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Novel Single Valued Neutrosophic Prioritized Aczel Alsina Aggregation Operators and Their Applications in Multi-Attribute Decision Making

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Abstract: The Multi-attribute decision-making (MADM) approaches are utilized to aggregate ambiguous and imprecise information based on different aggregation operators (AOs). The aim of this article is to explore the notion of single-valued neutrosophic (SVN) set (SVNS), wich is the modified structure of an intuitionistic fuzzy sets and picture fuzzy sets. Some appropriate operations of Aczel Alsina tools under the system of SVN information are also presented. By using the theory of prioritization aggregation model, we developed a class of new approaches including SVN Aczel Alsina prioritized average (SVNAAPA) and SVN Aczel Alsina prioritized geometric (SVNAAPG) operators. We also presented a series of new methodologies in the light of SVN information such as SVN Aczel Alsina prioritized weighted average (SVNAAPWA), and SVN Aczel Alsina prioritized weighted geometric (SVNAAPWG) operators. To verify discussed aggregation approaches, we also presented some notabe characteristics. We established a MADM technique to solve complexities and difficulties during decision-making in our real-life problems. By utilizing a practical numerical example to select an appropriate research scientist for the vacant post of a public university. To find the validity and flexibility of our invented approaches, sensitive analysis, and comparative study by comparing the results of existing approaches with currently proposed aggregation techniques.

Keywords: Neutrosophic values, Single valued neutrosophic values, Aczel Alsina Aggregation operators, and Multi-attribute decision-making approach.

1. Introduction

In order to choose the optimal option based on a set of criteria, decision-making is a common and daily activity in human existence. The last several years have seen extensive research and useful decision-making applications to management, economics, and other fields because of its outstanding ability to express information uncertainty. Fuzzy set theory has become more common in recent years as a way to resolve decision-making issues due to the uncertainty of decision data. Zadeh [1] anticipated the fuzzy set (FS) concepts, which have gained popularity among intellectuals. In order to deal with uncertain conditions, numerous theoretical advancements in FS have been made to date. However, in many circumstances, the notion of FS is effective. For instance, the FS theory is unable to deal with the knowledge supplied to a person in the form of positive membership value (PMV) and negative membership value (NMV). To address these issues, Atanassov [2] created the intuitionistic FS (IFS) theory by incorporating the concept of PMV into the drawbacks of FS. The limitation that the addition of the PMV and NMV is between $[0, 1]$ makes IFS significantly more useful than the current FS. When dealing with complex and unreliable data in decision-making scenarios, IFS is a thorough and powerful strategy. Many researchers have used IFS theory in a variety of fields [3], [4]. However, the IFS cannot handle such values if the sum of them exceeds the unit interval $[0, 1]$. To address such difficulties, Yager [5] investigated Pythagorean FS (PyFS). The PyFS is more effective for tackling complex, unreliable information in real-life situations. Yager [6] also modified and explored the theory of the PyFS in the framework of q-rung orthopair FS (q-ROFS), with the additional limitation that the sum of the qth power of PMV and NMV cannot be more than the unit interval $[0,1]$. The concepts of q-ROFS have received much use and have received more interest from researchers because of their structure. Many authors have applied the q-ROFS theory in ways that have been detrimental to a number of areas. Cuong [7], [8] extended the concepts of IFS, PyFS and q-ROFS using the characteristics PMV, abstinence membership value (AMV), and NMV such that the sum of PMV, AMV, and NMV restricted on interval $[0, 1]$. In order to convey ambiguous and conflicting data, Smarandache [9] anticipated the neutrosophic set (NS). A NS having PMV, AMV, and NMV are separately represented and lies in real standard or nonstandard subsets of $]^{-}0, 1^{+}[$. We may have faced many difficulties when we explored the results in nonstandard close intervals. In order to overcome this complexity, Wang et al. [10] gave the concepts of SVNS and provided the idea of interval NS [11]. Ye [12] explored the work of IFS, PFS, and NS using the system of simplified NSs to deal effectively with uncertain and inaccurate data during the decision-making process. Many research scientists explored the concepts of NS, and SVNS in the different fuzzy environments [13]–[15].

The AOs are reliable and convenient mathematical tools to easily handle inaccurate and uncertain information during aggregation. Due to the significance of AOs, several research scientists worked on different fuzzy environments. Xu [16] explored the idea of arithmetic and geometric tools using the framework of weighted averaging and geometric operators depending on IFS. Rahman et al. [17] gave some AOs of PyFSs by using the concepts of algebraic sum and algebraic product to handle imprecision information. Jan et al. [18] explored the notions of PyFS by applying the interval-valued PyFS (IVPyFS) structure to cope with ambiguous and uncertain information. Liu and Wang [19] presented AOs of q-ROFSs to solve real-life problems under a MADM approach. Garg [20] expanded the theory of IFS using the way of PFSs and anticipated some innovative AOs to handle the complexities of the fuzziness. Riaz and Farid [21] explored the theory of PFSs to handle unpredictable and imprecision information during the decision-making process by developing certain approaches. Jdid et al. [22] proposed a strong mechanism for checking the qualities of final products and developed some new mathematical approaches for the inspection of goods under their cost and benefits. A novel approach for the improvement of the sustainability and resilience of supply chain enterprises based on the theory of industry 5.0 was presented by Gamal et al. [23]. This theory has a great capability to provide strong decision under considering the decision-making process. Riaz and Hashmi [24] extended the ideology of FSs regarding Linear Diophantine FS to introduce some valuable AOs on the basis of the fundamental operations of PFSs. Liu and Jiang [25] explored the conception of distance measures in the form of interval-valued IFS (IVIFS) and used a number of AOs to deal with real-life problems under the MADM process. Mahnaz et al. [26] present detailed certain approaches to T-SFSs by utilizing the concepts of frank operators to cope with inaccurate and impression information. Ahmad et al. [27] provided some specific approaches of SFSs to deal with real-life issues under MADM techniques. Ali et al. [28] explored the theory of T-SFSs and explored the basic operation of T-SFSs. Ye [29] presented some AOs by using the correlation coefficients tools of SVNSs and interval-valued SVNSs. Wei and Zhang [30] present a few certain methodologies of Bonferroni power operators applying SVN information. Chen and Ye [31] anticipated certain approaches based on Dombi operations under the system of SVNSs. Mahmood and Ali [32] explored the theory of SVNS by utilizing complex SVNS (CSVNS) to develop certain approaches by using mathematical tools like prioritized Muirhead Mean operators. Fan et al. [33] invent a series of certain approaches to SVNS by utilizing innovative linguistic variables for the solution of complicatedness in different fuzziness. Hussain et al. [34] anticipated a series of complex IFS by using the theory of Hamy mean tools and established an application in the tourism industry. Garg [35] explored the theory of SVNS to overcome the loss of information during the aggregation process with the help of certain mathematical tools like Frank operators. Liu et al. [36] elaborated the theories of NS to develop a series of certain approaches to cope with vague and impression information in the fuzzy environment. Akram et al. [37] elaborated the structure of energy cell under the system of interval valued T-SFSs to develop certain models of Bonferroni mean operators. Ali Khan et al. [38] gave a series of certain approaches of PyFSs based on prioritized mathematical tools to express the ambiguous and vague information.

Aczel and Alsina [39] explored the theories of t-norm (TNM) and t-conorm (TCNM) to develop an innovative idea for Aczel Alsina tools in 1982. Farahbod and Eftekhari [40] compared other TNMs and TCNMs to evaluate and categorize more reliable TNMs and TCNMs after investigation. Recently several research scientists worked on different fuzzy environments to cope with uncertain and imprecise information. Senapati et al. [41] explored the idea of IFSs to establish a list of certain approaches by using the basic operations of Aczel Alsina tools to deal with real-life problems under a MADM approach. Senapati et al. [42] also utilized the basic operations of Aczel Alsina tools to develop a few certain approaches based on IVIFSs. Hussain et al. [43] generalized the concept of Aczel Alsina tools in framework of PyFSs and gave a series of certain approaches to aggregate ambiguous and uncertain information. Khan et al. [44] generalized the structure of q-ROFS and anticipated a series of certain approaches by using the basic operations of Aczel Alsina tools. Naeem et al. [45] expanded the concept of PFSs and anticipated detailed certain approaches by using basic operations of Aczel Alsina tools. Mahmood et al. [46] explored the meanings of IFS in terms of complex IFS to introduce a list of certain approaches using the basic operations of Aczel Alsina tools. Several research scientists also conceptualized the ideas of Aczel Alsina tools in different fuzzy environments seen in the references [47]–[49].

In order to handle vague information, we studied several aggregation models under considering different fuzzy circumstances. Sometimes decision-makers cannot approach an appropriate optimal option due to insufficient information on human opinions. To serve this purpose, the SVNS is a well-known aggregation model that provides the decision maker freedom in their decisions. Aczel Alsina aggregation tools have an attractive aggregation tool and play an essential role in the decision-making process. By inspiring the theory of prioritization and Aczel Alsian aggregation tools, we explored the theory of SVNSs. The main contribution of our research work is established as follows:

a) To expose the theory of SVNS with some specific properties.
b) To explore operations of Aczel Alsina aggregation tools under considering the system of SVN information.
c) By utilizing the degree of preferences of the attributes, we developed a class of new approaches based on Aczel Alsina aggregation models such as SVNAAPA and SVNAAPG operators.
d) We also proposed a series of new methodologies under the system of SVNS, including SVNAAPWA and SVNAAPWG operators.
e) To illustrate the applicability and effectiveness of our invented approaches, some notable characteristics are also demonstrated.
f) We established an algorithm of the MADM technique to resolve several real-life applications.
g) We gave a practical numerical example to find a suitable candidate for the vacant post of general manager for a multinational company. To find validity and flexibility of our invented approaches, we discussed sensitive analysis and comparative study by contrasting the results of existing approaches.
h) Additionally, some remarkable points related to our research work are expressed in the conclusion.

The structure of this manuscript is given as follows: In section 2, we studied the notion of SVNSs and its primary operations. In section 3, we revised the concepts of prioritized AOs based on SVNVs and some existing AOs based on SVNVs. In section 4, we improved the fundamental OLs of SVNVs based on Aczel Alsina operations. Section 5 listed certain approaches of SVNAAPA and SVNAAPG operators based on Aczel Alsina operations. In section 6, we anticipated the AOs of SVNAAPWA and SVNAAPWG operators with the help of weight vectors based on Aczel Alsina operations. In section 7, we evaluate a MADM technique to select a suitable research scientist by utilizing the SVNAAPWA and SVNAAPWG operators for a public university and observe the effects on the results of alternatives for different parametric values. In section 8, find the validity and reliability of our discussed approaches by contrasting the outcomes of current AOs with the result of our invented approaches. In section 9, the entire article was condensed into one paragraph and discussed the advantages of our research work.

2. Preliminaries

This section will study the basic definition of the neutrosophic set (NS) and single-valued neutrosophic set (SVNS). We also study some fundamental OLs of SVN value (SVNV) for further development of this article. We also provide a list of all abbreviations in Table 1.

Definition 1: [9] Let X be a be a non-empty set and a NS A in X is characterized by positive membership value (PMV), abstinence membership value (AMV) and negative membership value (NMV).Then, all the terms of membership are restricted in such intervals, $\varphi_A(r) \in]0^-, 1^+[, \delta_A(r) \in]0^-, 1^+[$ and $\sigma_A(r) \in]0^-, 1^+[$.
Where a PMV is denoted by $\varphi_A(r)$, AMV is denoted by $\delta_A(r)$ and a NMV is denoted by $\sigma_A(r)$.

Table 1 shows abbreviations and their meanings.

Abbreviations	Meanings	Abbreviations	Meanings
MADM	Multi-attribute decision making	NMV	negative membership value
OLs	Operational laws	AMV	abstinence membership value
NS	neutrosophic set	FS	Fuzzy set
SVNS	Single-valued neutrosophic set	IFS	Intuitionistic fuzzy set
SVNV	Single-valued neutrosophic value	PyFS	Pythagorean fuzzy set
AOs	Aggregation operators	q-ROFS	q-rung orthopair fuzzy set
NMV	positive membership value	PFS	Picture fuzzy set
TNM	t-norm	TCNM	t-conorm
SVNAAPA	Single valued neutrosophic Aczel Alsina prioritized average.		
SVNAAPG	Single-valued neutrosophic Aczel Alsina prioritized geometric.		
SVNAAPWA	Single valued neutrosophic Aczel Alsina prioritized weighted average.		
SVNAAPWG	Single-valued neutrosophic Aczel Alsina prioritized weighted geometric.		

Definition 2: [10] A SVNS A is defined as:

$$A = \{(r, \varphi_A(r), \delta_A(r), \sigma_A(r)) | r \in X\}$$

Where $\varphi_A(r): X \to [0,1]$, $\delta_A(r): X \to [0,1]$ and $\sigma_A(r): X \to [0,1]$ represent the PMV, AMV, and NMV, respectively. A SVNS satisfies such condition:

$$0 \le \varphi_A(r) + \delta_A(r) + \sigma_A(r) \le 3$$

A SVNV is denoted by the $\alpha = (\varphi_\alpha, \delta_\alpha, \sigma_\alpha)$.

Definition 3: [10] Let $\alpha = (\varphi_\alpha, \delta_\alpha, \sigma_\alpha)$ be a SVNV. Then, a score function $\mathcal{G}(\alpha)$ can be particularized as:

$$\mathcal{G}(\alpha) = \frac{2 + (\varphi_\alpha - \delta_\alpha - \sigma_\alpha)}{3} \quad (1)$$

Here, $\mathcal{G}(\alpha) \in [0,1]$.

Definition 4: [10] Let $\alpha = (\varphi_\alpha, \delta_\alpha, \sigma_\alpha)$ be a SVNV. Then, an accuracy function $\mathfrak{Q}(\alpha)$ can be particularized as:

$$\mathfrak{Q}(\alpha) = \varphi_\alpha + \sigma_\alpha \quad (2)$$

Here, $\mathfrak{Q}(\alpha) \in [0,1]$.

