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# Properties of multiplication operation of neutrosophic fuzzy matrices

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**Abstract:** The article reveals the investigation of properties of multiplication operation of neutrosophic fuzzy matrices. We study commutative property, associative property, distributive property of them. We show with suitable example that neutrosophic fuzzy matrices do not obey commutative property with respect to multiplication operation. We prove that neutrosophic fuzzy matrices hold associative property property with respect to multiplication operation. We also prove that neutrosophic fuzzy matrices hold distributive property with respect to multiplication operation over addition. These results are further justified by providing suitable numerical examples. The important aspects of the article is that the investigation of commutative, associative and distributive properties of neutrosophic fuzzy matrices with respect to multiplication operation will fill up the gaps in the existing literature.

**Keywords:** *Neutrosophic Set; Fuzzy Matrix; Neutrosophic fuzzy Matrix; Properties of Neutrosophic Fuzzy Matrices.*

## 1. Introduction:

Classical methods often fail to deal real - life problems due to uncertainty. Thereafter, Zadeh [1] invented fuzzy set associating with membership value to resolve uncertainty. Sometimes, non - membership value is necessary to resolve uncertainty properly. In order to deal with such a situation, in 1986, Atanassov initiated the notion of intuitionistic fuzzy sets by associating truth and falsity-membership values. However, it fails to resolve indeterminate situation. Smarandache [2] talked about neutrosophic set after associating membership, non-membership and indeterminacy functions independently. It turned out to take decision for solving real life problem in complex situation. Pal et al. [3, 4] introduced minimal structures and continuity in neutrosophic topological spaces. Das and Das [5] investigated on neutrosophic separation axioms. Dhar [6] studied compactness and neutrosophic topological space via grills. Recently, Broumi et al. [7, 8] and Abdel-Basset et al. [9, 10, 11, 12] and some other authors [13, 14, 15, 16, 17, 18, 19, 20] have successfully applied neutrosophic sets to solve different problems.

Matrices have significant contribution in the field of science and technology. It is often seen that usual matrix theory cann't address all uncertainty. Thomas [21] invented fuzzy matrices.

Kandasamy and Smarandache [22, 23] referred neutrosophic relational maps and the classical algebraic structures converted to neutrosophic algebra after inserting the indeterminacy element $I$ to it. The importance of matrices can be found in the theory of vector spaces. This concept has been generalized to neutrosophic matrices by Khaled et al. [24]. Addition and multiplication operations of square neutrosophic fuzzy matrices have been defined and investigated by Dhar et al. [25].

**Gap in the literature:**

Das et al. [26] studied on the subtraction operation and investigated algebraic properties of neutrosophic fuzzy matrices. However, they did not investigate on commutative, associative and distributive properties of them with respect to multiplication operation. In this article, we study commutative, associative and distributive properties of neutrosophic fuzzy matrices with respect to multiplication operation.

**The innovative values of the article:**

We have invented notion of multiplication operation of neutrosophic fuzzy matrices which is quite different from usual multiplication operation of other matrices of real or complex entries. We have discussed commutative property, associative property and distributive property of them with multiplication operation. We have also discussed suitable examples to justify the introduction of the notion.

We frame the paper in different sections. The next section procures few known definitions and results. We investigate few properties in section 3. Then conclusion appears.

2. **Preliminaries and Definitions:**

Necessary concepts and results have been procured in this section.

**Definition 2.1.** [2] The neutrosophic set η is the form η = {<$x$:$T_\eta(x)$, $I_\eta(x)$, $F_\eta(x)$>, $x \in U$}, where $U$ is an universe set and the independent functions $T$, $I$, $F$ : $U \rightarrow$ $]^{-}0, 1^{+}[$ referrer respectively degree of membership, indeterminacy and non-membership of $x \in U$ and $^{-}0 \leq T_\eta(x) + I_\eta(x) + F_\eta(x) \leq 3^{+}$.

It will be difficult to apply the interval $]^{-}0, 1^{+}[$ in the applications of scientific and engineering problems. So we need to take [0, 1] in place of $]^{-}0, 1^{+}[$.

**Definition 2.2.** [25] The neutrosophic matrix is defined as $M_{m \times n}$= {($m_{ij}$) : $m_{ij} \in K(I)$}. Here $K(I)$ denotes a neutrosophic field.

**Definition 2.3.** (One may refer to [26]) $A_{4\times 3}=\begin{pmatrix} 5 & 0 & 2.1I \\ 3.5I & 3 & 5 \\ 7 & 4I & 0 \\ 8 & -5I & I \end{pmatrix}$

denotes a neutrosophic matrix involving the elements (entries) from the real and indeterminacy.

**Definition 2.4.** [22] Take $P$ = [0, 1]∪$I$. The $p\times q$ matrices $C_{p\times q}$ = {($c_{ij}$) : $c_{ij} \in$ [0, 1]∪$I$} is said to be fuzzy integral neutrosophic matrices. Evidently collection of fuzzy integral neutrosophic matrices contain collection of $p\times q$ matrices.

The fuzzy neutrosophic row and column matrices are the row vector $1\times q$ and column vector $p\times 1$ respectively.

**Definition 2.5.** (One may refer [26]) Let $M_{4\times3} = \begin{pmatrix} 0.5 & 0 & 0.1I \\ I & 0.3 & 0.5 \\ 0.7 & 0.4I & 0 \\ 0.8 & 0.5I & I \end{pmatrix}$ be a 4×3 integral fuzzy neutrosophic matrix.

**Definition 2.6.** [22] We denote $N_s$ as fuzzy neutrosophic set where $N_s$ = [0, 1]∪{$bI$ :$b$∈ [0, 1]}. Then $M_{m\times n}$ = {($c_{ij}$):$c_{ij}$∈ $N_s$, $i$ = 1 to $m$ and $j$ = 1 to $n$} is defined as fuzzy neutrosophic matrices.

**Example 2.7.** (One may refer to [26]) Take $N_s$ = [0, 1]∪{$dI$ : $d$∈[0, 1]}as fuzzy neutrosophic set and

$$P = \begin{pmatrix} 0.5 & 0 & 0.1I \\ I & 0.3 & 0.5 \\ 0 & I & 0.01 \end{pmatrix}$$

is a fuzzy neutrosophic matrix of order 3× 3.

