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Neutrosophic Ranked Set Sampling Scheme for Estimating Population Mean: An Application to Demographic Data

Rajesh Singh[1] and Anamika Kumari [2,*]

[1]Department of Statistics, Institute of Science, Banaras Hindu University, India; rsinghstat@gmail.com

[2]Department of Statistics, Institute of Science, Banaras Hindu University, India; anamikatiwari1410@gmail.com

[*]Correspondence: anamikatiwari1410@gmail.com

Abstract. The primary goal of this study is to address the limitations of classical statistics in handling ambiguous or indeterminate data. The best alternative to classical and fuzzy statistics for handling such data uncertainty is neutrosophic statistics, which is a generalization of both. A generalization of classical statistics, neutrosophic statistics addresses hazy, ambiguous, and unclear information. To achieve this, this manuscript recommends the neutrosophic ranked set sampling approach. We have introduced neutrosophic estimators for estimating the mean of the finite population using auxiliary information under neutrosophic ranked set sampling to address the challenges of estimation of the population mean of neutrosophic data. The proposed estimators outperform the other existing estimators and proposed estimators evaluated in this work using MSE and PRE criteria, and equations for bias and mean squared error produced for the suggested estimators up to the first order of approximation. Under neutrosophic ranked set sampling, the suggested estimator has demonstrated superiority over the class of estimators, unbiased estimators, and comparable estimators. Using the R programming language, a numerical illustration and a simulation study have been conducted to demonstrate the effectiveness of the suggested methodology. When computing results when working with ambiguous, hazy, and neutrosophic-type data, the provided estimators are particularly helpful. These estimators produce findings that are not single-valued but rather have an interval form where our population parameter may lie more frequently. Since we now have an estimated interval with the population mean's unknown value provided a minimum MSE, the estimators are more effective.

Keywords: Neutrosophic ranked set sampling; Neutrosophic Statistics; Ranked Set Sampling; Study Variable; Auxiliary Variable; Bias; Mean Squared Error.

1. Introduction

Sampling is a crucial practice for a variety of reasons, such as cost and time constraints. In sampling theory, the goal of estimation procedures is to enhance the effectiveness of estimators for population parameters while minimizing sampling errors. To achieve this, auxiliary information is utilized to improve estimator efficiency, and this information can be incorporated at various stages of the process. When highly correlated auxiliary information is not readily available, it can be gathered from previous surveys. Estimation techniques like ratio, product, and regression are commonly employed in this context.

For instance, Sisodia and Dwivedi [1] introduced a modified ratio estimator that incorporates the coefficient of variation of auxiliary information. Pandey and Dubey [2], Bahl and Tuteja [3], Upadhyaya and Singh [4], Singh et al. [5], Kadilar and Cingi [6], and Singh et al. [7] have also proposed population parameter estimation methods using auxiliary information. However, our focus here is on ranked set sampling.

Efforts in sampling continually strive for improvements in estimator efficiency, cost-effectiveness, simplicity, and time savings. Ranked Set Sampling (RSS) offers a superior alternative to Simple Random Sampling (SRS) in various fields, including medicine, agriculture, earth sciences, statistics, and mathematics, especially when measurements are cumbersome, time-consuming, or expensive. The RSS technique was initially described for population mean estimation by McIntyre [8], and the mathematical theory behind RSS was provided by Takahashi and Wakimoto [9]. Dell and Clutter [10] demonstrated that, under both perfect and imperfect ranking scenarios, the mean estimate in RSS remains unbiased.

Numerous researchers, such as Samawi and Muttlak [11], Stokes [12], Al-Shaleh and Al-Omari [13], Bouza [14], Ganesh and Ganeslingam [15], Bouza [16], Kadilar et al. [17], Singh et al. [18], Mandowara and Mehta [19], Al-Omari and Bouza [20] have contributed to the field of ranked set sampling. For recent work, one can prefer Singh and Vishwakarma [21], Bhushan and Kumar [22], and Singh and Kumari [23].

Classical ranked set sampling deals solely with precise data, assuming no uncertainty. However, data can be uncertain and imprecise in practice, containing sets or intervals. To address such situations, fuzzy logic is a valuable tool that handles data with imprecision. Fuzzy statistics are used to analyze data with fuzzy, ambiguous, unclear, uncertain, or imprecise characteristics. Yet, they do not account for the degree of indeterminacy inherent in the data.

Neutrosophic statistics, an extension of fuzzy logic, offers a way to measure both indeterminacy and determinate aspects of uncertain or imprecise data.
When dealing with data that contains indeterminacy, neutrosophic statistics are employed. Neutrosophic statistics expand upon classical statistics and encompass fuzzy and intuitionistic statistics. Neutrosophy is applicable when observations in a population or sample lack precision, are indeterminate or are vague. Some examples of neutrosophic data include district-wise water level measurements with intervals, variations in machinery part sizes due to measurement errors, and day-wise temperature measurements resulting in interval-type data.

Atanassov [24] and Atanassov [25] defined Neutrosophic statistics is a generalization of classical statistics as well as fuzzy. The concept of neutrosophy was initially introduced by Smarandache [26-32], and extensive literature on neutrosophic sets, logic, and statistics can be found in his works. In the realm of sampling theory, Tahir et al. [33] recently addressed the estimation of population parameters under a neutrosophic environment, introducing neutrosophic ratio-type estimators for population means under SRS. One can also prefer Singh & Mishra [34] and Singh et al. [35] for neutrosophic estimators under SRS framework. However, there has been little focus on neutrosophic ranked set sampling for estimating population parameters.

Efficiency improvements in estimators are a constant objective in sampling. In this context, we propose enhanced neutrosophic ranked set sampling (NRSS) estimators for population mean estimation, with a particular emphasis on minimizing mean square error (MSE) and enhancing precision.

Our study is designed as follows: Section 1 presents an introduction, and Section 2 outlines motivation, needs, and research gaps. Section 3 outlines the NRSS method. Section 4 presents existing NRSS estimators. Section 5 presents proposed NRSS estimators, Section 6 presents an empirical study using natural growth rate data, and Section 7 offers a simulation study. Section 8 is dedicated to a discussion, and Section 9 covers a conclusion.

2. Motivation, Need and Research Gap

This article's main objective is to introduce a less explored approach known as "neutrosophic ranked set sampling"for dealing with neutrosophic or interval-type data. This method can encompass various types of NRSS, similar to classical RSS. Our study focuses on sampling theory, marking the instance of proposing an RSS technique tailored to neutrosophic data, along with the development of NRSS estimators for population mean estimation. This is a significant step in expanding the field of sampling theory and comparing these estimators with existing neutrosophic methods such as ratio, product, and generalized estimators. RSS is considered a superior alternative to SRS, making it an attractive avenue for further exploration in the context of NRSS.

Several factors drive our exploration of NRSS and its associated estimators for population parameter estimation. A primary motivation is to introduce RSS and RSS estimators in a neutrosophic setting. Previous research in survey sampling has predominantly focused on clear, well-defined data, where classical sampling methods yield precise results, albeit with potential risks of inaccuracies, overestimations, or underestimations. However, classical methods fall short when handling set-type or undetermined data, characteristic of neutrosophic data, which is more prevalent in real-world scenarios than crisp data. As such, there is a growing need for additional neutrosophic statistical techniques. Traditional statistical approaches are ill-suited to compute accurate estimates of unknown parameters when dealing with indeterminate, vague, imprecise, set-type, or interval-type data. Neutrosophic statistics serve as a suitable replacement for classical statistics in such scenarios.

Inspired by the work of Tahir et al. [33] and driven by the need to bridge the gap between classical and neutrosophic statistics, our work introduces enhanced NRSS estimators for population mean estimation. Despite thorough research in the field, we found not many prior studies in survey sampling that addressed the estimation of population means in the presence of auxiliary variables using neutrosophic data under ranked set sampling other than Singh and Vishwakarma [36]. Following Singh and Vishwakarma's work, this research represents a significant step toward filling this gap and contributes to the evolving domain of neutrosophic statistics.

It has been well-established by multiple authors that RSS is a more suitable option than SRS when dealing with cumbersome, expensive, or time-consuming measurements. The challenges associated with measurements in a neutrosophic context exacerbate these issues. Therefore, our research introduces an NRSS method to enhance the accuracy of the population mean estimators in this unique context.

3. **Sampling Methodology**

Numerous methods can be used to display the neutrosophic observations, and the neutrosophic numbers may include an unknown interval [a, b]. We are describing neutrosophic values as $Z_{rssN} \equiv Z_{rssL} + Z_{rssU} I_{rssN}$ with $I_{rssN} \in [I_{rssL}, I_{rssU}]$, N is here to represent the neutrosophic number and I_{rssN} is the degree of indeterminacy. Hence, our neutrosophic observations will lie in an interval $Z_{rssN} \in [a, b]$, where 'a'and 'b'denote the neutrosophic data's lower and upper values.

In RSS, a small subset of randomly chosen population units are measured after they have been ranked solely based on observation or past experience. Within the framework of RSS, multiple independent random sets, each comprising m units drawn from the population, are chosen. Each unit within a set has an equal probability of selection. The constituents of each random set are then ranked based on the characteristics of the auxiliary variable. Subsequently, the selection process involves choosing the smallest unit from the first ordered set, followed by the next-smallest unit from the second ordered set, and so on. This sequential selection continues until the largest rank in the m^{th} set is reached. Throughout this iterative cycle, a total of rm (= n) units are measured, and this entire process is repeated r times.