Definition 5: [10] Let $\alpha = (\varphi_\alpha, \delta_\alpha, \sigma_\alpha)$ and $\beta = (\varphi_\beta, \delta_\beta, \sigma_\beta)$ be two SVNVs and $\mathcal{G}(\alpha) = \frac{2+(\varphi_\alpha - \delta_\alpha - \sigma_\alpha)}{3}$ and $\mathcal{G}(\beta) = \frac{2+(\varphi_\beta - \delta_\beta - \sigma_\beta)}{3}$ be the score values of α and β respectively. Let $\mathfrak{Q}(\alpha) = \varphi_\alpha + \sigma_\alpha$ and $\mathfrak{Q}(\beta) = \varphi_\beta + \sigma_\beta$ be the accuracy values of α and β respectively. Then,

i. If $\mathcal{G}(\alpha) < \mathcal{G}(\beta)$, then $\alpha < \beta$

ii. If $\mathcal{G}(\alpha) = \mathcal{G}(\beta)$ then,

a. If $\mathfrak{Q}(\alpha) < \mathfrak{Q}(\beta)$, then $\alpha < \beta$

b. If $\mathfrak{Q}(\alpha) = \mathfrak{Q}(\beta)$, then $\alpha = \beta$

Definition 6: [32] Let $\alpha = (\varphi_\alpha, \delta_\alpha, \sigma_\alpha)$ and $\beta = (\varphi_\beta, \delta_\beta, \sigma_\beta)$ be two SVNVs. Then, some basic operations of SVNVs are given as:

i. $\alpha \oplus \beta = (\varphi_\alpha + \varphi_\beta - \varphi_\alpha\varphi_\beta, \delta_\alpha\delta_\beta, \sigma_\alpha\sigma_\beta)$

ii. $\alpha \otimes \beta = (\varphi_\alpha\varphi_\beta, \delta_\alpha + \delta_\beta - \delta_\alpha\delta_\beta, \sigma_\alpha + \sigma_\beta - \sigma_\alpha\sigma_\beta)$

iii. $\mu\alpha = (1-(1-\varphi_\alpha)^\mu, (\delta_\alpha)^\mu, (\sigma_\alpha)^\mu)$, $\mu > 0$

iv. $(\alpha)^\mu = ((\varphi_\alpha)^\mu, 1-(1-\delta_\alpha)^\mu, 1-(1-\sigma_\alpha)^\mu)$, $\mu > 0$

Definition 7: [50] Let $ß = (ß_1, ß_2, \ldots, ß_n)$ be the collection of characteristics and there is a prioritization between the attributes which is represented by linear ordering $ß_1 > ß_2 > \cdots > ß_n$ shows that attribute $ß_\rho$ has a maximum priority than $ß_k$, if $\rho < k$. The values $ß_\rho(r)$ shows the performance of any alternative r under the attribute $ß_\rho$, and satisfies $ß_\rho(r) \in [0,1]$. The prioritized average operator (PA) is defined as if it satisfies such axiom:

$$PA\left(\tau_\rho(r)\right) = \sum_{\rho=1}^{n} \omega_\rho \tau_\rho(r) \quad (3)$$

Where $\boldsymbol{\omega}_\rho = \frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}, \boldsymbol{\varepsilon}_\rho = \prod_{k=1}^{\rho-1}\mathcal{G}(\boldsymbol{\tau_k}), \rho = \mathbf{2}, \mathbf{3}, \ldots, \boldsymbol{n}$. The initial value $\boldsymbol{\varepsilon_1} = \mathbf{1}$ and $\mathcal{G}(\boldsymbol{\tau_k})$ represents score values of $\boldsymbol{k^{th}}$ SVNVs. Then, PA is called the prioritized averaging (PA) operator.

Definition 8: [50] Let $\tau_\rho = (\varphi_\rho, \delta_\rho, \sigma_\rho), \rho = 1, 2, 3, \ldots, n$ be the collection of SVNVs, with PA $\omega_\rho = \frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}$ operator for each τ_ρ. Then, SVN prioritized averaging (SVNPA) operator is particularized as:

$$SVNPA(\tau_1, \tau_2, \ldots, \tau_n) = \varepsilon_1\tau_1 \oplus \varepsilon_2\tau_2 \oplus \ldots \oplus \varepsilon_n\tau_n$$

Where $\varepsilon_\rho = \prod_{k=1}^{\rho-1} \mathcal{G}(\tau_\rho), \rho = 2, 3, \ldots, n$. The initial value of $\varepsilon_1 = 1$ and $\mathcal{G}(\tau_k)$ represents the score value of k^{th} SVNVs.

Definition 9: [50] Let $\tau_\rho = (\varphi_\rho, \delta_\rho, \sigma_\rho), \rho = 1,2,3,\dots,n$ be the collection of SVNVs with PA $\omega_\rho = \frac{\varepsilon_\rho}{\sum_{\rho=1}^{n} \varepsilon_\rho}$ operator for each τ_ρ. Then, the SVN prioritized geometric (SVNPG) operator is particularized as:

$$SVNPG(\tau_1, \tau_2, \dots, \tau_n) = \tau_1^{\varepsilon_1} \otimes \tau_2^{\varepsilon_2} \otimes \dots \otimes \tau_n^{\varepsilon_n}$$

Where $\boldsymbol{\varepsilon}_\rho = \prod_{k=1}^{\rho-1} \mathcal{G}(\boldsymbol{\tau}_\rho), \rho = \mathbf{2}, \mathbf{3}, \dots, \boldsymbol{n}$. The initial value of $\boldsymbol{\varepsilon_1} = \mathbf{1}$ and $\mathcal{G}(\boldsymbol{\tau_k})$ represents the score values of $\boldsymbol{k^{th}}$ SVNVs.

3. Basic Operations of Aczel Alsina tools Based on Single-Valued Neutrosophic Information

In this section, we will demonstrate Aczel Alsina operations in the form of sum, product, scalar multiplication, and power rule using SVNV data.

Definition 10: Let $\tau = (\varphi, \delta, \sigma), \tau_1 = (\varphi_1, \delta_1, \sigma_1)$ and $\tau_2 = (\varphi_2, \delta_2, \sigma_2)$ be the three SVNVs, $\not{C} \geq 1$ and $\mu > 0$. Then, we illustrate some basic operations of Aczel Alsina tools in the following form:

i. $$\tau_1 \oplus \tau_2 = \begin{pmatrix} 1 - e^{-\left(\left(-ln(1-\varphi_1)\right)^{\not{C}} + \left(-ln(1-\varphi_2)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}}, \\ e^{-\left(\left(-ln(\delta_1)\right)^{\not{C}} + \left(-ln(\delta_2)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}}, \\ e^{-\left(\left(-ln(\sigma_1)\right)^{\not{C}} + \left(-ln(\sigma_2)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}} \end{pmatrix}$$

ii. $$\tau_1 \otimes \tau_2 = \begin{pmatrix} e^{-\left(\left(-ln(\varphi_1)\right)^{\not{C}} + \left(-ln(\varphi_2)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}}, \\ 1 - e^{-\left(\left(-ln(1-\delta_1)\right)^{\not{C}} + \left(-ln(1-\delta_2)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}}, \\ 1 - e^{-\left(\left(-ln(1-\sigma_1)\right)^{\not{C}} + \left(-ln(1-\sigma_2)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}} \end{pmatrix}$$

iii. $$\mu\tau = \begin{pmatrix} 1 - e^{-\left(\mu\left(-ln(1-\varphi_r)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}}, \\ e^{-\left(\mu\left(-ln(\delta_r)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}}, \\ e^{-\left(\mu\left(-ln(\sigma_r)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}} \end{pmatrix}$$

iv. $$\tau^\mu = \begin{pmatrix} e^{-\left(\mu\left(-ln(\varphi_r)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}}, \\ 1 - e^{-\left(\mu\left(-ln(1-\delta_r)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}}, \\ 1 - e^{-\left(\mu\left(-ln(1-\sigma_r)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}} \end{pmatrix}$$

Theorem 1: Let $\tau = (\varphi, \delta, \sigma), \tau_1 = (\varphi_1, \delta_1, \sigma_1)$ and $\tau_2 = (\varphi_2, \delta_2, \sigma_2)$ be the three SVNVs with $\not{C} \geq 1$ and $\mu > 0$. Then, a few fundamental OLs are defined as follows:

i. $\tau_1 \oplus \tau_2 = \tau_2 \oplus \tau_1$

ii. $\tau_1 \otimes \tau_2 = \tau_2 \otimes \tau_1$

iii. $\mu(\tau_1 \oplus \tau_2) = \mu\tau_1 \oplus \mu\tau_2, \mu > 0$

iv. $(\mu_1 + \mu_2)\tau = \mu_1\tau + \mu_2\tau, \mu_1, \mu_2 > 0$

v. $(\tau_1 \otimes \tau_2)^{\mu} = \tau_1^{\mu} \otimes \tau_2^{\mu}, \mu > 0$

vi. $\tau^{\mu_1} \otimes \tau^{\mu_2} = \tau^{(\mu_1+\mu_2)} \ , \mu_1, \mu_2 > 0$

4. Single Valued Neutrosophic Aczel Alsina Prioritized Aggregation Operators

In this section, we will narrate some certain approaches of SVNVs based on Aczel Alsina operations and elaborate on some characteristics of our aimed work. We also extend our work to the weighted averaging and weighted geometric operators.

Definition 11: Let $\tau_{\rho} = (\varphi_{\rho}, \delta_{\rho}, \sigma_{\rho}), \rho = 1, 2, 3, \dots, n$ be the collection of SVNVs with PA $\omega_{\rho} = \frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n} \varepsilon_{\rho}}$ operator for each τ_{ρ}. Then SVNAAPA operator is particularized as:

$$SVNAAPA(\tau_1, \tau_2, \dots, \tau_n) = \bigoplus_{\rho=1}^{n} \left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n} \varepsilon_{\rho}} \tau_{\rho} \right)$$
$$SVNAAPA(\tau_1, \tau_2, \dots, \tau_n) = \left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n} \varepsilon_{\rho}} \right) \tau_1 \oplus \left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n} \varepsilon_{\rho}} \right) \tau_2 \oplus \dots \oplus \left(\frac{\varepsilon_n}{\sum_{\rho=1}^{n} \varepsilon_{\rho}} \right) \tau_n \quad (4)$$

Where $\varepsilon_{\rho} = \prod_{k=1}^{\rho-1} \mathcal{G}(\tau_k), \rho = 2, 3, \dots, n.$

Theorem 2: Let $\tau_{\rho} = (\varphi_{\rho}, \delta_{\rho}, \sigma_{\rho}), \rho = 1,2,3, \dots, n$ be the collection of SVNVs with PA $\omega_{\rho} = \frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n} \varepsilon_{\rho}}$ operator for each τ_{ρ}. Then, the SVNAAPA operator is particularized as:

$$SVNAAPA = \sum_{\rho=1}^{n} \left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n} \varepsilon_{\rho}} \tau_{\rho} \right) = \begin{pmatrix} 1 - e^{-\left(\sum_{\rho=1}^{n} \left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n} \varepsilon_{\rho}} \right) \left(-ln(1-\varphi_{\rho}) \right)^{\text{¢}} \right)^{\frac{1}{\text{¢}}}}, \\ e^{-\left(\sum_{\rho=1}^{n} \left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n} \varepsilon_{\rho}} \right) \left(-ln(\delta_{\rho}) \right)^{\text{¢}} \right)^{\frac{1}{\text{¢}}}}, \\ e^{-\left(\sum_{\rho=1}^{n} \left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n} \varepsilon_{\rho}} \right) \left(-ln(\sigma_{\rho}) \right)^{\text{¢}} \right)^{\frac{1}{\text{¢}}}} \end{pmatrix} \quad (5)$$

Proof: We will proof this theorem with the help of a mathematical induction technique in the following way:

i. Take the value of $\rho = 2$ depends on Aczel Alsina operations of SVNVs, we get,

$$\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n} \varepsilon_{\rho}} \tau_1 \right) = \begin{pmatrix} 1 - e^{-\left(\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n} \varepsilon_{\rho}} \right) \left(-ln(1-\varphi_1) \right)^{\text{¢}} \right)^{\frac{1}{\text{¢}}}}, \\ e^{-\left(\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n} \varepsilon_{\rho}} \right) \left(-ln(\delta_1) \right)^{\text{¢}} \right)^{\frac{1}{\text{¢}}}}, \\ e^{-\left(\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n} \varepsilon_{\rho}} \right) \left(-ln(\sigma_1) \right)^{\text{¢}} \right)^{\frac{1}{\text{¢}}}} \end{pmatrix}$$

$$\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\tau_2\right)=\begin{pmatrix} 1-e^{-\left(\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(1-\varphi_2))^{\text{Ȼ}}\right)^{\frac{1}{\text{Ȼ}}}}, \\ e^{-\left(\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(\delta_2))^{\text{Ȼ}}\right)^{\frac{1}{\text{Ȼ}}}}, \\ e^{-\left(\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(\sigma_2))^{\text{Ȼ}}\right)^{\frac{1}{\text{Ȼ}}}} \end{pmatrix}$$

By using the above Definition 11, we have:

$$SVNAAPA(\tau_1,\tau_2)=\begin{pmatrix} \begin{pmatrix} 1-e^{-\left(\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(1-\varphi_1))^{\text{Ȼ}}\right)^{\frac{1}{\text{Ȼ}}}}, \\ e^{-\left(\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(\delta_1))^{\text{Ȼ}}\right)^{\frac{1}{\text{Ȼ}}}}, \\ e^{-\left(\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(\sigma_1))^{\text{Ȼ}}\right)^{\frac{1}{\text{Ȼ}}}} \end{pmatrix} \oplus \\ \begin{pmatrix} 1-e^{-\left(\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(1-\varphi_2))^{\text{Ȼ}}\right)^{\frac{1}{\text{Ȼ}}}}, \\ e^{-\left(\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(\delta_2))^{\text{Ȼ}}\right)^{\frac{1}{\text{Ȼ}}}}, \\ e^{-\left(\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(\sigma_2))^{\text{Ȼ}}\right)^{\frac{1}{\text{Ȼ}}}} \end{pmatrix} \end{pmatrix}$$

$$=\begin{pmatrix} 1-e^{-\left(\left(\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(1-\varphi_1))^{\text{Ȼ}}\right)\oplus\left(\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(1-\varphi_2))^{\text{Ȼ}}\right)\right)^{\frac{1}{\text{Ȼ}}}}, \\ e^{-\left(\left(\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(\delta_1))^{\text{Ȼ}}\right)\oplus\left(\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(\delta_2))^{\text{Ȼ}}\right)\right)^{\frac{1}{\text{Ȼ}}}}, \\ e^{-\left(\left(\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(\sigma_1))^{\text{Ȼ}}\right)\oplus\left(\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)(-ln(\sigma_1))^{\text{Ȼ}}\right)\right)^{\frac{1}{\text{Ȼ}}}} \end{pmatrix}$$

$$=\begin{pmatrix} 1-e^{-\left(\sum_{\rho=1}^{2}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln\left(1-\varphi_\rho\right)\right)^{\text{Ȼ}}\right)^{\frac{1}{\text{Ȼ}}}}, \\ e^{-\left(\sum_{\rho=1}^{2}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln\left(\delta_\rho\right)\right)^{\text{Ȼ}}\right)^{\frac{1}{\text{Ȼ}}}}, \\ e^{-\left(\sum_{\rho=1}^{2}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln\left(\sigma_\rho\right)\right)^{\text{Ȼ}}\right)^{\frac{1}{\text{Ȼ}}}} \end{pmatrix}$$

Hence, this is true for $\rho = 2$.

