Neutrosophic Sets and Systems, Vol. 68, 2024
University of New Mexico

On NCT- Set Theory

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Abstract: We introduce a new class of neutrosophic crisp set, and then it presented some operations via this sets like, $NCT-$intersection, $NCT-$union and algebraic $NCT-$difference to arrive at the algebraic ring construction. Also, we introduce the $NCT-$ points, which showed that a NCT –set is $NCT-$union of its $NCT-$points, and so did we introduced the concept of the function on this sets, called NCT – function and some of their important properties. Finally, we introduced the concept of the topology and some of the concepts entrusted to it, as an introduction to those spaces that can be studied in detail in the future.

Keywords: NCT – sets; $NCT-$points; $NCT-$function and $NCT-$topological spaces.

1. Introduction

The main focus of this research is the construction of a new type of neutrosophic crisp sets, and the first to know these sets neutrosophic and neutrosophic crisp sets is the scientist Florentin between the years 1999-2005 [1-3], when we looking at these sets, we notice that they are determined within spaces $\boldsymbol{X} \times [\mathbf{0}, \mathbf{1}]^{\mathbf{3}}$ and $\boldsymbol{P}(\boldsymbol{X}) \times \boldsymbol{P}(\boldsymbol{X}) \times \boldsymbol{P}(\boldsymbol{X})$, respectively when Zadeh [4] defined the fuzzy sets in 1965 , which were identified in space $\boldsymbol{X} \times [\mathbf{0}, \mathbf{1}]$, and through this, Salama and Florentin generalized these sets , which he called neutrosophic sets [5-7]. The researcher Almohammed [8] invested in the fuzzy sets by finding a new definition of the local function in 2020. Imran et al. [9-11] provided the view of new types of weakly neutrosophic crisp continuity, new concepts of weakly neutrosophic crisp separation axioms, and new concepts of neutrosophic crisp open sets. Molodtsov [12] found a new type of sets at the $\boldsymbol{E} \times \boldsymbol{P}(\boldsymbol{X})$ (where $\boldsymbol{X}$ is universal set and $\boldsymbol{E}$ the parameters of elements of $\boldsymbol{X}$) spaces and named them soft sets, where Al-Swidi and others [13-16] invested these sets by linking them with the fuzzy sets as well as defining new points, which in turn obtained equivalents for the separation axioms. Tomma et al. [17-19] gave the view of stable neutrosophic crisp topological space, necessary and sufficient conditions for a stability of the concepts of stable interior and stable exterior via neutrosophic crisp sets, and confused crisp set stable neutrosophic topological spaces. Al-Tamimi et al. [20] provided partner sets for generalizations of multi neutrosophic sets. Sfook et al. [21] introduced neutrosophic crisp grill topological spaces. Abdulsada et al. [22,23] provided the view of separation axioms of center topological space, and Center set theory of proximity space. Finally, the senses of new types of weakly neutrosophic crisp open mappings and new types of weakly neutrosophic crisp closed functions were informed by Al-Obaidi et al. [24,25]. In this research, we introduce a new concept of neutrosophic crisp set, and then it presented some operations via this sets like, $\boldsymbol{NCT}-$intersection, $\boldsymbol{NCT}-$union and algebraic

$\boldsymbol{NCT}$ −difference to arrive at the algebraic ring construction. Also, we introduce the $\boldsymbol{NCT}-$ points, which showed that a $\boldsymbol{NCT}$ –set is $\boldsymbol{NCT}$ −union of its $\boldsymbol{NCT}$ −points, and so did we introduced the class of the function on this sets, called $\boldsymbol{NCT}$ – function and some of their important properties.

2. NCT- Sets

We presented a new class of neutrosophic crisp sets, complete with a neutrosophic crisp point, all operations are binary, a ring-building qualification, and boolean algebra.

Definition 2.1. Let $X \neq \emptyset$. A neutrosophic crisp triple set NCT_A is an object having the form $NCT_A = \langle A_1, A_2, A_3 \rangle$. Where $A_1, A_2, A_3 \subseteq X$ satisfying $A_1 \subseteq A_2$ and $A_2 \cap A_3 = \emptyset$. And $NCT(X) = \{NCT_A = \langle A_1, A_2, A_3 \rangle : A_1 \subseteq A_2$ and $A_2 \cap A_3 = \emptyset\}$ is the collection of all $NCT-$ sets on X.
From this definition we see that if $A_3 = X$, then $A_1 = A_2 = \emptyset$ and if $A_2 = X$, then $A_3 = \emptyset$, finally if $A_1 = X$, then $A_2 = X$ and $A_3 = \emptyset$.

Definition 2.2. Let $NCT_A = \langle A_1, A_2, A_3 \rangle$ and $NCT_B = \langle B_1, B_2, B_3 \rangle$ are NCT −a non-empty set X over sets. Therefore:

1. NCT_A is a $NCT-$ subset of NCT_B if $A_3 \supseteq B_3$ and $A_1 \subseteq B_1, A_2 \subseteq B_2$. We write $NCT_A \sqsubseteq NCT_B$.
2. $NCT_A = NCT_B$ iff $NCT_A \sqsubseteq NCT_B$ and $NCT_B \sqsubseteq NCT_A$.
3. The NCT −complement of $NCT-$ set NCT_A is $CNCT_A = \langle A_3, A_2^c, A_1 \rangle$.
4. $NCT_A \sqcup NCT_B = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 \rangle$ is a crisp triple set that is neutrosophic in union (NCT −union set).
5. $NCT_A \sqcap NCT_B = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$ is the intersection a crisp triple set that is neutrosophic (NCT −intersection sets).
6. $NCT_A - NCT_B = NCT_A \sqcap CNCT_B$.
7. $NCT_A \ \triangle \ NCT_B = (NCT_A \sqcap CNCT_B) \sqcup (CNCT_A \sqcap NCT_B)$.

Now we will explain the concepts NCT −universel and NCT −null set which are among the basic concepts in our work.

Definition 2.3. Let $X \neq \emptyset$. Then:

1. $NCT_X = \langle X, X, \emptyset \rangle$ is NCT −universel set.
2. $NCT_\varphi = \langle \emptyset, \emptyset, X \rangle$ is NCT −null set. Clearly $CNCT_X = NCT_\varphi$ and $CNCT_\varphi = NCT_X$.