**Definition 2.8.** [21] A matrix with entries from unit fuzzy interval [0, 1] is said to be a fuzzy matrix and if the number of rows and column of that matrix are equal, then it is referred as fuzzy square matrix. An example is given below:

$$M = \begin{pmatrix} u & v \\ t & w \end{pmatrix}$$

Here $u$, $v$, $t$, $w$ belong to [0, 1].

**Definition 2.9.** [22] The entries of a neutrosophic fuzzy matrix (in short, NFM) $M$ are form $x + Iy$ (neutrosophic number). Here $x$, $y$ are taken from [0, 1], $I$ is an indeterminate where $I^n = I$, ($n$∈ ℕ). As for example

$$M= \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix}$$

is a neutrosophic fuzzy matrix.

**Definition 2.10.** [26] Let us take two matrices as below:

$$A = \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix} \text{ and } B = \begin{pmatrix} m_1 + In_1 & m_2 + In_2 \\ m_3 + In_3 & m_4 + In_4 \end{pmatrix}$$

The product of them as multiplication operation is as below

$$AB = \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix} \begin{pmatrix} m_1 + In_1 & m_2 + In_2 \\ m_3 + In_3 & m_4 + In_4 \end{pmatrix}$$

$$= \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} \text{ as defined in Definition 2.8 of [26].}$$

## 3. Main Results:

Here we investigate some properties of neutrosophic fuzzy matrices.

**3.1. Proposition.** Multiplication operation is not commutative in case of neutrosophic fuzzy matrices.

**Proof.** We consider $A = \begin{pmatrix} 0.1 + I0.2 & 0.2 + I0.4 \\ 0.3 + I0.5 & 0.4 + I0.6 \end{pmatrix}$, $B = \begin{pmatrix} 0.3 + I0.7 & 0.4 + I0.5 \\ 0.2 + I0.4 & 0.2 + I0.8 \end{pmatrix}$

$$AB = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

where, $C_{11} = \text{Max}\{\text{Min}(0.1, 0.3), \text{Min}(0.2, 0.2)\} + I\text{Max}\{\text{Min}(0.2, 0.7), \text{Min}(0.4, 0.4)\} = \text{Max}\{0.1, 0.2\} + I\text{Max}\{0.2, 0.4\} = 0.2 + I0.4$.

Similarly, one can show that

$C_{12} = 0.2 + I0.4,\ C_{21} = 0.3 + I0.5,\ C_{22} = 0.3 + I0.6.$

Thus, $AB = \begin{pmatrix} 0.2 + I0.4 & 0.2 + I0.4 \\ 0.3 + I0.5 & 0.3 + I0.6 \end{pmatrix}$……………(1)

Now,

$$BA = \begin{pmatrix} 0.3 + I0.7 & 0.4 + I0.5 \\ 0.2 + I0.4 & 0.2 + I0.8 \end{pmatrix} \begin{pmatrix} 0.1 + I0.2 & 0.2 + I0.4 \\ 0.3 + I0.5 & 0.4 + I0.6 \end{pmatrix}$$

$= \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$ and

$$D_{11} = \text{Max}\{\text{Min}(0.3, 0.1), \text{Min}(0.4, 0.3)\} + I\text{Max}\{\text{Min}(0.7, 0.2), \text{Min}(0.5, 0.5)\}$$
$$= \text{Max}\{0.1, 0.3\} + I\text{Max}\{0.2, 0.5\} = 0.3 + I0.5$$

Similarly, one can show that

$D_{12} = 0.2 + I0.5,\ D_{21} = 0.2 + I0.5,\ D_{22} = 0.2 + I0.6.$

Thus, $BA = \begin{pmatrix} 0.3 + I0.5 & 0.2 + I0.5 \\ 0.2 + I0.5 & 0.2 + I0.6 \end{pmatrix}$………………(2)

From (1) and (2), it follows that $AB \neq BA$.

**3.2. Proposition.** Multiplication operation is associative in case of neutrosophic fuzzy matrices.

**Proof.** We consider $A = \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix}$, $B = \begin{pmatrix} c_1 + Id_1 & c_2 + Id_2 \\ c_3 + Id_3 & c_4 + Id_4 \end{pmatrix}$

$C = \begin{pmatrix} m_1 + In_1 & m_2 + In_2 \\ m_3 + In_3 & m_4 + In_4 \end{pmatrix}$

Now,

$$AB = \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix} \begin{pmatrix} c_1 + Id_1 & c_2 + Id_2 \\ c_3 + Id_3 & c_4 + Id_4 \end{pmatrix}$$

$$= \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix},$$

where, $M_{11} = \text{Max}\{\text{Min}(x_1, c_1), \text{Min}(x_2, c_3)\} + I\,\text{Max}\{\text{Min}(y_1, d_1), \text{Min}(y_2, d_3)\}$.

$M_{12} = \text{Max}\{\text{Min}(x_1, c_2), \text{Min}(x_2, c_4)\} + I\,\text{Max}\{\text{Min}(y_1, d_2), \text{Min}(y_2, d_4)\}$.

$M_{21} = \text{Max}\{\text{Min}(x_3, c_1), \text{Min}(x_4, c_3)\} + I\,\text{Max}\{\text{Min}(y_3, d_1), \text{Min}(y_4, d_3)\}$.

$M_{22} = \text{Max}\{\text{Min}(x_3, c_2), \text{Min}(x_4, c_4)\} + I\,\text{Max}\{\text{Min}(y_3, d_2), \text{Min}(y_4, d_4)\}$.

$\therefore AB = \begin{pmatrix} X_1 + IY_1 & X_2 + IY_2 \\ X_3 + IY_3 & X_4 + IY_4 \end{pmatrix}$,

where, $X_1 = \text{Max}\{\text{Min}(x_1, c_1), \text{Min}(x_2, c_3)\}$.

$X_2 = \text{Max}\{\text{Min}(x_1, c_2), \text{Min}(x_2, c_4)\}$.