ii. Now, suppose that this is true for $\rho = k$. Then, we have:

$$SVNAAPA(\tau_1, \tau_2, \dots, \tau_n) = \sum_{\rho=1}^{k}\left(\frac{\varepsilon_k}{\sum_{\rho=1}^{n}\varepsilon_\rho}\tau_k\right) = \begin{pmatrix} 1 - e^{-\left(\sum_{\rho=1}^{k}\left(\frac{\varepsilon_k}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-\ ln(1-\varphi_k)\right)^{\not\subset}\right)^{\frac{1}{\not\subset}}}, \\ e^{-\left(\sum_{\rho=1}^{k}\left(\frac{\varepsilon_k}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\delta_k)\right)^{\not\subset}\right)^{\frac{1}{\not\subset}}}, \\ e^{-\left(\sum_{\rho=1}^{k}\left(\frac{\varepsilon_k}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\sigma_k)\right)^{\not\subset}\right)^{\frac{1}{\not\subset}}} \end{pmatrix}$$

Now, for $\rho = k + 1$. We get,

$$SVNAAPA(\tau_1, \tau_2, \dots, \tau_k, \tau_{k+1}) = \bigoplus_{\rho=1}^{k}\left(\left(\frac{\varepsilon_k}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\tau_k \oplus \left(\frac{\varepsilon_{k+1}}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\tau_{k+1}\right)$$

$$= \begin{pmatrix} \begin{pmatrix} 1 - e^{-\left(\sum_{\rho=1}^{k}\left(\frac{\varepsilon_k}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-\ ln(1-\varphi_k)\right)^{\not\subset}\right)^{\frac{1}{\not\subset}}}, \\ e^{-\left(\sum_{\rho=1}^{k}\left(\frac{\varepsilon_k}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\delta_k)\right)^{\not\subset}\right)^{\frac{1}{\not\subset}}}, \\ e^{-\left(\sum_{\rho=1}^{k}\left(\frac{\varepsilon_k}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\sigma_k)\right)^{\not\subset}\right)^{\frac{1}{\not\subset}}} \end{pmatrix} \oplus \\ \begin{pmatrix} 1 - e^{-\left(\left(\frac{\varepsilon_{k+1}}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\varphi_{k+1})\right)^{\not\subset}\right)^{\frac{1}{\not\subset}}}, \\ e^{-\left(\left(\frac{\varepsilon_{k+1}}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\delta_{k+1})\right)^{\not\subset}\right)^{\frac{1}{\not\subset}}}, \\ e^{-\left(\left(\frac{\varepsilon_{k+1}}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\sigma_{k+1})\right)^{\not\subset}\right)^{\frac{1}{\not\subset}}} \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1 - e^{-\left(\sum_{\rho=1}^{k+1}\left(\frac{\varepsilon_{k+1}}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-\ ln(1-\varphi_{k+1})\right)^{\not\subset}\right)^{\frac{1}{\not\subset}}}, \\ e^{-\left(\sum_{\rho=1}^{k+1}\left(\frac{\varepsilon_{k+1}}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\delta_{k+1})\right)^{\not\subset}\right)^{\frac{1}{\not\subset}}}, \\ e^{-\left(\sum_{\rho=1}^{k+1}\left(\frac{\varepsilon_{k+1}}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\sigma_{k+1})\right)^{\not\subset}\right)^{\frac{1}{\not\subset}}} \end{pmatrix}$$

Hence proved.

Example 1: Let $(0.54, 0.98, 0.27), (0.87, 0.55, 0.61)$, $(0.49, 0.33, 0.72)$ and $(0.11, 0.39, 0.27)$ be the four SVNVs with $\not\subset = 3$. Then, SVNAAPA can be calculated as:

$$SVNAAPA(\tau_1,\tau_2,\tau_3,\tau_4)=\begin{pmatrix}1-e^{-\left(\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\varphi_1)\right)^{\mathcal{C}}+\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\varphi_2)\right)^{\mathcal{C}}+\left(\frac{\varepsilon_3}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\varphi_3)\right)^{\mathcal{C}}+\left(\frac{\varepsilon_4}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\varphi_4)\right)^{\mathcal{C}}\right)^{\frac{1}{\mathcal{C}}}},\\ e^{-\left(\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\delta_1)\right)^{\mathcal{C}}+\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\delta_2)\right)^{\mathcal{C}}+\left(\frac{\varepsilon_3}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\delta_3)\right)^{\mathcal{C}}+\left(\frac{\varepsilon_4}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\delta_4)\right)^{\mathcal{C}}\right)^{\frac{1}{\mathcal{C}}}},\\ e^{-\left(\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\sigma_1)\right)^{\mathcal{C}}+\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\sigma_2)\right)^{\mathcal{C}}+\left(\frac{\varepsilon_3}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\sigma_3)\right)^{\mathcal{C}}+\left(\frac{\varepsilon_4}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\sigma_4)\right)^{\mathcal{C}}\right)^{\frac{1}{\mathcal{C}}}}\end{pmatrix}$$

$$=\begin{pmatrix}1-e^{-\left(\left(\frac{1}{1.7927}\right)\left(-ln(1-0.54)\right)^{3}+\left(\frac{0.4300}{1.7927}\right)\left(-ln(1-0.87)\right)^{3}+\left(\frac{0.2451}{1.7927}\right)\left(-ln(1-0.49)\right)^{3}+\left(\frac{0.1176}{1.7927}\right)\left(-ln(1-0.11)\right)^{3}\right)^{\frac{1}{3}}},\\ e^{-\left(\left(\frac{1}{1.7927}\right)\left(\left(-ln(0.98)\right)^{3}\right)+\left(\frac{0.4300}{1.7927}\right)\left(\left(-ln(0.55)\right)^{3}\right)+\left(\frac{0.2451}{1.7927}\right)\left(\left(-ln(0.33)\right)^{3}\right)+\left(\frac{0.1176}{1.7927}\right)\left(\left(-ln(0.39)\right)^{3}\right)\right)^{\frac{1}{3}}},\\ e^{-\left(\left(\frac{1}{1.7927}\right)\left(\left(-ln(0.27)\right)^{3}\right)+\left(\frac{0.4300}{1.7927}\right)\left(\left(-ln(0.61)\right)^{3}\right)+\left(\frac{0.2451}{1.7927}\right)\left(\left(-ln(0.72)\right)^{3}\right)+\left(\frac{0.1176}{1.7927}\right)\left(\left(-ln(0.27)\right)^{3}\right)\right)^{\frac{1}{3}}}\end{pmatrix}$$

$$SVNAAPA(\tau_1,\tau_2,\tau_3,\tau_4)=(0.7349, 0.5149, 0.3239)$$

Theorem 3: If all $\tau_\rho=(\varphi_\rho,\delta_\rho,\sigma_\rho),\rho=1,2,3,\dots,n$ are equal, that is, $\tau_\rho=\tau$ for all τ. Then, we have:

$$SVNAAPA(\tau_1,\tau_2,\tau_3,\dots,\tau_n)=\tau$$

Proof: Since $\tau_\rho=(\varphi_\rho,\delta_\rho,\sigma_\rho),\rho=1,2,3,\dots,n.$ Then,

$$SVNAAPA(\tau_1,\tau_2,\dots,\tau_n)=\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\tau_\rho\right)$$

$$=\begin{pmatrix}1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\varphi_\rho)\right)^{\mathcal{C}}\right)^{\frac{1}{\mathcal{C}}}},\\ e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\delta_\rho)\right)^{\mathcal{C}}\right)^{\frac{1}{\mathcal{C}}}},\\ e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\sigma_\rho)\right)^{\mathcal{C}}\right)^{\frac{1}{\mathcal{C}}}}\end{pmatrix}$$

$$=\left(1-e^{-\left(-ln\left(1-(\varphi_\rho)\right)^{\mathcal{C}}\right)^{\frac{1}{\mathcal{C}}}},e^{-\left(\left(-ln(\delta_\rho)\right)^{\mathcal{C}}\right)^{\frac{1}{\mathcal{C}}}},e^{-\left(\left(-ln(\sigma_\rho)\right)^{\mathcal{C}}\right)^{\frac{1}{\mathcal{C}}}}\right)=\tau$$

Thus, it is obvious that $SVNAAPA(\tau_1,\tau_2,\dots,\tau_n)=\tau$ holds.

Theorem 4: Let $\tau_\rho=(\varphi_\rho,\delta_\rho,\sigma_\rho),\rho=1,2,3,\dots,n$ be the collection of SVNVs, with PA $\omega_\rho=\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}$ operator for each τ_ρ. Let $\tau^-=min(\tau_1,\tau_2,\dots,\tau_n)$ and $\tau^+=max(\tau_1,\tau_2,\dots,\tau_n)$. So,

$$\tau^-\le SVNAAPA(\tau_1,\tau_2,\dots,\tau_n)\le\tau^+$$

Proof: Let $\tau_{\rho} = (\varphi_{\rho}, \delta_{\rho}, \sigma_{\rho}), \rho = 1,2,3, \dots, n$ be the collection of SVNVs, with PA $\omega_{\rho} = \frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n} \varepsilon_{\rho}}$ operator for each τ_{ρ}. Let $\tau^{-} = min_{\rho}(\tau_1, \tau_2, \dots, \tau_n) = (\varphi^{-}, \delta^{-}, \sigma^{-})$, and $\tau^{+} = max_{\rho}(\tau_1, \tau_2, \dots, \tau_n) = (\varphi^{+}, \delta^{+}, \sigma^{+})$. We have, $\varphi^{-} = min_{\rho}\{\varphi_{\rho}\}, \delta^{-} = max_{\rho}\{\delta_{\rho}\}$ and $\sigma^{-} = max_{\rho}\{\sigma_{\rho}\}$ and $\varphi^{+} = max_{\rho}\{\varphi_{\rho}\}, \delta^{+} = min_{\rho}\{\delta_{\rho}\}$, and $\sigma^{+} = min_{\rho}\{\sigma_{\rho}\}$. Hence, there is the following result for the inequalities:

$$1 - e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(1-\varphi^{-})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \leq 1 - e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(1-\varphi_{\rho})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \leq 1 - e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(1-\varphi^{+})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}}$$

$$e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(\delta^{-})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \leq e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(\delta_{\rho})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \leq e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(\delta^{+})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}}$$

Similarly,

$$e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(\sigma^{-})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \leq e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(\sigma_{\rho})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \leq e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(\sigma^{+})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}}$$

So,

$$\tau^{-} \leq SVNAAPA(\tau_1, \tau_2, \dots, \tau_n) \leq \tau^{+}$$

Theorem 5: Let τ_{ρ} and $\tau'_{\rho}, \rho = 1, 2, 3, \dots, n$ be two sets of SVNVs, if $\tau_{\rho} \leq \tau'_{\rho}$ For all τ. So, $SVNAAPA(\tau_1, \tau_2, \dots, \tau_n) \leq SVNAAPA(\tau'_1, \tau'_2, \dots, \tau'_n)$

Proof: Let τ_{ρ} and $\tau'_{\rho}, \rho = 1, 2, 3, \dots, n$ be two sets of SVNVs, we can write in the following form:

$$1 - e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(1-\varphi_{\rho})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \leq 1 - e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(1-\varphi'_{\rho})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}}$$

$$e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(\delta_{\rho})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \geq e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(\delta'_{\rho})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}}$$

$$e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(\sigma_{\rho})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \geq e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)\left(-ln(\sigma'_{\rho})\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}}$$

Hence, it is proved that $SVNAAPA(\tau_1, \tau_2, \dots, \tau_n) \leq SVNAAPA(\tau'_1, \tau'_2, \dots, \tau'_n)$.

Definition 12: Let $\tau_{\rho} = (\varphi_{\rho}, \delta_{\rho}, \sigma_{\rho}), \rho = 1, 2, 3, \dots, n$ be the collection of SVNVs, with PA $\omega_{\rho} = \frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n} \varepsilon_{\rho}}$ operator for each τ_{ρ}. Then, the SVNAAPG operator is particularized as:

$$SVNAAPG(\tau_1, \tau_2, \dots, \tau_n) = \bigotimes_{\rho=1}^{n}\left(\tau_n^{\left(\frac{\varepsilon_{\rho}}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)}\right) \tag{6}$$

$$SVNAAPG(\tau_1, \tau_2, \dots, \tau_n) = \tau_1^{\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)} \otimes \tau_2^{\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)} \otimes \dots \otimes \tau_{\text{¢}}^{\left(\frac{\varepsilon_n}{\sum_{\rho=1}^{n}\varepsilon_{\rho}}\right)}$$

Where $\varepsilon_{\rho} = \prod_{k=1}^{\rho-1} \mathcal{G}(\tau_k), \rho = 2, 3, \dots, n$. The initial value of $\varepsilon_1 = 1$ and $\mathcal{G}(\tau_k)$ represents the score values of k^{th} SVNVs.

Theorem 6: Let $\tau_\rho = (\varphi_\rho, \delta_\rho, \sigma_\rho), \rho = 1,2,3,\dots,n$ be the collection of SVNVs, with PA $\omega_\rho = \frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}$ operator for each τ_ρ. Then, the SVNAAPG operator is particularized as:

$$SVNAAPG = \prod_{\rho=1}^{n}\left(\tau_\rho^{\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)}\right) = \begin{pmatrix} e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\varphi_\rho)\right)^{₡}\right)^{\frac{1}{₡}}}, \\ 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\delta_\rho)\right)^{₡}\right)^{\frac{1}{₡}}}, \\ 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\sigma_\rho)\right)^{₡}\right)^{\frac{1}{₡}}} \end{pmatrix} \quad (7)$$

Proof: Proof is similar to the theorem 2.

