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Solving Neutrosophic Fuzzy Transportation Problem Of Type-II

V.Hemalatha[1],M. Mullai[2,*], P.K.Santhi[3], R.Meenakshi[4]

[1] Department of Mathematics,Cardamom Planters Association College, Bodinayakanur, Tamilnadu, India; hemakathir07@gmail.com

[2,*] Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India; mullaim@alagappauniversity.ac.in

[3]Department of Mathematics, Jayaraj Annapackiam College for Women (Autonomous), Periyakulam, Theni(Dist)Tamilnadu, India; pksanthimat@annejac.ac.in

[4] Department of Mathematics, Seethalakshmi Achi College for Women, Pallathur, Tamilnadu, India; meenakshi.sac@rediffmail.com

[*]Correspondence: mullaim@alagappauniversity.ac.in;

Abstract. Transportation problems offer a structured approach to optimize the allocation of resources, minimize transportation costs, and improve overall efficiency in supply chain and logistics management, leading to several advantages for businesses and organizations. Fuzzy transportation problems are particularly relevant in supply chain and logistics management when dealing with uncertain demand, fluctuating costs, or imprecise data and the intuitionistic fuzzy transportation problem is a more advanced modeling technique that takes into account the nuanced handling of uncertainty and imprecision using intuitionistic fuzzy sets(IFS). It provides a more realistic approach to decision-making in situations where classical or fuzzy models may not capture the subtleties of uncertainty in data. In this article, we demonstrate a novel approach to resolving transportation problems in a neutrosophic atmosphere. Neutrosophic set is an extension of fuzzy and IFS and it is classified by three independent membership grades: truth, indeterminacy, and falsity membership grade. These sets are better suited to handle imprecise parameters. Transportation cost and demand are taken as neutrosophic numbers. Vogel's approximation method is used to get the optimum solution of this neutrosophic transportation problem. Also, we performed a numerical instance to figure out the successful outcome of our suggested technique.

Keywords: Neutrosophic Sets; Neutrosophic Triangle Fuzzy Numbers; Ordering of Triangle Fuzzy Numbers; Neutrosophic Minimum Total Cost; Vogel's Approximation Method.

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1. Introduction

In the current landscape of intense market competition, several firms are actively seeking more effective strategies to enhance their ability to generate and provide value to their consumers, therefore fortifying their overall position. The task of efficiently and securely delivering items to clients while minimizing costs has grown more complex. In order to address this formidable task, transportation models provide a robust foundation. The optimization issue discussed is well recognized within the field of operational research and was first formulated by Hitchcock in 1941.The primary objective of the transportation problem is to ascertain the optimal shipment schedule that reduces the overall shipping cost, while simultaneously meeting the constraints of supply limitations and demand needs. The classical transportation problem pertains to a distinct category of linear programming problems.

In [1-8], many authors developed the concept of transportation in fuzzy, intuitionistic fuzzy and neutrosophic environments. According to these findings, in this paper, alternative simple methods are proposed for solving neutrosophic fuzzy transportation promblems and for solving neutrosophic fully fuzzy transportation problems. Vogel's approximation method is used to find initial basic feasible solution for neutrosophic fuzzy and neutrosophic fully fuzzy transportation problems.

The transportation problem, while the cost for shipping a single unit of a good from a particular source to a target is quantified by neutrosophic numbers, however the availability and demand can be illustrated with real numbers, is usually referred to as the neutrosophic transportation problem.

The transportation problem, when the characteristics such as the cost of transmitting a unit amount of a product from a particular source to a specific destination, the availability, and the demand, are presented as neutrosophic numbers, is commonly known as the neutrosophic fully fuzzy transportation problem. Problem pertaining to the distribution of goods and services, wherein the cost involved in transporting a singular unit of a particular item from a designated origin to a specified destination is quantified using neutrosophic fuzzy numbers, while the accessibility and requirement is indicated using real numbers, is referred to as the neutrosophic transportation problem.

The only difference between the classical methods and the neutrosophic fuzzy methods is that in the neutrosophic fuzzy numbers, the arithmetic operations of neutrosophic fuzzy numbers are used instead of arithmetic operations of real numbers.