The three most significant relationships NCT −union, NCT −intersection and NCT −complement listed in the following properties.

Proposition 2.4. Let $NCT_S = \langle S_1, S_2, S_3 \rangle$ and $NCT_J = \langle J_1, J_2, J_3 \rangle$ are $NCT-$ a nonempty set X over sets. Then:

1. $NCT_S \sqcap NCT_S = NCT_S$.
2. $NCT_S \sqcup NCT_\varphi = NCT_S$.
3. $NCT_S \sqcup NCT_X = NCT_X$.
4. $NCT_S \sqcap NCT_\varphi = NCT_\varphi$.
5. $C(NCT_S \sqcup NCT_J) = CNCT_S \sqcap CNCT_J$.
6. $C(NCT_S \sqcap NCT_J) = CNCT_S \sqcup CNCT_J$.
7. $NCT_S \sqcup NCT_S = NCT_S$.

Proving the above proposition directly by applying Definitions 2.1, 2.2 and 2.3.
For any $NCT_A = \langle A_1, A_2, A_3 \rangle$, we get the following two properties , but the opposite is not necessarily true $NCT_\varphi \sqsubseteq NCT_A \sqcap CNCT_A$ and $NCT_A \sqcup CNCT_A \sqsubseteq NCT_X$ if $X = \{q_1, q_2, q_3\}$ and $NCT_A =$

$\langle\{q_1\},\{q_1,q_2\},\{q_3\}\rangle$, then $NCT_A \sqcap CNCT_A = \langle\emptyset,\emptyset,\{q_1,q_3\}\rangle \not\sqsubseteq NCT_\varphi = \langle\emptyset,\emptyset,X\rangle$. And, if $X = \{q_1,q_2,q_3\}$ and $NCT_A = \langle\{q_1\},\{q_1,q_2\},\{q_3\}\rangle$, then $NCT_X = \langle X,X,\emptyset\rangle \not\sqsubseteq (NCT_A \sqcup CNCT_A) = \langle\{q_1,q_3\},X,\emptyset\rangle$.

The following proposition shows the algebraic properties (associative and distributive laws) of these $NCT-$ sets via $NCT-$ union and $NCT-$intersection relations.

Proposition 2.5. Let $NCT_O = \langle O_1,O_2,O_3\rangle$, $NCT_Q = \langle Q_1,Q_2,Q_3\rangle$ and $NCT_J = \langle J_1,J_2,J_3\rangle$ over a nonempty set X, be three $NCT-$sets. Then:

1. $NCT_O \sqcap (NCT_Q \sqcup NCT_J) = (NCT_O \sqcap NCT_Q) \sqcup (NCT_O \sqcap NCT_J)$.
2. $NCT_O \sqcap (NCT_Q \sqcap NCT_J) = (NCT_O \sqcap NCT_Q) \sqcap NCT_J$.
3. $NCT_O \sqcup (NCT_Q \sqcup NCT_J) = (NCT_O \sqcup NCT_Q) \sqcup NCT_J$.
4. $NCT_O \sqcup (NCT_Q \sqcap NCT_J) = (NCT_O \sqcup NCT_Q) \sqcap (NCT_O \sqcup NCT_J)$.

New we defined the $NCT-$union and $NCT-$intersection relations on any collection of $NCT-$sets.

Definition 2.6. Let $\{NCT_{A_i} : i \in I\}$ be a $NCT-$sets in X, and $NCT_{A_i} = \langle A_{i1},A_{i2},A_{i3}\rangle$. Then

1. $\sqcup_{i\in I} NCT_{A_i} = \langle\cup_{i\in I} A_{i1},\cup_{i\in I} A_{i2},\cap_{i\in I} A_{i3}\rangle$.
2. $\sqcap_{i\in I} NCT_{A_i} = \langle\cap_{i\in I} A_{i1},\cap_{i\in I} A_{i2},\cup_{i\in I} A_{i3}\rangle$.

Proposition 2.7. Let NCT_A, NCT_B, NCT_C and $\{NCT_{A_i} : i \in I\}$ in X.Then

1. $NCT_A \sqsubseteq NCT_B$ and $NCT_B \sqsubseteq NCT_C$, implies $NCT_A \sqsubseteq NCT_C$.
2. $NCT_{A_i} \sqsubseteq NCT_B$ $\forall i \in I$, then $\sqcup_{i\in I} NCT_{A_i} \sqsubseteq NCT_B$.
3. $NCT_B \sqsubseteq NCT_{A_i}$ $\forall i \in I$, then $NCT_B \sqsubseteq \sqcap_{i\in I} NCT_{A_i}$.
4. $NCT_A \sqsubseteq NCT_B$ iff $CNCT_B \sqsubseteq CNCT_A$.

Proof: Obvious.

Now we present the definition of points $NCT-$points.

Definition 2.8. Let $X \neq \emptyset$ and $p \in X$. So $NCT-$points ($NCTP$) are structure:

1. $NCT_{\tilde{p}} = \langle\{p\},\{p\},\{p\}^c\rangle$.
2. $NCT_{\tilde{\tilde{p}}} = \langle\emptyset,\{p\},\{p\}^c\rangle$.
3. $NCT_{\tilde{\tilde{\tilde{p}}}} = \langle\emptyset,\emptyset,\{p\}^c\rangle$.

It's simple to see that $NCT-$points are $NCT-$sets. Also, the cardinal number of all $NCT-$points is $3n$, where n is the cardinal number of universal sets X.

Definition 2.9. Let $X \neq \emptyset$ and $p \in X$ and $NCT_A = \langle A_1,A_2,A_3\rangle$. Then the $NCT-$belong as follows:

1. $NCT_{\tilde{p}}$ € NCT_A iff $p \in A_1$ and $NCT_{\tilde{p}} \breve{€}$ NCT iff $p \notin A_1$.
2. $NCT_{\tilde{\tilde{p}}}$ € NCT_A iff $p \in A_2$ and $NCT_{\tilde{\tilde{p}}} \breve{€}$ NCT iff $p \notin A_2$.
3. $NCT_{\tilde{\tilde{\tilde{p}}}}$ € NCT_A iff $p \notin A_3$ and $NCT_{\tilde{\tilde{\tilde{p}}}} \breve{€}$ NCT iff $p \notin A_3$.