$X_3 = \text{Max}\{\text{Min}(x_3, c_1), \text{Min}(x_4, c_3)\}$.

$X_4 = \text{Max}\{\text{Min}(x_3, c_2), \text{Min}(x_4, c_4)\}$.

$$Y_1 = \text{Max}\{\text{Min}(y_1, d_1), \text{Min}(y_2, d_3)\}.$$
$$Y_2 = \text{Max}\{\text{Min}(y_1, d_2), \text{Min}(y_2, d_4)\}.$$
$$Y_3 = \text{Max}\{\text{Min}(y_3, d_1), \text{Min}(y_4, d_3)\}.$$
$$Y_4 = \text{Max}\{\text{Min}(y_3, d_2), \text{Min}(y_4, d_4)\}.$$

$$\therefore (AB)C = \begin{pmatrix} X_1 + IY_1 & X_2 + IY_2 \\ X_3 + IY_3 & X_4 + IY_4 \end{pmatrix} \begin{pmatrix} m_1 + In_1 & m_2 + In_2 \\ m_3 + In_3 & m_4 + In_4 \end{pmatrix}$$

$$= \begin{pmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{pmatrix},$$

where, $N_{11} = \text{Max}\{\text{Min}(\text{X}_1, \text{m}_1), \text{Min}(\text{X}_2, \text{m}_3)\} + I\,\text{Max}\{\text{Min}(Y_1, n_1), \text{Min}(Y_2, n_3)\}$.

$$N_{12} = \text{Max}\{\text{Min}(X_1, m_2), \text{Min}(\text{X}_2, \text{m}_4)\} + I\,\text{Max}\{\text{Min}(Y_1, n_2), \text{Min}(Y_2, n_4)\}.$$
$$N_{21} = \text{Max}\{\text{Min}(\text{X}_3, \text{m}_1), \text{Min}(\text{X}_4, \text{m}_3)\} + I\,\text{Max}\{\text{Min}(Y_3, n_1), \text{Min}(Y_4, n_3)\}.$$
$$N_{22} = \text{Max}\{\text{Min}(\text{X}_3, \text{m}_2), \text{Min}(\text{X}_4, \text{m}_4)\} + I\text{Max}\{\text{Min}(Y_3, n_2), \text{Min}(Y_4, n_4)\}.$$

Again, $BC = \begin{pmatrix} c_1 + Id_1 & c_2 + Id_2 \\ c_3 + Id_3 & c_4 + Id_4 \end{pmatrix} \begin{pmatrix} m_1 + In_1 & m_2 + In_2 \\ m_3 + In_3 & m_4 + In_4 \end{pmatrix}$

$= \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}$, where

$$Q_{11} = \text{Max}\{\text{Min}(c_1, m_1), \text{Min}(c_2, m_3)\} + I\,\text{Max}\{\text{Min}(d_1, n_1), \text{Min}(d_2, n_3)\},$$
$$Q_{12} = \text{Max}\{\text{Min}(c_1, m_2), \text{Min}(c_2, m_4)\} + I\,\text{Max}\{\text{Min}(d_1, n_2), \text{Min}(d_2, n_4)\},$$
$$Q_{21} = \text{Max}\{\text{Min}(c_3, m_1), \text{Min}(c_4, m_3)\} + I\text{Max}\{\text{Min}(d_3, n_1), \text{Min}(d_4, n_3)\},$$
$$Q_{22} = \text{Max}\{\text{Min}(c_3, m_2), \text{Min}(c_4, m_4)\} + I\,\text{Max}\{\text{Min}(d_3, n_2), \text{Min}(d_4, n_4)\}.$$

$\therefore BC = \begin{pmatrix} G_1 + IH_1 & G_2 + IH_2 \\ G_3 + IH_3 & G_4 + IH_4 \end{pmatrix}$, where

$$G_1 = \text{Max}\{\text{Min}(\text{c}_1, \text{m}_1), \text{Min}(\text{c}_2, \text{m}_3)\}.$$
$$G_2 = \text{Max}\{\text{Min}(\text{c}_1, \text{m}_2), \text{Min}(\text{c}_2, \text{m}_4)\}.$$
$$G_3 = \text{Max}\{\text{Min}(\text{c}_3, \text{m}_1), \text{Min}(\text{c}_4, \text{m}_3)\}.$$
$$G_4 = \text{Max}\{Min(c_3, m_2), Min(c_4, m_4)\}.$$
$$H_1 = \text{Max}\{\text{Min}(\text{d}_1, \text{n}_1), \text{Min}(\text{d}_2, \text{n}_3)\}.$$
$$H_2 = \text{Max}\{\text{Min}(d_1, n_2), \text{Min}(d_2, n_4)\}.$$
$$H_3 = \text{Max}\{\text{Min}(d_3, n_1), \text{Min}(d_4, n_3)\}.$$
$$H_4 = \text{Max}\{\text{Min}(d_3, n_2), \text{Min}(d_4, n_4)\}.$$

$$\therefore A(BC) = \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix} \begin{pmatrix} G_1 + IH_1 & G_2 + IH_2 \\ G_3 + IH_3 & G_4 + IH_4 \end{pmatrix}$$

$= \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$, where

$$R_{11} = \text{Max}\{\text{Min}(x_1, G_1), \text{Min}(x_2, G_3)\} + I\,\text{Max}\{\text{Min}(y_1, H_1), \text{Min}(y_2, H_3)\}.$$

$R_{12} = \text{Max}\{\text{Min}(x_1, G_2), \text{Min}(x_2, G_4)\} + I\ \text{Max}\{\text{Min}(y_1, H_2), \text{Min}(y_2, H_4)\}$.

$R_{21} = \text{Max}\{\text{Min}(x_3, G_1), \text{Min}(x_4, G_3)\} + I\ \text{Max}\{\text{Min}(y_3, H_1), \text{Min}(y_4, H_3)\}$.

$R_{22} = \text{Max}\{\text{Min}(\text{x}_3, \text{G}_2), \text{Min}(\text{x}_4, \text{G}_4)\} + I\ \text{Max}\{\text{Min}(y_3, H_2), \text{Min}(y_4, H_4)\}$.