2. Preliminaries

Definition:1 [9]

Let x be any element belongs to the universal set X. A neutrosophic set A in X is demonstrated by truth T_A, indeterminacy I_A and falsity-membership function F_A. Here, $T_A(x)$, $I_A(x)$ and $F_A(x)$ are nothing but the real standard or non-standard elements of [0,1]. i.e.,

$$T_A : X \to [0,1]$$

$$I_A : X \to [0,1]$$

$$F_A : X \to [0,1]$$

and no restriction on the sum of $T_A(X)$,$I_A(X)$ and $F_A(X)$, and also
$0 \leq supT_A(X) + supI_A(X) + supF_A(X) \leq 3$.

Definition:2 [9]

A single valued triangular neutrosophic number $\langle(a,b,c); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$, is a unique neutrosophic set on the real number **R**, of which the truth, indeterminacy and falsity-membership functions are given as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{(x-a)w_{\tilde{a}}}{(b-a)}, & (a \leq x < b) \\ \frac{(c-x)w_{\tilde{a}}}{(c-b)}, & (b \leq x \leq c) \\ 0, & otherwise \end{cases}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} \frac{(b-x+u_{\tilde{a}}(x-a))}{(b-a)}, & (a \leq x < b) \\ \frac{(x-b+u_{\tilde{a}}(c-x))}{(c-b)}, & (b \leq x \leq c) \\ 0, & otherwise \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} \frac{(b-x+y_{\tilde{a}}(x-a))}{(b-a)}, & (a \leq x < b) \\ \frac{(x-b+y_{\tilde{a}}(c-x))}{(c-b)}, & (b \leq x \leq c) \\ 0, & otherwise \end{cases}$$

Definition:3 [9]

Let $w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \in [0,1]$ and $a_1, a_2, a_3, a_4 \in \mathbb{R}$ such that $a_1 \leq a_2 \leq a_3 \leq a_4$. Then a single valued trapezoidal neutrosophic number, $\tilde{a} = \langle(a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ is a unique neutrosophic set on the real line $\mathbb{R}$, of which the truth, indeterminacy, and falsity-membership functions are given as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} w_{\tilde{a}}(\frac{x-a_1}{a_2-a_1}), & for\ a_1 \leq x \leq a_2 \\ w_{\tilde{a}}, & for\ a_2 \leq x \leq a_3 \\ w_{\tilde{a}}(\frac{a_4-x}{a_4-a_3}), & for\ a_3 \leq x \leq a_4 \\ 0, & otherwise, \end{cases}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} \frac{a_2 - x + u_{\tilde{a}}(x-a_1)}{a_2-a_1}, & for\ a_1 \le x \le a_2 \\ u_{\tilde{a}}, & for\ a_2 \le x \le a_3 \\ \frac{x - a_3 + u_{\tilde{a}}(a_4-x)}{a_4-a_3}, & for\ a_3 \le x \le a_4 \\ 1, & otherwise, \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} \frac{a_2 - x + y_{\tilde{a}}(x-a_1)}{a_2-a_1}, & for\ a_1 \le x \le a_2 \\ y_{\tilde{a}}, & for\ a_2 \le x \le a_3 \\ \frac{x - a_3 + y_{\tilde{a}}(a_4-x)}{a_4-a_3}, & for\ a_3 \le x \le a_4 \\ 1, & otherwise, \end{cases}$$

where $w_{\tilde{a}}, u_{\tilde{a}}$, and $y_{\tilde{a}}$ implies the maximum truth, minimum indeterminacy and minimum falsity membership degree, respectively. A single valued trapezoidal neutrosophic number $\tilde{a} = \langle(a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ may approximately identical to $[a_2, a_3]$ and it is denoted to be an ill-defined quantity about a.

2.1. *Arithmetic Operations on Triangular Neutrosophic Fuzzy Numbers*

Let $\tilde{A}^N = (a_1, a_2, a_3; a'_1, a_2, a'_3; a''_1, a_2, a''_3)$ and $\tilde{B}^N = (b_1, b_2, b_3; b'_1, b_2, b'_3; b''_1, b_2, b''_3)$ be two triangular neutrosophic fuzzy numbers.Then

i) $\tilde{A}^N \oplus \tilde{B}^N =$
$(a_1 + b_1, a_2 + b_2, a_3 + b_3; a'_1 + b'_1, a_2 + b_2, a'_3 + b'_3; a''_1 + b''_1, a_2 + b_2, a''_3 + b''_3)$

