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On Neutro Variable Q-subalgebras

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Abstract. This paper presents the unprecedented opinion of neutro Q algebra. Neutro Q algebras have a complex structure and are based on the three axioms of neutro algebras. We investigated the important properties of neutro Q algebras of orders 2 and 3. we try to find a connection between neutro Q algebras and groups and neutro groups. The opinions of very thin neutro Q algebras and valued-strong neutro Q algebras are introduced in this study and are computed in the numbers of 3-strong neutro Q algebras.

Keywords: Neutro Q algebra, very thin neutro Q algebra, valued-strong Q algebra.

1. Introduction

Florentin Smarandache [9], according to the basic realism in the application of mathematical problems in the real world, to the introduction of the theory of neuto algebra the payment. From his point of view, whichever is a real point of view, it was noticed that all the real problems of the world are not based on pure mathematical rules. They look at this issue in such a way that in every finite or infinite set, there are elements that remain unknown or cannot accept or apply some mathematical principles. In classical algebra, a set of elements must apply to a series of specific rules, and this issue is mandatory, and this issue contradicts the real world because, in the real world, there are elements that do not follow any rules or always remain in an unknown state. They stay so the theory of neuto algebra it as it may bea new step in this field so that we know that we don't have to look at the real issues of the world in a pure and forced way. The neutro algebras are very important in the real world and some researchers have investigated these soups such as neutro groups [1], neutro BCK algebras [2], neutro hyper BCK subalgebras [3], on neutro D subalgebras [4], semihypergroups [6], on neutro BE algebras and anti BE algebras [7], CI algebra [8], neutro algebraic structures [10] antialgebraic and semigroups [11].

In this survey, we introduce a new extension of Q algebras, whichever is a generalization of groups. Our goal in presenting this topic is to design the principles of the topic in such a way as to challenge the topic of group theory. Our idea is to suggest that in neutro Q algebras, we can confuse the number of null elements and the number of invertible elements and check that it is not necessary entire elements to be participative. In final, we define the notation of very thin neutro Q algebras and k-strong neutro Q algebras and bring up the relation between groups, neutro groups, and neutro Q algebras.

2. Preliminaries

In what follows, we present the topics that we need in our research.

Definition 2.1. [9] Let $X \neq \emptyset$. Then (X, κ) is a neutro-algebra, if κ be a neutro operation or an operation, whichever is satisfied in the neutro axioms.

Definition 2.2. [5] Let $X \neq \emptyset$, $\kappa : X^2 \to X$ and ι be a steady. Then, (X, κ, ι) is titled a *Q-algebra* if,

(D-1) $\kappa(x, x) = \iota$,
(D-2) $\kappa(x, \iota) = x$,
(D-3) $\kappa(\kappa(x, y), z) = \kappa(\kappa(x, z), y)$.

3. Neutro Q algebras

In this section, characterize neutro Q algebras and inspect their properties.

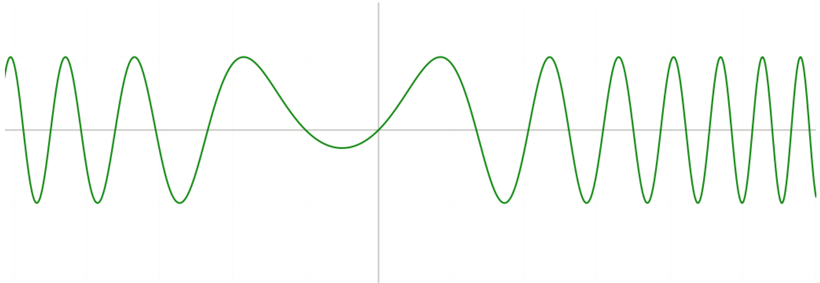
Definition 3.1. Let $X \neq \emptyset$, $\iota \in X$ be a steady and $\kappa : X^2 \to X$. Then $(X, \kappa\ , \iota)$ is a neutro Q algebra, if

(NQ-1) $\big(\exists x \in X$ with the aim that $x\kappa\ x = \iota\big)$ and $\big(\exists y \in X$ with the aim that $y\kappa\ y \neq \iota$ or indeterminate $\big)$;
(NQ-2) $\big(\exists x \in X$ with the aim that $x\kappa\ \iota = x\big)$ and $\big(\exists y \in X$ with the aim that $y\kappa\ \iota \neq y$ or indeterminate $\big)$;
(NQ-3) $\big(\exists x, y, z \in X$, with the aim that $(x\kappa\ y)\kappa\ z = (x\kappa\ z)\kappa\ y\big)$ and $\big(\exists r, s, t \in X$, with the aim that $(r\kappa\ s)\kappa\ t \neq (r\kappa\ t)\kappa\ s$ or indeterminate $\big)$.

