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Neutrosophic Fuzzy Ideals in Γ Rings

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Abstract: Fuzzy sets are a major oversimplification and extension of classical sets. Fuzzy sets have become a recognized research topic in many fields. This paper proposes a new type of set theory is neutrosophic set. As a novel study field, new hybrid sets created from neutrosophic sets are gaining prominence. The neutrosophic set is used to describe indeterminacy and uncertainty in any information. The neutrosophic set extension has been explored by many researchers. Here we introduce properties of Neutrosophic Fuzzy (NF) ideals in Γ Rings. Some new neutrosophic operations are explored.

Keywords: Γ Rings; Fuzzy set; Neutrosophic fuzzy set; Neutrosophic fuzzy ideal; Neutrosophic $\Gamma -$ endomorphism.

1. Introduction

In 1965, Zadeh proposed the fuzzy set as a method to deal with imprecise data [21]. Many applications have been found for fuzzy sets in various fields of research, these include intuitionistic fuzzy sets, picture fuzzy sets, orthopair fuzzy sets, and neutrosophic sets. Also, various algebraic structures have been discussed in fuzzy versions by many researchers. One of the algebraic structures is the gamma ring. In 1964 Nobusawa [9] first proposed the gamma ring concept. This is rather common when compared to a ring. Barnes [3] weakened the requirements of Nobusawa's gamma ring. As a continuation of his research, researchers are interested in gamma rings with apartness [6,7,10]. Gamma ring structure is used to investigate the number of Generalizations that are identical to the corresponding parts of Kyuno's ring theory [8]. Uddin[19] generalized the results of gamma endomorphism in gamma rings. Ardakani [2] discussed derivations of prime and semi-prime gamma rings. Atanassov created Intuitionistic fuzzy set to address the issue of non-determinacy brought on by a single membership function in the fuzzy set. The intuitionistic fuzzy set is highly helpful in that it offers a flexible model to explain the uncertainty and ambiguity inherent in decision-making. In 2010 Palaniappan et.al [11, 12, 13] proposed the intuitionistic fuzzy ideals and intuitionistic fuzzy prime ideals in Γ-Rings. Neutrosophic logic was introduced by Florentin Smarandache in 1995. Neutrosophic set is a generalization of the intuitionistic fuzzy set discussed by Smarandache[17]. Neutrosophic set is a set where each element of the universe has a degree of truth, indeterminacy, and falsity respectively, and which lies between 0 and 1. There are several applications in various fields. Salama [15] states the characteristic function of a Neutrosophic set. In 2010 Wang introduced the single-valued Neutrosophic sets [20]. Many authors exhibited NF ideals [5,14,16,18]. Agboola primarily focused on neutrosophic canonical hypergroups and neutrosophic hyperrings [1]. Chalapathi stated about neutrosophic rings [4]. During this paper, we introduced the notion of NF ideals in the gamma ring structure.

2. Prerequisites:

The required definitions are incorporated in this section.

Definition 2.1: [9] Consider (N,Γ) is an abelian group where $N=\{p,q,r\}$ and $\Gamma=\{\alpha,\beta,\gamma \ldots.\}$ and for all $p,q,r\,\epsilon\,N$ and $\alpha,\beta\epsilon\Gamma$,

(1) $p\alpha q\,\epsilon\,N$

(2) $(p+q)\alpha\,r = p\alpha\,r + q\alpha\,r, p(\alpha+\beta)q = p\alpha q + p\beta q, p\alpha(q+r) = p\alpha q + p\alpha r,$

(3) $(p\alpha q)\beta\,r = p\alpha(q\beta r)$. Then N is a Γ Ring.

Later the improved by Barnes [3]

(1') $p\alpha q\,\epsilon\,N\ \ \alpha p\beta\epsilon\Gamma,$

(2') $(p+q)\alpha\,r = p\alpha\,r + q\alpha\,r, p(\alpha+\beta)q = p\alpha q + p\beta q, p\alpha(q+r) = p\alpha q + p\alpha r,$

(3') $(p\alpha q)\beta\,r = p(\alpha q\beta)r = p\alpha(q\beta r),$

(4') $p\alpha q = 0\ for\ all\ p,q\epsilon N\ implies\ \alpha = 0$

Definition 2.2: [16] A fuzzy set φ in a Γ Ring N is called fuzzy ideal of N if x, y ϵ R

(i) $\varphi(x-y) \geq \min\{\varphi(x),\varphi(y)\}$

(ii) $\varphi(x\alpha y) \geq \max\{\varphi(x),\varphi(y)\}$

Definition 2.3: [18] A NF set $\mathcal{A}$ on the universe of discourse X characterized by a truth membership function $\mathcal{U}_{\mathcal{A}}$ (x), a indeterminacy function $\mathcal{V}_{\mathcal{A}}$ (x) and a falsity membership function$\mathcal{W}_{\mathcal{A}}(x)$ is defined as $\mathcal{A}$= {<x, $\mathcal{U}_{\mathcal{A}}$ (x), $\mathcal{V}_{\mathcal{A}}$ (x), $\mathcal{W}_{\mathcal{A}}(x) >: x\epsilon X\}$,

Where $\mathcal{U}_{\mathcal{A}}$,$\mathcal{V}_{\mathcal{A}}$,$\mathcal{W}_{\mathcal{A}}$: X→ [0,1] and $0 \leq \mathcal{U}_{\mathcal{A}}(x) + \mathcal{V}_{\mathcal{A}}(x) + \mathcal{W}_{\mathcal{A}}(x) \leq 3$

Definition 2.4: [20] Let X be a non-void set and let $\mathcal{A} = <\mathcal{U}_{\mathcal{A}}, \mathcal{V}_{\mathcal{A}}, \mathcal{W}_{\mathcal{A}}>$ and $\mathcal{B} = <\mathcal{U}_{\mathcal{B}}, \mathcal{V}_{\mathcal{B}}, \mathcal{W}_{\mathcal{B}}>$ be two NS sets in X. Then

Complement: $\mathcal{C}(\mathcal{A})$

$\mathcal{U}_{\mathcal{C}(\mathcal{A})}(x) = 1-\mathcal{U}_{\mathcal{A}}(x), \mathcal{V}_{\mathcal{C}(\mathcal{A})}(x) = 1-\mathcal{V}_{\mathcal{A}}(x)$, $\mathcal{W}_{\mathcal{C}(\mathcal{A})}(x) = 1-\mathcal{W}_{\mathcal{A}}(x)$.

Containment: $\mathcal{A} \subseteq \mathcal{B}$

$\inf \mathcal{U}_{\mathcal{A}}(x) \leq \inf \mathcal{U}_{\mathcal{B}}(x)$, $\sup\mathcal{U}_{\mathcal{A}}(x) \leq \sup \mathcal{U}_{\mathcal{B}}(x)$, $\inf \mathcal{W}_{\mathcal{A}}(x) \geq \inf \mathcal{W}_{\mathcal{B}}(x)$, $\sup \mathcal{W}_{\mathcal{A}}(x) \geq \sup \mathcal{W}_{\mathcal{B}}(x)$,

Union: $\mathcal{C} = \mathcal{A} \cup \mathcal{B}$

$\mathcal{U}_{\mathcal{C}}(x) = \mathcal{U}_{\mathcal{A}}(x) + \mathcal{U}_{\mathcal{B}}(x) - \mathcal{U}_{\mathcal{A}}(x) * \mathcal{U}_{\mathcal{B}}(x)$, $\mathcal{V}_{\mathcal{C}}(x) = \mathcal{V}_{\mathcal{A}}(x) + \mathcal{V}_{\mathcal{B}}(x) - \mathcal{V}_{\mathcal{A}}(x) * \mathcal{V}_{\mathcal{B}}(x)$,

$\mathcal{W}_{\mathcal{C}}(x) = \mathcal{W}_{\mathcal{A}}(x) + \mathcal{W}_{\mathcal{B}}(x) - \mathcal{W}_{\mathcal{A}}(x) * \mathcal{W}_{\mathcal{B}}(x)$,

Intersection: $\mathcal{C} = \mathcal{A} \cap \mathcal{B}$

$\mathcal{U}_{\mathcal{C}}(x) = \mathcal{U}_{\mathcal{A}}(x) * \mathcal{U}_{\mathcal{B}}(x)$, $\mathcal{V}_{\mathcal{C}}(x) = \mathcal{V}_{\mathcal{A}}(x) * \mathcal{V}_{\mathcal{B}}(x)$, $\mathcal{W}_{\mathcal{C}}(x) = \mathcal{W}_{\mathcal{A}}(x) * \mathcal{W}_{\mathcal{B}}(x)$ for all x in X.

Definition 2.5: A function $\theta: G_1 \rightarrow G_2$ where G_1 and G_2 are Γ Rings is said to be a Γ -homomorphism if $\theta(p+q) = \theta(p) + \theta(q), \theta(p\alpha q) = \theta(p)\alpha\theta(q)$ for all $p,q,\epsilon\,N, \alpha\epsilon\Gamma$.

Definition 2.6: A function $\theta: G_1 \rightarrow G_2$ Where θ is a Γ -homomorphism and G_1 and G_2 are Γ Rings is said to be a Γ – endomorphism if $G_2 \subseteq G_1$.

3. NF ideals of Γ Ring:

Definition 3.1: Let N be a Γ Ring. A NF set $\mathcal{A}$ in N is said to be NF ideal of N if (i)$\mathcal{U}_{\mathcal{A}}(p-q) \geq \{\mathcal{U}_{\mathcal{A}}(p) \wedge \mathcal{U}_{\mathcal{A}}(q)\}$, $\mathcal{V}_{\mathcal{A}}(p-q) \leq \{\mathcal{V}_{\mathcal{A}}(p) \vee \mathcal{V}_{\mathcal{A}}(q)\}$, and $\mathcal{W}_{\mathcal{A}}(p-q) \leq \{\mathcal{W}_{\mathcal{A}}(p) \vee \mathcal{W}_{\mathcal{A}}(q)\}$

(ii)$\mathcal{U}_{\mathcal{A}}(p\alpha q) \geq \mathcal{U}_{\mathcal{A}}(q)[resp. \mathcal{U}_{\mathcal{A}}(p\alpha q) \geq \mathcal{U}_{\mathcal{A}}(p)]$, $\mathcal{V}_{\mathcal{A}}(p\alpha q) \leq \mathcal{V}_{\mathcal{A}}(q)[resp. \mathcal{V}_{\mathcal{A}}(p\alpha q) \leq \mathcal{V}_{\mathcal{A}}(p)]$, and $\mathcal{W}_{\mathcal{A}}(p\alpha q) \leq \mathcal{W}_{\mathcal{A}}(q)[resp. \mathcal{W}_{\mathcal{A}}(p\alpha q) \leq \mathcal{W}_{\mathcal{A}}(p)]$ for all p, q $\epsilon\, N, \alpha\, \epsilon\, \Gamma$.