The method of NRSS consists of selecting $m_N \in [m_L, m_U]$ bivariate random samples of size $m_N \in [m_L, m_U]$ from a population of size N, and then ranking inside each sample concerning for auxiliary variable $X_N \in [X_L, X_U]$ associated with $Y_N \in [Y_L, Y_U]$. The book "Introduction to Neutrosophic Statistics"by Smarandache [32] will be the basis for the ranking of the neutrosophic number. To show the process of ranking, we are utilizing here two sets as $X_{1N} \in [X_{1L}, X_{1U}]$ and $X_{2N} \in [X_{2L}, X_{2U}]$, also their mid-points are as $X_{1midN} = [X_{1L} + X_{1U}]/2$ and $X_{2midN} = [X_{2L} + X_{2U}]/2$. The ordering of neutrosophic numbers $m_N \in [m_L, m_U]$ can be done as $X_{1N} \in [X_{1L}, X_{1U}]$ will be less than $X_{2N} \in [X_{2L}, X_{2U}]$ if $X_{1midN} \leq X_{2midN}$, also if both are same that is $X_{1midN} = X_{2midN}$ then we will compare or see by $X_{1L} \leq X_{2L}$. Further, if again $X_{1L} = X_{2L}$ then this implies $X_{1U} = X_{2U}$ and hence $X_{1N} \in [X_{1L}, X_{1U}] = X_{2N} \in [X_{2L}, X_{2U}]$, so the neutrosophic number ranking will be carried out in this manner. In the whole NRSS structure, first we count the smallest unit of the first data set size $m_N \in [m_L, m_U]$, for the first measurement unit in the entire NRSS structure, and then we scrap the remaining units. In a similar manner, we count the second-smallest observation from the second data set as the second observation and discard the remaining observations. This process counts the total $m_N \in [m_L, m_U]$ neutrosophic bivariate units for up to the m^{th} term. After r cycles of these steps, the total $n_N = m_N r \in [n_L, n_U]$ bivariate NeRSS units are obtained.

Consider a neutrosophic random sample of size $n_N \in [n_L, n_U]$ using RSS, which is acquired from a finite population of 'N' units $(U_1, U_2, ..., U_N)$. The neutrosophic study and auxiliary variable are $Y_N \in [Y_L, Y_U]$ and $X_N \in [X_L, X_U]$. Let $\overline{y}_{[n]N} \in [\overline{y}_{[n]L}, \overline{y}_{[n]U}]$ and $\overline{x}_{(n)N} \in [\overline{x}_{(n)L}, \overline{x}_{(n)U}]$ be the sample means of the neutrosophic study and auxiliary variables respectively, and also, let $\overline{Y}_N \in [\overline{Y}_L, \overline{Y}_U]$ and $\overline{X}_N \in [\overline{X}_L, \overline{X}_U]$ be the population means of the neutrosophic study and auxiliary variables, respectively. The correlation coefficient between both neutrosophic study and auxiliary variables is $\rho_{yxN} \in [\rho_{yxL}, \rho_{yxU}]$, $C_{xN} \in [C_{xL}, C_{xU}]$ and $C_{yN} \in [C_{yL}, C_{yU}]$ be the coefficient of variation of neutrosophic variables Y_N and X_N.

Let the neutrosophic mean error terms are $\epsilon_{0N} \in [\epsilon_{0L}, \epsilon_{0U}]$ and $\epsilon_{1N} \in [\epsilon_{1L}, \epsilon_{1U}]$. To obtain the bias and MSE of the estimators, we write

$\overline{y}_{[n]N} = \overline{Y}_N(1 + \epsilon_{0N})$, $\overline{x}_{(n)N} = \overline{X}_N(1 + \epsilon_{1N})$

$E(\epsilon_{0N}^2) = (\eta c_{yN}^2 - D_{y[N]}^2) = V_{yrN}$

$E(\epsilon_{1N}^2) = (\eta c_{xN}^2 - D_{x[N]}^2) = V_{xrN}$

$E(\epsilon_{0N}, \epsilon_{0N}) = (\eta C_{yx} - D_{yx[N]}^2) = V_{yxrN}$

where,

$\eta_N = \frac{1}{n_N r}$,

$D_{y[N]}^2 = \frac{1}{m_N^2 r \overline{Y}_N^2} \sum_{i=1}^{m_N} (\mu_{[iyN]} - \overline{Y}_N)^2$

$D_{x[N]}^2 = \frac{1}{m_N^2 r \overline{X}_N^2} \sum_{i=1}^{m_N} (\mu_{[ixN]} - \overline{X}_N)^2$

$D_{yx[N]} = \frac{1}{m_N^2 r \overline{Y}_N \overline{X}_N^2} \sum_{i=1}^{m_N} (\mu_{[iyN]} - \overline{Y}_N)(\mu_{[ixN]} - \overline{X}_N)$

where $\mu_{[iy]}$ and $\mu_{(ix)}$ are the means of the i^{th} ranked set and are given by

$$\mu_{[iyN]} = \frac{1}{r} \sum_{j=1}^{r} y_{j[N]}, \mu_{(ixN)} = \frac{1}{r} \sum_{l=1}^{r} x_{j[N]}.$$

$\eta_N \in [\eta_L, \eta_U]$; $S_{xN} \in [S_{xL}, S_{xU}]$; $S_{yN} \in [S_{yL}, S_{yU}]$; $S_{xyN} \in [S_{xyL}, S_{xyU}]$ $e_{0N}^2 \in [e_{0L}^2, e_{0U}^2]$;$e_{1N}^2 \in [e_{1L}^2, e_{1U}^2]$;$e_{0N} e_{1N} \in [e_{0L} e_{1L}, e_{0U} e_{0U}]$;

$C_{xN} \in [C_{xL}, C_{xU}]$; $C_{yN} \in [C_{yL}, C_{yU}]$; $C_{xyN} \in [C_{xyL}, C_{xyU}]$ $D_{y[N]}^2 \in [D_{y[L]}^2, D_{y[U]}^2]$; $D_{x[N]}^2 \in [D_{x[L]}^2, D_{x[U]}^2]$; $D_{yx[N]} \in [D_{yx[L]}, D_{yx[U]}]$; $\rho_{yxN} \in [\rho_{yxL}, \rho_{yxU}]$; $V_{yrN} \in [V_{yrL}, V_{yrU}]$
$V_{xrN} \in [V_{xrL}, V_{xrU}]$; $V_{yxrN} \in [V_{yxrL}, V_{yxrU}]$; $\mu_{[iyN]} \in [\mu_{[iyL]}, \mu_{[iyU]}]$; $\mu_{[ixN]} \in [\mu_{[ixL]}, \mu_{[ixU]}]$.

4. **Existing Estimators**

Using the NRSS technique, the usual unbiased estimator for the population mean $\overline{Y}$ is provided by

$$\overline{y}_{[n]N} = \frac{1}{n_N} \sum_{i=1}^{n_N} y_{[i]N} \tag{1}$$

The variance of the estimator $\overline{y}_{[n]N}$ is given by

$$var(\overline{y}_{[n]N}) = \overline{Y}_N^2 V_{yrN} \tag{2}$$

The ratio estimator under NRSS for the population mean $\overline{Y}$

$$\overline{y}_{rN} = \overline{y}_{[n]N} \left(\frac{\overline{X}_N}{\overline{x}_{[n]N}} \right) \tag{3}$$

The MSE of the estimator $\overline{y}_{rN}$ is given by

$$MSE(\overline{y}_{rN}) = \overline{Y}_N^2(V_{yrN} + V_{xrN} - 2V_{yxrN}) \tag{4}$$

Using NRSS, the regression estimator for the population mean $\overline{Y}$ is provided by

$$\overline{y}_{regN} = \overline{y}_{[n]N} + \beta(\overline{X}_N - \overline{y}_{[n]N}) \tag{5}$$

The MSE of the estimator $\overline{y}_{regN}$ is given by

$$MSE(\overline{y}_{regN}) = \overline{Y}_N^2 \left(V_{yrN} - \frac{V_{yxrN}^2}{V_{xrN}} \right) \tag{6}$$

Using NRSS, the exponential estimator for the population mean $\overline{Y}$ is provided by

$$\overline{y}_{expN} = \overline{y}_{[n]N} \exp\left(\frac{\overline{X}_N - \overline{x}_{[n]N}}{\overline{X}_N + \overline{x}_{[n]N}} \right) \tag{7}$$

The MSE of the estimator $\overline{y}_{expN}$ is given by

$$MSE(\overline{y}_{expN}) = \overline{Y}^2 \left(V_{yrN} + \frac{V_{yrN}}{4} - V_{yxrN} \right) \tag{8}$$

Vishwakarma and Singh (2021) gave NRSS generalized class of estimators

$$\overline{y}_{vsN} = \overline{y}_{[n]N} \left(\frac{A_N\overline{X}_N + B_N}{A_N\overline{x}_{[n]N} + B_N} \right)^{\delta} \tag{9}$$

The MSE of the estimator $\overline{y}_{vsN}$ is given by

$$MSE(\overline{y}_{vsN}) = \overline{Y}_N^2 \left(V_{yrN} - \frac{V_{yxrN}^2}{V_{xrN}} \right) \tag{10}$$

5. Proposed Estimators

No single estimator is universally effective in all situations. Consequently, prioritizing estimators that yield minimal Mean Squared Error (MSE) and high precision is desirable. The objective of this section is to develop estimators that demonstrate effective performance across a broader range of scenarios. We have chosen to incorporate Mishra et al.'s [37] estimator within the NRSS and have additionally introduced two novel estimators for the mean of a finite population under NRSS, leveraging auxiliary variables for improved accuracy.

$$1.) P_{1N} = \overline{y}_{[n]N} \left(g_{1N} + 1\right) + g_{2N} \log\left(\frac{\overline{x}_{[n]N}}{\overline{X}_N} \right) \tag{11}$$

where the constants g_{1N} and g_{2N} ensure that the estimators' MSE is kept to a minimal. Expressing the estimator P_{1N} given in equation (11) in terms of $\epsilon's$ we get

$$P_{1N} = \overline{Y}_N \left(1 + \epsilon_{01}\right) \left(g_{1N} + 1\right) + g_{2N} log \left(\frac{\overline{X}_N \left(1 + \epsilon_{1N}\right)}{\overline{X}_N} \right) \tag{12}$$