Example 3.2. (i) $(\mathbb{R}, -, \iota)$ is a not a neutro Q algebra, while $(\mathbb{R}, \kappa\ , \iota)$ is a neutro Q algebra, where for $x, y \in \mathbb{R}$, characterize $x\kappa\ y = Sin(x + xy)$ as shown in Figure 1.

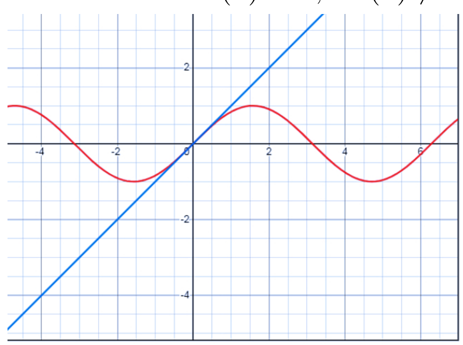
By Figure 1, one can see that there would be $x \in \mathbb{R}$, with the aim that $x\kappa\ x = \iota$ and there would be $y \in \mathbb{R}$, with the aim that $y\kappa\ y \neq \iota$. In addition, for any given $x, y, z \in \mathbb{R}, (x\kappa\ y)\kappa\ z = (x\kappa\ z)\kappa\ y$ if and only if $sin((sin(x+xy)+zsin(x+xy)) = sin((sin(x+xz)+ysin(x+xz))$. It

FIGURE 1. $x\kappa\ x = sin(x + x^2)$



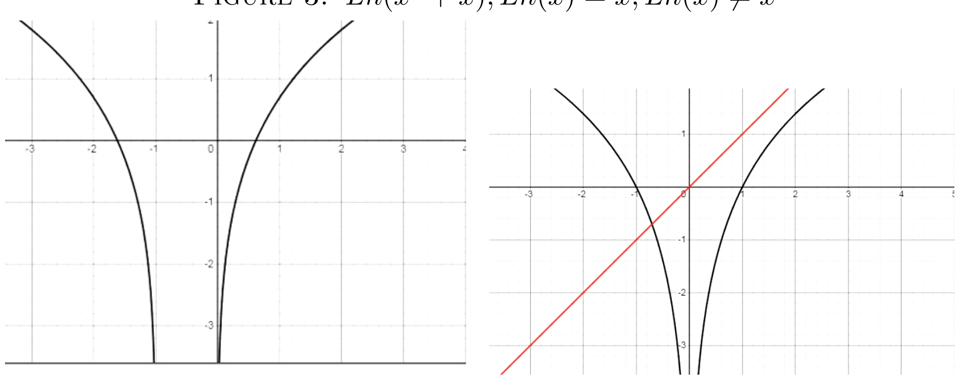
follows that if $z = y$, then $(x\kappa\ y)\kappa\ z = (x\kappa\ z)\kappa\ y$ and one can find $x, y, z \in \mathbb{R}$ with the aim that $(x\kappa\ y)\kappa\ z \neq (x\kappa\ z)\kappa\ y$. In addition, $x\kappa\ \iota = \iota$, implies that $sinx = \iota$, whichever $x = k\pi, k \in \mathbb{Z}$ and $x\kappa\ \iota \neq \iota$, implies that $x \neq k\pi, k \in \mathbb{Z}$. Moreover, $x\kappa\ \iota = x$, implies that $sinx = x$ and $x\kappa\ \iota \neq x$, implies that $sinx \neq x$. The axiom NQ-2 is valid by Figure 2.

FIGURE 2. $sin(x) = x, sin(x) \neq x$



(ii) $(\mathbb{R}, \kappa\ , \iota)$ is a neutro Q algebra, whichever for $x, y \in \mathbb{R}$, characterize $x\kappa\ y = Ln(x^2 + y)$. It is clear that $\iota\kappa\ \iota$ is indeterminate, $x\kappa\ \iota = Ln(x^2)$ and $x\kappa\ x = Ln(x^2 + x)$.

