy. An element of a neutrosophic set has three associated parameters: the degree of truth T, the degree of indeterminacy I, and the degree of falsehood F of x in A, respectively, and their images form standard or non-standard subsets within the range from 0 to 1. For this study, the following functions are defined:

- T= for the membership functions to true, where T ∈[0,1].
- I= for the membership functions to indeterminate, where I ∈[0,1].
- F= for the membership functions to false, where F ∈[0,1] [7].

A neutrosophic number is of the form (T, I, F) as shown in Table 2.

Table 2: Definition of linguistic terms according to the method and single-valued neutrosophic numbers. Source: Developed based on [7].

Single-Valued Neutrosophic Number	Interpretation for the studio
(1,0,0)	Excellent Variable Behavior
(0.9, 0.1, 0.1)	Very Good Variable Behavior
(0.8,0,15,0.20)	Good Variable Behavior
(0.70,0.25,0.30)	Acceptable Variable Behavior

Single-Valued Neutrosophic Number	Interpretation for the studio
(0.60,0.35,0.40)	Moderately Good Variable Behavior
(0.50,0.50,0.50)	Uncertain
(0.40,0.65,0.60)	Moderately Poor Variable Behavior
(0.30,0.75,0.70)	Poor Variable Behavior
(0.20,0.85,0.80)	Very Poor Variable Behavior
(0.10,0.90,0.90)	Extremely Poor Variable Behavior
(0,1,1)	Terrible Variable Behavior

Fuzzy C-means, also known as fuzzy C-means clustering, is a clustering technique used in machine learning. Unlike traditional K-means clustering, which assigns each data point to a single cluster, Fuzzy C-means allows data points to belong to multiple clusters with varying degrees of membership [7-13]. In this method, each data point is assigned a value between 0 and 1 for each cluster, indicating the degree to which it belongs to that cluster. A value of 0 means it does not belong, and 1 indicates complete membership. The process is iterative. Initially, random membership values are assigned to the data points. Then, the algorithm refines these values until they no longer change significantly [14].

The Neutrosophic C-means (NCM) algorithm is based on neutrosophic theory, extending the Fuzzy C-Means (FCM) by incorporating three types of memberships for each data point in relation to each cluster: truth T, indeterminacy I, and falsehood F. Below is an explanation of how these memberships are defined and the objective function of NCM [15,16].

The objective function of NCM is a generalization of that of FCM, incorporating the neutrosophic degrees. The aim is to minimize:

$$J(T,I,F,C) = \sum_{i=1}^{N}\sum_{j=1}^{C}\left(\omega_1 T_{ij}^m \parallel x_i - c_j \parallel^2 + \omega_2 I_{ij}^m d_{max} + \omega_3 F_{ij}^m \parallel x_i - c_j \parallel^2\right) \quad (1)$$

Where:

- tij is the degree of truth that point i belongs to cluster j.
- Iij is the degree of indeterminacy for point i in cluster j.
- Fij is the degree of falsehood that point i belongs to cluster j.
- $\omega_1,\omega_2,\omega_3$ are weights associated with truth, indeterminacy, and falsehood, respectively.
- m is a parameter that controls the diffusivity of the membership, similar to the "fuzziness" parameter in FCM.
- d_{max} is the maximum distance between any point and the centers of the clusters, used to normalize the influence of indeterminacy.
- x_i is the feature vector of point i.
- c_j is the centroid of cluster j.
- N is the total number of data points.
- C is the number of clusters

Update of Cluster Centers (c_j):

$$c_j = \frac{\sum_{i=1}^{N}(T_{ij}^m + F_{ij}^m)\, x_i}{\sum_{i=1}^{N}(T_{ij}^m + F_{ij}^m)} \quad (2)$$

Updating Membership Degrees:

Truth (T_{ij}): $$T_{ij} = \frac{1}{\sum_{k=1}^{C}\left(\frac{\|x_i - c_i\|}{\|x_i - c_k\|}\right)^{\frac{2}{m-1}}} \quad (3)$$

Indeterminacy (I_{ij}): $$I_{ij} = 1 - T_{ij} - F_{ij} \quad (4)$$

Falsehood (F_{ij}): $$F_{ij} = \frac{\|x_i - c_j\|^2 / d_{max}}{\sum_{k=1}^{C}\left(\frac{\|x_i - c_k\|^2}{d_{max}}\right)} \quad (5)$$

2.1 Methodology to follow

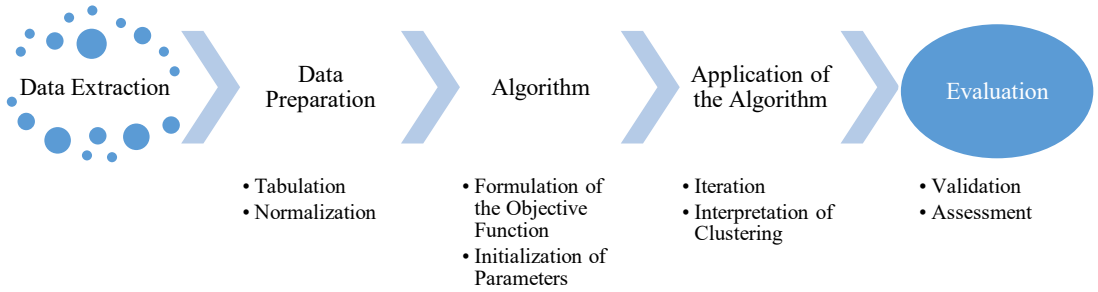

Figure 1: Data processing. Source: own elaboration.

3 Results

3.1 Extraction of relevant data

For this purpose, Brix degrees, pH, acidity, and alcoholic degrees during the fermentation of yuca and chonta chichas are established as relevant, measured every 72 hours, as well as specific values for different types of chichas, such as burned, white, wiwis, and chonta with and without enzymatic preparations, as explained below:

- ❖ Brix Degrees: Brix degree values throughout the fermentation process for yuca and chonta chichas, measured at various points over 72 hours.
- ❖ pH: pH measurements conducted at similar intervals during the fermentation process.
- ❖ Acidity: Acidity levels expressed in Dornic degrees, also taken during the fermentation process.
- ❖ Alcoholic Degrees: Volumetric alcohol percentages obtained from the chichas at different measurement points.

Organized Data for Rheological Analysis: These data include both rheological parameters (density ρ, viscosity μ, consistency index k, flow behavior n, shear stress τ, deformation rate γ) and physicochemical parameters (Brix degrees °Bx, pH, acidity, and alcoholic degrees % vol.) for each type of chicha and treatments with and without enzymatic preparations (CPE and SPE). Table 3 provides a comprehensive view of the parameters that influence the rheological behavior of the chichas, obtained using measurement instruments.

Table 3: Data Frame for Analysis with Instruments: Source: Developed from measurements taken from chicha samples during the production process.

Beverage	Treatment	ρ (g/cm³)	μ (cP)	k (Pa.s)	n	τ (Pa)	γ (1/s)	°Bx	pH	Acidity (°D)	Alc. % vol.
Burnt Chicha	SPE	1.0107	111	0.0852	0.0009	0.0855	77	49	4.02	0.55	3.2
Burnt Chicha	CPE	1.0319	18	0.0138	0.0004	0.0138	77	18	3.50	0.68	5.2
White Chicha	SPE	0.9935	113	0.0780	0.0142	0.0823	fifty	25	5.70	0.58	2.2
White Chicha	CPE	1.0124	113	0.1072	0.0305	0.1203	fifty	13	5.25	0.68	4.1
Chicha Wiwis	SPE	1.0112	38	0.0294	0.0004	0.0294	77	43	4.50	0.59	2.9
Chicha Wiwis	CPE	1.0280	292	0.2203	0.0021	0.2223	76	122	3.72	0.71	4.1
Chicha de Chonta	SPE	1,011	113	0.0097	0.0002	0.0097	fifty	35	3.96	0.30	2.5
Chicha de Chonta	CPE	1.0207	113	0.0122	0.0020	0.0123	fifty	8	4.25	0.44	4.9

For sample collection, in the case of experts, 17 individuals working in the production workshops and enterprises in the Pastaza area, who belong to the agricultural producers' association where the chichas are made and marketed, are chosen. Their sensory evaluation will be provided in this case through a questionnaire using neutrosophic linguistic terms aligned with the parameters to be measured by instruments, as outlined in the following survey:

- ❖ Density (ρ) [g/cm³]: Very Low | Mostly Low | Uncertain | Mostly High | Very High
- ❖ Viscosity (μ) [cP]: Very Fluid | Mostly Fluid | Uncertain | Mostly Viscous | Very Viscous
- ❖ Consistency (k) [Pa.s]: Very Weak | Mostly Weak | Uncertain | Mostly Consistent | Very Consistent
- ❖ Flow Index (n): Very Fluid | Mostly Fluid | Uncertain | Mostly Thick | Very Thick
- ❖ Shear Stress (τ) [Pa]: Very Low | Mostly Low | Uncertain | Mostly High | Very High

- ❖ Shear Rate (γ) [1/s]: Very Slow | Mostly Slow | Uncertain | Mostly Fast | Very Fast
- ❖ Brix Degrees (°Bx): Very Diluted | Mostly Diluted | Uncertain | Mostly Concentrated | Very Concentrated
- ❖ pH: Very Acidic | Mostly Acidic | Uncertain | Mostly Basic | Very Basic
- ❖ Acidity (°D): Very Low | Mostly Low | Uncertain | Mostly High | Very High
- ❖ Alcohol (% vol): Very Low | Mostly Low | Uncertain | Mostly High | Very High

From the application of the questionnaire, a database with an entry of 136 data points was obtained, which were tabulated for preparation. As an example, Table 4 is presented showing the sensory evaluation of a type of chicha with and without enzymes:

Table 4: Extraction of sensory evaluation data using neutrosophic linguistic terms for rheological study. Source: Developed from questionnaires provided by 17 experts.

Beverage	Treatment	ρ (g/cm³)	μ (cP)	k (Pa.s)	n	τ (Pa)	γ (1/s)	°Bx	pH	Acid (°D)	Alc. %vol
Burnt Chicha	SPE	Very low	Very viscous	Very consistent	Mostly fluid	very high	Mostly slow	Mostly concentrated	Very acidic	very high	Mostly high
Burnt Chicha	SPE	Very high	Very fluid	Very weak	Very thick	Mostly high	Mostly slow	Very diluted	Very basic	very high	Mostly low
Burnt Chicha	SPE	Mostly high	Uncertain	Mostly consistent	Very thick	Mostly high	Uncertain	Mostly diluted	Mostly acidic	Very low	Uncertain
Burnt Chicha	SPE	Mostly high	Very fluid	Uncertain	Very fluid	Mostly low	Mostly slow	Very diluted	Very acidic	Uncertain	Mostly high
Burnt Chicha	SPE	Mostly high	Very fluid	Very weak	Very thick	very high	Uncertain	Very concentrated	Very basic	Mostly high	Very low
Burnt Chicha	SPE	Very high	Very fluid	Very weak	Very fluid	Uncertain	very slow	Very diluted	Mostly acidic	Very low	Very low
Burnt Chicha	SPE	Mostly high	Mostly viscous	Very weak	Mostly thick	Very low	Uncertain	Mostly diluted	Mostly basic	Uncertain	Mostly high
Burnt Chicha	SPE	Very high	Very fluid	Very consistent	Mostly fluid	Mostly low	Uncertain	Mostly concentrated	Mostly acidic	Uncertain	Mostly high
Burnt Chicha	SPE	Mostly high	Very viscous	Mostly consistent	Mostly fluid	Mostly high	Mostly fast	Very concentrated	Uncertain	Very low	Mostly high
Burnt Chicha	SPE	Uncertain	Uncertain	Very consistent	Very fluid	Uncertain	Uncertain	Mostly diluted	Mostly acidic	very high	Very low
Burnt Chicha	SPE	Mostly high	Uncertain	Mostly consistent	Mostly fluid	Uncertain	Uncertain	Very concentrated	Mostly acidic	Uncertain	Very low
Burnt Chicha	SPE	Very high	Very fluid	Very consistent	Very thick	Very low	very slow	Uncertain	Uncertain	very high	Mostly high
Burnt Chicha	SPE	Mostly low	Mostly viscous	Very consistent	Uncertain	Mostly high	Mostly fast	Very diluted	Very basic	Very low	very high
Burnt Chicha	SPE	Mostly high	Mostly viscous	Uncertain	Very fluid	very high	Very fast	Uncertain	Very basic	very high	very high
Burnt Chicha	SPE	Mostly low	Very viscous	Very consistent	Mostly fluid	Uncertain	very slow	Mostly diluted	Mostly acidic	Mostly low	Mostly low
Burnt Chicha	SPE	Uncertain	Very fluid	Mostly weak	Very fluid	Very low	very slow	Mostly concentrated	Uncertain	Mostly high	Uncertain
Burnt Chicha	SPE	Very low	Uncertain	Uncertain	Very fluid	very high	Very fast	Very concentrated	Uncertain	Uncertain	very high
Burnt Chicha	CPE	Uncertain	Mostly viscous	Very consistent	Uncertain	Mostly high	Uncertain	Very diluted	Mostly basic	Mostly high	Very low
Burnt Chicha	CPE	Mostly high	Very fluid	Very weak	Very thick	Uncertain	Mostly slow	Very concentrated	Very basic	very high	very high
Burnt Chicha	CPE	Mostly low	Mostly viscous	Mostly weak	Very fluid	very high	Mostly fast	Mostly diluted	Very acidic	Uncertain	Mostly low
Burnt Chicha	CPE	Mostly low	Mostly viscous	Mostly weak	Mostly fluid	very high	Mostly slow	Very concentrated	Very basic	very high	Mostly low
Burnt Chicha	CPE	Very high	Uncertain	Mostly weak	Mostly thick	very high	Very fast	Very diluted	Very basic	Mostly low	Uncertain
Burnt Chicha	CPE	Mostly low	Mostly fluid	Uncertain	Very fluid	Uncertain	very slow	Mostly diluted	Mostly basic	Very low	very high
Burnt Chicha	CPE	Very high	Very viscous	Very consistent	Very fluid	Uncertain	very slow	Very concentrated	Very acidic	Uncertain	Very low
Burnt Chicha	CPE	Very low	Very viscous	Very consistent	Uncertain	Mostly low	Mostly fast	Mostly concentrated	Mostly basic	Mostly low	Very low
Burnt Chicha	CPE	Mostly high	Uncertain	Very weak	Very thick	Mostly high	very slow	Mostly diluted	Very acidic	Mostly low	Mostly high
Burnt Chicha	CPE	Very high	Mostly viscous	Very weak	Mostly thick	Very low	very slow	Mostly diluted	Mostly acidic	Uncertain	Very low
Burnt Chicha	CPE	Very low	Very fluid	Mostly weak	Mostly fluid	very high	Mostly fast	Mostly concentrated	Very acidic	very high	Uncertain
Burnt Chicha	CPE	Mostly low	Mostly viscous	Very weak	Mostly thick	Mostly low	Mostly fast	Uncertain	Mostly basic	Mostly low	Uncertain
Burnt Chicha	CPE	Mostly low	Uncertain	Uncertain	Mostly fluid	Very low	Very fast	Very diluted	Very basic	Mostly high	Very low
Burnt Chicha	CPE	Very low	Very viscous	Very consistent	Very thick	Mostly low	very slow	Uncertain	Very acidic	Mostly low	Very low
Burnt Chicha	CPE	Mostly low	Mostly viscous	Mostly weak	Mostly fluid	Uncertain	very slow	Very concentrated	Very acidic	very high	Mostly high

Beverage	Treatment	ρ (g/cm³)	μ (cP)	k (Pa.s)	n	τ (Pa)	γ (1/s)	°Bx	pH	Acid (°D)	Alc. %vol
Burnt Chicha	CPE	Very low	Very viscous	Very weak	Uncertain	very high	Very fast	Mostly concentrated	Uncertain	very high	Very low
Burnt Chicha	CPE	Very high	Very fluid	Uncertain	Uncertain	Very low	Uncertain	Uncertain	Very basic	Mostly high	Mostly high

3.2 Data preparation

To effectively apply the Neutrosophic C-means method, data entries in neutrosophic linguistic terms were converted into qualitative data according to the following transformation rules:

Table 5: Transformation rules. Source: own elaboration.

Variable	Burnt Chicha without enzymatic treatment	Burnt Chicha with enzymatic treatment	White Chicha without enzymatic treatment	White Chicha with enzymatic treatment	Wiwis Chicha without enzymatic treatment	Wiwis Chicha with enzymatic treatment	Chonta chicha without enzymatic treatment	Chonta chicha with enzymatic treatment
ρ (g/cm³)	Mostly low to Uncertain (about 1 g/cm³)	Mostly low to Uncertain (about 1 g/cm³)	Mostly low to Uncertain (about 1 g/cm³)	Mostly low to Uncertain (about 1 g/cm³)	Mostly low to Uncertain (about 1 g/cm³)	Mostly low to Uncertain (about 1 g/cm³)	Mostly low to Uncertain (about 1 g/cm³)	Mostly low to Uncertain (about 1 g/cm³)
μ (cP)	Mostly fluid to Uncertain (about 1 cP)	Lower viscosity than untreated (about 0.5 cP)	Mostly fluid to Uncertain (about 1 cP)	Lower viscosity than untreated (about 0.5 cP)	Mostly fluid to Uncertain (about 1 cP)	Lower viscosity than untreated (about 0.5 cP)	Mostly fluid to Uncertain (about 1 cP)	Lower viscosity than untreated (about 0.5 cP)
k (Pa.s)	Mostly weak to Uncertain (about 0.001 Pa.s)	Lower consistency than without treatment (around 0.0005 Pa.s)	Mostly weak to Uncertain (about 0.001 Pa.s)	Lower consistency than without treatment (around 0.0005 Pa.s)	Mostly weak to Uncertain (about 0.001 Pa.s)	Lower consistency than without treatment (around 0.0005 Pa.s)	Mostly weak to Uncertain (about 0.001 Pa.s)	Lower consistency than without treatment (around 0.0005 Pa.s)
n	Mostly Fluent to Uncertain (about 1)	Mostly Fluent to Uncertain (about 1)	Mostly Fluent to Uncertain (about 1)	Mostly Fluent to Uncertain (about 1)	Mostly Fluent to Uncertain (about 1)	Mostly Fluent to Uncertain (about 1)	Mostly Fluent to Uncertain (about 1)	Mostly Fluent to Uncertain (about 1)
τ (Pa)	Mostly low to Uncertain (about 0.1 Pa)	Lower shear stress than untreated (about 0.05 Pa)	Mostly low to Uncertain (about 0.1 Pa)	Lower shear stress than untreated (about 0.05 Pa)	Mostly low to Uncertain (about 0.1 Pa)	Lower shear stress than untreated (about 0.05 Pa)	Mostly low to Uncertain (about 0.1 Pa)	Lower shear stress than untreated (about 0.05 Pa)
γ (1/s)	Mostly slow to Uncertain (about 0.1 s^{-1})	Mostly slow to Uncertain (about 0.1 s^{-1})	Mostly slow to Uncertain (about 0.1 s^{-1})	Mostly slow to Uncertain (about 0.1 s^{-1})	Mostly slow to Uncertain (about 0.1 s^{-1})	Mostly slow to Uncertain (about 0.1 s^{-1})	Mostly slow to Uncertain (about 0.1 s^{-1})	Mostly slow to Uncertain (about 0.1 s^{-1})
°Bx	Mostly concentrated to Very concentrated (between 12 °Bx and 18 °Bx)	Mostly concentrated to Very concentrated (between 12 °Bx and 18 °Bx)	Mostly diluted to Mostly concentrated (between 5 °Bx and 12 °Bx)	Mostly diluted to Mostly concentrated (between 5 °Bx and 12 °Bx)	Mostly diluted to Mostly concentrated (between 5 °Bx and 12 °Bx)	Mostly diluted to Mostly concentrated (between 5 °Bx and 12 °Bx)	Mostly diluted to Mostly concentrated (between 5 °Bx and 12 °Bx)	Mostly diluted to Mostly concentrated (between 5 °Bx and 12 °Bx)
pH	Mostly Acid to Uncertain (between 3.0 and 4.0)	Mostly Acid to Uncertain (between 3.0 and 4.0)	Mostly acidic to Uncertain (between 3.5 and 4.5)	Mostly acidic to Uncertain (between 3.5 and 4.5)	Mostly acidic to Uncertain (between 3.5 and 4.5)	Mostly acidic to Uncertain (between 3.5 and 4.5)	Mostly acidic to Uncertain (between 3.5 and 4.5)	Mostly acidic to Uncertain (between 3.5 and 4.5)
Acidity (°D)	Mostly high to Very high (between 0.6°D and 1.2°D)	Mostly high to Very high (between 0.6°D and 1.2°D)	Mostly low to Mostly high (between 0.2°D and 0.6°D)	Mostly low to Mostly high (between 0.2°D and 0.6°D)	Mostly low to Mostly high (between 0.2°D and 0.6°D)	Mostly low to Mostly high (between 0.2°D and 0.6°D)	Mostly low to Mostly high (between 0.2°D and 0.6°D)	Mostly low to Mostly high (between 0.2°D and 0.6°D)
Alc. % vol.	Mostly low to Uncertain (between 1% vol and 2% vol)	Mostly low to Uncertain (between 1% vol and 2% vol)	Mostly low to Uncertain (between 1% vol and 2% vol)	Mostly low to Uncertain (between 1% vol and 2% vol)	Mostly low to Uncertain (between 1% vol and 2% vol)	Mostly low to Uncertain (between 1% vol and 2% vol)	Mostly low to Uncertain (between 1% vol and 2% vol)	Mostly low to Uncertain (between 1% vol and 2% vol)

- ρ (g/cm³) - Density: "Very low" to "Mostly low" could be considered as "Uncertain", so (0.50,0.50,0.50).
- μ (cP) - Viscosity: "Very fluid" to "Mostly fluid" could range from "Moderately good" to "Good", so (0.60,0.35,0.40) to (0.80,0.15,0.20).
- k (Pa.s) - Consistency Index: "Very weak" to "Mostly weak" can be assigned as "Moderately poor", so (0.40,0.65,0.60).

- n - Flow Index: "Mostly fluid" aligns with "Moderately good", so (0.60,0.35,0.40).
- τ (Pa) - Shear Stress: "Very high" to "Mostly high" could translate to "Very Good" or "Excellent", so (0.90, 0.1, 0.1) to (1.0,0,0).
- γ (1/s) - Rate of Deformation: "Mostly slow" would map to "Moderately poor", so (0.40,0.65,0.60).
- °Bx - Brix Degrees: "Very concentrated" could be considered "Excellent", so (1.0,0,0).
- pH: "Very acidic" to "Mostly acidic" could be "Poor", so (0.30,0.75,0.70).
- Acidity (°D): "Very high" to "Mostly high" can be viewed as "Very Good", so (0.90, 0.1, 0.1).
- Alc. % vol.: "Mostly high" could be assigned as "Good", so (0.80,0.15,0.20).

Table 6: Sample of data preparation. Source: own elaboration.

Bever-age	Treat-ment	ρ (g/cm³)	μ (cP)	k (Pa.s)	n	τ (Pa)	γ (1/s)	°Bx	pH	Acidity (°D)	Alc. % vol
Burnt Chicha	SPE	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0.6, 0.35, 0.4)	(0.9, 0.1, 0.1)	(0.4, 0.65, 0.6)	(0.5, 0.5, 0.5)	(0.3, 0.75, 0.7)	(0.9, 0.1, 0.1)	(0.8, 0.15, 0.2)
Burnt Chicha	SPE	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0.9, 0.1, 0.1)	(0.4, 0.65, 0.6)	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0.9, 0.1, 0.1)	(0.5, 0.5, 0.5)
Burnt Chicha	SPE	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0.9, 0.1, 0.1)	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0.3, 0.75, 0.7)	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)
Burnt Chicha	SPE	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0.4, 0.65, 0.6)	(0.5, 0.5, 0.5)	(0.3, 0.75, 0.7)	(0.5, 0.5, 0.5)	(0.8, 0.15, 0.2)

3.3 Algorithm

- Selection of the number of clusters: Based on the variety of treatments and types of beverages present in the data, start with a moderate number, such as 4 clusters, and adjust if necessary in future iterations.
- Initialization of membership degrees: Initialize the membership degrees of each data point for each cluster randomly, ensuring that the sum of the degrees of truth, indeterminacy, and falsehood for each point and cluster equals 1.
- Fuzzification Constant (m): 2
- Initialization of Cluster Centers: k-means++ algorithm with Euclidean distance
- Termination Criteria: Improvement in the objective function less than 0.001, maximum of 100 iterations
- Additional Parameters: m: 2 and δ: 0.1

4 Application of the algorithm and interpretation

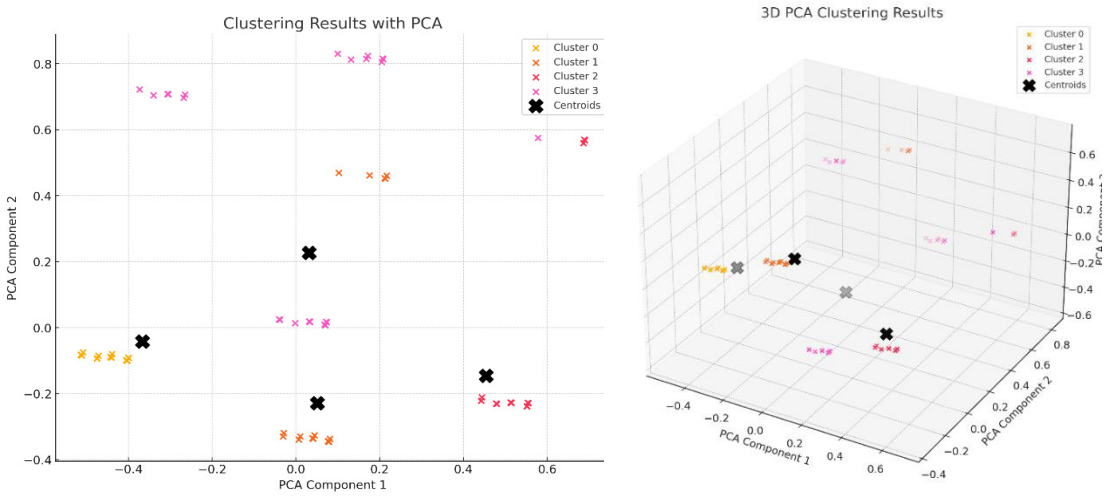

Figure 2: Clusters plotting using Principal Component Analysis to reduce dimensionality and plot the data points in a 2D and 3D space, colored according to their cluster. Source: Developed by the author.

In the graphs, each point represents a data point and is colored according to the cluster it belongs to. The black 'X'-shaped markers represent the centroids. Here is the cluster analysis by their distribution: Cluster 0: 39 points, Cluster 1: 31 points, Cluster 2: 27 points, Cluster 3: 39 points. As can be seen, the distribution of points is quite

balanced among the clusters, with a slight variation in sizes, which is desirable in many clustering contexts. The Silhouette Score is 0.357, indicating a reasonable level of separation and cohesion between the clusters. A value closer to 1 indicates ideal separation, while values closer to 0 or negative indicate overlapping or poorly defined clusters. The score of 0.357 suggests that the clusters are moderately well-defined.

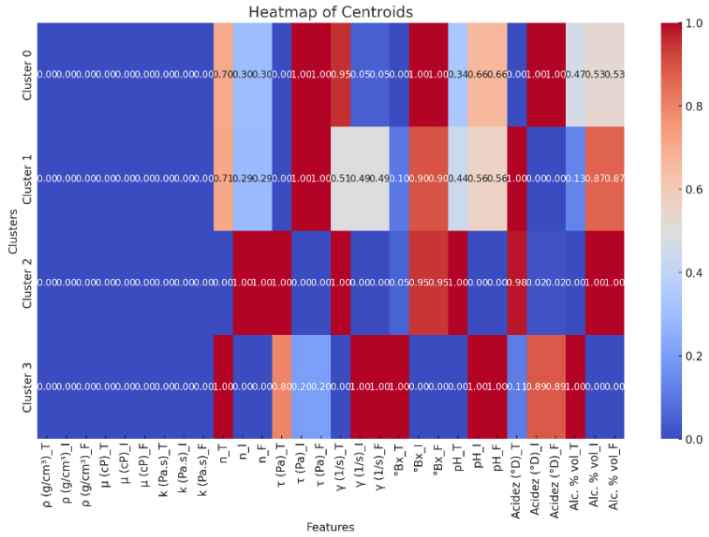

Figure 3: Heatmap of the Centroids. Source: Developed by the author.

The heatmap displays the normalized characteristics of the centroids for each cluster. Darker cells indicate lower values, while lighter cells represent higher values. As observed, the physicochemical properties have significantly different values, which influences the formation of clusters.

Observing the averages of each attribute that defines the centroids, it can be stated that the values for ρ (g/cm³), μ (cP), and k (Pa.s) are uniform across all clusters, indicating that these properties do not vary significantly between clusters or are not determinants in the formation of clusters under the current model.

Variables such as n and τ (Pa) show more significant variation between clusters, indicating differences in the physical properties of the beverages that impact their grouping. The values related to °Bx, pH, Acidity (°D), and Alc. % vol also shows variations associated with differences in the chemical composition or the manufacturing process of the beverages. The analysis shows that the clusters formed reflect variations in certain physical and chemical properties of the beverages. The evaluation using the Silhouette Score reveals that the clusters are reasonably well-defined, although there is room for improvement in the separation between them.

Next, a violin plot is presented that shows the distribution of the first two principal components for each cluster. These visualizations are useful for understanding how points are dispersed within each cluster and for comparing variability between clusters.

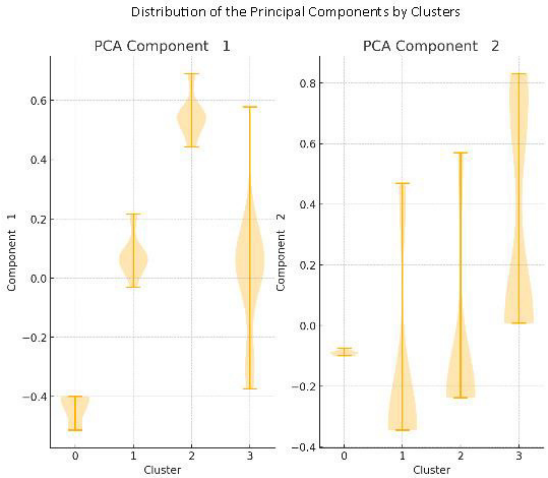

Figure 4: Violin plots for clusters. Source: own elaboration.

As observed in Figure 4:

- Component 1: Some clusters have a wider distribution, indicating a greater dispersion of data, while others are narrower.
- Component 2: Similar to Component 1, variability and dispersion vary among the clusters.

These differences highlight distinctive characteristics between groups, given by the manufacturing process that imparts different properties to each beverage. Finally, for a hierarchical view of how the clusters might be related, Figure 5 shows a dendrogram.

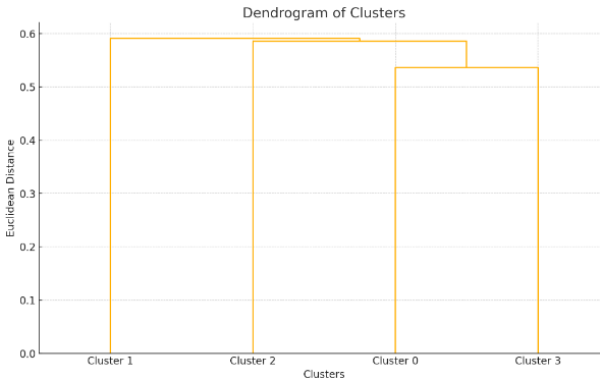

Figure 5: Dendrogram. Source: own elaboration.

The dendrogram shows clusters 1 and 2 as being closest to each other, indicated by the lower height of the linkage that connects them (low Euclidean distance). This demonstrates that the data points in these two clusters are more similar to each other compared to points in other clusters. Regarding the merging of Cluster 1 and Cluster 2 with Cluster 0: After Cluster 1 and Cluster 2, the next to join in the hierarchy is Cluster 0. The height at which Cluster 0 joins the group formed by Cluster 1 and Cluster 2 is noticeably higher than the height at which these two clusters join together. This indicates that Cluster 0 while sharing some similarities with Clusters 1 and 2, is generally less similar to these clusters than they are to each other. Finally, Cluster 3 joins the group formed by the other three clusters at the highest point on the dendrogram. This indicates that Cluster 3 is the least similar to any of the other clusters.

The dendrogram reveals a data structure where Cluster 1 and Cluster 2 represent subgroups within a broader category of data due to their proximity. Cluster 0 and especially Cluster 3 contain data points that are significantly different and represent wider variations within the dataset. The great height at which Cluster 3 joins the rest suggests substantial differences in the data characteristics of this cluster compared to the other clusters.

5 Discussion

Table 3 presents the evaluation of each treatment using measurement instruments. These were conducted at a temperature of 17.9°C with a relative humidity of 59.6%, where yuca and chonta chichas containing enzymatic preparations stood out, yielding the following results: burned chicha with a density of ρ=1.0319 g/cm³, apparent viscosity μ= 0.18 cP, consistency index k= 0.0138 Pa.s, flow behavior n= 0.0004, shear stress τ= 0.0138 Pa, and deformation rate γ=77 (1/s). Followed by white chicha with a density ρ= 1.0124g/cm³, apparent viscosity μ=1.13 cP, consistency index k=0.1072 Pa.s, flow behavior n=0.0305 adi, shear stress τ= 0.1203 Pa, and deformation rate γ=50 (1/s).

Subsequently, for wiwis chicha, it presents a density ρ=1.0280 g/cm³, apparent viscosity μ=2.92 cP, consistency index k=0.2203 Pa.s, flow behavior n=0.0021 adi, shear stress τ=0.2223 Pa, and deformation rate γ=76 (1/s). Finally, chonta chicha has a density ρ=1.0207 g/cm³, apparent viscosity μ=1.13 cP, consistency index k=0.0122 Pa.s, flow behavior n=0.0020 adi, shear stress τ= 0.0123 Pa, and deformation rate γ=50 (1/s).

Thus, fermented beverages of yuca and chonta exhibited rheological indices characteristic of a non-Newtonian fluid with pseudoplastic properties (n>0). Therefore, the presence of enzymes in the chichas positively influenced their production, improving the quality and sensory characteristics of the beverages; specifically, increasing the content of soluble solids for better fermentation processes.

When comparing these data with the results obtained with the Neutrosophic C Means algorithm compared to instrumental data, it can be said that according to experts, the data presented show:

- Different properties of fermented beverages, with a specific focus on yuca and chonta chichas with enzymes.
- Burned Chicha: Properties such as high density and low viscosity, indicate a more liquid and less

viscous flow.

- White Chicha: Lower density compared to Burned Chicha, but with significantly higher viscosity and higher values of consistency, shear stress, and a slower flow.
- Wiwis and Chonta Chicha: Both exhibit non-Newtonian fluid characteristics with pseudoplastic properties. Notably, Wiwis Chicha has a relatively high viscosity and high shear stress compared to Chonta Chicha.

The clusters identified through the analysis reflect groupings based on characteristics similar to those reported instrumentally, although the specific measurements in clustering are neutrosophic representations and not absolute values. Each cluster represents a group with similar physicochemical properties. The distribution of chichas in clusters shows similarities with the characteristics highlighted by the instruments, such as density and viscosity. For example, chichas with higher density and viscosity might be grouped, reflecting their behavior under controlled conditions.

Instrumental data suggest that the addition of enzymes improves important characteristics such as consistency and flow ability. Clusters reflect these improvements, and it is possible to identify which clusters correspond to chichas treated with enzymes. Neutrosophic analysis reveals more subtle variations in degrees of truth, indeterminacy, and falsehood that are not captured by direct instrumental measurements.

Thus, the comparison between instrumental results and those obtained through neutrosophic clustering helps validate or refine the understanding of the properties of the beverages. While instrumental measures provide concrete data under specific conditions, clustering reveals broader patterns or relationships between different samples and conditions.

6 Conclusion

It can be said that the use of enzymatic preparations in yuca and chonta chichas has significantly improved the rheological characteristics of the beverages, making them more fluid and enhancing their texture and smoothness on the palate. This suggests that enzymes help break down structural components of yuca and chonta, facilitating a more desirable texture profile and a better sensory experience.

Traditionally, some fermented beverages may require chewing to initiate fermentation. The use of enzymes not only improves quality but also modernizes and sanitizes the production process by eliminating the need for chewing, which is crucial for product acceptance in broader markets and for complying with food safety and hygiene regulations.

The chichas analyzed exhibit typical non-Newtonian fluid behaviors of a pseudoplastic type. This is relevant for the food industry, as these properties affect the handling, processing, and sensory qualities of the beverages. The ability to predict and control this behavior is crucial for ensuring the consistency and quality of the final product.

The application of advanced rheological techniques, such as the use of concentric cone geometries and parallel plate setups with rotational rheometers, allows for precise evaluation of properties such as viscosity and density. This precision is vital to ensure that modifications in the production process (such as the addition of enzymes) have the desired effect and to maintain strict quality control.

The results point to the need to continue exploring and optimizing the use of enzymes in the production of traditional fermented beverages. Future research could focus on comparing different types of enzymes and their concentrations to determine the most effective ones for each type of beverage.

Standardization of processes and adaptation to modern regulations can help expand the market for these traditional beverages, improving not only quality but also acceptability in different cultural and geographic contexts.

These studies demonstrate the value of combining modern scientific methods with ancestral traditions, which can result in the preservation and innovation in the production of traditional beverages with significant improvements in quality and market acceptance.

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Comparison of Fuzzy Cognitive Maps and SEM in Estimating the Perception of Corporate Social Responsibility

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Abstract: This study examines the impact of the perception of corporate social responsibility (CSR) on the perceptions of clients, employees, and managers in small and medium enterprises (SMEs) in Ecuador. Utilizing a combination of Structural Equation Modeling (SEM) and Fuzzy Cognitive Maps (FCM), the research addresses how CSR policies influence the various internal stakeholders of companies, identifying complex structures of interaction and perception dynamics that could impact organizational performance. This neutrosophic framework not only enhances the understanding of CSR but also promotes the development of workplace welfare policies that effectively cater to a diverse range of staff needs and expectations. SEM was used to estimate and validate the structural relationships, while FCM helped illustrate the dynamics within the CSR network, highlighting how different perceptions affect and are affected by other variables in the system. The study's findings indicate that the perception of CSR by employees and managers has a direct and significant effect on client perception, corroborating the importance of effective CSR management in enhancing customer satisfaction and overall business performance.

Keywords: Corporate Social Responsibility, Economic development, neutrosophic, SMEs, Fuzzy Cognitive Maps

1. Introduction

Corporate social responsibility (CSR) in small and medium-sized enterprises (SMEs) in Ecuador, and generally in Latin America, has evolved from a purely philanthropic perspective to a more comprehensive approach that includes economic, social, and environmental sustainability. Internal CSR in SMEs in Ecuador represents a comprehensive approach that goes beyond legal compliance and aims to enhance the well-being of employees and the local community. This practice is not only an ethical matter but also strategic, as it significantly influences the reputation and economic performance of the companies [1].

Firstly, implementing internal CSR practices in SMEs can lead to increased productivity and employee retention. When a company invests in a healthy work environment, professional development programs, and fair compensation, it fosters loyalty and motivation among staff. This results in a reduction of costs associated with employee turnover and absenteeism, and an increase in operational efficiency. Satisfied and committed employees are more likely to contribute to the growth and innovation within the company.

Furthermore, SMEs that adopt internal CSR tend to improve their image and relationship with customers and suppliers. In Ecuador, where the business fabric is largely community-based and consumers are increasingly aware of business practices, SMEs that demonstrate a commitment to the well-being of their team and community can gain a competitive edge. This approach can lead to greater customer loyalty, with customers preferring to buy products and services from socially responsible companies [2], [3].

Internal CSR also facilitates access to financing for SMEs. Financial institutions and development agencies are beginning to consider social responsibility practices as an important factor in risk assessment and lending decisions. An SME that shows a strong commitment to CSR can be seen as less risky and more sustainable in the long term, which may improve financing conditions.

By engaging in CSR, SMEs in Ecuador better align with government policies and the Sustainable Development Goals (SDGs) promoted by the UN. This not only contributes to the economic and social development of the country but also opens opportunities for collaborations with the public sector and international organizations that provide additional resources and visibility.

Empirical research has shown that conventional economic development models, based on technological advancement and profit maximization, have led to significant global inequalities, particularly in Latin American

countries. In response to the challenge of reducing these disparities, there has been a strong push to promote sustainable development. This concept aims fundamentally to conserve the environment and improve human well-being, using economic development and the political and institutional framework as means.

This vision necessitates examining the four dimensions of sustainable development. On one hand, there is the environmental dimension, where, according to the approach, the environment can be considered an essential and irreplaceable asset of the production process (strong sustainability), and in some cases, as in Ecuador, it is referred to as "natural heritage." This dimension is complemented by the socio-cultural dimension, focused on the living conditions of the local population, highlighting the importance of local policies that enhance ancestral knowledge, culture, social and human capital, through the transformation of consumption patterns.

The economic dimension proposes new business models and strategies for companies to address environmental conservation and the social well-being of their communities. Notably, the circular and collaborative economy, along with strategies for association and corporate social responsibility, especially in SMEs, aim to increase competitiveness and collaboration, which are the ultimate goals of sustainable development [4].

In this framework, corporate social responsibility (CSR) fosters the commitment of companies to act ethically and contribute to social well-being, integrating social, environmental, and ethical aspects into their business operations and decisions. Companies that adopt CSR commit to minimizing negative impacts and maximizing positive impacts on the communities where they operate. Thus, CSR promotes sustainable development goals, especially those related to responsible production and consumption, as well as the efficient use of ecosystem services, but from a territorial perspective, focusing on the regions and communities affected by business activities.

CSR is oriented toward commitment and ethics with all stakeholders, not only external ones such as the community, the environment, customers, and other businesses and suppliers but also internal ones, including employees and investors. CSR regarding employees seeks to ensure fair working conditions, promote diversity and equality of opportunity, and offer well-being and professional development programs. With customers, it strives to provide safe and quality products and services, promote fair and transparent business practices, and ensure data privacy and security [5].

Regarding the community, it strengthens initiatives and projects that contribute to local development, fostering civic participation and volunteering. In relation to the environment, it seeks to minimize the impact of operations, adopt sustainable resource management practices, reduce greenhouse gas emissions, and promote environmental conservation. Although in large companies CSR practices are an integral part of the organizational and strategic culture, in SMEs, it may be believed that these practices require additional financial resources and do not contribute to the company's performance or influence customer perception of the organization's reputation.

Several authors note that the limitations to implementing CSR policies in SMEs are related to a lack of resources and funding, a lack of awareness, the inclusion of family members as employees, and a lack of strategic planning. Even with CSR practices, a lack of communication and promotion limits the positive effects on stakeholders. However, CSR is as important for SMEs as it is for large corporations. Although SMEs may have fewer resources than large companies, implementing socially responsible practices can bring significant benefits both for the company and for society [6]–[9].

1.1 Application in Neutrosophy

In the realm of internal corporate social responsibility (CSR), especially in SMEs in Ecuador, neutrosophic studies gain relevance by enhancing a threshold of possibilities amidst the diverse indeterminacies that the SME field entails. In the context of Ecuadorian SMEs, these businesses analyze and assess the different degrees of truth and falsehood of internal opinions and perceptions regarding CSR policies. This allows for a deeper understanding of employee concerns and expectations, facilitating the creation of a work environment that balances operational and economic needs with employee well-being and motivation.

The application of neutrosophy in evaluating labor welfare policies enables SMEs in Ecuador to address the ambiguity and uncertainty inherent in employees' perceptions of these policies. By considering neutral states and their degrees, SMEs identify and develop wellness programs that are not only perceived as beneficial but also truly respond to a wide range of staff needs and expectations, thus enhancing their commitment and satisfaction, which is essential for the economic and social sustainability of the company.

Neutrosophy offers Ecuadorian SMEs a framework to develop CSR strategies that are inherently adaptive, considering the varying degrees of acceptance and resistance within the organization. This capability is particularly valuable in an economic context where market conditions are unstable, and SMEs need to quickly adapt their CSR practices to both internal and external variations. Such adaptability strengthens economic resilience and promotes sustainable development, ensuring that internal practices are in line with principles of equity and social responsibility.

The application of neutrosophy in managing internal CSR in Ecuadorian SMEs presents an opportunity to address the complexities and dynamics of internal needs and expectations more holistically and balanced. By integrating this approach, SMEs not only improve their internal environment and economic efficiency but also strengthen their commitment to sustainable development and social responsibility.

Against this backdrop, the present research aims to analyze how the perception of corporate social responsibility (CSR) impacts the perception of customers, employees, and managers in small and medium-sized enterprises (SMEs), using Structural Equation Models (SEM) and Fuzzy Cognitive Maps (FCM) to understand the structural relationships and dynamics among these groups.

2 Methodology

2.1 Structural Equation Modeling (SEM)

Structural Equation Modeling (SEM) is an advanced statistical technique that allows for the examination of complex relationships between observed and latent variables. This methodology combines elements of multiple regression analysis and factor analysis to estimate a series of interrelated equations. It is widely used in social sciences, marketing, behavioral studies, education, and more [10], [11].

To perform calculations in Structural Equation Modeling (SEM), various equations and mathematical procedures are used, which are outlined below:

1. Measurement Model: Defines how latent (unobserved) variables are related to observed variables (indicators) [12]. Having a latent variable η and observed variables $x_1, x_2, \dots, x_n$, the equations of the measurement model are:

$$x_1 = \lambda_{11\eta} + \delta_1 \quad (1)$$

$$x_2 = \lambda_{21\eta} + \delta_2 \quad (2)$$

$$x_n = \lambda_{n1\eta} + \delta_n \quad (3)$$

Where:

- $\lambda_{i1\eta}$ are the load factors, indicating how much the latent variable η contributes to each observed variable x_i.
- δ_i are the measurement errors for each observed variable.

In matrix form, this can be expressed as:

$$x = \Lambda_\eta + \delta \quad (4)$$

- x is the vector of observed variables.
- Λ is the load factor matrix.
- η is the vector of latent variables.
- δ is the vector of measurement errors.

2. Structural Model

The structural model defines the relationships between the latent variables [13]. If $\eta_1, \eta_2, \dots, \eta_m$ are latent variables, the equations of the structural model are:

$$\eta_1 = \gamma_{11}\xi_1 + \gamma_{12}\xi_2 + \cdots + \gamma_{1p}\xi_p + \zeta_1 \quad (5)$$

$$\eta_2 = \gamma_{21}\xi_1 + \gamma_{22}\xi_2 + \cdots + \gamma_{2p}\xi_p + \zeta_2 \quad (6)$$

$$\eta_m = \gamma_{m1}\xi_1 + \gamma_{m2}\xi_2 + \cdots + \gamma_{mp}\xi_p + \zeta_m \quad (7)$$

Where:

- γ_{ij} are the structural coefficients, which indicate how each exogenous variable ξ_j affects the latent variable η.
- ζ_i are the errors in the structural equations for each latent variable.

In matrix form, the structural model is expressed as:

$$\eta = \mathrm{B}_\eta + \Gamma\xi + \zeta \quad (8)$$

- η is the vector of endogenous latent variables.
- B is the coefficient matrix that describes the relationships between the endogenous latent variables.
- Γ is the coefficient matrix that describes the relationships between the exogenous latent variables ξ and endogenous latent variables η.
- ζ is the vector of errors in the structural equations.