In order to show $(AB)C = A(BC)$, we have to show $N_{11} = R_{11}$, $N_{12} = R_{12}$, $N_{21} = R_{21}$ and $N_{22} = R_{22}$.

In order to calculate $N_{11}$, we have to find $X_1, X_2$, $Y_1$ and $Y_2$.

Now, $X_1 = \text{Max}\{\text{Min}(x_1, c_1), \text{Min}(x_2, c_3)\}$

= Max$\{x_1,\ x_2\}$ (say)

= $x_1$ (say).

$X_2 = \text{Max}\{\text{Min}(x_1, c_2), \text{Min}(x_2, c_4)\}$ = Max$\{x_1,\ x_2\}$ (say)

= $x_1$.

$Y_1 = \text{Max}\{Min(y_1, d_1), \text{Min}(y_2, d_3)\}$ = Max$\{y_1,\ y_2\}$ [(say)

= $y_1$ (say).

$Y_2 = \text{Max}\{Min(y_1, d_2), Min(y_2, d_4)\}$ = Max$\{y_1,\ y_2\}$ (say)

$\therefore N_{11} = \text{Max}\{\text{Min}(\text{X}_1, \text{m}_1), \text{Min}(\text{X}_2, \text{m}_3)\} + I\text{Max}\{\text{Min}(Y_1, n_1), \text{Min}(Y_2, n_3)\}$

= Max{Min$(x_1, m_1)$, Min$(x_1, m_3)$} + $I$Max{Min$(y_1,\ n_1)$, Min$(y_1,\ n_3)$}

= Max$\{x_1,\ x_1\}$ + $I$Max$\{y_1,\ y_1\}$ [Assuming Min$(x_1, m_1) = x_1$, Min$(x_1,\ m_3) = x_1$,
Min$(y_1,\ n_1) = y_1$, Min$(y_1,\ n_3) = y_1$]

= $x_1 + Iy_1$

In order to calculate $R_{11}$, we have to find $G_1, G_3$, $H_1$ and $H_3$.

Now, $G_1 = \text{Max}\{\text{Min}(\text{c}_1, \text{m}_1), \text{Min}(\text{c}_2, \text{m}_3)\}$

= Max$\{m_1,\ m_3\}$ (say)

= $m_1$ (say).

$G_3 = \text{Max}\{\text{Min}(\text{c}_3, \text{m}_1), \text{Min}(\text{c}_4, \text{m}_3)\}$

= Max$\{m_1,\ m_3\}$ (say).

$H_1 = \text{Max}\{\text{Min}(\text{d}_1, \text{n}_1), \text{Min}(\text{d}_2, \text{n}_3)\}$ = Max$\{\text{d}_1,\ d_2\}$ (say) = $d_1$ (say).

$H_3 = \text{Max}\{Min(d_3, n_1), Min(d_4, n_3)\}$ = Max$\{d_3,\ d_4\}$ (say).

= $d_3$ (say).

$\therefore R_{11} = \text{Max}\{\text{Min}(x_1, G_1), \text{Min}(x_2, G_3)\} + I\text{Max}\{\text{Min}(y_1, H_1), \text{Min}(y_2, H_3)\}$

= $\text{Max}\{\text{Min}(x_1, m_1), \text{Min}(x_2, m_1)\} + I\text{Max}\{\text{Min}(y_1, d_1), \text{Min}(y_2, d_3)\}$

= Max$\{x_1,\ x_2\}$ $+ I\text{Max}\{y_1, y_2\}$(say)

= $x_1 + Iy_1$.

$\therefore N_{11} = R_{11}$. Assuming in all other cases, we can show that $N_{11} = R_{11}$.

Similarly we can show that $N_{12} = R_{12}$, $N_{21} = R_{21}$ and $N_{22} = R_{22}$.

Thus $(AB)C = A(BC)$.

This property is supported with a numerical example as shown below.

**3.3. Numerical Example.**

Let us consider

$$A = \begin{pmatrix} 0.3 + I0.2 & 0.4 + I0.5 \\ 0.4 + I0.1 & 0.5 + I0.6 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.2 + I0.6 & 0.3 + I0.2 \\ 0.4 + I0.7 & 0.5 + I0.2 \end{pmatrix}$$

$$C = \begin{pmatrix} 0.8 + I0.3 & 0.3 + I0.2 \\ 0.5 + I0.6 & 0.4 + I0.7 \end{pmatrix}$$

$AB = \begin{pmatrix} 0.3 + I0.2 & 0.4 + I0.5 \\ 0.4 + I0.1 & 0.5 + I0.6 \end{pmatrix}\begin{pmatrix} 0.2 + I0.6 & 0.3 + I0.2 \\ 0.4 + I0.7 & 0.5 + I0.2 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$ (say) and

$C_{11}$ = Max{Min(0.3, 0.2), Min(0.4, 0.4)} + *I*Max{Min(0.2, 0.6), Min(0.5, 0.7)}

= Max{0.2, 0.4} + *I*Max{0.2, 0.5}

= 0.4 + *I*0.5.

Similarly, one can show that

$C_{12} = 0.4 + I0.2,\ C_{21} = 0.4 + I0.6,\ C_{22} = 0.5 + I0.2.$

$\therefore\ AB = \begin{pmatrix} 0.4 + I0.5 & 0.4 + I0.2 \\ 0.4 + I0.6 & 0.5 + I0.2 \end{pmatrix}$

$\therefore (AB)C = \begin{pmatrix} 0.4 + I0.5 & 0.4 + I0.2 \\ 0.4 + I0.6 & 0.5 + I0.2 \end{pmatrix}\begin{pmatrix} 0.8 + I0.3 & 0.3 + I0.2 \\ 0.5 + I0.6 & 0.4 + I0.7 \end{pmatrix} = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$ (say) and

$D_{11}$ = Max{Min(0.4, 0.8), Min(0.4, 0.5)} + *I*Max{Min(0.5, 0.3), Min(0.2, 0.6)}

= Max{0.4, 0.4} + *I*Max{0.3, 0.2}

= 0.4 + *I*0.3.