FIGURE 3. $Ln(x^2 + x), Ln(x) = x, Ln(x) \neq x$



By Figure 3, the axioms NQ-1 and NQ-2 are valid. In addition, for $x, y, z \in \mathbb{R}, (x\kappa\ y)\kappa\ z = (x\kappa\ z)\kappa\ y$ iff $Ln(Ln^2(x^2+y)+z) = Ln(Ln^2(x^2+z)+z)$. It is clear that for $y = z$, we have $(x\kappa\ y)\kappa\ z = (x\kappa\ z)\kappa\ y$ and for $y = z$, we have $(x\kappa\ y)\kappa\ z \neq (x\kappa\ z)\kappa\ y$ or indeterminate.

(iii) Let $X = \{\iota, 1, 2, 3, 4, 5\}$. Characterize κ on X in kind:

κ	ι	1	2	3	4	5
ι	ι	ι	ι	ι	ι	5
1	1	ι	1	1	1	5
2	2	2	ι	2	2	3
3	3	3	3	ι	3	ι
4	4	4	4	4	ι	1
5	ι	2	ι	2	ι	5

Computations show that $(\{\iota, 1, 2, 3, 4\}, \kappa\ )$ is a Q algebra and $(\{\iota, 1, 2, 3, 4, 5\}, \kappa\ )$ is a neutro Q algebra.

Theorem 3.3. *Any Q algebra, as it may be lengthen to a neutro Q algebra.*

Proof. Let $(X, \kappa\ , \iota)$ be a Q algebra and $\alpha \notin X$. Then $(X \cup \{\alpha\}, \kappa\ ', \iota)$ is a neutro Q algebra, whichever entire $x, y \in X, x\kappa\ 'y = x\kappa\ y$, $\alpha\kappa\ '\alpha = \alpha, \alpha\kappa\ '\iota = x$, whichever $x \notin \{\iota, \alpha\}$ and $x\kappa\ '\alpha \neq x$. Since $(\alpha\kappa\ '\iota)\kappa\ '\alpha = x\kappa\ '\alpha$, $(\alpha\kappa\ '\alpha)\kappa\ '\iota = \alpha\kappa\ '\iota = x$ and $x\kappa\ '\alpha \neq x$, we acquire that $(\alpha\kappa\ '\iota)\kappa\ '\alpha \neq (\alpha\kappa\ '\alpha)\kappa\ '\iota$. Hence $(X \cup \{\alpha\}, \kappa\ ', \iota)$ is a neutro Q algebra. □

Theorem 3.4. *Let $|X| = 2$, $\iota \in X$ and $(X, \kappa\ , \iota)$ is a neutro Q algebra.*

(i) *If $\iota\kappa\ \iota = \iota$, then exists $x \in X$ with the aim that $\iota\kappa\ x = x$ or is indeterminate.*

(ii) *If $\iota\kappa\ \iota \neq \iota$, then one can find $x \in X$ with the aim that $\iota\kappa\ x \in \{\iota, x\}$ or is indeterminate.*

Proof. (i) Since, $\iota \in X$, by NQ-1 one can find $x \in X$, with the aim that $x\kappa\ x = \iota$ and one can find $y \in X$ with the aim that $y\kappa\ y \neq \iota$ or is indeterminate. Assume that $\iota\kappa\ \iota = \iota$. In this case one can find just one $\iota \neq x \in X$ with the aim that $x\kappa\ x \neq \iota$ or is indeterminate. Since $|X| = 2$, we acquire $x\kappa\ x = x$ or is indeterminate. Moreover, by NQ-2, one can find $x \in X$ with the aim that $x\kappa\ \iota \neq x$, because of $\iota\kappa\ \iota = \iota$. Since $|X| = 2$, we acquire $x\kappa\ \iota = \iota$ or is indeterminate. In addition by NQ-3, $\iota = \iota\kappa\ \iota = (\iota\kappa\ \iota)\kappa\ \iota = (\iota\kappa\ \iota)\kappa\ \iota = \iota$, because of $\iota\kappa\ \iota = \iota$. Now bring up the consecutive cases:

Case 1:

$$(x\kappa\ x)\kappa\ \iota = (x\kappa\ \iota)\kappa\ x \Rightarrow x\kappa\ \iota = \iota\kappa\ x \Rightarrow \iota\kappa\ x = \iota \text{ or is indeterminate.}$$