Example 2: Let $(0.59, 0.73, 0.34), (0.45, 0.56, 0.67), (0.78, 0.89, 0.9)$ and $(0.51, 0.82, 0.65)$ be the four SVNVs with $\varepsilon_1 = 1, \varepsilon_2 = 0.5067, \varepsilon_3 = 0.2060$ and $\varepsilon_4 = 0.0680$ and $₡ = 3$. Then, SVNAAPG can be calculated as:

$$SVNAAPG(\tau_1,\tau_2,\tau_3,\tau_4) == \begin{pmatrix} e^{-\begin{pmatrix}\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\varphi_1)\right)^{₡}+\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\varphi_2)\right)^{₡}+ \\ \left(\frac{\varepsilon_3}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\varphi_3)\right)^{₡}+\left(\frac{\varepsilon_4}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\varphi_4)\right)^{₡}\end{pmatrix}^{\frac{1}{₡}}}, \\ 1-e^{-\begin{pmatrix}\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\delta_1)\right)^{₡}+\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\delta_2)\right)^{₡}+ \\ \left(\frac{\varepsilon_3}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\delta_3)\right)^{₡}+\left(\frac{\varepsilon_4}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\delta_4)\right)^{₡}\end{pmatrix}^{\frac{1}{₡}}}, \\ 1-e^{-\begin{pmatrix}\left(\frac{\varepsilon_1}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\sigma_1)\right)^{₡}+\left(\frac{\varepsilon_2}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\sigma_2)\right)^{₡}+ \\ \left(\frac{\varepsilon_3}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\sigma_3)\right)^{₡}+\left(\frac{\varepsilon_4}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\sigma_4)\right)^{₡}\end{pmatrix}^{\frac{1}{₡}}} \end{pmatrix}$$

$$= \begin{pmatrix} e^{-\begin{pmatrix}\left(\frac{1}{1.7807}\right)\left(-ln(0.59)\right)^{3}+\left(\frac{0.5067}{1.7807}\right)\left(-ln(0.45)\right)^{3}+ \\ \left(\frac{0.2060}{1.7807}\right)\left(-ln(0.78)\right)^{3}+\left(\frac{0.0680}{1.7807}\right)\left(-ln(0.51)\right)^{3}\end{pmatrix}^{\frac{1}{3}}}, \\ 1-e^{-\begin{pmatrix}\left(\frac{1}{1.7807}\right)\left(-ln(1-0.73)\right)^{3}+\left(\frac{0.5067}{1.7807}\right)\left(-ln(1-0.56)\right)^{3}+ \\ \left(\frac{0.2060}{1.7807}\right)\left(-ln(1-0.89)\right)^{3}+\left(\frac{0.0680}{1.7807}\right)\left(-ln(1-0.82)\right)^{3}\end{pmatrix}^{\frac{1}{3}}}, \\ 1-e^{-\begin{pmatrix}\left(\frac{1}{1.7807}\right)\left(-ln(1-0.34)\right)^{3}+\left(\frac{0.5067}{1.7807}\right)\left(-ln(1-0.67)\right)^{3}+ \\ \left(\frac{0.2060}{1.7807}\right)\left(-ln(1-0.9)\right)^{3}+\left(\frac{0.0680}{1.7807}\right)\left(-ln(1-0.65)\right)^{3}\end{pmatrix}^{\frac{1}{3}}} \end{pmatrix}$$

$$SVNAAPG(\tau_1,\tau_2,\tau_3,\tau_4) = (0.5368, 0.7579, 0.7092)$$

Theorem 7: If all $\tau_\rho = (\varphi_\rho, \delta_\rho, \sigma_\rho), \rho = 1,2,3,\dots,n$ are equal, that is, $\tau_n = \tau$ for all τ. Then,

$$SVNAAPG(\tau_1,\tau_2,\dots,\tau_n) = \tau.$$

Proof: Since $\tau_\rho = (\varphi_\rho, \delta_\rho, \sigma_\rho), \rho = 1,2,3,\dots,n$. Then,

$$SVNAAPG(\tau_1,\tau_2,\dots,\tau_n) = \prod_{\rho=1}^{n}\left(\tau_\rho^{\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}}\right)$$

$$SVNAAPG(\tau_1,\tau_2,\dots,\tau_n) = \begin{pmatrix} e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\varphi_\rho)\right)^{\complement}\right)^{\frac{1}{\complement}}}, \\ 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\delta_\rho)\right)^{\complement}\right)^{\frac{1}{\complement}}}, \\ 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\sigma_\rho)\right)^{\complement}\right)^{\frac{1}{\complement}}} \end{pmatrix}$$

$$= \left(e^{-\left(\left(-ln(\varphi_{\complement})\right)^{\complement}\right)^{\frac{1}{\complement}}}, 1-e^{-\left(\left(-ln(1-\delta_{\complement})\right)^{\complement}\right)^{\frac{1}{\complement}}}, 1-e^{-\left(\left(-ln(1-\sigma_{\complement})\right)^{\complement}\right)^{\frac{1}{\complement}}} \right) = \tau$$

Thus, it is obvious that $SVNAAPG(\tau_1,\tau_2,\dots,\tau_n) = \tau$ holds.

Theorem 8: Let $\tau_\rho = (\varphi_\rho,\delta_\rho,\sigma_\rho), \rho = 1,2,3,\dots,n$ be the collection of SVNVs, with PA $\omega_\rho = \frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}$ operator for each τ_ρ. Let $\tau^- = min(\tau_1,\tau_2,\dots,\tau_n)$ and $\tau^+ = max(\tau_1,\tau_2,\dots,\tau_n)$. Then,

$$\tau^- \le SVNAAPG(\tau_1,\tau_2,\dots,\tau_n) \le \tau^+.$$

Proof: Let $\tau_\rho = (\varphi_\rho,\delta_\rho,\sigma_\rho), \rho = 1,2,3,\dots,n$ be the collection of SVNVs with PA $\omega_\rho = \frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}$ operator for each τ_ρ. Let $\tau^- = min_\rho(\tau_1,\tau_2,\dots,\tau_n) = (\varphi^-,\delta^-,\sigma^-)$, and $\tau^+ = max_\rho(\tau_1,\tau_2,\dots,\tau_n) = (\varphi^+,\delta^+,\sigma^+)$. We have, $\varphi^- = min_\rho\{\varphi_\rho\}$, $\delta^- = max_\rho\{\delta_\rho\}$ and $\sigma^- = max_\rho\{\sigma_\rho\}$ and $\varphi^+ = max_\rho\{\varphi_\rho\}$, $\delta^+ = min_\rho\{\delta_\rho\}$, and $\sigma^+ = min_\rho\{\sigma_\rho\}$. Hence, there is the following result for the inequalities:

$$e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\varphi^-)\right)^{\complement}\right)^{\frac{1}{\complement}}} \le e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\varphi_\rho)\right)^{\complement}\right)^{\frac{1}{\complement}}} \le e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\varphi^+)\right)^{\complement}\right)^{\frac{1}{\complement}}}$$

$$1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\delta^-)\right)^{\complement}\right)^{\frac{1}{\complement}}} \le 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\delta_\rho)\right)^{\complement}\right)^{\frac{1}{\complement}}} \le 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\delta^+)\right)^{\complement}\right)^{\frac{1}{\complement}}}$$

$$1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\sigma^-)\right)^{\complement}\right)^{\frac{1}{\complement}}} \le 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\sigma_\rho)\right)^{\complement}\right)^{\frac{1}{\complement}}} \le 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(1-\sigma^+)\right)^{\complement}\right)^{\frac{1}{\complement}}}$$

So, $\tau^- \le SVNAAPG(\tau_1,\tau_2,\dots,\tau_n) \le \tau^+$ holds.

Theorem 9: Let τ_ρ and $\tau'_\rho, \rho = 1,2,3,\dots,n$ be two sets of SVNVs, if $\tau_\rho \le \tau'_\rho$ for all τ. Then, $SVNAAPG(\tau_1,\tau_2,\dots,\tau_n) \le SVNAAPG(\tau'_1,\tau'_2,\dots,\tau'_n)$.

Proof: Let τ_ρ and $\tau'_\rho, \rho = 1,2,3,\dots,n$ be two sets of SVNVs. Then, we can write in the following way:

$$e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\varphi_\rho)\right)^{\complement}\right)^{\frac{1}{\complement}}} \le e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln(\varphi'_\rho)\right)^{\complement}\right)^{\frac{1}{\complement}}}$$

And,

$$1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln\left(1-\delta_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}} \geq 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln\left(1-\delta'_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}}$$

Similarly, we get:

$$1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln\left(1-\sigma_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}} \geq 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}\right)\left(-ln\left(1-\sigma'_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}}$$

From this, we can conclude that $\boldsymbol{SVNAAPG(\tau_1,\tau_2,\dots,\tau_n)} \leq \boldsymbol{SVNAAPG(\tau'_1,\tau'_2,\dots,\tau'_n)}$ holds.

5. Single Valued Neutrosophic Aczel Alsina Prioritized Weighted Aggregation Operators

In this section, we demonstrate many AOs of SVNAAPWA and SVNAAPWG based on Aczel Alsina operations with some specific characteristics by using our methodology.

Definition 13: Let $\tau_\rho = (\varphi_\rho, \delta_\rho, \sigma_\rho), \rho = 1,2,3,\dots,n$ be the collection of SVNVs with associated weight vectors (WVs) $\Psi = (\Psi_1, \Psi_2, \dots, \Psi_n)^T$ of τ_ρ, $\rho = 1,2,3,\dots,n$ such that $\Psi_\rho \in [0,1]$ and $\sum_{\rho=1}^{n}\Psi_\rho = 1$ with PA $\omega_\rho = \frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}$ operator for each τ_ρ. Then, the SVNAAPWA operator is particularized as:

$$SVNAAPWA(\tau_1,\tau_2,\dots,\tau_n) = \sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\tau_\rho\right) \quad (8)$$

$$SVNAAPWA(\tau_1,\tau_2,\dots,\tau_n) = \left(\frac{\Psi_1\varepsilon_1}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\tau_1\right) \oplus \left(\frac{\Psi_2\varepsilon_2}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\tau_2\right) \oplus \dots \oplus \left(\frac{\Psi_n\varepsilon_n}{\sum_{\rho=1}^{n}\Psi_n\varepsilon_n}\tau_n\right)$$

Where $\varepsilon_\rho = \prod_{k=1}^{\rho-1} \mathcal{G}(\tau_k), \rho = 2,3,\dots,n$. The initial value is $\varepsilon_1 = 1$ and $\mathcal{G}(\tau_k)$ be the score value of k^{th} SVNVs.

Theorem 9: Let $\tau_\rho = (\varphi_\rho, \delta_\rho, \sigma_\rho), \rho = 1,2,3,\dots,n$ be the collection of SVNVs with WVs $\Psi = (\Psi_1, \Psi_2, \dots, \Psi_n)^T$ and $\Psi_{\mathbb{C}} \in [0,1]$, $\sum_{\rho=1}^{n}\Psi_{\mathbb{C}} = 1$ associated with PA $\omega_\rho = \frac{\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}$ operator. Then SVNAAPWA operator is particularized as:

$$SVNAAPWA = \sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\tau_\rho\right) = \begin{pmatrix} 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(1-\varphi_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}}, \\ e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(\delta_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}}, \\ e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(\sigma_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}} \end{pmatrix} \quad (9)$$

Proof: We will proof this theorem with the help of a mathematical induction technique in the following way:

i. Take the value of $\rho = 2$ depends on Aczel Alsina operations of SVNVs, we get,

$$\left(\frac{\Psi_1\varepsilon_1}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\tau_1\right)=\left(\begin{array}{c}1-e^{-\left(\left(\frac{\Psi_1\varepsilon_1}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(1-\varphi_1))^{₡}\right)^{\frac{1}{₡}}},\\ e^{-\left(\left(\frac{\Psi_1\varepsilon_1}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\delta_1))^{₡}\right)^{\frac{1}{₡}}},\\ e^{-\left(\left(\frac{\Psi_1\varepsilon_1}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\sigma_1))^{₡}\right)^{\frac{1}{₡}}}\end{array}\right)$$

$$\left(\frac{\Psi_2\varepsilon_2}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\tau_2\right)=\left(\begin{array}{c}1-e^{-\left(\left(\frac{\Psi_2\varepsilon_2}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(1-\varphi_2))^{₡}\right)^{\frac{1}{₡}}},\\ e^{-\left(\left(\frac{\Psi_2\varepsilon_2}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\delta_2))^{₡}\right)^{\frac{1}{₡}}},\\ e^{-\left(\left(\frac{\Psi_2\varepsilon_2}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\sigma_2))^{₡}\right)^{\frac{1}{₡}}}\end{array}\right)$$

By using the above Definition 13, we have:

$$SVNAAPWA(\tau_1,\tau_2)=\left(\begin{array}{c}\left(\begin{array}{c}1-e^{-\left(\left(\frac{\Psi_1\varepsilon_1}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(1-\varphi_1))^{₡}\right)^{\frac{1}{₡}}},\\ e^{-\left(\left(\frac{\Psi_1\varepsilon_1}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\delta_1))^{₡}\right)^{\frac{1}{₡}}},\\ e^{-\left(\left(\frac{\Psi_1\varepsilon_1}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\sigma_1))^{₡}\right)^{\frac{1}{₡}}}\end{array}\right)\oplus\\ \left(\begin{array}{c}1-e^{-\left(\left(\frac{\Psi_2\varepsilon_2}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(1-\varphi_2))^{₡}\right)^{\frac{1}{₡}}},\\ e^{-\left(\left(\frac{\Psi_2\varepsilon_2}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\delta_2))^{₡}\right)^{\frac{1}{₡}}},\\ e^{-\left(\left(\frac{\Psi_2\varepsilon_2}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\sigma_2))^{₡}\right)^{\frac{1}{₡}}}\end{array}\right)\end{array}\right)$$

$$=\left(\begin{array}{c}1-e^{-\left(\sum_{\rho=1}^{2}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(1-\varphi_\rho))^{₡}\right)^{\frac{1}{₡}}},\\ e^{-\left(\sum_{\rho=1}^{2}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\delta_\rho))^{₡}\right)^{\frac{1}{₡}}},\\ e^{-\left(\sum_{\rho=1}^{2}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\sigma_\rho))^{₡}\right)^{\frac{1}{₡}}}\end{array}\right)$$

Hence, this is true for $\rho = 2$.

ii. Now suppose that this is true for $\rho = k$. Then, we have:

$$SVNAAPWA(\tau_1,\tau_2,\dots,\tau_n)=\sum_{\rho=1}^{n}\left(\frac{\Psi_k\varepsilon_k}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\tau_k\right)$$

$$= \begin{pmatrix} 1 - e^{-\left(\sum_{\rho=1}^{k}\left(\frac{\Psi_k \varepsilon_k}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-\ ln(1-\varphi_k)\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}}, \\ e^{-\left(\sum_{\rho=1}^{k}\left(\frac{\Psi_k \varepsilon_k}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-ln(\delta_k)\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}}, \\ e^{-\left(\sum_{\rho=1}^{k}\left(\frac{\Psi_k \varepsilon_k}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-ln(\sigma_k)\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}} \end{pmatrix}$$

Now, for $\rho = k + 1$. We get,

$$SVNAAPWA(\tau_1, \tau_2, \tau_k, \ldots, \tau_{k+1}) = \sum_{\rho=1}^{k+1}\left(\left(\frac{\Psi_k \varepsilon_k}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\tau_k\right) \oplus \left(\frac{\Psi_{k+1}\varepsilon_{k+1}}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\tau_{k+1}\right)\right)$$

$$= \begin{pmatrix} \begin{pmatrix} 1 - e^{-\left(\sum_{\rho=1}^{k}\left(\frac{\Psi_k \varepsilon_k}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-\ ln(1-\varphi_k)\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}}, \\ e^{-\left(\sum_{\rho=1}^{k}\left(\frac{\Psi_k \varepsilon_k}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-ln(\delta_k)\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}}, \\ e^{-\left(\sum_{\rho=1}^{k}\left(\frac{\Psi_k \varepsilon_k}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-ln(\sigma_k)\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}} \end{pmatrix} \oplus \\ \begin{pmatrix} 1 - e^{-\left(\left(\frac{\Psi_{k+1} \varepsilon_{k+1}}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-ln(1-\varphi_{k+1})\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}}, \\ e^{-\left(\left(\frac{\Psi_{k+1} \varepsilon_{k+1}}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-ln(\delta_{k+1})\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}}, \\ e^{-\left(\left(\frac{\Psi_{k+1} \varepsilon_{k+1}}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-ln(\sigma_{k+1})\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}} \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1 - e^{-\left(\sum_{\rho=1}^{k+1}\left(\frac{\Psi_{k+1} \varepsilon_{k+1}}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-\ ln(1-\varphi_{k+1})\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}}, \\ e^{-\left(\sum_{\rho=1}^{k+1}\left(\frac{\Psi_{k+1} \varepsilon_{k+1}}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-ln(\delta_{k+1})\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}}, \\ e^{-\left(\sum_{\rho=1}^{k+1}\left(\frac{\Psi_{k+1} \varepsilon_{k+1}}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-ln(\sigma_{k+1})\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}} \end{pmatrix}$$