We now take the properties of belonging to the three points.

Proposition 2.10. Let $\{NCT_{A_i} : i \in I\}$ is $NCT-$set in X. Then

1. $NCT_{\tilde{p}}$ € $\sqcap_{i\in I} NCT_{A_i}$ iff $NCT_{\tilde{p}}$ € NCT_{A_i} for each $i \in I$.
2. $NCT_{\tilde{\tilde{p}}}$ € $\sqcap_{i\in I} NCT_{A_i}$ iff $NCT_{\tilde{\tilde{p}}}$ € NCT_{A_i} for each $i \in I$.
3. $NCT_{\tilde{\tilde{\tilde{p}}}}$ € $\sqcap_{i\in I} NCT_{A_i}$ iff $NCT_{\tilde{\tilde{\tilde{p}}}}$ € NCT_{A_i} for each $i \in I$.
4. $NCT_{\tilde{p}}$ € $\sqcup_{i\in I} NCT_{A_i}$ iff $\exists i \in I$ such that $NCT_{\tilde{p}}$ € NCT_{A_i}.
5. $NCT_{\tilde{\tilde{p}}}$ € $\sqcup_{i\in I} NCT_{A_i}$ iff $\exists i \in I$ such that $NCT_{\tilde{\tilde{p}}}$ € NCT_{A_i}.
6. $NCT_{\tilde{\tilde{\tilde{p}}}}$ € $\sqcup_{i\in I} NCT_{A_i}$ iff $\exists i \in I$ $\ni NCT_{\tilde{\tilde{\tilde{p}}}}$ € NCT_{A_i}.

Proposition 2.11. Let NCT_A and NCT_B is $NCT-$set in X. Then:

i. $NCT_A \sqsubseteq NCT_B$ iff $\forall NCT_{\tilde{P}}$ with $NCT_{\tilde{P}} \in NCT_A \Rightarrow NCT_{\tilde{P}} \in NCT_B$, for each $NCT_{\tilde{\tilde{P}}}$ with $NCT_{\tilde{\tilde{P}}} \in NCT_A \Rightarrow NCT_{\tilde{\tilde{P}}} \in NCT_B$ and for each $NCT_{\tilde{\tilde{\tilde{P}}}}$ we have $NCT_{\tilde{\tilde{\tilde{P}}}} \in NCT_A \Rightarrow NCT_{\tilde{\tilde{\tilde{P}}}} \in NCT_B$.

ii. $NCT_A = NCT_B$ iff for each $NCT_{\tilde{P}}$ we have $NCT_{\tilde{P}} \in NCT_A \Leftrightarrow NCT_{\tilde{P}} \in NCT_B$ and for each $NCT_{\tilde{\tilde{P}}}$ we have $NCT_{\tilde{\tilde{P}}} \in NCT_A \Leftrightarrow NCT_{\tilde{\tilde{P}}} \in NCT_B$ and for each $NCT_{\tilde{\tilde{\tilde{P}}}}$ we have $NCT_{\tilde{\tilde{\tilde{P}}}} \in NCT_A \Leftrightarrow NCT_{\tilde{\tilde{\tilde{P}}}} \in NCT_B$.

Proposition 2.12. Let $NCT_A = \langle A_1, A_2, A_3 \rangle$ be triple in X . Then $NCT_A = (\{NCT_{\tilde{P}} : NCT_{\tilde{P}} \in NCT_A\}) \sqcup \left(\{NCT_{\tilde{\tilde{P}}} : NCT_{\tilde{\tilde{P}}} \in NCT_A\}\right) \sqcup \left(\left\{NCT_{\tilde{\tilde{\tilde{P}}}} : NCT_{\tilde{\tilde{\tilde{P}}}} \in NCT_A\right\}\right)$.

Proposition 2.13. Let $NCT_S = \langle S_1, S_2, S_3 \rangle$, $NCT_J = \langle J_1, J_2, J_3 \rangle$ and $NCT_R = \langle R_1, R_2, R_3 \rangle$ be $NCT-$sets. Then:

1. $NCT_S \sqcup NCT_J \sqsupseteq (NCT_S \sqcap NCT_J) \sqcup (NCT_J - NCT_S) \sqcup (NCT_S - NCT_J)$.
2. $(NCT_S - NCT_J) \sqcup (NCT_S - NCT_R) = NCT_S - (NCT_J \sqcap NCT_R)$.
3. $(NCT_S \sqcup NCT_J) - (NCT_R - NCT_S) = NCT_S \sqcup (NCT_J - NCT_R)$.
4. $(NCT_S \sqcap NCT_J) - (CNCT_S \sqcup NCT_R) = NCT_S \sqcap (NCT_J - NCT_R)$.
5. Not necessary if $NCT_S \sqsubseteq NCT_J$ and $NCT_S \sqsubseteq CNCT_J$, then $NCT_S = NCT_\varphi$.
6. Not necessary if $NCT_S \sqsubseteq NCT_J$ and $CNCT_S \sqsubseteq NCT_J$ then $NCT_S = NCT_X$.
7. $(NCT_S \sqcup NCT_J) - NCT_J \sqsupseteq NCT_S - NCT_J$.
8. $NCT_S - NCT_J \sqsubseteq NCT_S - (NCT_S \sqcap NCT_J)$.
9. $\sqcap \{NCT_J \in NCT_S(X)\} = NCT_\varphi$.
10. $NCT_S \sqcap CNCT_J \sqsubseteq (NCT_S \sqcup NCT_J) \sqcap CNCT_J$.
11. $(NCT_S \sqcup NCT_J) - (NCT_J \sqcap NCT_S) \sqsupseteq (NCT_S - NCT_J) \sqcup (NCT_J - NCT_S)$.
12. $NCT_S - (NCT_J \sqcup NCT_R) = (NCT_S - NCT_J) - NCT_R$.
13. $NCT_S \ \varDelta NCT_\varphi = NCT_S$.
14. $NCT_\varphi = NCT_S \ \varDelta NCT_S$ if and only if $S_1 \cup S_3 = X$.
15. $NCT_S \ \varDelta \ NCT_J = NCT_J \ \varDelta \ NCT_S$.