ii)$\tilde{A}^N \ominus \tilde{B}^N =$
$(a_1 - b_3, a_2 - b_2, a_3 - b_1; a'_1 - b'_3, a_2 - b_2, a'_3 - b'_1; a''_1 - b''_3, a_2 - b_2, a''_3 - b''_1)$

iii)$\tilde{A}^N \otimes \tilde{B}^N = (m_1, m_2, m_3; m'_1, m_2, m'_3; m''_1, m_{2,}, m''_3)$, where
$m_1 = min\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$, $m_2 (= m'_2 = m''_2) = a_2b_2$,
$m_3 = max\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$, $m'_1 = min\{a'_1b'_1, a'_1b'_3, a'_3b'_1, a'_3b'_3\}$,
$m'_3 = max\{a'_1b'_1, a'_1b'_3, a'_3b'_1, a'_3b'_3\}$, $m''_1 = min\{a''_1b''_1, a''_1b''_3, a''_3b''_1, a''_3b''_3\}$,
$m''_3 = max\{a''_1b''_1, a''_1b''_3, a''_3b''_1, a''_3b''_3\}$,

$$\text{iv)}\lambda\tilde{A}^N = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3; \lambda a'_1, \lambda a_2, \lambda a'_3; \lambda a''_1, \lambda a_2, \lambda a''_3); \lambda \ge 0, \\ (\lambda a_3, \lambda a_2, \lambda a_1; \lambda a'_3, \lambda a_2, \lambda a'_1; \lambda a''_3, \lambda a_2, \lambda a''_1); \lambda < 0, \end{cases}$$

2.2. *Ordering of Triangular Neutrosophic Fuzzy Numbers*

Let $\tilde{A}^N = (a_1, a_2, a_3; a'_1, a_2, a'_3; a''_1, a_2, a''_3)$ and $\tilde{B}^N = (b_1, b_2, b_3; b'_1, b_2, b'_3; b''_1, b_2, b''_3)$ be two triangular neutrosophic fuzzy numbers.Then

i)$\tilde{A}^N \succeq \tilde{B}^N if R(\tilde{A}^N) \ge R(\tilde{B}^N)$
ii)$\tilde{A}^N \approx \tilde{B}^N if R(\tilde{A}^N) = R(\tilde{B}^N)$
where $R(\tilde{A}^N) = \frac{[a_1 + 2a_2 + a_3 + a'_1 + 2a_2 + a'_3 + a''_1 + 2a_2 + a''_3]}{12}$
and $R(\tilde{B}^N) = \frac{[b_1 + 2b_2 + b_3 + b'_1 + 2b_2 + b'_3 + b''_1 + 2b_2 + b''_3]}{12}$

2.3. *Arithmetic Operations on Trapezoidal Neutrosophic Fuzzy Numbers*

Let $\tilde{A}^N = (a_1, a_2, a_3, a_4; a'_1, a'_2, a'_3, a'_4; a''_1, a''_2, a''_3, a''_4)$ and $\tilde{B}^N = (b_1, b_2, b_3, b_4; b'_1, b'_2, b'_3, b'_4; b''_1, b''_2, b''_3, b''_4)$ be two triangular neutrosophic fuzzy numbers.Then

i) $\tilde{A}^N \oplus \tilde{B}^N = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; a'_1 + b'_1, a'_2 + b'_2, a'_3 + b'_3, a'_4 + b'_4;$
$a''_1 + b''_1, a''_2 + b''_2, a''_3 + b''_3, a''_4 + b''_4)$

ii)$\tilde{A}^N \ominus \tilde{B}^N = (a_1 - b_4, a_2 - b_2, a_3 - b_2, a_4 - b_1; a'_1 - b'_4, a'_2 - b'_2, a'_3 - b'_2, a'_4 - b'_1;$
$a''_1 - b''_4, a''_2 - b''_2, a''_3 - b''_2, a''_4 - b''_1)$

iii)$\tilde{A}^N \otimes \tilde{B}^N = (m_1, m_2, m_3, m_4; m'_1, m'_2, m'_3, m'_4; m''_1, m''_{2,}, m''_3, m''_4)$, where

$m_1 = min\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$, $m_2 = min\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$

$m_3 = max\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$, $m_4 = max\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$,