3. Covariance Matrix

The modeled covariance matrix $\Sigma(\theta)$ depends on the parameter vector θ, and is calculated as follows:

$$\Sigma(\theta) = \Lambda\Phi\Lambda^{\intercal} + \Theta_\delta \quad (9)$$

Where:

- Λ is the load factor matrix.
- Φ is the covariance matrix of the latent variables.
- $\Theta_\delta$is the covariance matrix of measurement errors.

4. Parameter Estimation

Model parameters (factor loadings, structural coefficients, variances, and covariances) are commonly estimated using:
- Maximum Likelihood (ML)
- Generalized Least Squares (GLS)
- Asymptotically Weighted Least Squares (WLS)

5. Discrepancy Function

The discrepancy function F measures how different the observed covariance matrices S and the modeled $\Sigma(\theta)$ are.

$$F\left(S, \Sigma(\theta)\right) = trace\left(S\Sigma^{-1}(\theta)\right) - \log det\left(S\Sigma^{-1}(\theta)\right) - p \quad (10)$$

Where p is the number of observed variables.

6. Adjustment Indices

To evaluate how well the model fits the data, fit indices are calculated such as:

- Chi-squared (x^2): $x^2 = (N-1)F$, (11)

Where N is the sample size.
- RMSEA (Root Mean Square Error of Approximation)
- CFI (Comparative Fit Index)
- TLI (Tucker-Lewis Index)
- SRMR (Standardized Root Mean Square Residual)
- Likert

2.2 Fuzzy Cognitive Maps (FCM)

Fuzzy Cognitive Maps (FCMs) are fuzzy feedback models used to represent causality. They integrate theoretical tools from cognitive maps, fuzzy logic, neural networks, semantic networks, expert systems, and nonlinear dynamic systems [14]. This technique allows for modeling the system with feedback and fuzzy degrees of causality within the range [0,1]. In the diagram, each node represents a fuzzy set or event occurring to some degree. The nodes are causal concepts and can model events, actions, values, goals, or processes. This technique also offers benefits in visual modeling, simulation, and prediction [15].

In FCMs, there are three possible types of causal relationships between concepts:

- Positive causality (Wij> 0), indicating positive causality between the concepts Ci and Cj. That is, an increase (decrease) in the value of Ci leads to an increase (decrease) in the value of Cj.
- Negative causality (Wij < 0), indicating negative causality between the concepts Ci and Cj. That is, an increase (decrease) in the value of Ci leads to a decrease (increase) in the value of Cj.
- Non-existence of relationships (Wij = 0), indicating the absence of causal relationships between Ci and Cj.

An FCM can be represented through a directed graph in which the nodes represent concepts and the edges indicate causal relationships. The intensity of the causal relationship is represented by fuzzy values [16]. The values of the concepts are calculated at each step of the simulation. Depending on the initial vector, the FCM will converge to a fixed point, limit cycle, or chaotic attractor.

In this paper, the calculation will be developed as follows:

1. Selection of the most relevant indicators that affect the relevance of the scientific research of the IES.
2. Preparation of the adjacency matrix.
3. Static analysis: calculated for the absolute values of the adjacency matrix:
   - *Outdegree*, denoted by od(vi), is the sum for each row of the absolute values of a variable of the fuzzy adjacency matrix. It is a measure of the cumulative strength of the connections existing in the variable.
   - *Indegree,* denoted by id(vi), which is the sum for each column of the absolute values of a variable of the fuzzy adjacency matrix. Measures the cumulative input force of the variable.
   - The centrality or total degree of the variable is the sum of od(vi), with id(vi), as indicated below:

$$\mathrm{td(vi)} = \mathrm{od(vi)} + \mathrm{id(vi)} \quad (12)$$

Finally, the variables are classified according to the following criteria, see[17]:

a) The *transmitting variables* are those with $od(v_i) > 0$ and $id(v_i) = 0$
b) The *receiving variables* are those with $od(v_i) = 0$ and $id(v_i) > 0$
c) Ordinary variables satisfy both $od(v_i) \neq 0$ and $id(v_i) \neq 0$

They are ordered in ascending order according to the degree of centrality.

When a group of experts (k) participates, the adjacency matrix is formulated through an aggregation operator, such as the arithmetic mean. The simplest method involves finding the arithmetic mean of each connection for each expert. For k experts, the final Fuzzy Cognitive Map (FCM) adjacency matrix (E) is obtained as follows:

$$E = \frac{E_1 + E_2 + \cdots + E_k}{k} (13)$$

This ease of aggregation allows the creation of collective mental models with relative ease.

2.3 Methodology

To verify the theoretical causality between corporate social responsibility considering external actors (customers) and internal actors (employees and managers), and the performance of SMEs, a structural equation model was estimated. This was subsequently compared using Fuzzy Cognitive Maps for a better perspective on the evaluation.

The evaluation was conducted for 351 SMEs across three sectors—commerce, services, and manufacturing—which represent the most significant sectors in the economic activity of the canton. For the application of the instrument, a customer, an employee, and a manager were selected from each of the companies.

Each unit of analysis had a specific instrument. The first one, concerning customers, included seven items querying about the information provided by the company regarding the use, consumption, and handling of its products; complaint resolution; the service offered; awareness of the company's activities; the quality of products and services; interest in community issues, and sustainability, social, and ecological commitment programs.

Regarding employees, the aspects addressed were related to criteria for hiring, training, promotion, and evaluation of staff; discrimination; working conditions; the opportunity to express ideas, suggestions, proposals, or complaints; the promotion of commitment; environmental training; strengthening the culture of environmental responsibility; processes for reducing and optimizing water and energy consumption; compliance with current environmental regulations and norms, and the implementation of measures that reduce environmental impact in the development of new products.

Lastly, consultations with managers or administrators were related to two areas. The first was related to the degree of development of organizational culture and strategic planning, linked to mission and vision, principles and values, environmental policies, and social responsibility policies. The second was related to performance, measured as financial results, quality of labor relations and products, and interest in community and environmental problems (Table 1).

Table 1: Table 1: Statements (items) of each instrument. Source: Own elaboration.

Analysis unit		
Customers	**Employees**	**Managers**
The company provides clear and precise information about the use, consumption, and handling of its products (frequency).	Application of criteria for hiring, training, promotion, and evaluation (degree).	Social responsibility is reflected in the company's mission and vision (degree).
The company resolves consumer complaints (frequency).	Application of mechanisms to prevent all forms of discrimination (degree).	The degree of alignment of principles and values with company management.
The quality of customer service provided by the company (satisfaction).	Application of mechanisms to improve working conditions in terms of physical environment, hygiene, ventilation, and lighting (degree).	Existence of your company's social responsibility policies (degree).
Identifies the activity or activities carried out by this company (degree).	Application of mechanisms to encourage workers to express their ideas, suggestions, proposals, or complaints (degree).	Existence of the organization's environmental management policies (degree).
The quality of the products and services offered by the company (degree).	Opportunities provided by the organization to increase employee commitment (frequency).	The degree to which financial results reflect the organization's performance.
The company's interest in community issues (degree).	Promotion of environmental education among employees and strengthening of the culture of responsibility (degree).	The company's compliance with labor conditions and regulations (degree).

Analysis unit		
The company's interest in sustainability programs, and social and ecological commitment (degree).	Reduction and optimization of water and energy consumption (degree).	The company's interest in environmental issues (degree).
	Compliance with current environmental regulations and standards (degree).	The organization's interest in community issues (degree).
	Implementation of measures to reduce the environmental impact in the development of new products (use of energy, recycling, pollution) (degree).	Quality of products and services (degree).

For the assessment, a Likert scale ranging from 1 to 5 was proposed, where one is considered "none," in terms of non-compliance, reach, or non-evidence of the situation outlined in the statement, while the maximum value of 5 corresponds to the category of "excellent" or complete development.

This instrument was designed considering the literature review as well as its validity and reliability. In the first case, a group of experts evaluated the relevance, clarity, and pertinence of each of the statements, while in the second case, Cronbach's Alpha and exploratory factor analysis were applied using the principal components method.

Exploratory factor analysis, like confirmatory analysis and structural equation modeling, does not start from the idea that variables are dependent or independent but rather classifies variables into observed variables collected in the instrument (statements) and latent or unobserved variables, which are given by the factors generated from the statements.

In the case of structural equation models, latent variables can be exogenous, if they do not receive the effect of any other variable and are those generated from the grouping of observed variables into a factor; but they can also be endogenous, which receive the effect of observed variables and other exogenous latent variables.

Generally, graphical representation is done through a path diagram where latent variables, whether exogenous or endogenous, are represented with a circle or oval, and observed variables with a square. The residuals from the estimates are also latent variables because they are generated in the system and, therefore, are schematized as such.

Exploratory factor analysis, therefore, allowed the identification of the underlying structure or latent variables in a set of observable variables. Its main objective is to reduce the dimensionality of the data and summarize the information into a smaller number of unobservable factors.

It starts with the extraction of factors or the determination of the number of latent variables generated from the statements, for which the Kaiser method was used, considering factors that report eigenvalues greater than 1 and explaining a significant percentage of the total variance. This allows selecting factors that, together, explain a substantial amount of variance, generally looking for a cumulative percentage higher than 70%.

Additionally, Bartlett's test of sphericity was applied to assess whether the correlation matrix among the observable variables is suitable for factor analysis. Its objective is to determine if there is enough correlation between the variables to justify the use of factor analysis.

It proposes the null hypothesis that the population correlation matrix is an identity matrix, meaning that there is no correlation between the variables. To consider the grouping valid, the null hypothesis is sought to be rejected, justifying a factor analysis.

After extracting the factors, a rotation is performed to facilitate the interpretation of these. The rotation used was orthogonal by the varimax method. The rotation seeks to simplify the factor structure and have clearer and more distinct factor loadings.

Once the factors are obtained and the rotation is done, they are interpreted based on factor loadings. Factor loadings represent the correlations between observable variables and factors. Factors with high loadings indicate that observable variables are strongly associated with that factor. Thus, an item is part of the factor or latent variable if its loading is equal to or greater than 0.50.

Once the exploratory factor analysis and the factors or latent variables are identified, confirmatory factor analysis is conducted, representing the so-called measurement model, which identifies errors in indicators and constructs, allowing the confirmation of the latent structures, to then define and estimate the structural model.

Initially, the measurement model starts by establishing the hypothesized theoretical model, which includes the latent variables, observable variables, and the relationships between them. Measurement equations are defined that relate the observable variables to the latent variables.

For this study, the endogenous latent variable is given by the client factor (CLI) which considers their perception of corporate social responsibility. This factor is caused by the factor of social responsibility valuation by employees (TH) and managers (AD), as shown in (Figure 1).

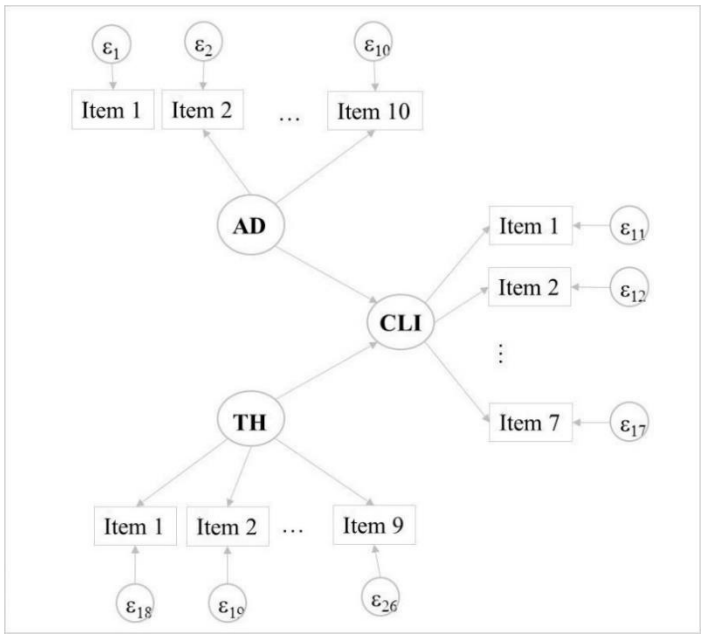

Figure 1: Path Diagram of the Measurement Model and the Structural Model. Source: Developed by the author.

The measurement model does not establish relationships among variables, latent or otherwise; these relationships are part of the structural model. The measurement model is limited to validating latent variables in terms of the statements that constitute them.

The confirmatory factor analysis or measurement model can be estimated using maximum likelihood, which assumes normality and is robust to violations of assumptions, as used in this study. However, other methods such as asymptotically distribution-free (weighted least squares), quasi-maximum likelihood, and handling missing values can also be utilized.

Once estimated, it is necessary to evaluate the fit of the model, internal consistency, convergent validity, and discriminant validity. Initially, the fit of the model to the observed data is assessed using various fit indices. These indices provide a measure of how well the theoretical model fits the observed data.

The indices considered in this study included the chi-square goodness of fit (or divided by degrees of freedom), the Comparative Fit Index (CFI), the Tucker-Lewis Index (TLI), the Incremental Fit Index (IFI), the Root Mean Square Error of Approximation (RMSEA), the Standardized Root Mean Square Residual (SRMR), and the Coefficient of Determination (CD) [18], whose criteria are detailed in Table 2.

Table 2: Acceptance Criteria for Fit Indices of the Measurement Model. Source: Own elaboration.

Index		Criterion
Probability coefficient	Chi-squared Chi-square/degrees of freedom	The model fits when the value is not significant. The Chi-square value relative to the degrees of freedom can also be estimated, and it must be less than 3.
Errors	Root Mean Square Error of Approximation (RMSEA)	It should take values equal to or less than 0.05 and be statistically significant (probability less than or equal to 0.10).
Base comparison	Comparative Fit Index (CFI) Tucker-Lewis Index (TLI)	Optimal values should exceed 0.90. This value would indicate that the model is 90% better than one in which the items are not correlated.
Residual size	Standardized Root Mean Square Error (SRMR)	It should take values less than 0.08

If the model fit is not adequate, modifications can be made to the theoretical model, such as adding or removing relationships between variables or allowing correlations between measurement errors. These modifications are based on theoretical considerations and the analysis of fit statistics.

Secondly, internal consistency must be ensured, which corresponds to the reliability of the latent variable. For this purpose, Cronbach's Alpha is used, and its value must be higher than 0.70. Cronbach's Alpha measures the average correlation among all possible item combinations in a set of variables. This coefficient was complemented by the Raykov reliability coefficient, which must also exceed 0.70 to consider that there is internal consistency.

Thirdly, convergent validity was assessed, which is related to the degree of confidence that the items measure the latent variable. In this case, standardized factorial loads should be at least 0.50 to consider convergent validity (item weight on the latent variable).

Furthermore, for convergent and discriminant validity, the Average Variance Extracted (AVE) index must be calculated, and its value must be above 0.50. If this is not the case, it may be because there are items with loads below 0.50, which should be removed and the estimation and validation performed again.

If it is still impossible to verify convergent validity, the correlation between the predicted latent variables can be estimated and the unit values (variances) substituted by the square root of the AVE of each variable; if this value is greater than the correlations, convergence is achieved.

Once the measurement model is validated and adjusted due to high covariances between the items or latent variables, the validity of the latent variables and consistency can be ensured, allowing for the estimation of a system of equations to verify the theoretical causality between the endogenous latent variable and the exogenous ones, whether latent or observed.

3 Results and Discussion

As indicated, the process began with exploratory factor analysis to ensure the validity of the construct by identifying the factors or latent variables for each unit of analysis (customers, employees, and managers). For this, the different items in each of the instruments (Table 1) were considered, obtaining the results shown in Table 3.

Table 3: Results of the Exploratory Factor Analysis Note: Significant up to 1% (***), between 1% and 5% (**), and between 5% and 10% (*). Source: Own elaboration

	Items								
	1	2	3	4	5	6	7	8	9
Customers: Customer latent variable (CLI) – Single factor									
Factor loadings	0.786	0.695	0.696	0.631	0.717	0.723	0.632		
Kaiser, Meyer and Olin test (KMO)	0.846	0.878	0.879	0.899	0.891	0.765	0.739		
Bartlett's sphericity test	756.53***								
Employees: Employee latent variable (TH) – Single factor									
Factor loadings	0.674	0.663	0.702	0.742	0.736	0.725	0.660	0.699	0.632
Kaiser, Meyer and Olin test (KMO)	0.915	0.904	0.928	0.881	0.873	0.923	0.852	0.894	0.884
Bartlett's sphericity test	1142.72***								
Managers: Manager latent variable (AD) – A single factor									
Factor loadings	0.528	0.742	0.763	0.733	0.654	0.666	0.690	0.590	0.558
Kaiser, Meyer and Olin test (KMO)	0.901	0.871	0.901	0.854	0.912	0.891	0.870	0.854	0.841
Bartlett's sphericity test	926.25***								

For each instrument, the results indicate that the statements can be grouped into a single factor, indicating the presence of a single latent variable per unit of analysis. In all three cases, the factor loadings exceed the minimum acceptable value of 0.50, and there is excellent data suitability, as the values of the Kaiser-Meyer-Olkin (KMO) test exceed 0.80.

Regarding Bartlett's test of sphericity in the three analyses, the null hypothesis is rejected, demonstrating that there is a correlation among the observable variables (items) justifying their grouping into factors or latent variables.

Following the exploratory factor analysis, as outlined in the methodology, the confirmatory factor analysis was conducted. After estimating, the fit of the model was validated, and the internal consistency, convergent validity, and discriminant validity were evaluated.

Using modification indices, the fit of the original model was improved by incorporating the highest covariances between the customer instrument's items related to the company's interest in community problems and environmental programs. For employees, the items were mechanisms to express ideas and generate commitment.

Finally, for managers, the statements with the highest covariance were related to the company's environmental policy and the community's environmental issues. The results of the evaluation of the initial model's fit, as well as the one incorporating the indicated observed variables' covariances, are presented in Table 4.

Table 4: Evaluation of the Fit of the Original and Modified Models. Note: Significant up to 1% (***), between 1% and 5% (**), and between 5% and 10% (*). Source: Own elaboration

Criteria	Initial model	Modified model (covariances)
Chi-square/degrees of freedom	2,469	1982
Root Mean Square Error of Approximation (RMSEA)	0.066***	0.054***
Comparative Fit Index (CFI)	0.872	0.916
Tucker-Lewis Index (TLI)	0.859	0.906
Standardized Root Mean Square Error (SRMR)	0.058	0.057
Determination coefficient	0.99	0.99

In both models, the criteria are met, but the inclusion of covariances helps to reduce the Chi-square to degrees of freedom ratio, indicating a better model fit. It also reduces the value of the Root Mean Square Error of Approximation (RMSEA) and the Standardized Root Mean Square Residual (SRMR). The Comparative Fit Index (CFI) and the Tucker-Lewis Index (TLI) are improved, while the Coefficient of Determination remains stable.

In addition to the model fit evaluation, internal consistency is also necessary through Cronbach's Alpha; the results indicate that in all cases the coefficient exceeds 0.70. For the latent variable of clients, the value was 0.814, for the latent variable or factor of employees it was 0.864, and for managers, it was 0.835.

Regarding the RayKov reliability factor, which should also exceed 0.70, superior values are also achieved, with 0.775 for clients, 0.852 for employees, and 0.823 for managers, thus validating internal consistency.

For convergent validity, while all observed variable coefficients in the modified estimation are above 0.50 and significant, the Average Variance Extracted (AVE) values do not exceed 0.50; for clients, it is 0.40, for employees 0.41, and managers 0.38.

Therefore, discriminant validity was analyzed by estimating the correlation matrix between the predicted latent variables, where substituting the diagonal variance (one) with the square root of the AVE for each latent variable shows that these values are higher than the correlations, thereby potentially verifying the model's discriminant validity (Fornell-Larcker criterion).

Once all validations of the modified model were resolved, the estimation of the Structural Equation Modeling (SEM) was carried out, incorporating the establishment of direct or indirect relationships and covariance in the measurement model.

The impact of the valuation of corporate social responsibility by employees and managers is statistically significant in customer perception, confirming the theoretical causality. In the case of the latent variable associated with managers, the coefficient was positive (0.33), as well as in the latent variable of employees (0.43), which are the direct effects of the estimation.

Also, when analyzing each construct, there are direct effects given by the coefficients of the estimations located on the straight arrows emanating from each latent variable towards the observed variable.

Regarding covariance relationships, and not causality, in the case of communication and commitment mechanisms in employees, it is 0.25; in the interest in community problems and environmental programs in the customer variable, it is 0.50, and in the case of the company's environmental policies and community environmental issues, it is 0.28.

Finally, the values associated with the observed variables correspond to the constants of each of the estimations for the latent variables or constructs. Specifically, the direct and indirect effects determined in the structural equation model are summarized in Table 5, for each latent, endogenous, or exogenous variable.

Table 5: Direct and Indirect Effects. Note: Significant up to 1% (***), between 1% and 5% (**), and between 5% and 10% (*). Values in parentheses correspond to standard errors. Source: Own elaboration.

	Direct effects			**Indirect effects**		
Variables	CLI	AD	TH	CLI	AD	TH
		Customer Instrument				
Information		1.00				0.33*** 0.43***
						(0.12) (0.08)
Complaints		0.92***				0.30*** 0.39***
		(0.08)				(0.11) (0.08)

	Direct effects	Indirect effects	
Service quality	0.78***	0.26***	0.33***
	(0.07)	(0.09)	(0.07)
Knowledge activity	0.71***	0.23***	0.30***
	(0.07)	(0.08)	(0.06)
Product Quality	0.82***	0.27***	0.35***
	(0.07)	(0.09)	(0.07)
Community interest	1.03***	0.34***	0.44***
	(0.10)	(0.12)	(0.09)
Sustainability interest	0.98***	0.32***	0.42***
	(0.12)	(0.12)	(0.09)
	Managers Instrument		
Strategic Plan	1.00		
Principles and values	1.26***		
	(0.16)		
CSR Policy	1.48***		
	(0.19)		
Environmental policy	1.58***		
	(0.21)		
Financial performance	1.15***		
	(0.16)		
Employee relationship	1.30***		
	(0.18)		
Environment relationship	1.39***		
	(0.19)		
Community relationship	1.06***		
	(0.16)		
Products Quality	0.80***		
	(0.12)		
	Employees Instrument		
Hiring	1.00		
Nondiscrimination	1.00***		
	(0.10)		
Labor conditions	0.88***		
	(0.09)		
Possibility of expression	1.01***		
	(0.10)		
Commitment	0.91***		
	(0.09)		
Environmental training	1.07***		
	(0.10)		
Water optimization	0.84***		
	(0.09)		
Environmental standard compliance	0.94***		
	(0.09)		
Proposal Environmental Measures	0.91***		
	(0.10)		
	Structural		
Latent managers (AD)	0.33***		
	(0.12)		
Latent employees (TH)	0.43***		
	(0.08)		

As observed, all direct effects are statistically significant and positive, indicating that each item directly impacts the latent variable. For the latent variable representing customers, the items related to the perception of the company's interest in environmental and community issues report the greatest effect, followed by the handling of complaints or claims.

Regarding the latent variable related to employees, the most influential factors in their valuation of CSR are the training provided on environmental topics and the opportunity to express their ideas. For the variable related to the managers' valuation, the company's environmental and CSR policies have the greatest impact. Although all direct effects are similar, the least significant is the quality of the product.

The indirect effects, also statistically significant and positive, relate to the influence of the items from the instrument applied to customers on the latent variables of employees and managers. It is evident in both cases that the effects are greater concerning interest in the community and the environment, as well as the information received by the customer.

Additionally, the fit of the estimation of the structural equation model was validated, similarly to what was done in the confirmatory factor analysis, using the criteria indicated in the methodology. The results are presented in Table 6.

Table 6: Evaluation of the Structural Equation Model Fit. Note: Significant up to 1% (***), between 1% and 5% (**), and between 5% and 10% (*). Source: Own elaboration.

Criteria	SEM
Chi-square/degrees of freedom	1982
Root Mean Square Error of Approximation (RMSEA)	0.054***
Comparative Fit Index (CFI)	0.916
Tucker-Lewis Index (TLI)	0.906
Standardized Root Mean Square Error (SRMR)	0.057
Determination coefficient	0.967

In all cases, the established criteria are validated, indicating that the estimated structural equation model is correctly fitted and can be used to verify the causality between the assessment of corporate social responsibility by employees and managers and the perception of the clients.

For a better analysis of the effects determined in the structural equation model, a comparison was made using Fuzzy Cognitive Maps. Studies of centrality regarding the variables Clients, Managers, and Employees were conducted as shown in Tables 7, 8, and 9. The presented results demonstrate the differences in how each group of components interacts and contributes to the overall structure of the model.

Table 7: Customer centrality analysis. Source: own elaboration.

Components	Indegree	Outdegree	Centrality	Type
Complaints	2.44	0.9199999999999999	3.36	ordinary
Quality service	2.04	1.67	3.71	ordinary
Knowledge activity	1.7200000000000002	1.88	3.6	ordinary
Product Quality	1.65	0.99	2.63999999999999997	ordinary
Interested community	1.5899999999999999	2.01	3.59999999999999996	ordinary
Sustainability interest	1.38000000000000001	0.97	2.35	ordinary
Information	0.49	2.87	3.3600000000000003	ordinary

The centrality values indicate the relative importance of each component within the customer group. Quality Service has the highest centrality at 3.71, standing out as a critical factor from the customers' perspective. Information also shows a significant centrality of 3.36, driven mainly by its high outdegree (2.87), indicating that this component has many outgoing connections, influencing other components.

Complaints and Knowledge Activity have similar centralities of 3.36 and 3.6, respectively, suggesting their active role in interacting with other aspects relevant to customers. The centrality demonstrates that customers value service quality, complaint management, and information as critical components.

Table 8: Manager Centrality analysis. Source: own elaboration.

Components	Indegree	Outdegree	Centrality	Type
strategic plan	2.15	4.13	6.279999999999999	ordinary
Quality Products	3.4	3.83	7.23	ordinary

Components	Indegree	Outdegree	Centrality	Type
community relationship	3.8199999999999994	0	3.8199999999999994	receiver
Principles and values	2.3600000000000003	3.16	5.5200000000000005	ordinary
Corporate social responsibility policies	2.49	4.4399999999999995	6.93	ordinary
Environmental policy	2.99	3.619999999999999	6.609999999999999	ordinary
Financial performance	3.64	1.75	5.390000000000001	ordinary
Environment relationship	4	3.4200000000000004	7.42	ordinary
Employee relationship	3.51	4.01	7.52	ordinary

Employee Relationship and Environment Relationship have the highest centralities at 7.52 and 7.42, respectively. This indicates that these components are extremely central from the managers' perspective, affecting and being affected by many other components. Quality Products and Corporate Social Responsibility Policies are also prominent with centralities of 7.23 and 6.93. This shows that managers view relationships with employees and the environment, along with product quality and social responsibility policies, as central to their management.

Table 9: Employee centrality analysis. Source: own elaboration.

Components	Indegree	Outdegree	Centrality	Type
Hiring	1.99	3.0799999999999996	5.069999999999999	ordinary
Nondiscrimination	2.94999999999999997	3.61	6.56	ordinary
Labor conditions	3.77	0.32	4.09	ordinary
Possibility of expression	3.13	2.63	5.76	ordinary
Commitment	2.73999999999999998	4.829999999999999	7.56999999999999985	ordinary
Environmental training	2.04	3.9800000000000004	6.0200000000000005	ordinary
Water optimization	2.63999999999999997	3.47	6.109999999999999	ordinary
Environmental standard compliance	3	2.7800000000000002	5.78	ordinary
Proposal Environmental Measures	3.6799999999999997	1.24	4.92	ordinary

Commitment has the highest centrality at 7.57, followed by Nondiscrimination at 6.56, and Environmental Training at 6.02. These components are critical from the employees' perspective. For employees, commitment, nondiscrimination, and environmental training are of utmost importance.

The correlation analysis between Structural Equation Modeling (SEM) and Fuzzy Cognitive Maps (FCM) in this context reveals that, although they use different theoretical and mathematical frameworks, both can effectively complement each other to provide a richer and more dynamic understanding of how corporate social responsibility influences the performance of SMEs. SEM provides a solid framework for estimating and validating the structural relationships between the perceptions of customers, employees, and managers, while FCM offers a flexible tool to explore how these perceptions influence and are influenced within the system, providing a more dynamic and adaptive view of the interaction between these factors.

4 Conclusions

This study applied Structural Equation Modeling (SEM) and Fuzzy Cognitive Maps (FCM) to analyze how the perception of corporate social responsibility (CSR) impacts the perception of customers, employees, and managers in small and medium-sized enterprises (SMEs) in Ecuador.

Exploratory factor analysis confirmed the presence of a single latent variable in each interest group (customers, employees, and managers), demonstrating that the items of each instrument can be effectively grouped under a single factor. This result is crucial as it simplifies the model structure and facilitates the interpretation of how CSR influences each group. The factor loadings obtained exceeded the acceptable minimum value of 0.50, and Bartlett's test of sphericity rejected the null hypothesis in all three cases, thus validating the grouping of items into latent factors due to the existence of significant correlation among the observable variables.

The direct and statistically significant effects indicate that each item of the instruments directly impacts its corresponding latent variable. Specifically, items related to the company's interest in environmental and community problems, and the handling of complaints, showed the greatest effects on customer perception. The evaluation of CSR by employees and managers had a significant impact on customer perception, with positive coefficients of 0.33 and 0.43, respectively. This underscores that a positive evaluation of CSR by employees and managers enhances customers' perception of the company.

Centrality analyses in the Fuzzy Cognitive Maps showed that for customers, aspects such as "Quality Service" and "Information" are critical, highlighting their importance in interacting with other variables. For managers, "Employee Relationship" and "Environment Relationship" were central, indicating their key role in CSR management. For employees, "Commitment", "Nondiscrimination", and "Environmental Training" had the highest centralities, reflecting their importance in the perception and practice of CSR within SMEs.

The integration of SEM and FCM not only allowed for the estimation and validation of structural relationships between the perceptions of different stakeholder groups but also explored how these perceptions interact and influence each other within the system. This dual approach provides a richer and more dynamic view of the influence of CSR on the performance of SMEs, offering a robust tool for understanding and improving CSR practices in the context of Ecuadorian SMEs.

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Application of Neutrosophic Statistics in the Integrated Management of Ceratitis Capitata in Ecuador

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Abstract. This study has addressed the management of Ceratitis capitata (Mediterranean fly) in Ecuador, focused on mitigating the impact on fruit crops under the influence of climate change and variations in agricultural practices. By using neutrosophic statistics, the study has modeled uncertainty in climatic and agricultural variables to improve the prediction and adaptability of management strategies. Integrated strategies have been proposed, which have included biological control, cultural management, and the use of insecticides, highlighting the effectiveness of adapting management practices based on neutrosophic analysis. The results indicated that the adaptations of strategies, provided by neutrosophic statistics, allowed an effective response to environmental variations, to maintain the effectiveness of pest control in the long term. In conclusion, the combination of management strategies with neutrosophic statistics is vital for effective control of Ceratitis capitata, by facilitating a proactive and adaptive approach in the face of changing conditions, to strengthen agricultural resilience.

Keywords: Integrated pest management, neutrosophic statistics, agricultural resilience.

1 Introduction

Fruit flies are phytophagous insects that cause significant economic losses worldwide [1]. They affect both the epicarp and mesocarp of the fruit through oviposition and the plant tissues through larval feeding, rendering the fruits unsuitable for industrial use [2]. Additionally, they cause indirect damage through pathogenic microorganisms that infect the wounds caused by these flies and their larvae [3].

Economically, fruit fly infestations have resulted in exorbitant costs [4], as evidenced in the United States where costs range between $300,000 and $200 million per event. In California, outbreaks of these pests have cost nearly $500 million over the past 25 years, and a specific outbreak in Florida in 1997 reached an eradication cost of $25 million.

The Mediterranean fruit fly (MFF; Ceratitis capitata Wiedemann), native to sub-Saharan Africa, is one of the most destructive pests of fruit trees worldwide. This species is distinguished by its wide range of hosts (approximately 300), its tolerance for low temperatures, and its adaptability to various climates, making it especially problematic. Its presence in a country can result in significant trade barriers for agricultural product exports due to its quarantine pest status.

In Ecuador, despite the relevance of Ceratitis capitata, there is a lack of studies on this pest, and a national program has not been effectively implemented to reduce its incidence or minimize its economic and social risks. The country hosts 11,250 hectares of crops such as mango, cucurbits, papaya, blackberry, tomato, cherimoya, dragon fruit, and bell pepper, all susceptible to MFF [5], highlighting the need for a better understanding of the risk it poses.

It should be noted that global trade and climate changes influence the distribution and impact of pests and diseases [6], facilitating their emergence in new areas [7]. Modern technologies, such as ILCYM 4.0 software, allow for the construction of process-based models to predict the population dynamics of insects in various ecosystems and generate pest risk maps using Geographic Information Systems (GIS). These tools support decision-making to optimize pest sampling and management [8], reducing pesticide use.

The main goal of this study is to develop integrated strategies to mitigate the impact of Ceratitis capitata on

fruit crops in Ecuador. This objective extends to considering the challenges posed by climate change and fluctuations in agricultural practices, using neutrosophic statistics to address the uncertainties associated with environmental and management variables affecting pest dynamics.

2 Materials and Methods

2.1 Neutrosophy

Definition 1. Let X be a universe of discourse. A Neutrosophic Set (NS) is characterized by three membership functions,$u_A(x), r_A(x), v_A(x): X \rightarrow]^-0.1+[$,that satisfy the condition $-0 \leq inf u_A(x) + inf r_A(x) + inf v_A(x)\ sup\ u_A(x) + sup\ \ r_A(x) + \sup v_A(x) \leq 3^+$ for all $x \in X$. $u_A(x), r_A(x)$ and $v_A(x)$ denote the true, indeterminate, and false membership functions of x in A, respectively[9] [10], and their images are standard or non-standard subsets of $]-0,1+[$.

Definition 2. Let X be a universe of discourse. A Single Value Neutrosophic Set (SVNS) A over X is an object of the form:

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\} \quad (1)$$

Where $u_A, r_A, v_A: X \rightarrow [0,1]$, satisfy condition $0 \leq u_A(x), r_A(x), v_A(x) \leq 3$ for all $x \in X$. $u_A(x), r_A(x)$ and $v_A(x)$ denote the true, indeterminate, and false membership functions of x in A, respectively. For convenience, a Single Valued Neutrosophic Number (SVNN) will be expressed as A = (a, b, c), where a, b, c [0,1] and satisfies $0 \leq a + b + c \leq 3$.

The SVNS arose with the idea of applying neutrosophic sets for practical purposes. Some operations between SVNN are expressed below:

Given A1 = (a1, b1, c1) and A2 = (a2, b2, c2) two SVNN, the sum of A1 and A2 is defined as:

$$A_1\ A_2 = (a_1 + a_2 - a_1a_2, b_1b_2, c_1c_2) \quad (2)$$

Given A1 = (a1, b1, c1) and A2 = (a2, b2, c2) two SVNNs, the multiplication between A1 and A2 is defined as:

$$A_1\ A_2 = (a_1a_2, b_1{+}b_2 - b_1b_2, c_1 + c_2 - c_1c_2) \quad (3)$$

The product for a positive scalar with an SVNN, A = (a, b, c) is defined by:

$$A = (1 - (1 - a), b, c) \quad (4)$$

2.2 Neutrosophic Statistics

Neutrosophic probabilities and statistics are a generalization of classical and imprecise probabilities and statistics [11]. The neutrosophic probability of event E is defined as the probability that event E occurs, the probability that event E does not occur, and the probability of indeterminacy (not knowing whether event E occurs or not). In classical probability, nsup≤1, while in neutrosophic probability nsup≤3+. The function modeling the neutrosophic probability of a random variable x is called a neutrosophic distribution:

$$NP(x) = (T(x), I(x), F(x))$$

where T(x) represents the probability that the value x occurs, F(x) represents the probability that the value x does not occur, and I(x) represents the indeterminate or unknown probability of the value x. Neutrosophic statistics is the analysis of neutrosophic events and deals with neutrosophic numbers, neutrosophic probability distribution, neutrosophic estimation, and neutrosophic regression [12].

It refers to a dataset, which is entirely or partly composed of data with some degree of indeterminacy and the methods to analyze them. Neutrosophic statistical methods allow the interpretation and organization of neutrosophic data (data that may be ambiguous, vague, imprecise, incomplete, or even unknown) to reveal underlying patterns [13].

In conclusion, neutrosophic logic, neutrosophic sets, and neutrosophic probabilities and statistics have broad applications in various research fields and constitute a novel area of study in full development. Neutrosophic descriptive statistics encompass all techniques for summarizing and describing the characteristics of neutrosophic numerical data.

Neutrosophic numbers are numbers of the form where a and b are real or complex numbers, while "I" is the indeterminacy part of the neutrosophic number N. The study of neutrosophic statistics refers to a neutrosophic random variable where X_l and $X_u I_N$ represent the lower and upper levels respectively that the studied variable can reach, within an indeterminate interval $[I_l, I_u]$. Accordingly, the neutrosophic mean of the variable $((\bar{x}_N))$ is formulated by:

$$X_N = X_l + X_u I_N;\ I_N \in [I_l, I_u] \quad (5)$$

$$\text{Where, } \bar{x}_a = \frac{1}{n_N}\sum_{i=1}^{n_N} X_{il}, \quad \bar{x}_b = \frac{1}{n_N}\sum_{i=1}^{n_N} X_{iu}, \; n_N \in [n_l, n_u], \tag{6}$$

it is a neutrosophic random sample. Nevertheless, for the calculation of neutrosophic number squares (NNS), it can be computed as follows[14]:

$$\sum_{i=1}^{n} N(X_i - \bar{X}_{iN})^2 = \sum_{i=1}^{n} N\left[\begin{matrix}\min\begin{pmatrix}(a_i + b_i I_L)(\bar{a} + \bar{b} I_L), (a_i + b_i I_L)(\bar{a} + \bar{b} I_U) \\ (a_i + b_i I_U)(\bar{a} + \bar{b} I_L), (a_i + b_i I_U)(\bar{a} + \bar{b} I_U)\end{pmatrix} \\ \max\begin{pmatrix}(a_i + b_i I_L)(\bar{a} + \bar{b} I_L), (a_i + b_i I_L)(\bar{a} + \bar{b} I_U) \\ (a_i + b_i I_U)(\bar{a} + \bar{b} I_L), (a_i + b_i I_U)(\bar{a} + \bar{b} I_U)\end{pmatrix}\end{matrix}\right], I \in [I_L, I_U] \tag{7}$$

Where $a_i = X_l b_i = X_u$. The variance of the neutrosophic sample can be calculated by

$$S_N^2 = \frac{\sum_{i=1}^{n_N}(X_i - \bar{X}_{iN})^2}{n_N}; \; S_N^2 \in [S_L^2, S_U^2] \tag{8}$$

The neutrosophic coefficient (CV_N) measures the consistency of the variable [15]. The lower the CV_N value, the more consistent the performance of the factor compared to other factors. The CV_N can be calculated as follows:

$$CV_N = \frac{\sqrt{S_N^2}}{\bar{X}_N} \times 100; \; CV_N \in [CV_L, CV_U] \tag{9}$$

3 Results

3.1 Data collection. Current presence of the Mediterranean fly in Ecuador.

Using data from the Ecuadorian Agency for Agricultural Quality Assurance, 926 presence points of MFF in mango orchards were recorded during the year 2020, with detailed UTM coordinates. Most of the records were located in the province of Imbabura, followed by Guayas and Santa Elena, as shown in Table 1.

Table 1: Sampling of the presence of Ceratitis capitata in Ecuador. Source: Agrocalidad.

Provinces (sampling)	Number of flies collected	Percentage (%)	Provinces (sampling)
Guayas	2461	20.35%	Guayas
Imbabura	8721	72.12%	Imbabura

In Table 2, Imbabura shows the highest incidence with 72.12%, followed by Guayas at 20.35% and Santa Elena at 7.53%. These data were collected by Agrocalidad agents in 2020 in mango plants. Meanwhile, the current and future establishment index of Ceratitis Capitata has identified an optimal area (A) in the tropical and subtropical zones of Ecuador with an average temperature of 22 °C. The inter-Andean regions, such as the Ecuadorian highlands, were categorized as an impossible area (D) with temperatures < 9 °C.

Table 2: Favorable areas according to the activity of Ceratitis capitata. Source: own elaboration.

Areas	Temperatures °C
Optimal	16-32
Favorable	10-35
Unfavorable	<10
Impossible	>40 years <10

The current analysis indicates that MFF could achieve from 4 to 25 annual generations in areas with temperatures ranging from 9 to 22°C; and from 0 to 4 generations in areas (C, D) with temperatures < 9°C, according to Table 3. The Current Generational Index (2020) and Future (2050) for Ceratitis capitata in Ecuador show an increase in regions affected by climate change. This forecasts the establishment of the insect in tropical and subtropical regions while indicating insect-free zones in the highlands of the Sierra.

Table 3: Average annual temperatures for regions in Ecuador. Source: own elaboration.

Areas	Temperatures °C
Andean Plateau	<9
coastal lowlands,	9-14
Jungle,	14-18
Galapagos Islands	18-22
	>22

3.2 Neutrosophic Statistics.

Once the distribution of the flies collected in the main affected provinces has been analyzed, the next step is to determine the elements that favor propagation. To do this, it is necessary to identify the variables that affect the development of the fly Ceratitis capitata. Therefore, it is essential to consider both the biological aspects of the insect and the environmental factors that influence its life cycle. Based on previous information and related scientific literature, the following variables are key in the development and expansion of this species.

To analyze and reformulate the impact of the most important variables on the development of Ceratitis capitata, five critical variables are selected and merged (see Table 4), using neutrosophic statistics. Then, the ranges of action are described, which allows the expression of uncertainty in the obtained information.

Table 4: Critical variables that affect the development of Ceratitis capitata. Source: own elaboration.

Cod.	Variable	Neutrosophic scale	Intervals	Description
H	Humidity and precipitation	The analysis covers from optimal humidity (65-80% RH) to non-suitable conditions (too low <65% RH or too high >80% RH).	Covers from optimal humidity (65-80% RH) to unsuitable conditions (too low <65% RH or too high >80% RH).	Humidity primarily affects the larval development and survival of pupae. Adequate moisture is crucial for pupation and the emergence of adults, while extremes can cause desiccation or diseases.
DH	Availability and diversity of hosts	From high diversity and availability of hosts (optimal) to low diversity (inadequate).	From high diversity and host availability (optimal) to low diversity (inappropriate).	The availability of a wide range of fruits and vegetables provides continuous opportunities for oviposition and feeding, essential for maintaining high annual generations.
PS	Agricultural practices and pest control	It evaluates from integrated and sustainable management practices (good) to inappropriate practices that increase the pest (bad).	Evaluates from integrated and sustainable management practices (good) to inadequate practices that increase the pest (bad).	Sustainable practices and integrated pest management help control populations without creating resistance or damaging the ecosystem, while inappropriate practices can increase the fly population.
CCA	Climate change and altitude	From moderate changes that can be manageable (moderately good) to extremes that displace the fly from optimal areas (moderately bad).	From moderate changes that can be manageable (fairly good) to extremes that displace the fly from optimal areas (fairly bad).	Climate change can alter the optimal geographic zones for Ceratitis capitata by modifying temperature and humidity conditions, while altitude locally affects these conditions.
T	Temperature	It evaluates from optimal conditions (16-32°C) to non-suitable conditions (<16°C or >32°C) for the development of the fly.	Evaluates from optimal conditions (16-32°C) to unsuitable conditions (<16°C or >32°C) for fly development.	Temperature is crucial for all development phases of Ceratitis capitata, from the rate of development to reproduction. An optimal range allows for up to 25-26 generations per year, while temperatures outside this range can inhibit development and reduce survival.

Observations:

- Variable: Elements that influence the life and proliferation of Ceratitis capitata.
- Neutrosophic Scale: Adapts from an evaluation of "Extremely Good" to "Extremely Bad," using a scale of truth, indeterminacy, and falsehood according to the provided neutrosophic coefficients.

Once the variables and their ranges of action have been defined, the process moves to the projected estimation of the variables over 20 years. This forecast projection includes indeterminacies and the ranges where the analyzed variable would impact various regions (see Tables 5 and 6).

Table 5: Estimated projection of variability intervals of H, DH, and PS in the coming years. Source: own elaboration.

Year	H		DH		PS	
A1	(0.3,0.75,0.80)	(0.50,0.55,0.5)	(0.10,0.90,0.95)	(0.3,0.75,0.80)	(0.10,0.90,0.95)	(0.3,0.75,0.80)
A2	(0.10,0.90,0.95)	(0.3,0.75,0.80)	(0.3,0.75,0.80)	(0.50,0.55,0.5)	(0.10,0.90,0.95)	(0.3,0.75,0.80)
A3	(0.3,0.75,0.80)	(0.3,0.75,0.80)	(0.10,0.90,0.95)	(0.10,0.90,0.95)	(0.3,0.75,0.80)	(0.50,0.55,0.5)
A4	(0.10,0.90,0.95)	(0.10,0.90,0.95)	(0.3,0.75,0.80)	(0.7,0.2,0.25)	(0.10,0.90,0.95)	(0.50,0.55,0.5)
A5	(0.50,0.55,0.5)	(0.50,0.55,0.5)	(0.50,0.55,0.5)	(0.50,0.55,0.5)	(0.10,0.90,0.95)	(0.3,0.75,0.80)
A6	(0.10,0.90,0.95)	(0.50,0.55,0.5)	(0.3,0.75,0.80)	(0.50,0.55,0.5)	(0.3,0.75,0.80)	(0.7,0.2,0.25)
A7	(0.3,0.75,0.80)	(0.50,0.55,0.5)	(0.10,0.90,0.95)	(0.3,0.75,0.80)	(0.10,0.90,0.95)	(0.50,0.55,0.5)
A8	(0.10,0.90,0.95)	(0.50,0.55,0.5)	(0.50,0.55,0.5)	(0.7,0.2,0.25)	(0.10,0.90,0.95)	(0.50,0.55,0.5)
A9	(0.3,0.75,0.80)	(0.50,0.55,0.5)	(0.10,0.90,0.95)	(0.50,0.55,0.5)	(0.10,0.90,0.95)	(0.50,0.55,0.5)
A10	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.10,0.90,0.95)	(0.50,0.55,0.5)
A11	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.3,0.75,0.80)	(0.50,0.55,0.5)
A12	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.50,0.55,0.5)	(0.7,0.2,0.25)	(0.3,0.75,0.80)	(0.50,0.55,0.5)
A13	(0.3,0.75,0.80)	(0.7,0.2,0.25)	(0.3,0.75,0.80)	(0.92,0.1,0.12)	(0.3,0.75,0.80)	(0.7,0.2,0.25)
A14	(0.3,0.75,0.80)	(0.7,0.2,0.25)	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.50,0.55,0.5)	(0.92,0.1,0.12)
A15	(0.3,0.75,0.80)	(0.7,0.2,0.25)	(0.3,0.75,0.80)	(0.50,0.55,0.5)	(0.50,0.55,0.5)	(0.92,0.1,0.12)
A16	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.3,0.75,0.80)	(0.7,0.2,0.25)
A17	(0.3,0.75,0.80)	(0.92,0.1,0.12)	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.3,0.75,0.80)	(0.7,0.2,0.25)
A18	(0.3,0.75,0.80)	(0.7,0.2,0.25)	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.50,0.55,0.5)	(0.92,0.1,0.12)
A19	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.3,0.75,0.80)	(0.7,0.2,0.25)
A20	(0.3,0.75,0.80)	(0.92,0.1,0.12)	(0.3,0.75,0.80)	(0.92,0.1,0.12)	(0.3,0.75,0.80)	(0.92,0.1,0.12)

Table 6: Estimated projection of CCA and T variability intervals in the coming years. Source: own elaboration.