Proof.

The converse of part (1) is not true in general for example, if $X = \{o_1, o_2, o_3\}$, $NCT_A = \langle \{o_1\}, \{o_1, o_2\}, \{o_3\} \rangle$ and $NCT_B = \langle \{o_2\}, \{o_2, o_3\}, \{o_1\} \rangle$, then: $NCT_A - (NCT_A \sqcap NCT_B) = \langle \{o_1\}, \{o_1\}, \{o_3\} \rangle \not\sqsubseteq NCT_A - NCT_B = \langle \{o_1\}, \{o_1\}, \{o_2, o_3\} \rangle$.

The converse of part 2 is not true generally, for instance, if $X = \{e_1, e_2, e_3\}$, $NCT_A = \langle \{e_1\}, \{e_1, e_2\}, \{e_3\} \rangle$ and $NCT_B = \langle \{o_2\}, \{o_2, o_3\}, \{o_1\} \rangle$,then: $NCT_A \sqcup NCT_B = \langle \{o_1, o_2\}, X, \emptyset \rangle \not\sqsubseteq (NCT_A \sqcap NCT_B) \sqcup (NCT_B - NCT_A) \sqcup (NCT_A - NCT_B) = \langle \{o_1\}, \{o_1, o_3\}, \emptyset \rangle$.

Part 7 , Let $X = \{o_1, o_2, o_3\}$, $NCT_A = \langle \emptyset, \emptyset, \{o_1, o_2\} \rangle$ and $NCT_B = \langle \emptyset, \emptyset, \{o_1\} \rangle$, then $NCT_A \sqsubseteq NCT_B$,$NCT_A \sqsubseteq CNCT_B$ and $NCT_A \neq NCT_\varphi$.

Part 8 , Let $X = \{o_1, o_2, o_3\}$, $NCT_A = \langle X, X, \{o_1, o_2\} \rangle$ and $NCT_B = \langle X, X, \{o_1\} \rangle$, then $NCT_A \sqsubseteq NCT_B$, $CNCT_A \sqsubseteq NCT_B$ and $NCT_A \neq NCT_X$.

Since Proposition 2.13, part 12 assures that the NCT −null set serves as an identification element for Proposition 13.2, part 13 ensures that every member of $NC^*(X) = \{NCT_A = \langle A_1, A_2, A_3 \rangle : NCT_A \ is \ NCT-set \ and \ A_1 \cup A_3 = X\}$.

Finally, the fact that part 14 has its own inverse demonstrates that the symbol is commutative. All of this lends credence to the contention that $(NC^*(X) \ , \varDelta)$ is a commutative group.

Theorem 2.14. Let X is non-null and $NC^*(X) = \{NCT_S = \langle S_1, S_2, S_3 \rangle : NCT_A \ is \ NCT-set \ and \ S_1 \cup S_3 = X\}$ on X. So $(NC^*(X), \varDelta, \sqcap)$ form a ring.

Proof.

According to propositions 2.4 and propositions 2.5, the groups $(NC^*(X),\sqcap)$ and $(NC^*(X)\ ,\varDelta)$ are semigroups and commutative groups. Only the distribution on the left must be examined $\sqcap$ operation on $\varDelta$.

$(NCT_S \sqcap NCT_W)\varDelta(NCT_S \sqcap NCT_P) = \{(NCT_S \sqcap NCT_W) \sqcap (CNCT_S \sqcup CNCT_P\ )\} \sqcup \{(NCT_S \sqcap NCT_P\ ) \sqcap (CNCT_S \sqcup CNCT_W)\} = \{[(NCT_S \sqcap NCT_W) \sqcap CNCT_S] \sqcup [(NCT_S \sqcap NCT_W) \sqcap CNCT_P\ ]\} \sqcup \{[(NCT_S \sqcap NCT_P) \sqcap CNCT_S] \sqcup [(NCT_S \sqcap NCT_P) \sqcap CNCT_W]\} = \{(NCT_S \sqcap NCT_W) \sqcap CNCT_P\ \} \sqcup \{(NCT_S \sqcap NCT_P) \sqcap CNCT_W\} = NCT_S \sqcap (NCT_W \varDelta NCT_P)$. Therefore $(NC^*(X),\sqcap\ ,\varDelta)$ is a ring.

Now we introduce the concept of NCT-function.

Definition 2.15. Let $f : X \to Y$ be a function. Define NCT –function $f_{NCT} : NCT(X) \to NCT(Y)$ by:

1. If $NCT_A = \langle A_1, A_2, A_3\rangle \in\ NCT_A(X)$,then $f_{NCT}(NCT_A) = \langle f(A_1), f(A_2), f-(A_3)\rangle$, where $f-(A_3) = Y - (f(X - A_3))$.
2. If $NCT_B = \langle R_1, R_2, R_3\rangle \in NCT_B(Y)$, then ${f_{NCT}}^{-1}(NCT_B) = \langle f^{-1}(R_1), f^{-1}(R_2), f^{-1}(R_3)\rangle$.

We now take the most important properties of the NCT-function that we will adopt in our research.

Proposition 2.16. Let $f_{NCT} : NCT(X) \to NCT(Y)$ be a $NCT-$function and NCT_A, $NCT_{A_i}(i \in I) \in NCT(X), NCT_B\ , NCT_{B_j}(j \in J) \in NCT(Y)$. Then:

1. $NCT_{A_1} \sqsubseteq NCT_{A_2} \Rightarrow f_{NCT}(NCT_{A_1}) \sqsubseteq f_{NCT}(NCT_{A_2})$.
2. $NCT_{B_1} \sqsubseteq NCT_{B_2} \Rightarrow {f_{NCT}}^{-1}(NCT_{B_1}) \sqsubseteq {f_{NCT}}^{-1}(NCT_{B_2})$.
3. If $NCT_A \sqsubseteq {f_{NCT}}^{-1}(f_{NCT}(NCT_A))$ and f is 1-1, then $NCT_A = {f_{NCT}}^{-1}(f_{NCT}(NCT_A))$.
4. If $f_{NCT}\left({f_{NCT}}^{-1}(NCT_B)\right) \sqsubseteq NCT_B$ and f is onto, then $f_{NCT}\left({f_{NCT}}^{-1}(NCT_B)\right) = NCT_B$
5. ${f_{NCT}}^{-1}\left(\sqcup NCT_{B_j}\right) = \sqcup {f_{NCT}}^{-1}(NCT_{B_j})$.
6. ${f_{NCT}}^{-1}\left(\sqcap NCT_{B_j}\right) = \sqcap {f_{NCT}}^{-1}(NCT_{B_j})$.
7. $f_{NCT}(\sqcup NCT_{A_i}) = \sqcup f_{NCT}(NCT_{A_i})$.
8. $f_{NCT}(\sqcap NCT_{A_i}) \sqsubseteq (\sqcap f_{NCT}(NCT_{A_i}))$ and if f is 1-1, then $f_{NCT}(\sqcap NCT_{A_i}) = \sqcap f_{NCT}(NCT_{A_i})$.
9. ${f_{NCT}}^{-1}(NCT_Y) = NCT_X$.
10. ${f_{NCT}}^{-1}(NCT_\varphi) = NCT_\varphi$.
11. $f_{NCT}(NCT_X) = NCT_Y$, if f is onto.
12. $f_{NCT}(NCT_\varphi) = NCT_\varphi$.

Proof.

Let $NCT_{A_i} = \langle A_{i1}, A_{i2}, A_{i3}\rangle$, $NCT_{B_j} = \langle B_{j1}, B_{j2}, B_{j3}\rangle$, $(i \in I, j \in J)$, $NCT_A = \langle A_1, A_2, A_3\rangle$ and $NCT_B = \langle B_1, B_2, B_3\rangle$.

1. Let $NCT_{A_1} \sqsubseteq NCT_{A_2}$. Since $A_{11} \subseteq A_{21}, A_{12} \subseteq A_{22}$ and $A_{23} \subseteq A_{13}$,then $f(A_{11}) \subseteq f(A_{21}), f(A_{12}) \subseteq f(A_{22})$ and $X - A_{13} \subseteq X - A_{23} \Rightarrow f(X - A_{13}) \subseteq f(X - A_{23}) \Rightarrow\ Y - f(X - A_{23}) \subseteq Y - f(X - A_{13}) \Rightarrow f-(A_{23}) \subseteq f-(A_{13})$. Hence $f_{NCT}(NCT_{A_1}) \sqsubseteq f_{NCT}(NCT_{A_2})$.
2. It is similar to (1.).
3. ${f_{NCT}}^{-1}(f_{NCT}(NCT_A)) = {f_{NCT}}^{-1}(f_{NCT}(\langle A_1, A_2, A_3\rangle)) = {f_{NCT}}^{-1}(\langle f(A_1), f(A_2), f-(A_3)\rangle) = \langle f^{-1}(f(A_1)), f^{-1}(f(A_2)), f^{-1}(f-(A_3))\rangle \sqsupseteq \langle A_1, A_2, A_3\rangle = NCT_A$.
4. It is similar to (3.).
5. ${f_{NCT}}^{-1}\left(\sqcup NCT_{B_j}\right) = {f_{NCT}}^{-1}(\langle \cup B_{j1}, \cup B_{j2}, \cap B_{j3}\rangle) = \langle f^{-1}(\cup B_{j1}), f^{-1}(\cup B_{j2}), f^{-1}(\cap B_{j3})\rangle = \langle \cup f^{-1}(B_{j1}), \cup f^{-1}(B_{j2}), \cap f^{-1}(B_{j3})\rangle = \sqcup {f_{NCT}}^{-1}(NCT_{B_j})$.
6. It is similar to (5.).

7. $f_{NCT}(\sqcup NCT_{A_i}) = f_{NCT}(\langle \cup A_{i1}, \cup A_{i2}, \cap A_{i3}\rangle) = \langle f(\cup A_{i1}), f(\cup A_{i2}), f-(\cap A_{i3})\rangle = \langle \cup f(A_{i1}), \cup f(A_{i2}), \cap f-(A_{i3})\rangle = \sqcup F(NCT_{A_i})$. Noties that $f-(\cap A_{i3}) = Y - f(X - \cap A_{i3}) = Y - f(\cup (X - A_{i3}) = Y - \cup f(X - A_{i3}) = \cap (Y - f(X - A_{i3})) = \cap f - (A_{i3})$.
8. $f_{NCT}(\sqcap NCT_{A_i}) = f_{NCT}(\langle \cap A_{i1}, \cap A_{i2}, \cup A_{i3}\rangle) = \langle f(\cap A_{i1}), f(\cap A_{i2}), f-(\cup A_{i3})\rangle \sqsubseteq \langle \cap f(A_{i1}), \cap f(A_{i2}), \cup f-(A_{i3})\rangle = \sqcap F(NCT_{A_i})$. Noties that $f-(\cup A_{i3}) = Y - f(X - \cup A_{i3}) = Y - f(\cap (X - A_{i3}) \supseteq Y - \cap f(X - A_{i3}) = \cup (Y - f(X - A_{i3})) = \cup f - (A_{i3})$.
9. ${f_{NCT}}^{-1}(NCT_Y) = {f_{NCT}}^{-1}(\langle Y, Y, \emptyset\rangle) = \langle f^{-1}(Y), f^{-1}(Y), f^{-1}(\emptyset)\rangle = \langle X, X, \emptyset\rangle = NCT_X$.

(10.), (11.), (12.) are like (9.).

3. Neutrosophic Crisp Triple Topological Space

In this section, we investigate some of the properties generated by $NCT-$sets, such as interior, exterior and boundary $NCT-$ points, which are the structure for all topological concepts, as well as closures.

Definition 3.1. The pair (NCT_X, τ^X_{NCT}) is called neutrosophic crisp triple topological space ($NCTT$) over NCT (X) , if you achieve the following:

1. $NCT_X, NCT_\varphi \in \tau^X_{NCT}$ ($\in$ is the classical belonging).
2. τ^X_{NCT} is closed under the finite $NCT-$intersection.
3. τ^X_{NCT} is closed under the $NCT-$union of every subfamily of τ_{NCT} .