Taking expectations by focusing on first-order approximation, we obtain MSE,

$$MSE\left(P_{1N}\right) = \overline{Y}_N{}^2 V_{yrN} + g_{1N}^2 A_{1N} + g_{2N}^2 B_{1N} - 2g_{1N} C_{1N} - 2g_{2N} D_{1N} + 2g_{1N} g_{2N} E_{1N} \quad (13)$$

where,

$$A_{1N} = \overline{Y}_N{}^2 (1 + V_{yrN})$$

$$B_{1N} = V_{xrN}$$

$$C_{1N} = \overline{Y}_N{}^2 V_{yrN}$$

$$D_{1N} = \overline{Y}\ V_{yxrN}$$

$$E_{1N} = \overline{Y}\left(V_{yxrN} - \frac{1}{2} V_{xrN}\right)$$

To find out the minimum MSE for P_{1N}, we partially differentiate equation (13) w.r.t. g_{1N} & g_{2N} and equating to zero we get

$$g_{1N}{}^* = \frac{B_{1N} C_{1N} - D_{1N} E_{1N}}{E_{1N}^2 - A_{1N} B_{1N}} \quad (14)$$

$$g_{2N}{}^* = \frac{A_{1N} D_{1N} - C_{1N} E_{1N}}{E_{1N}^2 - A_{1N} B_{1N}} \quad (15)$$

Putting the optimum value of g_{1N} & g_{2N} in the equation (13), we obtain a minimum value of MSE of P_{1N} as

$$MinMSE(P_{1N}) = C_{1N} + \frac{B_{1N} C_{1N}^2 + A_{1N} D_{1N}^2 - 2C_{1N} D_{1N} E_{1N}}{E_{1N}^2 - A_{1N} B_{1N}} \quad (16)$$

where $MSE(P_{1N}) \in [MSE(P_{1L}), MSE(P_{1U})]$

$$2.) P_{2N} = g_{3N} \overline{y}_{[n]N} + g_{4N} exp\left(\frac{\overline{X}_N - \overline{x}_{[n]N}}{\overline{X}_N + \overline{x}_{[n]N}}\right)\left(1 + log \frac{\overline{x}_{[n]N}}{\overline{X}_N}\right) \quad (17)$$

Expressing P_{2N} given in equation (17) in terms of $\epsilon's$ we get

$$P_{2N} = g_{3N} \overline{Y}_N \left(1 + \epsilon_{0N}\right) + g_{4N}\ exp\left(\frac{-\epsilon_{1N}}{2 + \epsilon_{1N}}\right)\left(1 + \log\left(1 + \epsilon_{1N}\right)\ \right) \quad (18)$$

$$P_{2N} - \overline{Y}_N = \left(g_{3N} - 1\right) \overline{Y}_N + g_{3N} \overline{Y}_N \epsilon_{0N} + g_{4N}\left(1 + \frac{\epsilon_{1N}}{2} - \frac{5\epsilon_{1N}{}^2}{8}\right) \quad (19)$$

$$Bias(P_{2N}) = \overline{Y}_N \left(g_{3N} - 1\right) + g_{4N}\left[1 - \frac{5}{8} V_{xrN}\right] \quad (20)$$

CASE 1: IF SUM OF WEIGHTS IS FIXED($g_{3N}+g_{4N}=1$)

The MSE of the estimator P_{2N} is shown as

$$MSE(P_{2N})=\overline{Y}_N^2\left[V_{yrN}+g_{4N}^2V_{xrN}-2g_{4N}V_{yxrN}\right] \tag{21}$$

To find out the minimum value of MSE for P_{2N}, we partially differentiate equation (21) w.r.t. g_{4N}, and equating to zero we get

$$g_{4N}{}^*=\frac{V_{yxrN}}{V_{xrN}} \tag{22}$$

Putting the optimum value of g_{4N} in the equation (21), we obtain a minimum MSE of P_{2N} as

$$MinMSE(P_{2N})=\overline{Y}_N{}^2\left(V_{yrN}-\frac{V_{yxrN}{}^2}{V_{xrN}}\right) \tag{23}$$

where $MSE(P_{2N})\in[MSE(P_{2L}),MSE(P_{2U})]$

CASE 2: IF THE SUM OF WEIGHTS IS ADJUSTABLE ($g_{3N}+g_{4N}\neq 1$)

$$P_{2N}-\overline{Y}_N=(g_{3N}-1)\,\overline{Y}_N+g_{3N}\overline{Y}_N\epsilon_{0N}+g_{4N}\left(1+\frac{\epsilon_{1N}}{2}-\frac{5\epsilon_1{}^2}{8}\right) \tag{24}$$

Squaring on both sides we get

$$\begin{aligned}(P_{2N}-\overline{Y}_N)^2=\overline{Y}_N{}^2+\overline{Y}_N{}^2g_{3N}^2(1+\epsilon_{01}^2)+g_{4N}^2\left(1-\epsilon_{1N}^2\right)-2g_{3N}\overline{Y}_N{}^2-2g_{4N}\overline{Y}_N\left(1-\frac{5\epsilon_{1N}^2}{8}\right)\\+2g_{3N}g_{4N}\overline{Y}_N\left(1-\frac{5\epsilon_{1N}{}^2}{8}+\frac{\epsilon_{0N}\epsilon_{1N}}{2}\right)\end{aligned} \tag{25}$$

Taking expectations by focusing on first-order approximation, we obtain mean square error (MSE),

$$MSE\left(P_{2N}\right)=\overline{Y}_N{}^2V_{yrN}+g_{3N}^2A_{2N}+g_{4N}^2B_{2N}-2g_{3N}C_{2N}-2g_{4N}D_{2N}+2g_{3N}g_{4N}E_{2N} \tag{26}$$

where,

$$\begin{aligned}A_{2N}&=\overline{Y}_N{}^2(1+V_{yrN})\\B_{2N}&=1-V_{xrN}\\C_{2N}&=\overline{Y}_N{}^2\\D_{2N}&=\overline{Y}_N\left(1-\frac{5}{8}V_{xrN}\right)\\E_{2N}&=\overline{Y}_N\left(1-\frac{5}{8}V_{xrN}+\frac{1}{2}V_{yxrN}\right)\end{aligned}$$

To find out the minimum MSE for P_{2N}, we partially differentiate equation (26) w.r.t. g_{3N} & g_{4N} and equating to zero we get

$$g_{3N}{}^*=\frac{B_{2N}C_{2N}-D_{2N}E_{2N}}{A_{2N}B_{2N}-E_{2N}^2} \tag{27}$$

$$g_{4N}{}^{*} = \frac{A_{2N}D_{2N} - C_{2N}E_{2N}}{A_{2N}B_{2N} - E_{2N}^{2}} \tag{28}$$

Putting the optimum value of g_{3N} & g_{4N} in the equation (26), we obtain a minimum MSE of P_{2N} as

$$MinMSE(P_{2N}) = C_{2N} + \frac{B_{2N}C_{2N}^{2} + A_{2N}D_{2N}^{2} - 2C_{2N}D_{2N}E_{2N}}{E_{2N}^{2} - A_{2N}B_{2N}} \tag{29}$$

where $MSE(P_{2N}) \in [MSE(P_{2L}), MSE(P_{2U})]$

$$3.)P_{3N} = g_{5N}\overline{y}_{[n]N} + \ g_{6N}\left(\frac{\overline{X}_N}{\overline{x}_{[n]N}}\right) exp\left(\frac{\overline{X}_N - \overline{x}_{[n]N}}{\overline{X}_N + \overline{x}_{[n]N}}\right) \tag{30}$$

Expressing P_{3N} given in equation (30) in terms of $\epsilon's$ we get

$$P_{3N} = g_{5N}\overline{Y}_N\left(1+\epsilon_{0N}\right) + g_{6N}(1+\epsilon_{1N})^{-1}\ exp\left(\frac{-\epsilon_1}{2+\epsilon_{1N}}\right) \tag{31}$$

$$P_{3N} - \overline{Y}_N = \left(g_{5N}-1\right)\overline{Y}_N + g_{5N}\overline{Y}_N\epsilon_{01} + g_{6N}\left(1 - \frac{3\epsilon_{1N}}{2} + \frac{15\epsilon_{1N}{}^{2}}{8}\right) \tag{32}$$

$$Bias(P_{3N}) = \overline{Y}_N\left(g_{5N}-1\right) + g_{6N}\left[1 + \frac{15}{8}V_{xrN}\right] \tag{33}$$

CASE 1: IF SUM OF WEIGHTS IS FIXED $(g_{5N} + g_{6N} = 1)$

The MSE of the estimator P_{3N} is shown as

$$MinMSE(P_{3N}) = \overline{Y}_N{}^{2}\left[V_{yrN} + g_{6N}^{2}V_{xrN} - 2g_{6N}V_{yxrN}\right] \tag{34}$$

To find out the minimum value of MSE for P_{3N}, we partially differentiate equation (34) w.r.t. g_{6N} and equating to zero we get

$$g_{6N}{}^{*} = \frac{V_{yxrN}}{V_{xrN}} \tag{35}$$

Putting the optimum value of g_{6N} in the equation (34), we obtain a minimum MSE of P_{3N} as

$$MinMSE(P_{3N}) = \overline{Y}_N{}^{2}\left(V_{yrN} - \frac{V_{yxrN}^{2}}{V_{xrN}}\right) \tag{36}$$

where $MSE(P_{3N}) \in [MSE(P_{3L}), MSE(P_{3U})]$

CASE 2: IF THE SUM OF WEIGHTS IS ADJUSTABLE $(g_{5N} + g_{6N} \neq 1)$

$$P_{3N} - \overline{Y}_N = \left(g_{5N}-1\right)\overline{Y}_N + g_{5N}\overline{Y}\epsilon_{0N} + g_{6N}\left(1 - \frac{3\epsilon_{1N}}{2} + \frac{15\epsilon_{1N}{}^{2}}{8}\right) \tag{37}$$

Squaring on both sides we get

$$\left(P_{3N}-\overline{Y}_N\right)^2=\overline{Y}_N^{\,2}+\overline{Y}_N^{\,2}g_{5N}{}^2(1+\epsilon_{0N}^2)+g_{6N}{}^2\left(1+6\epsilon_{1N}^2\right)-2g_{5N}\overline{Y}_N^{\,2}-2g_{6N}\overline{Y}_N\left(1+\frac{15\epsilon_{1N}{}^2}{8}\right)+2g_{5N}g_{6N}\overline{Y}_N\left(1+\frac{15\epsilon_{1N}{}^2}{8}-\frac{3\epsilon_{0N}\epsilon_{1N}}{2}\right) \quad (38)$$