Similarly, one can show that

$D_{12} = 0.4 + I0.2,\ D_{21} = 0.5 + I0.3,\ D_{22} = 0.4 + I0.2.$

$\therefore (AB)C = \begin{pmatrix} 0.4 + I0.3 & 0.4 + I0.2 \\ 0.5 + I0.3 & 0.4 + I0.2 \end{pmatrix}$…………………(3)

$BC = \begin{pmatrix} 0.2 + I0.6 & 0.3 + I0.2 \\ 0.4 + I0.7 & 0.5 + I0.2 \end{pmatrix}\begin{pmatrix} 0.8 + I0.3 & 0.3 + I0.2 \\ 0.5 + I0.6 & 0.4 + I0.7 \end{pmatrix} = \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix}$ (say) and

$E_{11}$ = Max{Min(0.2, 0.8), Min(0.3, 0.5)} + *I*Max{Min(0.6, 0.3), Min(0.2, 0.6)}

= Max{0.2, 0.3} + *I*Max{0.3, 0.2}

= 0.3 + *I*0.3.

Similarly, one can show that

$E_{12} = 0.3 + I0.2,\ E_{21} = 0.5 + I0.3,\ E_{22} = 0.4 + I0.2.$

$\therefore\ BC = \begin{pmatrix} 0.3 + I0.3 & 0.3 + I0.2 \\ 0.5 + I0.3 & 0.4 + I0.2 \end{pmatrix}$

$\therefore\ A(BC) = \begin{pmatrix} 0.3 + I0.2 & 0.4 + I0.5 \\ 0.4 + I0.1 & 0.5 + I0.6 \end{pmatrix}\begin{pmatrix} 0.3 + I0.3 & 0.3 + I0.2 \\ 0.5 + I0.3 & 0.4 + I0.2 \end{pmatrix} = \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix}$ (say) and

$F_{11}$ = Max{Min(0.3, 0.3), Min(0.4, 0.5)} + *I*Max{Min(0.2, 0.3), Min(0.5, 0.3)}

= Max{0.3, 0.4} + *I*Max{0.2, 0.3}

= 0.4 + *I*0.3.

Similarly, one can show that

$F_{12} = 0.4 + I0.2,\ F_{21} = 0.5 + I0.3,\ F_{22} = 0.4 + I0.2.$

$\therefore\ A(BC) = \begin{pmatrix} 0.4 + I0.3 & 0.4 + I0.2 \\ 0.5 + I0.3 & 0.4 + I0.2 \end{pmatrix}$………..(4)

From (3) and (4), it follows that $(AB)C = A(BC)$.

**3.4. Proposition.** Distributive property with respect to multiplication over addition holds in case of neutrosophic fuzzy matrices.

We consider $A = \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix}, B = \begin{pmatrix} c_1 + Id_1 & c_2 + Id_2 \\ c_3 + Id_3 & c_4 + Id_4 \end{pmatrix}$

$C = \begin{pmatrix} m_1 + In_1 & m_2 + In_2 \\ m_3 + In_3 & m_4 + In_4 \end{pmatrix}$

$$\therefore D = B + C$$

$= \begin{pmatrix} c_1 + Id_1 & c_2 + Id_2 \\ c_3 + Id_3 & c_4 + Id_4 \end{pmatrix} + \begin{pmatrix} m_1 + In_1 & m_2 + In_2 \\ m_3 + In_3 & m_4 + In_4 \end{pmatrix}$

where

$D_{11}$ = Max$\{c_1, m_1\}$ + $I$Max$\{d_1, n_1\}$ = $x_{11} + Iy_{11}$(say),

$D_{12}$ = Max$\{c_2, m_2\}$ + $I$Max$\{d_2, n_2\}$ = $x_{12} + Iy_{12}$(say),

$D_{21}$ = Max$\{c_3, m_3\}$ + $I$Max$\{d_3, n_3\}$ = $x_{21} + Iy_{21}$(say),

$D_{22}$ = Max$\{c_4, m_4\}$ + $I$Max$\{d_4, n_4\}$ = $x_{22} + Iy_{22}$(say).

$\therefore D = \begin{pmatrix} x_{11} + Iy_{11} & x_{12} + Iy_{12} \\ x_{21} + Iy_{21} & x_{22} + Iy_{22} \end{pmatrix}$

$\therefore AD = \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix} \begin{pmatrix} x_{11} + Iy_{11} & x_{12} + Iy_{12} \\ x_{21} + Iy_{21} & x_{22} + Iy_{22} \end{pmatrix} = \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix}$,

where

$E_{11}$ = Max{Min($x_1, x_{11}$), Min($x_2, x_{21}$)} + $I$Max{Min($y_1, y_{11}$), Min($y_2, y_{21}$)},

$E_{12}$ = Max{Min($x_1, x_{12}$), Min($x_2, x_{22}$)} + $I$Max{Min($y_1, y_{12}$), Min($y_2, y_{22}$)},

$E_{21}$ = Max{Min($x_3, x_{11}$), Min($x_4, x_{21}$)} + $I$Max{Min($y_3, y_{11}$), Min($y_4, y_{21}$)},

$E_{22}$ = Max{Min($x_3, x_{12}$), Min($x_4, x_{22}$)} + $I$Max{Min($y_3, y_{12}$), Min($y_4, y_{22}$)}.

In order to calculate $E_{11}$, we have to calculate $x_{11}$, $x_{21}$, $y_{11}$ and $y_{21}$.

$x_{11}$ = Max$\{c_1, m_1\}$ = $c_1$ (say).

$y_{11}$ = Max$\{d_1, n_1\}$ = $d_1$ (say).

$x_{21}$ = Max$\{c_3, m_3\}$ = $c_3$ (say).

$y_{21}$ = Max$\{d_3, n_3\}$= $c_3$ (say).