$m'_1 = min\{a'_1b'_1, a'_1b'_4, a'_4b'_1, a'_4b'_4\}$, $m'_2 = min\{a'_2b'_2, a'_2b'_3, a'_3b'_2, a'_3b'_3\}$,

$m'_3 = max\{a'_2b'_2, a'_2b'_3, a'_3b'_2, a'_3b'_3\}$, $m'_4 = max\{a'_1b'_1, a'_1b'_4, a'_4b'_1, a'_4b'_4\}$,

$m''_1 = min\{a''_1b''_1, a''_1b''_4, a''_4b''_1, a'_4b'_4\}$, $m''_2 = min\{a''_2b''_2, a''_2b''_3, a''_3b''_2, a''_3b''_3\}$,

$m''_3 = max\{a''_2b''_2, a''_2b''_3, a''_3b''_2, a''_3b''_3\}$, $m''_4 = max\{a''_1b''_1, a''_1b''_4, a''_4b''_1, a''_4b''_4\}$,

$$\text{iv)}\lambda\tilde{A}^N = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4; \lambda a'_1, \lambda a'_2, \lambda a'_3, \lambda a'_4; \lambda a''_1, \lambda a''_2, \lambda a''_3, \lambda a''_4); \lambda \geq 0, \\ (\lambda a_4 \lambda a_3, \lambda a_2, \lambda a_1; \lambda a'_4 \lambda a'_3, \lambda a'_2, \lambda a'_1; \lambda a''_4 \lambda a''_3, \lambda a_2, \lambda a''_1); \lambda < 0, \end{cases}$$

2.4. *Ordering of Trapezoidal Neutrosophic Fuzzy Numbers*

Let $\tilde{A}^N = (a_1, a_2, a_3, a_4; a'_1, a'_2, a'_3, a'_4; a''_1, a''_2, a''_3, a''_4)$ and $\tilde{B}^N = (b_1, b_2, b_3, b_4; b'_1, b'_2, b'_3, b'_4; b''_1, b''_2, b''_3, b''_4)$ be two triangular neutrosophic fuzzy numbers.Then

i)$\tilde{A}^N \succeq \tilde{B}^{(N)} if R(\tilde{A}^{(N)}) \geq R(\tilde{B}^{(N)})$

ii)$\tilde{A}^N \approx \tilde{B}^{(N)} if R(\tilde{A}^{(N)}) = R(\tilde{B}^{(N)})$

where $R(\tilde{A}^N) = \frac{[a_1+a_2+a_3+a_4+a'_1+a'_2+a'_3+a'_4+a''_1+a''_2+a''_3+a''_4]}{12}$

and $R(\tilde{B}^N) = \frac{[b_1+b_2+b_3+b_4+b'_1+b'_2+b'_3+b'_4+b''_1+b''_2+b''_3+b''_4]}{12}$

3. A New Method For Solving Neutrosophic Fuzzy Transportation Problem Of Type-II

3.1. *Notations:*

c_{ij}=Unit neutrosophic transportation cost;

a_i=Neutrosophic availability:

d_j=Neutrosophic demand:

x_{ij}=Neutrosophic quantity.

3.2. *Algorithm for proposed method*

The stepwise procedure of proposed method is carried out as follows.

Step (1): Construct a neutrosophic fuzzy balanced transportation problem as in below table.

Sources	**Destination D1**	**Destination D2**	**...**	**Destination Dn**	**Availabilities**
S_1	$a_{11}, b_{11}, c_{11}; a'_{11}, b_{11}, c'_{11}; a''_{11}, b_{11}, c''_{11}$	$a_{12}, b_{12}, c_{12}; a'_{12}, b_{12}, c'_{12}; a''_{12}, b_{12}, c''_{12}$	...	$a_{1n}, b_{1n}, c_{1n}; a'_{1n}, b_{1n}, c'_{1n}; a''_{1n}, b_{1n}, c''_{1n}$	a
S_2	$a_{21}, b_{21}, c_{21}; a'_{21}, b_{21}, c'_{21}; a''_{21}, b_{21}, c''_{21}$	$a_{22}, b_{22}, c_{22}; a'_{22}, b_{22}, c'_{22}; a''_{22}, b_{22}, c''_{22}$	...	$a_{2n}, b_{2n}, c_{2n}; a'_{2n}, b_{2n}, c'_{2n}; a''_{2n}, b_{2n}, c''_{2n}$	b
. . .	. . .	. . .	. . .	. . .	. . .
S_n	$a_{n1}, b_{n1}, c_{n1}; a'_{n1}, b_{n1}; c'_{n1}; a''_{n1}, b_{n1}; c''_{n1}$	$a_{n2}, b_{n2}, c_{n2}; a'_{n2}, b_{n2}, c'_{n2}; a''_{n2}, b_{n2}, c''_{n2}$	...	$a_{nn}, b_{nn}, c_{nn}; a'_{nn}, b_{nn}, c'_{nn}; a''_{nn}, b_{nn}, c''_{nn}$	z
Demand	A	B	.....	Z	