Case 2:

$$(\iota\kappa\ x)\kappa\ \iota = (\iota\kappa\ \iota)\kappa\ x \Rightarrow (\iota\kappa\ x)\kappa\ \iota = \iota\kappa\ x \text{ or is indeterminate.}$$

In this case, if $\iota\kappa\ x = \iota$, then $\iota = \iota\kappa\ \iota = \iota\kappa\ x$ and so $\iota\kappa\ x = \iota$ or is indeterminate. If $\iota\kappa\ x = x$, then $x\kappa\ \iota = \iota\kappa\ x$ and so $\iota\kappa\ x = \iota$ or is indeterminate. They follow that NQ-3 is not valid and it is a contradication. Hence we must bring up the consecutive cases:

Case 1:

$$(x\kappa\ x)\kappa\ x \neq (x\kappa\ \iota)\kappa\ x \Rightarrow x\kappa\ \iota \neq \iota\kappa\ x \Rightarrow \iota\kappa\ x \neq \iota \Rightarrow \iota\kappa\ x = x \text{ or is indeterminate.}$$

Case 2:

$$(\iota\kappa\ x)\kappa\ \iota \neq (\iota\kappa\ \iota)\kappa\ x \Rightarrow (\iota\kappa\ x)\kappa\ \iota \neq \iota\kappa\ x \text{ or is indeterminate.}$$

In this case, if $\iota\kappa\ x = \iota$, then $\iota = \iota\kappa\ \iota \neq \iota\kappa\ x$ and so $\iota\kappa\ x \neq \iota$ or ($\iota\kappa\ x = x$ or is indeterminate). If $\iota\kappa\ x = x$, then $x\kappa\ \iota \neq \iota\kappa\ x$ and so $\iota\kappa\ x \neq \iota$ or ($\iota\kappa\ x = x$ or is indeterminate). They concludee that $\iota\kappa\ x = x$ or is indeterminate.

(ii) Since $|X| = 2$ and $\iota\kappa\ \iota \neq \iota$, we acquire there would be just one $\iota \neq x \in X$ such that $\iota\kappa\ \iota = x$ and so $x\kappa\ x = \iota$ or is indeterminate. Moreover, by NQ-2, there would be $x \in X$ with the aim that $x\kappa\ \iota = x$, because of $\iota\kappa\ \iota \neq \iota$. Since $|X| = 2$, we acquire $x\kappa\ \iota = x$ or is indeterminate. In addition by NQ-3, $(\iota\kappa\ \iota)\kappa\ \iota = x\kappa\ \iota = x$, because of $\iota\kappa\ \iota = x$. Now, bring up the consecutive cases:

Case 1:

$$(x\kappa\ x)\kappa\ \iota = (x\kappa\ \iota)\kappa\ x \Rightarrow \iota\kappa\ \iota = x\kappa\ x \Rightarrow x = \iota, \text{whichever is contradication.}$$

Case 2:

$$(\iota\kappa\ x)\kappa\ \iota = (\iota\kappa\ \iota)\kappa\ x \Rightarrow (\iota\kappa\ x)\kappa\ \iota = x\kappa\ x \Rightarrow (\iota\kappa\ x)\kappa\ \iota = \iota \text{ or is indeterminate.}$$

In this case, if $\iota\kappa\ x = \iota$, then $\iota\kappa\ \iota = \iota$, whichever is a contradication. If $\iota\kappa\ x = x$, then $x\kappa\ \iota = \iota$, whichever is a contradication. Thus, $(x\kappa\ x)\kappa\ \iota \neq (x\kappa\ \iota)\kappa\ x$ and $(\iota\kappa\ x)\kappa\ \iota \neq (\iota\kappa\ \iota)\kappa\ x$. It concludes that $\iota\kappa\ x \in \{\iota, x\}$ or is indeterminate. □

Corollary 3.5. *Let X be a set and $(X, \kappa\ , \iota)$ be a neutro Q algebra. Then $|X| \geq 2$.*

Theorem 3.6. *Let $(X, \kappa\ )$ be a neutro Q algebra. Then there would be $x, y \in X$ with the aim that $x\kappa\ (x\kappa\ y) \neq x\kappa\ y$ or indeterminate.*