$\therefore E_{11}$ = Max{Min($x_1, x_{11}$), Min($x_2, x_{21}$)} + $I$Max{Min($y_1, y_{11}$), Min($y_2, y_{21}$)}

= Max{Min($x_1, c_1$), Min($x_2, c_3$)} + $I$Max{Min($y_1, d_1$), Min($y_2, d_3$)}

= Max$\{c_1, c_3\}$ + $I$Max$\{d_1, d_3\}$ [Assuming Min$(x_1, c_1\}$ = $c_1$, Min$(x_2, c_3)$ = $c_3$

Min$(y_1, d_1)$ = $d_1$, Min$(y_2, d_3)$ = $d_3$]

= $c_1 + Id_1$ (say).

Now $AB = \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix} \begin{pmatrix} c_1 + Id_1 & c_2 + Id_2 \\ c_3 + Id_3 & c_4 + Id_4 \end{pmatrix}$ = $F$ (say), where,

$F_{11}$ = Max{Min($x_1$, $c_1$), Min($x_2$, $c_3$)} + $I$Max{Min($y_1, d_1$), Min($y_2$, $d_3$)},

$F_{12}$ = Max{Min($x_1$, $c_2$), Min($x_2$, $c_4$)} + $I$Max{Min($y_1, d_2$), Min($y_2$, $d_4$)},

$F_{21}$ = Max{Min($x_3$, $c_1$), Min($x_4$, $c_3$)} + $I$Max{Min($y_3, d_1$), Min($y_4$, $d_3$)},

$F_{22}$ = Max{Min($x_3$, $c_2$), Min($x_4$, $c_4$)} + $I$Max{Min($y_3, d_2$), Min($y_4$, $d_4$)}.

Now $F_{11}$ = Max{Min($x_1$, $c_1$), Min($x_2$, $c_3$)} + $I$Max{Min($y_1, d_1$), Min($y_2$, $d_3$)}

= Max{$c_1, c_3$} + $I$Max{$d_1, d_3$}

= $c_1$ + $Id_1$ (say).

Now $AC$ = $\begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix}\begin{pmatrix} m_1 + In_1 & m_2 + In_2 \\ m_3 + In_3 & m_4 + In_4 \end{pmatrix}$ = G,

where,

$G_{11}$ = Max{Min($x_1$, $m_1$), Min($x_2$, $m_3$)} + $I$Max{Min($y_1, n_1$), Min($y_2$, $n_3$)},

$G_{12}$ = Max{Min($x_1$, $m_2$), Min($x_2$, $m_4$)} + $I$Max{Min($y_1, n_2$), Min($y_2$, $n_4$)},

$G_{21}$ = Max{Min($x_3$, $m_1$), Min($x_4$, $m_3$)} + $I$Max{Min($y_3, n_1$), Min($y_3$, $n_3$)},

$G_{22}$ = Max{Min($x_3$, $m_2$), Min($x_4$, $m_4$)} + $I$Max{Min($y_3, n_2$), Min($y_4$, $n_4$)}.

Now Max{$c_1, m_1$} = $c_1$ & Min($x_1, c_1$) = $c_1$imply Min($x_1$, $m_1$) = $m_1$.

Max{$c_3, m_3$} = $c_3$ & $M$in($x_2, c_3$) = $x_2$ imply Min($x_2$, $m_3$) = $m_3$.

Max{$d_1, n_1$} = $d_1$& Min($y_1, d_1$) = $y_1$ imply Min($y_1, n_1$) = $n_1$.

Max{$d_3, n_3$} = $d_3$& Min($y_2, d_3$) = $d_3$ imply Min($y_2, n_3$) = $n_3$.

So $G_{11}$ = Max{Min($x_1$, $m_1$), Min($x_2$, $m_3$)} + $I$Max{Min($y_1, n_1$), Min($y_2$, $n_3$)}

= Max{$m_1, m_3$} + $I$Max{$n_1$, $n_3$}

= $m_1$ + $In_1$ (say).

$\therefore F_{11} + G_{11}$ = Max{$c_1$, $m_1$} + $I$Max{$d_1$, $n_1$}

= $c_1$ + $Id_1$[Since Max{$c_1, m_1$} = $c_1$ and Max{$d_1, n_1$} = $d_1$]

, $\therefore E_{11} = F_{11} + G_{11}$.

Similarly we can show that $E_{12} = F_{12}$+ $G_{12}$, $E_{21} = F_{21} + G_{21}$ and $E_{22} = F_{22} + G_{22}$.

$$\therefore A(B + C) = AB + AC.$$

We can also show that $A(B - C) = AB - AC$.

Thus $A(B \pm C) = AB \pm AC$.

This property is supported by numerical examples as given below.

**3.5. Numerical Example.** We consider the matrices

$$A = \begin{pmatrix} 0.2 + I0.1 & 0.3 + I0.5 \\ 0.4 + I0.3 & 0.5 + I0.7 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.4 + I0.5 & 0.1 + I0.2 \\ 0.3 + I0.6 & 0.7 + I0.3 \end{pmatrix}$$

$$C = \begin{pmatrix} 0.7 + I0.2 & 0.2 + I0.3 \\ 0.6 + I0.5 & 0.4 + I0.8 \end{pmatrix}$$

$$\therefore D = B + C$$

$$= \begin{pmatrix} 0.4 + I0.5 & 0.1 + I0.2 \\ 0.3 + I0.6 & 0.7 + I0.3 \end{pmatrix} + \begin{pmatrix} 0.7 + I0.2 & 0.2 + I0.3 \\ 0.6 + I0.5 & 0.4 + I0.8 \end{pmatrix}$$

$$= \begin{pmatrix} 0.7 + I0.5 & 0.2 + I0.3 \\ 0.6 + I0.6 & 0.7 + I0.8 \end{pmatrix}$$

$$E = AD = A(B + C)$$

$$= \begin{pmatrix} 0.2 + I0.1 & 0.3 + I0.5 \\ 0.4 + I0.3 & 0.5 + I0.7 \end{pmatrix} \begin{pmatrix} 0.7 + I0.5 & 0.2 + I0.3 \\ 0.6 + I0.6 & 0.7 + I0.8 \end{pmatrix}$$

$$= \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix}$$

where, $E_{11} = \text{Max}\{\text{Min}(0.2, 0.3), \text{Min}(0.3, 0.6)\} + I\text{Max}\{\text{Min}(0.1, 0.5), \text{Min}(0.5, 0.6)\}$

$$= \text{Max}\{0.2, 0.3\} + \quad I\text{Max}\{0.1, 0.5\} = 0.3 + I0.5.$$

Similarly, one can show that

$E_{12} = 0.3 + I0.5$, $E_{21} = 0.5 + I0.6$, $E_{22} = 0.5 + I0.7$.