Year	CCA		T	
A1	(0.10,0.90,0.95)	(0.3,0.75,0.80)	(0.10,0.90,0.95)	(0.50,0.55,0.5)
A2	(0.50,0.55,0.5)	(0.7,0.2,0.25)	(0.10,0.90,0.95)	(0.50,0.55,0.5)
A3	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.3,0.75,0.80)	(0.92,0.1,0.12)
A4	(0.10,0.90,0.95)	(0.3,0.75,0.80)	(0.3,0.75,0.80)	(0.50,0.55,0.5)
A5	(0.3,0.75,0.80)	(0.50,0.55,0.5)	(0.3,0.75,0.80)	(0.50,0.55,0.5)
A6	(0.3,0.75,0.80)	(0.7,0.2,0.25)	(0.3,0.75,0.80)	(0.50,0.55,0.5)
A7	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.3,0.75,0.80)	(0.7,0.2,0.25)
A8	(0.3,0.75,0.80)	(0.50,0.55,0.5)	(0.3,0.75,0.80)	(0.7,0.2,0.25)
A9	(0.3,0.75,0.80)	(0.50,0.55,0.5)	(0.3,0.75,0.80)	(0.7,0.2,0.25)
A10	(0.3,0.75,0.80)	(0.7,0.2,0.25)	(0.50,0.55,0.5)	(0.92,0.1,0.12)
A11	(0.3,0.75,0.80)	(0.7,0.2,0.25)	(0.3,0.75,0.80)	(0.50,0.55,0.5)
A12	(0.3,0.75,0.80)	(0.50,0.55,0.5)	(0.3,0.75,0.80)	(0.50,0.55,0.5)
A13	(0.50,0.55,0.5)	(0.7,0.2,0.25)	(0.3,0.75,0.80)	(0.7,0.2,0.25)
A14	(0.3,0.75,0.80)	(0.50,0.55,0.5)	(0.50,0.55,0.5)	(0.92,0.1,0.12)
A15	(0.3,0.75,0.80)	(0.50,0.55,0.5)	(0.50,0.55,0.5)	(0.92,0.1,0.12)
A16	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.50,0.55,0.5)	(0.92,0.1,0.12)
A17	(0.3,0.75,0.80)	(0.7,0.2,0.25)	(0.50,0.55,0.5)	(0.92,0.1,0.12)
A18	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.3,0.75,0.80)	(0.7,0.2,0.25)
A19	(0.50,0.55,0.5)	(0.92,0.1,0.12)	(0.50,0.55,0.5)	(0.92,0.1,0.12)
A20	(0.3,0.75,0.80)	(0.92,0.1,0.12)	(0.3,0.75,0.80)	(0.92,0.1,0.12)

Neutrosophic Statistical Analysis of the Sample: Temperature emerges as the primary variable influencing the biological development of Ceratitis capitata, with direct effects on the rate of development, survival, and reproduction. However, the interaction of temperature with other factors such as humidity, availability of hosts, and agricultural practices is also fundamental. These combined factors determine the fly's ability to establish, survive, and expand, particularly under the climate change scenarios projected for the future. Therefore, calculations of the neutrosophic mean ($\bar{x}_N$,), neutrosophic standard deviation (S_N), and neutrosophic coefficient of variation (CV_N) are conducted to understand the relationship between the presence of this pest and the significance of the variables, and how Ceratitis capitata would impact Ecuador in the coming years (see Table 7).

Table 7: Estimated projection of $\bar{x}_N$, S_N, and CV_N in the next years. Source: own elaboration.

Items	$\bar{x}_N$		S_N		CV_N	
H	[(0.3,0.65,0.7);(0.4,0.55,0.65)]	0.37	4,511 + 7,163 I	0.016	1,271 + 1,038 I	0.125
DH	[(0.3,0.65,0.7);(0.4,0.55,0.65)]	0.45	5,305 + 7,528 I	0.008	1,378 + 1,046 I	0.087
PS	[(0.6,0.35,0.4);(0.7,0.25,0.3)]	0.67	3,448 + 6,825 I	0.030	1,112 + 1,019 I	0.172
CCA	[(0.6,0.35,0.4);(0.7,0.25,0.3)]	0.74	5,019 + 7,861 I	0.006	1,338 + 1,028 I	0.077
T	[(0.8,0.15,0.2);(0.9,0.05,0.1)]	0.88	5.113 + 8 I	0.006	1,346 + 1,026 I	0.076

The impact of Ceratitis capitata (Mediterranean fruit fly) in Ecuador is favored by temperature variations and how these affect the development and expansion of the annual generations of this pest in agriculture, especially in mango crops. The increase in temperatures may allow Ceratitis capitata to increase the number of generations per year under climate change scenarios projected for the year 2050, compared with current data. This underscores the importance of temperature as a determining factor in population dynamics and the potential for infestation of this species. The results for each variable in Table 7 showed that:

i. Temperature (Average: 0.88, Standard Deviation: 0.076)
   - ❖ Stability: The low standard deviation indicates that the temperature remains relatively constant over the years, varying slightly around the average of 0.76 on the de-neutrosophied scale. This suggests that thermal conditions for Ceratitis capitata are stable and predictable, which is crucial for planning phenological and biological controls.
   - ❖ Favorable Conditions: An average close to 0.8 implies that temperatures are often within optimal ranges for the insect's development, potentially increasing the frequency of its annual generations.

ii. Humidity (Average: 0.37, Standard Deviation: 0.125)
   - ❖ Moderate Variability: Humidity shows little variation from year to year, suggesting that moisture conditions do not fluctuate drastically and are predictably adequate for the fly.
   - ❖ Sufficiency: An average close to 0.37 on the de-neutrosophied scale suggests that humidity conditions are generally adequate for the survival and reproduction of Ceratitis capitata.

iii. Hosts (Average: 0.45, Standard Deviation: 0.087)
   - ❖ Optimal and Stable: The high average indicates a constant and sufficient abundance of hosts available for Ceratitis capitata, which is critical for its feeding and reproduction.
   - ❖ Impact on Management: The stability in host availability requires that management strategies continuously focus on crop rotation and other techniques to minimize infestation risks.

iv. Agricultural Practices (Average: 0.67, Standard Deviation: 0.172)
   - ❖ Management Efficacy: A high average and low variability in this variable suggest that agricultural management and pest control practices are effective and consistently applied, contributing to the effective management of the Ceratitis capitata population. However, the standard deviation indicates that effective methods to combat this pest have an impact on increasing costs.

v. Climate Change (Average: 0.74, Standard Deviation: 0.077)
   - ❖ Uncertainty and Risk: Although the average is moderately high, the relevance of climate change on this scale suggests that there is a significant potential impact on local conditions that could influence the population dynamics of Ceratitis capitata. Fluctuations, although small, could mean years with more extreme conditions in the future.

The consistency in the variables of temperature, humidity, and host availability suggests that the environment in Ecuador is generally favorable for Ceratitis capitata, which could complicate its management if adequate and effective agricultural practices are not applied. Management strategies must, therefore, be dynamic and adapt to the small annual changes in these conditions, especially considering the influence of climate change. Anticipation and adaptation to these conditions through the use of predictive models and sustainable agricultural practices are key to controlling the population of this pest and minimizing the economic impact on agriculture.

In summary, the analyses show a moderate increase in the number of generations of Ceratitis capitata due to the projected temperature rise for 2050. This trend suggests that climate change could exacerbate the impact of this pest in Ecuador, affecting agriculture and requiring attention to management and control strategies.

4 Long-term stock projection

The following table (Table 8) provides a comprehensive framework of actions and strategies to effectively address the issue of Ceratitis capitata. By integrating these measures, farmers and resource managers can enhance the resilience of agricultural systems against this pest and thus ensure the sustainability of agricultural production in the region. These strategies should be reviewed and adapted regularly to reflect changing conditions and advancements in agricultural science and technology. Additionally, it helps to visualize and understand how each

environmental and management factor affects the biology of Ceratitis capitata, especially regarding temperature variability due to climate change.

Table 8: Strategies to mitigate the impact of Ceratitis capitata in the coming years. Source: own elaboration.

No.	Strategy type	Action/alternative	Description	Scope	Objective	Measurement Metrics
S1	Monitoring and Surveillance.	Implementation of traps.	Use traps with specific attractants to monitor and reduce adult populations.	Local to National.	Adult population reduction.	Number of flies captured.
		Early warning systems.	Apply predictive models and alert systems based on meteorological and phenological data.	Regional to National.	Outbreak forecasting.	Alerts issued / Responses.
S2	Integrated Pest Management.	Biological control.	Encourage the use of natural enemies such as parasitoids and predators of the fly.	Local to Regional	Effective biological control.	Rate of parasitism / Predation.
		Microbial control.	Use entomopathogens such as fungi (Beauveria bassiana) and bacteria (Bacillus thuringiensis).	Local to Regional	Reduction of larvae and adults.	Reduction in larval population.
S3	Sterile Insect Techniques.	Release of sterile insects.	Release sterile males to interrupt reproduction and reduce the fly population.	Regional to National	Interruption of reproduction.	Number of sterile males released.
S4	Cultural and Agronomic Management.	Crop rotation.	Rotate crops that are not hosts to break the pest's life cycle.	Local to Regional	Host reduction.	Area of non-host crops.
		Removal of crop residues.	Clean fields to eliminate potential pest hosts.	Local	Minimizing resources for the pest.	Tons of waste removed.
		Physical barriers.	Use nets and covers to protect crops and prevent infestation.	Local	Prevention of infestation.	Hectares protected.
S5	Responsible Use of Insecticides.	Targeted application.	Employ specific insecticides in high-risk areas in a targeted manner.	Local to Regional	Efficient pest control.	Treated area vs. infested area.
		Biorational insecticides.	Promote less toxic and specific products such as insect growth regulators.	Local to National	Sustainability in control.	Reduction in chemical use.
S6	Adaptation to Climate Change.	Adjustment of practices based on climate predictions.	Modify management practices based on climate predictions to anticipate changes in pest activity.	National	Adaptation to climate change.	Changes implemented / Effectiveness.
		Research and development.	Promote research on climate models and their impact on pest dynamics.	National	Improvement in knowledge and action.	Publications / New strategies adopted.
S7	Education and Cooperation.	Farmer training.	Provide training on effective and sustainable pest management.	Local to National	Education and awareness.	Number of farmers trained.
		International collaboration.	Participate in international programs to share knowledge and strategies.	International	Global cooperation.	Joint programs and projects.
S8	Development of Resistant Varieties.	Genetic improvement.	Develop crop varieties that are resistant or less attractive to the fly.	National	Genetic resistance in crops.	Number of varieties developed.

5 Conclusion

Climate change is a critical factor influencing the distribution, phenology, and population dynamics of Ceratitis capitata. The analysis of future scenarios using neutrosophic statistics shows how the uncertainty and variability inherent in climate projections can be managed to anticipate changes in pest activity. Adapting management strategies to the context of climate change, by adjusting practices according to climate predictions, is essential for maintaining the effectiveness of fly control in the future.

Neutrosophic statistics prove to be a valuable tool in analyzing fluctuations and uncertainties in variables affecting Ceratitis capitata. By providing a framework to incorporate truth, indeterminacy, and falsehood into the

assessment of environmental and management conditions, it allows for a deeper understanding and a richer description of the complex dynamics at play. Using neutrosophic statistics helps to better model and understand how variations in factors such as temperature, humidity, and host availability can influence fly populations.

The proposed strategies, including monitoring and surveillance, integrated pest management, the use of sterile insect techniques, and agronomic and cultural practices, prove to be essential for effectively controlling the population of Ceratitis capitata. The coordinated implementation of these strategies addresses various cycles and aspects of the pest's biology, thereby reducing the impact on crops and minimizing economic losses. Moreover, the combination of biological, physical, chemical, and educational approaches ensures a sustainable solution that is adaptable to local and changing conditions.

6 References

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Optimizing Pest Management in Sugarcane Cultivation: An Integrative Approach using Neutrosophic Statistics and Plithogenic Analysis

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Abstract. Pest management in modern agriculture, particularly in sugarcane cultivation, faces the challenge of balancing efficacy, costs, and environmental sustainability. Among the notable pests, the spittlebug (Mahanarva andigena) emerged as a significant threat that reduced yields of sugar and other derivatives. Traditionally, it was controlled with chemical pesticides, which posed environmental and human health risks. Consequently, the study conducted analyzed alternatives by using neutrosophic statistics and plithogenic analysis. The results included evaluating factors such as product concentration, application frequency, environmental conditions, and pest resistance to determine the efficacy of treatments with entomopathogenic microorganisms. Eficax stood out as the best treatment, showing significant improvements in plant growth and a high mortality rate of spittlebugs. In conclusion, the neutrosophic statistical analysis provided an efficient and economically viable solution for responsible agricultural management.

Keywords: Integrated pest management, neutrosophic plithogenic statistics, agricultural sustainability, biological control.

1 Introduction

Sugarcane (Saccharum officinarum L.) is widely cultivated in tropical and subtropical regions around the world. In Ecuador, this crop extends to several provinces including Guayas, Cañar, and Loja. It is a crop of great global relevance, as it provides sugar, an essential energy source for households and the food industry. Globally, approximately 20.42 million hectares are allocated to this cultivation, with more than 110,000 hectares in Ecuador dedicated not only to sugar production but also to ethanol and other derivatives.

However, sugarcane production faces significant challenges, such as a reduction of 40% to 60% in agricultural yields due to pests, particularly the spittlebug Mahanarva andigena, an insect that severely affects the crop [1]. The nymphs and adults of this insect cause substantial damage by sucking the sap and transmitting toxins that necrose the plant, which can lead to complete loss of the plant and significantly reduce the sucrose content.

Traditionally, control of this pest has been carried out using chemical pesticides such as Carbaryl and Acephate. Despite their effectiveness, these compounds have disadvantages such as high costs, difficulties in application due to the dense growth of the cane, and risks to human health and the environment. Additionally, pollution and the development of resistance to pests are concerns.

Given these issues, there has been growing interest in more sustainable alternatives, such as entomopathogenic fungi [2]. Recent studies have demonstrated the efficacy of species like Beauveria bassiana and Metarhizium spp. in controlling different life stages of the spittlebug [3], offering a promising and environmentally friendly alternative [4].

This study focuses on evaluating three commercial products based on these microorganisms in order to identify the most effective one for controlling the spittlebug in sugarcane [5]. It also aims to determine and evaluate factors in product formulation and environmental conditions that affect the efficacy and costs of entomopathogenic treatments against the spittlebug in sugarcane. For this purpose, neutrosophic statistics and phylogenetic analysis are employed to optimize the management of indeterminate data associated with pests and environmental impacts.

2 Materials and methods

2.1 Neutrosophic Statistics

Neutrosophic probabilities and statistics are a generalization of classical and imprecise probabilities and statistics [6]. The neutrosophic probability of event E is the probability that event E occurs, the probability that event E does not occur, and the probability of indeterminacy (not knowing whether event E occurs or not). In classical

probability, nsup≤1, while in neutrosophic probability, nsup≤3+. The function that models the neutrosophic probability of a random variable x is called the neutrosophic distribution: $NP(x) = \big(T(x), I(x), F(x)\big)$

Where T(x) represents the probability that the value x occurs, F(x) represents the probability that the value x does not occur, and I(x) represents the indeterminate or unknown probability of the value x. Neutrosophic statistics is the analysis of neutrosophic events and deals with neutrosophic numbers, neutrosophic probability distribution, neutrosophic estimation, and neutrosophic regression.

It refers to a data set, which is formed totally or partially by data with some degree of indeterminacy and to the methods for analyzing them. Neutrosophic statistical methods allow interpreting and organizing neutrosophic data (data that can be ambiguous, vague, imprecise, incomplete, or even unknown) to reveal underlying patterns [7].

Ultimately, neutrosophic logic, neutrosophic sets, and neutrosophic probabilities and statistics have wide applications in various research fields and constitute a novel area of study in full development [8] [9]. Neutrosophic descriptive statistics comprise all techniques for summarizing and describing the characteristics of neutrosophic numerical data.

Neutrosophic numbers are numbers of the form where a and b are real or complex numbers, while "I" is the indeterminacy part of the neutrosophic number N. The study of neutrosophic statistics refers to a neutrosophic random variable where X_l and $X_u I_N$ represent the lower and upper levels respectively that the studied variable can reach, in an indeterminate interval $[I_l, I_u]$. Thus, the neutrosophic mean of the variable $(\bar{x}_N)$ is followed by formulating:

$$X_N = X_l + X_u I_N;\ I_N \in [I_l, I_u] \tag{1}$$

$$\text{Where, } \bar{x}_a = \frac{1}{n_N}\sum_{i=1}^{n_N} X_{il},\ \ \bar{x}_b = \frac{1}{n_N}\sum_{i=1}^{n_N} X_{iu},\ n_N \in [n_l, n_u], \tag{2}$$

it is a neutrosophic random sample. However, for the calculation of neutrosophic squares (NNS), it can be calculated as follows [10, 11]:

$$\sum_{i=1}^{n} N(X_i - \bar{X}_{iN})^2 = \sum_{i=1}^{n} N\left[\begin{matrix}\min\begin{pmatrix}(a_i + b_i I_L)(\bar{a} + \bar{b} I_L), (a_i + b_i I_L)(\bar{a} + \bar{b} I_U)\\ (a_i + b_i I_U)(\bar{a} + \bar{b} I_L), (a_i + b_i I_U)(\bar{a} + \bar{b} I_U)\end{pmatrix}\\ \max\begin{pmatrix}(a_i + b_i I_L)(\bar{a} + \bar{b} I_L), (a_i + b_i I_L)(\bar{a} + \bar{b} I_U)\\ (a_i + b_i I_U)(\bar{a} + \bar{b} I_L), (a_i + b_i I_U)(\bar{a} + \bar{b} I_U)\end{pmatrix}\end{matrix}\right], I \in [I_L, I_U] \tag{3}$$

Where $a_i = X_l b_i = X_u$. The variance of the neutrosophic sample can be calculated by

$$S_N^2 = \frac{\sum_{i=1}^{n_N}(X_i - \bar{X}_{iN})^2}{n_N};\ S_N^2 \in [S_L^2, S_U^2] \tag{4}$$

The neutrosophic coefficient (NCV) [12] measures the consistency of the variable. The lower the NCV value, the more consistent the performance of the factor compared to other factors. The NCV can be calculated as follows:

$$CV_N = \frac{\sqrt{S_N^2}}{\bar{X}_N} \times 100;\ CV_N \in [CV_L, CV_U] \tag{5}$$

The Neutrosophic Argumentation coefficient evaluates the criteria through Linguistic Terms with SVNN of consensus of justification of the expert opinion, (see Table 1).

Table 1: Linguistic Expression to Determine the Level of Importance of the Factor on the Variable. Source: Own elaboration.

Linguistic Expression	Scale	Plithogenic number (T, I, F)	$\mathcal{S}([\mathbf{T}, \mathbf{I}, \mathbf{F}])$	Description
Very low importance (VLI)	0	(0.05, 0.90, 0.95)	0.07	The factor has a negligible impact on the effective control of spittlebugs.
Low importance (LI)	1	(0.15, 0.75, 0.85)	0.18	The factor has a reduced impact, not critical for the effective control of spittlebugs.
Moderately low (MLI)	2	(0.25, 0.65, 0.75)	0.28	Moderate impact but considered low priority in the control of spittlebug.
Moderately important (MI)	3	(0.50, 0.50, 0.65)	0.45	Factor with a medium level of importance; significant but not decisive impact.
Important (I)	4	(0.65, 0.35, 0.50)	0.60	An important factor that considerably influences the control of spittlebugs.
Very important (VI)	5	(0.80, 0.20, 0.30)	0.77	A very important factor, whose proper management is key to the effective control of spittlebug.

Linguistic Expression	Scale	Plithogenic number (T, I, F)	$\mathcal{S}([\mathbf{T},\mathbf{I},\mathbf{F}])$	Description
Extremely Important (EI) or Critical (C)	6	(0.95, 0.05, 0.10)	0.93	Critical factors whose improper management could completely fail the control of spittlebug.

Mathematical modeling using neutrosophic logic to plithogenic logic is a methodology that focuses on including indeterminacy and contradiction in the evaluation of sets and systems [13,14]. Plithogenic logic has the following characteristics according to the methodology analyzed in the study materials [15, 16].

3 Results

3.1 Case study. Preliminary results in classical statistics.

Study Site: The research was conducted in an area of 7140 m² in the Palo Quemado parish, located along the Toachi River and part of the Sigse mountain range, in the Sigchos canton, Cotopaxi province. This area has an altitude ranging from 990 to 1270 meters above sea level and records temperatures between 23°C and 24°C, with annual precipitation ranging from 105 to 306 mm.

Implementation: A completely randomized block design with a factorial arrangement of seven and four repetitions was adopted. Each experimental unit occupied 25 m², totaling 28 units.

Construction of cages: Cages of 1m x 1m x 1m, lined with 2mm mesh fabric, were constructed. These cages included a sleeve on the front for the insertion of spittlebugs.

Isolation and infestation: A sugarcane plant between 3 and 4 months old was selected in each plot, after previously weeding and defoliating. After ensuring the absence of spittlebugs for seven days of isolation, 14 spittlebugs per cage were introduced using an entomological net.

Application of commercial products: Products were applied according to the specifications of their technical sheets, with two application frequencies: every 15 days for 2 and 3 weeks. The treatments are detailed below:

- T1-T6: Varied in product and number of applications. Combinations of Beauveria bassiana, Metarhizium anisopliae, Lecanicillum lecanii, and Purpureocillium were used.
- T7: Control without treatment.

Identification with iNaturalist: The spittlebugs captured at various life stages were preserved in 70% alcohol and transported to the laboratory at the Technical University of Cotopaxi. Photographs of various parts of the insect were taken and uploaded to the iNaturalist app for taxonomic identification.

Context: In the study of biological pest control in agriculture, specifically the management of spittlebugs in sugarcane crops through the use of entomopathogenic microorganisms, a model was developed to analyze the effects of different treatments on the health and growth of the crop.

Plant Growth: The results show that treatments T1 and T4 (Eficax), and T3 and T6 (Biometarhizium) demonstrated greater plant growth in the first 15 days. After 30 days, T1 and T4 reached maximum heights of 1.13 m and 1.17 m respectively. At 45 days, T4 showed the highest heights, with maximums of 1.22 m and minimums of 1.19 m. By day 60, T4 significantly stood out by reaching an average height of 1.44 m, followed by T6 with 1.23 m (see Figure 1).

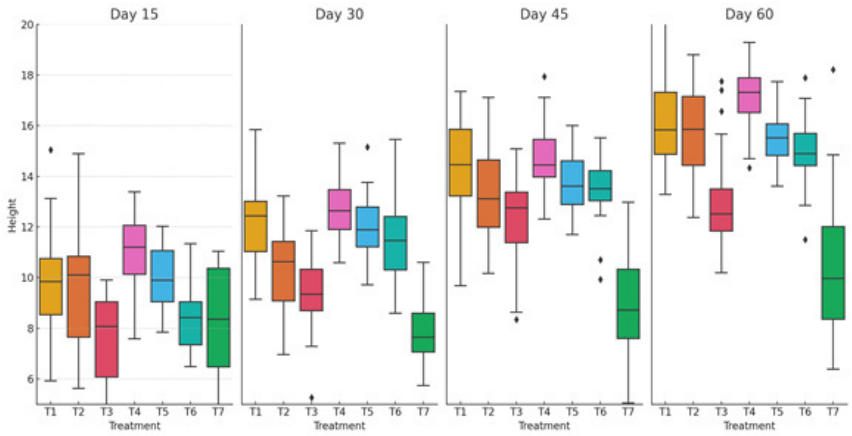

Figure 1: Boxplot graph for sugarcane plant height (cm). Source: Own elaboration.

Statistical Analysis: The variability in the data was acceptable with a variation coefficient of 11.86% (see Table 2). The analysis of variance revealed a significant difference between the treatments ($p < 0.0001$), indicating significant effects of the studied factors on the treatments. The interaction between factors did not show statistical significance, except in the direct comparison of the treatments with the control.

Table 2: Variance Analysis for spittlebug mortality. Source: Own elaboration.

F.V.	GL	SC	CM	F	p-value	
Block	3	5.79	1.93	1.87	0.1775	ns
Treatments	6	433.86	72.31	68.25	0.0001	**
Factor A	2	64.33	32.17	31.21	0.0001	**
Factor B	1	57.04	57.04	55.35	0.0001	**
A x B Factor	2	0.33	0.17	0.16	0.8521	ns
Factors vs. Witness	1	312.15	312.15	294.61	0.0001	**
Error	18	22.25	1.06			
Total	27	0.56				
CV	11.86					

Spittlebug Mortality: The Tukey test at 5% indicated that T4 was the most effective treatment for controlling Spittlebugs, followed by T6, with statistically significant differences between the groups (see Table 3).

Table 3: Tukey Test at 5% for Mortality of Sugarcane Spittlebugs Caused by Entomopathogens. Source: Own elaboration.

Factors vs. Witness	Mean				
T4	13.75	A			
T6	11.25		B		
T1	10.5		B	C	
T5	9.75		B	C	
T3	8.5			C	D
T2	6.5				D
T7	0.5				E

Control Costs: The control costs varied among the treatments, with T4 being the most expensive at $77.00, followed by T6 at $76.15 and T5 at $76.03 (see Table 4). The costs were calculated by adding up the costs of the product, labor, and the frequency of applications.

Table 4: Control Costs of the 3 Commercial Products Based on Entomopathogenic Microorganisms. Source: Own elaboration.

Product	Unit price	Treatment cost	Labor ($25 per day)	Cost/treatment
T1 (Eficax 2 applications)	20.6	1.5	50	51.5
T2 (Solubiomix 2 applications)	13.75	0.68	50	50.68
T3 (Biometarhizium 2 applications)	15.39	0.77	50	50.77
T4 (Eficax 3 applications)	20.6	2	75	77
T5 (Solubiomix 3 applications)	13.75	1.03	75	76.03
T6 (Biometarhizium 3 applications)	15.39	1.15	75	76.15
T7 (No application)	0	0	0	0
Total study cost				382.13

In conclusion, treatment T4 with three applications of Eficax proved to be the most effective both in terms of plant growth and Spittlebug mortality, although it was also the most costly. However, factors that were not considered or were quantified as of low importance due to their complexity need to be analyzed. For this, it is necessary to apply neutrosophic statistics.

3.2 Extension to Neutrosophic Statistics.

When analyzing the results obtained from classical statistics, the following factors in the analyzed sample should be reconsidered for this analysis:

- Concentration and formulation of the products (F1): The effectiveness of treatments with entomopathogenic microorganisms largely depends on the concentration and the way these biological agents are formulated.
- Frequency and timing of application (F2): The timing and frequency of applying the treatments can significantly affect the effectiveness of Spittlebug control.

- Environmental conditions (F3): Entomopathogenic microorganisms are sensitive to environmental conditions such as temperature, humidity, and UV radiation. Suboptimal conditions can reduce the efficacy of the microorganisms by decreasing their viability or activity.
- Pest resistance (F4): As with any control method, there is a risk that the Spittlebug may develop resistance to certain entomopathogenic microorganisms if these are used repeatedly and exclusively.
- Compatibility with other control methods (F5): The interaction of entomopathogenic microorganisms with other control methods (chemical, cultural, mechanical) can influence the overall effectiveness of Spittlebug management.

These factors must be considered in integrated pest management to ensure effective control of the Spittlebug in sugarcane crops. This approach aims to maximize the efficacy of biological treatments and minimize negative impacts on the environment and agricultural economy. These factors are common due to the complexity of ecological systems and variations in biological responses in pest control and management. Therefore, given the variability of the data and criteria obtained, the use of neutrosophic statistics is conditioned by experts to denote the indeterminate random components within the plithogenic set.

Development of the modeling: A group of experts analyzed the data obtained during the information collection phase. Through consensus, the following categorizations for the modeling were defined, as well as the relationship between subset and factor:

Plithogenic Set:	Spittlebug Control
Subsets or Dimensions:	• Efficacy of treatments (F1, F2, F4, F5) • Impact on plant growth (F3, F5) • Cost-effectiveness (F1,F5)
Variable:	Effectiveness in the control of Spittlebug. Code (ECS)
Factors (F):	F1, F2, F3, F4 and F5.
Sample:	60 days
Scale	[0, 6] (See Table 1)

Neutrosophic statistical analysis: When modeling the variable using neutrosophic statistics, absolute frequencies are obtained to determine the level of effectiveness in controlling the Spittlebug over a sample of 60 days of analyzed research (Table 5). To determine the level of incidence of each factor in its dimension, an indeterminacy measure for modeling was set with a scale of $0 \leq F_{V_n} \geq 1$, to determine the relative level of membership within the subset, as well as within the plithogenic set based on the analyzed frequency.

Table 5: Neutrosophic frequency for each plithogenic subset in the development of education. Source: Own elaboration.

Days	F1	F2	F3	F4	F5
1	(0 ; 2)	(2 ; 4)	(3 ; 5)	(1 ; 2)	(3 ; 5)
2	(3 ; 4)	(1 ; 3)	(0 ; 3)	(3 ; 6)	(3 ; 3)
3	(2 ; 4)	(1 ; 3)	(2 ; 3)	(1 ; 3)	(0 ; 3)
4	(2 ; 3)	(3 ; 5)	(0 ; 3)	(0 ; 1)	(3 ; 5)
5	(3 ; 6)	(1 ; 4)	(0 ; 3)	(1 ; 1)	(0 ; 1)
6	(3 ; 4)	(2 ; 3)	(0 ; 0)	(3 ; 5)	(2 ; 2)
7	(1 ; 4)	(1 ; 2)	(3 ; 4)	(1 ; 1)	(1 ; 1)
8	(3 ; 6)	(1 ; 4)	(3 ; 5)	(0 ; 1)	(0 ; 2)
9	(3 ; 4)	(0 ; 2)	(2 ; 5)	(1 ; 4)	(3 ; 6)
10	(3 ; 5)	(2 ; 3)	(2 ; 2)	(0 ; 1)	(2 ; 4)
11	(0 ; 0)	(3 ; 5)	(0 ; 1)	(2 ; 5)	(2 ; 5)
12	(3 ; 3)	(3 ; 6)	(1 ; 1)	(2 ; 2)	(2 ; 2)
13	(1 ; 1)	(2 ; 5)	(0 ; 3)	(0 ; 1)	(1 ; 1)
14	(1 ; 2)	(2 ; 3)	(0 ; 2)	(2 ; 3)	(3 ; 5)
15	(0 ; 2)	(1 ; 4)	(2 ; 5)	(1 ; 3)	(0 ; 3)
16	(0 ; 3)	(0 ; 1)	(0 ; 0)	(0 ; 2)	(3 ; 3)
17	(2 ; 4)	(2 ; 5)	(3 ; 6)	(0 ; 0)	(0 ; 2)
18	(3 ; 6)	(2 ; 4)	(0 ; 2)	(1 ; 3)	(0 ; 2)
19	(1 ; 4)	(2 ; 4)	(3 ; 6)	(2 ; 5)	(0 ; 3)
20	(3 ; 6)	(3 ; 6)	(3 ; 6)	(1 ; 1)	(2 ; 4)
0-60	(93; 186)	(92; 191)	(76; 169)	(75; 167)	(102; 189)

From the neutrosophic frequencies observed in the application of the treatments, it is noted that for a sample of 60 days analyzed, there is a level of indeterminacy ranging from 46.03% to 55.09%. Among the impacting factors are environmental conditions and pest resistance with representativity levels of (76; 169) and (75; 167) respectively. Therefore, the application of these treatments must be analyzed by including these neutrosophic elements.

Comparative Analysis: The modeling of data on the level of ECS indicates that factors 3 and 4 require studies with a deeper level of investigation. To determine the level of incidence within each subset, it is necessary to analyze the means of the sampled data (see Table 6). To understand which factor is representative, neutrosophic mean values, $\bar{x} = \in [\bar{x}_L; \bar{x}_U]$, and variations in the values of the neutrosophic standard deviation, $S_N \in [S_L; S_U]$, are calculated. Therefore, to determine which factor requires a higher level of accuracy when diagnosing each subset, it is necessary to calculate the $CV_N \in [CV_L\ ;\ CV_U]$ values and the associated levels of indeterminacy.

Table 6: Neutrosophic Statistical Analysis of the ECS Level. Source: Own elaboration.

Factors	$\bar{x}_N$	S.N.	CVN	
F1	1.55 + 3.1 I	0.861 + 2.466 I	0.555 + 0.795 I	I ∈ (0,0.795)
F2	1,533 + 3,183 I	0.76 + 2.458 I	0.496 + 0.772 I	I ∈ (0,0.772)
F3	1,267 + 2,817 I	0.86 + 2.472 I	0.679 + 0.878 I	I ∈ (0,0.878)
F4	1.25 + 2.783 I	0.776 + 2.306 I	0.621 + 0.829 I	I ∈ (0,0.829)
F5	1.7 + 3.15 I	0.831 + 2.096 I	0.489 + 0.665 I	I ∈ (0,0.665)

The control of the Spittlebug in Ecuadorian regions does not only depend on choosing an effective treatment without considering the external factors which, due to their indeterminate characteristics, are analyzed or projected onto other studies. The neutrosophic variable analyzing the ECS level depends on the interaction and balance of these factors. From the analysis of Table 6, the factors influencing the ECS levels and the associated level of indetermination are observed. The results show that the CV_N values range from 0.489 to 0.679 with an indetermination measure of [0.665; 0.878] generated by a sample of [0; 60] questionnaires and statistical information obtained from the analyzed sample. From the comparative analysis between factors, it is observed that:

Factor F2 and F5: These are among the factors with the greatest importance and the least complexity in the group analyzed in the neutrosophic statistical studies [0.772; 0.665], classified within a neutrosophic area of MI and C. Therefore, key management should be carried out in the effective control of the Spittlebug when applying the optimal treatment. Additionally, it is considered that too-spaced applications may allow the pest population to recover, while very frequent applications can be economically unviable. Concurrently, applying treatments at key moments in the pest's life cycle can significantly increase the efficacy of the treatment. It is important to note that the use of chemical insecticides could negatively affect the entomopathogenic microorganisms applied, whereas appropriate cultural practices can enhance conditions for their effectiveness.

Factor F3 and F4: These are identified as factors of lesser complexity within the analyzed group with a classification of I and MI. These factors are seen in a neutrosophic area as influential and key in pest control. The levels of indetermination analyzed are above the analyzed group [0.878; 0.829] due to an evolutionary effect shared by living beings, which should be considered in the neutrosophic statistical analysis. Moreover, for environmental conditions, it is suggested to analyze whether extreme temperatures can deactivate or kill the microorganisms before they can infect the Spittlebug. Meanwhile, pest resistance suggests rotating different types of entomopathogenic microorganisms and combining them with other control methods to help mitigate this risk.

The final factor to be analyzed is *factor F1*, where the effectiveness of the treatments is analyzed with an indetermination level of [0.795] and a classification between a neutrosophic area of MI and C. This involves including neutrosophic elements related to the proper concentration, ensuring there are enough active microorganisms to infect and control Spittlebug populations. It also analyzes and proposes a suitable formulation that helps maintain the viability of these microorganisms in the field.

These neutrosophic factors have a particularly significant impact on the ECS level regarding the development of sugarcane in different regions of Ecuador. Hence, the neutrosophic variable depends on the interaction and balance of these factors, establishing the following hierarchy of neutrosophic importance F5, F2, F1, F4, and F3. Subsequently, the degrees of contradiction of these factors and the areas where they converge within the plithogenic set are identified.

3.3 Neutrosophic Statistical Analysis of the Dimension through Integration into Plithogenic Logic.

To determine where factors or elements converge within the plithogenic ECS set, it is necessary to identify the associated characteristics and their relationship with the development of sugarcane. For this purpose, Table 7 proposes the key elements to consider.

Table 7: Characteristics of the Plithogenic Set in the Control of the Spittlebug. Source: Own elaboration.

CODE	Dimension	CODE	Sub-dimension or factor	Scale	Plithogenic number (T, I, F)	d_n (x; V_n)	Attribute value
V1	Efficacy of treatments	v11	Spittlebug mortality.	[0 ; 6]	(0.95, 0.05, 0.10)	0.93	EI
		v12	Repellency.	[0 ; 6]	(0.65, 0.35, 0.50)	0.60	I
		v13	Reduction of infestation.	[0 ;6]	(0.80, 0.20, 0.30)	0.77	VI
V2	Impact on plant growth	v21	Plant height.	[0 ; 6]	(0.55, 0.50, 0.65)	0.45	I
		v23	General plant health.	[0 ; 6]	(0.80, 0.20, 0.30)	0.77	MI
V3	Cost-effectiveness	v31	Cost of treatment per hectare.	[0 ; 6]	(0.80, 0.20, 0.30)	0.77	VI
		v32	Cost-benefit relation.	[0 ; 6]	(0.95, 0.05, 0.10)	0.93	EI

The plithogenic set is defined for three subsets V1, V2, and V3. Therefore, a plithogenic set is defined as consisting of 7 attributes, each of these attributes containing possible values, with their respective plithogenic particularities and possible values in the linguistic expression to determine the level of importance of the factor on the variable.

The multi-attribute of dimension 3 has a cardinality of 3x2x2 = 12.

The degrees of contradiction among the values for each attribute are defined as follows:

- $c_N(v_{11}, v_{12}) = 0.33; c_N(v_{12}, v_{13}) = 0.16$
- $c_N(v_{22}, v_{21}) = 0.32$
- $c_N(v_{32}, v_{31}) = 0.16$

As can be seen, the dominant values for each attribute are: v_{11} and v_{32}.

When v_{11} and v_{32} are activated, all other nodes are activated, which means that the incidence value caused by the *effectiveness of the treatments* in the development of sugarcane influences the *cost-effectiveness* for producers and is in turn influenced by the predominant factors. Therefore, to propose potential solutions, it is necessary to determine the intersected area of the predominant attributes in the plithogenic set. Thus, it is proposed to evaluate the level of importance of the treatment v_{11} and v_{32} and how it affects the effectiveness of the control of Spittlebug.

Spittlebug mortality. (v_{11}) and Cost-benefit ratio. (v_{32})

Neutrosophic Plithogenic Intersection	$S([T, I, F])$	Assessment
$(a_1, a_2, a_3) \wedge_p (b_1, b_2, b_3) = (a_1 \wedge_D b_1, \frac{1}{2}[(a_2 \wedge_D b_2) + (a_2 \vee_D b2)], a_3 \vee_D b3)$ $(a_1, a_2, a_3) \wedge_p (b_1, b_2, b_3) = (0.95; 0.05; 0.05)$	0.9633	It is located at a sublevel above EI and closer to 1 true.

There is a stronger relationship between the subsets of *treatment efficacy* and *cost-effectiveness* [in their attributes (v_{11}) and (v_{32})] than between the previous ones and the *impact on plant growth*, consequently, to the most predominant factors. A relationship is obtained with a degree beyond EI according to the plithogenic neutrosophic union and intersection operator. Therefore, solutions should be focused on addressing the factors that converge in the attributes (v_{11}) and (v_{32}) that affect the control of Spittlebug and the development of sugarcane in Ecuador.

3.4 Partial solutions

To address the challenges identified in the neutrosophic statistical analysis of Spittlebug control in sugarcane crops, two integrative solutions are proposed that focus on improving the efficacy of treatments and their cost-benefit relationship:

1. Optimization of the formulation and concentration of entomopathogenic treatments (F1): Given the critical dependency of the effectiveness of entomopathogenic treatments on their concentration and formulation, a key strategy is to develop improved formulations that maximize stability and biological activity under a range of environmental conditions. This includes:
   - Research and development: Collaborate with agricultural research institutions to innovate in formulations that extend the viability of entomopathogens in the field, considering factors such as resistance to UV radiation and extreme temperatures.
   - Controlled field tests: Implement field trials to determine the optimal concentration and formulation that produces the best results against Spittlebug, adjusting doses according to the specific needs of

the crop and environmental characteristics.

2. Integration of environmental management and monitoring strategies (F3, F5): Incorporate an integrated pest management system that combines the use of entomopathogens with environmental and cultural monitoring strategies to adapt to changing conditions and minimize pest resistance:
   - Early warning systems: Use sensor technologies and predictive models to monitor environmental conditions that may affect the efficacy of biological treatments and adjust applications in real time.
   - Rotation of biological and chemical agents: Alternate between different entomopathogenic microorganisms and, when necessary, use selective chemical insecticides to reduce the risk of Spittlebug resistance. This should be complemented with cultural practices that improve plant health and reduce dependence on chemical interventions.

Implementation and Evaluation:

- Training and education of farmers: Conduct workshops to educate farmers on the importance of new formulations, integrated pest management, and the effective use of early warning system data.
- Ongoing evaluation: Establish a protocol for the continuous evaluation of the effectiveness of the implemented strategies, using neutrosophic statistics to adjust practices based on indeterminate outcomes and ensure an adaptive response to field conditions.

These solutions aim not only to improve the efficacy of Spittlebug control but also to ensure the sustainability and profitability of agricultural practices in the context of prudent environmental and economic management.

4 Conclusion

The study on the control of Spittlebug in sugarcane crops demonstrates how neutrosophic statistics, applied through analyses of indeterminacy and plithogenic variables, allow for a deeper understanding of the interaction between multiple agricultural and environmental factors. This methodology is crucial for assessing the efficacy, costs, and environmental effects of treatments with entomopathogenic microorganisms, considering both the variability and the inherent uncertainty in agricultural systems. Neutrosophic statistics facilitate more informed and adaptive agronomic decisions, thereby optimizing integrated pest management and promoting a sustainable approach to agriculture.

The use of Plithogenics in this study on Spittlebug control illustrates how interactions between different factors and treatments can be analyzed to select the best pest management strategy. The selection of Eficax as the most effective treatment, based on detailed plithogenic analysis, demonstrates its superiority in terms of plant growth and Spittlebug mortality, while also fitting into a sustainable economic model. This plithogenic approach, which incorporates variability and indeterminacy into decision-making, allows agricultural managers to optimize their resources and enhance the effectiveness of treatments. Thus, they focus on solutions that provide the best results both in agronomic and environmental terms.

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Complex Analysis of Family and Social Problems in the Context of Parenting Competencies: Implementation of Neutrosophic Cognitive Maps (NCM) for the Evaluation and Resolution of Conflicts

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Abstract. At the intersection of family and social problems, complex challenges arise that demand an innovative and profound approach - This article explores how parental competencies, essential for family dynamics, can be better understood and strengthened through the implementation of Neutrosophic Cognitive Maps (NCM). Using a multidimensional approach, real cases are analyzed where family conflicts and tensions are intertwined with broader social problems, revealing the need for advanced analytical tools that allow a detailed and accurate evaluation. NCM emerges as a robust methodology, capable of capturing the ambiguity and uncertainty inherent in human interactions, offering a new perspective for intervention and conflict resolution in complex family contexts. The application of Neutrosophic Cognitive Maps not only facilitates a more nuanced understanding of parenting competencies, but which also provides a framework for developing effective conflict resolution strategies. By integrating emotional, cognitive and social factors, NCMs allow for a holistic representation of family dynamics, capturing subtleties and variations that escape traditional approaches. This study, Through a rigorous and detailed analysis, it demonstrates how the adoption of this innovative tool can transform the way family problems are addressed, offering more adaptive and resilient solutions. Ultimately, the results underline the importance of incorporating advanced cognitive technologies in the field of psychology and family sociology to effectively confront contemporary challenges.

Keywords: Neutrosophic Logic, Neutrosophic Cognitive Maps, NCM, Parental Competencies.

1 Introduction

The complexity of family and social problems has been a constant concern for researchers and professionals in the social and behavioral sciences. At the heart of these interactions, parental competencies play a crucial role, largely determining the dynamics and well-being in families [1]. Parenting competencies, understood as the set of skills and abilities that parents use to raise and educate their children, are intrinsically linked to the social contexts in which they develop. However, the evaluation and resolution of The conflicts that emerge in these environments are usually arduous tasks, due to the multiplicity of intervening factors and the dynamic nature of human relationships [2]. In this context, Neutrosophic Cognitive Maps (NCM) are presented as an innovative methodological tool and promising. NCMs allow us to capture and analyze the uncertainty, ambiguity and paradoxes that characterize family and social interactions. Unlike traditional approaches, NCMs integrate multiple perspectives and degrees of truth, offering a more faithful and complete representation of the complex and multifaceted reality of parenting skills [3].

The implementation of NCM in the analysis of family and social problems opens new possibilities for intervention and conflict resolution - By incorporating neutrosophic elements, greater flexibility and adaptability is achieved in the modeling of human interactions, allowing the identification of patterns and dynamics that otherwise they would go unnoticed. This ability to capture subtleties and variations in parenting competencies is crucial for developing more effective and personalized intervention strategies . The need for advanced analytical tools becomes even more evident when we consider the increasing complexity of contemporary family and social contexts . Family structures have evolved, and with them, the challenges and conflicts they face. Family problems can no

longer be addressed with simplistic or one-dimensional solutions; require integrative approaches that consider the entire family system and its social environment [4].

In this sense, NCMs provide a theoretical and practical framework to address these complexities in a holistic manner [5].The ability of NCMs to handle neutrosophic information – that which contains degrees of truth, falsity and indeterminacy – is particularly useful in the context of competencies. parental, where situations are often marked by uncertainty and ambivalence. This methodology not only facilitates a deeper understanding of family dynamics, but also allows the identification of more precise and effective interventions. The present research focuses on the application of Neutrosophic Cognitive Maps in the analysis of parental competencies and the resolution of family and social conflicts. Through case studies and detailed analysis, the advantages and limitations of this methodology are explored, as well as its potential to transform professional practice in the field of family psychology and sociology[6]. Real situations are examined where family conflicts and tensions are intertwined with broader social problems, demonstrating the usefulness of NCM to address these problems in a comprehensive and adaptive manner. Ultimately, this study seeks to contribute to the development of more effective and resilient intervention strategies, based on a deep and nuanced understanding of parental competencies and their interaction with the social context. By integrating NCM into the analysis of family and social problems, the aim is not only to improve the evaluation and resolution of conflicts, but also to encourage a more adaptive and sensitive approach to the complex and changing reality of contemporary families.

The methodological innovation represented by the implementation of Neutrosophic Cognitive Maps offers a renewed and powerful perspective to face current challenges in the field of social and behavioral sciences - This research not only expands the theoretical and practical horizon of parenting competencies, but which also opens new avenues for intervention and support for families in diverse and complex social contexts.

2. Related Works.

2.1 Parental Competencies: Analysis and Assessment.

Parenting skills represent a fundamental axis in the comprehensive development of children and adolescents, influencing not only their emotional well-being, but also their academic and social performance - These skills, understood as the set of skills, knowledge and attitudes that parents must be deployed to ensure positive parenting, they are essential for the establishment of a healthy and safe family environment – However, their evaluation and development present significant challenges, as they are deeply rooted in the cultural, social and economic context of each family [6]. The concept of parenting competencies covers a wide range of aspects, from the ability to provide affection and emotional support, to the ability to establish clear limits and rules. It is not enough to have good intentions; Parents must be able to translate these intentions into concrete actions that promote the optimal development of their children. Here lies one of the main complexities: the ability of parents to adapt and respond appropriately to the changing needs of their children over the course of their lives.