Any member of τ_{NCT} is called $NCT-$open and the complement is called $NCT-$closed.

i- For any NCT- set NCT_A the NCT – interior of NCT_A is of the form $NCT-$int $(NCT_A) = \sqcup\{ NCT_P ; \exists NCT_H \in \tau_{NCT} \ni NCT_P \in NCT_H \sqsubseteq NCT_A \}$. From this definition we can show that $NCT-$int $(NCT_A) = \sqcup\{ NCT_H \in \tau^X_{NCT} ; NCT_H \sqsubseteq NCT_A \}$.

- $NCT_A \in \tau_{NCT}$ iff $NCT-$int $(NCT_A) = NCT_A$.

ii- For any NCT-set NCT_A the $NCT-$closure of NCT_A is of the form $NCT-cl(NCT_A) = \sqcup \{ NCT_P; \forall NCT_H \in \tau^X_{NCT} \ni NCT_P \in NCT_H \ni NCT_A \sqcap NCT_H \neq NCT_\varphi \}$.

- From above we can show that $NCT - cl$ $(NCT_A) = \sqcap\{ NCT_F ; CNCT_F \in \tau_{NCT} \ni NCT_A \sqsubseteq NCT_F \}$.
- NCT_F is $NCT-$closed iff $NCT - cl(NCT_F) = NCT_F$.

iii- For any $NCT-$set NCT_A the $NCT-$exterior of NCT_A is of the form $NCT-$ext $(NCT_A) = NCT-$int $(CNCT_A)$.

iv- For any $NCT-$set NCT_A the $NCT-$ boundary of NCT_A is of the form $NCT-$fr $(NCT_A) = \sqcup\{ NCT_P ; NCT_P$ not NCT$-$interior and NCT$-$exterior point of NCT-A}.

So, from definition and properties above we can concluded.

1- $NCT_X = NCT-$int $(NCT_A) \sqcup$ NCT$-$ext$(NCT_A) \sqcup$ NCT$-$fr (NCT_A), for any $NCT-$set NCT_A.

2- $NCT-$int $(NCT_A) = C(NCT$-cl $(CNCT_A))$ and $NCT - cl(NCT_A) = C(NCT-int(CNCT_A))$.

Lemma 3.2. If NCT_H is $NCT-$open set and any $NCT-$set NCT_A , then $NCT_H \sqcap$ NCT$-$cl$(NCT_A) \sqsubseteq$ NCT$-$cl$(NCT_A \sqcap NCT_H)$.

Proof.

Let $NCT_P \in NCT_H \sqcap$ NCT$-$cl(NCT_A) , if possible, that $NCT_P \breve{\notin}$ NCT$-$cl$(NCT_A \sqcap NCT_H)$ .Then there is some $NCT-$open set NCT_K containing NCT_P and $NCT_K \sqcap NCT_A \sqcap NCT_H = NCT_\varphi$, but $NCT_K \sqcap NCT_H \in \tau_{NCT}$ and $NCT_P \in NCT_K \sqcap NCT_H$, this shows that $NCT_P \breve{\notin}$ NCT$-$cl(NCT_A) , which contradiction . This obligates us to $NCT_P \in$ NCT$-$cl$(NCT_A \sqcap NCT_H)$.

Proposition 3.3. Let $(NCT_X \ ,\tau_{NCT}^X \ )$be any $NCTT$ over $NCT(X)$,then the following properties are hold:

1- If $NCT_H \in \tau_{NCT}^X$ or $NCT_K \ \in \tau_{NCT}^X$, then $NCT-\text{int}\ (NCT-cl(NCT_K \sqcap \ NCT_H\ ))\ = NCT-int(NCT-cl\ (NCT_K)) \sqcap \ \text{NCT}-\text{int}(\text{NCT}-\text{cl}(NCT_H)))$.

2- $NCT-int\ (\ NCT-cl\ (\ NCT-int\ (NCT-c\ 1\ \ (\ NCT_A\ ))))\ =\ NCT-int\ (NCT-cl\ (NCT_A))$.

3- $NCT-cl\ (\ NCT-int\ (\ NCT-cl\ \ (NCT-int\ (NCT_A\ ))))\ =\ NCT-cl\ (NCT\ int\ (NCT_A))$.

4- $NCT-int\ (\ NCT-cl\ (\ NCT-int\ \ (NCT_A \sqcap NCT_B)))\ =\ NCT-int\ (NCT-cl\ (NCT-int\ (NCT_A))) \sqcap \text{NCT}-\text{int}\ (\ \text{NCT}-\text{cl}(\text{NCT}-\text{int}\ (NCT_B))$.

5- $NCT-cl\ (NCT-int\ (NCT-cl\ (NCT_A\ \ \sqcup NCT_B\ )))\ =\ NCT-cl\ (NCT-int\ (NCT-cl\ (NCT_A))) \sqcup NCT-cl(\text{NCT}-\text{int}\ (\ \text{NCT}-\text{cl}\ (NCT_B)))$.

Proof.

(1) Since $NCT_H \sqsubseteq NCT-cl(NCT_H)$ and $NCT_K \sqsubseteq NCT-cl(NCT_K)$. So, $NCT-int\ (NCT-cl\ (NCT_K \sqcap \ NCT_H) \ \sqsubseteq NCT-int(NCT-cl\ (NCT_K) \sqcap \ \text{NCT}-\text{cl}(\ NCT_H)) = \ NCT-int\ (NCT-cl\ (NCT_K)) \sqcap \ \ \text{NCT}-\text{int}(\text{NCT}-\text{cl}(NCT_{H)}))$. Conversely, If $NCT_H \in \tau_{NCT}^X$ or $NCT_K \ \in \tau_{NCT}^X$, then by Lemma 3.2. For if $NCT_H \in \tau_{NCT}$, $NCT_H \sqcap NCT-int(\ \text{NCT}-\text{cl}(\ NCT_K)) \sqsubseteq NCT-int(NCT-cl\ (NCT_H \sqcap \ \ NCT_K))$. (1)

Imply that $NCT-int\ (NCT-cl\ (NCT_H \sqcap NCT-int(\ \text{NCT}-\text{cl}(\ NCT_K)))) \sqsubseteq NCT-int(NCT-cl(NCT-int(NCT-cl\ (NCT_H \sqcap \ \ NCT_K))))$. (2)