$\therefore E = \begin{pmatrix} 0.3 + I0.5 & 0.3 + I0.5 \\ 0.5 + I0.6 & 0.5 + I0.7 \end{pmatrix}$ ………………………(5)

$$AB = \begin{pmatrix} 0.2 + I0.1 & 0.3 + I0.5 \\ 0.4 + I0.3 & 0.5 + I0.7 \end{pmatrix} \begin{pmatrix} 0.4 + I0.5 & 0.1 + I0.2 \\ 0.3 + I0.6 & 0.7 + I0.3 \end{pmatrix}$$

$$= \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix},$$

where, $F_{11} = \text{Max}\{\text{Min}(0.2, 0.4), \text{Min}(0.3, 0.3)\} + I\text{Max}\{\text{Min}(0.1, 0.5), \text{Min}(0.5, 0.6)\}$

$$= \text{Max}\{0.2, 0.3\} + \quad I\text{Max}\{0.1, 0.5\} = 0.3 + I0.5$$

Similarly, one can show that

$F_{12} = 0.3 + I0.3$, $F_{21} = 0.4 + I0.6$, $F_{22} = 0.5 + I0.3$.

$$\therefore AB = \begin{pmatrix} 0.3 + I0.5 & 0.3 + I0.3 \\ 0.4 + I0.6 & 0.5 + I0.3 \end{pmatrix}$$

Now, $AC = \begin{pmatrix} 0.2 + I0.1 & 0.3 + I0.5 \\ 0.4 + I0.3 & 0.5 + I0.7 \end{pmatrix} \begin{pmatrix} 0.7 + I0.2 & 0.2 + I0.3 \\ 0.6 + I0.5 & 0.4 + I0.8 \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}$,

where, $G_{11} = \text{Max}\{\text{Min}(0.2, 0.7), \text{Min}(0.3, 0.6)\} + I\text{Max}\{\text{Min}(0.1, 0.2), \text{Min}(0.5, 0.5)\}$

$$= \text{Max}\{0.2, 0.3\} + \quad I\text{Max}\{0.1, 0.5\} = 0.3 + I0.5.$$

Similarly, one can show that

$G_{12} = 0.3 + I0.5$, $G_{21} = 0.5 + I0.5$, $G_{22} = 0.4 + I0.7$.

$$\therefore AC = \begin{pmatrix} 0.3 + I0.5 & 0.3 + I0.5 \\ 0.5 + I0.5 & 0.4 + I0.7 \end{pmatrix}$$

$$\therefore AB + AC = \begin{pmatrix} 0.3 + I0.5 & 0.3 + I0.3 \\ 0.4 + I0.6 & 0.5 + I0.3 \end{pmatrix} + \begin{pmatrix} 0.3 + I0.5 & 0.3 + I0.5 \\ 0.5 + I0.5 & 0.4 + I0.7 \end{pmatrix}$$

$=$

$\begin{pmatrix} 0.3 + I0.5 & 0.3 + I0.5 \\ 0.5 + I0.6 & 0.5 + I0.7 \end{pmatrix}$ …………………..(6)

From (5) and (6) it follows that

$$A(B+C) = AB + AC$$

**3.6. Numerical Example.** Let us take $A = \begin{pmatrix} 0.7+I0.3 & 0.2+I0.4 \\ 0.5+I0.6 & 0.4+I0.3 \end{pmatrix}$

$$B = \begin{pmatrix} 0.1+I0.2 & 0.5+I0.3 \\ 0.3+I0.4 & 0.7+I0.5 \end{pmatrix}$$

$$C = \begin{pmatrix} 0.5+I0.4 & 0.2+I0.1 \\ 0.2+I0.3 & 0.3+I0.7 \end{pmatrix}$$

$$B - C = \begin{pmatrix} 0.1+I0.2 & 0.5+I0.3 \\ 0.3+I0.4 & 0.7+I0.5 \end{pmatrix} - \begin{pmatrix} 0.5+I0.4 & 0.2+I0.1 \\ 0.2+I0.3 & 0.3+I0.7 \end{pmatrix}$$

$$= \begin{pmatrix} 0.1+I0.2 & 0.2+I0.1 \\ 0.2+I0.3 & 0.3+I0.5 \end{pmatrix}$$

$$A(B-C) = \begin{pmatrix} 0.7+I0.3 & 0.2+I0.4 \\ 0.5+I0.6 & 0.4+I0.3 \end{pmatrix} \begin{pmatrix} 0.1+I0.2 & 0.2+I0.1 \\ 0.2+I0.3 & 0.3+I0.5 \end{pmatrix}$$

$$= \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix},$$

where, $D_{11} = \text{Max}\{\text{Min}(0.7, 0.1), \text{Min}(0.2, 0.2)\} + I\text{Max}\{\text{Min}(0.3, 0.2), \text{Min}(0.4, 0.3)\}$

$$= \text{Max}\{0.1, 0.2\} + I\text{Max}\{0.2, 0.3\} = 0.2 + I0.3.$$

Similarly, one can show that

$D_{12} = 0.2 + I0.4$, $D_{21} = 0.2 + I0.3$, $D_{22} = 0.3 + I0.3$.