In practice, assessing parenting competencies can be a complicated process. Traditional assessment tools, such as questionnaires and interviews, often do not capture the entirety of family dynamics. Additionally, subjective and emotional factors that influence perception of parenting competencies can bias the results. For this reason, it is crucial to develop more sophisticated assessment methods that integrate different dimensions and perspectives [7]

One of the most innovative proposals in this field is the use of Neutrosophic Cognitive Maps (NCM). NCMs allow a richer and more nuanced representation of parenting competencies, capturing the uncertainty and ambiguity that characterize many family interactions. Through From this methodology, a more complete and precise vision of parents' strengths and areas of improvement can be obtained, facilitating the identification of more effective intervention strategies - It is important to highlight that the development of parenting skills is not a linear or uniform process. Parents face a variety of challenges and obstacles that can affect their ability to optimally exercise their competencies. Factors such as stress, lack of financial resources, and interpersonal tensions can have a significant impact. Therefore, interventions aimed at improving parenting skills must be holistic and adaptive, considering the unique context of each family [8].

In terms of public policy, it is essential that parenting support programs not only focus on training and education of parents, but also provide comprehensive support that addresses external factors that may influence their performance. This includes access to mental health services, economic support, and community support networks. Only through a multidimensional approach can sustainable improvements in parenting skills be achieved. The impact of parenting skills transcends the family sphere, also affecting the community and society as a whole. Children who grow up in positive and supportive family environments tend to be more resilient, have better academic results, and develop stronger social skills. Therefore, investing in the development of parenting skills is not only a matter of social justice, but also an effective strategy to promote well-being and long-term social progress. Parenting competencies are a crucial component for the healthy and balanced development of children and adolescents. The evaluation and improvement of these competencies requires innovative and adaptive approaches that consider the

complexity and diversity of family dynamics. Neutrosophic Cognitive Maps represent a promising tool in this regard, offering a way to capture and analyze family interactions more deeply and accurately. At the same time, it is It is essential that parenting support policies and programs are comprehensive and holistic, providing parents with the resources and support necessary to play their role effectively and sustainably[9].

2.2. Neutrosophic Cognitive Maps.

In the vast field of social and behavioral sciences, the need for analytical tools that capture the complexity and uncertainty of human interactions is increasingly evident – Neutrosophic Cognitive Maps (NCM), an innovative methodology that integrates the principles of neutrosophic logic, have emerged as a promising solution to address this need. NCMs allow modeling situations that include degrees of truth, falsity and indeterminacy, offering a more faithful and nuanced representation of reality [9]. The concept of NCM is based in neutrosophic set theory, developed by Florentin Smarandache, which extends classical logic to handle uncertainty, ambiguity and paradox. This theory introduces a third neutral value (I), in addition to the traditional values of truth (T) and falsity (F), allowing a more flexible and adaptive representation of information. NCMs apply these principles to the field of cognitive maps, allowing a graphical and analytical representation of the causal relationships and dynamics of complex systems [10].

In the context of parenting competencies and family and social issues, NCMs offer a powerful tool for assessment and intervention – Family interactions are often marked by ambivalence and contradiction, and traditional approaches may be insufficient to capture this complexity. NCMs, by incorporating neutrosophic elements, allow a richer and more detailed representation of these dynamics, facilitating a deeper and more precise understanding.

The implementation of NCM in the analysis of parenting competencies involves the identification and modeling of causal relationships between different factors and behaviors [11]. For example, an NCM can represent how effective communication between parents and children influences the emotional development of the child, or how economic stress can affect parents' ability to set clear boundaries. By capturing these nuances, NCMs provide a solid foundation for the development of more effective and personalized intervention strategies. One of the main advantages of NCMs is their ability to manage indeterminacy and uncertainty, aspects that are inherent to human interactions. In familiar situations, there are often unknown factors or variables that cannot be clearly defined . NCMs allow these elements to be incorporated explicitly into the model, offering a more complete and realistic representation of the situation [12]. This is particularly useful in the assessment of parenting competencies, where perception and subjectivity play a crucial role.

Furthermore, NCMs facilitate the identification of patterns and dynamics that may not be evident through traditional analysis methods - By graphically representing causal relationships and interactions between different factors[13, 14] . NCMs allow the underlying structure of the system to be visualized and detect areas of conflict or dysfunction. This capacity for in-depth analysis is essential for the design of more precise and effective interventions. The use of NCM in the evaluation of parenting skills and the resolution of family and social problems not only offers benefits at the individual level, but also to community and societal level. By providing a more accurate and detailed understanding of family dynamics, NCMs can inform the development of more effective and adaptive policies and support programs. This, in turn, can contribute to improving the overall well-being of families families and strengthen the social fabric.

Neutrosophic Cognitive Maps represent an innovative and powerful tool for evaluation and intervention in the field of parenting skills and family and social problems - By allowing a richer and more nuanced representation of reality, capturing indeterminacy and uncertainty, NCM offer a renewed and profound perspective to address the complex challenges of human interactions – The adoption and development of this methodology promises to significantly transform professional practice in the social and behavioral sciences, providing new avenues for understanding and supporting families in diverse and changing contexts [14].

In this study, neutrosophic cognitive maps will be used, so we explain them below.

Definition 1[14] : Let X be a universe of discourse - A Neutrosophic Set (NS) is characterized by three membership functions, $u_A(x)$, $r_A(x)$, $v_A(x)$: which satisfy the condition -$0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3+$.

Definition 2 [15, 16]: Let X be a universe of discourse - A single-valued neutrosophic set (SVNS) A in X is a set of the form :

$$A = \{ \langle x, u_A(x), r_A(x), v_A(x) \rangle : x \in X\} \quad (1)$$

Where u_A , r_A , $v_A : \in A(x)$ are the true, indeterminate, and falsity membership functions of x in A, respectively . For convenience, a single-valued neutrosophic number (SVNN) will be expressed as A = (a, b, c), where a, b, c $\in [0,1]$ and satisfies $0 \leq a + b + c \leq 3$.

Other important definitions are related to graphs [17-20]

Definition 3 : A *neutrosophic graph* contains at least one indeterminate edge , represented by dotted lines
Definition 4 : A *neutrosophic directed graph* is a directed graph that contains at least one indeterminate edge, which is represented by dotted lines .
Definition 5: A *neutrosophic cognitive map (NCM) is a* neutrosophic directed graph , whose nodes represent concepts and whose edges represent causal relationships between the edges .
If there are k vertices C_1 , C_2 ,…, C_k , each can be represented by a vector (x_1 , x_2 ,…, x_k) where xi ∈ {0,1, I} depending on the state of the vertex C_i at a specific time or situation:

- x_i = 0: Vertex C_i is in an activated state.
- x_i = 1: Vertex C_i is in deactivated state.
- x_i = I: The state of vertex C_i is indeterminate.

Definition 6 : An NCM that has edges with weights in {−1, 0, 1, I} is called *a simple neutrosophic cognitive map* [20].
Connections between vertices: a directed edge from C_m to C_n is called a connection and represents causality from C_m to C_n.
Associate weights to each vertex: Each vertex in the NCM is associated with a weight within the set { 0 , 1, -1, I} . The weight of the edge $C_m C_n$, denoted as α_{mn} , indicates the influence of C_m on C_n and can be:
-α_{mn} = 0: Cm has no effect on C_n.
-α_{mn} = 1: An increase (decrease) of C_m results in an increase (decrease) of C_n.
-α_{mn} = -1: An increase (decrease) of C_m results in a decrease (increase) of C_n.
-α_{mn} = I: The effect of C_m on C_n is indeterminate.
Definition 7: If C_1 , C_2 ,…, C_k are the vertices of an NCM. The neutrosophic matrix N(E) is defined as N(E) = α_{mn}), where α_{mn} denotes the weight of the directed edge $C_m C_n$, with α_{mn} ∈ [-1,0,1, I]. N (E) is called *the neutrosophic adjacency matrix* of the NCM.
Definition 8: Let C_1 , C_2 ,…, C_k be the vertices of an NCM. Let A = (a_1 , a_2 ,…, a_k), where am ∈ {-1, 0, 1, I} . A is called *the neutrosophic instantaneous state vector* and means an on-off-indeterminate state position of the vertex at a given instant.
- a_m = 0 if C_m is disabled (has no effect),
- a_m = 1 if C_m is activated (takes effect),
- a_m = I if C_m is indeterminate (its effect cannot be determined).

Definition 9: Let C_1 , C_2 ,…, C_k be the vertices of an NCM. Let $\overrightarrow{C_1C_2}, \overrightarrow{C_2C_3}, \overrightarrow{C_3C_4}, \ldots, \overrightarrow{C_mC_n}$ the edges of the NCM be, then the edges constitute a *directed cycle*.
- The NCM is said to be *cyclic* if it has a directed cycle - It is said to be *acyclic* if it does not have any directed cycle.
Definition 10: An NCM containing cycles is said to have *feedback*. When there is feedback in the NCM it is said to be a *dynamic system*.
Definition 11: Let $\overrightarrow{C_1C_2}, \overrightarrow{C_2C_3}, \overrightarrow{C_3C_4}, \ldots, \overrightarrow{C_{k-1}C_k}$ be a cycle. when C_m is activated and its causality flows along the edges of the cycle and then is the cause of C_m itself, then the dynamical system is circulating. This is valid for each vertex C_m with m = 1, 2,… , k. The equilibrium state of this dynamic system is called the *hidden pattern*.
Definition 12: If the equilibrium state of a dynamical system is a single state, then it is called a *fixed point*. An example of a fixed point is when a dynamical system starts being activated by C_1. If the NCM is assumed to be set at C_1 and C_k , which means the state remains as (1, 0,…, 0, 1), then this neutrosophic state vector is called fixed point.
Definition 13: If the NCM establishes a repeating neutrosophic state vector of the form:
$A_1 \rightarrow A_2 \rightarrow \cdots \rightarrow A_m \rightarrow A_1$LCM *limit cycle*.

3. Results and discussion

After the identification of eight family and social problems that show deficiencies and have an impact on parenting skills, as specified in Table 1, progress was made towards the development of an NCM. The objective was to illustrate the causal connections between these elements through a map neutrosophic cognitive, whose process involved defining the interactions between the various factors. The visual representation of this analysis is detailed in Figure 1.

Table 1: Family and social problems that show deficiencies and have an impact on parenting skills

D 1. Domestic violence: Physical, emotional, or verbal violence within the home can deeply affect both parents and children, altering family dynamics and negatively affecting parenting skills.

D 2. Child abuse: Physical, emotional or sexual abuse of children can generate deep trauma and emotional difficulties in child development, affecting parents' ability to provide a safe and supportive environment.
D 3. Mental health problems: Conditions such as depression, anxiety, bipolar disorder or other mental illnesses can interfere with parenting skills, hindering parents' ability to adequately care for their children.
D 4. Divorce and separation: The breakup of the couple and the transition to single-parent families can introduce additional stresses and challenges in raising children, affecting emotional stability and family cohesion.
D 5. Addictions: Parental abuse of substances such as alcohol and drugs can cause significant dysfunction in the family, affecting the ability to maintain a safe and predictable environment for children.
D 6. Unemployment and economic difficulties: Lack of financial resources can increase family stress and limit access to educational and social services and opportunities, affecting the family's overall well-being and the ability to adequately raise children.
D 7. Lack of support networks: The absence of family, community or social support networks can leave parents without emotional or practical resources to face the daily challenges of parenting, affecting the quality of care they can offer.
D 8. Educational problems: Academic difficulties or lack of access to quality education can limit children's future opportunities, creating additional tensions in the home and affecting the self-esteem of both parents and children.

These problems may vary in their impact depending on the cultural, social and economic context of each family, but all have the potential to negatively influence parental skills and the healthy development of children .

The process began by developing an NCM to represent the causal connections between the eight identified family and social problems, which affect parenting competencies according to Table 2 . This stage involved defining the interactions between various factors and visualizing them in a detailed neutrosophic cognitive map in Figure 1.

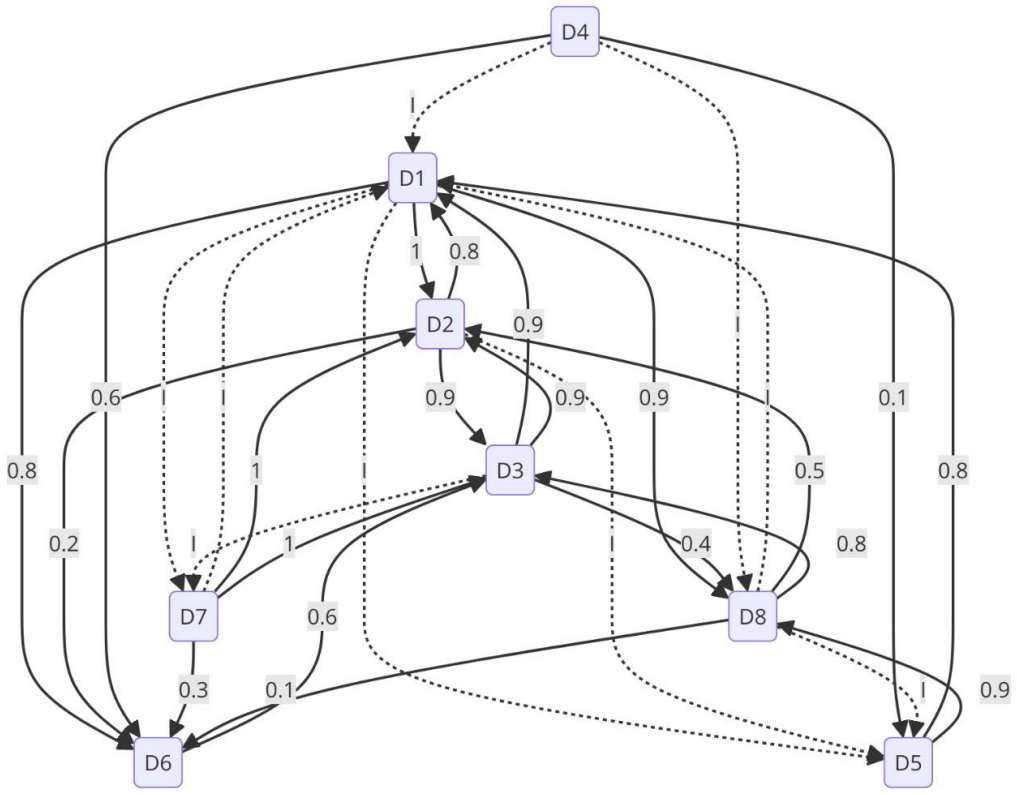

Figure 1: Neutrosophic cognitive maps

The NCM is developed through the collection and representation of relevant knowledge – The adjacency matrix obtained, which is based on the neutrosophic values provided by specialists, is detailed in Table 2 as an essential tool to analyze and interpret causal connections within the framework of the study.

Table 2: Adjacency matrix

	D1	D2	D3	D4	D5	D6	D7	D8
D1	0	1	0	0	I	0.8	I	0.9
D2	0.8	0	0.9	0	I	0.2	0	0
D3	0.9	0.9	0	0	0	0	I	0.4
D4	I	0	0	0.1	0.6	0	0	I
D5	0.8	0	0	0	0	0	0.9	0
D6	0	0	0.6	0	0	0	0	0
D7	I	1	1	0	0	0.3	0	0
D8	I	0.5	0.8	0	I	0.1	0	0

Following this perspective, the calculated centrality measures are presented below (Table 3). These metrics provide a quantitative analysis of the relative relevance of nodes within the network framework, which is crucial for understanding the dynamics and impact of the various components in the analyzed system.

Table 3: Centrality analysis

Node	*od(vi)*	*id(vi)*	*td(vi)*
D1	1.8+2I	0.8+3I	2.6+5I
D2	1+I	2.5	3.5+I
D3	0.4+I	1.6	2+I
D4	0.6+2I	0	0.6+2I
D5	0	0.6+3I	0.6+3I
D6	0.6	1.3	1.9
D7	2,3+I	0+2I	2,3+I
D8	0.5+2I	0.4+I	0.9+2I

In the context of static analysis in the NCM, initial results are obtained that incorporate the element of indeterminacy "I" within their neutrosophic values. To refine these results it is essential to carry out a process known as deneutrosophication, recommended by[21] . This process consists of replacing the indeterminacy parameter I, which ranges between 0 and 1, considering in this case "I" as 0.5. The importance of this method lies in its ability to produce more defined and precise results, which significantly simplifies the understanding of the interconnections present in the analysis in question (Table 4).

Table 4: Deneutrosophicated values of centrality

Nod	*td(vi)*
D1	5.1
D2	4
D3	2.5
D4	2.6
D5	3.6
D6	1.9
D7	2.8
D8	1.9

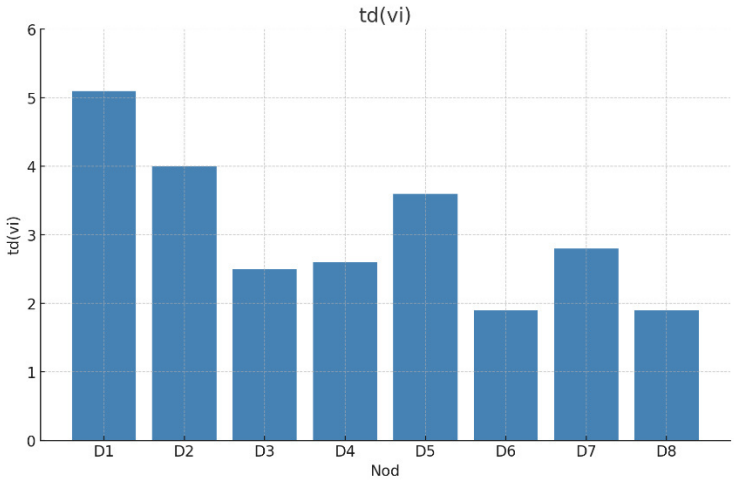

Figure 2: Deneutrosophicated centrality.

These values reflect the strength of the outgoing and incoming relationships, as well as the total centrality of each node within the cognitive map, after considering indeterminacy as a neutrosophic influence to a medium degree.

The family and social problems identified—such as domestic violence, child abuse, mental health problems, divorce and separation, addictions, unemployment and economic difficulties, lack of support networks, and educational problems—stand out as critical factors that profoundly affect parenting competencies and family well-being. These aspects not only compromise emotional stability within the home, but also have significant repercussions on parents' ability to raise their children in an effective and healthy manner. Violence Domestic violence, for example, not only physically affects those involved, but also creates an emotionally unstable environment that can harm family relationships and undermine essential parenting skills – Similarly, child abuse introduces deep trauma that negatively impacts development. emotional and psychological problems of children, affecting the ability of parents to provide a safe and protective environment. Mental health problems, such as depression and anxiety, add an additional layer of complexity to the family landscape, hindering the ability of parents to maintain a stable and consistent emotional presence that benefits the healthy growth of their children. In addition, divorce and the transition to single-parent families impose additional challenges, including managing new family dynamics and adapting to emotional changes that can influence competencies parental.

Addictions, whether to alcohol, drugs or other substances, also have a disruptive effect on the family structure, altering the ability of parents to provide a predictable and safe environment – Likewise, unemployment and economic difficulties not only generate stress financial, but also limit access to essential resources and educational opportunities, directly affecting the overall well-being of the family and the quality of care they can provide. The lack of support networks, both family and community, leaves parents without the emotional and practical support necessary to face the daily challenges of parenting. This lack can result in emotional and social isolation that

negatively impacts parents' ability to provide an appropriate and nurturing parenting environment. Lastly, educational problems, such as lack of access to quality education, can profoundly affect children's future prospects, creating additional tensions in the home and affecting the self-esteem of both parents and children. The detailed analysis of these problems along with the results of neutrosophic calculations reveals the complexity and interconnection of the factors that affect parenting competencies. The assessment of deneutrosophic centrality indicates which aspects have the most significant impact in this context, providing a solid basis for developing interventions and policies that effectively address effectively address family and social challenges. It is essential to adopt a holistic and comprehensive approach that recognizes the interdependence of these problems to promote healthier family environments and support the comprehensive development of children.

4. Conclusion

The family and social problems identified, such as domestic violence, child abuse, mental health problems, divorce and separation, addictions, unemployment and economic difficulties, lack of support networks, and educational problems, have been analyzed in depth, revealing their significant impact on parenting competencies and family well-being. These factors not only affect emotional stability within the home but also compromise the ability of parents to raise their children effectively and healthily. Domestic violence emerges as a critical factor that generates an emotionally unstable environment, affecting family relationships and weakening essential parenting skills. Similarly, child abuse introduces deep traumas that negatively affect the emotional and psychological development of children, hindering the ability of parental care to ensure a safe and protective environment. Challenges associated with mental health, such as depression and anxiety, further complicate the family landscape by hindering the emotional stability necessary for the healthy development of children. Additionally, the divorce process and the transition to single-parent family structures present new challenges, including managing changing family dynamics and adapting to the emotional needs of children.

Addictions, along with unemployment and economic hardship, not only create financial stress but also limit access to essential resources and educational opportunities, directly affecting the overall well-being of the family and the quality of the parenting environment offered to children. The lack of support networks, both family and community, leaves parents without the emotional and practical support necessary to face the daily challenges of parenting, which can lead to isolation that negatively affects the quality of parental care. Finally, educational problems, such as limited access to quality education, deeply impact children's future prospects, creating additional tensions in the home and affecting the self-esteem of both parents and children. The analysis of these problems, combined with the results of neutrosophic calculations to assess deneutrosophic centrality, reveals the complexity and interconnection of these factors in parenting competencies. This analysis provides a solid foundation for the development of effective interventions and policies that address these challenges holistically. It is essential to adopt a comprehensive approach that recognizes the interdependence of these problems to promote healthier family environments and support the comprehensive development of children. This involves implementing strategies that strengthen social support networks, improve access to mental health and educational services, and encourage positive and resilient parenting practices. Only through this integrated approach can we move towards more equitable societies and stronger, more cohesive families.

In future work, it would be valuable to explore the application of neutrosophic cognitive maps combined with multicriteria methods to analyze and address these complex problems. This combination could provide a deeper and more structured understanding of the interrelationships between the various factors affecting parenting competencies and family well-being, allowing for the design of more precise and effective interventions.

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Exploring the Impact of Educational Reforms and Their Dimensions on Teacher Performance Through the Analysis of Plithogenic Statistics

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Abstract. The exploration of the impact of educational reforms on teacher performance is a topic that has gained unusual relevance in recent decades. By analyzing these reforms through the prism of Plithogenic statistics, a window opens to a new approach that allows us to understand the inherent complexity of contemporary educational systems. Plithogenic statistics, which integrate heterogeneous and multivariate data, provide a powerful tool for unraveling how educational policies affect teacher performance. In this study, a multifaceted approach has been employed, ranging from longitudinal data analysis to qualitative evaluation of teacher perceptions, revealing an intricate interconnection between the reforms implemented and the results observed in the classrooms. The conclusions derived from this analysis are revealing and, in a certain sense, paradigmatic. Educational reforms, far from being mere administrative interventions, emerge as agents of dynamic change that influence the motivation, professional development and, ultimately, the performance of teachers. However, this impact is not homogeneous; varies significantly depending on the institutional context and the individual characteristics of educators. Through the application of Plithogenic methods, it has been possible to capture the subtle interaction of these factors, providing a more nuanced and holistic vision of educational reality. In summary, this study not only expands the understanding of educational reforms, but also highlights the importance of using advanced statistical approaches to capture the complexity of modern educational phenomena.

Keywords: Educational Reforms, Plithogenic Probability, Plithogenic Statistics, Multivariate Statistics, Plithogenicity, Neutrosophic Number .

1. Introduction

Education, a fundamental pillar of social and economic development, has been the subject of various reforms over time. In this context, the evaluation of the impact of these reforms on teaching performance emerges as an issue of increasing importance. Understanding how educational policies affect teachers is not only crucial for improving educational quality, but also for the design of future intervention strategies [1]. This study seeks to explore this relationship through the use of Plithogenic statistics, an innovative and complex approach that allows for a deeper and multidimensional analysis. Plithogenic statistics, at its core, represent a significant methodological advance, combining data of diverse nature to offer a holistic perspective [2]. This approach is not only limited to the quantification of results, but also integrates qualitative and contextual aspects, providing a richer and more nuanced understanding of educational phenomena. By applying this method to the study of educational reforms, the aim is to unravel the multiple layers of influence that they exert on teacher performance [3].

The choice of Plithogenic statistics is based on the need to overcome the limitations of traditional methods, which often oversimplify educational reality. Educational reforms, by their nature, are complex interventions that interact with a host of contextual and personal variables. In this way, a Plithogenic analysis allows us to capture the essence of these interactions, offering a more realistic and detailed vision of the impact of educational policies.

Teaching performance, understood as a multidimensional construct, covers aspects that go beyond the simple academic performance of students [4]. It includes motivation, continuous professional development, job satisfaction and the ability to adapt to new educational paradigms. Therefore, any study that seeks to evaluate the impact of educational reforms must necessarily consider this complexity and diversity of factors. This study focuses on a comprehensive analysis of the educational reforms implemented in recent years, using data collected from various sources. Through a Plithogenic approach, the direct and indirect effects of these reforms on teaching performance will be analyzed, considering variables such as professional training, availability of resources, working conditions

and institutional support. The integration of these elements will allow a more comprehensive and precise evaluation of the observed impacts.

One of the main hypotheses of this study is that educational reforms, although designed to improve the quality of education, can have varied and not always positive effects on teacher performance. The pressure to adapt to new regulations, the lack of adequate resources and increased responsibilities can generate tensions and challenges that negatively affect teachers. However, it is also possible that certain reforms provide tools and support that enhance professional development and teacher effectiveness. The use of Plithogenic statistics in this analysis will not only validate or refute this hypothesis, but will also open new avenues of research. This methodological approach, by considering multiple dimensions and variables, offers the possibility of discovering patterns and correlations that would not be evident through traditional methods. Thus, the findings of this study are expected to contribute significantly to the existing body of knowledge on education and educational reform [5].

Exploring the impact of educational reforms on teacher performance through the analysis of Plithogenic statistics represents an innovative and necessary effort. By adopting this approach, we seek not only to evaluate the effects of current educational policies, but also to provide valuable elements for the design of future reforms. Ultimately, the goal is to improve the quality of education through a deeper and more detailed understanding of the factors that influence teacher performance.

2. Related Words.

2.1 Educational Reforms.

Educational reforms, a recurring phenomenon in the history of education, continually seek to improve the quality of the educational system. Each country, with its unique particularities and contexts, has implemented reforms with varying degrees of success and failure. These reforms are often seen as the magic solution to deep-seated problems, from lack of resources to educational inequality. However, it is crucial to critically analyze and evaluate these reforms, understanding their motivations, implementations and results [6].

One of the main motivations behind educational reforms is the need to adapt to social and economic changes. In a globalized and constantly evolving world, educational systems must prepare students to face challenges that did not even exist a decade ago. The reforms seek, in this sense, to modernize the curriculum, integrate emerging technologies and promote critical skills such as critical thinking and creativity [7]. However, the success of these initiatives depends largely on the institutions' ability to adapt and the resources available for their implementation. The implementation of educational reforms is not without challenges. One of the main problems is resistance to change, both on the part of teachers, students and parents. Teachers, accustomed to certain methods and routines, may feel overwhelmed by the need to adapt to new policies and pedagogical approaches. Furthermore, the lack of adequate training and institutional support can turn these reforms into an additional burden rather than a substantial improvement [8]. On the other hand, students and their families may have difficulties understanding and accepting changes, especially if they affect fundamental aspects such as evaluation and curricular content.

Despite the challenges, it is undeniable that some educational reforms have had a positive impact. For example, the inclusion of emotional education in the curriculum has proven to be beneficial for the comprehensive development of students. Likewise, the promotion of inclusive education has allowed boys and girls with disabilities to have access to quality education, breaking down historical barriers and promoting a more equitable society. These examples show that, when implemented correctly, reforms can contribute significantly to the well-being and development of students. However, not all reforms have been equally successful. Some, driven by political or economic interests, have failed by not considering the realities and needs of the educational context. An example of this is the implementation of standardized assessment policies, which often ignore local particularities and individual differences of students. These policies can create excessive pressure and encourage a mechanistic approach to education, to the detriment of creativity and critical thinking. The key, therefore, lies in designing reforms that are sensitive to specific contexts and that involve all educational actors in their development and implementation.

The participation of the educational community is essential for the success of any reform. Teachers, as the main agents of change, must be included in the process of designing and executing educational policies. Their experience and knowledge of the classroom are invaluable in identifying what changes are necessary and how to implement them effectively. Likewise, students and their families must have a voice in these decisions, ensuring that the reforms respond to their needs and aspirations. Only through a participatory and collaborative approach can resistance be overcome and sustainable educational transformation achieved. The continuous and rigorous evaluation of educational reforms is another crucial aspect. It is not enough to implement changes and wait for results; It is necessary to constantly monitor and analyze the effects of these policies. This involves collecting accurate and relevant data, as well as carrying out longitudinal studies that allow us to understand the long-term impact. Only through constant evaluation can the necessary adjustments be made and ensure that the reforms meet their objectives.

Education reforms are powerful tools for improving education systems, but their success depends on careful planning, inclusive implementation, and continuous evaluation. By critically assessing these reforms, we can learn

from past successes and failures, designing more effective policies adapted to the needs of the educational community. Ultimately, the goal must always be to provide quality education for all, preparing them for the challenges of the future and promoting a more just and equitable society.

2.2 Plithogenic Statistics.

Plithogenic statistics (PS) represent an advanced and multifaceted methodological approach in data analysis, designed to handle and synthesize heterogeneous information from multiple sources. Unlike traditional statistical methods, which usually focus on isolated variables or simplified models, PS seeks to capture the complexity and interconnectivity of the phenomena studied. This approach allows for a deeper and more nuanced understanding of the data, offering a powerful tool for research in fields as diverse as education, economics, biomedicine, and more [10].

In the educational field, PS are particularly useful for evaluating the impact of educational reforms. The reforms, by their nature, affect a wide range of factors, from the academic performance of students to the motivation and professional development of teachers. By employing PS, researchers can analyze how these variables interact with each other and influence educational outcomes. This analysis can reveal patterns and correlations that would not be evident through traditional methods, providing valuable elements for the design and implementation of educational policies [11]. The analysis process with PS involves the integration of quantitative and qualitative data, allowing for a more holistic evaluation of the phenomena studied. For example, when evaluating educational reform, quantitative data such as standardized test scores and graduation rates can be considered, along with qualitative data such as teacher perceptions and student experiences. This combination of data provides a more complete and accurate picture of the impact of the reform, helping to identify both its strengths and weaknesses [12].

One of the key advantages of PS is its ability to handle large volumes of data and to identify complex relationships between variables. This is especially important in the context of educational reforms, where the effects can be multidimensional and often non-linear. For example, a reform that improves the resources available to teachers may have direct positive effects on students' academic performance, but may also indirectly influence teachers' motivation and job satisfaction, which in turn affects their effectiveness. in the classroom [13]. PSs allow these complex dynamics to be captured and provide a more detailed understanding of the underlying mechanisms.

The application of PS in educational research also offers opportunities for personalization and adaptation of educational policies. By identifying specific patterns and correlations at the subgroup level, PSs can help design more effective interventions tailored to the particular needs of different school communities. This is crucial in diverse educational contexts, where a policy that works well in one setting may not be equally effective in another [14]. Despite its numerous advantages, PS implementation also presents challenges. Requires a high level of technical competence and a deep understanding of advanced statistical methodologies. Additionally, collecting and integrating heterogeneous data can be complex and costly. However, the potential benefits of a more complete and nuanced understanding of educational phenomena justify these challenges, and investment in PS can pay significant dividends in terms of improving educational quality.

Plithogenic statistics offer a powerful and sophisticated approach to data analysis, allowing for deeper and more detailed evaluation of educational phenomena. By capturing the complexity and interconnectivity of the variables involved, SPs provide valuable elements for the design and implementation of effective educational policies. Despite the challenges associated with their implementation, PS represent an invaluable tool for researchers and policy makers in their efforts to improve the quality of education.

Plithogenic Statistics (PS) comprises the analysis and observations of the events under study. This allows an analysis of many output variables that are neutrosophic or indeterminate.

There are several subclasses of Plithogenic Statistics which are shown:

- Multivariate statistics,
- Neutrosophic Plithogenic Statistics,
- Plithogenic indeterminate statistics,
- Plithogenic intuitionistic fuzzy statistics,
- Fuzzy statistics of Plithogenic images,
- Plithogenic spherical fuzzy statistics,
- And in general: Plithogenic statistics (diffuse extension).

In a neutrosophic population, each element has a triple probability of affiliation (T_j, I_j, F_j), where$T_j, I_j, F_j \in [0,1]$ similar to that $0 \leq T_j + I_j + F_j \leq 3$.

If we assume that we must have the data set $(T_j, I_j, F_j)$for $j = 1, 2, \dots, n$, where n is the sample size, then the average probability of all the sample data is calculated using Equation 1[15-16]

$$\frac{1}{n}\sum_{j=1}^{n}(T_j, I_j, F_j) = \left(\frac{\sum_{j=1}^{n} T_j}{n}, \frac{\sum_{j=1}^{n} I_j}{n}, \frac{\sum_{j=1}^{n} F_j}{n}\right) \quad (1)$$

In this investigation, we also consider some operations in the form of neutrosophic numbers . These ways of representing indeterminacy, under certain conditions, are equivalent to working with intervals.

Definition 1 : ([17-18]) A *neutrosophic number* N is defined as a number as follows:

$$N = d + I \quad (2)$$

Where d is called *the determinate part* and I is called *the indeterminate part*.

Given $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ are two neutrosophic numbers, some operations between them are defined as follows:

$N_1 + N_2 = a_1 + a_2 + (b_1 + b_2)I$ (Addition) ;

$N_1 - N_2 = a_1 - a_2 + (b_1 - b_2)I$ (Difference),

$N_1 \times N_2 = a_1 a_2 + (a_1 b_2 + b_1 a_2 + b_1 b_2)I$ (Product),

$\frac{N_1}{N_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$ (Division).

3 Results and Discussion.

The research focused on a population of 339 teachers. Using non-probability sampling, it was applied at the discretion of the researcher. For data collection, the survey was used as a quantitative research method, and the data were collected using a previously prepared questionnaire. This questionnaire, developed according to the objectives and dimensions of the dependent variable, contains approximately 25 items. It was administered to both the control and experimental groups, before and after the interventions. The questionnaire was structured as follows:

1. **Quality of Learning (7):**
   - Effectiveness of the teaching methods introduced by the reform.
   - Impact of the reform on the academic performance of students.
   - Teachers' perception of the improvement in students' critical and creative skills.
   - Availability and quality of the educational resources provided.

2. **Teacher Professional Development (7):**
   - Effectiveness of the teacher education and training programs included in the reform.
   - Level of institutional and administrative support received by teachers.
   - Teachers' perception of their professional growth and development opportunities.
   - Teacher satisfaction with new educational methodologies and technologies.

3. **Inclusion and Equity (4):**
   - Impact of the reform on the inclusion of students with special needs.
   - Perception of equity in access to educational resources for all students.
   - Measures taken to reduce achievement gaps between different demographic groups.
   - Evaluation of cultural diversity and curricular adaptation to local realities.

4. **Community Participation and Commitment (7):**
   - Level of involvement of parents and the community in the implementation of the reform.
   - Students' perception of their participation in the educational process and decision making.
   - Effectiveness of communication channels between the school and the community.
   - General satisfaction of the different educational actors (teachers, students, parents) with the reform and its impact on school life.

The years of experience of the teachers and, consequently, the possible limitations they could have in understanding neutrosophic methods were considered. For this reason, they were asked to express their opinions

using a range of values rather than assigning a single value on a continuous numerical scale ranging from 0 (Never) to 10 (Always). These intervals are expressed in the form $I_i = [a_i L, a_i U]$ for each of the respondents.

The validation of the instruments for data collection was carried out through the judgment of experts with a doctorate degree. The reliability of the instruments was evaluated through Cronbach's Alpha analysis. Finally, the results indicated that the instrument used is reliable.

The last step was to administer the survey to the members of the experimental group. All this data was collected to be processed by the researchers. The steps followed are detailed below:

1. Different variables are specified. for the dimensions to measure:
$S = \{s_1, s_2, \ldots, s_{34}\}$denotes the set of students in the study group.
$\tilde{S} = \{\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_{34}\}$denotes the set of students in the control group.
$d = \{d_1, d_2, d_3, d_4\}$denotes the set of dimensions to be measured, such that:
d_1: Symbolizes the dimension " **Quality of Learning** ",
d_2: Symbolizes the dimension " **Teacher Professional Development** ",
d_3: Symbolizes the dimension " **Inclusion and Equity** ",
d_4: Symbolizes the dimension " **Community Participation and Commitment** ".
Each of these elements is a set of elements in itself, where:

$d_1 = \{d_{11}, d_{12}, \ldots, d_{17}\}$is the set of elements of the first dimension (d_{1j}represents the 1st item Dimension),

$d_2 = \{d_{21}, d_{22}, \ldots, d_{26}\}$is the set of elements of the second dimension (d_{2j}represents the 2nd item Dimension),

$d_3 = \{d_{31}, d_{32}, \ldots, d_{37}\}$is the set of elements of the third dimension (d_{3j}represents the 3rd article Dimension),

$d_4 = \{d_{41}, d_{42}, \ldots, d_{47}\}$It is the set of elements of the fourth dimension (d_{4j}represents the 4th Article Dimension).

In this way, the evaluations for each item are represented by:

$I_{ij}^K = \left[a_{ij}^{KL}, a_{ij}^{KU}\right]$, which is the evaluation of the ith teacher in the objective group for the k th article of the j th dimension.

The equivalent notation for the control group is $\tilde{I}_{ij}{}^K = \left[\tilde{a}_{ij}^{KL}, \tilde{a}_{ij}^{KU}\right]$.

2. The dimension scores were obtained for each respondent and each of the dimensions using the following expression:

$$D_{ji} = \sum_{k=1} I_{ij}{}^k \quad (3)$$

$D_{ji}$is the score of a variable or dimension j for respondent i. This score is obtained by the arithmetic sum of all the k items of the variable or dimension j, answered by respondent i, using the sum of intervals.

Equivalently, we have the results for the control group:

$$\widetilde{D}_{ji} = \sum_{k=1} \tilde{I}_{ij}{}^K \quad (4)$$

3. Since the dimensions and variables have different numbers of elements, the scores are transformed into a range from 0 to 100 using the following expression for the study group:

$$D_{ji}^* = \frac{D_{ji} - \min \text{ punt theor } D_j}{\max \text{ punt theoric } D_j - \min \text{ puntn theoric } D_j} * 100 \quad (5)$$

Where: $D_{ji}^*$is the transformed score for variable or dimension j of respondent i.
In the same way, we have Equation 6 for the control group.

$$\widetilde{D}_{ji}^* = \frac{\widetilde{D}_{ji} - \min \text{ punt theoric } \widetilde{D}_j}{\max \text{ punt theoric } \widetilde{D}_j - \min \text{ punt theoric } \widetilde{D}_j} * 100 \quad (6)$$

These transformations allow the scores of the variables or dimensions to have the same range of values despite their number of elements so that 0 represents the minimum level and 100 the maximum level. That is, these new scores are the proportions of the dimensions or value of the variable by the respondents.

$\overline{D}_j^*$de notes the average of the results for the jth dimension for the study group and is calculated by the following formula:

$$\overline{D}_j^* = \frac{\sum_{i=1}^{34} D_{ji}^*}{34} \quad (7)$$

equivalently for the control group:

$$\overline{\tilde{D}}_j^* = \frac{\sum_{i=1}^{34} \tilde{D}_{ji}^*}{34} \quad (8)$$

As the change occurs before and after passing the group study program , formula 9 is used:

$$\overline{\Delta}_j^* = \overline{D}_j^{*after} - \overline{D}_j^{*before} \quad (9)$$

Where $\overline{D}_j^{*after}$ denotes the scores of the study group after passing the program, while $\overline{D}_j^{*before}$ are the previous results.

While :

$$\overline{\tilde{\Delta}}_j^* = \overline{D}_j^* - \overline{\tilde{D}}_j^* \quad (10)$$

Denotes the difference between the average of the group to be studied with the control group.
Once the indices used to measure these results were defined, calculations were made that indicate the following, as can be seen in the following figures:

Figure 1 shows the percentages achieved in the interval for the **Quality of Learning dimension** .

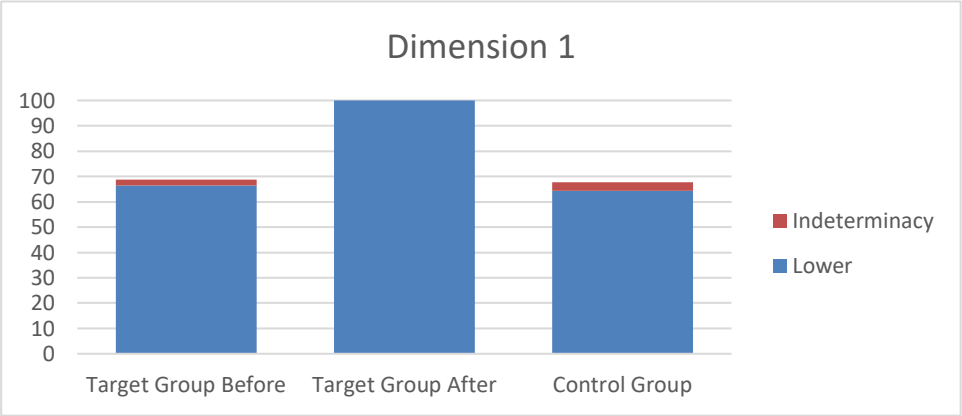

Figure 1. Results of the average of the target group before and after the educational reforms and of the control group for Dimension 1. In blue is the certain percentage and in red is the indeterminate percentage.

Figure 2 is the result of the " **Teacher Professional Development** " Dimension.

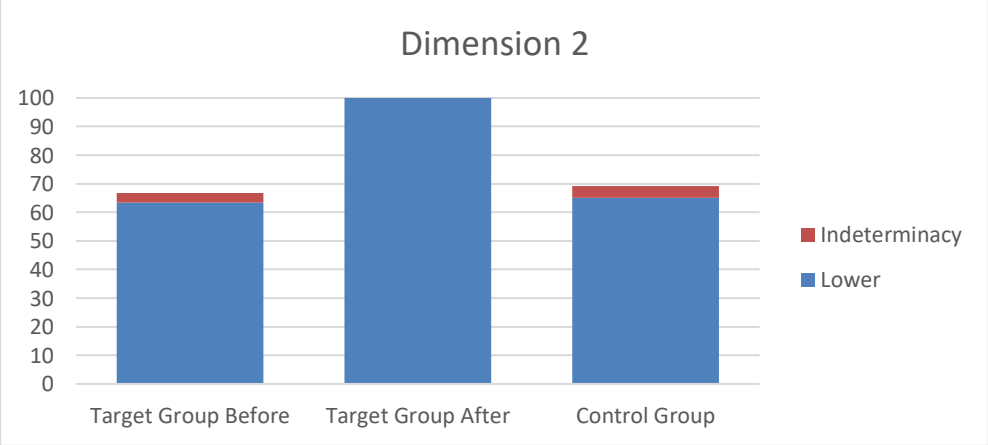

Figure 2. Results of the average of the target group before and after the educational reforms and of the control group for Dimension 2. In blue is the certain percentage and in red is the indeterminate percentage.

Figure 3 refers to the result of the Dimension: "Inclusion and Equity".

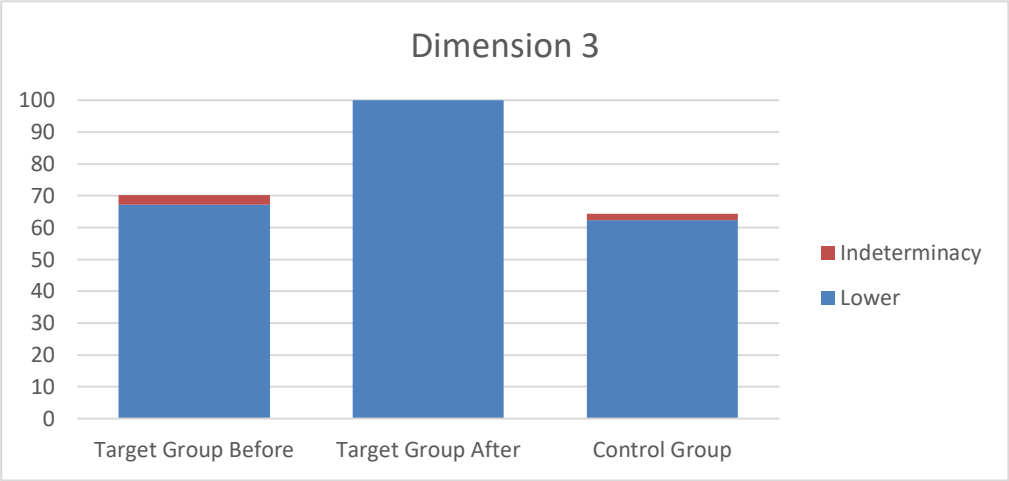

Figure 3. Results of the average of the target group before and after the educational reforms and of the control group for Dimension 3. In blue is the certain percentage and in red is the indeterminate percentage.

Figure 4 refers to the result of the Dimension: " **Community Participation and Commitment** ". .

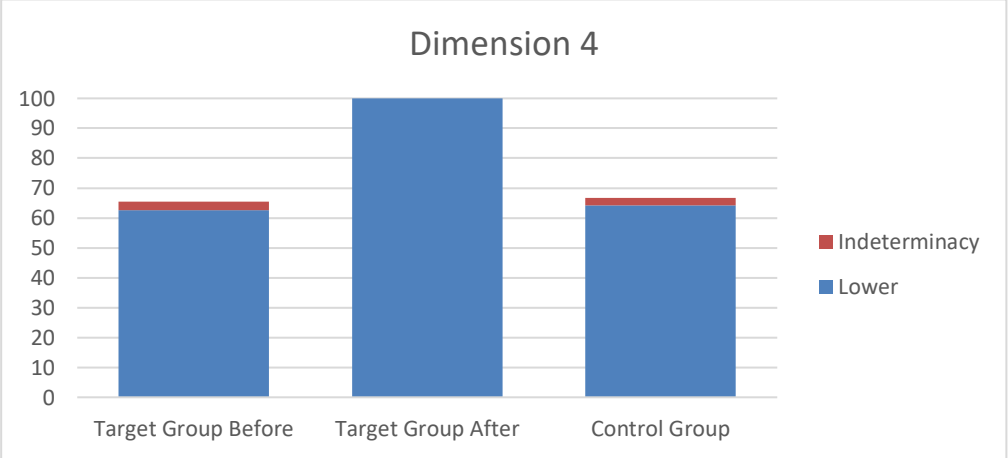

Figure 4. Results of the average of the target group before and after the educational reforms and of the control group for Dimension 4. In blue is the certain percentage and in red is the indeterminate percentage.

Thus using the difference between intervals we have:

- $\overline{\Delta}_1^* = [100,100] - [69\ .31, 62\ .36] = [30\ .69, 37\ .64]$,
- $\overline{\Delta}_2^* = [100,100] - [63\ .36, 62\ .74] = [36\ .64, 37\ .26]$,
- $\overline{\Delta}_3^* = [100,100] - [65\ .21, 72\ .23] = [34\ .79, 27\ .77]$,
- $\overline{\Delta}_4^* = [100,100] - [60\ .96, 62\ .39] = [39\ .04, 37\ .61]$.