Step (2): In general, The above table may be expressed as follows:

Minimize

$$R[\sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}; c''_{ij1}, c_{ij2}, c''_{ij3})x_{ij}]$$

subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i; i = 1, 2, ..., m,$$
$$\sum_{i=1}^{m} x_{ij} = b_j; j = 1, 2, ...,$$
$$x_{ij} \geq 0; i = 1, 2, ..., m; j = 1, 2, ..., n. \tag{1}$$

Step (3): Using the relation,

$$R[\sum_{i=1}^{m}\sum_{j=1}^{n}(a_{ij}, b_{ij}, c_{ij}; a'_{ij}, b_{ij}, c'_{ij}; a''_{ij}, b_{ij}, c''_{ij})] = \sum_{i=1}^{m}\sum_{j=1}^{n} R(a_{ij}, b_{ij}, c_{ij}; a'_{ij}, b_{ij}, c'_{ij}; a''_{ij}, b_{ij}, c''_{ij})$$

,the above problem can be stated as

Minimize

$$\sum_{i=1}^{m}\sum_{j=1}^{n} R[(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}; c''_{ij1}, c_{ij2}, c''_{ij3})x_{ij})]$$

subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i; i = 1, 2, ..., m,$$
$$\sum_{i=1}^{m} x_{ij} = b_j; j = 1, 2, ..., n,$$
$$x_{ij} \geq 0; i = 1, 2, ..., m; j = 1, 2, ..., n. \tag{2}$$

Step (4): The expression

$R(\lambda \times (a, b, c; a', b, c'; a'', b, c'')) = \lambda \times R(a, b, c; a', b, c'; a'', b, c'')$ can be used to rewrite the above problem as

Minimize

$$\sum_{i=1}^{m}\sum_{j=1}^{n} R(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}; c''_{ij1}, c_{ij2}, c''_{ij3}) \times x_{ij})$$

subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i; i = 1, 2, ..., m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j; j = 1, 2, ..., n,$$

$$x_{ij} \geq 0; i = 1, 2, ..., m; j = 1, 2, ..., n. \quad (3)$$

Step(5) : With the help of $R(a, b, c; a', b, c'; a'', b, c'') = \frac{a+2b+c+a'+2b+c'+a''+b+c''}{12}$, rewrite the above problem

Minimize

$$\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{[c_{ij1} + 2c_{ij2} + c_{ij3} + c'_{ij1} + 2c_{ij2} + c'_{ij3} + c''_{ij1} + 2c_{ij2} + c''_{ij3}]}{12} \times x_{ij}$$

subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i; i = 1, 2, ..., m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j; j = 1, 2, ..., n,$$

$$x_{ij} \geq 0; i = 1, 2, ..., m; j = 1, 2, ..., n. \quad (4)$$

Step(6) : Find the optimal solution by using Vogel's approximation method.

Step(7) :The minimum neutrosophic fuzzy transportation cost is

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij1}, c_{ij2}, c_{ij3}; c'_{ij1}, c_{ij2}, c'_{ij3}; c''_{ij1}, c_{ij2}, c''_{ij3}) \times x_{ij}$$

3.3. *Numerical example*

Step (1): The existing neutrosophic fuzzy balanced transportation problem can be given below.