$\therefore A(B-C) = \begin{pmatrix} 0.2+I0.3 & 0.2+I0.4 \\ 0.2+I0.3 & 0.3+I0.3 \end{pmatrix}$. ..............................(7)

Now,

$$AB = \begin{pmatrix} 0.7+I0.3 & 0.2+I0.4 \\ 0.5+I0.6 & 0.4+I0.3 \end{pmatrix} \begin{pmatrix} 0.1+I0.2 & 0.5+I0.3 \\ 0.3+I0.4 & 0.7+I0.5 \end{pmatrix} = \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix},$$

where, $E_{11} = \text{Max}\{\text{Min}(0.7, 0.1), \text{Min}(0.2, 0.3)\} + I\text{Max}\{\text{Min}(0.3, 0.2), \text{Min}(0.4, 0.4)\}$

$$= \text{Max}\{0.1, 0.2\} + I\text{Max}\{0.2, 0.4\} = 0.2 + I0.4.$$

Similarly, one can show that

$E_{12} = 0.5 + I0.4$, $E_{21} = 0.3 + I0.3$, $E_{22} = 0.5 + I0.3$.

$$\therefore AB = \begin{pmatrix} 0.2+I0.4 & 0.5+I0.4 \\ 0.3+I0.3 & 0.5+I0.3 \end{pmatrix}$$

$$\therefore AC = \begin{pmatrix} 0.7+I0.3 & 0.2+I0.4 \\ 0.5+I0.6 & 0.4+I0.3 \end{pmatrix} \begin{pmatrix} 0.5+I0.4 & 0.2+I0.1 \\ 0.2+I0.3 & 0.3+I0.7 \end{pmatrix}$$

$$= \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix},$$

where, $F_{11} = \text{Max}\{\text{Min}(0.7, 0.5), \text{Min}(0.2, 0.2)\} + I\text{Max}\{\text{Min}(0.3, 0.4), \text{Min}(0.4, 0.3)\}$

$$= \text{Max}\{0.5, 0.2\} + I\text{Max}\{0.3, 0.3\} = 0.5 + I0.3.$$

Similarly, one can show that

$F_{12} = 0.2 + I0.4,\ F_{21} = 0.5 + I0.4,\ F_{22} = 0.3 + I0.3.$

$$\therefore AC = \begin{pmatrix} 0.5 + I0.3 & 0.2 + I0.4 \\ 0.5 + I0.4 & 0.3 + I0.3 \end{pmatrix}$$

$$\therefore AB - AC = \begin{pmatrix} 0.2 + I0.4 & 0.5 + I0.4 \\ 0.3 + I0.3 & 0.5 + I0.3 \end{pmatrix} - \begin{pmatrix} 0.5 + I0.3 & 0.2 + I0.4 \\ 0.5 + I0.4 & 0.3 + I0.3 \end{pmatrix}$$

$=$

$\begin{pmatrix} 0.2 + I0.3 & 0.2 + I0.4 \\ 0.3 + I0.3 & 0.3 + I0.3 \end{pmatrix}$ ……….(8)

From (7) and (8) it follows that A(B – $C$) $= AB - AC$.

**3.7. Identity element for multiplication.**

We consider A $= \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix}, I_N = \begin{pmatrix} 1 + I1 & 0 + I0 \\ 0 + I0 & 1 + I1 \end{pmatrix}$.

Now A$I_N = \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix} \begin{pmatrix} 1 + I1 & 0 + I0 \\ 0 + I0 & 1 + I1 \end{pmatrix}$ = B (say),

where, $B_{11}$ = Max{Min($x_1$, 1), Min($x_2$, 0)} + $I$Max{Min($y_1$, 1), Min($y_2$, 0)}

= Max{$x_1$, 0} + $I$Max{$y_1$, 0} = $x_1 + Iy_1$.

Similarly, one can show

$B_{12}$ = Max{Min($x_1$, 0), Min($x_2$, 1)} + $I$Max{Min($y_1$, 0), Min($y_2$, 1)}

$= x_2 + Iy_2$.

$B_{21}$ = Max{Min($x_3$, 1), Min($x_4$, 0)} + $I$Max{Min($y_3$, 1), Min($y_4$, 0)}

$= x_3 + Iy_3$.

$B_{22}$ = Max{Min($x_3$, 0), Min($x_4$, 1)} + $I$Max{Min($y_3$, 0), Min($y_4$, 1)}

$= x_4 + Iy_4$.

∴A$I_N$ = $\begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix}$ = A …………..(9)

Again $I_N$A $= \begin{pmatrix} 1 + I1 & 0 + I0 \\ 0 + I0 & 1 + I1 \end{pmatrix} \begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix}$ = C (say),

where, $C_{11}$ = Max{Min(1, $x_1$), Min(0, $x_3$)} + $I$Max{Min(1, $y_1$), Min(0, $y_3$)}

= Max{$x_1$, 0} + $I$Max{$y_1$, 0} = $x_1 + Iy_1$.

$C_{12}$ = Max{Min(1, $x_2$), Min(0, $x_4$)} + $I$Max{Min(1, $y_2$), Min(0, $y_4$)}

$= x_2 + Iy_2$.

$C_{21}$ = Max{Min(0, $x_1$), Min(1, $x_3$)} + $I$Max{Min(0, $y_1$), Min(1, $y_3$)}

$= x_3 + Iy_3$.

$C_{22}$ = Max{Min(0, $x_2$), Min(1, $x_4$)} + $I$Max{min(0, $y_2$), Min(1, $y_4$)}

$= x_4 + Iy_4$.

∴ $I_N$A = $\begin{pmatrix} x_1 + Iy_1 & x_2 + Iy_2 \\ x_3 + Iy_3 & x_4 + Iy_4 \end{pmatrix}$ = A …………..(10)

From (9) and (10) it follows that

$AI_N = I_N A = A$

Thus $I_N = \begin{pmatrix} 1+I1 & 0+I0 \\ 0+I0 & 1+I1 \end{pmatrix}$ is the identity multiplication for neutrosophic fuzzy matrices.

**4. Conclusion:**

This paper has provided the properties of multiplication operation of neutrosophic fuzzy matrices. We have shown that commutative property is not satisfied here. However, we have proved that associative property is satisfied. We have also proved that distributive property with respect to multiplication operation over addition of neutrosophic fuzzy matrices is satisfied. The results have further been examined with suitable numerical examples. In future, the authors will investigate on determinant, adjont, inverse and other relevant topics of neutrosophic fuzzy matrices.

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