Example 3.2: Let N = {0, 1, 2, 3} and α = {0, 1, 2, 3} and define N and α as follows

-	0	1	2	3
0	0	1	2	3
1	1	1	3	2
2	2	3	3	2
3	3	2	2	2

α	0	1	2	3
0	0	1	2	3
1	1	1	3	2
2	2	3	3	2
3	3	2	2	2

$$\mathcal{U}_{\mathcal{A}}(x) = \begin{cases} 0.7 & if\ x = 0 \\ 0.8\ if\ x = 1 \\ 0.8\ if\ x = 2,3 \end{cases},\ \mathcal{V}_{\mathcal{A}}(x) = \begin{cases} 0.9\ if\ x = 0 \\ 0.7\ if\ x = 1 \\ 0.6\ if\ x = 2,3 \end{cases},\ \mathcal{W}_{\mathcal{A}}(x) = \begin{cases} 0.8\ if\ x = 0 \\ 0.5\ if\ x = 1 \\ 0.3\ if\ x = 2,3 \end{cases}$$

Clearly N is a NF ideal of N.

Definition 3.3: Consider NF ideal $\varphi =< \mathcal{U}_\varphi, \mathcal{V}_\varphi, \mathcal{W}_\varphi >$ of a Γ Ring N is normal if $\mathcal{U}_\varphi(0) = 1, \mathcal{V}_\varphi(0) = 0$, and $\mathcal{W}_\varphi(0) = 0$.

Theorem 3.4: Let $\varphi =< \mathcal{U}_\varphi, \mathcal{V}_\varphi, \mathcal{W}_\varphi >$ be a NF ideal of a Γ Ring N and let $\mathcal{U}_\varphi^+(p) = \mathcal{U}_\varphi(p) + 1 - \mathcal{U}_\varphi(0), \mathcal{V}_\varphi^+(P) = \mathcal{V}_\varphi(x) - \mathcal{V}_\varphi(0)$ and $\mathcal{W}_\varphi^+(p) = \mathcal{W}_\varphi(p) - \mathcal{W}_\varphi(0)$. If $\mathcal{U}_\varphi^+(p) + \mathcal{V}_\varphi^+(P) + \mathcal{W}_\varphi^+(p) \leq 3$ for all p$\in N$, then $\varphi^+ =< \mathcal{U}_\varphi^+, \mathcal{V}_\varphi^+, \mathcal{W}_\varphi^+ >$ is a normal NF ideal of N.

Proof: First of all, let us note that $\mathcal{U}_\varphi^+(0) = 1, \mathcal{V}_\varphi^+(0) = 0$ and $\mathcal{W}_\varphi^+(0) = 0$ and $\mathcal{U}_\varphi^+, \mathcal{V}_\varphi^+, \mathcal{W}_\varphi^+ \in [0,1]$ for every p$\in N$ so $\varphi^+ =< \mathcal{U}_\varphi^+, \mathcal{V}_\varphi^+, \mathcal{W}_\varphi^+ >$ is a normal NF set. To prove φ^+ is a NF ideal. Let p, $q \in N$ and $\alpha \in \Gamma$ then

$$\mathcal{U}_\varphi^+(p-q) = \mathcal{U}_\varphi(p-q) + 1 - \mathcal{U}_\varphi(0)$$
$$\geq \{\mathcal{U}_\varphi(p) \wedge \mathcal{U}_\varphi(q)\} + 1 - \mathcal{U}_\varphi(0)$$
$$= \{\mathcal{U}_\varphi(p) + 1 - \mathcal{U}_\varphi(0)\} \wedge \{\mathcal{U}_\varphi(q) + 1 - \mathcal{U}_\varphi(0)\}$$
$$= \mathcal{U}_\varphi^+(p) \wedge \mathcal{U}_\varphi^+(q)$$
$$\mathcal{V}_\varphi^+(p-q) = \mathcal{V}_\varphi(p-q) - \mathcal{V}_\varphi(0)$$
$$\leq \{\mathcal{V}_\varphi(p) \vee \mathcal{V}_\varphi(q)\} - \mathcal{V}_\varphi(0)$$
$$= \{\mathcal{V}_\varphi(p) - \mathcal{V}_\varphi(0)\} \vee \{\mathcal{V}_\varphi(q) - \mathcal{V}_\varphi(0)\}$$
$$= \mathcal{V}_\varphi^+(p) \vee \mathcal{V}_\varphi^+(q)$$
$$\mathcal{W}_\varphi^+(p-q) = \mathcal{W}_\varphi(p-q) - \mathcal{W}_\varphi(0)$$
$$\leq \{\mathcal{W}_\varphi(p) \vee \mathcal{W}_\varphi(q)\} - \mathcal{W}_\varphi(0)$$
$$= \{\mathcal{W}_\varphi(p) - \mathcal{W}_\varphi(0)\} \vee \{\mathcal{W}_\varphi(q) - \mathcal{W}_\varphi(0)\}$$
$$= \mathcal{W}_\varphi^+(p) \vee \mathcal{W}_\varphi^+(q) \text{ and}$$
$$\mathcal{U}_\varphi^+(p\alpha q) = \mathcal{U}_\varphi(p\alpha q) + 1 - \mathcal{U}_\varphi(0)$$
$$\geq \mathcal{U}_\varphi(q) + 1 - \mathcal{U}_\varphi(0) = \mathcal{U}_\varphi^+(q)$$
$$\mathcal{U}_\varphi^+(p\alpha q) \geq \mathcal{U}_\varphi^+(q)$$
$$\mathcal{V}_\varphi^+(p\alpha q) = \mathcal{V}_\varphi(p\alpha q) - \mathcal{V}_\varphi(0)$$

$\leq \mathcal{V}_{\varphi}(q) - \mathcal{V}_{\varphi}(0)=\mathcal{V}_{\varphi}^{+}(q)$

$\mathcal{V}_{\varphi}^{+}(p\alpha q) \leq \mathcal{V}_{\varphi}^{+}(q)$

$\mathcal{W}_{\varphi}^{+}(p\alpha q) = \mathcal{W}_{\varphi}(p\alpha q) - \mathcal{W}_{\varphi}(0)$

$\leq \mathcal{W}_{\varphi}(q) - \mathcal{W}_{\varphi}(0) = \mathcal{W}_{\varphi}^{+}(q)$

$\mathcal{W}_{\varphi}^{+}(p\alpha q) \leq \mathcal{W}_{\varphi}^{+}(q)$

Hence φ^{+} is a NF ideal of a Γ Ring N.

Definition 3.5: Let $X =< \mathcal{U}_X, \mathcal{V}_X, \mathcal{W}_X >$ and $Y =< \mathcal{U}_Y, \mathcal{V}_Y, \mathcal{W}_Y >$ be two NF subsets of a Γ Ring N. Then the Neutrosophic sum of X and Y is $X \oplus Y =< \mathcal{U}_{X\oplus Y}, \mathcal{V}_{X\oplus Y}, \mathcal{W}_{X\oplus Y} >$ in N given by

$$\mathcal{U}_{X\oplus Y}(P) = \begin{cases} \bigvee_{p=q+r} \{\mathcal{U}_X(q) \wedge \mathcal{U}_Y(r)\} & if\ p = q + r, \\ 0 & otherwise \end{cases}$$

$$\mathcal{V}_{X\oplus Y}(P) = \begin{cases} \bigwedge_{p=q+r} \{\mathcal{V}_X(q) \vee \mathcal{V}_X(r)\} & if\ p = q + r, \\ 1 & otherwise \end{cases}$$

$$\mathcal{W}_{X\oplus Y}(P) = \begin{cases} \bigwedge_{p=q+r} \{\mathcal{W}_X(q) \vee \mathcal{W}_Y(r)\} & if\ p = q + r, \\ 1 & otherwise \end{cases}$$

Theorem 3.6: If $X =< \mathcal{U}_X, \mathcal{V}_X, \mathcal{W}_X >$ and $Y =< \mathcal{U}_Y, \mathcal{V}_Y, \mathcal{W}_Y >$ be two NF subsets of a Γ Ring N then the Neutrosophic sum $X \oplus Y =< \mathcal{U}_{X\oplus Y}, \mathcal{V}_{X\oplus Y}, \mathcal{W}_{X\oplus Y} >$ is a NF ideal of Γ Ring.

Proof: For any p, $q \in N$, *we have*

$\mathcal{U}_{X\oplus Y}(p) \wedge \mathcal{U}_{X\oplus Y}(q) = \bigvee \{\mathcal{U}_X(x) \wedge \mathcal{U}_Y(y): p = x + y\} \wedge \bigvee \{\mathcal{U}_X(c) \wedge \mathcal{U}_Y(d): q = c + d\}$

$= \bigvee \{(\mathcal{U}_X(x) \wedge \mathcal{U}_Y(y)) \wedge (\mathcal{U}_X(c) \wedge \mathcal{U}_Y(d)): p = x + y, q = c + d\}$

$= \bigvee \{(\mathcal{U}_X(x) \wedge \mathcal{U}_Y(y)) \wedge (\mathcal{U}_X(-c) \wedge \mathcal{U}_Y(-d)): p = x + y, q = -c - d\}$

$= \bigvee \{(\mathcal{U}_X(x) \wedge \mathcal{U}_X(-c)) \wedge (\mathcal{U}_Y(y) \wedge \mathcal{U}_Y(-d)): p = x + y, q = -c - d\}$

$\leq \bigvee \{(\mathcal{U}_X(x-c) \wedge \mathcal{U}_Y(y-d)): p - q = \{(x-c) + (y-d)\}$

$= \mathcal{U}_{X\oplus Y}(p-q)$

$\mathcal{U}_{X\oplus Y}(p) \wedge \mathcal{U}_{X\oplus Y}(q) \leq \mathcal{U}_{X\oplus Y}(p-q)$

$\mathcal{V}_{X\oplus Y}(p) \vee \mathcal{V}_{X\oplus Y}(q) = \bigwedge \{\mathcal{V}_X(x) \vee \mathcal{V}_Y(y): p = x + y\} \vee \bigwedge \{\mathcal{V}_X(c) \vee \mathcal{V}_Y(d): q = c + d\}$

$= \bigwedge \{(\mathcal{V}_X(x) \vee \mathcal{V}_Y(y)) \vee (\mathcal{V}_X(c) \vee \mathcal{V}_Y(d)): p = x + y, q = c + d\}$