Example 3: Let $\tau_1 = (0.98, 0.45, 0.32), (0.56, 0.76, 0.3), (0.11, 0.23, 0.66)$ and $(0.45, 0.6, 0.29)$ be the four SVNVs with WVs $(0.3783, 0.4180, 0.1045, 0.0992)$ and $\cancel{C} = 3$. Then, SVNAAPWA can be calculated as:

$$SVNAAPWA = \sum_{\rho=1}^{n}\left(\frac{\Psi_\rho \varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\tau_\rho\right) = \begin{pmatrix} 1 - e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho \varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-ln(1-\varphi_\rho)\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}}, \\ e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho \varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-ln(\delta_\rho)\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}}, \\ e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho \varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho \varepsilon_\rho}\right)\left(-ln(\sigma_\rho)\right)^{\cancel{C}}\right)^{\frac{1}{\cancel{C}}}} \end{pmatrix}$$

$$SVNAAPWA(\tau_1,\tau_2,\tau_3,\tau_4)=\begin{pmatrix} 1-e^{-\left(\left(\frac{\Psi_1\varepsilon_1}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(1-\varphi_1))^{\complement}+\left(\frac{\Psi_2\varepsilon_2}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(1-\varphi_2))^{\complement}+\left(\frac{\Psi_3\varepsilon_3}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(1-\varphi_3))^{\complement}+\left(\frac{\Psi_4\varepsilon_4}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(1-\varphi_4))^{\complement}\right)^{\frac{1}{\complement}}}, \\ e^{-\left(\left(\frac{\Psi_1\varepsilon_1}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\delta_1))^{\complement}+\left(\frac{\Psi_2\varepsilon_2}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\delta_2))^{\complement}+\left(\frac{\Psi_3\varepsilon_3}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\delta_3))^{\complement}+\left(\frac{\Psi_4\varepsilon_4}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\delta_4))^{\complement}\right)^{\frac{1}{\complement}}}, \\ e^{-\left(\left(\frac{\Psi_1\varepsilon_1}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\sigma_1))^{\complement}+\left(\frac{\Psi_2\varepsilon_2}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\sigma_2))^{\complement}+\left(\frac{\Psi_3\varepsilon_3}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\sigma_3))^{\complement}+\left(\frac{\Psi_4\varepsilon_4}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\sigma_4))^{\complement}\right)^{\frac{1}{\complement}}} \end{pmatrix}$$

$$=\begin{pmatrix} 1-e^{-\left(\left(\frac{0.2000}{0.5287}\right)(-ln(1-0.98))^3+\left(\frac{0.2210}{0.5287}\right)(-ln(1-0.56))^3+\left(\frac{0.0553}{0.5287}\right)(-ln(1-0.11))^3+\left(\frac{0.0524}{0.5287}\right)(-ln(1-0.45))^3\right)^{\frac{1}{3}}}, \\ e^{-\left(\left(\frac{0.2000}{0.5287}\right)(-ln(0.45))^3+\left(\frac{0.2210}{0.5287}\right)(-ln(0.76))^3+\left(\frac{0.0553}{0.5287}\right)(-ln(0.23))^3+\left(\frac{0.0524}{0.5287}\right)(-ln(0.6))^3\right)^{\frac{1}{3}}}, \\ e^{-\left(\left(\frac{0.2000}{0.5287}\right)(-ln(0.32))^3+\left(\frac{0.2210}{0.5287}\right)(-ln(0.3))^3+\left(\frac{0.0553}{0.5287}\right)(-ln(0.66))^3+\left(\frac{0.0524}{0.5287}\right)(-ln(0.29))^3\right)^{\frac{1}{3}}} \end{pmatrix}$$

$$SVNAAPWA(\tau_1,\tau_2,\tau_3,\tau_4)=(0.9416, 0.4416, 0.3196)$$

Theorem 10: If all $\tau_\rho=(\varphi_\rho,\delta_\rho,\sigma_\rho), \rho=1,2,3,\dots,n$ are equal, that is, $\tau_n=\tau$ for all τ. Then, we have:

$$SVNAAPWA(\tau_1,\tau_2,\dots,\tau_n)=\tau$$

Proof: Since $\tau_\rho=(\varphi_\rho,\delta_\rho,\sigma_\rho), \rho=1,2,3,\dots,n$. Then,

$$SVNAAPWA(\tau_1,\tau_2,\dots,\tau_n)=\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\tau_\rho$$

$$=\begin{pmatrix} 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(1-\varphi_\rho))^{\complement}\right)^{\frac{1}{\complement}}}, \\ e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\delta_\rho))^{\complement}\right)^{\frac{1}{\complement}}}, \\ e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)(-ln(\sigma_\rho))^{\complement}\right)^{\frac{1}{\complement}}} \end{pmatrix}$$

$$=\left(1-e^{-\left(-ln(1-(\varphi_\rho))^{\complement}\right)^{\frac{1}{\complement}}}, e^{-\left((-ln(\delta_\rho))^{\complement}\right)^{\frac{1}{\complement}}}, e^{-\left((-ln(\sigma_\rho))^{\complement}\right)^{\frac{1}{\complement}}}\right)=\tau$$

Thus, it is obvious that $SVNAAPWA(\tau_1,\tau_2,\dots,\tau_n)=\tau$ holds.

Theorem 11: Let $\tau_\rho=(\varphi_\rho,\delta_\rho,\sigma_\rho), \rho=1,2,3,\dots,n$ be the collection of SVNVs with associated WVs $\Psi=(\Psi_1,\Psi_2,\dots,\Psi_n)^T$ of τ_ρ, $\rho=1,2,3,\dots,n$ such that $\Psi_\rho\in[0,1]$ and $\sum_{\rho=1}^{n}\Psi_\rho=1$ with PA $\omega_\rho=\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\varepsilon_\rho}$ operator for each τ_ρ. Let $\tau^-=min(\tau_1,\tau_2,\dots,\tau_n)$, and $\tau^+=max(\tau_1,\tau_2,\dots,\tau_n)$, then $\tau^-\leq SVNAAPWA(\tau_1,\tau_2,\dots,\tau_n)\leq\tau^+$.

Proof: Let $\tau_\rho = (\varphi_\rho, \delta_\rho, \sigma_\rho), \rho = 1,2,3, \dots, n$ be the collection of SVNVs with associated WVs $\Psi = (\Psi_1, \Psi_2, \dots, \Psi_n)^T$ of τ_ρ, $\rho = 1, 2, 3, \dots, n$ such that $\Psi_\rho \in [0,1]$ and $\sum_{\rho=1}^{n} \Psi_\rho = 1$ with PA $\omega_\rho = \frac{\Psi_\rho \varepsilon_\rho}{\sum_{\rho=1}^{n} \varepsilon_\rho}$ operator for each τ_ρ. Let $\tau^- = min(\tau_1, \tau_2, \dots, \tau_n) = (\varphi^-, \delta^-, \sigma^-)$, and $\tau^+ = max(\tau_1, \tau_2, \dots, \tau_n) = (\varphi^+, \delta^+, \sigma^+)$. We have, $\varphi^- = min_\rho\{\varphi_\rho\}, \delta^- = max_\rho\{\delta_\rho\}$ and $\sigma^- = max_\rho\{\sigma_\rho\}$ and $\varphi^+ = max_\rho\{\varphi_\rho\}, \delta^+ = min_\rho\{\delta_\rho\}$, and $\sigma^+ = min_\rho\{\sigma_\rho\}$. Hence, there is the following result for the inequalities:

$$1 - e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(1-\varphi^-)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \leq 1 - e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(1-\varphi_\rho)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \leq 1 - e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(1-\varphi^+)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}}$$

And,

$$e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(\delta^-)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \leq e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(\delta_\rho)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \leq e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(\delta^+)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}}$$

Similarly,

$$e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(\sigma^-)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \leq e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(\sigma_\rho)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \leq e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(\sigma^+)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}}$$

So, $\tau^- \leq SVNAAPWA(\tau_1, \tau_2, \dots, \tau_n) \leq \tau^+$ holds.

Theorem 12: Let τ_ρ and $\tau'_\rho, \rho = 1, 2, 3, \dots, n$ be two sets of SVNVs, if $\tau_\rho \leq \tau'_\rho$ for all τ. Then, $SVNAAPWA(\tau_1, \tau_2, \dots, \tau_n) \leq SVNAAPWA(\tau'_1, \tau'_2, \dots, \tau'_n)$.

Proof: Let τ_ρ and $\tau'_\rho, \rho = 1, 2, 3, \dots, n$ be two sets of SVNVs, we can say that:

$$1 - e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(1-\varphi_\rho)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \leq 1 - e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(1-\varphi'_\rho)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}}$$

$$e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(\delta_\rho)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \geq e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(\delta'_\rho)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}}$$

Similarly,

$$e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(\sigma_\rho)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}} \geq e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(\sigma'_\rho)\right)^{\text{¢}}\right)^{\frac{1}{\text{¢}}}}$$

Definition 14: Let $\tau_\rho = (\varphi_\rho, \delta_\rho, \sigma_\rho), \rho = 1, 2, 3, \dots, n$ be the collection of SVNVs with associated WVs $\Psi = (\Psi_1, \Psi_2, \dots, \Psi_n)^T$ of τ_ρ, $\rho = 1, 2, 3, \dots, n$ such that $\Psi_\rho \in [0,1]$ and $\sum_{\rho=1}^{n} \Psi_\rho = 1$ with PA $\omega_\rho = \frac{\varepsilon_\rho}{\sum_{\rho=1}^{n} \varepsilon_\rho}$ operator. Then, the SVNAAPWG operator is particularized as:

$$SVNAAPWG(\tau_1, \tau_2, \dots, \tau_n) = \prod_{\rho=1}^{n}\left(\tau_\rho^{\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}}\right) \tag{10}$$

$$SVNAAPWG(\tau_1, \tau_2, \dots, \tau_n) = \tau_1^{\left(\frac{\Psi_1 \varepsilon_1}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)} \otimes \tau_2^{\left(\frac{\Psi_2 \varepsilon_2}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)} \otimes \dots \otimes \tau_n^{\left(\frac{\Psi_n \varepsilon_n}{\sum_{\rho=1}^{n} \Psi_n \varepsilon_n}\right)}$$

Where $\varepsilon_\rho = \prod_{k=1}^{\rho-1} \mathcal{G}(\tau_k), \rho = 2, 3, \dots, n$. The initial value is $\varepsilon_1 = 1$ and $\mathcal{G}(\tau_k)$ be the score value of k^{th} SVNVs.

Theorem 10: Let $\tau_\rho = (\varphi_\rho, \delta_\rho, \sigma_\rho), \rho = 1, 2, 3, \dots, n$ be the collection of SVNVs with WVs $\Psi = (\Psi_1, \Psi_2, \dots, \Psi_n)^T$ and $\Psi_\rho \in [0,1]$, $\sum_{\rho=1}^{n} \Psi_\rho = 1$ associated with PA $\omega_\rho = \frac{\varepsilon_\rho}{\sum_{\rho=1}^{n} \varepsilon_\rho}$ operator. Then, the SVNAAPWG operator is particularized as:

$$SVNAAPWG = \prod_{\rho=1}^{n} \left(\tau_\rho^{\frac{\Psi_\rho \varepsilon_\rho}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}} \right) = \begin{pmatrix} e^{-\left(\sum_{\rho=1}^{n} \left(\frac{\Psi_\rho \varepsilon_\rho}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(\varphi_\rho)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}}, \\ 1 - e^{-\left(\sum_{\rho=1}^{n} \left(\frac{\Psi_\rho \varepsilon_\rho}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(1-\delta_\rho)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}}, \\ 1 - e^{-\left(\sum_{\rho=1}^{n} \left(\frac{\Psi_\rho \varepsilon_\rho}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(1-\sigma_\rho)\right)^{\not{C}}\right)^{\frac{1}{\not{C}}}} \end{pmatrix} \tag{11}$$

Proof: Proof is similar to Theorem 9.

Example 3: Let $\tau_1 = (0.39, 0.69, 0.41), (0.19, 0.43, 0.71), (0.8, 0.37, 0.99)$ and $(0.55, 0.51, 0.25)$ be the four SVNVs with WVs $(0.5307, 0.3423, 0.0599, 0.0671)$ and $\not{C} = 3$. Then, SVNAAPWG can be calculated as:

$$SVNAAPWG = \prod_{\rho=1}^{n} \left(\tau_\rho^{\frac{\Psi_\rho \varepsilon_\rho}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}} \right) = \begin{pmatrix} e^{-\left(\sum_{\rho=1}^{n} \left(\left(\frac{\Psi_\rho \varepsilon_\rho}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(\varphi_\rho)\right)^{\not{C}}\right)\right)^{\frac{1}{\not{C}}}}, \\ 1 - e^{-\left(\sum_{\rho=1}^{n} \left(\left(\frac{\Psi_\rho \varepsilon_\rho}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(1-\delta_\rho)\right)^{\not{C}}\right)\right)^{\frac{1}{\not{C}}}}, \\ 1 - e^{-\left(\sum_{\rho=1}^{n} \left(\left(\frac{\Psi_\rho \varepsilon_\rho}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(1-\sigma_\rho)\right)^{\not{C}}\right)\right)^{\frac{1}{\not{C}}}} \end{pmatrix}$$

$$SVNAAPWG(\tau_1, \tau_2, \tau_3, \tau_4) = \begin{pmatrix} e^{-\begin{pmatrix} \left(\frac{\Psi_1 \varepsilon_1}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(\varphi_1)\right)^{\not{C}} + \left(\frac{\Psi_2 \varepsilon_2}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(\varphi_2)\right)^{\not{C}} \\ + \left(\frac{\Psi_3 \varepsilon_3}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(\varphi_3)\right)^{\not{C}} + \left(\frac{\Psi_4 \varepsilon_4}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(\varphi_4)\right)^{\not{C}} \end{pmatrix}^{\frac{1}{\not{C}}}}, \\ 1 - e^{-\begin{pmatrix} \left(\frac{\Psi_1 \varepsilon_1}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(1-\delta_1)\right)^{\not{C}} + \left(\frac{\Psi_2 \varepsilon_2}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(1-\delta_2)\right)^{\not{C}} + \\ \left(\frac{\Psi_3 \varepsilon_3}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(1-\delta_3)\right)^{\not{C}} + \left(\frac{\Psi_4 \varepsilon_4}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(1-\delta_4)\right)^{\not{C}} \end{pmatrix}^{\frac{1}{\not{C}}}}, \\ 1 - e^{-\begin{pmatrix} \left(\frac{\Psi_1 \varepsilon_1}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(1-\sigma_1)\right)^{\not{C}} + \left(\frac{\Psi_2 \varepsilon_2}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(1-\sigma_2)\right)^{\not{C}} + \\ \left(\frac{\Psi_3 \varepsilon_3}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(1-\sigma_3)\right)^{\not{C}} + \left(\frac{\Psi_4 \varepsilon_4}{\sum_{\rho=1}^{n} \Psi_\rho \varepsilon_\rho}\right)\left(-ln(1-\sigma_4)\right)^{\not{C}} \end{pmatrix}^{\frac{1}{\not{C}}}} \end{pmatrix}$$