Sources	Destination D1	Destination D2	Destination D3	Destination D4	Availabilities
S 1	2,4,5;1,4,6;0.1,4,6.1	2,5,7;1,5,8;0.1,5,8.1	4,6,8;3,6,9;2.1,6,9.1	4,7,8;3,7,9;2.1,7,9.1	11
S 2	4,6,8;3,6,9;2.1,6,9.2	3,7,12;2,7,13;1.2,7,13.2	10,15,20;8,15,22;7.2,15,22.1	11,12,13;10,12,14;9.2,12,14.2	11
S 3	3,4,6;1,4,8;0.2,4,8.5	8,10,13;5,10,16;4.1,10,16.2	2,3,5;1,3,6;0.2,3,6.2	6,10,14;5,10,15;4.2,10,15.1	11
S 4	2,4,6;1,4,7;0.1,4,7.2	3,9,10;2,9,12;0.2,9,12.1	3,6,10;2,6,12;0.1,6,12.3	3,4,5;2,4,8;0.1,4,8.2	12
Demand	16	10	8	11	

Step(2): The above problem can be transformed into the neutrosophic fuzzy linear programming problem.

Minimize

$[(2, 4, 5; 1, 4, 6; 0.1, 4, 6.1)x_{11} \oplus (2, 5, 7; 1, 5, 8; 0.1, 5, 8.1)x_{12} \oplus$

$(4, 6, 8; 3, 6, 9; 2.1, 6, 9.1)x_{13} \oplus (4, 7, 8; 3, 7, 9; 2.1, 7, 9.1)x_{14} \oplus$
$(4, 6, 8; 3, 6, 9; 2.1, 6, 9.2)x_{21} \oplus (3, 7, 12; 2, 7, 13; 1.2, 7, 13.2)x_{22} \oplus$
$(10, 15, 20; 8, 15, 22; 7.2, 15, 22.1)x_{23} \oplus (11, 12, 13; 10, 12, 14; 9.2, 12, 14.2)x_{24} \oplus$
$(3, 4, 6; 1, 4, 8; 0.2, 4, 8.5)x_{31} \oplus (8, 10, 13; 5, 10, 16; 4.1, 10, 16.2)x_{32} \oplus$
$(2, 3, 5; 1, 3, 6; 0.2, 3, 6.2)x_{33} \oplus (6, 10, 14; 5, 10, 15; 4.2, 10, 15.1)x_{34} \oplus$
$(2, 4, 6; 1, 4, 7; 0.1, 4, 7.2)x_{41} \oplus (3, 9, 10; 2, 9, 12; 0.2, 9, 12.1)x_{42} \oplus$
$(3, 6, 10; 2, 6, 12; 0.1, 6, 12.3)x_{43} \oplus (3, 4, 5; 2, 4, 8; 0.1, 4, 8.2)x_{44}]$

subject to the constraints

$x_{11} + x_{12} + x_{13} + x_{14} = 11,$
$x_{21} + x_{22} + x_{23} + x_{24} = 11,$
$x_{31} + x_{32} + x_{33} + x_{34} = 11,$
$x_{41} + x_{42} + x_{43} + x_{44} = 12,$
$x_{11} + x_{21} + x_{31} + x_{41} = 16,$
$x_{12} + x_{22} + x_{32} + x_{42} = 10,$
$x_{13} + x_{23} + x_{33} + x_{43} = 8,$
$x_{14} + x_{24} + x_{34} + x_{44} = 11,$
$x_{ij} \geq 0; i = 1, 2, 3, 4; j = 1, 2, 3, 4.$

Step(3): By step (3) in the algorithm, we have

Minimize

R $[(2, 4, 5; 1, 4, 6; 0.1, 4, 6.1)x_{11} \oplus (2, 5, 7; 1, 5, 8; 0.1, 5, 8.1)x_{12} \oplus$
$(4, 6, 8; 3, 6, 9; 2.1, 6, 9.1)x_{13} \oplus (4, 7, 8; 3, 7, 9; 2.1, 7, 9.1)x_{14} \oplus$
$(4, 6, 8; 3, 6, 9; 2.1, 6, 9.2)x_{21} \oplus (3, 7, 12; 2, 7, 13; 1.2, 7, 13.2)x_{22} \oplus$
$(10, 15, 20; 8, 15, 22; 7.2, 15, 22.1)x_{23} \oplus (11, 12, 13; 10, 12, 14; 9.2, 12, 14.2)x_{24} \oplus$
$(3, 4, 6; 1, 4, 8; 0.2, 4, 8.5)x_{31} \oplus (8, 10, 13; 5, 10, 16; 4.1, 10, 16.2)x_{32} \oplus$
$(2, 3, 5; 1, 3, 6; 0.2, 3, 6.2)x_{33} \oplus (6, 10, 14; 5, 10, 15; 4.2, 10, 15.1)x_{34} \oplus$
$(2, 4, 6; 1, 4, 7; 0.1, 4, 7.2)x_{41} \oplus (3, 9, 10; 2, 9, 12; 0.2, 9, 12.1)x_{42} \oplus$
$(3, 6, 10; 2, 6, 12; 0.1, 6, 12.3)x_{43} \oplus (3, 4, 5; 2, 4, 8; 0.1, 4, 8.2)x_{44}]$