Proof. Let entire $x, y \in X, x\kappa\ (x\kappa\ y) = x\kappa\ y$. Then by $x = y$, we acquire $x\kappa\ (x\kappa\ x) = x\kappa\ x$. Since $(X, \kappa\ )$ is a neutro Q algebra, using NQ-2, we obtain that $x = x\kappa\ \iota = x\kappa\ (x\kappa\ x) = x\kappa\ x = \iota$. It follows that $|X| = 1$, whichever is a contradication by Corollary 3.5. □

Theorem 3.7. *Let $(X, \kappa\ , \iota)$ be a neutro Q algebra.*

(i) *If there would be $x, y \in X$ with the aim that $x\kappa\ (x\kappa\ y) \neq x\kappa\ y$ or indeterminate and $\iota\kappa\ \iota = \iota$, then $|X| \geq 3$.*

(*ii*) *If one can find* $x, y \in X$ *with the aim that* $x\kappa\ (x\kappa\ y) = x\kappa\ y$ *or indeterminate and* $\iota\kappa\ \iota \neq \iota$, *then* $|X| \geq 3$.

Proof. Since $(X, \kappa\ )$ is a neutro Q algebra, by Corollary 3.5, we acquire $|X| \geq 2$. Suppose that $|X| = 2$ and $X = \{\iota, x\}$.

(*i*) Because $\iota\kappa\ \iota = \iota$, we acquire $x\kappa\ x = x$ and $x\kappa\ \iota = \iota$. It follows that $x\kappa\ (x\kappa\ x) \neq x\kappa\ x$.Thus $x\kappa\ x \neq x\kappa\ x$, whichever is a contradication and so $|X| \geq 3$.

(*ii*) Because $\iota\kappa\ \iota \neq \iota$, we acquire $x\kappa\ x = \iota$ and $x\kappa\ \iota = x$. It follows that $x\kappa\ (x\kappa\ x) = x\kappa\ x$.Thus $x\kappa\ \iota = \iota$ and so $x = \iota$, whichever is a contradication and so $|X| \geq 3$. □

Definition 3.8. Let $(X, \kappa\ , \iota)$ be a neutro Q algebra. Then we will call it is a neutro-commutative, if $\big(\exists x, y \in X$ with the aim that $x\kappa\ y = y\kappa\ x\big)$ and $\big(\exists r, s \in X$ with the aim that $r\kappa\ s \neq s\kappa\ r$ or indeterminate $\big)$.

Theorem 3.9. *Let* $(X, \kappa\ , \iota)$ *be a neutro* Q *algebra and* $|X| = 2$.

(*i*) *If* $\iota\kappa\ \iota = \iota$, *then* $(X, \kappa\ , \iota)$ *is neutro-commutative.*

(*ii*) *If* $\iota\kappa\ \iota \neq \iota$, *then* $(X, \kappa\ , \iota)$ *is not neutro-commutative, necessarily.*

(*iii*) *If* $\iota\kappa\ \iota \neq \iota$ *and* $\iota\kappa\ x = \iota$, *then* $(X, \kappa\ , \iota)$ *is neutro-commutative.*

Proof. Let $x \in X$.

(*i*) If $\iota\kappa\ \iota = \iota$, then by Theorem 3.4, we acquire $\iota\kappa\ x = x$ and $x\kappa\ \iota = \iota$.

(*ii, iii*) If $\iota\kappa\ \iota = x$, then by Theorem 3.4, we acquire $x\kappa\ \iota = x$ and $\iota\kappa\ x \in \{\iota, x\}$. Now, if $\iota\kappa\ x = x$, $(X, \kappa\ , \iota)$, then is not neutro-commutative and if $\iota\kappa\ x = \iota$, then $(X, \kappa\ , \iota)$ is neutro-commutative. □

Theorem 3.10. *Let* X *be nonevoid set,* $\iota \in X$ *and* $|X| \geq 2$. *Then* $(X, \kappa\ , \iota)$ *is a neutro* Q *algebra iff* $(X, \kappa\ , \iota)$ *is not a group.*

Proof. Let $(X, \kappa\ , \iota)$ is a group. Then for any $x \in X, x\kappa\ \iota = x$, and so the axiom ($NQ$-2) is not valid. Thus $(X, \kappa\ , \iota)$ is a not neutro Q algebra.