$= \bigwedge \{(\mathcal{V}_X(x) \vee \mathcal{V}_Y(y)) \vee (\mathcal{V}_X(-c) \vee \mathcal{V}_Y(-d)): p = x + y, q = -c - d\}$

$= \bigwedge \{(\mathcal{V}_X(x) \vee \mathcal{V}_X(-c)) \vee (\mathcal{V}_Y(y) \vee \mathcal{V}_Y(-d)): p = x + y, q = -c - d\}$

$\geq \bigwedge \{(\mathcal{V}_X(x-c) \vee \mathcal{V}_Y(y-d)): p - q = (x-c) + (y-d)\}$

$= \mathcal{V}_{X\oplus Y}(p-q)$

$\mathcal{V}_{X\oplus Y}(p) \vee \mathcal{V}_{X\oplus Y}(q) \geq \mathcal{V}_{X\oplus Y}(p-q)$

$\mathcal{W}_{X\oplus Y}(p) \vee \mathcal{W}_{X\oplus Y}(q) = \bigwedge\{\mathcal{W}_X(x)\bigvee \mathcal{W}_Y(y): p = x+y\}\bigvee\bigwedge\ \{\mathcal{W}_X(c)\bigvee\mathcal{W}_Y(d): q = c+d\}$

$= \bigwedge\{(\mathcal{W}_X(x)\bigvee\mathcal{W}(y))\bigvee(\mathcal{W}_X(c)\bigvee\mathcal{W}(d)): p = x+y, q = c+d\}$

$= \bigwedge\{(\mathcal{W}_X(x)\bigvee\mathcal{W}_Y(y))\bigvee(\mathcal{W}_X(-c)\bigvee\mathcal{W}_Y(-d)): p = x+y, q = -c-d\}$

$= \bigwedge(\mathcal{W}_X(x)\bigvee\mathcal{W}_X(-c))\bigvee(\mathcal{W}_Y(y)\bigvee\mathcal{W}_Y(-d)): p = x+y, q = -c-d\}$

$\geq \bigwedge\{(\mathcal{W}_X(x-c)\bigvee\mathcal{W}_Y(Y-d)): p-q = \{(x-c)+(y-d)\}$

$= \mathcal{W}_{X\oplus Y}(p-q)$

$\mathcal{W}_{X\oplus Y}(p) \vee \mathcal{W}_{X\oplus Y}(q) \geq \mathcal{W}_{X\oplus Y}(p-q)$

$\mathcal{U}_{X\oplus Y}(p) = \bigvee\{\mathcal{U}_X(x)\bigwedge\mathcal{U}_Y(y): p = x+y\}$

$\leq \bigvee\{\mathcal{U}_X(x\alpha q)\bigwedge\mathcal{U}_Y(Y\alpha q): p\alpha q = x\alpha q + Y\alpha q\}$

$= \bigvee\{\mathcal{U}_X(U)\bigwedge\mathcal{U}_Y(V): p\alpha q = U+V\}$

$= \mathcal{U}_{X\oplus Y}(p\alpha q)$

$\mathcal{U}_{X\oplus Y}(p\alpha q) \geq \mathcal{U}_{X\oplus Y}(p)$

$\mathcal{V}_{X\oplus Y}(p) = \bigwedge\{\mathcal{V}_X(x)\bigvee\mathcal{V}_Y(y): p = x+y\}$

$\geq \bigwedge\{\mathcal{V}_X(x\alpha q)\bigvee\mathcal{V}_Y(Y\alpha q): p\alpha q = x\alpha q + Y\alpha q\}$

$= \bigwedge\{\mathcal{V}_X(U)\bigvee\mathcal{V}_Y(V): p\alpha q = U+V\} = \mathcal{V}_{X\oplus Y}(p\alpha q)$

$\mathcal{V}_{X\oplus Y}(p\alpha q) \leq \mathcal{V}_{X\oplus Y}(p)$

$\mathcal{W}(p) = \bigwedge\{\mathcal{W}_X(x)\bigvee\mathcal{W}(y): p = x+y\}$

$\geq \bigwedge\{\mathcal{W}_X(x\alpha q)\bigvee\mathcal{W}_Y(Y\alpha q): p\alpha q = x\alpha q + Y\alpha q\}$

$= \bigwedge\{\mathcal{W}_X(U)\bigvee\mathcal{W}_Y(V): p\alpha q = U+V\} = \mathcal{W}_{X\oplus Y}(p\alpha q)$

$\mathcal{W}_{X\oplus Y}(p\alpha q) \leq \mathcal{W}_{X\oplus Y}(p)$

We conclude that $X\oplus Y$ is a NF ideal of N.

Definition 3.7: Suppose that $X =< \mathcal{U}_X, \mathcal{V}_X, \mathcal{W}_X >$ and $Y =< \mathcal{U}_Y, \mathcal{V}_Y, \mathcal{W}_Y >$ be two NF subsets of a Γ Ring N. Then $X \circ Y =< \mathcal{U}_{X\circ Y}, \mathcal{V}_{X\circ Y}, \mathcal{W}_{X\circ Y} >$ in N given by

$$\mathcal{U}_{X\circ Y}(p) = \bigvee\begin{cases}\bigwedge_{1\leq i\leq k}\{\mathcal{U}_X(x_i)\bigwedge\mathcal{U}_Y(y_i)\}: p = \sum_1^k x_i\alpha y_i, x_i y_i \epsilon N, \alpha\epsilon\Gamma, k\epsilon Z^+, \\ 0 \qquad otherwise\end{cases}$$

$$\mathcal{V}_{X\circ Y}(p) = \bigwedge\begin{cases}\bigvee_{1\leq i\leq k}\{\mathcal{V}_X(x_i)\bigvee\mathcal{V}_Y(y_i)\}: p = \sum_1^k x_i\alpha y_i, x_i y_i \epsilon N, \alpha\epsilon\Gamma, k\epsilon Z^+ \\ 1 \qquad otherwise\end{cases}$$

$$\mathcal{W}_{X\circ Y}(p) = \bigwedge\begin{cases}\bigvee_{1\leq i\leq k}\{\mathcal{W}_X(x_i)\bigvee\mathcal{W}_Y(y_i)\}: p = \sum_1^k x_i\alpha y_i, x_i y_i \epsilon N, \alpha\epsilon\Gamma, k\epsilon Z^+ \\ 1 \qquad otherwise\end{cases}$$

Theorem 3.8: If $=< \mathcal{U}_X, \mathcal{V}_X, \mathcal{W}_X >$ and $Y =< \mathcal{U}_Y, \mathcal{V}_Y, \mathcal{W}_Y >$ be two NF subsets of a Γ Ring N then the composition $X \circ Y =< \mathcal{U}_{X\circ Y}, \mathcal{V}_{X\circ Y}, \mathcal{W}_{X\circ Y} >$ is a NF ideal of N.

Proof: For any $p, q \in N$ we have

$$\mathcal{U}_{X\circ Y}(p-q) = \bigvee\{\wedge_{1\le i\le k}\mathcal{U}_X(u_i)\bigwedge\mathcal{U}_Y(v_i): p-q = \sum_1^k u_i\alpha v_i, u_i, v_i\epsilon N, \alpha\epsilon\Gamma, k\epsilon Z^+\} \geq$$

$$\bigvee\{(\wedge_{1\le i\le m}\mathcal{U}_X(x_i)\bigwedge\mathcal{U}_Y(y_i))\bigwedge(\wedge_{1\le i\le n}\mathcal{U}_X(-c_i)\bigwedge\mathcal{U}_Y(d_i))$$

$$: p = \sum_1^m x_i\alpha y_i, -q = \sum_1^n -c_i\alpha d_i, x_i, y, -c_i, d_i\epsilon N, \alpha\epsilon\Gamma \text{ and } m.n\epsilon Z^+\}$$

$$= \bigvee\{(\bigwedge_{1\le i\le m}\mathcal{U}_X(x_i)\bigwedge\mathcal{U}_Y(y_i))\bigwedge(\bigwedge_{1\le i\le n}\mathcal{U}_X(-c_i)\bigwedge\mathcal{U}_Y(d_i))$$

$$: p = \sum_1^m x_i\alpha y_i, q = \sum_1^n c_i\alpha d_i, x_i, y, -c_i, d_i\epsilon N, \alpha\epsilon\Gamma \text{ and } m.n\epsilon Z^+\}$$

$$= \bigvee\{\bigwedge_{1\le i\le m}\mathcal{U}_X(x_i)\bigwedge\mathcal{U}_Y(y_i): p = \sum_1^m x_i\alpha y_i x_i, y_i\epsilon N, \alpha\epsilon\Gamma \text{ and } m\epsilon Z^+\}\bigwedge$$

$$\bigvee\{\bigwedge_{1\le i\le m}\mathcal{U}_X(c_i)\bigwedge\mathcal{U}_Y(d_i): q = \sum_1^n c_i\alpha d_i, x_i, y_i, c_i, d_i\epsilon N, \alpha\epsilon\Gamma \text{ and } n\epsilon Z^+\}$$

$$\mathcal{U}_{X\circ Y}(p-q) \geq \mathcal{U}_{X\circ Y}(p)\wedge\mathcal{U}_{X\circ Y}(q)$$

$$\mathcal{V}_{X\circ Y}(p-q) = \bigwedge\{\bigvee_{1\le i\le k}\mathcal{V}_X(u_i)\bigvee\mathcal{V}_Y(v_i) : p-q = \sum_1^k u_i\alpha v_i, u_i, v_i\epsilon N, \alpha\epsilon\Gamma, k\epsilon Z^+\}$$

$$\leq \bigwedge\{(\bigvee_{1\le i\le k}\{(\mathcal{V}_X(x_i)\bigvee\mathcal{V}_Y(y_i))\bigvee(\bigvee_{1\le i\le n}(\mathcal{V}_X(-c_i)\bigvee\mathcal{V}_Y(d_i))$$

$$: p = \sum_1^m x_i\alpha y_i, -q = \sum_1^n -c_i\alpha d_i, x_i, y_i, -c_i, d_i\in N, \alpha\in\Gamma \text{ and m, n } \in Z^+\}$$

$$= \bigwedge\{(\bigvee_{1\le i\le k}\{(\mathcal{V}_X(x_i)\bigvee\mathcal{V}_Y(y_i))\bigvee(\bigvee_{1\le i\le n}(\mathcal{V}_X(c_i)\bigvee\mathcal{V}_Y(d_i))$$

$$: p = \sum_1^m x_i\alpha y_i,, q = \sum_1^n c_i\alpha d_i, x_i, y_i, -c_i, d_i\in N, \alpha\in\Gamma \text{ and m, n } \in Z^+\}$$

$$= \bigwedge\{\bigvee_{1\le i\le k}\{(\mathcal{V}_X(x_i)\bigvee\mathcal{V}_Y(y_i))\}: p = \sum_1^m x_i\alpha y_i, x_i, y_i, \text{ci}, \text{di}\in N, \alpha\in\Gamma \text{ and m } \in Z^+\}\bigvee\bigwedge$$

$$\{\bigvee_{1\le i\le m}\{\mathcal{V}_X(c_i)\bigvee\mathcal{V}_y(d_i)\}: \text{q} = \sum_1^n \text{ci}\alpha\text{di}, \text{ci}, \text{di}\in\alpha\in\Gamma \text{ and m, n } \in Z^+\}.$$

$$=\mathcal{V}_{X\circ Y}(p)\bigvee\mathcal{V}_{X\circ Y}(q)$$

$\mathcal{V}_{X\circ Y}(p-q)\le \mathcal{V}_{X\circ Y}(p)\bigvee \mathcal{V}_{X\circ Y}(q)$