By utilizing first-order approximations for expectations, we can derive mean square error (MSE)

$$MSE\left(P_{3N}\right)=\overline{Y}_N^{\,2}V_{yrN}+g_{5N}^2A_{3N}+g_{6N}^2B_{3N}-2g_{5N}C_{3N}-2g_{6N}D_{3N}+2g_{5N}g_{6N}E_{3N} \quad (39)$$

where,

$$A_{3N}=\ \overline{Y}_N^{\,2}(1+V_{yrN})$$

$$B_{3N}=\ 1+6V_{xrN}$$

$$C_{3N}=\ \overline{Y}_N^{\,2}$$

$$D_{3N}=\ \overline{Y}_N\left(1+\frac{15}{8}V_{xrN}\right)$$

$$E_{3N}=\ \overline{Y}_N\left(1+\frac{15}{8}V_{xrN}-\frac{3}{2}V_{yxrN}\right)$$

To find out the minimum MSE for P_{3N}, we partially differentiate equation (39) w.r.t. g_{5N} & g_{6N} and equating to zero we get

$$g_{5N}{}^*=\frac{B_{3N}C_{3N}-D_{3N}E_{3N}}{A_{3N}B_{3N}-E_{3N}^2} \quad (40)$$

$$g_{6N}{}^*=\frac{A_{3N}D_{3N}-C_{3N}E_{3N}}{A_{3N}B_{3N}-E_{3N}^2} \quad (41)$$

Putting the optimum value of g_{5N} & g_{6N} in the equation (39), we obtain a minimum MSE of P_{3N} as

$$MinMSE((P_{3N}))=C_{3N}+\frac{B_{3N}C_{3N}^2+A_{3N}D_{3N}^2-2C_{3N}D_{3N}E_{3N}}{E_{3N}^2-A_{3N}B_{3N}} \quad (42)$$

where $MSE(P_{3N})\in[MSE(P_{3L}),MSE(P_{3U})]$
$P_{iN}\in[P_{iL},P_{iU}];i=1,2,3$, $A_{iN}\in[A_{iL},A_{iU}];i=1,2,3$, $B_{iN}\in[B_{iL},B_{iU}];i=1,2,3$
$C_{iN}\in[C_{iL},C_{iU}];i=1,2,3$, $D_{iN}\in[D_{iL},D_{iU}];i=1,2,3$, $E_{iN}\in[E_{iL},E_{iU}];i=1,2,3$

6. Numerical Illustrations

Here, we evaluate the performance of the recommended estimators in comparison to the other existing estimators considered in this paper. We have taken real-life natural growth rate data from the sample registration system (SRS) (2020). The data mentioned in the sample registration system (SRS) (2020) have four neutrosophic variables for every state, but in our research, we use birth rate vs natural growth rate. Here, the birth rate is the neutrosophic auxiliary variable $X_N \in [X_L, X_U]$ and natural growth rate is a neutrosophic study variable $Y_N \in [Y_L, Y_U]$.

Table 1: The Data of Natural Growth Rate as per SRS 2020

State	BRl	BRu	NGRl	NGRu	State	BRl	BRu	NGRl	NGRu
Andhra Pradesh	15	16	9	10.1	Uttar Pradesh	22.1	26.1	19.3	16.7
Assam	14.3	21.9	8.9	15.5	Uttarakhand	15.6	17	10.5	10.3
Bihar	21	26.2	15.7	20.7	West Bengal	11.2	16.1	10.8	5.4
Chhattisgarh	17.3	23.4	11	15	Arunachal Pradesh	15	17.8	11.8	10.6
NCT of Delhi	14.1	15.5	10.6	11.4	Goa	11.7	12.4	6.9	5.3
Gujrat	17.1	21.1	12	15.1	Himachal Pradesh	10	15.7	8.7	5.6
Haryana	17.7	21.2	12.3	14.7	Manipur	12.8	13.5	9.5	8
Jammu & Kashmir	11.1	16.1	7	11.3	Meghalaya	12.9	25.1	19.6	8.5
Jharkhand	17.6	23.4	13.1	17.9	Mizoram	11.7	16.8	13	7.1
Karnataka	15	17.5	10.2	10.5	Nagaland	11.8	12.9	9	8.4
Kerala	13.1	13.3	6.1	6.3	Sikkim	14	18.2	14.5	9.7
Madhya Pradesh	18.8	26	13.1	19.2	Tripura	10.7	13.4	8	4.2
Maharashtra	14.6	15.3	9.1	10.1	Andaman & Nicobar	10	11.5	5.4	4.7
Odisha	13.1	18.7	6.6	11.2	Chandigarh	12.8	18.1	14	9
Punjab	13.6	14.9	6.6	7.9	Dadar Nagar Haveli	18	21.4	18.1	13.3
Rajasthan	20.8	24.4	15.7	18.6	Ladakh	10.8	15.2	10	6.5

Tamil Nadu	13.6	14	6.8	8.5	Lakshadweep	13.1	19.9	12.7	8.1
Telangana	15.9	16.8	9.6	11.7	Puducherry	13.1	13.1	7	5.6

Further, we have drawn total $n_N = m_N r = 12$ samples from the given population of size 36 by utilizing the method of NRSS with set size $m_N = [3, 3]$ and replication $r = 4$. The NRSS method for the study and auxiliary variables is used to draw the NRSS sample simultaneously, as explained in Section 2. The formula for Percent Relative Efficiency (PRE) is defined, as

$$PRE\,(Estimators) = \frac{MSE(\overline{y}_{[n]N})}{MSE(estimator)} \times 100 \tag{43}$$

Table 2: The MSE and PRE of the Estimators

Estimators	MSE	I_N	PRE
$\overline{y}_{[n]N}$	[0.51461, 0.95605]	[0, 0.46]	[100, 100]
$\overline{y}_{rN}$	[0.11421, 0.16089]	[0, 0.29]	[451, 594]
$\overline{y}_{regN}$	[0.06610, 0.11572]	[0, 0.42]	[778, 826]
$\overline{y}_{expN}$	[0.26370, 0.43450]	[0, 0.39]	[195, 220]
$\overline{y}_{vsN}$	[0.06610, 0.11572]	[0, 0.42]	[778, 826]
t_{p1}	[0.06550, 0.11467]	[0, 0.42]	[786, 834]
t_{p2}	[0.01237, 0.03204]	[0, 0.61]	[2983, 4158]
t_{p3}	[0.01551, 0.03096]	[0, 0.49]	[3088, 3317]

7. Simulation Studies

We perform simulation studies to check the recommended estimator's efficiency with other existing estimators like the conventional, ratio, regression estimator, etc. This is done via the following steps

1. It is well known that a neutrosophic normal distribution (NND) will be followed by neutrosophic random variables (NRV), i.e.$(X_N, Y_N) \sim NN[(\mu_{xN}, \sigma^2_{xN}), (\mu_{yN}, \sigma^2_{yN})]$, $X_N \in [X_L, X_U]$, $Y_N \in [Y_L, Y_U]$, $\mu_{xN} \in [(\mu_{xL}, \mu_{xU})]$, $\mu_{yN} \in [(\mu_{yL}, \mu_{yU})]$, $\sigma^2_{xN} \in [\sigma^2_{xL}, \sigma^2_{xU}]$, $\sigma^2_{yN} \in [\sigma^2_{yL}, \sigma^2_{yU}]$.

We have generated 4-variate random observations of size N=1000 from a 4-variate normal distribution with mean $(\mu_{xL},\ \mu_{yL}\ \ \mu_{xU}, \mu_{yU}) = (50, 50, 60, 60)$ and covariance matrix $\begin{bmatrix} \sigma^2_{xL} & \rho_{xyL}\sigma_{xL}\sigma_{yL} & 0 & 0 \\ \rho_{xyL}\sigma_{xL}\sigma_{yL} & \sigma^2_{yL} & 0 & 0 \\ 0 & 0 & \sigma^2_{xU} & \rho_{xyL}\sigma_{v}\sigma_{yL} \\ 0 & 0 & \rho_{xyU}\sigma_{xU}\sigma_{yU} & \sigma^2_{yU} \end{bmatrix}$, where we have $\sigma^2_{xL} = 100$,$\sigma^2_{yL} =$ 100 , $\sigma^2_{xU} = 121$,$\sigma^2_{yU} = 121$.

2. For this N = 1000 simulated population, the parameters were computed.

3. A sample of size n with $m_N = 3$ and $r = 4, 6, 10$ has been selected from this simulated population.
4. To find the MSE of each estimator under study, use the sample data.
5. To get MSEs, the entire step 3–4 process was repeated 10,000 times. The MSE of each population mean estimator is the average of the 10,000 values that were obtained.
6. The PRE of each estimator in relation to $\overline{y}_{[n]N}$ has been calculated using the formula.
7. It can be done for some other population with parameters $(\mu_{xL},\ \mu_{yL}\ ,\ \mu_{xU}, \mu_{yU}) = (150, 150, 200, 200)$ where we have $\sigma^2_{xL} = 625$, $\sigma^2_{yL} = 625$, $\sigma^2_{xU} = 961$, $\sigma^2_{yU} = 961$.