Conversly, let $(X, \kappa\ , \iota)$ is a neutro Q algebra. Then using the axiom (NQ-2), the structure $(X, \kappa\ , \iota)$ has't the identity elemen, so $(X, \kappa\ , \iota)$ is not a group. □

Theorem 3.11. *Every group with involution, as it may be lengthen to a neutro* Q *algebra.*

Proof. Let $(X, \kappa\ , \iota)$ be a group with involution and $\alpha \notin X$. Then $(X \cup \{\alpha\}, \kappa\ ', \iota)$ is a neutro Q algebra, whichever entire $x, y \in X, x\kappa\ 'y = x\kappa\ y$, $\alpha\kappa\ 'x \neq x\kappa\ '\alpha$ and $\alpha\kappa\ '\iota$ is indeterminate. Suppose that $z \in X$ is an involution. Then $z\kappa\ '(\alpha\kappa\ 'z) = (z\kappa\ '\alpha)\kappa\ 'z = (z\kappa\ 'z)\kappa\ '\alpha = \iota\kappa\ '\alpha = \alpha$. It follows that $z\kappa\ '(z\kappa\ '(\alpha\kappa\ 'z)) = z\kappa\ '\alpha$ and so $(z\kappa\ 'z)\kappa\ '(\alpha\kappa\ 'z) = z\kappa\ '\alpha$, whichever is a contadication. Thus one can find $\alpha, z \in X$ with the aim that $(z\kappa\ '\alpha)\kappa\ 'z \neq (z\kappa\ 'z)\kappa\ '\alpha$ and so the axiom NQ-3 is valid. In addition, $z\kappa\ 'z = \iota$, implies that NQ-1 is valid Hence $(X \cup \{\alpha\}, \kappa\ ', \iota)$ is a neutro Q algebra. □

Example 3.12. Bring up the abelian group $(\mathbb{Z}_6, \oplus)$ and $X = \{\bar{\iota}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \sqrt{2}\}$. For every $x, y \in \mathbb{Z}_6$, characterize $x\kappa\ y = x \oplus y$ and $\sqrt{2}\kappa\ \bar{\iota}$ is an indeterminate. It is easey to see that $(X, \kappa\ , \bar{\iota})$ is a neutro Q algebra.

S. Dus et al. introduced the opinion of neutro groups. We recall this consept in kind:

Let X be a non-empty set, $\iota \in X$ be a steady and "κ " be a map on X. An algebra $(X, \kappa\ , \iota)$ of type $(2, \iota)$ is a neutro group, if

(NG-1) $\big(\exists x, y, z \in X$ with the aim that $(x\kappa\ y)\kappa\ z = x\kappa\ (y\kappa\ z)\big)$ and $\big(\exists r, s, t \in X$ with the aim that $(r\kappa\ s)\kappa\ t \neq r\kappa\ (s\kappa\ t)$ or indeterminate $\big)$;

(NG-2) $\big(\exists x \in X$ with the aim that $x\kappa\ \iota = \iota\kappa\ x = x\big)$ and $\big(\exists y \in X$ with the aim that $y\kappa\ \iota \neq y$ or indeterminate $\big)$;

(NG-3) $\big(\exists x, y \in X$, with the aim that $x\kappa\ y = y\kappa\ x = \iota\big)$ and $\big(\exists r \in X$, with the aim that entire $s \in X$, $r\kappa\ s \neq \iota$ or indeterminate $\big)$.

Each above neutro axiom has a degree of equality (T), degree of non-equality (F), and degree of indeterminacy (I), where $(T, I, F) \notin (1, \iota, \iota), (\iota, \iota, 1)$.

Theorem 3.13. *Let $(X, \kappa\ , \iota)$ be a neutro Q algebra whichever satisfies in (NG-1). Then $(X, \kappa\ , \iota)$ is a neutro group.*

Proof. Since $(X, \kappa\ , \iota)$ is a neutro Q algebra, one can find $x \in X$ with the aim that $x\kappa\ \iota = \iota$ and one can find $y \in X$ with the aim that $y\kappa\ y = \iota$. Using (NG-1), we acquire $\iota\kappa\ x = (y\kappa\ y)\kappa\ x = y\kappa\ (y\kappa\ x) = x\kappa\ \iota = \iota$ and so $\iota\kappa\ x = \iota$. It follows that one can find $x \in X$ with the aim that $x\kappa\ \iota = \iota = \iota\kappa\ x$. □

Theorem 3.14. *Let $(X, \kappa\ , \iota)$ be a neutro Q algebra and $|X| = 2$. Then*

(i) if $\iota\kappa\ \iota = \iota$, then $(X, \kappa\ , \iota)$ is not a neutro group.