$\mathcal{W}_{X\circ Y}(p-q)=\bigwedge\{\bigvee_{1\le i\le k}\mathcal{W}_X(u_i)\bigvee \mathcal{W}_Y(v_i):p-q=\sum_1^k u_i\alpha v_i, u_i, v_i\epsilon N,\alpha\epsilon\Gamma, k\epsilon Z^+\}$

$\le \bigwedge\{\bigvee_{1\le i\le k}\{(\mathcal{W}_X(x_i)\bigvee \mathcal{W}_Y(y_i))\bigvee(\bigvee_{1\le i\le n}(\mathcal{W}_X(-c_i)\bigvee \mathcal{W}_Y(d_i))$

$: p=\sum_1^m x_i\alpha y_i$, $-q=\sum_1^n -c_i\alpha d_i$, x_i, y_i, $-c_i, d_i\in N$, $\alpha\in\Gamma$ and m, n $\in Z^+\}$

$=\bigwedge\{\bigvee_{1\le i\le k}\{(\mathcal{W}_X(x_i)\bigvee \mathcal{W}_Y(y_i))\}\bigvee(\bigvee_{1\le i\le n}(\mathcal{W}_X(c_i)\bigvee \mathcal{W}_Y(d_i)))$

$: p=\sum_1^m x_i\alpha y_i$, $q=\sum_1^n c_i\alpha d_i$, $x_i, y_i, c_i, d_i\in\alpha\in\Gamma$ and m, n $\in Z^+\}$

$=\bigwedge\{\bigvee_{1\le i\le k}\{(\mathcal{W}_X(x_i)\bigvee \mathcal{W}_Y(y_i))\}$: p $=\sum_1^m x_i\alpha y_i, x_i, y_i$, ci, di$\in$ N, $\alpha\in\Gamma$ and m, n $\in Z^+\}\bigvee\bigwedge$

$\{\bigvee_{1\le i\le m}\{\mathcal{W}_X(c_i)\bigvee \mathcal{W}_y(d_i)\}$: q $=\sum_1^n$ ciαdi, x_i, y_i, ci, di$\in$N, $\alpha\in\Gamma$ and m, n $\in Z^+\}$.

$=\mathcal{W}_{X\circ Y}(p)\bigvee \mathcal{W}_{X\circ Y}(q)$

$\mathcal{W}_{X\circ Y}(p-q)\le \mathcal{W}_{X\circ Y}(p)\bigvee \mathcal{W}_{X\circ Y}(q)$

$\mathcal{U}_{X\circ Y}(p)=\bigvee\{\left(\bigwedge_{1\le i\le m}\mathcal{U}_X(x_i)\bigwedge \mathcal{U}_Y(y_i)\right):p=\sum_1^m x_i\alpha y_i x_i, y_i\epsilon N,\alpha\epsilon\Gamma\ and\ m\epsilon Z^+\}$

$\le \bigvee\{\left(\bigwedge_{1\le i\le m}\mathcal{U}_X(x_i)\bigwedge \mathcal{U}_Y(y_i\alpha q)\right):p\alpha q=\sum_1^m x_i\alpha(y_i\alpha q)x_i, y_i\alpha q\epsilon N,\alpha\epsilon\Gamma\ and\ m\epsilon Z^+\}$

$=\bigvee\{\left(\wedge_{1\le i\le m}\mathcal{U}(u_i)\bigwedge \mathcal{U}_Y(v_i)\right):p\alpha q=\sum_1^m u_i\alpha v_i, u_i, v_i\epsilon N,\alpha\epsilon\Gamma\ and\ m\epsilon Z^+\}=\mathcal{U}_{X\circ Y}(p\alpha q)$.

$\mathcal{U}_{X\circ Y}(p)\le \mathcal{U}_{X\circ Y}(p\alpha q)$ and similiarly we get $\mathcal{U}_{X\circ Y}(q)\le \mathcal{U}_{X\circ Y}(p\alpha q)$

$\mathcal{V}_{X\circ Y}(p)=\bigwedge\{\bigvee_{1\le i\le m}\{\mathcal{V}_X(x_i)\bigvee \mathcal{V}_Y(y_i)\}:p=\sum_1^m x_i\alpha y_i, x_i, y_i\epsilon N,\alpha\epsilon\Gamma, k\epsilon Z^+\}$

$\ge \bigwedge\{\bigvee_{1\le i\le m}\{(\mathcal{V}_X(x_i)\bigvee \mathcal{V}_Y(y_i\alpha q))\}:\ p\alpha q=\sum_1^m x_i\alpha(y_i\alpha q)x_i, y_i\alpha q\epsilon N,\alpha\epsilon\Gamma\ and\ m\epsilon Z^+\}$

$=\bigwedge\{\bigvee_{1\le i\le m}\{(\mathcal{V}_X(u_i)\bigvee \mathcal{V}_Y(v_i))\}$

$:p\alpha q=\sum_1^m u_i\alpha v_i,\ u_i, v_i\epsilon M,\alpha\epsilon\Gamma\ and\ m\epsilon Z^+\}$ $=\mathcal{V}_{X\circ Y}(p\alpha q)$

$\mathcal{V}_{X\circ Y}(p) \geq \mathcal{V}_{X\circ Y}(p\alpha q)$ and similiarly we get $\mathcal{V}_{X\circ Y}(q) \geq \mathcal{V}_{X\circ Y}(p\alpha q)$

$$\mathcal{W}_{X\circ Y}(p) = \bigwedge\{ \bigvee_{1\leq i\leq m} \{\mathcal{W}_X(x_i) \bigvee \mathcal{W}_Y(y_i)\} : p = \sum_1^m x_i\alpha y_i, x_i, y_i \epsilon N, \alpha\epsilon\Gamma, k\epsilon Z^+\}$$

$$\geq \bigwedge\{ \bigvee_{1\leq i\leq m} \{(\mathcal{W}_X(x_i) \bigvee \mathcal{W}_Y(y_i\alpha q))\} : \ p\alpha q = \sum_1^m x_i\alpha(y_i\alpha q)x_i, y_i\alpha q\epsilon N, \alpha\epsilon\Gamma \ and \ m\epsilon Z^+\}$$

$$= \bigwedge\{ \bigvee_{1\leq i\leq m} \{(\mathcal{W}_X(u_i) \bigvee \mathcal{W}_Y(v_i))\} :$$

$$: p\alpha q = \sum_1^m u_i\alpha v_i, \ u_i, v_i\epsilon N, \alpha\epsilon\Gamma \ and \ m\epsilon Z^+\} = \mathcal{W}_{X\circ Y}(p\alpha q)$$

$\mathcal{W}_{X\circ Y}(p) \geq \mathcal{W}_{X\circ Y}(p\alpha q)$ and similiarly we get $\mathcal{W}_{X\circ Y}(p) \geq \mathcal{W}_{X\circ Y}(p\alpha q)$

Therefore $X \circ Y$ is a NF ideal of N.

Definition 3.9: If $\{ \varphi_i\}_{i\epsilon j}$ be an arbitrary family of NF set in X, where $\varphi_i = \langle \wedge\mathcal{U}_{\eta_i}, \vee \mathcal{V}_{\eta_i}, \vee \mathcal{W}_{\eta_i}, \rangle$ for each $i \in J$. The

$(i) \cap\varphi_i = \langle \wedge\mathcal{U}_{\eta_i}, \vee \mathcal{V}_{\eta_i}, \vee \mathcal{W}_{\eta_i} \rangle$ $(ii) \cup\varphi_i = \langle \vee \mathcal{U}_{\eta_i}, \wedge \mathcal{V}_{\eta_i}, \wedge \mathcal{W}_{\eta_i} \rangle$

Theorem 3.10: If $\{ \varphi_i\}_{i\epsilon j}$ be an arbitrary family of NF set in N, then $\cup\varphi_i = \langle \vee \mathcal{U}_{\varphi_i}, \wedge \mathcal{V}_{\varphi_i}, \wedge \mathcal{W}_{\varphi_i} \rangle$ is a NF ideal of N.

Proof: Let $p, q \in N$ and $\alpha\epsilon \ \Gamma$ then

$\left(\cup_{i\in J} \mathcal{U}_{\varphi_i}\right)$ (p - q) = $\vee_{i\in J} \mathcal{U}_{\varphi_i}(p-q)$

$\geq \vee_{i\in J}(\mathcal{U}_{\varphi_i}(p) \wedge \mathcal{U}_{\varphi_i}(q))$= $\vee_{i\in J}(\mathcal{U}_{\varphi_i}(p)) \wedge \vee_{i\in J}(\mathcal{U}_{\varphi_i}(q))$

=$\left(\cup_{i\in J} \mathcal{U}_{\varphi_i}\right)(p) \wedge \left(\cup_{i\in J} \mathcal{U}_{\varphi_i}\right)(q)$

$\left(\cup_{i\in J} \mathcal{V}_{\varphi_i}\right)$ (p - q) = $\wedge_{i\in J} \mathcal{V}_{\varphi_i}(p-q)$

$\leq \wedge_{i\in J}(\mathcal{V}_{\varphi_i}(p) \vee \mathcal{V}_{\varphi_i}(q))$=$\left(\wedge_{i\in J} \mathcal{V}_{\varphi_i}\right)(p) \vee \left(\wedge_{i\in J} \mathcal{V}_{\varphi_i}\right)(q)$

=$\left(\cup_{i\in J} \mathcal{V}_{\varphi_i}\right)(p) \vee \left(\cup_{i\in J} \mathcal{V}_{\varphi_i}\right)(q)$