Table 3: MSEs and PREs of the recommended and existing estimators under NRSS for Population 1

n=12	ρ=0.9			ρ=0.8		
Estimators	MSE	I_N	PRE	MSE	I_N	PRE
$\overline{y}_{[n]N}$	[4.74611, 8.39614]	[0, 0.43]	[100, 100]	[5.28228, 8.15711]	[0, 0.35]	[100, 100]
$\overline{y}_{rN}$	[1.45809, 1.7075]	[0, 0.14]	[326, 492]	[2.85339, 3.37538]	[0, 0.15]	[185, 242]
$\overline{y}_{regN}$	[1.27631, 1.45609]	[0, 0.12]	[372, 577]	[2.41149, 2.65902]	[0, 0.09]	[219, 307]
$\overline{y}_{expN}$	[2.07258, 2.97855]	[0, 0.30]	[229, 282]	[3.03606 , 3.68283]	[0, 0.17]	[174, 221]
$\overline{y}_{vsN}$	[1.27631, 1.45609]	[0, 0.12]	[372, 577]	[2.41149, 2.65902]	[0, 0.09]	[219, 307]
t_{p1}	[1.27201, 1.44744]	[0, 0.12]	[373, 580]	[2.40421, 2.64772]	[0, 0.09]	[220, 308]
t_{p2}	[0.62571, 0.99329]	[0, 0.37]	[759, 845]	[0.81364, 1.50894]	[0, 0.46]	[541, 649]
t_{p3}	[0.46767, 0.53091]	[0, 0.11]	[1015, 1581]	[0.9161, 1.0529]	[0, 0.12]	[577, 775]
	ρ=0.7			ρ=0.6		
	MSE	I_N	PRE	MSE	I_N	PRE
$\overline{y}_{[n]N}$	[5.80652, 8.11845]	[0, 0.28]	[100, 100]	[6.14863, 8.47812]	[0, 0.27]	[100, 100]
$\overline{y}_{rN}$	[4.21099, 5.02332]	[0, 0.16]	[138, 162]	[5.45556, 6.86807]	[0, 0.20]	[113, 123]
$\overline{y}_{regN}$	[3.46994, 3.75404]	[0, 0.07]	[167, 216]	[4.25988, 4.95358]	[0, 0.14]	[144, 171]
$\overline{y}_{expN}$	[3.97544, 4.50379]	[0, 0.11]	[146, 180]	[4.76362, 5.58763]	[0, 0.14]	[129, 152]
$\overline{y}_{vsN}$	[3.46994, 3.75404]	[0, 0.07]	[167, 216]	[4.25988, 4.95358]	[0, 0.14]	[144, 171]
t_{p1}	[3.45901, 3.74]	[0, 0.07]	[168, 217]	[4.24612, 4.93587]	[0, 0.13]	[145, 172]
t_{p2}	[0.89482, 1.7252]	[0, 0.48]	[471, 649]	[0.92517, 1.84276]	[0, 0.49]	[460, 665]
t_{p3}	[1.34764, 1.57163]	[0, 0.14]	[431, 517]	[1.74662, 2.15988]	[0, 0.19]	[352, 393]
n=18	ρ=0.9			ρ=0.8		
	MSE	I_N	PRE	MSE	I_N	PRE
$\overline{y}_{[n]N}$	[3.28385, 5.96403]	[0, 0.44]	[100, 100]	[3.68678, 5.79281]	[0, 0.36]	[100, 100]

$\overline{y}_{rN}$	[1.03602,1.21419]	[0, 0.14]	[317, 491]	[2.02872, 2.39945]	[0, 0.15]	[182, 241]
$\overline{y}_{regN}$	[0.9431, 1.08339]	[0, 0.12]	[348, 550]	[1.78499, 1.98643]	[0, 0.10]	[207, 292]
$\overline{y}_{expN}$	[1.45551, 2.11756]	[0, 0.31]	[226, 282]	[2.1514, 2.61751]	[0, 0.17]	[171, 221]
$\overline{y}_{vsN}$	[0.9431, 1.08339]	[0, 0.12]	[348, 550]	[1.78499, 1.98643]	[0, 0.10]	[207, 292]
t_{p1}	[0.94105, 1.07914]	[0, 0.12]	[349, 553]	[1.78141, 1.98074]	[0, 0.10]	[207, 292]
t_{p2}	[0.453, 0.74575]	[0, 0.39]	[725, 800]	[0.58369, 1.12502]	[0, 0.48]	[515, 632]
t_{p3}	[0.34604, 0.4]	[0, 0.13]	[949, 1491]	[0.67659, 0.79081]	[0, 0.14]	[545, 733]
	ρ=0.7			ρ=0.6		
$\overline{y}_{[n]N}$	[4.06532, 5.77401]	[0, 0.29]	[100, 100]	[4.32649, 6.02582]	[0, 0.28]	[100, 100]
$\overline{y}_{rN}$	[2.96637, 3.57305]	[0, 0.16]	[137, 162]	[3.85889, 4.88325]	[0, 0.20]	[112, 123]
$\overline{y}_{regN}$	[2.54775, 2.79559]	[0, 0.08]	[160, 207]	[3.14193, 3.68093]	[0, 0.14]	[138, 164]
$\overline{y}_{expN}$	[2.81002, 3.20285]	[0, 0.12]	[145, 180]	[3.38185, 3.97694]	[0, 0.14]	[128, 152]
$\overline{y}_{vsN}$	[2.54775, 2.79559]	[0, 0.08]	[160, 207]	[3.14193, 3.68093]	[0, 0.14]	[138, 164]
t_{p1}	[2.54239, 2.78846]	[0, 0.08]	[160, 207]	[3.13511, 3.67194]	[0, 0.14]	[138, 164]
t_{p2}	[0.63681, 1.28576]	[0, 0.50]	[449, 638]	[0.6594, 1.36624]	[0, 0.51]	[441, 656]
t_{p3}	[0.98574, 1.17705]	[0, 0.16]	[412, 491]	[1.28237 , 1.61483]	[0, 0.20]	[337, 373]
n=30	ρ=0.9			ρ=0.8		
	MSE	I_N	PRE	MSE	I_N	PRE
$\overline{y}_{[n]N}$	[2.02919, 3.74983]	[0, 0.45]	[100, 100]	[2.28937, 3.65813]	[0, 0.37]	[100, 100]
$\overline{y}_{rN}$	[0.64949, 0.76453]	[0, 0.15]	[312, 490]	[1.27051, 1.51332]	[0, 0.16]	[180, 242]
$\overline{y}_{regN}$	[0.6084, 0.70205]	[0, 0.13]	[334, 534]	[1.15219, 1.29443]	[0, 0.10]	[199, 283]
$\overline{y}_{expN}$	[0.90713, 1.33037]	[0, 0.31]	[224, 282]	[1.34678, 1.65497]	[0, 0.18]	[170, 221]
$\overline{y}_{vsN}$	[0.6084, 0.70205]	[0, 0.13]	[334, 534]	[1.15219, 1.29443]	[0, 0.10]	[199, 283]
t_{p1}	[0.60762, 0.70039]	[0, 0.13]	[334, 535]	[1.1508, 1.29218]	[0, 0.10]	[199, 283]
t_{p2}	[0.28907, 0.48705]	[0, 0.40]	[702, 770]	[0.36999, 0.72973]	[0, 0.49]	[501, 619]
t_{p3}	[0.2232, 0.26151]	[0, 0.14]	[909, 1434]	[0.43531, 0.51667]	[0, 0.15]	[526, 708]
	ρ=0.7			ρ=0.6		
	MSE	I_N	PRE	MSE	I_N	PRE
$\overline{y}_{[n]N}$	[2.53419, 3.64756]	[0, 0.30]	[100, 100]	[2.70872, 3.79625]	[0, 0.28]	[100, 100]
$\overline{y}_{rN}$	[1.85449, 2.25458]	[0, 0.17]	[137, 162]	[2.41511, 3.07899]	[0, 0.21]	[112, 123]
$\overline{y}_{regN}$	[1.63839, 1.81847]	[0, 0.09]	[155, 201]	[2.03392, 2.38552]	[0, 0.14]	[133, 159]
$\overline{y}_{expN}$	[1.76129, 2.02175]	[0, 0.12]	[144, 180]	[2.12632, 2.50541]	[0, 0.15]	[127, 152]
$\overline{y}_{vsN}$	[1.63839, 1.81847]	[0, 0.09]	[155, 201]	[2.03392, 2.38552]	[0, 0.14]	[133, 159]
t_{p1}	[1.63631, 1.81563]	[0, 0.09]	[155, 201]	[2.03124, 2.38195]	[0, 0.14]	[133, 159]
t_{p2}	[0.40188, 0.83762]	[0, 0.52]	[435, 631]	[0.41655, 0.8877]	[0, 0.53]	[428, 650]
t_{p3}	[0.63275, 0.76768]	[0, 0.17]	[401, 475]	[0.82381, 1.05238]	[0, 0.21]	[329, 361]

Table 4: MSEs and PREs of the recommended and existing estimator under NRSS for Population 2

n=12	ρ=0.9			ρ=0.8		
Estimators	MSE	I_N	PRE	MSE	I_N	PRE
$\overline{y}_{[n]N}$	[29.57415, 66.14823]	[0, 0.55]	[100, 100]	[33.01424, 64.78497]	[0, 0.49]	[100, 100]
$\overline{y}_{rN}$	[9.15248, 13.43196]	[0, 0.31]	[323, 492]	[17.79146, 26.83172]	[0, 0.33]	[186, 241]
$\overline{y}_{regN}$	[8.07017, 11.47311]	[0, 0.29]	[366, 577]	[15.07181, 21.11836]	[0, 0.28]	[219, 307]
$\overline{y}_{expN}$	[12.91778, 23.47267]	[0, 0.44]	[229, 282]	[18.97746, 29.18226]	[0, 0.34]	[174, 222]
$\overline{y}_{vsN}$	[8.07017, 11.47311]	[0, 0.29]	[366, 577]	[15.07181, 21.11836]	[0, 0.28]	[219, 307]
t_{p1}	[8.06334, 11.45174]	[0, 0.29]	[367, 578]	[15.06036, 21.09007]	[0, 0.28]	[219, 307]
t_{p2}	[3.96677, 7.94098]	[0, 0.50]	[746, 833]	[5.0761, 12.12343]	[0, 0.58]	[534, 650]
t_{p3}	[2.96821, 4.26557]	[0, 0.30]	[996, 1551]	[5.75332, 8.46915]	[0, 0.32]	[574, 765]
	ρ=0.7			ρ=0.6		
	MSE	PRE	MSE	PRE		
$\overline{y}_{[n]N}$	[36.2907, 66.5611]	[0, 0.45]	[100, 100]	[38.4289, 67.3345]	[0, 0.42]	[100, 100]
$\overline{y}_{rN}$	[26.3081, 40.3418]	[0, 0.34]	[138, 165]	[33.9908, 54.1438]	[0, 0.37]	[113, 124]
$\overline{y}_{regN}$	[21.6871, 30.9702]	[0, 0.29]	[167, 215]	[26.6243, 39.3420]	[0, 0.32]	[144, 171]
$\overline{y}_{expN}$	[24.8384, 37.1523]	[0, 0.33]	[146, 179]	[29.7571, 44,3811]	[0, 0.32]	[129, 152]
$\overline{y}_{vsN}$	[21.6871, 30.9702]	[0, 0.29]	[167, 215]	[26.6243, 39.3420]	[0, 0.32]	[144, 171]
t_{p1}	[21.67, 30.9336]	[0, 0.29]	[167, 215]	[26.6026, 39.2976]	[0, 0.32]	[144, 171]
t_{p2}	[5.6044, 13.5728]	[0, 0.58]	[490, 648]	[5.7578, 14.4769]	[0, 0.60]	[465, 667]
t_{p3}	[8.4746, 12.8287]	[0, 0.33]	[428, 519]	[10.9482, 17.1972]	[0, 0.36]	[351, 392]
n=18	ρ=0.9			ρ=0.8		
	MSE	PRE	MSE	PRE		
$\overline{y}_{[n]N}$	[20.46536, 47.09541]	[0, 0.56]	[100, 100]	[23.0424, 46.00735]	[0, 0.49]	[100, 100]
$\overline{y}_{rN}$	[6.46258, 9.56084]	[0, 0.32]	[317, 493]	[12.65581, 19.08902]	[0, 0.33]	[182, 241]
$\overline{y}_{regN}$	[5.91444, 8.54838]	[0, 0.30]	[346, 551]	[11.15622, 15.77654]	[0, 0.29]	[207, 292]
$\overline{y}_{expN}$	[9.0553, 16.73269]	[0, 0.45]	[226, 281]	[13.44981, 20.74272]	[0, 0.35]	[171, 222]
$\overline{y}_{vsN}$	[5.91444, 8.54838]	[0, 0.30]	[346, 551]	[11.15622, 15.77654]	[0, 0.29]	[207, 292]
t_{p1}	[5.91119, 8.53782]	[0, 0.30]	[346, 552]	[11.15058, 15.76229]	[0, 0.29]	[207, 292]
t_{p2}	[2.85023, 5.91261]	[0, 0.51]	[718, 797]	[3.63777, 9.02249]	[0, 0.59]	[510, 633]
t_{p3}	[2.17347, 3.19023]	[0, 0.31]	[942, 1476]	[4.23969, 6.33831]	[0, 0.33]	[543, 726]