(ii) if $\iota\kappa\ \iota \neq \iota$ and $\iota\kappa\ x = x$, then $(X, \kappa\ , \iota)$ is a neutro group.

(iii) if $\iota\kappa\ \iota \neq \iota$ and $\iota\kappa\ x = \iota$, then $(X, \kappa\ , \iota)$ is not a neutro group.

Proof. (i) Since $\iota\kappa\ \iota = \iota$, by Theorem 3.4, for $x \in X$, we acquire $x\kappa\ x = x, x\kappa\ \iota = \iota$, and $\iota\kappa\ x = x$. It follows that the axioms NG-1 and NG-2 are valid. Let $x \in X$, then have the consecutive cases:

Case 1:

$$(x\kappa\ \iota)\kappa\ x = \iota\kappa\ x = x = x\kappa\ x = x\kappa\ (\iota\kappa\ x).$$

Case 2:

$$(x\kappa\ x)\kappa\ \iota = x\kappa\ \iota = x\kappa\ (x\kappa\ \iota).$$

Case 3:

$$(\iota\kappa\ x)\kappa\ x = x\kappa\ x = x = \iota\kappa\ x = \iota\kappa\ (x\kappa\ x).$$

Case 4:

$$(\iota\kappa\ \iota)\kappa\ x = \iota\kappa\ x = x = \iota\kappa\ x = \iota\kappa\ (\iota\kappa\ x).$$

Case 5:

$$(\iota\kappa\ x)\kappa\ \iota = x\kappa\ \iota = \iota = \iota\kappa\ \iota = \iota\kappa\ (x\kappa\ \iota).$$

They follow that the axiom NG-3 is not valid and so $(X, \kappa\ , \iota)$ is not a neutro group.

(ii) Let $\iota\kappa\ \iota \neq \iota$. Using theorem 3.4, for $x \in X$, acquire $x\kappa\ x = \iota, x\kappa\ \iota = x$ and $\iota\kappa\ x \in \{\iota, x\}$. If $\iota\kappa\ x = x$, then the axioms NG-2 and NG-3 are valid. Now, bring up the consecutive cases:

Case 1:

$$(x\kappa\ \iota)\kappa\ x = x\kappa\ x = \iota = x\kappa\ x = x\kappa\ (\iota\kappa\ x).$$

Case 2:

$$(x\kappa\ x)\kappa\ \iota = \iota\kappa\ \iota = x \neq \iota = x\kappa\ x = x\kappa\ (x\kappa\ \iota).$$

They follow that NG-1 is valid and so $(X, \kappa\ , \iota)$ is a neutro group.

(iii) Let $\iota\kappa\ \iota \neq \iota$. Using theorem 3.4, for $x \in X$, acquire $x\kappa\ x = \iota, x\kappa\ \iota = x$ and $\iota\kappa\ x \in \{\iota, x\}$. If $\iota\kappa\ x = \iota$, then the axiom NG-2 is not valid and so $(X, \kappa\ , \iota)$ is not a neutro group. □

Definition 3.15. Let $(X, \kappa\ , \iota)$ be a neutro Q algebra. We say that $(X, \kappa\ , \iota)$ is a very thin neutro Q algebra, if for any $\iota \neq x \in X$, $\iota\kappa\ \iota \neq \iota, x\kappa\ x = \iota$ and $x\kappa\ \iota = x$.