$\left(\cup_{i\in J} \mathcal{W}_{\varphi_i}\right)$ (p - q) = $\wedge_{i\in J} \mathcal{W}_{\varphi_i}(p-q)$

$\leq \wedge_{i\in J}(\mathcal{W}_{\varphi_i}(p) \vee \mathcal{W}_{\varphi_i}(q))$=$\left(\wedge_{i\in J} \mathcal{W}_{\varphi_i}\right)(p) \vee \left(\wedge_{i\in J} \mathcal{W}_{\varphi_i}\right)(q)$

=$\left(\cup_{i\in J} \mathcal{W}_{\varphi_i}\right)(p) \vee \left(\cup_{i\in J} \mathcal{W}_{\varphi_i}\right)(q)$

Also $\left(\cup_{i\in J} \mathcal{U}_{\varphi_i}\right)$ (p α q) = $\vee_{i\in J} \mathcal{U}_{\varphi_i}(p\alpha q) \geq \vee_{i\in J} \mathcal{U}_{\varphi_i}(q)$=$\left(\cup_{i\in J} \mathcal{U}_{\varphi_i}\right)(q)$

$\left(\cup_{i\in J} \mathcal{V}_{\varphi_i}\right)$ (p α q) = $\wedge_{i\in J} \mathcal{V}_{\varphi_i}(p\ \alpha q) \leq \wedge_{i\in J} \mathcal{V}_{\varphi_i}$(q)= $\left(\cup_{i\in J} \mathcal{V}_{\varphi_i}\right)(q)$

$\left(\cup_{i\in J} \mathcal{W}_{\varphi_i}\right)$ (p α q) = $\wedge_{i\in J} \mathcal{W}_{\varphi_i}(p\ \alpha q) \leq \wedge_{i\in J} \mathcal{W}_{\varphi_i}$(q)= $\left(\cup_{i\in J} \mathcal{W}_{\varphi_i}\right)(q)$

Similiarly for right ideals

$\left(\cup_{i\in J} \mathcal{U}_{\varphi_i}\right)$ (p α q) = $\vee_{i\in J} \mathcal{U}_{\varphi_i}(p\alpha q) \geq \vee_{i\in J} \mathcal{U}_{\varphi_i}(p)$=$\left(\cup_{i\in J} \mathcal{U}_{\varphi_i}\right)(p)$

$\left(\cup_{i\in J} \mathcal{V}_{\varphi_i}\right)$ (p α q) = $\wedge_{i\in J} \mathcal{V}_{\varphi_i}(p\ \alpha q) \leq \wedge_{i\in J} \mathcal{V}_{\varphi_i}$(p)= $\left(\cup_{i\in J} \mathcal{V}_{\varphi_i}\right)(p)$

$\left(\cup_{i\in J} \mathcal{W}_{\varphi_i}\right)$ (p α q) = $\wedge_{i\in J} \mathcal{W}_{\varphi_i}(p\ \alpha q) \leq \wedge_{i\in J} \mathcal{W}_{\varphi_i}$(p)= $\left(\cup_{i\in J} \mathcal{W}_{\varphi_i}\right)(p)$

Hence$\cup_{i\in J} \varphi_i$ is a NF ideal of N.

Definition 3.11: Let $X =< \mathcal{U}_X, \mathcal{V}_X, \mathcal{W}_X >$ and $Y =< \mathcal{U}_Y, \mathcal{V}_Y, \mathcal{W}_Y >$ be two NF subsets of a Γ Ring N then the product of of X and Y is $X\Gamma Y =< \mathcal{U}_{X\Gamma Y}, \mathcal{V}_{X\Gamma Y}, \mathcal{W}_{X\Gamma Y} >$ in N given by

$$\mathcal{U}_{X\Gamma Y}(P) = \begin{cases} \bigvee_{p=q\alpha r} \{\mathcal{U}_X(q) \bigwedge \mathcal{U}_Y(r)\} & if\ p = q\alpha r \\ 0 & otherwise \end{cases}$$

$$\mathcal{V}_{X\Gamma Y}(P) = \begin{cases} \bigwedge_{p=q\alpha r} \{\mathcal{V}_X(q) \bigvee \mathcal{V}_Y(r)\} & if\ p = q\alpha r \\ 1 & otherwise \end{cases}$$

$$\mathcal{W}_{X\Gamma Y}(P) = \begin{cases} \bigwedge_{p=q\alpha r} \{\mathcal{W}_X(q) \bigvee \mathcal{W}_Y(r)\} & if\ p = q\alpha r \\ 1 & otherwise \end{cases}$$

Theorem 3.12: Assume that $X =< \mathcal{U}_X, \mathcal{V}_X, \mathcal{W}_X >$ and $Y =< \mathcal{U}_Y, \mathcal{V}_Y, \mathcal{W}_Y >$ be NF subsets of a Γ Ring N then $X \cap Y$ is a NF left (resp. right) ideal of N. If X is a NF left ideal and Y is a NF right ideal then $X\Gamma Y \subseteq X \cap Y$

Proof: Suppose X and Y are Neutrosophic contained in M and let $p, q \in N, \alpha \in \Gamma$.

$\mathcal{U}_{X\cap Y}(p - q) = \mathcal{U}_{X\cap Y}(p) \wedge \mathcal{U}_{X\cap Y}(q)$

$\geq [\{\mathcal{U}_X(p) \wedge \mathcal{U}_X(\text{q})\} \wedge \{ \mathcal{U}_Y(p) \wedge \mathcal{U}_Y(q)\}]$

$= [\mathcal{U}_X(p) \wedge \mathcal{U}_Y(p)] \wedge [\mathcal{U}_X(\text{q}) \wedge \mathcal{U}_Y(q)]$

$= \mathcal{U}_{X\cap Y}(p) \wedge \mathcal{U}_{X\cap Y}(q)$

$\mathcal{V}_{X\cap Y}(p - q) = \mathcal{V}_{X\cap Y}(p) \vee \mathcal{V}_{X\cap Y}(q)$

$\leq [\{\mathcal{V}_X(p) \vee \mathcal{V}_X(\text{q})\} \vee \{ \mathcal{V}(p) \vee \mathcal{V}_Y(q)\}]$

$= [\mathcal{V}_X(p) \vee \mathcal{V}_Y(p)] \wedge [\mathcal{V}_X(\text{q}) \vee \mathcal{V}_Y(q)]$

$= \mathcal{V}_{X\cap Y}(p) \vee \mathcal{V}_{X\cap Y}(q)$

$\mathcal{W}_{X\cap Y}(p - q) = \mathcal{W}_{X\cap Y}(p) \vee \mathcal{W}_{X\cap Y}(q)$

$\leq [\{\mathcal{W}_X(p) \vee \mathcal{W}_X(\text{q})\} \square \{ \mathcal{W}(p) \vee \mathcal{W}_Y(q)\}]$

$= [\mathcal{W}_X(p) \vee \mathcal{W}_Y(p)] \wedge [\mathcal{W}_X(\text{q}) \vee \mathcal{W}_Y(q)]$

$= \mathcal{W}_{X\cap Y}(p) \vee \mathcal{W}_{X\cap Y}(q)$

$\mathcal{U}_X(p\alpha q) \geq \mathcal{U}_X(\text{q})$, $\mathcal{V}_X(p\alpha q) \leq \mathcal{V}_X(\text{q})$, and $\mathcal{W}_X(p\alpha q) \leq \mathcal{W}_X(\text{q})$,

$\mathcal{U}_Y(p\alpha q) \geq \mathcal{U}_Y(\text{q})$, $\mathcal{V}_Y(p\alpha q) \leq \mathcal{V}_Y(\text{q})$, and $\mathcal{W}_Y(p\alpha q) \leq \mathcal{W}_Y(\text{q})$,

Clearly X and Y are NF ideal of N, we have,

Now,

$\mathcal{U}_{X\cap Y}(p\alpha q) = \mathcal{U}_X(p\alpha q) \wedge \mathcal{U}_Y(p\alpha q)$

$\geq \mathcal{U}_X(\text{q}) \wedge \mathcal{U}_Y(\text{q}) = \mathcal{U}_{X\cap Y}(\text{q})$

$\mathcal{V}_{X\cap Y}(p\alpha q) = \mathcal{V}_X(p\alpha q) \vee \mathcal{V}_Y(p\alpha q$

$\leq \mathcal{V}_X(\text{q}) \vee \mathcal{V}_Y(\text{q}) = \mathcal{V}_{X\cap Y}(p\alpha q)$

$\mathcal{W}_{X\cap Y}(p\alpha q) = \mathcal{W}_X(p\alpha q) \vee \mathcal{W}_Y(p\alpha q)$

$\leq \mathcal{W}_X(\text{q}) \vee \mathcal{W}_Y(\text{q}) = \mathcal{W}_{X\cap Y}(\text{q})$

Therefore $X \cap Y$ is a NF ideal of N.

To Prove $\mathcal{U}_{X\Gamma Y}(\text{p}) = 0$ and $\mathcal{V}_{X\Gamma Y}(\text{p}) = 1$, $\mathcal{W}_{X\Gamma Y}(\text{p}) = 1$.

Suppose $X\Gamma Y(\text{p}) \neq (0,1)$

The definition of $X\Gamma Y$,

$\mathcal{U}_X(p) = \mathcal{U}_X(q\alpha r) \geq \mathcal{U}_X(q)$, $\mathcal{V}_X(p) = \mathcal{V}_X(q\alpha r) \leq \mathcal{V}_X(q)$ and $\mathcal{W}_X(p) = \mathcal{W}_X(q\alpha r) \leq \mathcal{W}_X(q)$

$\mathcal{U}_Y(p) = \mathcal{U}_Y(q\alpha r) \geq \mathcal{U}_Y(q)$, $\mathcal{V}_Y(p) = \mathcal{V}_Y(q\alpha r) \leq \mathcal{V}_Y(q)$ and $\mathcal{W}_Y(p) = \mathcal{W}_Y(q\alpha r) \leq \mathcal{W}_Y(q)$

Since X is a NF right ideal and Y is a NF left ideal of N, we have

$\mathcal{U}_X(p) = \mathcal{U}_X(q\alpha r) \geq \mathcal{U}_X(q)$,$\mathcal{V}_X(p) = \mathcal{V}_X(q\alpha r) \leq \mathcal{V}_X(q)$and $\mathcal{W}_X(p) = \mathcal{W}_X(q\alpha r) \leq \mathcal{W}_X(q)$

$\mathcal{U}_Y(p) = \mathcal{U}_Y(q\alpha r) \geq \mathcal{U}_Y(r)$,$\mathcal{V}_Y(p) = \mathcal{V}_Y(q\alpha r) \leq \mathcal{V}_Y(r)$and $\mathcal{W}_Y(p) = \mathcal{W}_Y(q\alpha r) \leq \mathcal{W}_Y(r)$