	ρ=0.7			ρ=0.6		
	MSE	PRE	MSE	PRE		
$\overline{y}_{[n]N}$	[25.40823, 47.41032]	[0, 0.46]	[100, 100]	[27.04054, 47.85793]	[0, 0.43]	[100, 100]
$\overline{y}_{rN}$	[18.53928, 28.71605]	[0, 0.35]	[137, 165]	[24.0593, 38.54122]	[0, 0.37]	[112, 124]
$\overline{y}_{regN}$	[15.92344, 23.08036]	[0, 0.31]	[160, 205]	[19.63708, 29.23447]	[0, 0.32]	[138, 164]
$\overline{y}_{expN}$	[17.55694, 26.47846]	[0, 0.33]	[145, 179]	[21.13014, 31.59581]	[0, 0.33]	[128, 151]
$\overline{y}_{vsN}$	[15.92344, 23.08036]	[0, 0.31]	[160, 205]	[19.63708, 29.23447]	[0, 0.32]	[138, 164]
t_{p1}	[15.91501, 23.06175]	[0, 0.30]	[160, 206]	[19.62635, 29.21191]	[0, 0.32]	[138, 164]
t_{p2}	[3.98907, 10.09363]	[0, 0.60]	[470, 637]	[4.10388, 10.73474]	[0, 0.61]	[446, 659]
t_{p3}	[6.18868, 9.58314]	[0, 0.35]	[411, 495]	[8.02644, 12.83379]	[0, 0.37]	[337, 373]
n=30	ρ=0.9			ρ=0.8		
	MSE	PRE	MSE	PRE		
$\overline{y}_{[n]N}$	[12.6555, 29.7430]	[0, 0.57]	[100, 100]	[14.3085, 29.0534]	[0, 0.50]	[100, 100]
$\overline{y}_{rN}$	[4.0533, 6.0421]	[0, 0.32]	[312, 492]	[7.9305, 12.0457]	[0, 0.34]	[180, 241]
$\overline{y}_{regN}$	[3.8106, 5.5601]	[0, 0.31]	[332, 535]	[7.2011, 10.2805]	[0, 0.29]	[199, 283]
$\overline{y}_{expN}$	[5.6476, 10.5702]	[0, 0.46]	[224, 281]	[8.4200, 13.1170]	[0, 0.35]	[170, 221]
$\overline{y}_{vsN}$	[3.8106, 5.5601]	[0, 0.31]	[332, 535]	[7.2011, 10.2805]	[0, 0.29]	[199, 283]
t_{p1}	[3.8093, 5.5560]	[0, 0.31]	[332, 535]	[7.19898, 10.2749]	[0, 0.29]	[199, 283]
t_{p2}	[1.8154, 3.8495]	[0, 0.52]	[697, 773]	[2.3060, 5.8458]	[0, 0.60]	[497, 620]
t_{p3}	[1.3995, 2.0815]	[0, 0.32]	[904,1429]	[2.7238, 4.1296]	[0, 0.34]	[525, 704]
	ρ=0.7			ρ=0.6		
	MSE	PRE	MSE	PRE		
$\overline{y}_{[n]N}$	[15.83871, 29.97289]	[0, 0.47]	[100, 100]	[16.9295, 30.15039]	[0, 0.43]	[100, 100]
$\overline{y}_{rN}$	[11.59369, 18.12673]	[0, 0.36]	[137, 165]	[15.06837, 24.32259]	[0, 0.38]	[112, 124]
$\overline{y}_{regN}$	[10.23995, 14.99098]	[0, 0.31]	[155, 200]	[12.712, 18.94614]	[0, 0.32]	[133, 159]
$\overline{y}_{expN}$	[11.00532, 16.73579]	[0, 0.34]	[144, 179]	[13.28795, 19.90633]	[0, 0.33]	[127, 151]
$\overline{y}_{vsN}$	[10.23995, 14.99098]	[0, 0.31]	[155, 200]	[12.712, 18.94614]	[0, 0.32]	[133, 159]
t_{p1}	[10.23667, 14.98357]	[0, 0.31]	[155, 200]	[12.70778, 18.93718]	[0, 0.32]	[133, 159]
t_{p2}	[2.51734, 6.55744]	[0, 0.61]	[457, 629]	[2.59353, 6.98028]	[0, 0.62]	[432, 653]
t_{p3}	[3.96677, 6.23344]	[0, 0.36]	[399, 481]	[5.15061, 8.35123]	[0, 0.38]	[329, 361]

Table 5: PREs of the NRSS estimators over estimators under NSRS for Population 1

n=12	ρ=0.9	ρ=0.8	ρ=0.7	ρ=0.6
Estimators	PRE	PRE	PRE	PRE
$\overline{y}_{[n]N}$	[120, 183]	[120, 165]	[120, 149]	[120, 142]
$\overline{y}_{rN}$	[119, 122]	[119, 125]	[120, 126]	[119, 131]
$\overline{y}_{rgN}$	[120, 121]	[120, 121]	[119, 121]	[120, 121]
$\overline{y}_{expN}$	[119, 148]	[119, 131]	[119, 121]	[119, 121]
$\overline{y}_{vsN}$	[120, 121]	[120, 121]	[119, 121]	[120, 121]
t_{p1}	[120, 121]	[120, 121]	[119, 121]	[120, 121]
t_{p2}	[121, 170]	[122, 193]	[123, 205]	[122, 210]
t_{p3}	[121, 121]	[121, 125]	[122, 128]	[121, 131]
n=18	ρ=0.9	ρ=0.8	ρ=0.7	ρ=0.6
	PRE	PRE	PRE	PRE
$\overline{y}_{[n]N}$	[112, 174]	[112, 156]	[112, 142]	[112, 133]
$\overline{y}_{rN}$	[112, 115]	[112, 118]	[113, 120]	[112, 124]
$\overline{y}_{regN}$	[112, 113]	[112, 113]	[113, 113]	[112, 113]
$\overline{y}_{expN}$	[112, 139]	[112, 122]	[112, 115]	[112, 113]
$\overline{y}_{vsN}$	[112, 113]	[112, 113]	[113, 113]	[112, 113]
t_{p1}	[112, 113]	[112, 113]	[112, 113]	[112, 113]
t_{p2}	[113, 165]	[112, 188]	[114, 197]	[113, 204]
t_{p3}	[113, 114]	[113, 118]	[113, 121]	[113, 124]
n=30	ρ=0.9	ρ=0.8	ρ=0.7	ρ=0.6
	PRE	PRE	PRE	PRE
$\overline{y}_{[n]N}$	[107, 169]	[107, 150]	[107, 136]	[107, 127]
$\overline{y}_{rN}$	[106, 110]	[106, 112]	[107, 115]	[106, 118]
$\overline{y}_{regN}$	[107, 107]	[106, 107]	[107, 107]	[107, 107]
$\overline{y}_{expN}$	[107, 133]	[107, 116]	[107, 110]	[107, 107]
$\overline{y}_{vsN}$	[107, 107]	[106, 107]	[107, 107]	[107, 107]
t_{p1}	[107, 107]	[106, 107]	[107, 107]	[107, 107]
t_{p2}	[107, 159]	[107, 183]	[108, 192]	[107, 199]
t_{p3}	[107, 109]	[107, 112]	[107, 116]	[107, 119]