On the other hand, the results for $\overline{\overline{\Delta}}_j^*$ are as shown below:

- $\overline{\overline{\Delta}}_1^* = [100,100] - [62\ .16, 69\ .71] = [37\ .84, 30\ .29]$,
- $\overline{\overline{\Delta}}_2^* = [100,100] - [62\ .34, 66\ .35] = [37\ .66, 33\ .65]$,
- $\overline{\overline{\Delta}}_3^* = [100,100] - [66\ .31, 65\ .12] = [33\ .69, 34\ .88]$,
- $\overline{\overline{\Delta}}_4^* = [100,100] - [62\ .47, 61\ .14] = [37\ .53, 38\ .86]$.

As can be seen, the values always showed improvements of around 30% or more, both when the target group was compared with itself before and after the program, and when compared with the control group.

To obtain a result that encompasses all the dimensions in a single final value, formula 11 will be used:

$$\min([a_1, b_1], [a_2, b_2]) = [\min(a_1, a_2), \min(b_1, b_2)] \quad (11)$$

In this case,

$D^* = \min([69.31, 62.36], [63.36, 62.74], [65.21, 72.23], [60.96, 62.39]) = [60.96, 62.36]$ It is the result of the target group before the educational reforms.

After passing the educational reforms the general result is $[100, 100]$. For the control group this is

$\tilde{D}^* = \min([62.16, 69.71], [62.34, 66.35], [66.31, 65.12], [62.47, 61.14]) = [62.16, 61.14]$.

Finally, we obtained the result for the " Teaching Performance " test, before and after for the objective group and the control group. These are shown in Figure 5:

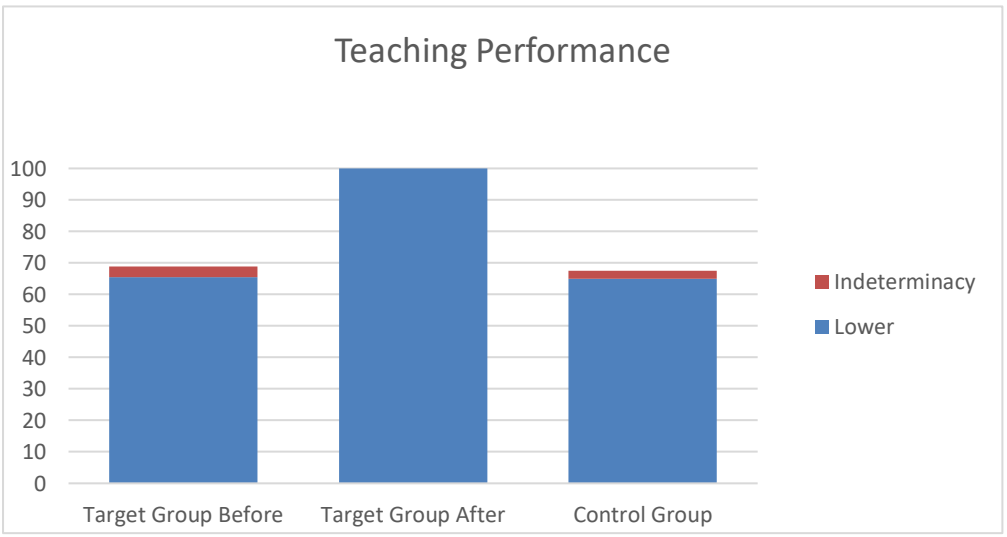

Figure 5. Average results of the target group before and after the educational reforms and of the control group for " teaching performance ". in blue is the certain percentage and in red is the indeterminate percentage.

In this case, we will calculate the difference in absolute value to avoid negative numbers in the calculation of the relationship between educational reforms and teacher performance. That is, equation 12 will be used.

$$[a_1, b_1] \ominus [a_2, b_2] = [\text{abs}(a_1 - b_2), \text{abs}(b_1 - a_2)] \quad (12)$$

In this case, it is:

$[62.16, 61.14] \ominus [60.96, 62.36] = [0.2, 0.18]$ which is the result of comparing " teaching performance " with the aggregation of the four dimensions that represent " educational reforms . " This represents a difference of less than 4% between both results.

On the other hand, $[100, 100] \ominus [100, 100] = [0, 0]$ for both variables after the program. This suggests a high and positive correlation between " educational reforms " and " teacher performance

4. Conclusion

The conclusions derived from our study on the comparison between "teaching performance" and the aggregation of the four dimensions that represent "educational reforms" reveal significant findings. We find that the result of this comparison, [62. 16, 61. 14] ⊖ [60. 96, 62. 36] = [0. 2, 0. 18], indicates a minimum difference of less than 4%. This finding suggests a close relationship between the reforms implemented and the impact on teacher performance. On the other hand, when analyzing [100, 100] ⊖ [100, 100] for both variables after the program, we observe a result of [0, 0]. This situation denotes a high and positive correlation between educational reforms and teaching performance, where both maintain consistent and positive values after the implementation of the program.

This study highlights the importance of educational reforms as potential catalysts for improving teacher performance. The results show that the dimensions incorporated in the reforms, such as quality of learning, teacher professional development, inclusion and equity, as well as community participation, have a measurable positive impact

on educational outcomes. Furthermore, the relevance of evaluating educational interventions using robust quantitative methods such as those used in this study is highlighted. The use of equations such as equation 12, which calculates differences in absolute value to avoid negative numbers in the relationships between variables, provides an accurate and nuanced measure of the impact of educational policies. The practical implications of these findings are substantial for educational policy makers. We suggest that future research delve into the longitudinal evaluation of educational reforms, considering not only the short-term results but also their sustainability and long-term effects on the educational system. In terms of practical recommendations, it is suggested that policymakers strengthen support and ongoing training for teachers, thereby ensuring that educational reforms are implemented effectively and sustainably. This could include robust professional development programs and the integration of innovative educational technologies that support teaching and learning.

In conclusion, this study provides a deeper understanding of how educational reforms impact teacher performance, highlighting both their achievements and potential areas for further improvement. It is essential to continue researching and refining our educational policies to promote an educational environment that is equitable, inclusive, and that maximizes the potential of all students and educators.

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Impact of the development and implementation of a multiplatform mobile application in the gastronomic sector: an analysis through the neutrosophic PEST-SWOT approach

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Abstract. The dizzying technological advance has permeated all areas, including gastronomy. In this context, the development and implementation of a multiplatform mobile application has generated a significant impact in this sector. This article explores how this technological innovation has transformed the operational and commercial dynamics of gastronomy, using the neutrosophic PEST-SWOT approach. Through detailed analysis, political, economic, social and technological (PEST) factors are examined and combined with an exhaustive study of strengths, weaknesses, opportunities and threats (SWOT) from a neutrosophic perspective, revealing the complexity and dynamism inherent in this interaction. Neutrosophic analysis allows for a deep understanding of the uncertainties and ambiguities surrounding the implementation of mobile applications in gastronomy, offering a holistic and nuanced view. The findings show that while there are significant challenges, such as adapting to rapid technological changes and data management, there are also vast opportunities to improve operational efficiency and user experience. This study highlights the importance of an adaptive strategy that is aware of the multiple dimensions of the dining environment, suggesting that success lies in the ability to navigate and balance these complex forces.

Keywords: Gastronomy, SWOT Analysis, PEST Analysis, Neutrosophic Single Value Numbers, PEST-SWOT Neutrosophic Analysis, Multiplatform Mobile Application.

1 Introduction

In the current digital era, the gastronomic sector has not been immune to technological transformations . The implementation of cross-platform mobile applications has revolutionized the way restaurants and other related businesses operate and connect with their customers. These applications have not only improved operational efficiency but have also transformed the user experience, offering a range of services ranging from table reservations to home delivery. This study focuses on analyzing the impact of these technological innovations in the gastronomic sector. The development of cross-platform mobile applications has allowed gastronomic businesses to quickly adapt to changing market demands [1]. The flexibility of these applications, which can run on various operating systems such as iOS and Android, has significantly expanded the reach of dining services. Customers can now access menus, place orders and make payments from the convenience of their mobile devices, which has resulted in increased customer satisfaction and loyalty. However, the impact of these applications goes beyond mere customer convenience. In operational terms, these tools have optimized inventory management, personnel scheduling, and logistics coordination. The data collected through these applications provides valuable information that can be used to improve efficiency and reduce costs [2]. Analyzes of sales and customer preferences allow businesses to adjust their offers and marketing strategies more accurately and effectively.

The analysis of this impact is carried out using the neutrosophic PEST-SWOT approach, which combines the analysis of political, economic, social and technological factors (PEST) with a detailed study of strengths, weaknesses, opportunities and threats (SWOT). This neutrosophic approach is particularly useful in addressing the uncertainties and ambiguities surrounding the implementation of new technologies in complex environments such

as the gastronomic sector. It allows for a holistic evaluation that considers both the positive and negative aspects of these innovations. From a policy perspective, the regulation of mobile technologies and data protection are crucial aspects that affect the implementation of these applications. Government policies and privacy regulations must be carefully considered to ensure that applications meet legal requirements and protect user information. On the economic front, the cost of developing and implementing mobile applications can be significant, but the long-term benefits in terms of operational efficiency and increased revenue can justify this investment.

Socially, mobile applications have changed the way customers interact with food businesses. The ability to order online and access personalized services has increased consumer expectations. Applications must be intuitive and offer a fluid user experience to meet these new demands [3]. Technologically, the development of these applications requires a robust infrastructure and constant updating to keep up with the latest innovations and ensure the security and functionality of the system. The strengths of these applications include improving operational efficiency, reducing costs, and increasing customer satisfaction [4]. However, there are also weaknesses, such as dependence on technology and risks associated with cybersecurity. Opportunities come in the form of new markets and the ability to offer innovative services, while threats include intense competition and rapid changes in consumer technological preferences [5].

The development and implementation of multi-platform mobile applications in the gastronomic sector has had a profound and multifaceted impact. This study, through the neutrosophic PEST-SWOT approach, provides a detailed understanding of these impacts and offers recommendations to maximize the benefits and mitigate the associated risks. The ability of gastronomic businesses to adapt to these technologies and take advantage of their advantages will be crucial to their success in the future.

2. Related Works.

2.1 Cross-Platform App Developers.

The digital era has brought with it countless innovations that have transformed our daily lives, and among them, multiplatform mobile applications stand out for their transversal impact in various sectors. These applications, designed to work on different operating systems such as iOS and Android, have revolutionized the way we interact with technology and the world around us. Its development has not only democratized access to tools and services, but has also generated a new paradigm in the software industry [6].

The main advantage of cross-platform mobile applications is their ability to reach a wide and diverse audience. Instead of developing and maintaining separate versions for each operating system, developers can create a single application that works efficiently across multiple platforms. This not only reduces development and maintenance costs, but also ensures a consistent and consistent user experience. Thus, users can enjoy the same functionalities and features, regardless of the device they use [7]. However, this approach also presents significant challenges. The need to adapt an application for different technical environments can result in compromises in terms of performance and functionality. Native apps, developed specifically for a particular operating system, typically offer superior performance and deeper integration with device features. On the other hand, cross-platform apps must balance these differences, which can sometimes result in a less optimized experience [8].

The mobile app market is highly competitive, and the quality of the user experience can be the deciding factor in the success or failure of an app. Users expect applications to be fast, intuitive and error-free. A failure in any of these aspects can lead to user dissatisfaction and, ultimately, uninstallation of the application. Therefore, cross-platform app developers must invest in extensive testing and constant optimization to ensure their products meet user expectations. Despite these challenges, cross-platform mobile apps have proven to be a powerful tool for innovation and digital transformation. In sectors such as education, health and commerce, these applications have facilitated access to services and resources that were previously inaccessible for many people. For example, in education, cross-platform applications allow students to access study materials, complete assignments, and participate in virtual classes from any device, democratizing access to quality education [9].

In the healthcare sector, these applications have enabled medical professionals to manage patient records, conduct virtual consultations, and monitor patients' health remotely. This not only improves the efficiency and quality of healthcare, but also facilitates access to healthcare services for people living in rural areas or with limited mobility. The ability to access these services from any device significantly increases equity in access to healthcare [10].

Commerce has also benefited greatly from cross-platform mobile applications. Consumers can shop, compare prices and access deals from their mobile devices, facilitating a more convenient and personalized shopping experience. Businesses, for their part, can reach a broader audience and manage their operations more efficiently. This has led to increased competition and the need to constantly innovate to attract and retain customers. However, the success of a cross-platform mobile app is not only measured by its reach and functionality, but also by its ability to adapt to changing user needs [11]. In a constantly evolving technological environment, developers must be

willing to continually update and improve their applications. This involves not only fixing bugs and adding new features, but also anticipating trends and adapting to new technologies.

Cross-platform mobile applications represent a significant innovation in the software industry, with the potential to transform entire sectors and improve people's lives. However, their development and maintenance present unique challenges that require constant investment in quality and optimization. By balancing these challenges with the opportunities, they offer, developers can create applications that not only meet user needs but also drive innovation and digital transformation. The future of mobile apps is undoubtedly promising, and their impact will continue to expand as technology advances.

2.2. SWOT Analysis.

SWOT analysis is an essential technique for evaluating the status of a company or project, examining both its internal characteristics (Weaknesses and Strengths) and its external environment (Threats and Opportunities) in a structured matrix. This process is broken down into four phases: analysis external, internal analysis, creation of the SWOT matrix and determination of the strategy to follow. The survival and prosperity of the organization are deeply linked to the environment that surrounds it, which presents both opportunities and threats. These are the key components of the external analysis. Simultaneously, the internal factors of the organization, such as its weaknesses and strengths, depend directly on its internal management [12].

Each of these four aspects can be classified as positive, driving the development of the organization, or negative, representing obstacles that impede said development - Opportunities are positive factors in the environment that, once identified, can be used to promote the growth of the organization The organization or project. On the contrary, threats are negative external influences that must be addressed with tactics and strategies to overcome them Internally, weaknesses are negative elements that need to be overcome through proper management, while strengths are positive aspects that must be exploited and enhanced. The SWOT analysis identifies strengths and weaknesses in areas such as the availability of capital resources, personnel, assets, product quality, internal and market structure, and consumer perception. The results of this analysis are placed in a matrix and are evaluated by experts, whose combined assessment offers a clear vision of the most promising strategies and tactics for the organization or project [13].

2.3. PEST Analysis.

The PEST analysis examines the external factors that influence a company, covering Political, Economic, Social and Technological components. This analysis allows us to understand how legislative regulations, economic conditions, sociocultural trends and technological advances impact the organization. For example, political factors include environmental protection laws, antitrust regulations and government stability, while economic factors encompass all variables that affect the market Sociocultural aspects refer to the configuration and behavior of consumers, and technological factors consider the development and adoption of new technologies [14]. The PEST-SWOT methodology is developed in two main stages. First, an exhaustive analysis of external factors is carried out from political, economic, social and technological perspectives In the second stage, the principles of SWOT analysis are applied to evaluate the internal characteristics of the company - Combining both approaches, a comprehensive and detailed vision of the business situation is obtained, identifying external opportunities and threats, as well as internal strengths and weaknesses, which facilitates the formulation of more effective and holistic strategies for the development and sustainability of the company [15].

2.4. PEST Analysis.

Unlike traditional PEST-SWOT methods, in this work the evaluations are carried out based on Triangular Neutrosophic Numbers of Single Value. Below are the fundamental explanations on this topic.

Definition 1 ([17]) : The neutrosophic set NS is characterized by three membership functions, which are the truth membership function T_A, the indeterminacy membership functionI_A and membership function to falsehood F_A, where Uis the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]\bar{a}0, 1^+[$, and $\bar{a}0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

See that by definition, $T_A(x), I_A(x)$ and $F_A(x)$ are standard or non-standard real subsets of $]\bar{a}0, 1^+[$ and, therefore, $T_A(x), I_A(x)$and $F_A(x)$can be subintervals of $[0, 1]$. $\bar{\iota}0$and1^+ They belong to the set of hyperreal numbers.

Definition 2 ([17]): The single-valued neutrosophic set $F_A: U \to [0, 1]$(SVN N) A is U, $T_A: U \to [0, 1]$ where $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$and $I_A: U \to [0, 1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The single-valued neutrosophic number (SVN N) is symbolized by

$N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$and $0 \leq t + i + f \leq 3$.

Definition 3 ([17]): The single- $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$valued triangular neutrosophic number, is a neutrosophic set in $\mathbb{R}$, whose membership functions of truth, indeterminacy and falsity are defined as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}, x=a_2} \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), a_2 < x \leq a_3 \\ 0, \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \leq x \leq a_2 \\ \beta_{\tilde{a},}\, x = a_2 \\ \frac{(x-a_2+\beta_{\tilde{a}}(a_3-x))}{a_3-a_2},\ a_2 < x \leq a_3 \\ 1, \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\gamma_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \leq x \leq a_2 \\ \gamma_{\tilde{a},} x = a_2 \\ \frac{(x-a_2+\gamma_{\tilde{a}}(a_3-x))}{a_3-a_2},\ a_2 < x \leq a_3 \\ 1, \text{otherwise} \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0, 1]$,$a_1,\ a_2, a_3 \in \mathbb{R}$ and $a_1 \leq a_2 \leq a_3$.

Definition 4 ([17]) : Given$\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and$\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$ two triangular neutrosophic numbers of a single value andλ any non-zero number on the real line . Then, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,

2. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,

3. Investment: $\tilde{a}^{-1} = \langle ({a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3 \neq 0$.

4. Multiplication by a scalar number:

$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda < 0 \end{cases}$$

5. Division of two triangular neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

6. Multiplication of two triangular neutrosophic numbers:

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle,\ a_3 > 0 \text{ and } b_3 > 0 \\ \langle (a_1 b_3, a_2 b_2, a_3 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle (a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

Where,$\wedge$ It's a norm$\vee$ It is a conorm t .

3. Results and discussion

The study is carried out on the factors that impact the development and implementation of a multiplatform mobile application in the gastronomic sector. For this purpose, experts on the subject and specialized literature were consulted. In this way the following factors were identified:

1. **Technological Compatibility:** Ensure that the application works smoothly on different mobile operating systems such as iOS and Android.
2. **User Experience (UX):** Design an intuitive and easy-to-use interface that improves the user experience when interacting with the application.
3. **Data Security:** Implement robust security and privacy measures to protect users' personal and financial information.
4. **Development Cost:** Evaluate the budget necessary to develop and maintain the application, considering the costs associated with different platforms and updates.
5. **Integration with Existing Systems:** Ensure that the application can be effectively integrated with existing management and operational systems in restaurants and gastronomic establishments.
6. **Speed and Performance:** Optimize application performance to ensure fast loading times and smooth response to user interactions.
7. **Marketing and Promotion:** Plan effective strategies to promote the application and attract potential users, using digital marketing and public relations techniques.
8. **Feedback and Continuous Improvement:** Establish mechanisms to receive feedback from users and use this data to continually improve the application and its functionalities.
9. **Regulatory Compliance:** Comply with local and international regulations and standards related to data protection, electronic commerce and computer security.

Considering these factors during the development and implementation of a multiplatform mobile application in the gastronomic sector can help maximize its effectiveness and ensure its acceptance by the users and establishments involved.

The construction of a comprehensive model on the impact of the development and implementation of a multiplatform mobile application in the gastronomic sector may face several obstacles that require attention and consideration – The main obstacles to include are:

1. **Technological Differences:** Adapting the application to work optimally on different mobile operating systems, such as iOS and Android, can be complex due to variations in development technologies and standards.
2. **High Costs:** Cross-platform application development can be more expensive initially due to the need for compatibility with multiple operating systems and devices.
3. **Performance Optimization:** Achieving optimal performance on all platforms can be a challenge, as native applications often offer better performance than cross-platform ones in certain cases.
4. **Uneven User Experience:** Maintaining a consistent and satisfying user experience across different de-vices and operating systems may require additional design and development effort.
5. **Security and Privacy:** Ensuring the security of user data across platforms can be complicated due to differences in security policies across operating systems and mobile devices.
6. **Complex Integration:** Integrating the application with existing management systems in restaurants and gastronomic establishments can be challenging due to differences in technological infrastructure and operational processes.
7. **Updates and Maintenance:** Keeping the application updated and compatible with new versions of operating systems and mobile devices can require significant resources and constant attention.
8. **Market Adoption:** Convincing users and dining establishments to adopt and use the application can be difficult due to competition in the application market and established consumer preferences.

Overcoming these obstacles requires a multidisciplinary approach that includes diverse perspectives, fosters collaboration among multiple actors, and employs comprehensive data collection along with meticulous analysis of the complexity and uncertainty inherent in socioeconomic impact assessment. Based on the PEST analysis, we can categorize the mentioned factors as threats and opportunities in relation to the four components of this analysis.

Threats

1.1. Political

- T 1: Changes in data protection and privacy regulations.
- T 2: Political instability and legislative changes in technological regulations.

1.2. Economic.
- T 3: Fluctuations in application development and maintenance costs.
- T 4: Increase in digital marketing and app promotion costs.

1.3. Social
- T 5: Resistance of some users to adopt new technologies in gastronomy.
- T 6: Impact on local culinary culture and the acceptance of new digital platforms.

1.4. Technological
- T 7: Limitations in interoperability between mobile operating systems.
- T 8: Shortage of experts in mobile application development for multiple platforms.

Opportunities

2.1. Political
- O 1: Government support for digitalization initiatives in the gastronomic sector.

2.2. Economic
- O 2: Creation of new business models and markets through the mobile application.
- O 3: Increase in the competitiveness of the gastronomic sector through technological innovations.

2.3. Social
- O 4: Generation of local employment in urban and rural areas through the digital economy.
- O 5: Contribution to accessibility and culinary diversity for local and international consumers.

2.4. Technological
- O 6: Advances in geolocation technologies and personalization of gastronomic experiences.
- O 7: Opportunities for research and development of new functionalities and continuous improvements.

Weaknesses
- W 1: Shortage of UX/UI specialists to design intuitive and attractive interfaces.
- W 2: Complexity in integration with existing management systems in restaurants and gastronomic establishments.

Strengths
- S 1: Taking advantage of the growing popularity of food delivery and gastronomic tourism.
- S 2: Potential to improve operational efficiency and order management in gastronomic establishments.
- S 3: Improvement in customer experience and loyalty through loyalty programs and personalized recommendations.

A team made up of eleven experts was in charge of analyzing various combinations between an external and an internal factor . Each of them were asked to carry out evaluations using the linguistic terms detailed in Table 1.

Table 1. Linguistic terms for evaluations and their associated SVTNNs . See [14-17] .

Linguistic Terms	SVTNN
Very low (VL)	$\langle(0,0,1);0\ .00,1\ .00,1\ .00\rangle$
Low (L)	$\langle(0,1,3);0\ .17,0\ .85,0\ .83\rangle$
Medium Low (MDL)	$\langle(1,3,5);0\ .33,0\ .75,0\ .67\rangle$
Medium (M)	$\langle(3,5,7);0\ .50,0\ .50,0\ .50\rangle$
Medium High (MDH)	$\langle(5,7,9);0\ .67,0\ .25,0\ .33\rangle$
Height (H)	$\langle(7,9,10);0\ .83,0\ .15,0\ .17\rangle$
Very high (VH)	$\langle(9,10,10);0\ .00,1\ .00,1\ .00\rangle$

Specifically, there are the following sets:

$W = \{W_1, W_2\}$ denotes the set of Weaknesses,

$S = \{S_1, S_2, S_3\}$denotes the set of Strengths,

$T = \{T_1, T_2, T_3, T_4, T_5, T_6, T_7\}$denotes the set of Threats,

$O = \{O_1, O_2, O_3, O_4, O_5\}$denotes the set of Opportunities .

The steps are the following:

1. Each expert was asked to evaluate the possible combinations between the elements of SO, ST, WO and WT. This evaluation is carried out in terms of how the development and implementation of a multi-platform mobile application in the gastronomic sector would have a socioeconomic impact.
2. Linguistic terms are replaced by the equivalent single-valued triangular neutrosophic numbers (SVTNN) in Table 4.
3. A single SVTNN is obtained by calculating the median of the SVTNNs of all experts for each pair of items.
4. The arithmetic mean of the SVTNN is calculated for each quadrant SO, ST, WO and WT.
5. The final result of each quadrant is converted to a crisp value using precision Equation 4. This converts them into values on a numerical scale out of 10 that allows the results to be compared.

$$A(\tilde{a}) = \frac{\cdot}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \quad (4)$$

Tables 1, 2, 3 and 5 summarize the results obtained after applying the previous steps .

Table 2. Calculation results for the SW quadrant. The medians of all experts are shown.

		Opportunities				
		O_1	O_2	O_3	O_4	O_5
Strengths	S_1	H	V . H .	H	H	V . H .
	S_2	V . H .	H	V . H .	V . H .	H
	S_3	H	MDH	H	H	V . H .

Table 3. Calculation results for the ST quadrant . The medians of all experts are shown.

		Threats						
		T_1	T_2	T_3	T_4	T_5	T_6	T_7
Strengths	S_1	MDH	H	MDH	MDH	H	H	V . H .
	S_2	H	VVH	H	H	V . H .	V . H .	H
	S_3	V . H .	MDH	H	V . H .	V . H .	MDH	V . H .

Table 4. Calculation results for the WO quadrant. The medians of all experts are shown.

		Opportunities				
		O_1	O_2	O_3	O_4	O_5
Weaknesses	w_1	MDH	MDH	MDH	MDH	MDH
	w_2	MDH	MDH	MDH	MDH	MDH

Table 5. Calculation results for the WT quadrant. The medians of all experts are shown.

		Threats						
		T_1	T_2	T_3	T_4	T_5	T_6	T_7
Weaknesses	w_1	V . H .	H	H	V . H .	H	V . H .	H
	w_2	MDH	MDH	MDH	MDH	MDH	MDH	MDH

From Tables 1 to 5, we have the following results:

- Potentials (Opportunities+Strengths): $\langle((7.6667, 9.2667, 9.9333); 0.67, 0.25, 0.33\rangle$,
- Risks (Strengths+Threats): $\langle(5.5190, 6.5714, 9.7519); 0.57, 0.25, 0.33\rangle$,
- Challenges (Weaknesses+Opportunities) $\langle(6, 7, 9); 0.57, 0.35, 0.23\rangle$:
- Limitations (Weaknesses+Threats): $\langle(6.0, 7.0, 6.5); 0.60, 0.40, 0.50\rangle$.

As a last step, these values are converted into a crisp scale with a maximum of 10 using Equation 4 . From here we have the following results:

1. Potentials (Opportunities+Strengths): 9. 6563 .
2. Risks (Strengths + Threats): 6. 4657 .
3. Challenges (Weaknesses + Opportunities): 7 .8241 .
4. Limitations (Weaknesses + Threats): 6 .7652 .

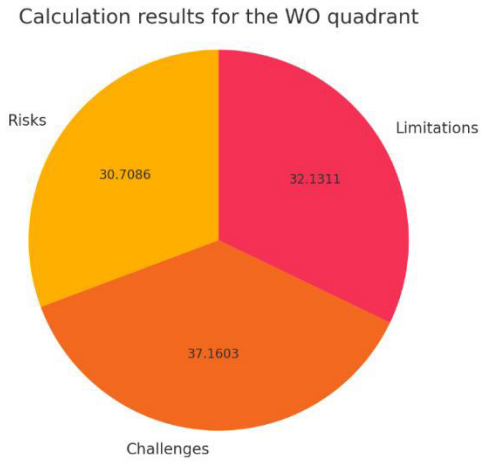

Figure 1. Calculation results for the WO quadrant.

After analyzing the SWOT analysis and the neutrosophic values associated with the development and implementation of a multiplatform mobile application in the gastronomic sector, several significant conclusions can be drawn that impact both the opportunities and challenges faced by this technological initiative. The political, economic, social and technological threats identified reveal a complex panorama for the integration of a mobile application in gastronomy. Regulatory changes in data protection (T1) and political instability (T2) suggest potential risks in terms of legal certainty and adaptability to future regulations. Economically, fluctuations in development costs (T3) and increased digital marketing (T4) can affect the long-term financial viability of the project.

Socially, the resistance of some users (T5) to the adoption of new technologies and the impact on local culinary culture (T6) pose challenges in the acceptance and integration of the application in the market. Technologically, limitations in interoperability (T7) and a shortage of specialized talent (T8) are critical obstacles that could compromise the efficiency and operational effectiveness of the system. On the other hand, the opportunities and strengths identified indicate significant potential for the development and expansion of the gastronomic sector through technological innovations. Government support (O1) and the creation of new business models (O2) represent strategic advantages that can drive mobile application adoption and scalability. Socially, the generation of local employment (O4) and the improvement in culinary accessibility (O5) promote inclusion and diversification of the market.

Technologically, advances in geolocation and personalization (O6) offer opportunities to improve the user experience and differentiate the gastronomic offer in a competitive market. Strengths such as leveraging growth in food delivery and gastronomic tourism (S1), improving operational efficiency (S2), and building customer loyalty through loyalty programs (S3) reinforce the project's strategic position.

The neutrosophic analysis provides a balanced perspective by considering both the risks and opportunities inherent in mobile application development in the gastronomic sector. The potentials (9.6563) indicate a favorable balance between opportunities and strengths, suggesting an environment conducive to innovation and growth . However, risks (6.4657) and challenges (7.8241), which combine strengths and threats with weaknesses and opportunities respectively, highlight the need to mitigate risks and make the most of the identified opportunities. Limitations (6.7652), which group weaknesses and threats, highlight critical areas where attention and specific strategies are required to overcome obstacles. It is essential to adopt a strategic approach that capitalizes on identified strengths and opportunities, while proactively managing risks and overcoming technological and operational challenges. The success of the development and implementation of a multiplatform mobile application in the gastronomic sector depends on a careful evaluation and management of the factors identified in the SWOT analysis. Effective technology integration, agile response to regulatory changes and creating tangible value for users and

gastronomic businesses will be crucial to achieving and maintaining a sustainable competitive advantage in today's digital market.

Machine learning has been increasingly recognized as a powerful tool for enhancing SWOT (Strengths, Weaknesses, Opportunities, and Threats) analysis, facilitating more accurate and data-driven decision-making processes. For instance, Abdel-Basset et al. [17] introduced a novel group decision-making model utilizing triangular neutrosophic numbers, showcasing the integration of advanced computational techniques in strategic analysis. Furthermore, recent advancements in sustainable machine intelligence, as demonstrated by Abd El-Khalik [18, 19], highlight the potential of machine learning in improving predictive models for various applications, including thermal comfort prediction in built environments). These studies underscore the transformative impact of machine learning on traditional analytical frameworks, suggesting its viability for enhancing SWOT analysis methodologies.

4. Conclusion

After a careful analysis of the SWOT analysis and the neutrosophic values associated with the development and implementation of a multi-platform mobile application in the gastronomic sector, fundamental conclu-sions emerge that outline both the promising opportunities and the significant challenges that this technological project faces. The threats identified in the political, economic, social and technological spheres reveal a complex panorama that could hinder the effective integration of the application in the gastronomic industry. Aspects such as regulatory changes in data protection and political instability pose considerable risks in terms of legal certainty and adaptability to future regulations, thus affecting the long-term operational stability of the project. On the economic front, fluctuations in development costs and the rise of digital marketing present additional challenges that could compromise the financial viability of the project. Socially, the resistance of some users to adopting new technologies and the potential impact on local culinary culture underscore the importance of marketing and educational strategies to overcome these barriers to acceptance. Technologically, limitations in interoperability between systems and a shortage of specialized talent represent critical obstacles that require innovative and collaborative solutions.

On the other hand, the identified opportunities and strengths indicate significant potential to positively transform the gastronomic sector through technological innovations. Government support and the creation of new business models represent strategic advantages that can catalyze the adoption and scalability of the mobile application in the market. Socially, the generation of local employment and the improvement in culinary accessibility promote the inclusion and diversification of the market, while technologically, advances in geolocation and personalization offer new ways to improve the user experience and differentiate the gastronomic offer in a competitive environment.

The neutrosophic analysis sheds light on the delicate balance between risks and opportunities inherent in mobile application development in the gastronomic sector. Although the potentials indicate a positive balance between opportunities and strengths, it is crucial to proactively address the identified risks and challenges. Risks combined with strengths and threats with opportunities outline critical areas where meticulous and strategic management is required. Overcoming technological and operational limitations, as well as capitalizing on identified strengths, will be crucial to ensure the sustainable success of the project. In summary, the success of the development and implementation of a multiplatform mobile application in the gastronomic sector will depend on the ability to effectively evaluate and manage the factors identified in the SWOT analysis. Effective integration of technology, adaptive response to regulatory changes, and creating tangible value for both users and food businesses are imperative to maintaining a competitive advantage in today's dynamic digital marketplace.

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Exploring the Impact of Performance Audits on the Management of Public Organizations Through the Analysis of Plithogenic Statistics

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Abstract. Exploring the impact of performance audits on public company management becomes a crucial field of study, highlighting how these critical assessments not only reveal operational effectiveness but also shape strategic and policy decisions. Plithogenic statistics analysis, in particular, emerges as an innovative approach that goes beyond traditional methods, introducing the inherent complexity of multiple interdependent variables and their dynamic effects on organizational outcomes. This statistical framework not only captures the inherent fluctuations in the data, but also unravels the root causes of varying performances, providing deep insights that challenge static perceptions of public administration. From a practical perspective, plithogenic analysis not only quantifies current performance, but also anticipates future trends, equipping managers with powerful tools to adjust strategies and policies more precisely. By considering the complex interaction between multiple factors, from resource management to operational efficiency, this statistical approach allows for a more holistic and nuanced assessment of the impacts of performance audits. Thus, a dynamic landscape is revealed where each piece of data reflects not only superficial results, but also the hidden connections that define the effectiveness and long-term sustainability of modern public companies.

Keywords: Public Companies, Plithogenic Probability, Plithogenic Statistics, Multivariate Statistics, Plitogenicity, Neutrosophic Number.

1 Introduction

Exploring the impact of performance audits on the administration of public companies through the analysis of plithogenic statistics represents a significant challenge and opportunity in the field of organizational management [1]. Performance audits, fundamental to evaluating the efficiency and effectiveness of government operations, not only seek to meet standards of transparency and accountability, but also play a crucial role in policy formulation and strategic decision making. In this context, the plithogenic approach emerges as an advanced methodology that allows capturing the inherent complexity of multiple variables that impact organizational performance [2].

Public companies face a dynamic and often complex environment, where operational effectiveness and efficient resource management are imperative to meet the expectations of citizens and stakeholders. Performance audits, by critically evaluating every aspect of the operation of these entities, provide a window into continuous improvement and process optimization. However, the simple act of evaluating numbers and figures is not enough; It is crucial to understand the interrelationships and synergistic effects between various areas and internal policies that affect overall performance [3]. Plithogenic analysis, by introducing a statistical framework that embraces the complexity and dynamism of interdependent variables, allows for a deeper and more precise evaluation of the impact of these audits. Rather than simply measuring performance in absolute terms, this approach examines how factors such as resource allocation, the effectiveness of regulatory policies, and organizational responsiveness interact to influence observed outcomes. This involves not only a retrospective look at the past, but also a forward-looking perspective that can inform future strategies and policy decisions [4].

Additionally, plithogenic analysis is notable for its ability to identify hidden patterns and causal relationships that may not be evident using traditional statistical analysis methods. By modeling the inherent uncertainty and variability in the data, this statistical approach offers a truer representation of the complex reality in which public

companies operate. This not only strengthens the evidence base for informed decision-making, but also provides valuable elements for the implementation of more effective and adaptive policies. In a world where the demand for transparency and efficiency in public management is growing, performance audits and plithogenic analysis emerge as indispensable tools. They not only help ensure that public resources are used effectively and efficiently, but also promote a culture of continuous improvement and accountability in government institutions. This study seeks to explore these issues from a multidimensional perspective, highlighting how the integration of advanced methodologies can transform the way performance is evaluated and managed in the public sector.

2 Related Words.

2. 1 Performance Audits.

Performance audits are a fundamental tool in business and government management, intended to evaluate the efficiency and effectiveness of the operations and policies implemented in an organization. This process is not limited to simply reviewing figures and regulatory compliance, but goes further, seeking to identify areas of improvement and opportunities to optimize resources and processes. In the business context, performance audits allow organizations to ensure that their strategies and operations are aligned with established objectives, providing a clear and objective view on the current state and future projections of the company [5]. One of the key aspects of performance audits is their ability to provide a holistic assessment of management and organizational functioning. This involves not only reviewing financial performance, but also analyzing the effectiveness of internal processes, the quality of the products or services offered, and the satisfaction of customers or users. Through this comprehensive assessment, companies can identify areas of inefficiency or risk, as well as opportunities to implement strategic changes that drive long-term competitiveness and sustainability.

In government, performance audits play a crucial role in accountability and transparency. By evaluating how public resources are used and how established objectives are met, these audits provide a solid basis for making informed and responsible policy decisions. In addition, they contribute to strengthening public trust in government institutions by demonstrating an effective commitment to efficiency and responsible management of public resources. Importantly, performance audits not only focus on the past and retrospective evaluation, but also have a prospective focus [6]. This means that not only are past results reviewed, but potential future challenges and opportunities are also anticipated. This future-oriented perspective allows organizations and government entities to better pre-pare to face changes and adapt quickly to new economic, political or social conditions.

However, performance audits face certain challenges and criticisms. Among them, the complexity in data collection and analysis, as well as the appropriate interpretation of the results obtained. Additionally, the effectiveness of audits can be compromised by factors such as resistance to organization-al change or lack of internal resources and capabilities to implement improvement recommendations. These challenges underline the importance of having appropriate methodologies and tools, as well as a strong commitment from senior management or policy makers, to ensure that performance audits are truly effective and generate added value [7].

Another crucial aspect to consider is the need to adapt performance audits to the specific characteristics of each organization or government entity. Not all companies or public institutions face the same challenges or have the same objectives, so it is essential to design personalized audits that adjust to the particular needs and realities of each case. This involves not only selecting the appropriate metrics and indicators, but also ensuring that the audit process is transparent, objective and participatory, in-volving all relevant stakeholders in the evaluation and continuous improvement process. Performance audits represent an essential tool for both private companies and government entities, providing a critical and systematic evaluation of organizational performance. Through this process, organizations can identify areas of strength and weakness, as well as opportunities to improve operational efficiency and strategic resource management [8]. However, to maximize the impact of performance audits, it is crucial to address the associated challenges and criticisms, adapting approaches and methodologies according to the specific needs of each organizational or government context. In this way, performance audits can not only fulfill their function of accountability and transparency, but al-so actively contribute to continuous improvement and the achievement of long-term organizational objectives [9].

2.2 Plithogenic Statistics.

To address the topic of Plithogenic Statistics (PS), it is crucial to understand its multidimensional nature and its application in various fields of research. Plithogenic Statistics is emerging as an innovative approach that seeks to capture the inherent complexity and interrelationships between variables in complex data sets. Unlike conventional statistical methods that focus on linearity and normality of distributions, PS incorporate the notion of plitogenicity, which reflects the diversity and interdependence between the elements analyzed [10]. In essence, PS allow us to model phenomena where the interactions between variables are significant and cannot be easily simplified into linear relationships. This approach is particularly relevant in disciplines such as evolutionary biology, complex economics, and dynamic sociology, where the systems studied exhibit nonlinear and emergent

behaviors. When considering Plithogenic Statistics, the need for analytical methods that can capture the emergence of systemic properties from the interaction between multiple factors is recognized, thus avoiding the oversimplification that limits the deep understanding of complex phenomena [11].

From a methodological perspective, Plithogenic Statistics is distinguished by its ability to handle large and heterogeneous data sets, where the relationships between variables can be non-linear and non-stationary. This involves the use of advanced techniques such as deep neural networks, complex net-work analysis and unsupervised machine learning methods [12]. The practical application of PS can transform the way we interpret and model complex systems, offering arguments that go beyond the limitations of traditional approaches based on linear models and Gaussian distributions. In the context of contemporary scientific research, Plithogenic Statistics represents a bridge between theory and observed reality, facilitating the exploration of phenomena that challenge conventional statistical simplifications. This approach allows for the capture of heterogeneity, nonlinearity, and temporal dynamics in empirical data, thereby fostering a deeper and more nuanced understanding of the complexity inherent in natural and social systems [13].

However, it is important to highlight the challenges associated with the implementation of Plithogenic Statistics. Interpretation of results can be complex due to the inherently nonlinear nature of the modeled relationships. Furthermore, the appropriate choice of techniques and validation of models require a deep understanding of the specific context of the problem under investigation, as well as careful management of biases and underlying assumptions. In terms of potential impact, PSs offer new perspectives for addressing complex problems ranging from predicting economic trends to understanding evolutionary dynamics in biological systems. By integrating plithogenic concepts into statistical practice, it opens the door to a more robust and true-to-life analysis, capable of revealing hidden patterns and subtle connections that could be overlooked with more traditional approaches [14, 15].

Plithogenic Statistics represents a significant evolution in the field of statistical analysis, promoting a more inclusive and mathematically rigorous paradigm for studying complex phenomena. As we move toward a deeper understanding of dynamic and adaptive systems, PSs offer a powerful tool to explore and model the true complexity of the natural and social world, overcoming the limitations of conventional statistical methods and opening new frontiers for research. interdisciplinary and scientific innovation.

There are several subclasses of Plithogenic Statistics which are shown[15]:

- Multivariate statistics,
- Neutrosophic Plithogenic Statistics,
- Plithogenic indeterminate statistics,
- Plithogenic intuitionistic fuzzy statistics,
- Fuzzy statistics of plithogenic images,
- Plithogenic spherical fuzzy statistics,
- and in general: Plithogenic statistics

In a neutrosophic population, each element has a triple probability of affiliation (T_j, I_j, F_j), where$T_j, I_j, F_j \in [0,1]$ similar to that $0 \leq T_j + I_j + F_j \leq 3$.

If we assume that we must have the data set $(T_j, I_j, F_j)$for $j = 1, 2, \dots, n$, where n is the sample size, then the average probability of all the sample data is calculated using Equation 1.

$$\frac{1}{n}\sum_{j=1}^{n}(T_j, I_j, F_j) = \left(\frac{\sum_{j=1}^{n} T_j}{n}, \frac{\sum_{j=1}^{n} I_j}{n}, \frac{\sum_{j=1}^{n} F_j}{n}\right) \quad (1)$$

In this investigation, we also consider some operations in the form of *neutrosophic numbers.* These ways of representing indeterminacy, under certain conditions, are equivalent to working with intervals.

Definition 1: ([16,17]) A *neutrosophic number* N is defined as a number as follows:

$$N = d + I \quad (2)$$

Where d is called *the determinate part* and I is called *the indeterminate part.*

Furthermore, the arithmetic operations between intervals are important in this paper, which are summarized below ([18,19, 20]):

Given$I_1 = [a_1, b_1]$ and$I_2 = [a_2, b_2]$ we have the following operations between them:

$I_1 \leq I_2$ If and only if$a_1 \leq a_2$ and $b_1 \leq b_2$.

$I_1 + I_2 = [a_1 + a_2, b_1 + b_2]$ (Addition) ;

$I_1 - I_2 = [a_1 - b_2, b_1 - a_2]$ (Subtraction),

$I_1 \cdot I_2 = [\min\{a_1 \cdot b_1, a_1 \cdot b_2, a_2 \cdot b_1, a_2 \cdot b_2\}, \max\{a_1 \cdot b_1, a_1 \cdot b_2, a_2 \cdot b_1, a_2 \cdot b_2\}]$ (Product),

$I_1/I_2 = I_1 \cdot (1/I_2) = \{a/b: a \in I_1, b \in I_2\}$, always that $0 \notin I_2$(Division).

3 Results and Discussion.

The research focused on a population of 200 accountants and directors of companies that were audited, as well as experienced auditors. Using non-probability sampling, it was applied at the discretion of the researcher. For data collection, the survey was used as a quantitative research method, and the data was collected using a previously prepared questionnaire. This questionnaire, developed according to the objectives and dimensions of the dependent variable, contains approximately 20 items. It was administered to both the control and experimental groups, before and after the interventions. The questionnaire was structured as follows:

1. **Efficiency (5):** This dimension focuses on evaluating how the resources available in the organization are used to achieve established objectives. Process productivity, cost optimization and the elimination of unnecessary activities that may affect overall efficiency are analyzed.
2. **Process effectiveness (5):** This dimension focuses on the ability of organizational processes to meet quality standards and achieve expected results. The quality of the final product or service delivered, customer satisfaction and alignment with regulatory and regulatory requirements is evaluated.
3. **impact (5):** This dimension evaluates how operational activities and decisions affect the organization's long-term strategic objectives. The contribution to the achievement of the organizational mission and vision is analyzed, as well as the alignment with the global strategy and the adaptability to changes in the external environment.
4. **Compliance and responsibility (5):** This dimension refers to the organization's compliance with legal, ethical and social responsibility standards. Compliance with internal and external regulations, transparency in financial and operational management, and accountability to stakeholders and the community in general are evaluated.

Auditors and economists were evaluated considering their accumulated experience, and possible limitations they might face in understanding neutrosophic methods were taken into account. Therefore, they were asked to express their opinions using ranges of values rather than assigning a single number on a continuous scale ranging from 0 (Never) to 10 (Always). Each respondent defined their intervals as $I_i = [a_i^L, a_i^U$To ensure the validity of the instruments used to collect data, validation was carried out through the judgment of experts with doctorates. The reliability of these instruments was evaluated by analyzing Cronbach's Alpha coefficient, which confirmed that the instrument used is reliable. The last step of the process involved the administration of the survey to the members of the experimental group, with the collection of all the necessary data for subsequent analysis by the researchers. The detailed steps followed in this process are as follows:

1. Different variables are specified. for the dimensions to measure:

   $S = \{s_1, s_2, \ldots, s_{34}\}$denotes the set of economics and auditors of the study group.
   $\tilde{S} = \{\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_{34}\}$denotes the set of economics and auditors in the control group.
   $d = \{d_1, d_2, d_3, d_4\}$denotes the set of dimensions to be measured, such that:
   d_1: Symbolizes the dimension "**Operational efficiency"**,
   d_2: Symbolizes the dimension "Process **effectiveness"**,
   d_3: Symbolizes the "Strategic **Impact"** dimension,
   d_4: Symbolizes the "Compliance **and responsibility"** dimension.
   Each of these elements is a set of elements in itself, where:

   $d_1 = \{d_{11}, d_{12}, \ldots, d_{17}\}$is the set of elements of the first dimension (d_{1j}represents the 1st item Dimension),

   $d_2 = \{d_{21}, d_{22}, \ldots, d_{26}\}$is the set of elements of the second dimension (d_{2j}represents the 2nd item Dimension),

   $d_3 = \{d_{31}, d_{32}, \ldots, d_{37}\}$is the set of elements of the third dimension (d_{3j}represents the 3rd article Dimension),

   $d_4 = \{d_{41}, d_{42}, \ldots, d_{47}\}$It is the set of elements of the fourth dimension (d_{4j}represents the 4th Article Dimension).
   In this way, the evaluations for each item are represented by:

   $I_{ij}^K = \left[a_{ij}^{KL}, a_{ij}^{KU}\right]$, which is the evaluation of the ^{ith} economic in the target group for the k th item of the j th dimension.

The equivalent notation for the control group is $\tilde{I}_{ij}^{K} = \left[\tilde{a}_{ij}^{KL}, \tilde{a}_{ij}^{KU}\right]$.

2. The dimension scores were obtained for each respondent and each of the dimensions using the following expression:

$$D_{ji} = \sum_{k=1} I_{ijk} \quad (3)$$

D_{ji} is the score of a variable or dimension j for respondent i. This score is obtained by the arithmetic sum of all the k items of the variable or dimension j, answered by respondent i, using the sum of intervals.