$$= \begin{pmatrix} e^{-\left(\begin{matrix}\left(\frac{0.2000}{0.3769}\right)(-ln(0.39))^3+\left(\frac{0.1290}{0.3769}\right)(-ln(0.19))^3+ \\ \left(\frac{0.0226}{0.3769}\right)(-ln(0.8))^3 +\left(\frac{0.0253}{0.3769}\right)(-ln(0.55))^3\end{matrix}\right)^{\frac{1}{3}}}, \\ 1-e^{-\left(\begin{matrix}\left(\frac{0.2000}{0.3769}\right)(-ln(1-0.69))^3+\left(\frac{0.1290}{0.3769}\right)(-ln(1-0.43))^3+ \\ \left(\frac{0.0226}{0.3769}\right)(-ln(1-0.37))^3+\left(\frac{0.0253}{0.3769}\right)(-ln(1-0.51))^3\end{matrix}\right)^{\frac{1}{3}}}, \\ 1-e^{-\left(\begin{matrix}\left(\frac{0.2000}{0.3769}\right)(-ln(1-0.41))^3+\left(\frac{0.1290}{0.3769}\right)(-ln(1-0.71))^3+ \\ \left(\frac{0.0226}{0.3769}\right)(-ln(1-0.99))^3+\left(\frac{0.0253}{0.3769}\right)(-ln(1-0.25))^3\end{matrix}\right)^{\frac{1}{3}}} \end{pmatrix}$$

$$SVNAAPWG(\tau_1,\tau_2,\tau_3,\tau_4)=(0.2830, 0.6250, 0.8465)$$

Theorem 13: If all $\tau_\wp=(\varphi_\wp,\delta_\wp,\sigma_\wp), \wp=1,2,3,\dots,n$ are equal, that is, $\tau_n=\tau$ for all τ. Then, $SVNAAPWG(\tau_1,\tau_2,\dots,\tau_n)=\tau$.

Proof: Since $\tau_\wp=(\varphi_\wp,\delta_\wp,\sigma_\wp), \wp=1,2,3,\dots,n$. Then,

$$SVNAAPWG(\tau_1,\tau_2,\dots,\tau_n)=\prod_{\wp=1}^{n}\left(\tau_\wp^{\frac{\Psi_\wp\varepsilon_\wp}{\sum_{\wp=1}^{n}\Psi_\wp\varepsilon_\wp}}\right)$$

$$SVNAAPWG(\tau_1,\tau_2,\dots,\tau_n)=\begin{pmatrix} e^{-\left(\sum_{\wp=1}^{n}\left(\frac{\Psi_\wp\varepsilon_\wp}{\sum_{\wp=1}^{n}\Psi_\wp\varepsilon_\wp}\right)(-ln(\varphi_\wp))^{\text{₡}}\right)^{\frac{1}{\text{₡}}}}, \\ 1-e^{-\left(\sum_{\wp=1}^{n}\left(\frac{\Psi_\wp\varepsilon_\wp}{\sum_{\wp=1}^{n}\Psi_\wp\varepsilon_\wp}\right)(-ln(1-\delta_\wp))^{\text{₡}}\right)^{\frac{1}{\text{₡}}}}, \\ 1-e^{-\left(\sum_{\wp=1}^{n}\left(\frac{\Psi_\wp\varepsilon_\wp}{\sum_{\wp=1}^{n}\Psi_\wp\varepsilon_\wp}\right)(-ln(1-\sigma_\wp))^{\text{₡}}\right)^{\frac{1}{\text{₡}}}} \end{pmatrix}$$

$$=\left(e^{-\left((-ln(\varphi_{\text{₡}}))^{\text{₡}}\right)^{\frac{1}{\text{₡}}}}, 1-e^{-\left((-ln(1-\delta_{\text{₡}}))^{\text{₡}}\right)^{\frac{1}{\text{₡}}}}, 1-e^{-\left((-ln(1-\sigma_{\text{₡}}))^{\text{₡}}\right)^{\frac{1}{\text{₡}}}}\right)=\tau$$

Thus, it is obvious that $SVNAAPWG(\tau_1,\tau_2,\dots,\tau_n)=\tau$ holds.

Theorem 14: Let $\tau_\wp=(\varphi_\wp,\delta_\wp,\sigma_\wp), \wp=1,2,3,\dots,n$ be the collection of SVNVs with associated WVs $\Psi=(\Psi_1,\Psi_2,\dots,\Psi_n)^T$ of $\tau_\wp$, $\wp=1,2,3,\dots,n$ such that $\Psi_\wp\in[0,1]$ and $\sum_{\wp=1}^{n}\Psi_\wp=1$ with PA $\omega_\wp=\frac{\Psi_\wp\varepsilon_\wp}{\sum_{\wp=1}^{n}\Psi_\wp\varepsilon_\wp}$ operator for each $\tau_\wp$. Let $\tau^-=min(\tau_1,\tau_2,\dots,\tau_n)$ and $\tau^+=max(\tau_1,\tau_2,\dots,\tau_n)$. Then, $\tau^-\le SVNAAPWG(\tau_1,\tau_2,\dots,\tau_n)\le\tau^+$.

Proof: Let $\tau_\wp=(\varphi_\wp,\delta_\wp,\sigma_\wp), \wp=1,2,3,\dots,n$ be the collection of SVNVs with associated WVs $\Psi=(\Psi_1,\Psi_2,\Psi_3,\dots,\Psi_n)^T$ of $\tau_\wp$, $\wp=1,2,3,\dots,n$ such that $\Psi_\wp\in[0,1]$ and $\sum_{\wp=1}^{n}\Psi_\wp=1$ with PA $\omega_\wp=\frac{\Psi_\wp\varepsilon_\wp}{\sum_{\wp=1}^{n}\Psi_\wp\varepsilon_\wp}$ operator for each $\tau_\wp$. Let $\tau^-=min(\tau_1,\tau_2,\dots,\tau_n)=(\varphi^-,\delta^-,\sigma^-)$, and $\tau^+=max(\tau_1,\tau_2,\dots,\tau_n)=(\varphi^+,\delta^+,\sigma^+)$. We have, $\varphi^-=min_\wp\{\varphi_\wp\}$, $\delta^-=max_\wp\{\delta_\wp\}$ and $\sigma^-=max_\wp\{\sigma_\wp\}$ and $\varphi^+=max_\wp\{\varphi_\wp\}$, $\delta^+=min_\wp\{\delta_\wp\}$, and $\sigma^+=min_\wp\{\sigma_\wp\}$. Hence, there is the following result for the inequalities:

$$e^{-\left(\sum_{\wp=1}^{n}\left(\frac{\Psi_\wp\varepsilon_\wp}{\sum_{\wp=1}^{n}\Psi_\wp\varepsilon_\wp}\right)(-ln(\varphi^-))^{\text{₡}}\right)^{\frac{1}{\text{₡}}}}\le e^{-\left(\sum_{\wp=1}^{n}\left(\frac{\Psi_\wp\varepsilon_\wp}{\sum_{\wp=1}^{n}\Psi_\wp\varepsilon_\wp}\right)(-ln(\varphi_\wp))^{\text{₡}}\right)^{\frac{1}{\text{₡}}}}\le e^{-\left(\sum_{\wp=1}^{n}\left(\frac{\Psi_\wp\varepsilon_\wp}{\sum_{\wp=1}^{n}\Psi_\wp\varepsilon_\wp}\right)(-ln(\varphi^+))^{\text{₡}}\right)^{\frac{1}{\text{₡}}}}$$

$$1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(1-\delta^{-}\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}} \leq 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(1-\delta_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}}$$
$$\leq 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(1-\delta^{+}\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}}$$

$$1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(1-\sigma^{-}\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}} \leq 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(1-\sigma_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}}$$
$$\leq 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(1-\sigma^{+}\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}}$$

So, $\tau^{-} \leq SVNAAPWG(\tau_1, \tau_2, \dots, \tau_n) \leq \tau^{+}$ holds.

Theorem 15: Let τ_ρ and $\tau'_\rho, \rho = 1,2,3, \dots, n$ be two sets of SVNVs, if $\tau_\rho \leq \tau'_\rho$ for all τ. Then, $SVNAAPWG(\tau_1, \tau_2, \dots, \tau_n) \leq SVNAAPWG(\tau'_1, \tau'_2, \dots, \tau'_n)$.

Proof: Let τ_ρ and $\tau'_\rho, \rho = 1,2,3, \dots, n$ be two sets of SVNVs. Then, we use the following way to prove it:

$$e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(\varphi_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}} \leq e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(\varphi'_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}}$$

And,

$$1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(1-\delta_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}} \geq 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(1-\delta'_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}}$$

In the same way,

$$1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(1-\sigma_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}} \geq 1-e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln\left(1-\sigma'_\rho\right)\right)^{\mathbb{C}}\right)^{\frac{1}{\mathbb{C}}}}$$

From the above, we can conclude that:

$$SVNAAPWG(\tau_1, \tau_2, \dots, \tau_n) \leq SVNAAPWG(\tau'_1, \tau'_2, \dots, \tau'_n)$$

6. MADM Techniques of SVNAAPWA and SVNAAPWG Operations

In this section, we shall use the SVNAAPWA and SVNAAPWG operators to solve the MADM technique by using the information of SVNVs. Suppose that $ħ = (ħ_1, ħ_2, \dots, ħ_n)$ be the set of alternatives and $ß = (ß_1, ß_2, ß_3, \dots, ß_n)$ be the set of attributes with the degree of weights $\Psi = (\Psi_1, \Psi_2, \dots, \Psi_n)^T, \rho = (1, 2, 3, \dots, n)$ such that $\Psi_\rho \in [0,1]$ and $\sum_{\rho=1}^{n}\Psi_\rho = 1$. The decision maker also explores the theory of prioritization between attributes which is represented as linear ordering $\Sigma_1 > \Sigma_2 > \dots > \Sigma_n$. The following decision matrix $Ř = (Y_{\eta\rho})_{\kappa\times ή}$ contained information in the form of SVNVs.

$$Ř = (Y_{\eta\rho})_{\varkappa\times ή} = \begin{pmatrix} (\varphi_{\tau_{11}}, \delta_{\tau_{11}}, \sigma_{\tau_{11}}) & (\varphi_{\tau_{12}}, \delta_{\tau_{12}}, \sigma_{\tau_{12}}) & \cdots & (\varphi_{\tau_{1n}}, \delta_{\tau_{1n}}, \sigma_{\tau_{1ή}}) \\ (\varphi_{\tau_{21}}, \delta_{\tau_{21}}, \sigma_{\tau_{21}}) & (\varphi_{\tau_{22}}, \delta_{\tau_{22}}, \sigma_{\tau_{22}}) & \cdots & (\varphi_{\tau_{2n}}, \delta_{\tau_{2n}}, \sigma_{\tau_{2ή}}) \\ \vdots & \vdots & \ddots & \vdots \\ (\varphi_{\tau_{\varkappa1}}, \delta_{\tau_{\varkappa1}}, \sigma_{\tau_{\varkappa1}}) & (\varphi_{\tau_{\varkappa2}}, \delta_{\tau_{\varkappa2}}, \sigma_{\tau_{\varkappa2}}) & \vdots & (\varphi_{\tau_{\varkappa ή}}, \delta_{\tau_{\varkappa ή}}, \sigma_{\tau_{\varkappa ή}}) \end{pmatrix}$$

In this decision matrix $\left(\varphi_{\tau_{\varkappa\hat{\eta}}}, \delta_{\tau_{\varkappa\hat{\eta}}}, \sigma_{\tau_{\varkappa\hat{\eta}}}\right)$ represents the value of SVNV and $\varphi_{\tau_{\varkappa\hat{\eta}}} \in [0,1]$, $\delta_{\tau_{\varkappa\hat{\eta}}} \in [0,1]$ and $\sigma_{\tau_{\varkappa\hat{\eta}}} \in [0,1]$ such that $0 \leq \varphi_{\tau_{\varkappa\hat{\eta}}} + \delta_{\tau_{\varkappa\hat{\eta}}} + \sigma_{\tau_{\varkappa\hat{\eta}}} \leq 3$. There are two kinds of attributes: cost factor and beneficial factor. If the cost factor is involved in the decision matrix, then the decision matrix transforms to normalize matrix:

$$\check{R} = \left(Y_{\eta\rho}\right)_{\varkappa\times\hat{\eta}} = \begin{cases} \left(\varphi_{\tau_{\varkappa\hat{\eta}}}, \delta_{\tau_{\varkappa\hat{\eta}}}, \sigma_{\tau_{\varkappa\hat{\eta}}}\right) & if \quad benefit \quad factor \\ \left(\varphi_{\tau_{\varkappa\hat{\eta}}}, \delta_{\tau_{\varkappa\hat{\eta}}}, \sigma_{\tau_{\varkappa\hat{\eta}}}\right) & if \quad cost \quad factor \end{cases}$$

Now, we will describe the following steps of the algorithm for solving given a MADM technique by the decision maker.

6.1 Algorithm

Step 1: In the first step, the decision maker collects information and arranges a decision matrix under the system of SVNVs.

Step 2: We must convert the decision matrix into a normalizer matrix if the cost factor involves in the set of attributes; otherwise, there is no need.

Step 3: We utilized our proposed methodologies to solve a MADM technique by using the SVNAAWA and SVNAAWG operators.

Step 4: Shows the results of SVNAAWA and SVNAAWG operators in a table.

Step 5: Calculate score values by using the consequences of SVNAAWA and SVNAAWG operators. We evaluate suitable alternatives after ranking and ordering of the score values.