subject to the constraints

$x_{11} + x_{12} + x_{13} + x_{14} = 11,$
$x_{21} + x_{22} + x_{23} + x_{24} = 11,$
$x_{31} + x_{32} + x_{33} + x_{34} = 11,$
$x_{41} + x_{42} + x_{43} + x_{44} = 12,$
$x_{11} + x_{21} + x_{31} + x_{41} = 16,$
$x_{12} + x_{22} + x_{32} + x_{42} = 10,$
$x_{13} + x_{23} + x_{33} + x_{43} = 8,$
$x_{14} + x_{24} + x_{34} + x_{44} = 11,$
$x_{ij} \geq 0; i = 1, 2, 3, 4; j = 1, 2, 3, 4.$

Step(4): Using step(4), the above problem becomes

Minimize

$[R([(2,4,5;1,4,6;0.1,4,6.1)x_{11}) \oplus R((2,5,7;1,5,8;0.1,5,8.1)x_{12}) \oplus$
$R((4,6,8;3,6,9;2.1,6,9.1)x_{13}) \oplus R((4,7,8;3,7,9;2.1,7,9.1)x_{14}) \oplus$
$R((4,6,8;3,6,9;2.1,6,9.2)x_{21}) \oplus R((3,7,12;2,7,13;1.2,7,13.2)x_{22}) \oplus$
$R((10,15,20;8,15,22;7.2,15,22.1)x_{23}) \oplus R((11,12,13;10,12,14;9.2,12,14.2)x_{24}) \oplus$
$R((3,4,6;1,4,8;0.2,4,8.5)x_{31}) \oplus R((8,10,13;5,10,16;4.1,10,16.2)x_{32}) \oplus$
$R((2,3,5;1,3,6;0.2,3,6.2)x_{33}) \oplus R((6,10,14;5,10,15;4.2,10,15.1)x_{34}) \oplus$
$R((2,4,6;1,4,7;0.1,4,7.2)x_{41}) \oplus R((3,9,10;2,9,12;0.2,9,12.1)x_{42}) \oplus$
$R(3,6,10;2,6,12;0.1,6,12.3)x_{43} \oplus R((3,4,5;2,4,8;0.1,4,8.2)x_{44})]$

subject to the constraints

$x_{11} + x_{12} + x_{13} + x_{14} = 11,$
$x_{21} + x_{22} + x_{23} + x_{24} = 11,$
$x_{31} + x_{32} + x_{33} + x_{34} = 11,$
$x_{41} + x_{42} + x_{43} + x_{44} = 12,$
$x_{11} + x_{21} + x_{31} + x_{41} = 16,$
$x_{12} + x_{22} + x_{32} + x_{42} = 10,$
$x_{13} + x_{23} + x_{33} + x_{43} = 8,$
$x_{14} + x_{24} + x_{34} + x_{44} = 11,$
$x_{ij} \geq 0; i = 1,2,3,4; j = 1,2,3,4.$

Step(5): The relation in step(5) connect the later problem into below one

Minimize

$[R((2,4,5;1,4,6;0.1,4,6.1)x_{11} \oplus R(2,5,7;1,5,8;0.1,5,8.1)x_{12} \oplus$
$R(4,6,8;3,6,9;2.1,6,9.1)x_{13} \oplus R(4,7,8;3,7,9;2.1,7,9.1)x_{14} \oplus$
$R(4,6,8;3,6,9;2.1,6,9.2)x_{21} \oplus R(3,7,12;2,7,13;1.2,7,13.2)x_{22} \oplus$
$R(10,15,20;8,15,22;7.2,15,22.1)x_{23} \oplus R(11,12,13;10,12,14;9.2,12,14.2)x_{24} \oplus$
$R(3,4,6;1,4,8;0.2,4,8.5)x_{31} \oplus R(8,10,13;5,10,16;4.1,10,16.2)x_{32} \oplus$
$R(2,3,5;1,3,6;0.2,3,6.2)x_{33} \oplus R(6,10,14;5,10,15;4.2,10,15.1)x_{34} \oplus$
$R(2,4,6;1,4,7;0.1,4,7.2)x_{41} \oplus R(3,9,10;2,9,12;0.2,9,12.1)x_{42} \oplus$
$R(3,6,10;2,6,12;0.1,6,12.3)x_{43} \oplus R(3,4,5;2,4,8;0.1,4,8.2)x_{44}]$