Equivalently, we have the results for the control group:

$$\widetilde{D}_{ji} = \sum_{k=1} \tilde{I}_{ijk} \quad (4)$$

3. Since the dimensions and variables have different numbers of elements, the scores are transformed into a range from 0 to 100 using the following expression for the study group:

$$D_{ji}^{*} = \frac{D_{ji} - \text{min punt theoric } D_j}{\text{max punt theoric } D_j - \text{min punt theoric } D_j} * 100 \quad (5)$$

Where: D_{ji}^{*} is the transformed score for variable or dimension j of respondent i.

In the same way, we have Equation 6 for the control group.

$$\widetilde{D}_{ji}^{*} = \frac{\tilde{D}_{ji} - \text{min punt theoric } \tilde{D}_j}{\text{max punt theoric } \tilde{D}_j - \text{min punt theoric } \tilde{D}_j} * 100 \quad (6)$$

These transformations allow the scores of the variables or dimensions to have the same range of values despite their number of elements so that 0 represents the minimum level and 100 the maximum level. That is, these new scores are the proportions of the dimensions or value of the variable by the respondents.

$\overline{D}_j^{*}$ denotes the average of the results for the ^{jth} dimension for the study group and is calculated by the following formula:

$$\overline{D}_j^{*} = \frac{\sum_{i=1}^{34} D_{ji}^{*}}{34} \quad (7)$$

equivalently for the control group:

$$\overline{\widetilde{D}}_j^{*} = \frac{\sum_{i=1}^{34} \tilde{D}_{ji}^{*}}{34} \quad (8)$$

As the change occurs before and after passing the group study program, formula 9 is used:

$$\overline{\Delta}_j^{*} = \overline{D}_j^{*\text{after}} - \overline{D}_j^{*\text{before}} \quad (9)$$

Where $D_{ji\text{after}}^{*}$ denotes the scores of the study group after passing the program, while $\overline{D}_j^{*\text{before}}$ are the previous results.

While :

$$\overline{\widetilde{\Delta}}_j^{*} = \overline{D}_j^{*} - \overline{\widetilde{D}}_j^{*} \quad (10)$$

Denotes the difference between the average of the group to be studied with the control group.

Once the indices used to measure these results were defined, calculations were made that indicate the following, as can be seen in the following figures:

Figure 1 shows the percentages achieved in the interval for the **Operational Efficiency dimension**.

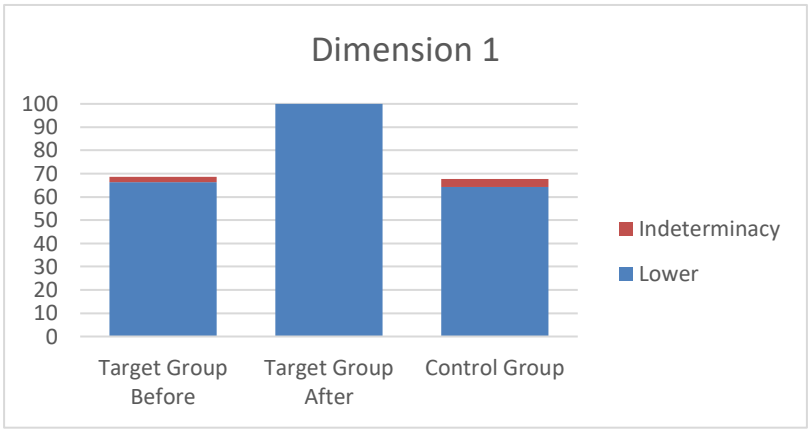

Figure 1. Results of the average of the target group before and after the performance audits and of the control group for Dimension 1. In blue is the certain percentage and in red is the indeterminate percentage.

Figure 2 is the result of Dimension "**Process effectiveness**".

Figure 2. Results of the average of the target group before and after the performance audits and of the control group for Dimension 2. In blue is the certain percentage and in red is the indeterminate percentage.

Figure 3 refers to the result of the Dimension: "**Strategic impact**".

Figure 3. Results of the average of the target group before and after the performance audits and of the control group for Dimension 3. In blue is the certain percentage and in red is the indeterminate percentage.

Figure 4 refers to the result of the Dimension: "**Compliance and responsibility**".

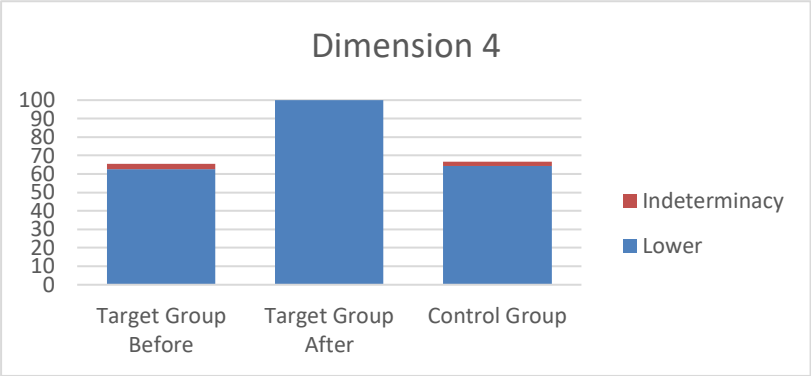

Figure 4. Results of the average of the target group before and after the performance audits and of the control group for Dimension 4. In blue is the certain percentage and in red is the indeterminate percentage.

Thus using the difference between intervals, we have:

- $\overline{\Delta}_1^* = [100,100] - [62.33,66.16] = [37.67,33.84]$,
- $\overline{\Delta}_2^* = [100,100] - [64.35,63.78] = [34.65,36.22]$,
- $\overline{\Delta}_3^* = [100,100] - [66.31,65.12] = [33.69,34.88]$,
- $\overline{\Delta}_4^* = [100,100] - [62.16,69.71] = [37.53,38.86]$.

On the other hand, the results for $\widetilde{\Delta}_j^*$ are as shown below:

- $\widetilde{\overline{\Delta}}_1^* = [100,100] - [61.16,70.71] = [38.84,29.29]$,
- $\widetilde{\overline{\Delta}}_2^* = [100,100] - [62.34,66.35] = [37.66,33.65]$,
- $\widetilde{\overline{\Delta}}_3^* = [100,100] - [62.33,64.12] = [37.67,35.88]$,
- $\widetilde{\overline{\Delta}}_4^* = [100,100] - [67.87,63.19] = [32.13,36.81]$.

As can be seen, the values always showed improvements of around 30% or more, both when the target group was compared with itself before and after the program, and when compared with the control group.

To obtain a result that encompasses all the dimensions in a single final value, formula 11 will be used:

$$\min([a_1,b_1],[a_2,b_2]) = [\min(a_1,a_2),\min(b_1,b_2)] \quad (11)$$

In this case,

$D^* = \min([62.33,66.16],[64.35,63.78],[66.31,65.12],[62.16,69.71]) = [62.16,63.78]$ It is the result of the target group before the educational reforms.

After passing the performance audits the overall result is [100,100]. For the control group this is $\widetilde{D}^* = \min([61.16,70.71],[62.34,66.35],[62.33,64.12],[67.87,63.19]) = [61.16,63.19]$

Finally, we obtained the result for the test of "the quality of management of public organizations", be-fore and after for the objective group and the control group. These are shown in Figure 5:

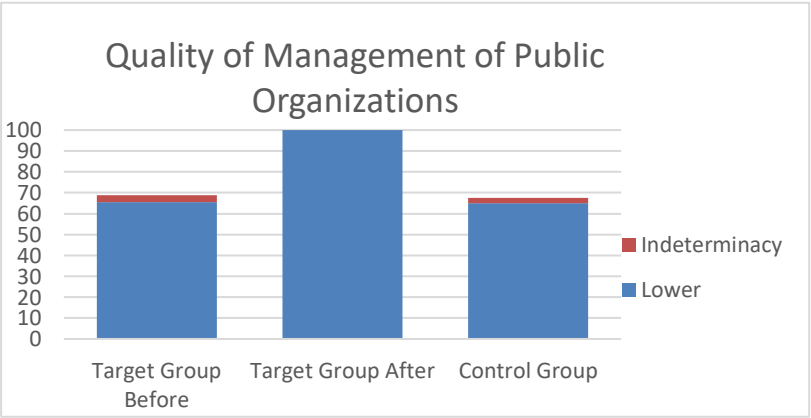

Figure 5. Average results of the target group before and after performance audits and of the control group for "the quality of management of public organizations". in blue is the certain percentage and in red is the indeterminate percentage.

In this case, we will calculate the difference in absolute value to avoid negative numbers in the calculation of the relationship between performance audits and the quality of management of public organizations. That is, equation 12 will be used.

$$[a_1, b_1] \ominus [a_2, b_2] = [\text{abs}(a_1 - b_2), \text{abs}(b_1 - a_2)] \quad (12)$$

In this case, it is:

$[62.16,63.78] \ominus [61.16,63.19] = [1.03, 2.62]$ which is the result of comparing "the quality of management of public organizations" with the aggregation of the four dimensions that represent "performance audits." This represents a difference of less than 5. 5% between both results.

On the other hand, $[100, 100] \ominus [100, 100] = [0, 0]$ for both variables after the program. This suggests a high and positive correlation between "performance audits" and "the quality of management of public organizations."

To address the evaluation of performance audits and their impact on the management of public organizations, it is essential to understand the critical dimensions involved in this process. Performance audits are structured around four fundamental dimensions: operational efficiency, process effective-ness, strategic impact, and compliance and accountability. Each of these dimensions plays a crucial role in the comprehensive evaluation of how resources are used, quality standards are achieved, strategic objectives are contributed, and legal and ethical regulations are met within a public organization. Operational efficiency, the first dimension considered, focuses on productivity and optimization of resources to achieve established objectives. It is essential to evaluate the elimination of redundant activities that may negatively affect the overall efficiency of the organization. This dimension not only seeks to reduce unnecessary costs, but also improve the effective use of available resources.

On the other hand, process effectiveness addresses the ability of organizational processes to meet quality standards and deliver satisfactory results. Here the quality of the final product or service is evaluated, as well as customer satisfaction and alignment with current regulations. This dimension is crucial to ensure that internal processes are aligned with the organization's strategic objectives and can adapt to changes in the external environment. Strategic impact, the third aspect evaluated, analyzes how operational decisions and actions affect the organization's long-term objectives. This involves evaluating the contribution to the achievement of the organizational mission and vision, as well as the ability to adapt to significant changes in the market or in the political and social context. Proper strategic alignment ensures that the organization can maintain its relevance and competitiveness in the long term.

Finally, compliance and responsibility refer to rigorous adherence to legal, ethical and social responsibility regulations. Transparency in financial and operational management is evaluated, as well as accountability to stakeholders and the community in general. This dimension not only ensures compliance with regulations, but also strengthens public trust in government management and promotes institutional integrity. In the context of the neutrosophic evaluation carried out, the management quality of public organizations was compared with the aggregation of these fourperformance audit dimensions. The results showed a minimum difference, less than 5. 5%, which suggests a significant and positive correlation between the effective implementation of performance audits and the improvement in the quality of management of public entities. This finding highlights the importance of using comprehensive and multidimensional approaches such as performance audits to improve efficiency, strategic effectiveness and regulatory compliance in the public sector.

Furthermore, a perfect correlation was observed between performance audits and management quality, indicating high consistency between the implementation of audit practices and the level of performance achieved by public organizations. This result reinforces the idea that performance audits are not only retrospective evaluation tools, but also strategic instruments to guide decision making and continually improve public management. However, it is crucial to recognize that the effective implementation of performance audits faces significant challenges, such as the need for adequate resources and specialized staff training. These obstacles can limit the ability of organizations to obtain full benefits from these evaluation practices. It is therefore imperative to invest in training and capacity development, as well as strengthening data infrastructures and information systems that support the effective implementation of performance audits. In conclusion, performance audits represent an essential tool to evaluate and improve the management of public organizations through key dimensions such as operational efficiency, process effectiveness, strategic impact and compliance and responsibility. The neutrosophic results obtained highlight a strong correlation between the effective implementation of these audits and the quality of public management, underlining the importance of adopting integrated and systematic approaches to strengthen governance and transparency in the public sector [21,22].

4 Conclusion

Audits are structured into four fundamental dimensions: operational efficiency, process effectiveness, strategic impact and compliance and responsibility. Each of these dimensions plays a crucial role in the comprehensive evaluation of how available resources are used, quality standards are achieved, strategic objectives are contributed to, and compliance with legal and ethical regulations is ensured within a public organization. Operational efficiency, as the first dimension, focuses on improving productivity and optimizing the use of resources to achieve organizational goals. This includes eliminating redundant activities that could negatively impact overall efficiency. It is not just about reducing costs, but about maximizing the effectiveness of available resources to improve the overall performance of the entity. On the other hand, process effectiveness evaluates the ability of organizational processes to meet quality standards and achieve satisfactory results. The quality of the final product or service delivered, customer satisfaction and alignment with current regulations is analyzed. It is essential that internal processes are strategically aligned and flexible enough to adapt to external changes.

Strategic impact, as a third crucial aspect, studies how operational decisions affect the organization's long-term objectives. This involves evaluating the contribution to the achievement of the organizational mission and vision, as well as the ability to adapt to significant changes in the political, social and economic environment. Effective strategic alignment ensures the organization's continued relevance and competitiveness in a dynamic environment. Finally, compliance and responsibility focus on rigorously adhering to legal regulations, ethics, and social responsibilities. This includes transparency in financial and operational management, as well as accountability to stakeholders and the community in general. Strengthening this dimension not only guarantees regulatory compliance, but also reinforces public trust in government administration and promotes institutional integrity. In the neutrosophic evaluation carried out, quality management in public organizations was compared with the integration of these four performance audit dimensions. The results showed a minimal difference, less than 5. 5%, indicating a positive and significant correlation between the effective implementation of performance audits and improvement in the management of public entities. This finding under-scores the importance of adopting comprehensive and multidimensional approaches such as performance audits to improve operational efficiency, strategic effectiveness and regulatory compliance in the public sector.

Furthermore, a perfect correlation was observed between performance audits and management quality, which reflects a high consistency between the implementation of audit practices and the level of performance achieved by public organizations. This result reinforces the idea that performance audits are not only retrospective evaluation tools, but also strategic instruments to guide decision-making and continually improve public management. However, it is crucial to recognize that effective implementation of performance audits faces significant challenges, such as appropriate resource allocation and specialized staff training. These obstacles can limit organizations' ability to take full advantage of these evaluation practices. Therefore, it is imperative to invest in training and capacity development, as well as strengthening data infrastructures and information systems that support effective implementation of performance audits. In conclusion, performance audits represent an essential tool to evaluate and improve the management of public organizations through key dimensions such as operational efficiency, process effectiveness, strategic impact and compliance and responsibility. The neutrosophic results obtained highlight a strong correlation between the effective implementation of these audits and the quality of public management, underscoring the importance of adopting comprehensive and systematic approaches to strengthen governance and transparency in the public sector.

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Use of Neutrosophic Cognitive Maps for the Analysis of Five Didactic Options for Instruction in the Numerical and Research Skills of Accounting Students

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Abstract. This article explores the innovative use of Neutrosophic Cognitive Maps to evaluate and compare five teaching approaches aimed at improving numerical and research skills among accounting students. This methodology, which integrates elements of neutrosophic theory, allows us to capture the inherent uncertainty and ambiguities in the perception and evaluation of educational options. Through a detailed and systematic analysis, the strengths and limitations of each approach are examined, highlighting how Neutrosophic Cognitive Maps offer a robust conceptual structure for discerning between the subtle complexities that affect effective learning. The results reveal profound elements about how different teaching strategies can influence the acquisition of key competencies in accounting. From the integration of advanced technologies to more traditional methods focused on conceptual development, each approach is evaluated not only for its surface effectiveness, but also for its ability to adapt to the cognitive and emotional diversity of students. This multidimensional approach underscores the importance of considering not only quantitative results, but also the underlying processes and subjective perceptions that shape the educational experience, offering a comprehensive framework for continually improving teaching in de-manding and dynamic academic environments.

Keywords: Didactic Approaches, Neutrosophic Cognitive Map, Neutrosophic Number, Neutrosophic Graph, Hidden Patterns.

1 Introduction

In the contemporary educational field, the design of effective teaching strategies represents a crucial challenge to improve learning and skill development among students, especially in disciplines such as accounting that require a deep mastery of numerical and research skills [1]. The evaluation of different pedagogical alternatives becomes essential not only to optimize the teaching-learning process, but also to adapt to the varied needs and profiles of students in diversified and dynamic academic environments. In this context, the emerging use of Neutrosophic Cognitive Maps represents an innovative and promising approach. These maps not only allow us to visualize and structure complex cognitive and affective interrelationships in the educational process, but also integrate principles of neutrosophic theory, which deals with the uncertainty, indeterminacy and vagueness inherent to human perceptions. This methodology, relatively new in educational research, offers a rich and nuanced conceptual framework to critically evaluate and compare different teaching strategies aimed at strengthening the essential skills required in the field of accounting [2]. The present study focuses specifically on the analysis of five teaching options designed to improve both numerical skills and research capabilities among accounting students. Through a meticulous and systematic approach, the strengths and weaknesses of each approach are explored, seeking to identify not only their superficial effectiveness, but also their ability to foster deep and sustainable learning in a constantly evolving educational context.

The choice of Neutrosophic Cognitive Maps as the main methodological tool is justified by its unique ability to model and represent the ambiguity and indeterminacy inherent in human perception [3]. This methodology not only facilitates the capture of multiple perspectives and divergent points of view on the effectiveness of the proposed educational strategies, but also offers a space for critical reflection on how these strategies can be adapted and optimized according to the specific needs of the students and the demands of the current educational

environment. As we move towards a more comprehensive and holistic approach in teaching complex competencies such as numerical and research skills in accounting, it is crucial to adopt methodologies that not only address the variety of learning styles and individual abilities, but also promote autonomy. intellectual and critical thinking. Neutrosophic Cognitive Maps offer precisely this opportunity by providing a flexible and adaptable framework that can be modulated according to the changing dynamics and demands of the contemporary educational context [4].

In this sense, this study not only seeks to contribute to the growing body of knowledge in the field of education and cognitive psychology, but also to provide practical guidelines and informed guidance for educators, curriculum designers and educational policy makers interested in improving the quality and the effectiveness of teaching in specialized disciplines such as accounting [5]. By embracing the complexity inherent in teaching numerical and inquiry skills, this study is positioned as a crucial starting point for future research and development in the field of innovative and adaptive pedagogy.

2 Related Words.

2. 1 Numerical Skills.

Developing numerical skills is essential in the current educational and professional landscape. These skills are not limited simply to the ability to handle numbers and perform basic arithmetic calculations, but encompass a complex set of competencies that allow one to interpret, analyze and use quantitative data effectively in various contexts. From solving everyday problems to making strategic decisions in companies and organizations, numerical skills are essential to face the challenges of the modern world. In a broader sense, numerical skills involve the ability to understand fundamental mathematical concepts such as algebra, geometry, and statistics [6]. These disciplines not only provide tools to perform accurate calculations, but also foster logical reasoning and the critical ability to evaluate quantitative information objectively. This capacity for critical analysis is crucial in fields as diverse as scientific research, financial planning and data management in the technological field. Furthermore, numerical skills are essential for developing a deep under-standing of complex and abstract phenomena. For example, in science and engineering, these skills allow us to model and simulate physical and natural systems, predicting outcomes and optimizing processes through the use of advanced mathematical tools. Likewise, in areas such as economics and business administration, numerical skills are essential for performing cost analysis, financial projections and risk assessments, providing a solid basis for making informed and strategic decisions [7].

From an educational perspective, the development of numerical skills not only implies the acquisition of technical knowledge, but also the ability to apply that knowledge in practical and real contexts. This requires innovative teaching methods that encourage active learning and discovery, allowing students to explore mathematical concepts through problem solving and experimentation. The integration of educational technologies and digital tools also plays a crucial role in facilitating interactive and adaptive learning, personalizing the learning experience according to individual needs and promoting a deeper and more meaningful understanding of numerical concepts [8]. However, despite the growing importance of numerical skills in the 21st century, significant challenges remain in their teaching and learning. One of these challenges lies in the negative perception that some students may have towards mathematics and related disciplines, which can generate emotional and psychological barriers that hinder the effective development of these skills. Addressing this gap requires inclusive pedagogical approaches that promote a positive and motivating learning culture, highlighting the practical relevance and real applications of numerical skills in daily life and in various professions. Additionally, the gap in numerical proficiency between different demographic and socioeconomic groups also represents a significant challenge to educational equity. It is crucial to implement educational policies and programs that ensure equal access to resources and learning opportunities in mathematics and related disciplines, empowering all students to reach their full academic and professional potential [9].

The development of numerical skills is not only fundamental for individual and professional success, but also plays a crucial role in building an informed, innovative and resilient society. By investing in the continuous improvement of the teaching and learning of these skills, we can strengthen the foundations for scientific, economic and social progress, ensuring that each individual has the necessary tools to effectively meet the challenges and seize the opportunities of the modern world. and ethics.

2.2 Neutrosophic Cognitive Maps.

Neutrosophic Cognitive Maps represent a significant evolution in the field of complex data representation and analysis. This unconventional methodology not only seeks to capture the complexity inherent in human perceptions, but also integrates principles of neutrosophic theory, which deals with truth, falsehood, and indeterminacy simultaneously. This innovative approach is especially relevant in contexts where ambiguity and uncertainty are key factors in decision making and understanding complex phenomena [10].

From a conceptual point of view, Neutrosophic Cognitive Maps allow you to visualize and structure relationships between concepts that may be ambiguous or contradictory according to different perspectives. This not only broadens the spectrum of analysis by including divergent opinions and perceptions, but also promotes a deeper and more holistic understanding of the issues investigated [11]. This ability to manage the vagueness inherent in human reality is crucial in disciplines such as philosophy, psychology and sociology, where subjective interpretations play a central role in the construction of knowledge. In practical terms, Neutrosophic Cognitive Maps find application in a variety of fields, from scientific research to strategic planning and business decision making. Its methodological flexibility allows researchers and practitioners to explore and analyze complex and multidimensional data in a structured and comprehensive manner. This methodology not only provides a visual representation of the complexity inherent in the systems and processes studied, but also facilitates the identification of hidden patterns and subtle connections that might be overlooked with more traditional approaches. However, like any emerging methodology, Neutrosophic Cognitive Maps face challenges and criticism. One of the main questions lies in the difficulty of quantifying and validating the indeterminacy and vagueness represented in these maps. Objectively evaluating the quality and reliability of data entered into maps can be complicated, especially when subjective or qualitative information is involved. Furthermore, interpretation of results can vary significantly depending on the theoretical framework and underlying assumptions of those using this methodology [12].

Despite these challenges, Neutrosophic Cognitive Maps offer considerable potential to advance the understanding and modeling of complex systems in an increasingly interconnected and dynamic world. By integrating principles from neutrosophic theory, these maps not only address reality in all its complexity and ambiguity, but also promote an inclusive and multidimensional approach to research and decision-making. This is especially valuable in contexts where the diversity of opinions and perspectives enriches the analysis process and contributes to more robust and adaptive solutions. In conclusion, Neutrosophic Cognitive Maps represent a powerful and promising tool for researchers and professionals seeking to navigate the complexity of the contemporary world. Its ability to represent and analyze vagueness and indeterminacy offers new opportunities to understand and address complex problems in fields as diverse as business management, public policy, and social science. As research in this area advances, it is crucial to continue exploring and refining this methodology to maximize its usefulness and accuracy in the information and knowledge age [13].

This section contains the basic concepts of neutrosophic cognitive maps and the algorithms associated with them.

Definition 1: ([14]) Let X be a universe of discourse. A *neutrosophic set* (NS) is characterized by three membership functions, $u_A(x), r_A(x), v_A(x) : X \to]^-_r0, 1^+[$which satisfy the condition $^-_r0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^+$ for all x∈X. $u_A(x), r_A(x)$and $v_A(x)$are the truthfulness, indeterminacy, and falsity membership functions of x in A, respectively, and their images are standard or non-standard subsets of $]^-_r0, 1^+[$.

Definition 2: ([14]) Let X be a universe of discourse. A *single-valued neutrosophic set* (SVNS) A on X is a set of the form:

$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\}$(1)

Where $u_A, r_A, v_A : X \to [0,1]$, satisfies the condition $0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3$ for all x∈X. $u_A(x), r_A(x)$and $v_A(x)$denotes the truthfulness, indeterminacy, and falsity membership functions of x in A, respectively. For convenience, a *single-valued neutrosophic number* (SVNN) will be expressed as $A = (a, b, c)$, where $a, b, c \in [0,1]$and satisfy $0 \leq a + b + c \leq 3$.

Other important definitions are related to graphics.

Definition 3: ([15, 17-18]) A *neutrosophic graph* is a graph that contains at least one indeterminate edge, which is represented by dotted lines.

Definition 4: ([15, 17-18]) A *neutrosophic directed graph* is a directed graph that contains at least one indeterminate edge, which is represented by dotted lines.

Definition 5: ([15, 17-18]) A *neutrosophic cognitive map* (NCM) is a neutrosophic directed graph, whose nodes represent concepts and whose edges represent causal relationships between the edges.

If $C_1, C_2, \dots, C_k$there are k nodes, each of them $C_i\,(i = 1, 2, \dots, k)$can be represented by a vector $(x_1, x_2, \dots, x_k)$where $x_i \in \{0, 1, I\}$. $x_i = 0$means that the node $C_i$is in an up state, $x_i = 1$it means that the node $C_i$is in a down state, and $x_i = I$it means that the node $C_i$is in an undetermined state, at a specific time, or in a specific situation.

If $C_m$and $C_n$are two nodes of the NCM, a directed edge from $C_m$to $C_n$is called *a connection* and represents causality from $C_m$to C_n. Each node in the NCM is associated with a weight within the set $\{-1, 0, 1, I\}$. If α_{mn}denotes the edge weight C_mC_n, $\alpha_{mn} \in \{-1, 0, 1, I\}$then we have the following:

$\alpha_{mn} = 0$YeahC_m does not affect C_n,

$\alpha_{mn} = 1$If an increase (decrease) in $C_m$produces an increase (decrease) in C_n,
$\alpha_{mn} = -1$If an increase (decrease) of $C_m$produces a decrease (increase) of C_n,
$\alpha_{mn} = I$If the effect ofC_m on $C_n$is indeterminate.

Definition 6: ([19]) An NCM that has edges with weights $\{-1, 0, 1, I\}$is called *a simple neutrosophic cognitive map.*

Definition 7: ([19]) If $C_1, C_2, \dots, C_k$they are the nodes of an NCM. The *neutrosophic matrix* N(E) is defined as $N(E) = (\alpha_{mn})$, whereα_{mn} denotes the weight of the directed edge C_mC_n, such that $\alpha_{mn} \in \{-1, 0, 1, I\}$. N(E)It is called *neutrosophic adjacency.* NCM *matrix.*

Definition 8: ([19]) Let be $C_1, C_2, \dots, C_k$the nodes of an NCM. We go $A = (a_1, a_2, \dots, a_k)$where $a_m \in \{-1, 0, 1, I\}$. A is called *instantaneous state. neutrosophic vector* and means a position of the on-off-indeterminate state of the node at a given instant.

$a_m = 0$if $C_m$it is disabled (has no effect),
$a_m = 1$if $C_m$it is activated (has an effect),
$a_m = I$YeahC_m It is indeterminate (its effect cannot be determined).

Definition 9: ([19]) Let $C_1, C_2, \dots, C_k$the nodes of an NCM be. Leave $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,…,$\overrightarrow{C_mC_n}$ be the edges of the NCM, then the edges constitute a *directed cycle.*

The NCM is called *cyclic* if it has a directed cycle. It is said *acyclic* if it does not have a directed cycle.

Definition 10: ([19]) An NCM containing cycles is said to have *feedback.* When there is feedback in the NCM it is said to be a *dynamic system.*

Definition 11: ([19]) Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,…,$\overrightarrow{C_{k-1}C_k}$ be a cycle. When $C_m$it is activated and its causality flows along the edges of the cycle and is then the cause of $C_m$itself, then the dynamic system circulates. This is true for each node $C_m$with $m = 1, 2, \dots, k$. The equilibrium state of this dynamic system is called the *hidden pattern.*

Definition 12: ([19]) If the equilibrium state of a dynamic system is a single state, then it is called *a fixed point.*

An example of a fixed point is when a dynamic system begins by being activated by C_l. If the NCM is supposed to sit onC_l and C_k, that is, the state remains like this $(1, 0, \dots, 0, 1)$, then this neutrosophic state vector is called *fixed point.*

Definition 13: ([19]) If the NCM is established with a neutrosophic state vector that repeats in the form:
$A_1 \to A_2 \to \cdots \to A_m \to A_1$, then the equilibrium is called the NCM *limit cycle.*

Method to determine hidden patterns

Let be $C_1, C_2, \dots, C_k$the nodes of the NCM with feedback. Let E be the associated adjacency matrix. A hidden pattern is found when activated and C_la vector input $A_1 = (1, 0, 0, \dots, 0)$is provided. The data must pass through the neutrosophic matrix N(E), which is obtained by multiplyingA_1 by the matrix N(E).

Leave$A_1N(E) = (\alpha_1, \alpha_2, \dots, \alpha_k)$ with the threshold operation of replace $\alpha_m$by 1 if $\alpha_m > p$andα_m for 0 if $\alpha_m < p$(p is a suitable positive integer) andα_m is replaced by I if it is not an integer. The resulting concept is updated; The vector $C_1$is included in the updated vector by transforming the first coordinate of the resulting vector to 1.

Yeah$A_1N(E) \to A_2$ It is assumed, then $A_2N(E)$considered and the same procedure is repeated. This procedure is repeated until a limit cycle or set point is reached.

Definition 14: ([20]) A *neutrosophic number* N is defined as a number as follows:
$N = d + I$(2)
where d is called *determined part* and they call me the *indeterminate part.*

Given $N_1 = a_1 + b_1I$and $N_2 = a_2 + b_2I$are two neutrosophic numbers, some operations between them are defined as follows:
$N_1 + N_2 = a_1 + a_1 + (b_1 + b_2)I$(Addition);
$N_1 - N_2 = a_1 - a_1 + (b_1 - b_2)I$(Difference),
$N_1 \times N_2 = a_1a_2 + (a_1b_2 + b_1a_2 + b_1b_2)I$ (Product),
$\frac{N_1}{N_2} = \frac{a_1+b_1I}{a_2+b_2I} = \frac{a_1}{a_2} + \frac{a_2b_1-a_1b_2}{a_2(a_2+b_2)}I$(Division).

3 Results and Discussion.

First, we specify the variables to take into account for the study, these are the following:
Here are five teaching options for instruction in numerical and research skills for accounting students:

1. **Problem-Based Learning (PBL):** This methodology involves presenting students with problematic cases or situations related to accounting. Students work in teams to identify and analyze relevant numerical data, applying research methods to arrive at evidence-based solutions or recommendations.

2. **Financial Simulations:** Use interactive simulations that imitate real financial scenarios. This includes the use of specialized software to perform financial analysis, cash flow projections, and evaluate investment performance. Students develop practical numerical skills as they investigate and make strategic decisions based on the results obtained.
3. **Case Studies in Auditing and Accounting**: Use case studies based on real auditing and accounting situations. Students analyze accounting documents, perform complex financial calculations, and apply investigative techniques to identify irregularities or areas for improvement in accounting practices. This encourages the development of advanced numerical skills and the ability to carry out detailed investigations.
4. **Use of Data Analysis Tools:** Introduce students to the use of data analysis software such as Excel, SPSS or data mining software. Through hands-on exercises, students learn to manipulate large data sets, perform statistical analysis, and present results effectively. This option promotes both numerical skills and research competencies through the exploration and management of complex data.
5. **Independent Research Projects:** Assign research projects that require the collection, analysis and presentation of actual financial data. Students choose topics of interest within the field of accounting, design appropriate research methodologies, and apply advanced numerical techniques to reach informed conclusions. This option encourages academic autonomy and deepening numerical and research skills.

These teaching options not only strengthen accounting students' numerical and research skills, but also prepare them to face real challenges in the professional world, integrating theory and practice effectively.

Let be $E = \{e_1, e_2, \cdots, e_n\}$the set of n experts. R_{ijk}symbolizes the relationship between the jth and kth criteria ($j, k \in \{1,2, \ldots ,5\}$, $j \neq k$) according to the expert e_i($i = 1,2, \ldots , n$) such that $R_{ijk} \in \{-5, -4, \cdots , -1,0,1, \cdots ,4,5, I\}$.

1. The numerical values of $R_{ijk}$are calculated, then $\hat{R}_{ijk} = R_{ijk}$, and if $R_{ijk} = I$then is kept $\hat{R}_{ijk} = I$.
2. For each fixed pair $j, k \in \{1,2, \ldots ,5\}$, it is calculated $\bar{R}_{jk}$as follows:
   - If the mode of $\hat{R}_{ijk}$for $i = 1,2, \ldots ,$nis unimodal, we take $\bar{R}_{jk} = mode_i(\hat{R}_{ijk})$and $\bar{R}_{kj} = 0$.
   - If the mode of $\hat{R}_{ijk}$for $i = 1,2, \ldots , n$is not unimodal, it is defined as follows:
     - ✧ If $\hat{R}_{ikj}$for $i = 1,2, \ldots ,$nis unimodal, take $\bar{R}_{kj} = mode_i(\hat{R}_{ikj})$and $\bar{R}_{jk} = 0$.
     - ✧ If $\hat{R}_{ikj}$for $i = 1,2, \ldots ,$nis not unimodal, it is taken $\bar{R}_{jk} = \bar{R}_{kj} = I$.
3. In this way, the adjacency matrix is formed with the elements $\bar{R}_{jk}$obtained from this algorithm.

To obtain the weights and form the NCM, 30 specialists in mathematics teaching were surveyed. Among them are scientists and academics with at least 10 years of experience. The Adjacency Matrix obtained is summarized in Table 1.

Table 1. The prediction adjacency matrix of the teaching options according to the 30 experts surveyed.

Variable	V_1	V_2	V_3	V_4	V_5
V_1	0	I	I	0	I
V_2	I	0	-1	0	1
V_3	I	0	0	0	0
V_4	1	0	1	0	1
V_5	I	0	-1	0	0

Figure 1 contains the NCM graph according to the adjacency matrix established in Table 1.

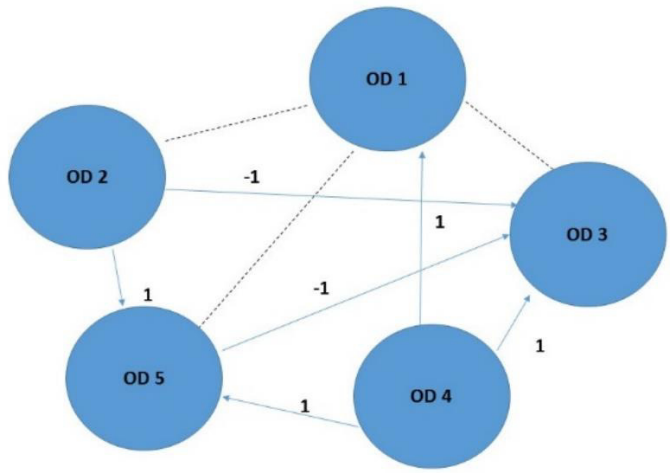

Figure 1: Neutrosophic Cognitive Map obtained from the experts.

All possible cases of convergence were studied when at least one of the variables was activated. This occurs in a total number of cases equal to $2^5 - 1 = 31$. Table 2 summarizes the results in absolute and relative frequencies for each of the three possible states of activated (1), deactivated (0) or indeterminate (I).

Table 2: Absolute frequency of convergence of the system in each of the possible values. Relative frequen-cies in percentage appear in parentheses.

Convergence To Value						
SDG	**0**	**%**	**1**	**%**	**Yo**	**%**
OD1	4	12.90	9	29.03	18	58.06
OD2	5	16.13	4	12.90	22	70.97
OD3	9	29.03	13	41.94	9	29.03
OD4	2	6.45	11	35.48	18	58.06
OD5	26	83.87	5	16.13	0	0.00

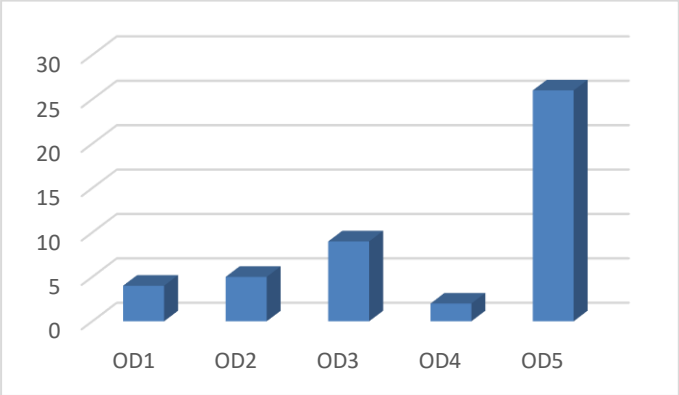

Figure 2: Absolute convergence frequency of the system (0).

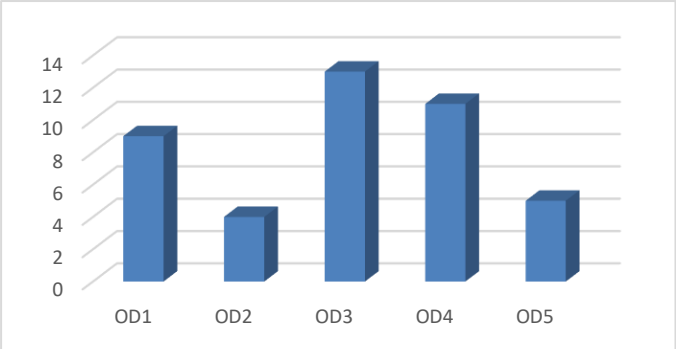

Figure 3: Absolute convergence frequency of system (1).

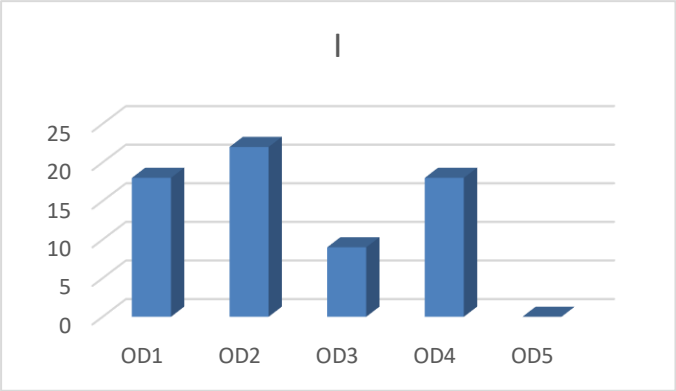

Figure 4: Absolute frequency of system convergence (I).

The results shown in Table 2 confirm the complexity of the work to be done to implement the didactic options for instruction in the numerical and research skills of accounting students.

Table 2 shows that the most robust teaching option is OD_3 or "Case **Studies in Auditing and Accounting:**" since it is activated in almost half of the possible initial conditions, while it is indeterminate in 29.3% of them.

The safest teaching option is OD_5 or "Independent **Research Projects"**. However, this will be activated in 16.3% of the possible initial conditions, which makes it difficult for it to be activated from the rest of the variables.

The rest of the options OD_2 or "Financial **Simulations"**, OD_4 or "Use **of Data Analysis Tools"** and OD_1 or "Problem**-Based Learning (PBL)"** show similar behaviors to each other, where there is indeterminacy for the majority. of the possible initial conditions of the states. This fact corroborates the complex and experimental nature of the project to be implemented.

A more detailed study shows that the most efficient proposal is to activate (or implement) the teaching options OD_3 and OD_4 ($x_0 = (0,0,1,1,0)$) or OD_3 and OD_5 ($x_0 = (0,0,1,0,1)$). If the three OD_3, OD_4 and OD_5 are activated ($x_0 = (0,0,1,1,1)$) The same results are obtained, however, this is not enough to activate the first two didactic options.

In the other cases of initial conditions it can be seen that no other option is activated.

Accounting education requires teaching methods that not only convey theoretical knowledge but also develop practical and research skills crucial for professional success. In this context, Neutrosophic Cognitive Maps have provided an exhaustive and detailed evaluation of various teaching options, revealing complex patterns and offering valuable elements about their effectiveness. This methodology has allowed us to identify that "Case Studies in Auditing and Accounting" (OD 3) stand out as the most robust option, activating in almost half of the possible initial conditions and showing indeterminacy in 29.3% of them. This combination of certainty and indeterminacy reflects the flexibility and applicability of this approach in various educational settings. The robustness of OD 3 suggests that case studies based on real situations are highly effective in fostering advanced numerical skills and detailed research competencies. By facing authentic problems, students can apply their theoretical knowledge to practical situations, developing critical and analytical thinking that is essential in accounting practice. This methodology not only strengthens technical skills, but also promotes a deep understanding of accounting and auditing principles, preparing students for real-world challenges.

On the other hand, "Independent Research Projects" (OD 5) are presented as the safest didactic option, although with an activation rate of 16.3% in the possible initial conditions. This low activation rate indicates that although OD 5 is very effective when implemented, its applicability may be limited by the need for specific conditions and an appropriate learning environment. However, when these conditions are met, independent research projects can foster academic autonomy and a deepening of numerical and research skills, essential for professional development in accounting. The remaining options, "Financial Simulations" (OD 2), "Use of Data Analysis Tools" (OD 4) and "Problem-Based Learning" (OD 1), show similar behaviors with high indeterminacy in most conditions. initials. This underlines the experimental and complex nature of these approaches. The high indeterminacy could suggest that its effectiveness depends largely on specific contextual and implementation factors, which may limit its general applicability but offer great benefits in suitable settings.

A more detailed analysis reveals that the combination of options OD 3 and OD 4 or OD 3 and OD 5 turns out to be the most efficient proposal. Simultaneously activating "Case Studies in Auditing and Accounting" with "Use of Data Analysis Tools" or "Independent Research Projects" maximizes the development of numerical and research skills in accounting students. This combination allows the complementary strengths of each approach to be leveraged, providing a rich and diversified learning experience that addresses both technical competencies and research skills. It is interesting to note that when the three options are activated (OD 3, OD 4 and OD 5) similar results are obtained, but the first two didactic options cannot be activated. This suggests that, although each methodology has its own strengths, their combination may not always be synergistic. This observation highlights the importance of careful planning and strategic implementation of teaching options to maximize their effectiveness.

In summary, Neutrosophic Cognitive Maps have proven to be an invaluable tool to evaluate and optimize didactic methodologies in teaching accounting. The robustness of OD 3, combined with the security of OD 5 and the flexibility of OD 4, offers a comprehensive and adaptable framework for developing numerical and research skills. By strategically integrating these approaches, educators can create dynamic and effective learning environments that prepare students to meet professional challenges with confidence and competence [21, 22].

This detailed evaluation and analysis not only provides guidance for the implementation of teaching methodologies in accounting, but also highlights the importance of considering the complexity and indeterminacy inherent in education. By addressing these factors proactively and strategically, we can significantly improve the quality of teaching and learning in this crucial discipline.

4 Conclusion

Accounting education requires a balanced combination of teaching methods that convey theoretical knowledge and develop practical and research skills essential for professional success. Neutrosophic Cognitive Maps have proven to be a powerful tool to evaluate the effectiveness of various teaching options, offering valuable elements and revealing complex patterns. Through this methodology, we have identified that "Case Studies in Auditing and Accounting" (OD 3) are the most robust option, due to their ability to activate in almost half of the possible initial conditions and show an indeterminacy of 29.3 %. The effectiveness of OD 3 lies in its ability to confront students with real problems, allowing them to apply their theoretical knowledge in practical situations. This not only encourages the development of advanced numerical skills and detailed research competencies, but also promotes critical and analytical thinking essential to accounting practice. The implementation of case studies based on real situations prepares students for the challenges of the professional world, strengthening both their technical skills and their understanding of accounting and auditing principles.

On the other hand, "Independent Research Projects" (OD 5) are presented as the safest didactic option, although with an activation rate of 16.3%. This low activation rate suggests that although OD 5 is highly effective when implemented, its applicability may be limited by the need for specific conditions and an appropriate learning environment. However, when these conditions are met, independent research projects foster academic autonomy and deepening numerical and research skills, essential for professional development in accounting. The remaining options, "Financial Simulations" (OD 2), "Use of Data Analysis Tools" (OD 4) and "Problem-Based Learning" (OD 1), show similar behaviors with high indeterminacy in most conditions. initials. This high indeterminacy suggests that the effectiveness of these approaches is highly dependent on specific contextual and implementation factors, which may limit their general applicability but offer great benefits in appropriate settings. A more detailed analysis reveals that the combination of options OD 3 and OD 4 or OD 3 and OD 5 turns out to be the most efficient proposal. Simultaneously activating "Case Studies in Auditing and Accounting" with "Use of Data Analysis Tools" or "Independent Research Projects" maximizes the development of numerical and research skills in accounting students. This combination allows the complementary strengths of each approach to be leveraged, providing a rich and diversified learning experience that addresses both technical competencies and research skills.

It is notable that when the three options are activated (OD 3, OD 4 and OD 5) similar results are obtained, but the first two didactic options cannot be activated. This suggests that, although each methodology has its own strengths, their combination may not always be synergistic. This observation highlights the importance of careful planning and strategic implementation of teaching options to maximize their effectiveness.

In summary, Neutrosophic Cognitive Maps have proven to be an invaluable tool to evaluate and optimize didactic methodologies in teaching accounting. The robustness of OD 3, combined with the security of OD 5 and the flexibility of OD 4, offers a comprehensive and adaptable framework for developing numerical and research skills. By strategically integrating these approaches, educators can create dynamic and effective learning environments that prepare students to meet professional challenges with confidence and competence. To improve the quality of teaching and learning in accounting, it is crucial to proactively and strategically address the complexity and indeterminacy inherent in education. The detailed assessment and analysis provided by Neutrosophic Cognitive Maps not only offers guidance for the implementation of teaching methodologies, but also highlights the importance of considering these factors in the design and implementation of educational programs. In this way, we can significantly advance the preparation of highly trained and adaptable accounting students, ready to meet the challenges of today's professional world.

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Neutrosophic PEST-SWOT Analysis on the Impact of ICT on the Academic Training of Systems Engineering Students

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Abstract. The integration of Information and Communications Technologies (ICT) in the education of future systems engineers has generated a significant impact that deserves to be analyzed from multiple perspectives. This article explores these repercussions using the neutrosophic PEST-SWOT approach, a methodology that combines political, economic, social and technological (PEST) analysis with strengths, weaknesses, opportunities and threats (SWOT) within a framework of neutrosophic indeterminacy and complexity. The adoption of ICT in educational curricula has transformed not only the way in which students access information and knowledge, but also the way in which they interact and collaborate, promoting a more dynamic and adaptive education to contemporary market demands. labor. This analysis reveals both the benefits and challenges associated with this transformation, offering a comprehensive vision that ranges from opportunities for educational innovation to the structural barriers that still persist. In this context, profound changes are observed in pedagogy and teaching strategies, where ICT acts as catalysts for more active and personalized learning. The PEST-SWOT neutrosophic approach allows us to unravel the complexity of these changes, highlighting how political and economic forces influence the implementation of new technologies, while social and technological variables outline the horizon of possibilities and challenges. Thus, the article not only presents a detailed overview of the repercussions of ICT in the training of systems engineers, but also offers a critical reflection on the future implications of these technologies in the educational field. Ultimately, this research highlights the need for adaptive educational policies and a robust technological infrastructure to maximize the benefits of ICT, fostering a resilient and cutting-edge learning environment.

Keywords: Information and Communications Technologies, SWOT Analysis, PEST Analysis, Single Value Neutrosophic Numbers, PEST-SWOT Neutrosophic Analysis.