Example 3.16. Let $X = \{\iota, 1, 2, 3, 4, 5\}$. Then (X, κ_1, ι) is a very thin neutro Q algebra and (X, κ_2, ι) is a very strong neutro Q algebra in kind:

κ_1	ι	1	2	3	4	5
ι	1	ι	ι	ι	ι	ι
1	1	ι	1	1	1	1
2	2	2	ι	2	2	2
3	3	3	3	ι	3	3
4	4	4	4	4	ι	4
5	5	5	5	5	5	ι

and

κ_2	ι	1	2	3	4	5
ι	ι	ι	ι	ι	ι	ι
1	5	5	1	1	1	1
2	4	2	5	2	2	2
3	2	3	3	5	3	3
4	3	4	4	4	5	4
5	1	5	5	5	5	5

Theorem 3.17. *Let $(X, \kappa\ , \iota)$ be a neutro commutative very thin neutro Q algebra and $|X| \geq 3$. Then $(X, \kappa\ , \iota)$ is a neutro group.*

Proof. Let $x \in X$. Since $(X, \kappa\ , \iota)$ is a very thin neutro Q algebra, acquire entire $\iota \neq x \in X, x\kappa\ x = \iota$ and $x\kappa\ \iota = x$. Since $(X, \kappa\ , \iota)$ is neutro commutative, one can find $y \in X$ with the aim that $\iota\kappa\ y \neq y$. They follow that NG-2 and NG-3 are valid. Because $(x\kappa\ \iota)\kappa\ x = x\kappa\ x = \iota = x\kappa\ x = x\kappa\ (\iota\kappa\ x)$ and $(x\kappa\ x)\kappa\ \iota = \iota\kappa\ \iota \neq \iota = x\kappa\ x = x\kappa\ (x\kappa\ \iota)$, acquire NG-1 is valid. Hence $(X, \kappa\ , \iota)$ is a neutro group. □

Definition 3.18. Let $(X, \kappa\ , \iota)$ be a neutro Q algebra and $k \in \mathbb{N}$. We say that $(X, \kappa\ , \iota)$ is k-strong neutro Q algebra, if one can find $x_1, x_2, \ldots, x_{k-1} \in X$ with the aim that $\iota\kappa\ \iota = x_i\kappa\ x_i = \iota, x_i\kappa\ \iota = \iota\kappa\ x_i = x_i$, implies that $i \in \{1, 2, \ldots, k-1\}$ and for any $i \notin \{1, 2, \ldots, k-1\}, x_i\kappa\ x_i \neq \iota, x_i\kappa\ \iota \neq x_i, \iota\kappa\ x_i \neq x_i$ and are't indeterminate.

Let X be a set and $k \in \mathbb{N}$. Denote $\mathcal{N}(X, Q, k)$ by the set of all k-strong neutro Q algebras $(X, \kappa\ , \iota)$ and $\mathcal{N}(X, G, k)$ by the set of all k-strong neutro groups $(X, \kappa\ , \iota)$.

Theorem 3.19. *Let* $|X| = 3$. *Then* $|\mathcal{N}(X, Q, 2)| = 2^3 \times 3^2$.

Proof. Suppose that $X = \{\iota, a, b\}$. If $(X, \kappa\ , \iota)$ is a 2-strong neutro Q algebras, we have $\iota\kappa\ \iota = a\kappa\ a = \iota$ and $a\kappa\ \iota = \iota\kappa\ a = a$. Noe, characterize a general kayley table in kind:

$\kappa_{\ i}$	ι	a	b
ι	ι	a	$c(\iota, b)$
a	a	ι	$c(a, b)$
b	$c(b, \iota)$	$c(b, a)$	$c(b, b)$

,

whichever for any $x, y \in X, c(x, y)$ is the number of possible cases for choosing of the $x\kappa\ _i y$. Simple cimputations show that $c(\iota, b) = 2, c(a, b) = 3, c(b, \iota) = 2, c(b, a) = 3$ and $c(b, b) = 2$. Thus $|\mathcal{N}(X, Q, 2)| = 2^3 \times 3^2$. □

Corollary 3.20. *Let* $X \neq \emptyset$. *Then* $\mathcal{N}(X, G, 3) \subseteq \mathcal{N}(X, Q, 3)$.

4. Conclusions

As an important result of this article, we can mention that we have dealt with the connection of the real world of objects with each other under a series of imagined principles of logic and we have shown that even for a finite number of elements we can create an infinite system of rules. Since the structure of neutro Q subalgebra is complex, we investigated the neutro Q subalgebras with order $2, 3$. We show that the Q subalgebras can't be groups and try to make some conditions to be neutro groups. After completing this research, which is an interesting start in the field of neutro algebras, we intend to discuss the relationship of sets of elements with other sets in a real way and under the principles of neutro. In fact, we want to apply these types of algebras in the real world, and by modeling compatible and non-compatible systems with these types of algebras, we want to show the importance of publishing these types of articles.

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