By the Definition of $X\Gamma Y$

$$\mathcal{U}_{X\Gamma Y}(p) = \bigvee_{\mathrm{p=q\alpha r}} \{\mathcal{U}_{\mathrm{X}}(\mathrm{q}) \bigwedge \mathcal{U}_{\mathrm{Y}}(\mathrm{r})\} \leq \mathcal{U}_{\mathrm{X}}(p) \bigwedge \mathcal{U}_{\mathrm{Y}}(p) = \mathcal{U}_{\mathrm{X}\cap Y}(p),$$

$$\mathcal{V}_{X\Gamma Y}(p) = \bigwedge_{p=q\alpha r} \{\mathcal{V}_X(q) \bigvee \mathcal{V}_Y(r)\} \geq \{\mathcal{V}_X(q) \bigvee \mathcal{V}_Y(r)\} = \mathcal{V}_{\mathrm{X}\cap Y}(p)$$

$$\mathcal{W}_{X\Gamma Y}(p) = \bigwedge_{p=q\alpha r} \{\mathcal{W}_X(q) \bigvee \mathcal{W}_Y(r)\} \geq \{\mathcal{W}_X(q) \bigvee \mathcal{W}_Y(r)\} = \mathcal{W}_{\mathrm{X}\cap Y}(p)$$

Consequently, $X\Gamma Y \subseteq X \cap Y$

Corollary 3.13: If $X =< \mathcal{U}_X, \mathcal{V}_X, \mathcal{W}_X >$ and $Y =< \mathcal{U}_Y, \mathcal{V}_Y, \mathcal{W}_Y >$ be two neutrosophic fuzzy subsets of a Γ Ring N, then X ∪ Y is a NF ideal of N.

Definition 3.14: A Γ Ring N is regular if there exists $p \in N, \forall\, x \in N$ and $\alpha, \beta \in \Gamma$ then x=x$\alpha p\beta x$

Result 3.15: A Γ Ring N is said to be regular $\Leftrightarrow if\ I\Gamma J = I \cap J$ for each right ideal I and for each left ideal J of N.

Theorem 3.16: A Γ Ring N is regular if for each NF right ideal X and for each NF left ideal Y of N, $X\Gamma Y = X \cap Y$.

Proof. Suppose that N is regular.

By theorem 3.12, $X\Gamma Y \subseteq X \cap Y$

Therefore, it is sufficient to prove $X \cap Y \subseteq X\Gamma Y$

Let x$\in N, \alpha, \beta \in \Gamma$

By definition, there exists p$\in N$ such that x = x$\alpha p\beta x$

$\mathcal{U}_X(x) = \mathcal{U}_X(\mathrm{x}\alpha p\beta x) \geq \mathcal{U}_X(\mathrm{x}\alpha p) \geq \mathcal{U}_X(x), \mathcal{V}_X(x) = \mathcal{V}_X(\mathrm{x}\alpha p\beta x) \leq \mathcal{V}_X(\mathrm{x}\alpha p) \leq \mathcal{V}_X(x).$

$\mathcal{W}_X(x) = \mathcal{W}_X(\mathrm{x}\alpha p\beta x) \leq \mathcal{W}_X(\mathrm{x}\alpha p) \leq \mathcal{W}_X(x).$

So, $\mathcal{U}_X(\mathrm{x}\alpha p) \geq \mathcal{U}_X(x), \mathcal{V}_X(\mathrm{x}\alpha p) \leq \mathcal{V}_X(x)\ and\ \mathcal{W}_X(\mathrm{x}\alpha p) \leq \mathcal{W}_X(x).$

Furthermore,

$$\mathcal{U}_{X\Gamma Y}(x) = \bigvee_{\mathrm{x=x}\alpha p\beta x} \{\mathcal{U}_X(\mathrm{x}\alpha p) \bigwedge \mathcal{U}_Y(x)\} \geq \{\mathcal{U}_{\mathrm{X}}(x) \bigwedge \mathcal{U}_{\mathrm{Y}}(x)\} = \mathcal{U}_{\mathrm{X}\cap Y}(x),$$

$$\mathcal{V}_{X\Gamma Y}(x) = \bigwedge_{\mathrm{x=x}\alpha p\beta x} \{\mathcal{V}_X(\mathrm{x}\alpha p) \bigvee \mathcal{V}(x)\} \leq \{\mathcal{V}_{\mathrm{X}}(x) \bigvee \mathcal{V}_{\mathrm{Y}}(x)\} = \mathcal{V}_{\mathrm{X}\cap Y}(x),$$

$$\mathcal{W}_{X\Gamma Y}(x) = \bigwedge_{\mathrm{x=x}\alpha p\beta x} \{\mathcal{W}_X(\mathrm{x}\alpha p) \bigvee \mathcal{W}(x)\} \leq \{\mathcal{W}_{\mathrm{X}}(x) \bigvee \mathcal{W}_{\mathrm{Y}}(x)\} = \mathcal{W}_{\mathrm{X}\cap Y}(x),$$

Thus X$\cap Y \subseteq X\Gamma Y$. Hence $X\Gamma Y =$ X$\cap Y$.

Definition 3.17: An ideal φ of the Γ Ring N is said to be prime if for any ideals X and Y of N,

$X\Gamma Y \subseteq \varphi \Longrightarrow X \subseteq \varphi$ or $Y \subseteq \varphi$.

Definition 3.18: Let φ be a NF ideal of a Γ Ring N. Then φ is said to be prime if φ is not a constant mapping and for any neutrosophic X, Y of a Γ Ring N, $X\Gamma Y \subseteq \varphi$ implies $X \subseteq \varphi$ or $Y \subseteq \varphi$.

Theorem 3.19: Let $\mathcal{J}$ be an ideal of a Γ Ring N. ∋ $\mathcal{J} \neq N$ Then $\mathcal{J}$ is a prime ideal of N iff $\left(\mathcal{U}_{\chi_{\mathcal{J}}}, \mathcal{V}_{\overline{\chi}_{\mathcal{J}}}, \mathcal{W}_{\overline{\chi}_{\mathcal{J}}}\right)$ is a NF prime ideal of N.

Proof: $(\Rightarrow)$ Suppose $\mathcal{J}$ is a prime ideal of N. and let $\varphi = \left(\mathcal{U}_{\chi_{\mathcal{J}}}, \mathcal{V}_{\overline{\chi}_{\mathcal{J}}}, \mathcal{W}_{\overline{\chi}_{\mathcal{J}}}\right)$. Since $\mathcal{J} \neq N$. φ is not a constant mapping on N. Let X and Y be two NF ideal of N such that $X\Gamma Y \subseteq \varphi$ and $X \not\subset \varphi$ or $Y \not\subset \varphi$, then $\exists\, p, q \in N$ such that

$\mathcal{U}_X(p) > \mathcal{U}_\varphi(p) = \mathcal{U}_{\chi_{\mathcal{J}}}(p), \mathcal{V}_X(p) < \mathcal{V}_\varphi(p) = \mathcal{V}_{\overline{\chi}_{\mathcal{J}}}(p)$ and $\mathcal{W}_X(p) < \mathcal{W}_\varphi(p) = \mathcal{W}_{\overline{\chi}_{\mathcal{J}}}(p)$,

$\mathcal{U}_Y(p) > \mathcal{U}_\varphi(p) = \mathcal{U}_{\chi_{\mathcal{J}}}(p), \mathcal{V}_Y(p) < \mathcal{V}_\varphi(p) = \mathcal{V}_{\overline{\chi}_{\mathcal{J}}}(p)$ and $\mathcal{W}_Y(p) < \mathcal{W}_\varphi(p) = \mathcal{W}_{\overline{\chi}_{\mathcal{J}}}(p)$

Thus $\mathcal{U}_X(p) \neq 0, \mathcal{V}_X(p) \neq 1, \mathcal{W}_X(p) \neq 1$ *and* $\mathcal{U}_Y(q) \neq 0, \mathcal{V}_Y(q) \neq 1, \mathcal{W}_Y$. But $\mathcal{U}_{\chi_{\mathcal{J}}}(p) = 0, \mathcal{V}_{\chi_{\mathcal{J}}}(p) = 0$ *and* $\mathcal{W}_{\chi_{\mathcal{J}}}(p) = 0$, so $p \notin \mathcal{J}, q \notin \mathcal{J}$. *Since* $\mathcal{J}$ is a prime ideal of N, by the Theorem 5[3] there exists r ∈ N and $\alpha, \beta \in \Gamma$. such that $p\alpha r\beta q \notin \mathcal{J}$. Let c= $p\alpha r\beta q$ then $\mathcal{U}_{\chi_{\mathcal{J}}}(c) = 0, \mathcal{V}_{\overline{\chi}_{\mathcal{J}}}(c)$ *and* $\mathcal{W}_{\overline{\chi}_{\mathcal{J}}}(c)$=1.Thus $X\Gamma Y(c) = (0,1)$. But $\mathcal{U}_{X\Gamma Y}(c) = \vee_{c=m\gamma n}[\ \mathcal{U}_X(m) \wedge \mathcal{U}_Y(n)] \geq \mathcal{U}_X(p\alpha r) \wedge \mathcal{U}_Y(q)]$ (since c=p$\alpha r\beta q$) $\geq \mathcal{U}_X(p) \wedge \mathcal{U}_Y(q) > 0$. (since $\mathcal{U}_X(p) \neq 0$ *and* $\mathcal{U}_Y(p) \neq 0$)

$\mathcal{V}_{X\Gamma Y}(c) = \ \wedge_{c=m\gamma n}[\mathcal{V}_X(m) \vee \mathcal{V}_Y(n)] \leq [\mathcal{V}_X(p\alpha r) \vee \mathcal{V}_Y(q)] \leq \mathcal{V}_X(p) \vee \mathcal{V}_X(q)<1$

(since $\mathcal{V}_X(p) \neq 1$ *and* $\mathcal{V}_Y(p) \neq 1$).

$$\mathcal{W}_{X\Gamma Y}(c) = \bigwedge_{c=m\gamma n} [\mathcal{W}_X(m) \vee \mathcal{W}_Y(n)] \ \leq [\mathcal{W}_X(p\alpha r) \vee \mathcal{W}_Y(q)] \leq \mathcal{W}_X(p) \vee \mathcal{W}_X(q) < 1$$

(since $\mathcal{W}_X(p) \neq 1$ *and* $\mathcal{W}(p) \neq 1$.)Then X𝛤$B(c) \neq (0,1)$. This contradicts the result. Then for any two NF ideals Xand Y $X\Gamma Y \subseteq \varphi$. implies $A \subseteq \varphi$ or $B \subseteq \varphi$. Hence φ is a NF ideals of N.