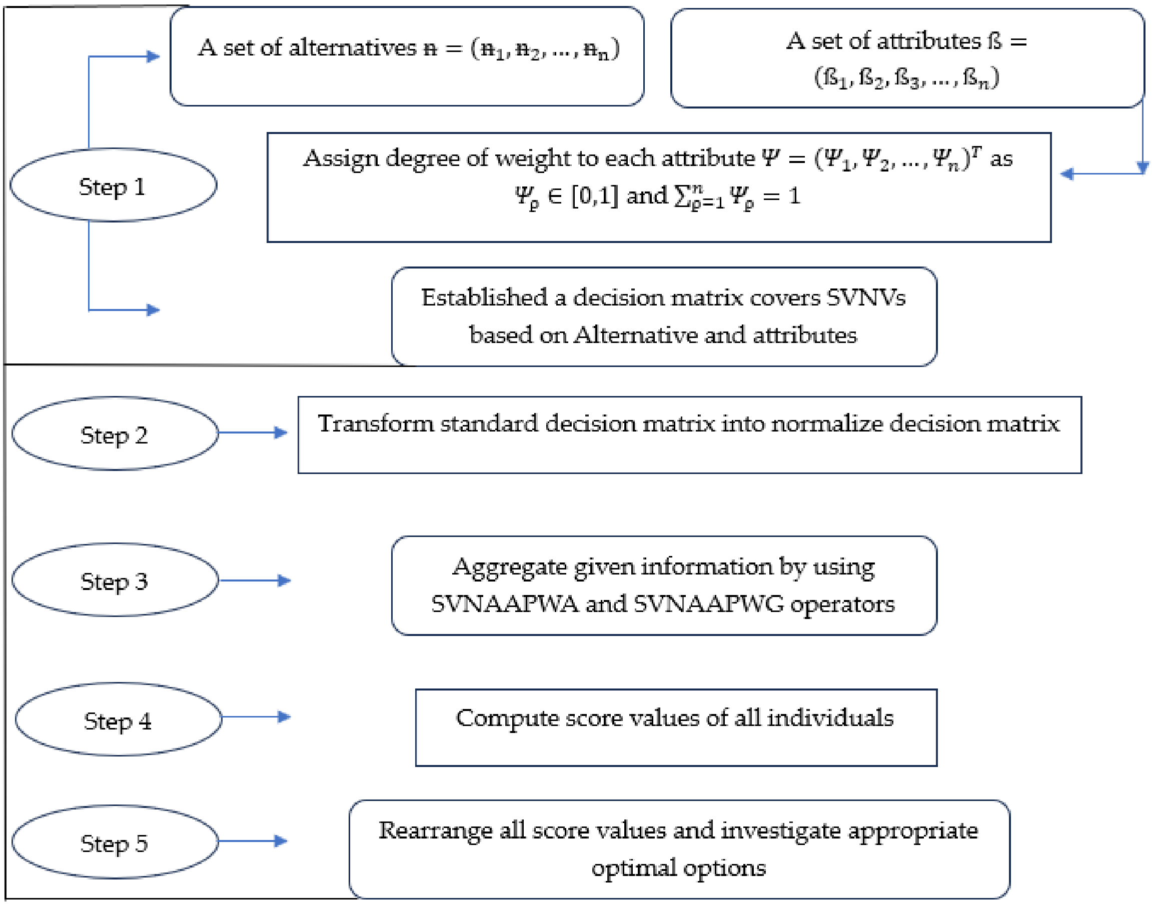

Figure 1 Flow chart of an algorithm.

6.2. Application

Research scientists are present in various alternative domains, such as mathematics, chemistry, biology, software engineering, environmental science, medicine, nano technology, human science, history, political science and so on. They develop a conceptual model for collecting information, and findings respond to inquiries about individuals and the universe. Research scientists are employed by various institutions, including universities and colleges, government agencies, organizations, and businesses engaged in production and innovation. Research scientists generally hold master's or doctoral degrees in their respective professions. Most research scientists hold postgraduate degrees in their specialized disciplines. While master's degrees are frequently sufficient for employment in the general financial industry, PhDs are typically necessary for research scientist careers at colleges and universities. Research scientists are generally interested. Their task involves analytical skills and sensitive, caring attention in order to put up a repeatable approach and recommend the right results. For their discoveries to be communicated in publications and oral presentations, research scientists must be effective communicators and editors.

6.3. Numerical Example

Consider a public university wanted to fill its vacant post with a research scientist, and the selection committee selects from five different applicants $\text{ñ}_i = (\text{ñ}_1, \text{ñ}_2, \ldots, \text{ñ}_n)$ based on the following four characteristics. ß_1 : represents the qualification/ academic history, ß_2: represents the publication and its citations, $ß_3$: experience in teaching related to the research field, $ß_4$: Personality/ digital skills/ communication skills/ moral value.

The decision maker selects a suitable candidate under above-discussed characteristics. Consider WVs $\Psi = (0.20, 0.30, 0.15, 0.35)$ associated with the collected information in the form of SVNVs. We aggregated information given by the decision maker in Table 2 by following the steps of the algorithm.

Table 2 shows the information in the form of SNNVSs given by the decision maker.

	$ß_1$	$ß_2$	$ß_3$	$ß_4$	$ß_5$
$ħ_1$	$(0.23, 0.45, 0.56)$	$(0.66, 0.65, 0.78)$	$(0.12, 0.97, 0.32)$	$(0.21, 0.78, 0.78)$	$(0.65, 0.54, 0.76)$
$ħ_2$	$(0.67, 0.78, 0.98)$	$(0.89, 0.12, 0.32)$	$(0.99, 0.88, 0.76)$	$(0.23, 0.32, 0.71)$	$(0.65, 0.55, 0.61)$
$ħ_3$	$(0.8, 0.39, 0.19)$	$(0.12, 0.34, 0.54)$	$(0.33, 0.9, 0.1)$	$(0.62, 0.56, 0.69)$	$(0.78, 0.61, 0.32)$
$ħ_4$	$(0.7, 0.39, 0.88)$	$(0.78, 0.1, 0.2)$	$(0.2, 0.4, 0.5)$	$(0.11, 0.77, 0.19)$	$(0.77, 0.22, 0.11)$

Step 1: The information gathered by the decision-maker using the system of SVNVs is represented in Table 2.

Step 2: As no cost factor is included in the attributes set data, we have not transformed the decision matrix into the normalized matrix.

Step 3: Applied the techniques of SVNAAPWA and SVNAAPWG operators to aggregate information given by the decision maker, which is depicted in Table 2.

$$SVNAAPWA = \sum_{\rho=1}^{n}\left(\frac{\Psi_\rho \varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\tau_\rho\right) = \begin{pmatrix} 1 - e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(1-\varphi_\rho)\right)^{₡}\right)^{\frac{1}{₡}}}, \\ e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(\delta_\rho)\right)^{₡}\right)^{\frac{1}{₡}}}, \\ e^{-\left(\sum_{\rho=1}^{n}\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(\sigma_\rho)\right)^{₡}\right)^{\frac{1}{₡}}} \end{pmatrix}$$

And,

$$SVNAAPWG = \prod_{\rho=1}^{n}\left(\tau_\rho^{\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}}\right) = \begin{pmatrix} e^{-\left(\sum_{\rho=1}^{n}\left(\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(\varphi_\rho)\right)^{₡}\right)\right)^{\frac{1}{₡}}}, \\ 1 - e^{-\left(\sum_{\rho=1}^{n}\left(\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(1-\delta_\rho)\right)^{₡}\right)\right)^{\frac{1}{₡}}}, \\ 1 - e^{-\left(\sum_{\rho=1}^{n}\left(\left(\frac{\Psi_\rho\varepsilon_\rho}{\sum_{\rho=1}^{n}\Psi_\rho\varepsilon_\rho}\right)\left(-ln(1-\sigma_\rho)\right)^{₡}\right)\right)^{\frac{1}{₡}}} \end{pmatrix}$$

Table 3 shows the results of our proposed work.

SVNAAPWA	SVNAAPWG
$(\mathbf{0.6067}, \mathbf{0.4812}, \mathbf{0.5002})$	$(0.2768, 0.6612, 0.9359)$
$(\mathbf{0.7990}, \mathbf{0.1950}, \mathbf{0.3747})$	$(0.3459, 0.5610, 0.6983)$
$(\mathbf{0.9468}, \mathbf{0.6972}, \mathbf{0.3062})$	$(0.1510, 0.9553, 0.6059)$
$(\mathbf{0.3138}, \mathbf{0.4944}, \mathbf{0.5376})$	$(0.2250, 0.7457, 0.7587)$

$(\mathbf{0.6877, 0.4317, 0.3256})$	$(0.5387, 0.5375, 0.6947)$

Step 4: After the computation of information using our proposed methodologies, we displayed all outcomes in Table 3.

Step 5: Investigate the results of the score values obtained by SVNAAPWA and SVNAAPWG operators shown in Table 3. Invested results of all individual by the SVNAAPWA and SVNAAPWG operators listed in Table 4.

Table 4 shows the score values of AOs of SVNAAPWA and SVNAAPWG

Operators	$\mathcal{G}(ꞥ_1)$	$\mathcal{G}(ꞥ_2)$	$\mathcal{G}(ꞥ_3)$	$\mathcal{G}(ꞥ_4)$	$\mathcal{G}(ꞥ_5)$	Ranking and ordering
SVNAAPWA	0.5418	0.7431	0.6478	0.4273	0.6435	$ꞥ_2 > ꞥ_3 > ꞥ_5 > ꞥ_1 > ꞥ_4$
SVNAAPWG	0.2266	0.3622	0.1966	0.2402	0.4355	$ꞥ_5 > ꞥ_2 > ꞥ_4 > ꞥ_1 > ꞥ_3$

We noticed the ranking and ordering of score values $ꞥ_2 > ꞥ_5 > ꞥ_3 > ꞥ_1 > ꞥ_4$ and $ꞥ_2 > ꞥ_5 > ꞥ_1 > ꞥ_3 > ꞥ_4$ for SVNAAPWA and SVNAAPWG, respectively. $ꞥ_2$ is a suitable applicant for the vecant post. Similarly, $ꞥ_5$ is the best applicants for a research scientist of a public university. We also show the outcomes of the score values acquired from the SVNAAWA and SVNAAWG operators as a graphical representation of the following Figure 2.

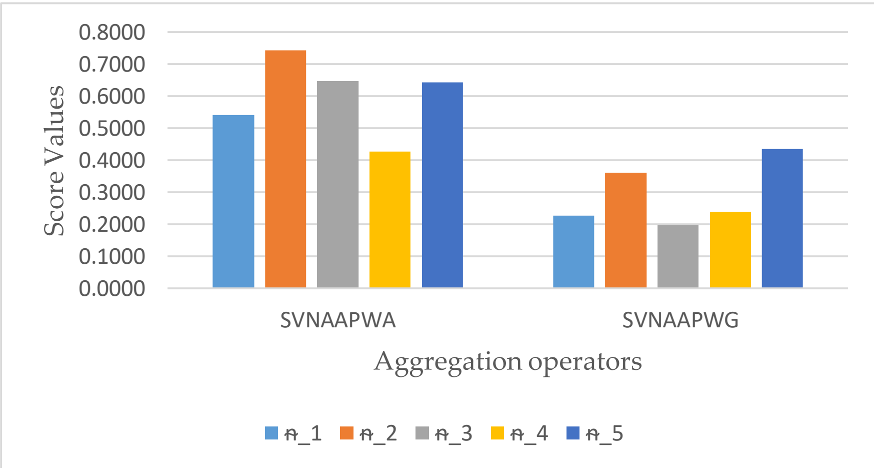

Figure 2 Covers the geometrical representation of all score values, which are listed in Table 4.

6.4. Behavior of Different Parameters of ₡ on our Purposed Methodologies

We modified several values of ₡ in step 4 of the recommended MADM approach to explore the impact of different parameter values ₡ on the ranking of all alternatives. The derived outcomes are displayed in Tables 5-6. From Table 5, we noticed when the valued of ₡ increases, score values gained through the SVNAAPWA and SVNAAPWG operators also increase. Moreover, we noticed that the ranking and ordering sequence of the score values remain the same when we change the parametric values of ₡ for our invented approaches SVNAAPWA and SVNAAPWG operators. To see this increasing sequence of the parameter value ₡ and outcomes obtained from our discussed approaches is shown the isotonicity property.

Table 5 shows the results of SVNAAPWA operators for the variation of Ȼ.

	$\mathcal{G}(ꞥ_1)$	$\mathcal{G}(ꞥ_2)$	$\mathcal{G}(ꞥ_3)$	$\mathcal{G}(ꞥ_4)$	$\mathcal{G}(ꞥ_5)$	Ranking and Ordering
Ȼ = 1	0.4352	0.6502	0.4841	0.2984	0.5522	$ꞥ_2 > ꞥ_5 > ꞥ_3 > ꞥ_1 > ꞥ_4$
Ȼ = 3	0.5418	0.7431	0.6478	0.4273	0.6435	$ꞥ_2 > ꞥ_3 > ꞥ_5 > ꞥ_1 > ꞥ_4$
Ȼ = 25	0.7041	0.8436	0.8061	0.6675	0.7905	$ꞥ_2 > ꞥ_3 > ꞥ_5 > ꞥ_1 > ꞥ_4$
Ȼ = 75	0.7281	0.8566	0.8222	0.6915	0.8075	$ꞥ_2 > ꞥ_3 > ꞥ_5 > ꞥ_1 > ꞥ_4$
Ȼ = 105	0.7315	0.8585	0.8244	0.6949	0.8101	$ꞥ_2 > ꞥ_3 > ꞥ_5 > ꞥ_1 > ꞥ_4$
Ȼ = 155	0.7343	0.8601	0.8262	0.6976	0.8122	$ꞥ_2 > ꞥ_3 > ꞥ_5 > ꞥ_1 > ꞥ_4$
Ȼ = 201	0.7356	0.8608	0.8271	0.6989	0.8132	$ꞥ_2 > ꞥ_3 > ꞥ_5 > ꞥ_1 > ꞥ_4$
Ȼ = 255	0.7365	0.8614	0.8277	0.6999	0.8139	$ꞥ_2 > ꞥ_3 > ꞥ_5 > ꞥ_1 > ꞥ_4$
Ȼ = 275	0.7368	0.8615	0.8279	0.7001	0.8141	$ꞥ_2 > ꞥ_3 > ꞥ_5 > ꞥ_1 > ꞥ_4$
Ȼ = 321	0.7372	0.8618	0.8282	0.7006	0.8145	$ꞥ_2 > ꞥ_3 > ꞥ_5 > ꞥ_1 > ꞥ_4$
Ȼ = 375	0.7376	0.8620	0.8284	0.7010	0.8148	$ꞥ_2 > ꞥ_3 > ꞥ_5 > ꞥ_1 > ꞥ_4$
Ȼ = 421	0.7379	0.8621	0.8286	0.7012	0.8150	$ꞥ_2 > ꞥ_3 > ꞥ_5 > ꞥ_1 > ꞥ_4$
Ȼ = 463	0.7381	0.8623	0.8287	0.7014	0.8152	$ꞥ_2 > ꞥ_3 > ꞥ_5 > ꞥ_1 > ꞥ_4$

Table 6 shows the results of SVNAAPWG operators for the variation of Ȼ.