Table 6: PREs of the NRSS estimators over estimators under NSRS for Population 2

n=12	ρ=0.9	ρ=0.8	ρ=0.7	ρ=0.6
Estimators	PRE	PRE	PRE	PRE
$\overline{y}_{[n]N}$	[121, 183]	[120, 165]	[121, 149]	[120, 142]
$\overline{y}_{rN}$	[120, 121]	[119, 125]	[120, 126]	[119, 131]
$\overline{y}_{regN}$	[120, 122]	[120, 121]	[119, 123]	[120, 121]
$\overline{y}_{expN}$	[120, 147]	[119, 131]	[121, 121]	[119, 121]
$\overline{y}_{vsN}$	[120, 122]	[120, 121]	[119, 123]	[120, 121]
t_{p1}	[119, 122]	[120, 121]	[119, 122]	[120, 121]
t_{p2}	[120, 167]	[121, 194]	[123, 205]	[123, 210]
t_{p3}	[120, 120]	[120, 125]	[122, 127]	[121, 131]
n=18	ρ=0.9	ρ=0.8	ρ=0.7	ρ=0.6
	PRE	PRE	PRE	PRE
$\overline{y}_{[n]N}$	[112, 176]	[112, 156]	[113, 142]	[112, 133]
$\overline{y}_{rN}$	[113, 115]	[112, 118]	[113, 120]	[112, 124]
$\overline{y}_{regN}$	[113, 113]	[112, 113]	[113, 114]	[112, 113]
$\overline{y}_{expN}$	[141, 112]	[122, 113]	[115, 113]	[113, 112]
$\overline{y}_{vsN}$	[113, 113]	[112, 113]	[112, 113]	[112, 113]
t_{p1}	[113, 113]	[112, 113]	[112, 113]	[112, 113]
t_{p2}	[113, 161]	[112, 189]	[114, 197]	[144, 205]
t_{p3}	[113, 115]	[112, 118]	[113, 121]	[113, 124]
n=30	ρ=0.9	ρ=0.8	ρ=0.7	ρ=0.6
	PRE	PRE	PRE	PRE
$\overline{y}_{[n]N}$	[107, 170]	[107, 150]	[107, 136]	[107, 127]
$\overline{y}_{rN}$	[107, 109]	[106, 112]	[107, 115]	[107, 118]
$\overline{y}_{regN}$	[107, 107]	[106, 107]	[107, 107]	[107, 107]
$\overline{y}_{expN}$	[107, 135]	[107, 116]	[107, 110]	[107, 107]
$\overline{y}_{vsN}$	[107, 107]	[106, 107]	[107, 107]	[107, 107]
t_{p1}	[107, 107]	[106, 107]	[107, 107]	[106, 107]
t_{p2}	[107, 156]	[106, 183]	[108, 192]	[108, 199]
t_{p3}	[107, 109]	[106, 113]	[107, 116]	[107, 119]

Table 7: PREs of the estimators (neutrosophic vs classical) for Population 1

n=12	PRE		PRE		PRE		PRE	
	ρ=0.9		ρ=0.8		ρ=0.7		ρ=0.6	
Estimators	neutrosophic	classical	neutrosophic	classical	neutrosophic	classical	neutrosophic	classical
$\overline{y}_{[n]N}$	[100, 100]	100	[100, 100]	100	[100, 100]	100	[100, 100]	100
$\overline{y}_{rN}$	[326, 492]	291	[185, 242]	167	[138, 162]	125	[113, 123]	103
$\overline{y}_{regN}$	[372, 577]	364	[219, 307]	215	[167, 216]	166	[144, 171]	142
$\overline{y}_{expN}$	[229, 282]	246	[174, 221]	179	[146, 180]	148	[129, 152]	129
$\overline{y}_{vsN}$	[372, 577]	364	[219, 307]	215	[167, 216]	166	[144, 171]	142
t_{p1}	[373, 580]	366	[220, 308]	215	[168, 217]	166	[145, 172]	143
t_{p2}	[759, 845]	559	[541, 649]	497	[471, 649]	506	[460, 665]	530
t_{p3}	[1015,1581]	923	[577, 775]	523	[431, 517]	390	[352, 393]	320
n=18	PRE		PRE		PRE		PRE	
	ρ=0.9		ρ=0.8		ρ=0.7		ρ=0.6	
	neutrosophic	classical	neutrosophic	classical	neutrosophic	classical	neutrosophic	classical
$\overline{y}_{[n]N}$	[100, 100]	100	[100, 100]	100	[100, 100]	100	[100, 100]	100
$\overline{y}_{rN}$	[317, 491]	285	[182, 241]	165	[137, 162]	124	[112, 123]	103
$\overline{y}_{regN}$	[348, 550]	342	[207, 292]	203	[160, 207]	158	[138, 164]	136
$\overline{y}_{expN}$	[226, 282]	243	[171, 221]	177	[145, 180]	146	[128, 152]	128
$\overline{y}_{vsN}$	[348, 550]	342	[207, 292]	203	[160, 207]	158	[138, 164]	136
t_{p1}	[349, 553]	343	[207, 292]	204	[160, 207]	158	[138, 164]	136
t_{p2}	[725, 800]	534	[515, 632]	482	[449, 638]	496	[441, 656]	522
t_{p3}	[949, 1491]	862	[545, 733]	493	[412, 491]	371	[337, 373]	307
n=30	PRE		PRE		PRE		PRE	
	ρ=0.9		ρ=0.8		ρ=0.7		ρ=0.6	
	neutrosophic	classical	neutrosophic	classical	neutrosophic	classical	neutrosophic	classical
$\overline{y}_{[n]N}$	[100, 100]	100	[100, 100]	100	[100, 100]	100	[100, 100]	100
$\overline{y}_{rN}$	[312, 490]	280	[180, 242]	163	[137, 162]	123	[112, 123]	102
$\overline{y}_{regN}$	[334, 534]	326	[199, 283]	195	[155, 201]	152	[133, 159]	131
$\overline{y}_{expN}$	[224, 282]	240	[170, 221]	175	[144, 180]	145	[127, 152]	127
$\overline{y}_{vsN}$	[334, 534]	326	[199, 283]	195	[155, 201]	152	[133, 159]	131
t_{p1}	[334, 535]	327	[199, 283]	195	[155, 201]	152	[133, 159]	132
t_{p2}	[702, 770]	521	[501, 619]	476	[435, 631]	491	[428, 650]	519
t_{p3}	[909,1434]	821	[526, 708]	474	[401, 475]	357	[329, 361]	296

Table 8: PREs of the estimators (neutrosophic vs classical) for Population 2

n=12	PRE		PRE		PRE		PRE	
	ρ=0.9		ρ=0.8		ρ=0.7		ρ=0.6	
Estimators	neutrosophic	classical	neutrosophic	classical	neutrosophic	classical	neutrosophic	classical
$\overline{y}_{[n]N}$	[100, 100]	100	[100, 100]	100	[100, 100]	100	[100, 100]	100
$\overline{y}_{rN}$	[323, 492]	322	[186, 241]	183	[138, 165]	136	[113, 124]	112
$\overline{y}_{regN}$	[366, 577]	369	[219, 307]	217	[167, 215]	168	[144, 171]	144
$\overline{y}_{expN}$	[229, 282]	227	[174, 222]	173	[146, 179]	146	[129, 152]	129
$\overline{y}_{vsN}$	[366, 577]	369	[219, 307]	217	[167, 215]	168	[144, 171]	144
t_{p1}	[367, 578]	369	[219, 307]	217	[167, 215]	168	[144, 171]	144
t_{p2}	[746, 833]	757	[534, 650]	641	[490, 648]	635	[465, 667]	654
t_{p3}	[996,1551]	1013	[574, 765]	574	[428, 519]	429	[351, 392]	352
n=18	PRE		PRE		PRE		PRE	
	ρ=0.9		ρ=0.8		ρ=0.7		ρ=0.6	
	neutrosophic	classical	neutrosophic	classical	neutrosophic	classical	neutrosophic	classical
$\overline{y}_{[n]N}$	[100, 100]	100	[100, 100]	100	[100, 100]	100	[100, 100]	100
$\overline{y}_{rN}$	[317, 493]	314	[182, 241]	180	[137, 165]	135	[112, 124]	112
$\overline{y}_{regN}$	[346, 551]	344	[207, 292]	205	[160, 205]	160	[138, 164]	137
$\overline{y}_{expN}$	[226, 281]	223	[171, 222]	171	[145, 179]	144	[128, 151]	128
$\overline{y}_{vsN}$	[346, 551]	344	[207, 292]	205	[160, 205]	160	[138, 164]	137
t_{p1}	[346, 552]	345	[207, 292]	205	[160, 206]	160	[138, 164]	137
t_{p2}	[718, 797]	725	[510, 633]	623	[470, 637]	622	[446, 659]	642
t_{p3}	[942,1476]	945	[543, 726]	543	[411, 495]	408	[337, 373]	337
n=30	PRE		PRE		PRE		PRE	
	ρ=0.9		ρ=0.8		ρ=0.7		ρ=0.6	
	neutrosophic	classical	neutrosophic	classical	neutrosophic	classical	neutrosophic	classical
$\overline{y}_{[n]N}$	[100, 100]	100	[100, 100]	100	[100, 100]	100	[100, 100]	100
$\overline{y}_{rN}$	[312, 492]	308	[180, 241]	178	[137, 165]	134	[112, 124]	111
$\overline{y}_{regN}$	[332, 535]	329	[199, 283]	197	[155, 200]	154	[133, 159]	133
$\overline{y}_{expN}$	[224, 281]	221	[170, 221]	169	[144, 179]	143	[127, 151]	127
$\overline{y}_{vsN}$	[332, 535]	329	[199, 283]	197	[155, 200]	154	[133, 159]	133
t_{p1}	[332, 535]	329	[199, 283]	197	[155, 200]	154	[133, 159]	133
t_{p2}	[697, 773]	703	[497, 620]	611	[457, 629]	611	[432, 653]	634
t_{p3}	[904,1429]	900	[525, 704]	520	[399, 481]	392	[329, 361]	325

8. Discussion

The study established mathematical expressions for novel NRSS estimators, approximating up to the first order. Subsequently to examine the properties of the proposed NRSS estimators, numerical illustrations and simulation studies were conducted. The former used real-world natural growth rate data, while the latter involved two artificial neutrosophic datasets with varying correlation coefficients and sample sizes. The results were encapsulated in Tables 2, 3, and 4, showcasing MSEs and PREs for both existing and proposed neutrosophic ranked set estimators. We have computed the PREs of the NRSS estimators over estimators under NSRS and these results are displayed in Tables 5 and 6.

In Table 2, the MSEs of the existing and proposed estimators are given along with PRE. The superiority of the suggested NRSS estimators over the existing NRSS estimators is displayed in Table 2 in the bolded text. We also see the MSE and PRE of the recommended estimator are lesser and higher than other existing estimators. It is evident from the table that recommended estimators outperformed existing ones, offering lower MSEs and higher PREs, and it has been established that t_{p3} is the best estimator available.