(⇐) Suppose $\varphi = \left(\mathcal{U}_{\chi_{\mathcal{J}}}, \mathcal{V}_{\overline{\chi}_{\mathcal{J}}}, \mathcal{W}_{\overline{\chi}_{\mathcal{J}}}\right)$ is a NF prime ideal of N. Since φ is not a constant mapping on N, $\varphi \neq N$. Let X, Y be two ideals of N such that $X\Gamma Y \subseteq \mathcal{J}$ and let $\overline{X} = \left(\mathcal{U}_{\chi_X}, \mathcal{V}_{\overline{\chi}_X}, \mathcal{W}_{\overline{\chi}_X}\right)$ and $\overline{Y} = \left(\mathcal{U}_{\chi_Y}, \mathcal{V}_{\overline{\chi}_Y}, \mathcal{W}_{\overline{\chi}_Y}\right)$ be two fuzzy ideals of N. Consider the product $\overline{X}\Gamma\overline{Y}$. let $p \in N$ if $\overline{X}\Gamma\overline{Y}(p) = (0,1)$ then $\overline{X}\Gamma\overline{Y} \subseteq \mathcal{U}$. Suppose $\overline{X}\Gamma\overline{Y} \neq (0,1)$ then $\mathcal{U}_{\overline{X}\Gamma\overline{Y}}(p) = \vee_{p=q\gamma r}[\ \mathcal{U}_{\chi_X}(q) \wedge \mathcal{U}_{\chi_Y}(r)]\ \neq 0$, $\mathcal{V}_{\overline{X}\Gamma\overline{Y}}(p) = \wedge_{p=q\gamma r}[\mathcal{V}_{\overline{\chi}_X}(q) \vee \mathcal{V}_{\overline{\chi}_Y}(r)]\ \neq 1\}$ and $\mathcal{W}_{\overline{X}\Gamma\overline{Y}}(p) = \wedge_{p=q\gamma r}\left[\mathcal{W}_{\overline{\chi}_X}(q) \vee \mathcal{W}_{\overline{\chi}_Y}(r)\right] \neq 1$.There exist $q, r \in N$. with p=qαr such that $\mathcal{U}_{\chi_X}(q) \neq 0$.$\mathcal{V}_{\overline{\chi}_X}(q) \neq 1$*and* $\mathcal{W}_{\overline{\chi}_X}(q) \neq 1$, $\mathcal{U}_Y(r) \neq 0, \mathcal{V}_{\overline{\chi}_Y}(r) \neq 1. \mathcal{W}_{\overline{\chi}_Y}(r) \neq 1$. So $\mathcal{U}_{\chi_x}(q) = 1$, $\mathcal{V}_{\overline{\chi}_x}(q) = 0$, $\mathcal{W}_{\overline{\chi}_Y}(q) = 0$ and $\mathcal{U}_{\chi_Y}(r) = 1$, $\mathcal{V}_{\overline{\chi}_Y}(r) = 0, \mathcal{W}_{\overline{\chi}_Y}(r) = 0$. This implies $q \in X$ *and* $r \in Y$. Thus p=q$\alpha r \in X\Gamma Y \subseteq \mathcal{J}$, So $\mathcal{U}_{\chi_{\mathcal{J}}}(p) = 1, \mathcal{V}_{\overline{\chi}_{\mathcal{J}}}(p) = 0$ *and* $\mathcal{W}_{\overline{\chi}_{\mathcal{J}}}(p)$=0. It follows that $\overline{X}\Gamma\overline{Y}(p) \subseteq \varphi$ .Since φ is a NF ideal of N, either $\overline{X} \subseteq \varphi$ or $\overline{Y} \subseteq \varphi$. Thus either X ⊆ φ or Y ⊆ φ. Hence $\mathcal{J}$ is a prime ideal of N.

Definition 3.20: (Neutrosophic Γ endomorphism) Mapping $\theta: N \rightarrow N$ of the Γ Ring N into itself is called a neutrosophic Γ-endomorphism of N. If for $p, q \in N$, $\alpha \epsilon$ Γ then

$(i)\mathcal{U}(p+q)\theta = \mathcal{U}(p\theta) + \mathcal{U}(q\theta), \mathcal{V}(p+q)\theta = \mathcal{V}(p\theta) + \mathcal{V}(q\theta)$ *and*

$\mathcal{W}(p+q)\theta = \mathcal{W}(p\theta) + \mathcal{W}(q\theta)$….. (1)

$(ii)\mathcal{U}(p\alpha q)\theta = \mathcal{U}(p\theta\alpha q\theta), \mathcal{V}(p\alpha q)\theta = \mathcal{V}(p\theta\alpha q\theta)$ *and*

$\mathcal{W}(p\alpha q)\theta = \mathcal{W}(p\theta\alpha q\theta)$……………(2)

Let Δ represent the group of Γ -endomorphism of the Γ Ring N. The multiplication and addition on the set as Δ follows, If $x, y \in \Delta$ then

$\mathcal{U}(p(x\alpha y)) = \mathcal{U}((px)\alpha y)\ p \in N, \alpha \epsilon\ \Gamma, \mathcal{V}(p(x\alpha y)) = \mathcal{V}((px)\alpha y)\ p \in N, \alpha \epsilon\ \Gamma$ and

$\mathcal{W}(p(x\alpha y)) = \mathcal{W}((px)\alpha y)\ p \in N,\ \alpha \epsilon\ \Gamma \ldots\ldots\ldots\ldots$ (3)

$\mathcal{U}(p(x+y)) = \mathcal{U}(px) + \mathcal{U}(py))\ p \in N,\ \mathcal{V}(p(x+y)) = \mathcal{V}(px) + \mathcal{V}(py) p \in N$

$\mathcal{W}(p(x+y)) = \mathcal{W}(px) + \mathcal{W}(py) p \in N \ldots\ldots\ldots$ (4)

Theorem 3.21: If Δ be the group of all neutrosophic Γ -endomorphism of a Γ Ring N. Then Δ is a Γ -endomorphism of a Γ Ring with unity with respect to usual operations**.**

Proof: Given Δ be the set of all Neutrosophic Γ -endomorphism of a Γ -ring M.

To Prove Δ is a Γ Ring with Unity and Let $x, y, z \in \Delta, \alpha \epsilon\ \Gamma$, p $\epsilon\ N$,

(i) $\mathcal{U}(x$ ((a + b) α c) = $\mathcal{U}((x(a+b)\alpha c))$

= $\mathcal{U}((xa + xb)\alpha c)$

= $\mathcal{U}((xa)\alpha c + (xb)\alpha c)$

=$\mathcal{U}(x(a\alpha c) + x(b\alpha c))$

= $\mathcal{U}(x(a\alpha c + b\alpha c))$

Hence $\mathcal{U}((a+b)\alpha c) = \mathcal{U}(a\alpha c + b\alpha c)$

$\mathcal{V}(x$((a+b) α c) = $\mathcal{V}((x(a+b)\alpha c))$

= $\mathcal{V}((xa + xb)\alpha c)$

= $\mathcal{V}((xa)\alpha c + (xb)\alpha c)$

=$\mathcal{V}(x(a\alpha c) + x(b\alpha c))$

= $\mathcal{V}(x(a\alpha c + b\alpha c))$

Hence $\mathcal{V}((a+b)\alpha c)) = \mathcal{V}(a\alpha c + b\alpha c)$

$\mathcal{W}(x$((a+b) α c) = $\mathcal{W}((x(a+b)\alpha c))$

= $\mathcal{W}((xa + xb)\alpha c)$

= $\mathcal{W}((xa)\alpha c + (xb)\alpha c)$

=$\mathcal{W}(x(a\alpha c) + x(b\alpha c))$

= $\mathcal{W}(x(a\alpha c + b\alpha c)$

Hence $\mathcal{W}((a+b)\alpha c)) = \mathcal{W}(a\alpha c + b\alpha c)$

Now $\mathcal{U}(x$ (a(α+β)c)= $\mathcal{U}((xa)(\alpha+\beta)$c a,c∈△, $\alpha, \beta \in \Gamma$, x $\epsilon\ N$

= $\mathcal{U}((xa)\alpha$c + $(xa)\beta c)$

= $\mathcal{U}(x(a\alpha$c +$a\beta c))$

$\mathcal{U}$((a(α+β)c)= $\mathcal{U}(a\alpha$c +$a\beta c)$

$\mathcal{V}(x$ (a(α+β)c)= $\mathcal{V}((xa)(\alpha+\beta)$c a,c∈△, $\alpha, \beta \in \Gamma$, x $\epsilon\ N$

= $\mathcal{V}((xa)\alpha$c + $(xa)\beta c)$

= $\mathcal{V}(x(a\alpha$c +$a\beta c))$

$\mathcal{V}$((a(α+β)c)= $\mathcal{V}(a\alpha$c +$a\beta c)$

$\mathcal{W}(x$ (a(α+β)c)= $\mathcal{W}((xa)(\alpha+\beta)$c a,c∈△, $\alpha, \beta \in \Gamma$, x $\epsilon\ N$

= $\mathcal{W}((xa)\alpha$c + $(xa)\beta c)$

= $\mathcal{W}(x(a\alpha$c +$a\beta c))$

$\mathcal{W}$((a(α+β)c)= $\mathcal{W}((a\alpha$c +$a\beta c))$

Again,

$\mathcal{U}$ (x (aα(b+c)))= $\mathcal{U}((xa)\alpha(b+c))$ a,b,c$\in\triangle, \alpha \in \Gamma$, x ϵ N

= $\mathcal{U}((xa)\alpha b) + \mathcal{U}((xa)\alpha c)$

= $\mathcal{U}(x(a\alpha b) + (x(a\alpha c))$

= $\mathcal{U}(x(a\alpha c + b\alpha c))$

Hence $\mathcal{U}$(aα(b + c)) = $\mathcal{U}((a\alpha b + a\alpha c))$

$\mathcal{V}$ (x (aα(b+c)))= $\mathcal{V}$ $((xa)\alpha(b+c))$ a,b,c$\in\triangle, \alpha \in \Gamma$, x ϵ N

= $\mathcal{V}$ $((xa)\alpha b) + \mathcal{V}$ $((xa)\alpha c)$

= $\mathcal{V}$ $(x(a\alpha b) + (x(a\alpha c))$

= $\mathcal{V}$ $(x(a\alpha c + b\alpha c))$

Hence $\mathcal{V}$ (aα(b + c)) = $\mathcal{V}$ $((a\alpha b + a\alpha c))$

$\mathcal{W}$(x (aα(b+c)))= $\mathcal{W}((xa)\alpha(b+c))$ a,b,c$\in\triangle, \alpha \in \Gamma$, x ϵ N