But $NCT-int(NCT-cl(NCT_K\ \ )) \in \tau_{NCT}^X$, again, by Lemma 3.2, we get that $NCT-int(NCT-cl(NCT_H)) \sqcap \ \ \text{NCT}-\text{int}(\text{NCT}-\text{int}(\text{NCT}-\text{cl}(NCT_K))) = NCT-int(NCT-cl(NCT_H)) \sqcap \ \ \text{NCT}-\text{int}(\text{NCT}-\text{cl}(NCT_K)) \sqsubseteq NCT-int(\ NCT-cl[NCT_H \sqcap \ \ \text{NCT}-\text{int}(\text{NCT}-\text{cl}(\ NCT_K))])$. (3)

From eq. (2) and (3), we get the following inequality $NCT-int(NCT-cl(NCT_H)) \sqcap \ \ \text{NCT}-\text{int}(\text{NCT}-\text{cl}(NCT_K)) \sqsubseteq NCT-int(\ NCT-cl(NCT-int(NCT-cl(NCT_H \sqcap \ \ NCT_K\ ))))$. (4)

But $NCT-int\ (NCT-cl(NCT-int\ (NCT-cl(NCT_H\ \ \sqcap\ \ NCT_K)))) \sqsubseteq NCT-int(\ NCT-cl(\ NCT_H \sqcap \ NCT_K\ ))$. (5)

From eq. (3) and (4) we get the result.

(2) Let $NCT_H = NCT-int(NCT-cl(NCT_A))$,to show that $NCT_H = NCT-int(NCT-cl(NCT_H))$ .Since $NCT-int(NCT-cl(NCT_H)) = NCT-int(NCT-cl(\ NCT-int(NCT-cl(NCT_A)))) \sqsubseteq NCT-int(NCT-cl(NCT-cl(NCT_A)))) = \ NCT-int(NCT-cl(NCT_A)) = \ NCT_H$. (6)

But $NCT-int(NCT_H) = NCT-int(NCT-int(NCT-cl(NCT_A))) = NCT-int(NCT-cl(NCT_A)) = NCT_H \sqsubseteq \ NCT-int(NCT-cl(NCT_H))$. (7)

From eq. (6) and (7) we get the result. Similarly, we can prove part (3). To proof 4 and 5, direct from part (1).

Definition 3.4. $NCT-$ function $f_{NCT}\colon(\ NCT_X\ \ ,\tau_{NCT}^X\ \ ) \longrightarrow (\ NCT_Y\ \ ,\tau_{NCT}^Y\ \ )$ is $NCT-$ open (NCT −closed) map, if the image of each set NCT −open (NCT −closed) in NCT_X is NCT −open (NCT −closed) in NCT_Y.

Example 3.5. let $X =\ \{1,2,3\}, Y =\ \{a,b,c\}$ and $f : X\ \ \longrightarrow Y$ s.t. $f\ (1)\ =\ a, f\ (2)\ = c$ and $f\ (3)\ = b$. For $\tau_{NCT}^X =\ \{NCT_X, NCT_\varphi, < \{1\},\{1,2\},\{3\} > \ \}$, $\tau_{NCT}^Y =\ \{NCT_Y, NCT_\varphi, < \{b\},\{a,b\},\varphi > \ \}$,then $f_{NCT}\colon(\ NCT_X\ \ ,\tau_{NCT}^X\ \ ) \longrightarrow (\ NCT_Y\ \ ,\tau_{NCT}^Y\ \ )$ is not $NCT-$open and not $NCT-$closed. For $\tau_{NCT}^X =\ \{\ \ NCT_X, NCT_\varphi, < \{1\},\{1,2\},\{3\} > \ \}$, $\tau_{NCT}^Y\ =\ \{\ \ NCT_Y, NCT_\varphi, < \{b\},\{a,c\},\{b\} > \ \}$,then $f_{NCT}\colon(\ NCT_X\ \ ,\tau_{NCT}^X\ \ ) \longrightarrow (\ NCT_Y\ \ ,\tau_{NCT}^Y\ \ )$ is NCT −open and NCT −closed.

Theorem 3.6. let $f_{NCT}\colon(\ NCT_X\ \ ,\tau_{NCT}^X\ \ ) \longrightarrow (\ NCT_Y\ \ ,\tau_{NCT}^Y\ \ )$ is NCT −closed (NCT −open) map, for any $NCT_S\ \sqsubseteq NCT_Y$ and any $NCT-$ open ($NCT-$ closed) NCT_U containing $f_{NCT}^{-1}(NCT_S\ \ ), \exists NCT$ −open (NCT −closed) NCT_V containing NCT_S s.t. $f_{NCT}^{-1}(NCT_V\ \ ) \sqsubseteq NCT_U$.

Proof.

Let $NCT_V = NCT_Y - f_{NCT}(\ NCT_X - \ NCT_U)$, since $f_{NCT}^{-1}(NCT_S\ ) \sqsubseteq NCT_U$, it follows that $NCT_S \sqsubseteq NCT_V$, and because f_{NCT} is $NCT-$closed map, NCT_V $NCT-$open in NCT_Y. Observing that $f_{NCT}^{-1}(NCT_V\ ) = NCT_X - f_{NCT}^{-1}\big(f_{NCT}(\ NCT_X - \ NCT_U)\big) \sqsubseteq NCT_X - (NCT_X - NCT_U))$ $=NCT_U$.

Theorem 3.7. The following four properties of $NCT-$ function f_{NCT}: $(\ NCT_X\ ,\tau_{NCT}^X\ ) \longrightarrow (\ NCT_Y\ ,\tau_{NCT}^Y\ )$ are equivalent:

- **i-** f_{NCT} is an $NCT-$open map.
- **ii-** $f_{NCT}(\ NCT - int(NCT_A)) \sqsubseteq NCT - int(f_{NCT}(NCT_A))$ for each NCT_A in NCT_X.
- **iii-** For each $NCT-$point NCT_P and $NCT-$open NCT_U containing it, there exist NCT_V $NCT-$open in NCT_Y containing $f_{NCT}(NCT_P\ )$ s.t. $NCT_V \sqsubseteq f_{NCT}(NCT_U)$.