Similarly, in Table 3 and Table 4, the MSEs of the recommended and existing estimators are given along with PRE through a simulation study based on artificial neutrosophic data for different values of the correlation coefficient and different sample sizes. Like Table 2, also in Tables 3 and 4, the superiority of the suggested NRSS estimators over the existing NRSS estimators is displayed by the bolded text. We also see the MSEs and PREs of the recommended estimators are lesser and higher, respectively than those of other existing estimators. Hence, Tables 3 and 4 mirrored these findings, with the proposed estimators continuing to outshine existing ones, demonstrating lower MSEs and higher PREs in the simulation study too.

From Tables 3 and 4, we see with the increase in values of sample sizes, and correlation coefficients, the MSE and PRE of the recommended estimator decrease and increase. Therefore, under NRSS, the suggested estimators exhibit sensitivity similar to that of classical ranked set sampling.

Tables 5 and 6 featured PRE values of the proposed NRSS estimators over NSRS counterparts. We see from Tables 5 and 6, that all PRE values exceeded 100 that is all the NRSS estimators are superior to corresponding estimators under NSRS as RSS is the best replacement for SRS. The comparison between classical RSS and NRSS using PREs is provided in Tables 7 and 8. Tables 7 and 8 demonstrate that the PREs of the suggested estimators obtained through classical RSS are lower than those obtained using NRSS, indicating that the latter method is more effective than the former.

The study highlighted that classical ranked set sampling was ill-suited for dealing with vague or indeterminate data. NRSS proved superior for estimating uncertain or interval data. The tables presented dependable results for neutrosophic data compared to classical results.

9. Conclusion

In this research paper, we've put forth some enhanced neutrosophic ranked set estimators designed for estimating population means while making use of auxiliary information. To assess their accuracy, we calculated both bias and MSE for these proposed estimators, focusing on first-order approximations. We compared our recommended estimators against existing ones, by using a natural population's data on natural growth rates and two simulated populations. Through a combination of numerical illustrations and simulated studies, we've found compelling evidence that our proposed estimators outperform existing ones within the framework of neutrosophic ranked set sampling. Among these estimators, t_{p3} emerged as the top performer. It's important to note that the sensitivity analysis of our recommended estimators under NRSS mirrors that of classical RSS.

Moreover, a comparison between the recommended estimators under NRSS and the estimators under NSRS revealed that NRSS is a more effective alternative to NSRS, much like classical RSS to classical SRS. Our study underscores the efficiency and reliability of NRSS for handling neutrosophic data, with our proposed NRSS delivering superior mean estimations compared to existing methods.

The current investigation is subject to certain constraints, notably concerning the applicability of neutrosophic ranked set sampling. This method proves to be proficient in estimating population parameters under conditions of equal allocation, perfect ranking, and adherence to a symmetric distribution. However, when these conditions are not met, the efficiency of the estimation diminishes, leading to suboptimal results.

Based on the numerical illustrations and simulation studies we've conducted, it's reasonable to recommend the use of our proposed estimators over the alternatives presented in this paper in various real-world scenarios, spanning fields like agriculture, mathematics, biology, poultry farming, economics, commerce, and the social sciences.

Furthermore, given the limited availability of neutrosophic RSS estimators, there's ample room for further exploration. Building upon this study, we can consider defining variations of neutrosophic ranked set sampling, such as unbalanced NRSS, median NRSS, extreme NRSS, double NRSS, and percentile neutrosophic ranked set sampling, akin to what exists in classical ranked set sampling. Additionally, we can explore the replacement of our proposed estimators with alternative methods or estimators.

Expanding beyond sampling theory, further research avenues in statistics, encompassing fields like control charts, inference, reliability analysis, non-parametric estimation, hypothesis testing, and some other fields of science, present promising opportunities for exploration.

Conflicts of Interest: The authors declare no conflict of interest.

References

[1] Sisodia, B.V.S. & Dwivedi V.K. (1981). A modified ratio estimator using co-efficient of variation of auxiliary variable. J Indian Soc Agric Stat, 33(1), 13–18.

[2] Pandey, B.N. & Dubey, V. (1988). Modified product estimator using coefficient of variation of the auxiliary variate. Assam Statistical Rev., 2(2), 64-66.

[3] Bahl, S. & Tuteja, R.K. (1991). Ratio and product type exponential estimators, Journal of information and optimization sciences, 12(1), 159- 164.

[4] Upadhyaya, L.N. & Singh, H.P. (1999). Use of transformed auxiliary variable in estimating the finite population mean. Biom J, 41(5), 627–636.

[5] Singh, H.P., Tailor, R., Tailor R, & Kakran, M.S. (2004). An Improved Estimator of population mean using Power transformation. J Indian Soc Agric Stat, 58(2), 223–230.

[6] Kadilar, C. & Cingi, H. (2004). Ratio estimators in simple random sampling. Appl Math Comput, 151, 893–902.

[7] Singh, P., Bouza, C., & Singh, R. (2019). Generalized exponential estimator for estimating the population mean using an auxiliary variable. Jour. of Sci. Res., 63, 273-280.

[8] McIntyre, G.A. (1952). A method for unbiased selective sampling using ranked sets. Crop Pasture Sci, 3, 385–390.

[9] Takahasi, K. & Wakimoto, K. (1968). On unbiased estimates of the population mean based on the sample stratified by means of ordering. Ann Inst Stat Math, 20, 1–31.

[10] Dell, T. & Clutter, J. (1972). Ranked set sampling theory with order statistics background. Biometrics, 28, 545–555.

[11] Samawi, H.M. & Muttlak, H.A. (1996). Estimation of ratio using rank set sampling. Biom J., 38(6), 753–764.

[12] Stokes, L. (1977). Ranked set sampling with concomitant variables. Commun Stat Theory Methods, 6, 1207–1211.

[13] Al-Saleh, M.F. & Al-Omari, A.I. (2002). Multistage ranked set sampling. J Stat Plann Inference, 102(2), 273–286.

[14] Bouza, C.N. (2002). Ranked set sub-sampling of the non-response strata for estimating the difference of means. Biomet J J Math Methods Biosci, 44(7), 903–915.

[15] Ganesh, S. & Ganeslingam, S. (2006). Ranked set sampling vs simple random sampling in the estimation of the mean and the ratio. Journal of Statistics and Management Systems, 9(2), 459-472.

[16] Bouza, C.N. (2008). Ranked set sampling for the product estimator. Rev Invest Oper, 29(3), 201–206.

[17] Kadilar, C., Unyazici. Y., & Cingi, H. (2009). Ratio estimator for the population mean using ranked set sampling. Stat. Pap., 50, 301-309.

[18] Singh, H.P., Tailor, R., & Singh, S. (2014). General procedure for estimating the population means using ranked set sampling. Journal of Statistical Computation and Simulation, 84 (5), 931-945.

[19] Mandowara, V.L. & Mehta, N. (2013). Efficient generalized ratio-product type estimators for finite population mean with ranked set sampling. Austr J Stat, 42(3), 137–148.

[20] Al-Omari, A.I. & Bouza, C.N. (2015). Ratio estimators of the population mean with missing values

using ranked set sampling. Environmetrics, 26(2), 67-76.
[21] Singh, A. & Vishwakarma, G.K. (2021). Improved predictive estimation for mean using the Searls technique under ranked set sampling. Communications in Statistics-Theory and Methods, 50(9), 2015-2038.
[22] Bhushan, S., Kumar, A., & Lone, S.A. (2022). On some novel classes of estimators using RSS. Alexandria Engineering Journal, 61, 5465-5474.
[23] Singh, R. & Kumari, A. (2023). Improved Estimators of Population Mean Using Auxiliary Variables in Ranked Set Sampling. Rev Invest Oper, 44(2), 271-280.
[24] Atanassov, K. (1986). Intuitionistic fuzzy sets. Fuzzy Sets Syst, 20(1), 87-96.
[25] Atanassov, K. (1999). New operations defined over the intuitionistic fuzzy sets. Fuzzy Sets Syst, 61(2), 137-142.
[26] Smarandache, F. (1998). Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis & synthetic analysis. ProQuest Inf Learn, 105, 118–123.
[27] Smarandache, F. (1999). A unifying field in logics: neutrosophic logic. In: Philosophy. American Research Press, 1-141.
[28] Smarandache, F. (2001). A unifying field in logics: Neutrosophic logic, neutrosophic set, neutrosophic probability, and statistics. arXiv preprint math/0101228.
[29] Smarandache, F. (2005). Neutrosophic set a generalization of the intuitionistic fuzzy set. Int J Pure Appl Math, 24(3), 287.
[30] Smarandache, F. (2010). Neutrosophic logic-a generalization of the intuitionistic fuzzy logic. Multispace Multi Struct Neutrosophic Transdiscipl, 4, 396.
[31] Smarandache, F. (2013). Introduction to neutrosophic measure, neutrosophic integral, and neutrosophic probability. In: Infinite study.
[32] Smarandache, F. (2014). Introduction to neutrosophic statistics. In: Infinite Study.
[33] Tahir, Z., Khan, H., Aslam, M., Shabbir, J., Mahmood, Y., & Smarandache, F. (2021). Neutrosophic ratio-type estimators for estimating the population mean. Compl Intell Syst, 7, 2991–3001.
[34] Singh, R. & Mishra, R. (2021). Neutrosophic transformed ratio estimators for estimating finite population mean in sample surveys. Advancements in Sampling Theory & its Applications, 1, 39-47.
[35] Singh, R., Smarandache, F., & Mishra, R. (2023). Generalized robust-type neutrosophic ratio estimators of pharmaceutical daily stock prices. Cognitive Intelligence with Neutrosophic Statistics in Bioinformatics, 417-429.
[36] Vishwakarma, G.K., & Singh, A. (2022). Generalized estimator for computation of population mean under neutrosophic ranked set technique: An application to solar energy data. Computational and Applied Mathematics, 41(4), 144.
[37] Mishra, P., Adichwal, N.K., & Singh, R. (2017). A New Log-Product-Type Estimator Using Auxiliary Information. Journal of Scientific Research, 61, 179-183.

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