= $\mathcal{W}((xa)\alpha b) + \mathcal{U}((xa)\alpha c)$

= $\mathcal{W}(x(a\alpha b) + (x(a\alpha c))$

= $\mathcal{W}(x(a\alpha c + b\alpha c))$

Hence $\mathcal{W}$(aα(b + c)) = $\mathcal{W}((a\alpha b + a\alpha c))$

$(ii)\mathcal{U}$ ((x(aαb)βc))= $\mathcal{U}((x(a\alpha b))\beta$c), a,b,c$\in\triangle, \alpha, \beta \in \Gamma$, x ϵ N

= $\mathcal{U}(((xa)\alpha b)\beta$c)

= $\mathcal{U}((xa)\alpha(b\beta$c))

= $\mathcal{U}(x\big(a\alpha(b\beta$c)\big))

= $\mathcal{U}(x\big(a\alpha(b\beta$c))\big)

Hence $\mathcal{U}$((aαb)βc)= $\mathcal{U}\big(a\alpha(b\beta$c)\big)

$\mathcal{V}$ (x((aαb)βc)))= $\mathcal{V}((x(a\alpha b))\beta$c), a,b,c$\in\triangle, \alpha, \beta \in \Gamma$, x ϵ N

= $\mathcal{V}(((xa)\alpha b)\beta$c)

= $\mathcal{V}((xa)\alpha(b\beta$c))

= $\mathcal{V}(x\big(a\alpha(b\beta$c)\big))

= $\mathcal{V}(x\big(a\alpha(b\beta$c)\big)

Hence$\mathcal{V}$((aαb)βc)= $\mathcal{V}\big(a\alpha(b\beta$c)\big)

$\mathcal{W}$((x(aαb)βc))) = $\mathcal{W}((x(a\alpha b))\beta$c), a,b,c $\in\triangle, \alpha, \beta \in \Gamma$, x ϵ N

= $\mathcal{W}(((xa)\alpha b)\beta$c)

= $\mathcal{W}(xa)\alpha(b\beta$c))

= $\mathcal{W}(x\big(a\alpha(b\beta$c)\big))

= $\mathcal{W}(x\big(a\alpha(b\beta$c)\big)

Hence $\mathcal{W}$((aαb)βc)= $\mathcal{W}\big(a\alpha(b\beta$c)\big)

(iii) For all a$\in\triangle$ then there exists unity element 1$\in\triangle$ such that

$\mathcal{U}$(x(1αa)) = $\mathcal{U}$(((x1)α)a) = $\mathcal{U}(xa), \alpha \in \Gamma$, x ϵ N, $\mathcal{V}$(x(1αa)) = $\mathcal{V}$(((x1)α)a) = $\mathcal{V}(xa), \alpha \in \Gamma$, x ϵ N,

And $\mathcal{W}$(x(1αa)) = $\mathcal{W}$(((x1)α)a) = $\mathcal{W}(xa), \alpha \in \Gamma$, x ϵ N,

And $\mathcal{U}$(x(aα1))= $\mathcal{U}$((xa)α)1) = xa, $\mathcal{V}$(x(aα1))= $\mathcal{V}$((xa)α)1) = xa, and

$\mathcal{W}$(x(aα1)) = $\mathcal{W}$((xa)α)1) = xa

Hence $\mathcal{U}$(aα1) = $\mathcal{U}$(1αa) = a, $\mathcal{V}$(aα1)= $\mathcal{V}$(1αa) = a, and $\mathcal{W}$(aα1)= $\mathcal{W}$(1αa) = a.

Thus $\triangle$ satisfies all the conditions of Γ Ring. Hence $\triangle$ is a Γ Ring with unity.

Theorem 3.22: Let Δ be the set of all neutrosophic Γ endomorphism of the Γ Ring N. If $x\epsilon\Delta$ then x has (Multiplicative inverse) in Δ if and only if x is one to one function.

Proof: Assume Δ be the set of all neutrosophic Γ -endomorphism of a Γ -ring M. If $x\epsilon\Delta$ then x has an inverse in Δ. To prove x is one to one function. Let x has an inverse y in Δ. $x\alpha y = y\alpha x = 1, \alpha\epsilon\Gamma$.

Then for each $p\epsilon N \cdot$ we get

$\mathcal{U}((py)\alpha x) = \mathcal{U}(p(y\alpha x)) = \mathcal{U}(p), \mathcal{V}((py)\alpha x) = \mathcal{V}(p(y\alpha x)) = \mathcal{V}(p)$ and

$\mathcal{W}((py)\alpha x) = \mathcal{W}(p(y\alpha x)) = \mathcal{W}(p)$ Clearly x is onto.

Furthermore $p_1, p_2\epsilon N$ such that

$\mathcal{U}(p_1x) = \mathcal{U}(p_2x), \mathcal{V}(p_1x) = \mathcal{V}(p_2x)$, and $\mathcal{W}(p_1x) = \mathcal{W}(p_2x)$,

$\mathcal{U}(p_1) = \mathcal{U}(p_1.1) = \mathcal{U}(p_1(x\alpha y)) = \mathcal{U}((p_1.x)\alpha y) = \mathcal{U}((p_2.x)\alpha y) = \mathcal{U}(p_2(x\alpha y)) = \mathcal{U}(p_2.1) = \mathcal{U}(p_2)$

$\mathcal{V}(p_1) = \mathcal{V}(p_1.1) = \mathcal{V}(p_1(x\alpha y)) = \mathcal{V}((p_1.x)\alpha y) = \mathcal{V}((p_2.x)\alpha y) = \mathcal{V}(p_2(x\alpha y)) = \mathcal{V}(p_2.1) = \mathcal{V}(p_2)$.

$\mathcal{W}(p_1) = \mathcal{W}(p_1.1) = \mathcal{W}(p_1(x\alpha y)) = \mathcal{W}((p_1.x)\alpha y) = \mathcal{W}((p_2.x)\alpha y) = \mathcal{W}(p_2(x\alpha y)) = \mathcal{W}(p_2)$.

Therefore x is one to one mapping.

Conversely, Let us assume that the Γ -endomorphism x is one to one mapping of N onto N. So that each element of N is of the form $px, p\epsilon N$. We define a mapping y of N into N as follows

$\mathcal{U}\left(((px)\alpha)y\right) = \mathcal{U}(p), p\epsilon N, \alpha\epsilon\Gamma$. If $p, q\epsilon N$ then

$\mathcal{U}\left(((px+qx)\alpha)y\right)$=$\mathcal{U}((((p+q)x)\alpha)y)$=$\mathcal{U}(p+q)$= $\mathcal{U}(((px)\alpha)y) + \mathcal{U}(((qx)\alpha)y)$ =

$\mathcal{U}(((px\alpha qx)\alpha)y) = \mathcal{U}(((p\alpha q)x\alpha)y) = \mathcal{U}(p\alpha q) = \mathcal{U}((px)\alpha)yx(((qx)\alpha)y)$

$\mathcal{V}\left(((px)\alpha)y\right) = \mathcal{V}(p), p\epsilon N, \alpha\epsilon\Gamma$. If $p, q\epsilon N$ then

$\mathcal{V}\left(((px+qx)\alpha)y\right)$=$\mathcal{V}((((p+q)x)\alpha)y)$=$\mathcal{V}((p+q)$=$\mathcal{V}(((px)\alpha)y + \mathcal{V}(((qx)\alpha)y)$=

$\mathcal{V}(((px\alpha qx)\alpha)y) = \mathcal{V}(((p\alpha q)x\alpha)y) = \mathcal{V}(p\alpha q) = \mathcal{V}((px)\alpha)yx(((qx)\alpha)y)$

$\mathcal{W}\left(((px)\alpha)y\right) = \mathcal{W}(p), p\epsilon N, \alpha\epsilon\Gamma$. If $p, q\epsilon N$ then

$\mathcal{W}\left(((px+qx)\alpha)y\right)$=$\mathcal{W}((((p+q)x)\alpha)y)$=$\mathcal{W}((p+q)$=$\mathcal{W}(((px)\alpha)y + \mathcal{W}(((qx)\alpha)y)$=

$\mathcal{W}(((px\alpha qx)\alpha)y) = \mathcal{W}(((p\alpha q)x\alpha)y) = \mathcal{W}(p\alpha q) = \mathcal{W}((px)\alpha)yx(((qx)\alpha)y)$

We see that y is a neutrosophic Γ endomorphism of N. Furthermore $\mathcal{U}((px)\alpha y) = \mathcal{U}(p(x\alpha y)) = \mathcal{U}(p)$ For every p in N and hence $x\alpha y = 1$ finally $p\epsilon N$, $\mathcal{U}(((px)\alpha)(y\alpha x)) = \mathcal{U}\left((p(x\alpha y))\alpha x\right) = \mathcal{U}(p(1)\alpha x) = \mathcal{U}(p(1\alpha x)) = \mathcal{U}(px)$, $\mathcal{V}((px)\alpha y) = \mathcal{V}(p(x\alpha y) = \mathcal{V}(p)$ For every p in N and hence $x\alpha y = 1$ finally $p\epsilon N$, $\mathcal{V}(((px)\alpha)(y\alpha x)) = \mathcal{V}\left((p(x\alpha y))\alpha x\right) = \mathcal{V}(p(1)\alpha x) = \mathcal{V}(p(1\alpha x)) = \mathcal{V}(px)$, and $\mathcal{W}((px)\alpha y) = \mathcal{W}(p(x\alpha y) = \mathcal{W}(p)$ For every p in N and hence $x\alpha y = 1$ finally $p\epsilon N$, $\mathcal{W}(((px)\alpha)(y\alpha x)) = \mathcal{W}\left((p(x\alpha y))\alpha x\right) = \mathcal{W}(p(1)\alpha x) = \mathcal{W}(p(1\alpha x)) = \mathcal{W}(px)$,. That is equivalent to the statement that $\mathcal{U}(q(y\alpha x)) = \mathcal{U}(q), \mathcal{V}(q(y\alpha x)) = \mathcal{V}(q)$ and $\mathcal{W}(q(y\alpha x)) = \mathcal{W}(q)$. For every $q\ \epsilon N$. Hence $y\alpha x = 1$ and y is the inverse of x in Δ.

4. Conclusions

In recent years, many algebraic structures have been considered neutrosophic structures. Using neutrosophic environments, we analyzed gamma rings. NF prime ideals are introduced in this article, along with their basic algebraic properties. In addition, some new neutrosophic operations are discussed.

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