Proof.
Since $NCT - int(NCT_A) \sqsubseteq NCT_A$, by Proposition 2.16 (1), we have $f_{NCT}(NCT - int(NCT_A)) \sqsubseteq f_{NCT}(NCT_A)$, by (i) $f_{NCT}(NCT - int(NCT_A))$ is $NCT-$ open in NCT_Y and because $NCT - int(f_{NCT}(NCT_A))$ is the $NCT-$union of $NCT-$open sets contained in $f_{NCT}(NCT_A)$. We must have $f_{NCT}(NCT - int(NCT_A)) \sqsubseteq NCT - int(f_{NCT}(NCT_A))$.
ii $\longrightarrow$ $\boldsymbol{i}$, let NCT_U is $NCT-$open in NCT_X , $NCT_U = NCT - int(NCT_U\ )$ and so $f_{NCT}(NCT_U) = f_{NCT}(NCT - int(NCT_U)) \sqsubseteq\ NCT - int\ \ (f_{NCT}(NCT_U))\ \ )) \sqsubseteq\ f_{NCT}(NCT_U)$, thus $f_{NCT}(NCT_U) = NCT - int(f_{NCT}(NCT_U))$ and therefore $f_{NCT}(NCT_U)$ is $NCT-$ open in NCT_Y , that is f_{NCT} is an $NCT-$open map.
$\boldsymbol{i} \longrightarrow \boldsymbol{iii}$, let $NCT_P^X \ €\ NCT_U\ \ \in \tau_{NCT}^X$, but $f_{NCT}(\ NCT_P^X\ €\ f_{NCT}(NCT_U)\ \ \in \tau_{NCT}^Y$.
$\boldsymbol{iii} \longrightarrow \boldsymbol{i}$, let $NCT_U\ \ \in \tau_{NCT}^X$, by iii, each $NCT_P^Y\ €f_{NCT}(\ NCT_U)$ has $NCT-$open NCT_V in NCT_Y s.t. $NCT_V \sqsubseteq\ f_{NCT}(NCT_U) = \sqcup\{NCT_V;\ \ NCT_P^Y\ €\ f_{NCT}(NCT_U)\}$is $NCT-$open in NCT_Y.

Proposition 3.8. f_{NCT}: $(\ NCT_X\ ,\tau_{NCT}^X\ ) \longrightarrow (\ NCT_Y\ ,\tau_{NCT}^Y\ )$ is $NCT-$ closed map iff $NCT - cl\ \ (f_{NCT}(NCT_A)) \sqsubseteq f_{NCT}(NCT - cl(NCT_A))$, for each NCT_A in NCT_X.
Proof.
Since $NCT - cl(NCT_A)$ is $NCT-$closed in NCT_X , and so $f_{NCT}(NCT - cl(NCT_A))$, is $NCT-$ closed in NCT_Y , since $f_{NCT}(NCT_A) \sqsubseteq f_{NCT}(NCT - cl(NCT_A))$, obtain $NCT - cl\ \ (f_{NCT}(NCT_A)) \sqsubseteq NCT - cl(f_{NCT}(NCT - cl(NCT_A)))= f_{NCT}(NCT - cl(NCT_A)$. Conversely, if the condition hold and NCT_A is $NCT-$ closed in NCT_X , then $f_{NCT}(NCT_A) \sqsubseteq NCT - cl(f_{NCT}(NCT_A)) \sqsubseteq f_{NCT}(NCT - cl(NCT_A))$ $=f_{NCT}(NCT_A)$, so that $f_{NCT}(NCT_A)$ is $NCT-$closed in NCT_Y.

4. Conclusions

First, Salama and Florentin have established several types of neutrosophic crisp points, but they do not cover spaces, the reason for this is the type of neutrosophic crisp space structure with its unique characteristics to be used for life's problems and scientific problems, and then we proposed four new types of neutrosophic crisp points to enhance these spaces as follows:

- **1-** New conceptual from the NCP points as $P_{N_1} = \langle P_1\ \ ,P_2\ \ ,P_3 \rangle \ni P_i \neq \emptyset$ for $i = 1$ or $i = 2$ or $i = 3$ moreover, there are different focus points empty.
- **2-** $P_{N_2} = \langle P_1\ \ ,P_2\ \ ,P_3 \rangle\ \ni P_i \neq \emptyset$ for $i = 1$ or $i = 2$ or $i = 3$ and other points are individual, and $p_i \subseteq A_i, \forall\ \ i = 1,2,3$ iff $P_{N_i} \in A$.
- **3-** $PN3\ = \ <\ A1, A2, A3\ >$ is called neutrosophic crisp point, if exactly only one subset $Ai\ \neq \emptyset$, for $i = 1, 2, 3$.
- **4-** $Ai\ \ = \emptyset$ for $i = 1$ or $i = 2$ or $i = 3$ and as for the remaining two sets, the first set is any mono set and the second is its complement.

Second, we can modify most of the important mathematical concepts on $NCT-$ sets to be inputs for future research studies, and here we define the concept of ideal and filter as follows.
I- The sub collection I_{NCT} of $T(X)$ is neutrosophic crisp triple ideal ($NCT-$ideal) if fulfill that:

1. If $NCT_A \in I_{NCT}$ and $NCT_B \sqsubseteq NCT_A$, so $NCT_B \in I_{NCT}$.
2. In general, $I_{NCT}$is closed beneath the finite $NCT -$ union. In general, 2 using this definition, we are able to alter all notions, namely the results and then in the publications.

II- The sub collection $F_{NCT}$of $T(X)$ is neutrosophic crisp triple filter (NCT −filter) if:

1. If $NCT_A \in F_{NCT}$ and $NCT_A \sqsubseteq NCT_B$, so $NCT_B \in F_{NCT}$.
2. $F_{NCT}$is closed under finite NCT −intersection, additionally characterize the vicinity connection on $NCT_A(X)$, The idea of a neutrosophic crisp triple set can also be generalized to the ideas, theories and concepts.

Funding: This examination has no source of financial support.

Ethical approval: None of the creators explored different avenues regarding human subjects or creatures during this exploration.

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Received: Mar 7, 2024. Accepted: May 30, 2024