	$\mathcal{G}(ꞥ_1)$	$\mathcal{G}(ꞥ_2)$	$\mathcal{G}(ꞥ_3)$	$\mathcal{G}(ꞥ_4)$	$\mathcal{G}(ꞥ_5)$	Ranking and Ordering
Ȼ = 1	0.3007	0.5041	0.2636	0.2610	0.4807	$ꞥ_2 > ꞥ_5 > ꞥ_1 > ꞥ_3 > ꞥ_4$
Ȼ = 3	0.2266	0.3622	0.1966	0.2402	0.4355	$ꞥ_5 > ꞥ_2 > ꞥ_4 > ꞥ_1 > ꞥ_3$
Ȼ = 25	0.1591	0.2445	0.1366	0.2212	0.3423	$ꞥ_5 > ꞥ_2 > ꞥ_4 > ꞥ_1 > ꞥ_3$
Ȼ = 75	0.1489	0.2348	0.1285	0.2183	0.3254	$ꞥ_5 > ꞥ_2 > ꞥ_4 > ꞥ_1 > ꞥ_3$
Ȼ = 105	0.1473	0.2334	0.1271	0.2179	0.3229	$ꞥ_5 > ꞥ_2 > ꞥ_4 > ꞥ_1 > ꞥ_3$
Ȼ = 155	0.1460	0.2323	0.1258	0.2175	0.3209	$ꞥ_5 > ꞥ_2 > ꞥ_4 > ꞥ_1 > ꞥ_3$
Ȼ = 201	0.1454	0.2318	0.1253	0.2173	0.3199	$ꞥ_5 > ꞥ_2 > ꞥ_4 > ꞥ_1 > ꞥ_3$
Ȼ = 255	0.1450	0.2314	0.1249	0.2172	0.3192	$ꞥ_5 > ꞥ_2 > ꞥ_4 > ꞥ_1 > ꞥ_3$
Ȼ = 275	0.1448	0.2313	0.1247	0.2171	0.3190	$ꞥ_5 > ꞥ_2 > ꞥ_4 > ꞥ_1 > ꞥ_3$
Ȼ = 321	0.1446	0.2311	0.1245	0.2171	0.3187	$ꞥ_5 > ꞥ_2 > ꞥ_4 > ꞥ_1 > ꞥ_3$
Ȼ = 375	0.1444	0.2309	0.1244	0.2170	0.3184	$ꞥ_5 > ꞥ_2 > ꞥ_4 > ꞥ_1 > ꞥ_3$
Ȼ = 421	0.1443	0.2308	0.1243	0.2170	0.3182	$ꞥ_5 > ꞥ_2 > ꞥ_4 > ꞥ_1 > ꞥ_3$
Ȼ = 463	0.1442	0.2308	0.1242	0.2169	0.3181	$ꞥ_5 > ꞥ_2 > ꞥ_4 > ꞥ_1 > ꞥ_3$

Further, we explored all the results obtained by the SVNAAPWA and SVNAAPWG operators in the graphical representation of Figure 3 and Figure 4.

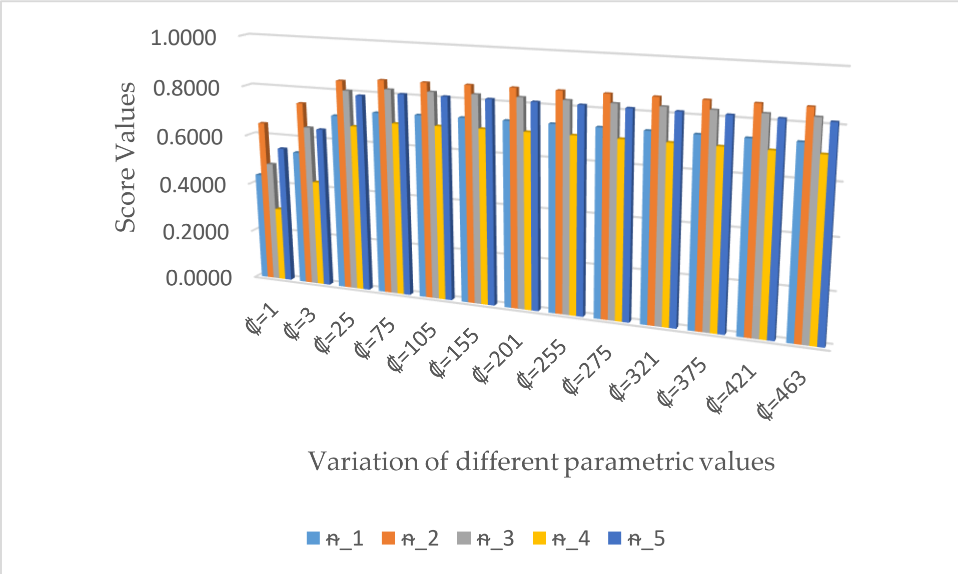

Figure 3 Graphical representation of score values depicted in Table 5.

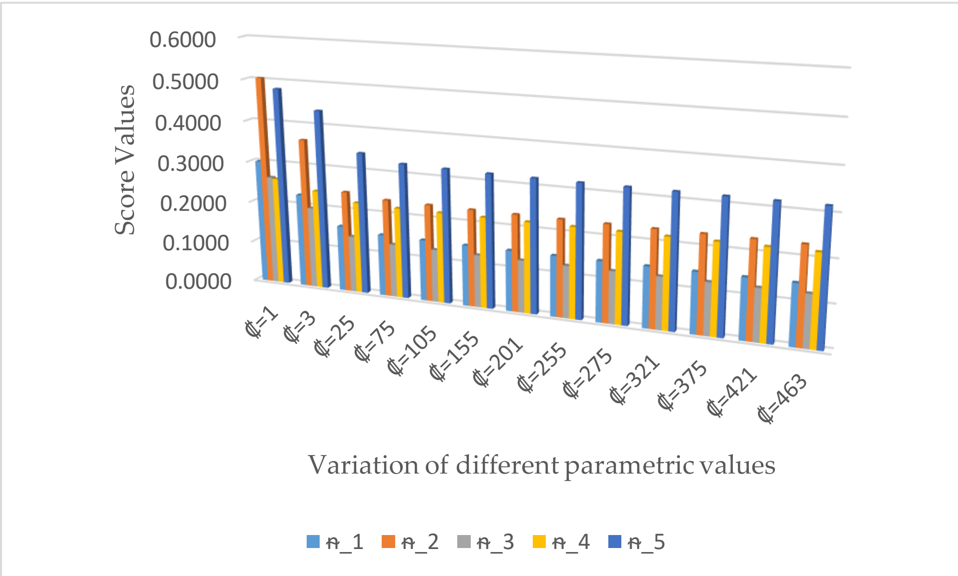

Figure 4 Graphical representation of score values depicted in Table 6.

7. Comparative Study

To show the effectiveness and applacability of our discussed approaches, we make a comparison of the outcomes of the current discussed approaches with the consequences of the existing approaches. For this purpose, we utilized a few numbers of used AOs on the data of SVNVs presented by the decision maker and shown in Table 2. AOs of SVN Dombi weighted average and SVN Dombi weighted geometric operators anticipated by Chen and Ye [31], AOs of SVN weighted average and SVN weighted geometric operators presented by Peng et al. [51], AOs of SVN Einstein weighted average and SVN Einstein weighted geometric operators anticipated by the Ye et al. [52], and AOs of complex SVNVs (CSVNVs) based on Prioritized Muirhead Mean tools given by the Mahmood and

Ali [32]. All the results obtained by the existing AOs [31], [32], [51], [52] are shown in the following Table 7.

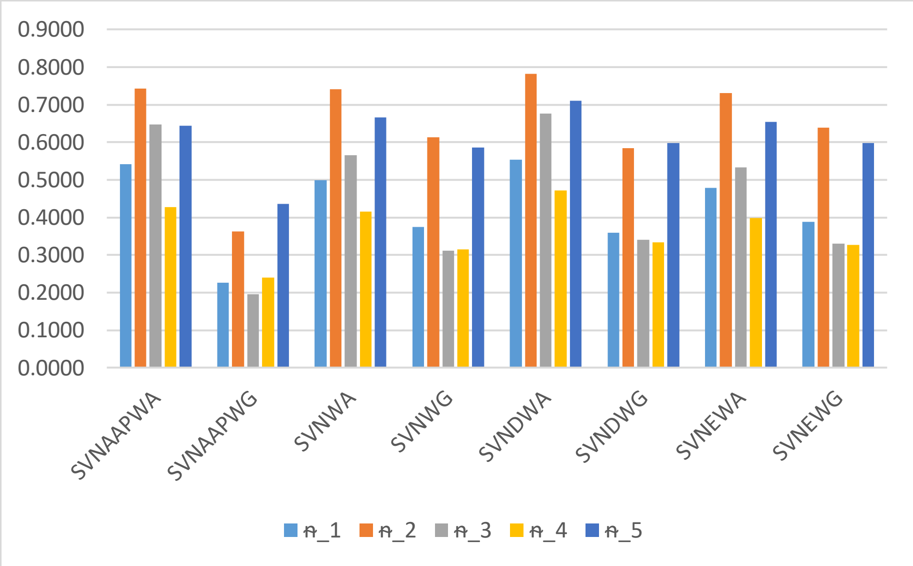

Figure 5. Shows the results of the comparative study in a graphical representation.

Table 7. Shows the results of a comparative study.

AOs	Score values	Ranking and ordering
Current work	$\mathcal{G}(ɦ_1) = 0.5418, \mathcal{G}(ɦ_2) = 0.7431, \mathcal{G}(ɦ_3) = 0.6478, \mathcal{G}(ɦ_4) = 0.4273, \mathcal{G}(ɦ_5) = 0.6435$	$ɦ_2 > ɦ_3 > ɦ_5 > ɦ_1 > ɦ_4$
Current work	$\mathcal{G}(ɦ_1) = 0.2266, \mathcal{G}(ɦ_2) = 0.3622, \mathcal{G}(ɦ_3) = 0.1966, \mathcal{G}(ɦ_4) = 0.2402, \mathcal{G}(ɦ_5) = 0.4355$	$ɦ_5 > ɦ_2 > ɦ_4 > ɦ_1 > ɦ_3$
SVNWA **[51]**	$\mathcal{G}(ɦ_1) = 0.4985, \mathcal{G}(ɦ_2) = 0.7415, \mathcal{G}(ɦ_3) = 0.5654, \mathcal{G}(ɦ_4) = 0.4160, \mathcal{G}(ɦ_5) = 0.6655$	$ɦ_2 > ɦ_5 > ɦ_3 > ɦ_1 > ɦ_4$
SVNWG **[51]**	$\mathcal{G}(ɦ_1) = 0.3751, \mathcal{G}(ɦ_2) = 0.6136, \mathcal{G}(ɦ_3) = 0.3120 , \mathcal{G}(ɦ_4) = 0.3158, \mathcal{G}(ɦ_5) = 0.5862$	$ɦ_2 > ɦ_5 > ɦ_1 > ɦ_3 > ɦ_4$
SVNDWA **[31]**	$\mathcal{G}(ɦ_1) = 0.5539, \mathcal{G}(ɦ_2) = 0.7815, \mathcal{G}(ɦ_3) = 0.6759, \mathcal{G}(ɦ_4) = 0.4714, \mathcal{G}(ɦ_5) = 0.7115$	$ɦ_2 > ɦ_5 > ɦ_3 > ɦ_1 > ɦ_4$
SVNDWG **[31]**	$\mathcal{G}(ɦ_1) = 0.3597, \mathcal{G}(ɦ_2) = 0.5842, \mathcal{G}(ɦ_3) = 0.3416 , \mathcal{G}(ɦ_4) = 0.3346, \mathcal{G}(ɦ_5) = 0.5980$	$ɦ_2 > ɦ_5 > ɦ_1 > ɦ_3 > ɦ_4$
SVNEWA **[52]**	$\mathcal{G}(ɦ_1) = 0.4797, \mathcal{G}(ɦ_2) = 0.7309, \mathcal{G}(ɦ_3) = 0.5333 , \mathcal{G}(ɦ_4) = 0.3979, \mathcal{G}(ɦ_5) = 0.6549$	$ɦ_2 > ɦ_5 > ɦ_3 > ɦ_1 > ɦ_4$
SVNEWG **[52]**	$\mathcal{G}(ɦ_1) = 0.3882, \mathcal{G}(ɦ_2) = 0.6397, \mathcal{G}(ɦ_3) = 0.3313 , \mathcal{G}(ɦ_4) = 0.3271, \mathcal{G}(ɦ_5) = 0.5977$	$ɦ_2 > ɦ_5 > ɦ_1 > ɦ_4 > ɦ_3$
Mahmood and Ali **[32]**	CSVNVs	Failed

From Table 7, we examined the results of existing approaches and concluded that invented methodologies are superior to other ones. Due to the parametric value of Aczel Alsina aggregation tools, Decision makers can acquire results of score values according to their preferences by setting

different parametric values of Aczel Alsina aggregation tools. We also observed the consistency and effectiveness of our invented approaches in Tables 5-6.
Following graphical representation shows the results of existing approaches obtained by the decision matrix of Table 2 and shown in Figure 5.

8. Conclusion

The decision information is more appropriately described in terms of SVNVs during the decision-making process due to the increasing uncertainties and complexity of practical situations. In this article, we exposed the notion of SVNSs to cope with ambiguous and vague information about human opinions. The SVNS is the modified version of an IFSs and PFSs, which provides freedom to decision-makers in the decision-making process and contains more extensive information than other frameworks of fuzzy systems. Aczel Alsina aggregation tools are superior to other aggregation tools. By using the theory of Aczel Alsina aggregation tools, we proposed a class of new approaches based on SVN information, including SVNAAPA and SVNAAPG operators. We also generalized the theory of SVNSs with properties of Aczel Alsina aggregation tools and presented a series of new approaches like SVNAAPWA and SVNAAPWG operators. To reveal the intensity and effectiveness of our invented methodologies, some notable characteristics are also explored. We established an algorithm for the MADM problem under the system of SVN information. We discussed a numerical example to find the most appropriate candidate for the vacant post of a general manger for the multinational company. To find the validity and flexibility of our methods, we evaluated the effects of the results on the alternatives for several parametric values. The advantages of our presented methodologies are also presented by comparing the findings of existing approaches with currently proposed AOs.

Sometimes decision-makers cannot find an appropriate optimal option due to insufficient information about weight vectors. We can use the concepts of power operators and entropy measures to handle this situation. We also apply our invented approaches to resolve different applications such as artificial intelligence, game theory, waste management, and social selection. Furthermore, we will explore our invented approaches in the framework of the bipolar soft set [53], [54], picture fuzzy sets [55], spherical fuzzy sets, and complex spherical fuzzy sets [56]. Next, we will apply our invented approaches to improve the healthcare system's reliability and establish a strong model for the waste materials under the system of NS [57].

Furthermore, we also attached a list of variables used throughout this article.

Symbols	Meanings	Symbols	Meanings
X	Non-empty set	$\mathfrak{Q}$	Accuracy function
φ	PMV	$\boldsymbol{\Psi}$	Weight vector
δ	AMV	ß	Attribute
σ	NMV	ћ	Alternative

$\mathscr{r}$	Element from non-empty set	$\not\subset$	Parametric values
α	SVNV	$\mathcal{G}$	Score function

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