1 Introduction

Information and Communications Technologies (ICT) have disrupted all areas of modern society, re-configuring the way people interact, work and, crucially, learn. In the field of education, especially in the training of systems engineers, the incorporation of ICT has meant a true revolution, transforming methodologies, contents and pedagogical objectives [1]. This introduction focuses on exploring the various repercussions of ICT on the academic training of these future professionals, a topic of vital importance to understand current and future changes in higher education. One of the most notable aspects of the integration of ICT in the education of systems engineers is the improvement in access to information and educational resources [2]. Previously, students relied heavily on textbooks and physical libraries, but today they have a vast amount of information at their disposal online, accessible from anywhere and at any time. This accessibility not only facilitates autonomous learning, but also encourages the development of research and knowledge self-management skills, essential for systems engineers. ICT has also enhanced collaboration and teamwork, fundamental elements in the training of engineers. Tools such as online learning platforms, project management software and instant communication applications allow students to work together, even remotely, sharing ideas, developing projects and solving problems together. This ability to work in distributed teams is especially relevant in an increasingly globalized and connected world [3].

Furthermore, the implementation of ICT in education has led to the personalization of learning. Learning management systems and educational platforms can adapt to the individual needs of students, providing specific content according to their pace and learning style. This not only enhances the educational experience, but also

helps identify and address each student's areas of difficulty, increasing their chances of academic success [4]. Another significant impact of ICT on the training of systems engineers is the introduction of new pedagogical methodologies. Project-based learning, flipped learning, and competency-based learning are approaches that have gained ground thanks to digital technologies. These methodologies promote more active and participatory learning, where students are not mere recipients of information, but protagonists of their own learning process. Learning assessment has also evolved with the help of ICT. Educational platforms allow for continuous and formative evaluation, offering immediate feedback to students. This not only helps educators monitor their students' progress more effectively, but also allows students to identify their strengths and weaknesses throughout the course, enabling continuous improvement [5].

However, the integration of ICT in the training of systems engineers is not without challenges. Among the most significant are technological barriers, such as the lack of adequate infrastructure and the digital divide that still persists in many regions. Furthermore, teacher training is crucial for the success of ICT implementation; Educators must be prepared and competent in using these tools in order to harness their full potential. The impact of ICT also extends to the development of new skills in students [6]. Digital literacy, the ability to work with large volumes of data and knowledge of cybersecurity are skills that are increasingly in demand in the labor market and that systems engineering programs must incorporate into their curricula. Thus, ICT not only changes the way of teaching, but also the content of what is taught. As ICT continues to evolve, it is essential that educational institutions constantly adapt and update their pedagogical strategies. The speed with which new technologies emerge means that both students and educators must be in a continuous process of learning and adaptation, ensuring that the education provided is aligned with the needs and demands of the real world [7].

The repercussions of ICT on the academic training of systems engineering students are profound and multifaceted. From access to information to personalization of learning and continuous assessment, ICT has transformed every aspect of the educational process. However, to maximize its benefits and overcome the challenges, a continuous commitment to updating and adaptation by all parties involved in the educational process is necessary.

2 Related Words.

2. 1 Academic training.

Academic training, in its essence, is the process through which individuals acquire knowledge, skills and competencies that allow them to develop both personally and professionally. This process, traditionally linked to formal educational institutions such as schools and universities, has evolved significantly in recent decades, influenced by technological, social and economic changes. However, beyond its formal definition, academic training is a complex and multifaceted phenomenon that deserves deep and critical reflection. From a historical perspective, academic training has been seen as the main route to social mobility and economic development. In the industrial era, for example, formal education was essential to prepare individuals for specific jobs that required technical skills and specialized knowledge. However, in the information age, the nature of work and the skills needed have changed dramatically [8]. The ability to adapt to new technologies, think critically and solve complex problems is now as important, if not more so, than specific technical knowledge.

One of the most controversial aspects of contemporary academic training is its accessibility. Despite advances in online education and efforts to democratize access to knowledge, significant disparities in access to quality education still persist. Economic, geographic, and social barriers continue to limit opportunities for many people, calling into question the equity of the current education system. How can we, as a society, ensure that all individuals have equal access to quality education?

The quality of academic training is also under constant scrutiny. Curricula and pedagogical methods must evolve to remain relevant in a rapidly changing world. However, many education systems are inflexible and slow to adapt to these changes. Competency-based education, personalized learning, and the use of advanced technologies in the classroom are just some of the innovations that could im-prove the quality of education, but their implementation is often met with resistance and logistical challenges. Another crucial aspect is teacher training. Teachers not only need to be well versed in their subject, but also in innovative pedagogical methodologies that encourage critical thinking and active learning [9]. Continuing training and professional development for educators is essential to ensure they can adapt to change and deliver high-quality education. However, investment in teacher training is often insufficient, limiting the positive impact these professionals can have on their students. The evaluation of learning is another topic of debate in academic training. Traditional assessment methods, such as standardized tests, often do not adequately reflect students' competencies and abilities. We need more holistic assessment approaches that consider a range of skills, from critical thinking to creativity and collaboration. This change will require a reevaluation of what we consider academic success and how we measure it. The relationship between education and the labor market is also constantly evolving. Educational institutions must work closely with industry to ensure that study programs are aligned with market needs. This not only improves the employability of graduates, but also ensures that companies can access a well-

trained talent pool ready to meet the challenges of the future. However, this collaboration is often insufficient or non-existent, resulting in a significant skills gap [10].

Lifelong learning is another essential component of modern academic training. In a world where technologies and industries change rapidly, education cannot end with obtaining a degree. Individuals must be prepared to continue learning and adapting throughout their professional lives. Educational institutions must foster a mindset of continuous learning and provide the necessary tools and resources to support this type of education. Academic training also has a significant impact on personal development. Beyond technical and professional skills, education helps individuals develop values, ethics, and a critical understanding of the world around them. This humanistic aspect of education is often overlooked in debates focused on productivity and the economy, but it is fundamental to the development of responsible and engaged citizens. Finally, we must consider the future of academic training in a global context. The internationalization of education, driven by technology and student mobility, offers unprecedented opportunities for cultural exchange and global learning [11]. However, it also poses challenges in terms of quality and equity. Educational institutions must navigate these challenges carefully to ensure they can offer a high-quality education to all students, regardless of their background.

Academic training is a fundamental pillar of our society that faces multiple challenges and opportunities. From accessibility and quality to relevance and internationalization, every aspect of education needs to be carefully considered and improved. Only through a continued commitment to innovation and equity can we ensure that education delivers on its promise of preparing individuals for an uncertain and everchanging future [12].

2.2 SWOT Analysis

SWOT analysis is considered a fundamental tool in evaluating the current state of a company or project, adopting a structured approach that examines both its internal characteristics (strengths and weaknesses) and its external environment (opportunities and threats). This process unfolds in several well-defined phases: first, a thorough analysis of the external environment is performed, identifying opportunities that could drive the organization's growth and threats that could hinder its progress [13]. At the same time, the organization's internal factors are evaluated, such as its weaknesses that require internal management and strengths that must be optimized for maximum benefit.

Each of these components can be classified as positive or negative, depending on its impact on organizational development. Opportunities, for example, represent positive external factors that, once identified, can be strategically exploited to drive growth and competitiveness. On the other hand, threats constitute negative external influences that require effective tactics and strategies to be successfully mitigated and overcome. Internally, weaknesses are negative aspects that need to be addressed and improved through efficient management, while strengths represent areas of excellence that must be enhanced and used to the maximum [14]. The SWOT analysis delves into various critical areas such as the availability of financial resources, the organizational structure, the quality of the product or service offered, market perception and customer satisfaction. These results are organized and presented in a matrix that is then evaluated by experts, whose combined evaluation provides a clear and objective vision of the most viable and effective strategies for the organization or project in question.

2.3 PEST Analysis.

The PEST analysis is a strategic tool that examines the external factors that affect a company, covering the Political, Economic, Social and Technological components. This approach allows us to understand how legislative regulations, economic conditions, sociocultural trends and technological advances directly impact the organization. For example, political factors include environmental laws, antitrust regulations, and government stability, while economic factors address variables that influence market and financial stability. Sociocultural aspects, for their part, focus on the configuration and behavior of consumers, while technological factors consider the development and adoption of new technologies 15].

The PEST-SWOT methodology is developed in two fundamental stages that complement and strengthen the strategic analysis. Firstly, an exhaustive analysis of external factors is carried out from political, economic, social and technological perspectives. This phase provides a deep understanding of the environment in which the company operates, identifying emerging opportunities and potential threats that could influence its performance and future development. In the second stage, the principles of SWOT analysis are applied to evaluate the internal characteristics of the organization, such as its strengths and weaknesses. This internal analysis provides a critical view of available resources, organizational structure, product or service quality, and other key aspects that impact the company's ability to compete and prosper in its target market [16,17].

By combining both stages, the PEST-SWOT methodology offers a comprehensive and detailed view of the business situation. This allows business leaders and strategists to formulate more effective and holistic strategies that capitalize on identified external opportunities, while addressing internal weaknesses and mitigating potential threats. This integrated approach not only facilitates proactive adaptation to changes in the business environment,

but also promotes longterm sustainability and continuous growth of the company in a dynamic and competitive context.

2.4 Basic concepts about neutrosophy

Unlike traditional PEST-SWOT methods, in this work the evaluations are carried out based on Triangular Neutrosophic Numbers of Single Value. Below are the fundamental explanations on this topic.

Definition 1 ([18]) : The neutrosophic set N is characterized by three membership functions, which are the truth membership function T_A, the indeterminacy membership functionI_A and membership function to falsehood F_A, where Uis the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]^{-}_{T}0, 1^{+}[$, and $^{-}_{T}0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^{+}$.

See that by definition, $T_A(x), I_A(x)$and $F_A(x)$are standard or non-standard real subsets of $]^{-}_{T}0, 1^{+}[$and, therefore, $T_A(x), I_A(x)$and $F_A(x)$can be subintervals of $[0, 1]$. $^{-}_{T}0$and1^{+} They belong to the set of hyperreal numbers.

Definition 2 ([19]) : The single-valued neutrosophic set $F_A: U \to [0, 1]$(SVN N) Ais U, $T_A: U \to [0, 1]$where $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$and $I_A: U \to [0, 1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The single-valued neutrosophic number (SVN N) is symbolized by

$N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$and $0 \leq t + i + f \leq 3$.

Definition 3 ([20]) : The single- $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$valued triangular neutrosophic number , , is a neutrosophic set in $\mathbb{R}$, whose membership functions of truth, indeterminacy and falsity are defined as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}}, x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), a_2 < x \leq a_3 \\ 0, \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \leq x \leq a_2 \\ \beta_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\beta_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \leq a_3 \\ 1, \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\gamma_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \leq x \leq a_2 \\ \gamma_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\gamma_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \leq a_3 \\ 1, \text{otherwise} \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0, 1]$,$a_1, a_2, a_3 \in \mathbb{R}$ and $a_1 \leq a_2 \leq a_3$.

Definition 4 ([21]) : Given$\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and$\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$ two triangular neutrosophic numbers of a single value andλ any non-zero number on the real line. Then, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
2. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
3. Investment: $\tilde{a}^{-1} = \langle (a_3{}^{-1}, a_2{}^{-1}, a_1{}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3 \neq 0$.
4. Multiplication by a scalar number:

$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda < 0 \end{cases}$$

5. Division of two triangular neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

6. Multiplication of two triangular neutrosophic numbers:

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1b_1, a_2b_2, a_3b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, \ a_3 > 0 \text{ and } b_3 > 0 \\ \langle (a_1b_3, a_2b_2, a_3b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle (a_3b_3, a_2b_2, a_1b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

Where,∧ It's a norm∨ It is a conorm t [22].

3 Results and Discussion.

The study is carried out on the ICT factors that impact the Academic Training of Systems Engineering Students. For this purpose, experts on the subject and specialized literature were consulted. In this way the following factors were identified:

Access to educational resources: ICT provides instant access to a wide range of online educational resources, including e-books, academic articles, educational videos and interactive simulations, which enrich the learning of technical and theoretical concepts.

Online Learning Platforms: The availability of online learning platforms allows Systems Engineering students to participate in courses, carry out practical activities and collaborate with other students and teachers from anywhere and at any time.

Simulations and virtual laboratories: ICT facilitates the creation and access to simulations and virtual laboratories that allow students to experience and practice real situations safely and efficiently, improving their understanding of complex concepts.

Communication and collaboration: Tools such as emails, videoconferences and instant messaging platforms facilitate communication and collaboration between students, teachers and external experts, promoting collaborative learning and team problem solving.

Adaptive learning: ICT allows the personalization of learning through the use of algorithms that adapt the content and educational methodology according to the pace and learning style of each student, thus optimizing the effectiveness of the educational process.

Development of technical skills: Systems Engineering students can develop advanced technical skills through the use of specialized software, programming environments and computer-aided de-sign (CAD) tools, facilitating their preparation for the job market.

Access to experts and external resources: ICT allows students to access experts and external resources in real time, through professional social networks, discussion forums and online communities, thus expanding their network of contacts and learning opportunities.

Continuous updating: The rapid evolution of ICT ensures that Systems Engineering students are up to date with the latest trends and technological advances, preparing them to face technical challenges and adapt to changes in the field proactively.

Assessment and feedback: ICT-based educational platforms offer tools to evaluate student progress continuously and provide immediate feedback, allowing adjustments in learning and ensuring a better understanding of concepts.

Interaction with industry: ICT facilitates collaboration between educational institutions and industry, through joint projects, professional internships and mentoring programs, providing students with practical experiences and opportunities for real application of their knowledge.

The development of a strategy to guarantee a positive impact on the Academic Training of Systems Engineering Students may face several obstacles that require attention and consideration - The main obstacles to include are:

Unequal access to technology: Lack of equitable access to devices such as computers and internet connectivity can limit student participation and learning, especially in rural or economically disadvantaged areas.

Poor infrastructure: The quality and availability of technological infrastructure in some educational institutions may be insufficient to support the intensive use of ICT, affecting the learning experience and the effectiveness of educational tools.

Lack of adequate training: The lack of sufficient training for teachers and students in the effective use of ICT tools and platforms can limit their effective integration into the academic curriculum, reducing their educational impact.

Data security and privacy: Concerns about cybersecurity and personal data privacy can inhibit the full adoption of ICT in the educational environment, especially when sensitive information of students and teachers is involved.

High costs: The cost associated with the acquisition of software, licenses, hardware and maintenance of technological infrastructure can be prohibitive for some educational institutions, limiting their ability to implement ICT effectively and sustainably.

Digital disconnection: The digital divide between generations or groups of students can result in significant differences in familiarity and skills to use ICT effectively, affecting participation and academic performance.

Lack of curricular integration: Lack of coherent integration of ICT into the academic curriculum can lead to its fragmented and occasional use instead of continuous and meaningful application in learning and teaching.

Resistance to change: Resistance on the part of some teachers and educational administrators to adopt new technologies and teaching methods can limit innovation and the transformative potential of ICT in the educational process.

Connectivity issues: Frequent interruptions in internet connectivity or lack of reliable access to online services can hinder the effective use of web-based educational tools and resources.

Information overload: The excess of information available through ICT can overwhelm students and teachers, making it difficult to identify and select relevant and reliable content for academic learning. Overcoming these obstacles requires a multidisciplinary approach that includes diverse perspectives, fosters collaboration among multiple actors, and employs comprehensive data collection along with meticulous analysis of the complexity and uncertainty inherent in socioeconomic impact assessment.

Based on the PEST analysis, we can categorize the mentioned factors as threats and opportunities in relation to the four components of this analysis.

• **Threats**
○ **Political**
• **T 1: Changes in data protection and privacy regulations:** Evolution in data protection laws may require constant adjustments to educational institutions' internal policies to ensure compliance and security of student information.
• **T 2: Political instability and legislative changes in technological regulations:** Political uncertainty can influence the stability of investment policies in educational technological infrastructure, affecting long-term planning and implementation of educational projects based on ICT.
○ **Economic**
• **T 3: Fluctuations in application development and maintenance costs:** Changes in software development and maintenance costs can challenge the financial ability of educational institutions to keep their educational platforms up to date.
• **T 4: Increase in digital marketing and app promotion costs:** Increased costs for promoting online educational platforms may limit the reach and adoption of these technologies by students and educators.
○ **Social**
• **T 5: Resistance of some users to adopt new educational technologies:** The reluctance of some students and teachers to adopt new digital tools can hinder the effective integration of ICT in the educational process.
• **T 6: Impact on traditional educational culture and acceptance of digital platforms:** The introduction of educational technologies may face cultural resistance and require a process of adaptation and acceptance by the educational community.
○ **Technological**
• **T 7: Limitations in interoperability between educational systems and technological platforms:** Difficulties in achieving fluid integration between different educational systems and technological platforms can affect the user experience and learning efficiency.
• **T 8: Shortage of experts in the development of educational applications and learning platforms:** The lack of professionals trained in the development of educational technologies can limit the capacity for innovation and continuous improvement in ICT-based educational environments.
• **Opportunities**
• **2.1 Political**
• **O 1: Government support for educational digitalization initiatives:** Political support can provide additional funds and resources for the implementation and expansion of innovative educational technologies in the Systems Engineering curriculum.
• **2.2 Economic**
• **O 2: Creation of new learning models and markets through digital platforms:** Educational technologies can open new markets and opportunities for the development of personalized and adaptive educational content.

• **O 3: Increase in educational competitiveness through technological innovations:** The adoption of ICT can improve the educational quality and training offer of institutions, increasing their attractiveness for potential students and academic collaborators.
• **2.3 Social**
• **O 4: Local employment generation in urban and rural areas through the digital economy:** Educational technologies can promote job creation in the technological and educational sector, benefiting both urban and rural communities.
• **O 5: Contribution to educational accessibility and diversity for local and international students:** Online educational platforms can improve access to higher education and encourage cultural and linguistic diversity in the learning of Systems Engineering.
• **2.4 Technological**
• **O 6: Advances in technologies for personalization and adaptation of learning:** Technological innovations allow educational content to be adapted according to the individual needs of students, thus improving the effectiveness and personalization of the learning process.
• **O 7: Opportunities for research and development of new educational tools:** The integration of ICT in Systems Engineering education can boost the research and development of new educational methodologies and tools, continuously improving the academic experience.
• **Weaknesses**
• **W 1: Shortage of UX/UI specialists to design intuitive and attractive interfaces:** The lack of specialized user experience designers can limit the usability and acceptance of educational platforms by students and teachers.
• **W 2: Complexity in integration with existing management systems in educational institutions:** The difficulty in integrating online educational platforms with pre-existing academic management systems can generate compatibility and data redundancy problems.
• **Strengths**
• **S 1: Leveraging the growing popularity of online education and educational technology:** The growing demand for online education and educational technology offers opportunities to expand and diversify educational offerings in Systems Engineering.
• **S 2: Potential to improve operational efficiency and educational management:** ICT-based educational platforms can optimize academic administration, resource management and coordination between departments within educational institutions.
• **S 3: Improved student experience and loyalty through tracking and personalization tools:** Tracking and personalization tools can improve the student experience, increase academic retention, and promote student satisfaction.

This structured analysis allows us to visualize in a clear and organized manner the various factors that influence the effective implementation of ICT in the academic training of Systems Engineering students, as well as the opportunities and challenges that arise in this dynamic and technological context.

A team made up of eleven experts was in charge of analyzing various combinations between an external and an internal factor. Each of them was asked to carry out evaluations using the linguistic terms detailed in Table 1.

Table 1. Linguistic terms for evaluations and their associated SVTNNs. See [14-17].

Linguistic Terms	SVTNN
Very low (VL)	$\langle(0,0,1);0.00,1.00,1.00\rangle$
Low (L)	$\langle(0,1,3);0.17,0.85,0.83\rangle$
Medium Low (MDL)	$\langle(1,3,5);0.33,0.75,0.67\rangle$
Medium (M)	$\langle(3,5,7);0.50,0.50,0.50\rangle$
Medium High (MDH)	$\langle(5,7,9);0.67,0.25,0.33\rangle$
Height (H)	$\langle(7,9,10);0.83,0.15,0.17\rangle$
Very high (VH)	$\langle(9,10,10);0.00,1.00,1.00\rangle$

Specifically, there are the following sets:

$W = \{W_1, W_2\}$ denotes the set of Weaknesses,

$S = \{S_1, S_2, S_3\}$denotes the set of Strengths,

$T = \{T_1, T_2, T_3, T_4, T_5, T_6, T_7\}$denotes the set of Threats,

$O = \{O_1, O_2, O_3, O_4, O_5\}$denotes the set of Opportunities.

The steps are the following:

1. Each expert was asked to evaluate the possible combinations between the elements of SO, ST, WO and WT. This evaluation is carried out in terms of how the development and implementation of a multi-platform mobile application in the gastronomic sector would have a socioeconomic impact.
2. Linguistic terms are replaced by the equivalent single-valued triangular neutrosophic numbers (SVTNN) in Table 1.
3. A single SVTNN is obtained by calculating the median of the SVTNNs of all experts for each pair of items.
4. The arithmetic mean of the SVTNN is calculated for each quadrant SO, ST, WO and WT.
5. The final result of each quadrant is converted to a crisp value using precision Equation 4. This con-verts them into values on a numerical scale out of 10 that allows the results to be compared [23, 24].

$$A(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}})$$

Tables 1, 2, 3 and 5 summarize the results obtained after applying the previous steps.

Table 2. Calculation results for the SW quadrant. The medians of all experts are shown.

		Opportunities				
		O_1	O_2	O_3	O_4	O_5
Strengths	S_1	V.H.	V.H.	V.H.	V.H.	H
	S_2	V.H.	H	V.H.	H	V.H.
	S_3	H	MDH	V.H.	V.H.	H

Table 3. Calculation results for the ST quadrant. The medians of all experts are shown.

		Threats						
		T_1	T_2	T_3	T_4	T_5	T_6	T_7
Strengths	S_1	MDVH	V.H.	MDVH	MDVH	V.H.	V.H.	VVH
	S_2	V.H.	VVVH	H	V.H.	VVH	VVH	H
	S_3	V.H.	MDH	H	V.H.	V.H.	MDH	H

Table 4. Calculation results for the WO quadrant. The medians of all experts are shown.

		Opportunities				
		O_1	O_2	O_3	O_4	O_5
Weaknesses	w_1	MDH	MDH	MDH	MDH	MDH
	w_2	MDH	MDH	MDH	MDH	MDH

Table 5. Calculation results for the WT quadrant. The medians of all experts are shown.

		Threats						
		T_1	T_2	T_3	T_4	T_5	T_6	T_7
Weaknesses	w_1	H	V.H.	V.H.	H	V.H.	H	V.H.
	w_2	MDVH	MDVH	MDVH	MDVH	MDVH	MDVH	MDVH

From Tables 1 to 5, we have the following results:

- Potentials (Opportunities+Strengths): $\langle(7.6467, 9.2637, 9.9813); 0.57, 0.55, 0.63\rangle$,
- Risks (Strengths+Threats): $\langle(5.5190, 6.5714, 9.7519); 0.57, 0.25, 0.33\rangle$,
- Challenges (Weaknesses+Opportunities) $\langle(6, 5,9); 0.57, 0.35, 0.13\rangle$:
- Limitations (Weaknesses+Threats): $\langle(6.2, 7.0, 5.5); 0.55, 0.43, 0.51\rangle$.

As a last step, these values are converted into a neat scale with a maximum of 10 using Equation 4. From here we have the following results:

$$A(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \quad (4)$$

	A	a1	a2	a3	α	β	γ
Potentials	**8.9079**	7.6467	9.2637	9.9813	0.5700	0.5500	0.6300
Risks	**7.2353**	5.5190	6.5714	9.7519	0.5700	0.2500	0.3300
Challenges	**5.8750**	6,0000	5,0000	9,0000	0.5700	0.3500	0.1300
Limitations	**6.1476**	6,2000	7,0000	5,5000	0.5500	0.4300	0.5100

1. Potentials (Opportunities+Strengths): 8.9079.
2. Risks (Strengths + Threats): 7.2353.
3. Challenges (Weaknesses + Opportunities): 5.8750.
4. Limitations (Weaknesses + Threats): 6.1476.

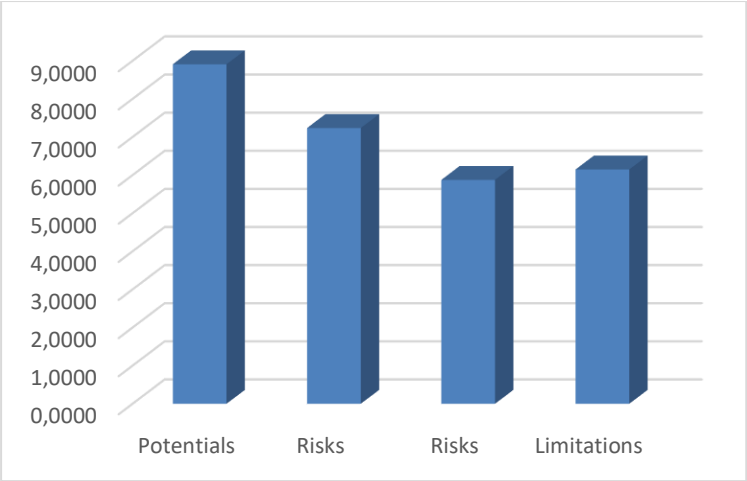

Figure 1. Calculation results for the WO quadrant.

To design an effective strategy that ensures a positive impact on the Academic Training of Systems Engineering Students, it is crucial to address in detail the critical factors identified through the PEST analysis and neutrosophic calculations. This exhaustive analysis reveals a series of threats, opportunities, weaknesses and strengths that shape the educational landscape in the context of Information and Communication Technologies (ICT). In the political sphere, threats arising from regulatory changes in data protection and technological legislation suggest the constant need for adaptation and regulatory compliance by educational institutions. This not only involves operational adjustments, but also strategic investments to ensure the security and privacy of student information in a digitally changing environment. Likewise, political instability can affect the long-term planning of ICT-based educational projects, complicating investment in infrastructure and the effective implementation of education-al innovations. From an economic perspective, fluctuations in app development and digital marketing costs stand out as significant challenges. Educational institutions face pressure to keep their educational platforms up to date while managing limited financial resources. Additionally, rising promotion costs may restrict the reach and adoption of advanced educational technologies, exacerbating disparities in access to digital education. Socially, resistance to adopting new educational technologies and the need to integrate digital platforms into traditional educational cultures are key obstacles. The reluctance of some students and teachers to adopt digital tools may hinder the effectiveness of ICT integration in the educational process, requiring cultural change and adaptation strategies to foster broader and more effective acceptance.

In the technological field, limitations in interoperability between educational systems and the short-age of experts in the development of educational applications are critical challenges. The lack of seamless integration between technological platforms can affect user experience and learning efficiency, while the lack of specialized talent limits the capacity for innovation and continuous improvement in digital educational environments.

However, political, economic, social and technological opportunities offer fertile ground for educational transformation. Government support for educational digitalization, for example, can provide additional resources and crucial financial support for the expansion of innovative educational technologies. Economically, the creation of new learning models through digital platforms and the increase in educational competitiveness are significant advantages that can drive the development of personalized and adaptive educational content. Socially, educational technologies have the potential to generate local employment and improve educational accessibility, benefiting both urban and rural communities. Furthermore, the ability of online educational platforms to improve access to higher education and promote cultural and linguistic diversity underlines their importance in global educational inclusion.

From a technological perspective, advances in personalization and adaptation of learning, together with the research and development of new educational tools, promise to continually improve the academic experience and prepare Systems Engineering students to meet the technological challenges of the future. Identified weaknesses, such as a shortage of UX/UI specialists and complexity in integration with existing management systems, represent critical barriers that must be addressed. The lack of specialized user experience designers can limit the usability and acceptance of educational platforms, while the difficulty in integrating these platforms with preexisting academic systems can generate incompatibilities and redundancies. On the other hand, current strengths, such as the growing use of online education and the improvement in operational efficiency through tracking and personalization tools, highlight the ability of ICT to optimize educational management and improve the student experience. These tools not only facilitate academic administration and coordination between departments, but also promote student retention and academic satisfaction through innovative teaching and learning methods. In conclusion, designing a comprehensive strategy to enhance the Academic Training of Systems Engineering Students involves navigating a complex terrain of threats, opportunities, weaknesses and strengths. Overcoming the identified challenges will demand a multidisciplinary approach that promotes collaboration between various educational and technological actors, thus ensuring an effective and sustainable implementation of educational technologies. This process not only requires investment in infrastructure and resources, but also a continuous commitment to educational innovation and cultural adaptation in educational institutions.

4 Conclusion

Designing an effective strategy that guarantees a positive impact on the Academic Training of Systems Engineering Students involves thoroughly addressing the critical factors revealed by the PEST analysis and neutrosophic calculations. This exhaustive analysis has highlighted a series of challenges and opportunities that outline the educational landscape in the context of Information and Communication Technologies (ICT). In the political sphere, threats arising from regulatory changes in data protection and technology legislation highlight the constant need for regulatory and strategic adjustments to safeguard student information in a constantly evolving digital environment. Furthermore, political instability can complicate long-term planning of ICT-based educational initiatives, impacting investment in infrastructure and the effective execution of innovative educational projects. From an economic perspective, fluctuations in digital marketing and app development costs represent significant challenges. Educational institutions face pressure to keep their educational platforms up to date while managing limited financial resources. Additionally, increased promotion costs may restrict the reach and adoption of advanced educational technologies, exacerbating disparities in access to digital education. Socially, resistance to adopting new educational technologies and the need to integrate digital platforms into traditional educational cultures are key obstacles. The reluctance of some students and teachers to adopt digital tools may hinder the effective integration of ICT into the educational process, requiring strategies to encourage broader and more effective acceptance. In the technological field, limitations in interoperability between educational systems and the shortage of experts in the development of educational applications are critical challenges. The lack of seamless integration between technological platforms can affect user experience and learning efficiency, while the lack of specialized talent limits the capacity for innovation and continuous improvement in digital educational environments. However, political, economic, social and technological opportunities offer fertile ground for educational transformation.

For example, government support for educational digitalization can provide additional resources and crucial financial support for the expansion of innovative educational technologies. Economically, the creation of new learning models through digital platforms and the increase in educational competitiveness are significant advantages that can drive the development of personalized and adaptive educational content. Socially, educational technologies have the potential to generate local employment and improve educational accessibility, benefiting both urban and rural communities. From a technological perspective, advances in personalization and adaptation of learning, together with the re-search and development of new educational tools, promise to continually improve the academic experience and prepare Systems Engineering students to meet the technological challenges of the future. Identified weaknesses, such as a shortage of UX/UI specialists and complexity in integration with existing

management systems, represent critical barriers that must be addressed. The lack of specialized user experience designers can limit the usability and acceptance of educational platforms, while the difficulty in integrating these platforms with pre-existing academic systems can generate incompatibilities and redundancies. On the other hand, current strengths, such as the growing use of online education and the improvement in operational efficiency through tracking and personalization tools, highlight the ability of ICT to optimize educational management and improve the student experience. These tools not only facilitate academic administration and coordination between departments, but also promote student retention and academic satisfaction through innovative teaching and learning methods. In conclusion, designing a comprehensive strategy to enhance the Academic Training of Systems Engineering Students involves navigating a complex terrain of threats, opportunities, weaknesses and strengths. Overcoming the identified challenges will demand a multidisciplinary approach that promotes collaboration between various educational and technological actors, thus ensuring an effective and sustainable implementation of educational technologies. This process not only requires investment in infrastructure and resources, but also a continuous commitment to educational innovation and cultural adaptation in educational institutions.

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Utilizing SVNL OWA Distance Measure and Neutrosophic TOPSIS in the Assessment of Management Leadership Styles

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Abstract. In the complex field of evaluating leadership styles, the integration and aggregation of data has become crucial. This article delves into the use of OWA-TOPSIS, a hybrid methodology that combines the Ordered Weighted Averages (OWA) Operator and the Method for Ordering Preferences by Similarity with the Ideal Solution (TOPSIS), to achieve an accurate and holistic assessment of leadership executive. By unifying diverse data and aggregating information from multiple sources, you achieve a more complete and nuanced view of management capabilities and styles. This approach not only improves the objectivity of the analysis, but also provides a powerful tool for decision-making in complex corporate environments, where leadership skills are critical to organizational success. The OWA-TOPSIS methodology stands out for its ability to handle ambiguous and subjective information, transforming it into a quantifiable and easily interpretable result. By applying this method, you can discern more clearly which leadership styles are most effective in different contexts and specific situations. Furthermore, the flexibility of this technique allows it to be adapted to various industries and sectors, making OWA-TOPSIS a versatile solution for leadership analysis. The research detailed in this article offers an innovative and insightful perspective on how to optimize management evaluation, thereby providing essential guidance for improving organizational effectiveness through informed leadership strategically aligned with corporate objectives.

Keywords: OWA; TOPSIS; SVNLOWAD Decision Making; Management Styles.

1 Introduction

The evaluation of leadership styles has been a central concern in the field of management and business administration for decades. Understanding how different leadership styles impact organizational performance, employee satisfaction, and decision-making effectiveness is critical for any organization seeking to improve its processes and achieve its strategic objectives [1]. In this sense, the precise and objective assessment of these leadership styles becomes a crucial tool for managers and human resources managers. Historically, various methodologies have been used to assess leadership, from qualitative surveys and interviews to more sophisticated quantitative techniques [2]. However, one of the persistent challenges has been the integration and aggregation of data from multiple sources and the interpretation of information that is often subjective and ambiguous. This is where hybrid methods such as OWA-TOPSIS emerge as innovative and effective solutions to address these challenges [3]. The Operator of Ordered Weighted Averages (OWA) is a technique that allows the aggregation of data taking into account the degree of importance of each criterion, while the Method for Ordering Preferences by Similarity with the Ideal Solution (TOPSIS) facilitates the comparison of alternatives depending on its proximity to an ideal solution [4]. The combination of these two methodologies offers a robust tool for the assessment of leadership styles, providing results that are both accurate and interpretable [5].

The use of OWA-TOPSIS in the evaluation of leadership styles allows us to overcome some of the limitations associated with traditional methods. For example, the ability to handle ambiguous and subjective information is particularly useful in contexts where employee perceptions and opinions play a crucial role in assessing leadership [6]. Furthermore, this methodology offers the necessary flexibility to adapt to different industries and organizational contexts, making it a versatile and widely applicable solution [7].

A key aspect in the implementation of OWA-TOPSIS is the correct determination of the weights assigned to each criterion. This requires a deep understanding of organizational priorities and close collaboration with relevant stakeholders [8]. Precision in the assignment of these weights ensures that the results adequately reflect the realities and needs of the organization, providing a solid basis for strategic decision making.

This article aims to explore in detail the application of OWA-TOPSIS in the assessment of management leadership styles. Through an exhaustive analysis of case studies and practical examples, it will be demonstrated

how this methodology can be used to obtain valuable elements and improve organizational effectiveness [9]. Furthermore, the advantages and limitations of this approach will be discussed, providing clear and practical guidance for its implementation in different contexts. The importance of effective leadership cannot be underestimated in the contemporary business environment. Leaders not only guide their teams toward achieving organizational goals, but also set the cultural tone and work climate within the company [10]. Therefore, having accurate and reliable tools to evaluate and improve leadership styles is essential for any organization that aspires to maintain its competitiveness and relevance in the market [11].

The OWA-TOPSIS methodology represents a significant advance in the evaluation of management leadership styles. By providing a systematic and objective way to analyze complex, multifaceted data, this technique offers organizations a powerful tool to optimize their leadership and, ultimately, their overall performance. As companies continue to face increasing challenges in an increasingly dynamic and competitive business environment, the ability to evaluate and adjust leadership styles effectively will be more crucial than ever [12].

This study aims to contribute to the field of business management and administration, providing a theoretical and practical framework for the application of OWA-TOPSIS in the assessment of leadership styles. The findings presented in this article are expected to serve as a valuable guide for practitioners and academics interested in improving leadership practices and strategic decision making in their organizations [13].

2 Related Words.

2. 1 Leadership styles.

Leadership styles, a topic widely discussed in management and organizational psychology literature, represent a crucial aspect for the functioning and success of any organization. From autocratic styles to more democratic and transformational ones, each approach has its own characteristics and effects on employee behavior and performance. The essential question is how to determine which of these styles is most appropriate in different contexts and how leaders can adapt to meet the changing needs of their teams and organizations. First, autocratic leadership, characterized by centralized decision-making and absolute control by the leader, has traditionally been valued in crisis situations or when clear and rapid direction is needed. However, this style can result in low morale and a lack of creativity among employees, who may feel that their opinions and contributions are not valued. Despite these disadvantages, there are times when strong and decisive leadership is essential to overcome significant obstacles [10].

On the other hand, democratic leadership, which encourages participation and collaboration among team members, can create a more inclusive and motivating work environment. This style promotes creativity and innovation, as employees feel empowered to share their ideas and solutions. However, in situations where rapid action is required, the consultation and consensus process can slow down decision-making, which could be detrimental. Transformational leadership, which focuses on inspiring and motivating employees to reach their full potential, has gained popularity in recent years [14]. Transformational leaders are visionaries who foster a strong sense of purpose and commitment among their followers. However, this style also requires a high level of charisma and interpersonal skills, which can be a challenge for some leaders. Additionally, overreliance on a transformational figure can be risky if that figure retires or moves to another organization. It is important to note that there is no single, perfect leadership style that fits all situations. The best leaders are those who can assess circumstances and adapt their approach to the needs of the moment. Flexibility and the ability to learn and evolve are essential characteristics of effective leadership. In this sense, leadership development programs should focus on strengthening these adaptive skills instead of promoting a single style as the ideal.

The organizational context also plays a crucial role in the effectiveness of leadership styles. In organizations with rigid hierarchical structures, an autocratic style may be more accepted and effective, while in organizations with flatter, more collaborative structures, a democratic or transformational approach may be more appropriate. Therefore, leaders must be aware of organizational culture and adjust their style accordingly. Diversity in leadership is also a factor that should not be underestimated. The inclusion of different perspectives and experiences can enrich the decision-making process and foster a more innovative and resilient environment. Leaders should strive to create diverse teams and be open to different leadership approaches and styles within their teams. In terms of leadership assessment, it is essential to use a combination of qualitative and quantitative metrics. 360-degree evaluations, which include feedback from subordinates, colleagues, and superiors, can provide a complete view of a leader's performance. Likewise, organizational performance indicators, such as productivity, employee satisfaction, and talent retention, should also be considered. Finally, ongoing training and professional development are essential for leadership growth and effectiveness. Leaders must be committed to their own development and seek opportunities to improve their skills and knowledge. This includes not only formal training, but also self-reflection and learning from past experiences [10].

Leadership styles are multifaceted and must be tailored to the specific needs of the organization and team. There is no single approach that guarantees success in all situations. The ability to continually evaluate, adapt, and evolve

is what distinguishes truly effective leaders. As organizations face increasingly complex and dynamic challenges, the importance of adaptive and diversified leadership becomes even more evident.

2.2 OWA-TOPSIS.

The OWA-TOPSIS Neutrosophic method combines the techniques of the Operator of Ordered Weighted Averages (OWA) and the Method for Ordering Preferences by Similarity with the Ideal Solution (TOPSIS) with the principles of neutrosophic logic, providing a robust approach for making decisions. decisions in complex and ambiguous environments [15]. Below is a detailed and argued analysis of this method, highlighting its components, benefits and applications. Firstly, it is essential to understand the fundamentals of each technique involved. The OWA is an aggregation operator that allows multiple decision criteria to be combined, weighting each one according to their relative importance. This operator is especially useful when subjective and objective judgments must be integrated, offering a flexible way to handle ambiguous and diverse information. On the other hand, TOPSIS is a multi-criteria decision method that orders the alternatives according to their proximity to an ideal solution, considering both the distance to the ideal solution and the worst possible solution. The integration of these two methods provides a powerful tool for the evaluation and comparison of alternatives in a complex decision space [16,17].

Neutrosophic logic, developed by Florentin Smarandache, adds an additional dimension to this approach, allowing uncertainty, imprecision and inconsistency in data to be handled more effectively than traditional techniques [18]. Instead of working with exact and precise values, neutrosophic logic allows for the inclusion of degrees of truth, falsehood, and indeterminacy, which is crucial in situations where information is incomplete or contradictory. By incorporating neutrosophic logic into OWA-TOPSIS, the method's ability to handle the complexity and ambiguity inherent in many decision problems is significantly improved. The OWA-TOPSIS Neutrosophic method implementation process follows several steps. First, the decision criteria are defined and weights are assigned using the OWA operator, considering the relative importance of each criterion. Alternatives are then evaluated based on these criteria, using neutrosophic logic to represent information in a richer and more flexible way [19]. Next, the TOPSIS method is applied to order the alternatives according to their proximity to the ideal neutrosophic solution, taking into account both the truth, falsity and indeterminacy of the data. One of the main advantages of this approach is its ability to integrate multiple sources of information and manage uncertainty effectively. This is particularly useful in contexts where data is incomplete or contradictory, such as in risk assessment, strategic planning, and decision making in dynamic environments. Additionally, the OWA-TOPSIS Neutrosophic method is highly adaptable, allowing its application in a wide variety of fields, from business management to engineering and social sciences [20].

In terms of practical applications, the OWA-TOPSIS Neutrosophic method has proven to be useful in the evaluation of investment projects, supplier selection, prioritization of strategic initiatives and quality management, among others [17]. For example, in the selection of investment projects, this method allows decision-makers to evaluate multiple projects based on a series of financial, technical and strategic criteria, integrating both objective data and subjective judgments and managing the uncertainty inherent in future projections. Despite its advantages, the OWA-TOPSIS Neutrosophic method also presents some challenges. Determining appropriate weights for criteria and accurately representing neutrosophic information requires a high level of expertise and a deep understanding of neutrosophic logic. Furthermore, the implementation of this method can be computationally intensive, especially when dealing with large volumes of data or considering numerous alternatives [18].

The OWA-TOPSIS Neutrosophic method represents a significant contribution to the theory and practice of multi-criteria decision making. By combining the strengths of OWA and TOPSIS with the flexibility of neutrosophic logic, this approach provides a robust and adaptable tool to meet the challenges of decision making in complex and ambiguous environments. As organizations and decision makers face increasingly complicated problems, the adoption of methods such as OWA-TOPSIS Neutrosophic becomes increasingly relevant, offering an effective way to integrate multiple perspectives and manage uncertainty systematically [19].

This section provides a brief overview of the fundamental principles related to SVNS and SVNLS, covering definitions, operating principles, and metrics for measuring distances.

Definition 1 [21]. Let x be an element in a finite set,

$$P = \{ x, T_P(x), I_P(x), F_P(x) | x \in X \}, \quad (1)$$

where the membership function for truth, $T_P(x)$,the membership function for indeterminacy $I_P(x)$, and the membership function for falsehood $F_P(x)$clearly adhere to condition (2):

$$0 \leq T_P(x), I_P(x), F_P(x) \leq 1; \ 0 \leq T_P(x) + I_P(x) + F_P(x) \leq 3 \quad (2)$$

For an SVNS, P in X, we call the triplet $\left(T_P(x), I_P(x), F_P(x) \right)$its single-valued neutrosophic value (SVNV), denoted simply $x = (T_x, I_x, F_x)$for computational convenience.

Definition 2 [22]. Let $x = (T_x, I_x, F_x)$and and $= (T_y, I_y, F_y)$there are two SVNVs. So

1) $x \oplus y = (T_x + T_y - T_x * T_y, I_x * T_y, F_x * F_y)$;
2) $\lambda * x = (1 - (1 - T_x)\lambda, (I_x)\lambda, (F_x)\lambda), \lambda > 0$;
3) $x^{\lambda} = ((T_x)\lambda, 1 - (1 - I_x)\lambda, 1 - (1 - F_x)\lambda), \lambda > 0$

2.3 The linguistic set

Let be $S = \{s_\alpha | \alpha = 1, \dots, l\}$a finite and fully ordered discrete term set with the odd value, l, wheres_α denotes a possible value for a linguistic variable. For example, if $l = 7$, then a set of linguistic terms S could be described as follows [23]:

$$S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\} = \{extremely\ poor, very\ poor, poor, fair, good, very\ good, extremely\ good\}. \quad (3)$$

Any linguistic variable, s_iy s_j, in S must satisfy the following rules:

1) $Neg(s_i) = s_{-i}$;
2) $s_i \leq s_j \Leftrightarrow i \leq j$;
3) $\max(s_i, s_j) = s_j,\ if\ i \leq j$;
4) $\min(s_i, s_j) = s_i,\ if\ i \leq j$.

To avoid loss of information during an aggregation process, the discrete term set S will be extended to a continuous term set. $S = \{s_\alpha | \alpha \in R\}$.Any two linguistic variables $s_\alpha, s_\beta \in S$satisfy the following operating laws [13,22] :

1) $s_\alpha \oplus s_\beta = s_\alpha + \beta$;
2) $\mu s_\alpha = s_{\mu\alpha}, \mu \geq 0$;
3) $\frac{s_\alpha}{s_\beta} = s_{\frac{\alpha}{\beta}}$

Definition 5 [22] . Let there be $x_i = \langle s_{\theta(xi)}, (T_{xi}, I_{xi}, F_{xi})\rangle$ $(i = 1, 2)$two SVNLN. Its distance measure is defined as in (6):

$$d(x_1, x_2 v) = \left[|s_{\theta(x1)} T_{x1} - s_{\theta(x2)} T_{x2}|^{\mu} + |s_{\theta(x1)} I_{x1} - s_{\theta(x2)} I_{x2}|^{\mu} + |s_{\theta(x1)} F_{x1} - s_{\theta(x2)} F_{x2}|^{\mu}\right]^{\frac{1}{\mu}} \quad (4)$$

In particular, equation (6) reduces the Hamming distance [23] of SVNLS and the Euclidean distance of SVNLS when $\mu = 1$and $\mu = 2$, respectively.

2.4 MADM Based on the SVNLOWAD-TOPSIS Method

For a given multi-attribute decision-making problem in SNVL environments, $A = \{A_1, \dots, A_m\}$denotes a set of discrete feasible alternatives, $C = \{C_1, \dots, C_n\}$represents a set of attributes, and $E = \{e_1, \dots, e_k\}$is a set of experts (or DM) with the weight vector $\omega = \{\omega_1, \dots, \omega_k\}$T, such that $\sum_{i=1}^{n} w_i = 1$ and $0 \leq \omega_i \leq 1$. Suppose the attribute weight vector $s\ v = (v_1, \dots, v_n)^T$, which satisfies $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$. The evaluation, $\alpha_{ij}^{(k)}$given by the expert, $e_{t(t = 1,\dots,k)}$on the alternative, $A_{i(i = 1,\dots,m)}$,relative to the attribute, $C_{j(j = 1,\dots,n)}$forms the individual decision matrix as shown in equation [24] (7):

$$D^k = \begin{array}{c} \\ A_1 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \cdots & C_n \end{array} \\ \begin{pmatrix} \alpha_{11}^{(k)} & \cdots & \alpha_{1n}^{(k)} \\ \vdots & \ddots & \vdots \\ \alpha_{m1}^{(k)} & \cdots & \alpha_{mn}^{(k)} \end{pmatrix} \end{array} \quad (5)$$

where $\alpha_{ij}^k = \langle s_{\theta(\alpha_{ij})}^k, (T_{\alpha_{ij}}^k, I_{\alpha_{ij}}^k, F_{\alpha_{ij}}^k)\rangle$is represented by an SVNLN, which satisfies$s_{\theta(\alpha ij)}^k \in \bar{S}, T_{\alpha_{ij}}^k, I_{\alpha_{ij}}^k, F_{\alpha_{ij}}^k \in [0,1] and\ 0 \leq T_{\alpha_{ij}}^k + I_{\alpha_{ij}}^k + F_{\alpha_{ij}}^k \leq 3$.