subject to the constraints

$x_{11} + x_{12} + x_{13} + x_{14} = 11,$
$x_{21} + x_{22} + x_{23} + x_{24} = 11,$
$x_{31} + x_{32} + x_{33} + x_{34} = 11,$
$x_{41} + x_{42} + x_{43} + x_{44} = 12,$
$x_{11} + x_{21} + x_{31} + x_{41} = 16,$
$x_{12} + x_{22} + x_{32} + x_{42} = 10,$
$x_{13} + x_{23} + x_{33} + x_{43} = 8,$
$x_{14} + x_{24} + x_{34} + x_{44} = 11,$
$x_{ij} \geq 0; i = 1,2,3,4; j = 1,2,3,4.$

Step(6): Using the expression $R(a,b,c;a',b,c';a'',b,c'') = \frac{a+2b+c+a'+2b+c'+a''+b+c''}{12}$, rewrite the

above problem as
Minimize
$(3.68x_{11} + 4.68x_{12} + 5.93x_{13} + 6.43x_{14} + 5.94x_{21} + 7.2x_{22} + 14.9x_{23} + 11.95x_{24} + 4.22x_{31} + 10.19x_{32} + 3.2x_{33} + 9.94x_{34} + 3.94x_{41} + 7.775x_{42} + 6.28x_{43} + 4.19x_{44})$
subject to the constraints
$x_{11} + x_{12} + x_{13} + x_{14} = 11,$
$x_{21} + x_{22} + x_{23} + x_{24} = 11,$
$x_{31} + x_{32} + x_{33} + x_{34} = 11,$
$x_{41} + x_{42} + x_{43} + x_{44} = 12,$
$x_{11} + x_{21} + x_{31} + x_{41} = 16,$
$x_{12} + x_{22} + x_{32} + x_{42} = 10,$
$x_{13} + x_{23} + x_{33} + x_{43} = 8,$
$x_{14} + x_{24} + x_{34} + x_{44} = 11,$
$x_{ij} \geq 0; i = 1, 2, 3, 4; j = 1, 2, 3, 4.$
Step(7): Solving the crisp linear programming problem by Vogel's approximation method, the obtained optimal solution is
$x_{11} = 1, x_{12} = 10, x_{13} = 0, x_{14} = 0, x_{21} = 11, x_{22} = 0, x_{23} = 0, x_{24} = 0,$
$x_{31} = 3, x_{32} = 0, x_{33} = 8, x_{41} = 1, x_{42} = 0, x_{43} = 0, x_{44} = 11.$
Step(8): Using the optimal solution, the minimum neutrosophic fuzzy transportation cost is
$(2, 4, 5; 1, 4, 6; 0.1, 4, 6.1) \times 1 \oplus (2, 5, 7; 1, 5, 8; 0.1, 5, 8.1) \times 10 \oplus$
$(4, 6, 8; 3, 6, 9; 2.1, 6, 9.2) \times 11 \oplus (3, 4, 6; 1, 4, 8; 0.2, 4, 8.5) \times 3 \oplus$
$(2, 3, 5; 1, 3, 6; 0.2, 3, 6.2) \times 8 \oplus (2, 4, 6; 1, 4, 7; 0.1, 4, 7.2) \times 1 \oplus$
$(3, 4, 5; 2, 4, 8; 0.1, 4, 8.2) \times 11 = (126, 204, 282; 78, 204, 352; 26.5, 204, 359.7)$

Conclusion:

In the proposed method, the new algorithm for finding optimal solution for the transportation problem under neutrosophic environment by Vogel's approximation method is established. The final results of the stated approach are investigated through a numerical example. Using this concept, the comparision between existing methods and proposed method and various applications in neutrosophic transportaion problems will be carried out in future.

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