Geng et al.[25] extended the TOPSIS method to adapt it to the SVNLS scenario, and the procedures of the extended model can be summarized as follows.

Step 1. Normalize the individual decision matrices:

In practical scenarios, MADM problems can encompass both benefit attributes and cost attributes. Let B and S the sets of benefit attributes and cost attributes, respectively. Therefore, the conversion rules specified in (8) apply:

$$\begin{cases} r_{ij}^{(k)} = \alpha_{ij}^{(k)} = \langle s_{\theta(\alpha_{ij})}^k, (T_{\alpha_{ij}}^k, I_{\alpha_{ij}}^k, F_{\alpha_{ij}}^k)\rangle, & \text{for} \quad j \in B, \\ r_{ij}^{(k)} = \langle s_{l-\theta(\alpha_{ij})}^k, (T_{\alpha_{ij}}^k, I_{\alpha_{ij}}^k, F_{\alpha_{ij}}^k)\rangle, & \text{for} \quad j \in S. \end{cases} \quad (6)$$

Thus, the standardized decision information, $R^k = (r_{ij}^{(k)})_{m\times n}$, is set as in (9):

$$R^k = (r_{ij}^{(k)})_{m\times n} = \begin{pmatrix} r_{11}^{(k)} & \cdots & r_{1n}^{(k)} \\ \vdots & \ddots & \vdots \\ r_{m1}^{(k)} & \cdots & r_{mn}^{(k)} \end{pmatrix} \quad (7)$$

Step 2 . Build the collective matrix:

All opinions from individual DMs are aggregated into a group opinion:

$$R = (r_{ij})_{m\times n} = \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{pmatrix} \quad (8)$$

Where $r_{ij} = \sum_{k=1}^{t} \omega_k \, r_{ij}^{(k)}$.

Step 3. Set the weighted SVNL decision information:

The weighted SVNL decision matrix, , is formed as shown in (11), using the operating laws given in Definition 2 above:

$$Y = (y_{ij})_{m\times n} = \begin{pmatrix} v_1 r_{11} & \cdots & v_n r_{1n} \\ \vdots & \ddots & \vdots \\ v_1 r_{m1} & \cdots & v_n r_{mn} \end{pmatrix} \quad (9)$$

The OWA operator is fundamental in aggregation techniques, widely studied by researchers . Its main advantage lies in organizing the arguments and facilitating the integration of the experts' attitudes in decision-making. Recent research has explored OWA in distance measurement, generating variations of OWAD[26] . Taking advantage of the benefits of OWA, the text proposes an SVNL OWA distance measure (SVNLOWAD). Given the desirable properties of the OWA operator, an SVNL OWA distance measure (SVNLOWAD) is proposed in the following text.

Definition 6 [26] . *Let $x_j, x_j^{'}$ $(j = 1, \ldots, n)$the two collections of SVNLN be.*

$$SVNLOWAD((x_1, x_1'), \ldots, (x_n, x_n')) = \sum_{j=1}^{n} w_j d(x_j, x_j'), \quad (10)$$

Therefore, step 4 of this method can be considered as follows:

Step 4 [27]. For each alternative, $A_i$the SVNLOWAD is calculated for the PIS, $A^+$and the NIS A^-, using equation (12):

$$SVNLOWAD(A_i, A^+) = \sum_{j=1}^{n} w_j \, \dot{d}\left(y_{ij}, y_j^+\right), i = 1, \ldots, m \quad (11)$$

$$SVNLOWAD(A_i, A^-) = \sum_{j=1}^{n} w_j \, \dot{d}\left(y_{ij}, y_j^-\right), i = 1, \ldots, m \quad (12)$$

where $\dot{d}(y_{ij}, y_j^+)$and $\dot{d}(y_{ij}, y_j^-)$are the j -th largest values of $\dot{d}(y_{ij}, y_j^+)$and , $\dot{d}(y_{ij}, y_j^-)$respectively.

Step 5 . In the classical TOPSIS approach, the relative closeness coefficient, C , is used to rank the alternatives. However, some researchers have highlighted cases where relative closeness does not achieve the desired objective of simultaneously minimizing the distance from the PIS and maximizing the distance from the NIS. Thus, following an idea proposed in references [18] , in equations (15)–(17), we introduce a modified relative closeness coefficient, $C'(Ai)$, used to measure the degree to which the alternatives, Ai (i = 1,..., m =1,...,), are close to the PIS and also far from the NIS, congruently:

$$C'(A_i) = \frac{SVNLOWAD(A_i, A^-)}{SVNLOWAD_{\max}(A_i, A^-)} - \frac{SVNLOWAD(A_i, A^+)}{SVNLOWAD_{\min}(A_i, A^+)}, \quad (13)$$

where

$$SVNLOWAD_{\max}(A_i, A^-) = \max_{1\le i\le m} SVNLOWAD(A_i, A^-), \quad (14)$$

and

$$SVNLOWAD_{\min}(A_i, A^+) = \min_{1\le i\le m} SVNLOWAD(A_i, A^+). \quad (15)$$

It is clear that $C'(A_i) \le 0$ $(i = 1, \ldots, m)$the higher the value of $C'(A_i)$, the better A_i the alternative. Furthermore, if an alternative A^*satisfies the conditions $SVNLOWAD(A^*, A^-) = SVNLOWAD_{\max}(A^*, A^-)$and $SVNLOWAD(A^*, A^+) = SVNLOWAD_{\min}(A^*, A^+)$, then $C'(A^*) = 0$y the alternative $A^*$is the most suitable candidate, since it has the minimum distance to the PIS and the maximum distance to the NIS.

Step 6. Rank and identify the most desirable alternatives based on the decreasing closeness coefficient $C'(A_i)$obtained using Equation (15).

3. Results and discussion.

The importance of leadership styles in organizations is a central topic in business management and organizational theory. Below, ten key aspects are explored that highlight its relevance:

1. **Influence on Organizational Culture:** Leadership styles shape the culture of an organization. A leader who promotes transparency, collaboration and innovation, for example, will establish an organizational culture that values these principles, directly influencing employee behavior and attitudes.
2. **Impact on Morale and Motivation:** Leadership style significantly affects employee morale and motivation. Transformational leaders, who inspire and motivate their teams through a shared vision, can raise employee morale and engagement, resulting in greater productivity and job satisfaction.
3. **Decision Making Effectiveness:** Different leadership styles impact decision making effectiveness. An autocratic leader can make decisions quickly in crisis situations, while a democratic leader can involve his or her team in the process, resulting in more inclusive and well-informed decisions.
4. **Talent Development:** Leaders who adopt a coaching or transformational leadership style focus on the development and growth of their employees. This not only enhances individual skills, but also prepares the organization for future leadership needs, ensuring successful succession.
5. **Adaptability and Organizational Resilience:** An adaptive leadership style can help an organization become more resilient in the face of change and challenges. Leaders who can adjust their style according to the needs of the context and the team contribute to greater flexibility and organizational responsiveness.
6. **Labor Relations:** Leadership style influences labor relations and team cohesion. A leader who practices a participatory and empathetic style fosters a positive and collaborative work environment, which can reduce conflict and improve communication.
7. **Innovation and Creativity:** Leaders who adopt styles that encourage creativity and innovation, such as transformational leadership or situational leadership, can drive the generation of new ideas and solutions. This is vital for the competitiveness and continued growth of the organization.
8. **Performance and Results:** Organizational performance is closely linked to leadership style. Effective leaders align individual goals with organizational goals, which can improve overall performance and results, from productivity to profitability.
9. **Customer Satisfaction:** A leadership style that promotes customer service and quality can have a positive impact on customer satisfaction. When employees are motivated and engaged, they are more likely to provide superior service, which improves customer loyalty and company reputation.
10. **Talent Retention:** Leadership styles influence talent retention. A positive work environment and effective leadership are key factors in retaining valuable employees, reducing turnover and costs associated with hiring and training new employees.

Leadership styles are fundamental to the success and sustainability of an organization. Leaders who can adapt and use different leadership styles depending on the needs of the context and the team have a significant advantage in creating a strong, flexible and growth-oriented organization. Understanding and developing effective leadership styles should be a priority for any organization that aspires to achieve its strategic objectives and maintain a competitive advantage in the market.

For the development of the study, three experts in the field participated, who collaborate in the selection of criteria of interest and in the evaluation of various leadership styles. In this case, three styles are considered for evaluation:

1. **Autocratic Leadership:**
   - Characteristics: The autocratic leader makes decisions unilaterally, without consulting team members. This style is characterized by centralized control and top-down communication.
   - Advantages: Allows quick decision making, which is useful in crisis situations or when immediate action is required. It can also be effective in environments where close supervision is needed.
   - Disadvantages: It can lead to low morale among employees, as their opinions and contributions are not valued. In the long term, it can result in a lack of creativity and innovation due to dependence on the leader for decision making.
2. **Democratic Leadership:**
   - Characteristics: Also known as participative leadership, this style involves the participation of employees in the decision-making process. The democratic leader encourages collaboration and two-way communication.
   - Advantages: Promotes a high level of commitment and job satisfaction among employees, as they feel valued and listened to. This style can also encourage creativity and innovation as multiple perspectives are considered.
   - Disadvantages: The decision-making process can be slower, which can be a drawback in situations

that require a quick response. Additionally, it can be difficult to reach consensus in large, diverse teams.

3. **Transformational Leadership:**
   - Characteristics: The transformational leader inspires and motivates employees to reach their full potential by creating a shared vision and fostering a strong sense of purpose. This style focuses on change and innovation.
   - Advantages: Can result in very high levels of commitment and motivation among employees, which in turn can lead to superior organizational performance. Transformational leaders are capable of bringing about significant and positive changes within the organization.
   - Disadvantages: Requires a high level of charisma and interpersonal skills on the part of the leader, which may not be easy to find or develop. Additionally, reliance on a transformational figure can be risky if that leader retires or moves to another organization.
4. **Situational leadership:**
   1. Adaptability: Situational leaders adjust their leadership style based on the team members' level of maturity and skills, as well as the specific demands of the task.
   2. Continuous diagnosis: They constantly evaluate the situation to determine the level of direction and support each team member needs at any given time.

Advantages:

- Flexibility: Allows the leader to adapt to different contexts and challenges, thus optimizing the team's effectiveness.
- Team Development: Fosters the individual and collective development of team members by providing the appropriate level of direction and support.
- Disadvantages:
- Complexity: Requires advanced diagnostic skills and adaptability on the part of the leader, which can be difficult to master.

The evaluation criteria are fundamental to evaluate the different styles. In this study the following criteria have been selected:

1. **Decision Making Efficiency:**
   - Description: This criterion evaluates the leader's ability to make timely and effective decisions that benefit the organization.
   - Indicators: Speed in decision making, quality of decisions, impact of decisions on organizational performance.
   - Example: An autocratic leader may be very effective in crisis situations where a quick decision is needed, while a democratic leader may be more appropriate in situations where it is important to consider multiple perspectives.
2. **Employee Motivation and Engagement:**
   - Description: This criterion measures the degree to which the leader can inspire and motivate his or her employees, as well as his or her ability to foster commitment and job satisfaction.
   - Indicators: Levels of job satisfaction, staff turnover rates, commitment to the organization, enthusiasm and team morale.
   - Example: A transformational leader who inspires his team with a shared vision can significantly increase employee engagement and motivation, while an autocratic style could have the opposite effect.
3. **Innovation and Creativity Capacity:**
   - Description: This criterion evaluates how the leadership style fosters innovation and creativity within the organization.
   - Indicators: Number of new ideas generated, implementation of innovative initiatives, work environment that encourages creativity.
   - Example: Democratic leadership, which encourages participation and the exchange of ideas, can be more effective in generating innovation compared to autocratic leadership, which could restrict creativity.

Experts assign a set of weighting values to the chosen criteria to reflect their relative relevance in the evaluation process. These weight values, provided by the experts, are as follows: C1: 0.15, C2: 0.25, C3: 0.25 and C4: 0.35.

The experts participating in the study provide evaluations for each alternative with respect to the mentioned criteria. These evaluations are expressed in terms of SVNL (Semantic Linguistic Numerical Variable) decision information using the linguistic term set $(S = s_1 = \text{“}extremely\ poor\text{”}, s_2\text{“}very\ poor\text{”}, s_3 = \text{”}poor\text{”}, s_4 = \text{“}fair\text{”}, s_5 = \text{“}good\text{”}, s_6 = \text{“}very\ good\text{”}, s_7 = \text{“}extremely\ good\text{”})$.

The standardized SVNL decision matrices are set out in Table 1 to Table 4.

Table 1: Evaluation of alternatives according to Criterion 1

Alternatives	EXPERT 1	EXPERT 2	EXPERT 3
1	S5(0,4,0,2,0,3)	S5(0,4,0,3,0,4)	S6(0,5,0,2,0,3)
2	S6(0,6,0,1,0,2)	S6(0,7,0,2,0,3)	S5(0,5,0,2,0,3)
3	S5(0,4,0,3,0,4)	S6(0,4,0,2,0,4)	S6(0,5,0,1,0,3)
4	S6(0,7,0,2,0,3)	S4(0,8,0,1,0,2)	S4(0,6,0,1,0,2)

Table 2: Evaluation of alternatives according to Criterion 2

Alternatives	EXPERT 1	EXPERT 2	EXPERT 3
1	S5(0,4,0,2,0,3)	S6(0,5,0,1,0,2)	S6(0,6,0,2,0,4)
2	S5(0,6,0,1,0,2)	S6(0,7,0,2,0,3)	S4(0,7,0,2,0,2)
3	S4(0,5,0,2,0,3)	S6(0,6,0,3,0,4)	S5(0,6,0,1,0,3)
4	S4(0,6,0,1,0,2)	S5(0,7,0,2,0,3)	S4(0,5,0,2,0,2)

Table 3: Evaluation of alternatives according to Criterion 3

Alternatives	EXPERT 1	EXPERT 2	EXPERT 3
1	S3(0,3,0,2,0,5)	S5(0,3,0,1,0,6)	S5(0,2,0,1,0,6)
2	S4(0,5,0,2,0,2)	S5(0,6,0,2,0,2)	S5(0,7,0,2,0,1)
3	S3(0,5,0,3,0,1)	S4(0,6,0,1,0,3)	S4(0,6,0,2,0,1)
4	S3(0,3,0,1,0,2)	S4(0,4,0,2,0,2)	S3(0,4,0,1,0,1)

Table 4: Evaluation of alternatives according to Criterion 4

Alternatives	EXPERT 1	EXPERT 2	EXPERT 3
1	S4(0,5,0,3,0,3)	S3(0,7,0,1,0,1)	S4(0,5,0,2,0,3)
2	S3(0,6,0,2,0,4)	S4(0,5,0,4,0,2)	S6(0,4,0,6,0,2)
3	S5(0,3,0,5,0,2)	S5(0,4,0,4,0,1)	S4(0,3,0,6,0,2)
4	S6(0,6,0,1,0,2)	S6(0,6,0,3,0,3)	S5(0,7,0,2,0,1)

The collective opinion is calculated and the SVNL collective decision matrix is obtained, which is presented in Table 5. This process is crucial to integrate the individual evaluations of the experts and obtain a joint perspective on the alternatives under study. The resulting matrix reflects the synthesis of individual opinions, allowing for a comprehensive and systematic evaluation of the alternatives under consideration.

Table 5: SVNL Collective Decision Matrix

Alternatives	C1	C2	C3	C4
1	S5.28(0.432,0.232,0.334)	S5.61(0.503,0.162,0.292)	S4.29(0.266,0.129,0.568)	S3.63(0.575,0.185,0.211)
2	S5.61(0.605,0.162,0.266)	S4.95(0.666,0.162,0.232)	S4.62(0.605,0.203,0.162)	S4.29(0.503,0.367,0.255)
3	S5.61(0.432,0.185,0.367)	S4.95(0.565,0.185,0.334)	S3.63(0.565,0.185,0.147)	S4.62(0.332,0.497,0.162)
4	S4.62(0.708,0.129,0.232)	S4.29(0.605,0.162,0.232)	S3.3(0.365,0.129,0.162)	S5.61(0.633,0.185,0.185)

The process of obtaining the weighted collective SVNL matrix is based on the specific operating rules of SVNL. The result of this calculation is presented in Table 6. This methodological approach ensures that the weights

assigned to each criterion, as well as the individual evaluations carried out by the experts, are adequately integrated. By applying these operational rules, a weighted and rigorous synthesis of individual opinions is achieved, providing a comprehensive and coherent vision of the alternatives evaluated in the study.

Table 6: Weighted collective SVNL decision matrix.

Alternatives	C1	C2	C3	C4
1	S1.06(0.107;0.75;0.8)	S1.06(0.1;0.76;0.83)	S1.06(0.074;0.6;0.87)	S1.06(0.29;0.51;0.54)
2	S1.12(0.17;0.69;0.77)	S1.12(0.152;0.76;0.8)	S1.12(0.207;0.67;0.63)	S1.12(0.244;0.67;0.58)
3	S1.12(0.107;0.71;0.82)	S1.12(0.117;0.78;0.85)	S1.12(0.188;0.66;0.62)	S1.12(0.149;0.76;0.48)
4	S0.92(0.218;0.66;0.75)	S0.92(0.13;0.76;0.8)	S0.92(0.107;0.6;0.63)	S0.92(0.33;0.51;0.51)

Decision makers, in order to incorporate their complex attitudes, determine the weight vector of the OWA operator: W = (0.25, 0.30, 0.35, 0.10). Then, we use equations (13) and (14) to calculate the measures SVNLOWAD $SVNLOWAD(A_i, A^+)$and $(SVNLOWAD(A_i, A^-)$between $A_i$the alternative and $PISA^+$and $NIS\ A^-$respectively.

This approach allows decision makers to explicitly weigh the relative importance of different criteria in the evaluation process. By employing the aforementioned equations, the relative distance between each alternative and the reference points (PIS and NIS) is quantified, providing an objective basis for comparing and ranking the alternatives based on their performance with respect to the established criteria.

For EL 1:

- SVNLOWAD(EL 1) = 0.127
- SVNLOWAD(EL 1) = 0.135
- C` (EL 1) = (0.127 + 0.135) / 2 = 0.131

For EL 2:

- SVNLOWAD (EL 2)= 0.037
- SVNLOWAD (EL 2) = 0.081
- C` (EL 2) = (0.037 + 0.081) / 2 = 0.059

For EL 3:

- SVNLOWAD (EL 3) = 0.07
- SVNLOWAD (THE 3) = 0.117
- C` (EL 3) = (0.07 + 0.117) / 2 = 0.0935

For EL 4:

- SVNLOWAD (EL 4) = 0.07
- SVNLOWAD (EL 4) = 0.005
- C` (EL 4) = (0.07 + 0.005) / 2 = 0.0375

Table 7: Relative distances between each alternative and the reference points.

	$SVNLOWAD\ (A_i, A^+)$	$(SVNLOWAD(A_i, A^-)$	C`
1	0.127	0.135	0.131

	$SVNLOWAD\ (A_i, A^+)$	$(SVNLOWAD(A_i, A^-)$	C`
2	0.037	0.081	0.059
3	0.07	0.117	0.0935
4	0.07	0.005	0.0375

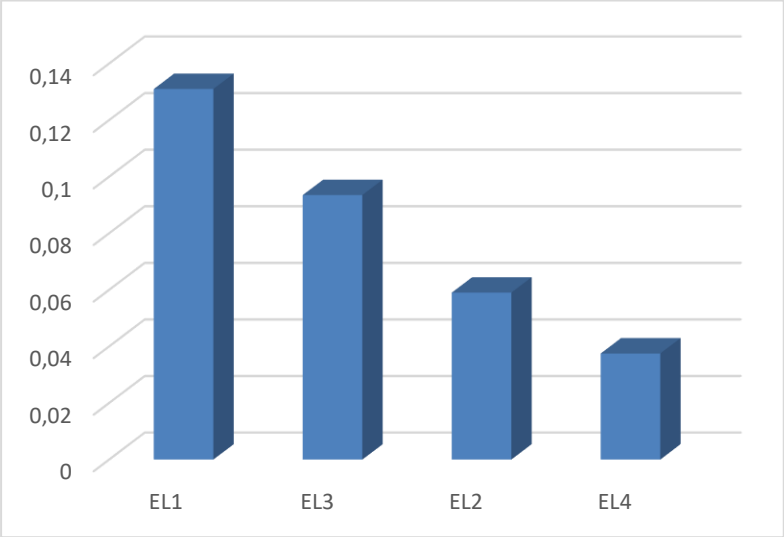

Figure 1: Relative distances between each alternative and the reference points.

Order of the leadership styles of the most prevalent leaders:

1. Autocratic Leadership
2. Transformational Leadership
3. Democratic Leadership

Leading teams effectively in the dynamic business world is a constant challenge that requires understanding various leadership styles: autocratic, democratic, transformational, and situational. Each style has unique characteristics that impact team dynamics and organizational outcomes. Autocratic leadership excels in crisis situations with its swift decision-making but may hinder long-term morale and creativity. Democratic leadership fosters participation and commitment but can slow decision-making and face challenges in reaching consensus in large teams. Transformational leadership inspires and motivates through a shared vision, enhancing commitment and performance, though it relies heavily on the leader's charisma. Situational leadership adapts to the specific needs of the team and task, offering flexibility in dynamic environments, but requires exceptional diagnostic and adaptive skills. The effectiveness of a leadership style depends on the context and organizational circumstances, with no universally superior approach. Successful leaders proactively assess situations, fostering an environment of collaboration, innovation, and continuous development to meet challenges and achieve goals sustainably.

4 Conclusion

Leading teams effectively in the dynamic business world requires understanding various leadership styles such as autocratic, democratic, transformational, and situational. Each style has unique characteristics that impact team dynamics and organizational outcomes. Autocratic leadership is efficient in crises but may reduce long-term morale and creativity. Democratic leadership fosters participation and commitment but can slow decision-making. Transformational leadership inspires and motivates through a shared vision but depends on the leader's charisma. Situational leadership adapts to the specific needs of the team and task, offering flexibility but requires exceptional diagnostic and adaptive skills. There is no universally superior approach; the key lies in the leader's ability to proactively assess and respond to situations.

Future work could explore the use of OWA operators and the neutrosophic TOPSIS method in information fusion to enhance decision-making in complex contexts. These approaches can provide a more robust and flexible framework for integrating diverse information sources and managing uncertainty, thereby optimizing leadership strategies and organizational effectiveness.

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Evaluation and Selection of Strategies for the Collection of Real Estate Taxes through Neutrosophic Cognitive Maps (NCM)

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Abstract. The evaluation and selection of strategies for real estate tax collection using neutrosophic cognitive maps (NCM) represents an innovative approach in urban fiscal management. This method combines elements of neutrosophic theory with cognitive evaluation techniques, allowing public administrators and urban planners to analyze the effectiveness of various fiscal tactics in complex and variable environments. NCMs facilitate a more accurate evaluation by considering not only numerical data, but also subjective perceptions and opinions of the stakeholders involved, such as owners, developers, and residents. This comprehensive approach not only optimizes efficiency in tax collection, but also promotes greater transparency and legitimacy in the administrative process, thus improving the quality of public service and citizen satisfaction. Additionally, NCMs provide a flexible platform to adapt collection strategies to changing real estate market dynamics and local tax policies. The ability of neutrosophic cognitive maps to handle the uncertainty and ambiguity inherent in modern urban management makes them an invaluable tool for urban planners and public policy makers. By integrating multiple perspectives and evaluating the acceptability of proposed strategies, these maps not only help foresee possible resistance or conflicts, but also facilitate the implementation of more equitable and effective tax policies. In summary, the application of NCM in urban fiscal management not only promotes administrative effectiveness but also strengthens local governance by improving citizen participation and democratic legitimacy in fiscal decision-making.

Keywords: Tax Collection, Neutrosophic Cognitive Maps, NCM.

1. Introduction

The collection of real estate taxes constitutes a crucial component of urban fiscal policy, essential to finance vital public services and promote sustainable development in dynamic urban environments. Currently, the efficient management of these taxes not only implies the effective application of tax regulations, but also the adoption of innovative strategies that can adapt to the complex dynamics of the real estate market and global economic fluctuations [1]. The effectiveness of these strategies is not only measured in terms of gross collection, but also in their ability to guarantee equity, transparency and efficiency in the use of public resources. Planning and implementing effective tax policies requires a comprehensive approach that incorporates detailed market analysis, socioeconomic impact assessment, and tax equity considerations [2]. In this context, strategies for real estate tax collection must be carefully designed to minimize tax evasion and maximize voluntary participation of taxpayers, while maintaining adequate incentives for sustainable real estate investment and development. Furthermore, it is crucial to consider the administrative and technological capacity of local authorities to implement and manage these strategies effectively and efficiently. Technological innovations and advances in data analytics have revolutionized urban tax management, offering new tools to improve the accuracy and effectiveness of real estate tax collection [3]. From the use of geographic information systems (GIS) for the evaluation of the tax base to the implementation of predictive models based on artificial intelligence for the detection of tax evasion, these technologies are transforming the way cities manage their tax revenues. However, the successful adoption of these

tools requires not only significant investments in technological infrastructure, but also the development of analytical and management capabilities within local tax administrations [4].

The complexity inherent in real estate tax collection also requires constant evaluation of current tax policies and their alignment with long-term urban development and community well-being objectives. This analysis must consider not only the immediate demands for municipal financing but also the need to establish a stable and predictable fiscal environment that encourages investment and continued economic growth. Furthermore, tax equity and social justice should be central considerations in the design of tax strategies, ensuring that the costs and benefits of real estate taxes are distributed equitably among all sectors of urban society [5]. Real estate tax collection strategies are critical to the financial health and sustainable development of modern cities. This article will critically examine the different tools and approaches used in urban fiscal management, exploring their potential impact on the equity, efficiency and transparency of the local tax system. By integrating theoretical and practical perspectives, we will seek to offer informed recommendations to improve the effectiveness of these strategies in the current and future context of urban planning and fiscal administration.

2. Related Works.

2.1 Property Tax Collection.

To address the complexity of property tax collection, it is crucial to consider a variety of interrelated factors that impact both the efficiency and equity of the tax system. The task of collecting property taxes is not simply limited to the application of tax rates; It involves an in-depth analysis of the local economic structure, market dynamics, and current fiscal policies [6]. This process is not only technical, but also involves political and social elements that shape taxpayer perceptions and responses. First, the tax administration must establish effective methods for valuing assets, ensuring that appraisals are fair and accurate. This involves using appropriate tools such as comparative appraisals, income valuation, and other approaches that consider market fluctuations and the specific characteristics of each property. Accuracy in valuation not only guarantees equity among taxpayers, but also strengthens the legitimacy of the tax system in the eyes of the public.

Furthermore, the effectiveness of collection depends largely on the government's ability to ensure voluntary compliance with tax obligations. This goes beyond mere coercion and requires proactive strategies that foster tax awareness and the perception that taxes are a fair contribution to collective development. Initiatives such as tax education and transparency in the use of public funds can play a crucial role in this regard, encouraging a culture of compliance that reduces tax evasion and avoidance [5].

However, complexity arises when considering the challenges inherent to the informal economy and the difficulty in capturing income from unregulated sectors. In many contexts, a significant portion of the economy operates below the fiscal radar, limiting the government's ability to raise adequate resources. Addressing this gap requires innovative strategies that balance the need to incentivize formalization with measures to combat evasion and under-registration [9].

Additionally, the collection of property taxes may be affected by external factors such as changes in global economic policies, fluctuations in the prices of goods, and financial crises that impact the purchasing power of taxpayers. These events can alter tax revenue expectations and require agile responses from tax authorities to adjust strategies and mitigate financial risks [10]. It is crucial to also recognize that the effectiveness of collection policies is not only measured in terms of revenue generated, but also in their ability to promote social and economic equity. A fair tax system not only distributes the tax burden proportionally, but also uses revenue to finance essential public services and welfare programs that reduce inequalities and promote sustainable development. On the other hand, debates about the structure of property taxes also often involve considerations about the redistribution of wealth and the correction of economic asymmetries. The ability to adequately tax high-value properties, for example, can be crucial for financing affordable housing policies and improving accessibility to basic services in urban and rural areas [11].

Estate tax collection is a multifaceted process that requires comprehensive and adaptive strategic planning. From accurately valuing assets to managing voluntary compliance and mitigating external risks, every aspect of the tax system must be addressed with a holistic approach that considers both technical and social and political aspects. Only through an integrated and flexible approach can it be ensured that property taxes fulfill their economic and social function effectively and equitably in the current and future context.

2.2 Neutrosophic Cognitive Maps.

In the broad field of social and behavioral sciences, the need for analytical tools capable of capturing the complexity and uncertainty inherent in human interactions is becoming increasingly evident. Neutrosophic Cognitive Maps (NCM), an innovative methodology that integrates the principles of neutrosophic logic, have emerged as a promising solution to address this growing demand. This methodology not only allows modeling

situations that involve degrees of truth, falsity and indeterminacy, but also offers a more precise and nuanced representation of reality [12]. The conceptual basis of NCM is based on the theory of neutrosophic sets, developed by Florentin Smarandache, which extends classical logic to handle uncertainty, ambiguity and paradoxes. This theory introduces a third neutral value (N), in addition to the traditional values of truth (T) and falsehood (F), which enables a more flexible and adaptive representation of information. NCMs apply these principles to the field of cognitive maps, thus allowing a graphical and analytical representation of the causal relationships and dynamics of complex systems [13-14].

In the specific context of parenting skills and family and social problems, NCMs are presented as a powerful tool for evaluation and intervention. Family interactions are often marked by ambivalence and contradiction, which may exceed the capabilities of traditional approaches to capture this complexity. By incorporating neutrosophic elements, NCMs allow for a more detailed and richer representation of these dynamics, facilitating a deeper and more precise understanding [15].

The application of Neutrosophic Cognitive Maps (NCM) in analyzing parenting competencies involves identifying and modeling causal relationships between various factors and behaviors, such as how parent-child communication quality affects emotional development or how economic stress impacts boundary-setting. NCMs' strength lies in their ability to handle indeterminacy and uncertainty, intrinsic to human interactions. This is particularly valuable in assessing parenting competencies, where perception and subjectivity are crucial. NCMs help visualize causal relationships and interactions, revealing patterns and dynamics not evident through conventional analysis, which is essential for designing effective interventions. The use of NCM in evaluating parenting skills and addressing family issues benefits individuals and the broader community by informing more effective policies and support programs. NCMs offer a nuanced and realistic representation of reality, capturing inherent indeterminacy and uncertainty, providing new perspectives for tackling complex human interaction challenges. The continued adoption and development of NCMs promise significant advancements in the social and behavioral sciences, supporting families in diverse and changing contexts. In this study, neutrosophic cognitive maps will be used, so we explain them below[16].

Definition 1: Let X be a universe of discourse. A Neutrosophic Set (NS) is characterized by three membership functions, $u_A(x)$, $r_A(x)$, $v_A(x) : X \rightarrow]-0,1+[$, which satisfy the condition $n\text{-}0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3+$ for all $\in x$ $u_A(x)$, $r_A(x)$, and $v_A(x)$ are the true, indeterminate, and falsity membership functions of x in A, respectively, and their images are standard or non-standard subsets of $]-0,1+[$. [15]

Definition 2: Let X be a universe of discourse. A single-valued neutrosophic set (SVNS) A on X is a set of the form [16] :

$$A = \{ \langle x, u_A(x), r_A(x), v_A(x) \rangle : x \in X\} \quad (1)$$

Where u_A, r_A, $v_A : \in u_A(x)$, $r_A(x)$ and $v_A(x)$ are the true, indeterminate and falsity membership functions of x in A, respectively. For convenience, a single-valued neutrosophic number (SVNN) will be expressed as A = (a, b, c), where a, b, c $\in [0,1]$ and satisfies $0 \leq a + b + c \leq 3$.

Other important definitions are related to graphs.

Definition 3 [17]: A *neutrosophic graph* contains at least one indeterminate edge, represented by dotted lines.

Definition 4[18] : A *neutrosophic directed graph* is a directed graph that contains at least one indeterminate edge, which is represented by dotted lines .

Definition 5: A *neutrosophic cognitive map (NCM) is a* neutrosophic directed graph, whose nodes represent concepts and whose edges represent causal relationships between the edges.

If there are k vertices $C_1, C_2, \ldots, C_k$, each can be represented by a vector $(x_1, x_2, \ldots, x_k)$ where xi ∈ {0,1, I} depending on the state of the vertex C_i at a specific time or situation[19, 20, 21]:

- $x_i = 0$: Vertex C_i is in an activated state.
- $x_i = 1$: Vertex C_i is in disabled state.
- $x_i = I$: The state of vertex C_i is indeterminate.

Definition 6 : An NCM that has edges with weights in {−1, 0, 1, I} is called *a simple neutrosophic cognitive map* .

Connections between vertices: A directed edge from C_m to C_n is called a connection and represents causality from C_m to C_n.

Associate weights to each vertex: Each vertex in the NCM is associated with a weight within the set {0, 1, -1, I}. The edge weight $C_m C_n$, denoted as α_{mn}, indicates the influence of C_m on C_n and can be:

-$\alpha_{mn} = 0$: Cm has no effect on C_n.

-$\alpha_{mn} = 1$: An increase (decrease) in C_m results in an increase (decrease) in C_n.

-$\alpha_{mn} = -1$: An increase (decrease) in C_m results in a decrease (increase) in C_n.

-$\alpha_{mn} = I$: The effect of C_m on C_n is indeterminate.

Definition 7: If $C_1, C_2 \ldots, C_k$ are the vertices of an NCM. The neutrosophic matrix N(E) is defined as N(E) = α_{mn}), where α_{mn} denotes the weight of the directed edge $C_m C_n$, with $\alpha_{mn} \in [-1,0,1, I]$. N (E) is called *the neutrosophic adjacency matrix* of the NCM.

Definition 8: Let $C_1, C_2, \ldots, C_k$ be the vertices of an NCM. Let A = $(a_1, a_2 \ldots, a_k)$, where am ∈ {-1, 0, 1, I}. A is called *the neutrosophic instantaneous state vector* and means an on-off-indeterminate state position of the vertex at a given instant.

- $a_m = 0$ if C_m is disabled (has no effect),
- $a_m = 1$ if C_m is activated (takes effect),
- $a_m = I$ if C_m is indeterminate (its effect cannot be determined).

Definition 9: Let $C_1, C_2, \ldots, C_k$ be the vertices of an NCM. Let $\overrightarrow{C_1C_2}, \overrightarrow{C_2C_3}, \overrightarrow{C_3C_4}, \ldots, \overrightarrow{C_mC_n}$ the edges be the NCM, then the edges constitute a *directed cycle.*

- The NCM is said to be *cyclical* if it has a directed cycle. It is said to be *acyclic* if it does not have any directed cycle.

Definition 10: An NCM containing loops is said to have *feedback* . When there is feedback in the NCM it is said to be a *dynamic system* .

Definition 11: Let $\overrightarrow{C_1C_2}, \overrightarrow{C_2C_3}, \overrightarrow{C_3C_4}, \ldots, \overrightarrow{C_{k-1}C_k}$ be a cycle. when C_m is activated and its causality flows around the edges of the cycle and is then the cause of C_m itself, then the dynamical system is circulating. This is valid for each vertex C_m with m = 1, 2 ,… , k. The equilibrium state of this dynamic system is called the *hidden pattern.*

Definition 12: If the equilibrium state of a dynamic system is a single state, then it is called a *fixed point* . An example of a fixed point is when a dynamical system begins by being activated by C_1. If the NCM is assumed to be set to C_1 and C_k, meaning that the state remains as (1, 0,…, 0, 1), then this neutrosophic state vector is called a fixed point.

Definition 13: If the NCM establishes a repeating neutrosophic state vector of the form:

$A_1 \rightarrow A_2 \rightarrow \cdots \rightarrow A_m \rightarrow A_1$LCM *limit cycle* .

3. Results And Discussion.

The collection of real estate taxes involves the implementation of various strategies designed to ensure the effectiveness and equity of the tax system. Below are some key strategies that can be considered:

Est 1 - Review and update appraisals: Conduct a periodic review of property appraisals to reflect changes in market values and ensure that taxes are calculated fairly and accurately.

Est 2 - Implementation of progressive tax rates: Establish graduated tax rates that increase with property value, ensuring that owners of more valuable real estate contribute proportionately more to the tax system.

Est 3 - Selective tax incentives: Offer tax incentives for those owners who make improvements to their properties that benefit the community, such as the restoration of historic buildings or the implementation of green technologies.

Est 4 - Expansion of the tax base: Expand the taxpayer base by incorporating currently untaxed or undervalued properties, ensuring that all owners contribute equitably according to the value of their assets.

Est 5 - Improvement in administration and compliance: Strengthen audits and controls to detect and mitigate tax evasion, ensuring that all owners comply with their tax obligations effectively.

Est 6 - Tax education and awareness: Implement educational programs aimed at both property owners and the community in general about the importance of real estate taxes and how they contribute to local development and the provision of public services.

Est 7 - Technological modernization: Use advanced technologies such as geographic information systems (GIS) and digital platforms to improve cadastral management, property evaluation and efficient tax administration.

Est 8 - Review of tax exemptions and exemptions: Evaluate and review existing tax exemptions to ensure that they are justified and aligned with economic and social development objectives, eliminating those that do not effectively contribute to these purposes.

The process began by developing an NCM to represent the causal connections between the eight key real estate tax collection strategies . This stage involved defining the interactions between various strategies and visualizing them in a neutrosophic cognitive map, detailed in Figure 1.

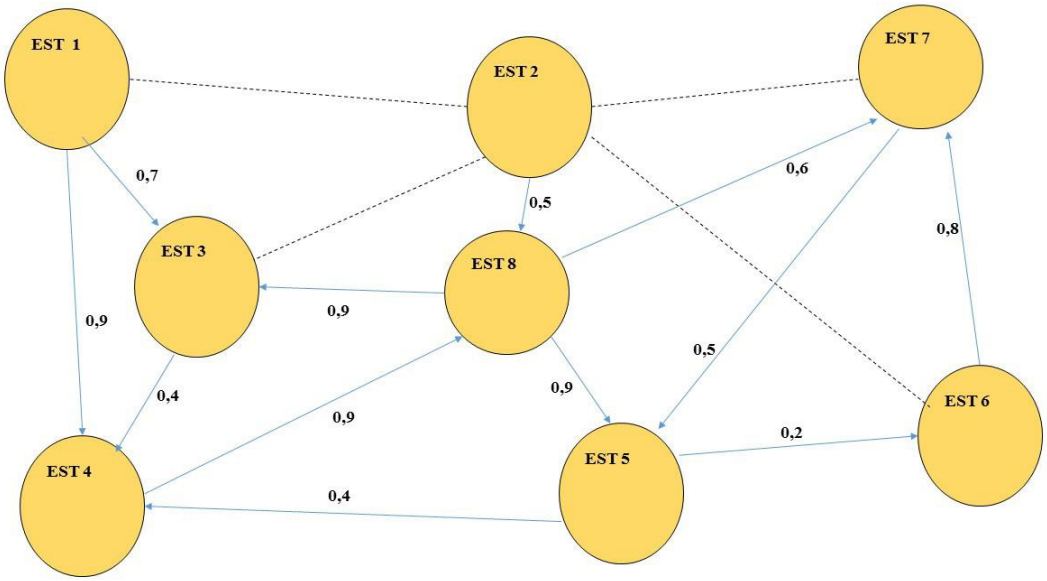

Figure 1: Neutrosophic cognitive map between collection strategies.

The NCM is developed through the collection and representation of relevant knowledge. The adjacency matrix obtained, which is based on the neutrosophic values provided by the specialists, is detailed in Table 1 as an essential tool to analyze and interpret the causal connections within the framework of the study.

Table 1: Adjacency matrix.

	STRATEGY1	**STRATEGY2**	**STRATEGY3**	**STRATEGY4**	**STRATEGY5**	**STRATEGY6**	**STRATEGY7**	**STRATEG8**
STRATEGY1	0	0	0.7	0.9	0	0	0	0
STRATEGY2	I	0	Io	0	0	I	I	0.5
STRATEGY3	0	0	0	0.4	0	0	0	0
STRATEGY4	0	I	0	0	0	0	0	0.9
STRATEGY5	0	0	0	0.4	0	0.2	0	0
STRATEGY6	I	0	0	0	0	0	0.8	0
STRATEGY7	0	0	0	0	0.5	0	0	0

	STRATEGY1	STRATEGY2	STRATEGY3	STRATEGY4	STRATEGY5	STRATEGY6	STRATEGY7	STRATEG8
STRATEG8	0	0	0.9	I	0	0	0.6	0

Following this perspective, the calculated centrality measures are presented below (Table 2). These metrics provide a quantitative analysis of the relative relevance of nodes within the network framework, which is crucial to understanding the dynamics and impact of the various components in the analyzed system.

Table 2: Centrality analysis Source: Own elaboration.

Node	*od(vi)*	*in(vi)*	*td(vi)*
STRATEGY1	1.6	0+2I	1.6+2I
STRATEGY2	0.5+4I	0+I	0.5+2I
STRATEGY3	0.4	1.6+I	2+I
STRATEGY4	0.9+I	1.7+I	2.6+2I
STRATEGY5	0.6+I	0.5	1.1+I
STRATEGY6	0.8	0.2+I	1+I
STRATEGY7	0.5	1.4+I	1.9+I
STRATEG8	1.5+I	1.4	2.9+I

In the context of static analysis in the NCM, initial results are obtained that incorporate the element of indeterminacy "I" within its neutrosophic values. To refine these results, it is essential to carry out a process known as deneutrosification, recommended by [22]. This process consists of replacing the indeterminacy parameter I, which ranges between 0 and 1, considering in this case "I" as 0.5. The importance of this method lies in its ability to produce more defined and precise results, which significantly simplifies the understanding of the interconnections present in the analysis in question (Table 3).

Table 3: Neutrosified centrality.

nod	*td(vi)*
STRATEGY1	2.6
STRATEGY2	1.5
STRATEGY3	2.5
STRATEGY4	3.6
STRATEGY5	1.6
STRATEGY6	1.5
STRATEGY7	2.4
STRATEG8	3.4

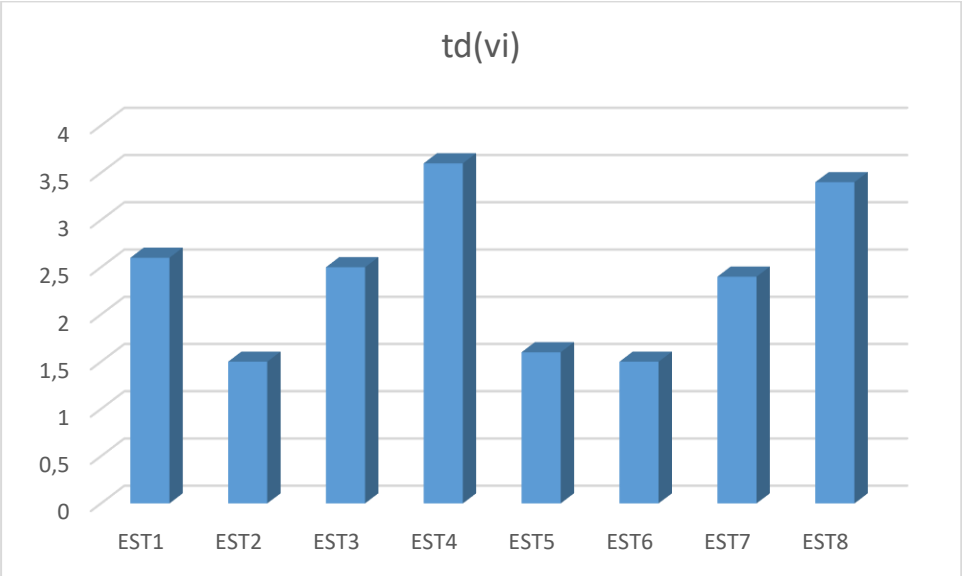

Figure 2: Deneutrosified centrality Source: own elaboration.

Table 4: Deneutrosified centrality ordered from highest to lowest. Source: self-made.

Factors	Deseutrosophized Centrality
Expansion of the tax base	**3.6**
Review of tax exemptions and exemptions	**3.4**
Review and update of appraisals	**2.6**
Selective tax incentives	**2.5**
Technological modernization	**2.4**
Improvement in administration and compliance	**1.6**
Implementation of progressive tax rates	**1.5**
Tax education and awareness	**1.5**

The most efficient strategies for collecting real estate taxes according to the study carried out are:

1. **Expanding the tax base:** Expand the taxpayer base by incorporating currently untaxed or undervalued properties, ensuring that all owners contribute equitably based on the value of their assets.
2. **Review of tax exemptions and exemptions:** Evaluate and review existing tax exemptions to ensure that they are justified and aligned with economic and social development objectives, eliminating those that do not effectively contribute to these purposes.
3. **Review and Update Appraisals:** Conduct a periodic review of property appraisals to reflect changes in market values and ensure that taxes are calculated fairly and accurately.

A recent study identifies the most efficient strategies for collecting real estate taxes, focusing on broadening the tax base, reviewing exemptions, and updating property appraisals. Broadening the tax base involves incorporating untaxed or undervalued properties to ensure equitable contribution, increasing revenue, and promoting fairness. Reviewing exemptions ensures alignment with economic and social goals, eliminating ineffective exemptions to maximize revenue and transparency. Regularly updating property appraisals ensures accurate and fair tax calculations, reflecting market changes and maintaining consistent revenue for public services. These strategies adapt to changing market conditions, reduce tax evasion, and enhance public confidence

in fiscal management. Implementing these measures comprehensively strengthens the tax system, promotes equity, and supports sustainable economic and social development, ensuring efficient use of tax revenues for community benefit.

4. Conclusion.

To conclude this study on the most efficient strategies for collecting real estate taxes, it is evident that the proposed actions have the potential to significantly transform local and regional tax systems. Broadening the tax base emerges as a crucial measure to increase tax revenues in an equitable and sustainable manner. By incorporating currently undervalued or untaxed properties, the disproportionate tax burden on some taxpayers can be reduced while promoting a fairer distribution of the tax burden based on the real value of real estate assets. Reviewing tax exemptions is also revealed as a fundamental strategy, requiring evaluation and adjustment to ensure alignment with economic and social development objectives. Eliminating ineffective exemptions not only strengthens the tax base but also promotes transparency and tax equity, essential for the legitimacy and public acceptance of the tax system. Periodically updating property appraisals is essential to maintaining the integrity and accuracy of the tax system, ensuring taxes are calculated fairly and accurately, optimizing tax collection, and improving the perception of fairness among taxpayers.

In terms of economic and social impact, these strategies can generate significant positive effects. A more efficient and equitable tax system provides the necessary resources to finance vital public services such as infrastructure and education while promoting a favorable environment for investment and local economic development. Optimizing real estate tax collection through these measures directly contributes to fiscal stability and the sustainable growth of communities. However, effective implementation requires a comprehensive approach adapted to local and regional realities, with policies designed to reflect real estate market dynamics and specific socioeconomic conditions. Robust monitoring and evaluation mechanisms are essential to ensure the long-term effectiveness and benefits of tax reforms. In summary, the strategies identified in this study represent a comprehensive set of actions aimed at strengthening tax administration and promoting equity in real estate tax contributions, ultimately contributing to sustainable economic development and social well-being. Future research could explore the use of neutrosophic approaches, such as Neutrosophic Cognitive Maps (NCMs) and neutrosophic TOPSIS, to manage uncertainties and indeterminacies in tax system reforms, providing new insights and methodologies for enhancing tax collection efficiency and equity.

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