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A New Approach to Neutrosophic Hypersoft Rough Sets.

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Abstract. The aim of this work is to introduce the new notion of Neutrosophic hypersoft rough set and study its properties. Neutrosophic hypersoft rough set is a generalization of Neutrosophic soft rough set. The notion of Neutrosophic hypersoft rough set has not been reported in the literature. The concept of Neutrosophic hypersoft approximation space is presented with illustrative examples and some of its properties are established. The notions of equality, reduct and core among Neutrosophic hypersoft rough sets are studied with suitable examples. Some directions for applications and future research in this area are also indicated.

Keywords: Hypersoft sets, Neutrosophic sets, Neutrosophic soft sets, Neutrosophic hypersoft sets, Neutrosophic hypersoft rough sets, Equality, Reduct and Core.

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1. Introduction

Extension of soft set to hypersoft set was discussed by Smarandache.F [13, 14]. Al-Quran.A et al. [1] presented a novel approach to Neutrosophic soft rough set in 2019. Some basic operations on hypersoft sets was studied by Mujahid.A et al. [10]. Jafar.M.N et al. [8] proposed trigonometry similarity measures of Neutrosophic hypersoft set and investigated the basic operations on them. They have also presented an application to revolvable energy source selection. Jafar.M.N et al. [7] have proposed different similarity measures with the help of distances and max-min operators defined on Neutrosophic hypersoft sets. They have proved some properties of this similarity measures and presented an application in site selection for solid waste management system. Jafar.M.N and Saeed.M [6] have also presented an algorithm based on a score function to solve a multi attribute multi criteria decision making problem. Aggregate operators on Neutrosophic hypersoft sets was studied by Saqlain.M et al. [12]. Huseyin.K [5] investigated on hybrid structure of hypersoft sets and rough sets. Das.M et al. [4] expanded the scope of rough set, soft set, and Neutrosophic set by combining Neutrosophic soft set with rough set theory. Broumi.S et al. [1,2] developed a hybrid structure called rough neutrosophic set and discussed its properties. Maji.P.K [9] defined some operations on Neutrosophic soft set and established some properties. Ozturk.T.Y et al. [11] have redefined some operations on Neutrosophic soft sets with illustrative examples. Yolcu.A et al. [15] have broadened the scope of rough, soft and Neutrosophic sets by developing the notion of Neutrosophic soft rough set. They have also presented examples and established some properties of the new structure.

From the above literature study it can be seen that hybrid structures combing Neutrosophic sets and soft sets, Neutrosophic sets and rough sets as well as Neutrosophic hyper sets and soft sets have been considered by different authors for different applications. No work in the literature exists combining Neutrosophic hypersoft sets and rough sets.

In this paper we propose to introduce the hybrid structure Neutrosophic hypersoft rough set ($\mathcal{NHSRS}$) and discuss some of its basic properties like union, intersection and complementation with illustrative examples. The notions of equality between $\mathcal{NHSRS}s$, the reduct and core of a $\mathcal{NHSRS}$ are studied.

The rest of the paper is organized as follows. Section 2, deals with the necessary preliminaries. In section 3, we present the definition of $\mathcal{NHSRS}$ and give an example. Some properties of Neutrosophic hypersoft approximation space are also established. In section 4, equality between $\mathcal{NHSRS}s$ is defined and some interesting results are also established. Section 5, deals with reduct and core of a $\mathcal{NHSRS}$. Some theoretical results connecting core and reduct are proved with necessary examples.

2. Preliminaries

The necessary fundamental definitions such as neutrosophic set, hypersoft set, rough set, some properties of neutrosophic hypersoft set and neutrosophic hypersoft rough set can be found in [11, 15].

3. Neutrosophic Hypersoft Rough Sets ($\mathcal{NHSRS}s$).

In this section we introduce the notion of $\mathcal{NHSRS}s$.

Definition 3.1. Let U be a non-empty universe set and $P_N(U)$ be the set of all neutrosophic sets over U. Let E denote the set of parameters. We assume that $E = \{\Delta_1, \Delta_2, ..., \Delta_n\}$, where $\Delta_i \cap \Delta_j = \emptyset$ for $i \neq j$. Let $\delta_j \subseteq \Delta_j$ $j \in \{1, 2, ..., n\}$. Then $\Pi_{j=1}^{n}\delta_j \subseteq \Pi_{j=1}^{n}\Delta_j$. The pair $\left(\mathbb{N},\ \Pi_{j=1}^{n}\delta_j\right) = P_{NH}(U)$, where $\mathbb{N}$ is mapping defined by $\mathbb{N} : \Pi_{j=1}^{n}\delta_j \to P_N(U)$ is called a Neutrosophic hypersoft set ($\mathcal{NHSS}$). The triplet $\left(U, \mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)$ is called a neutrosophic hypersoft approximation space. The $\underline{lower}$ and $\overline{upper}$ neutrosophic hypersoft approximation spaces of $K \in P_{NH}(U)$ with respect to $\left(U, \mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)$ are denoted by $\underline{apr}_{\mathcal{NHSS}}(K)$ and $\overline{apr}_{\mathcal{NHSS}}(K)$ respectively, defined by

$$\underline{apr}_{\mathcal{NHSS}}(K) = \left\{ \left(\Pi_{j=1}^{n}\delta_j, < \frac{\varkappa}{\underline{\mu}_{\Pi_{j=1}^{n}\delta_j}(\varkappa), \underline{\eta}_{\Pi_{j=1}^{n}\delta_j}(\varkappa), \underline{\nu}_{\Pi_{j=1}^{n}\delta_j}(\varkappa)} > \right), \forall \varkappa \in U \right\}.$$
$$\overline{apr}_{\mathcal{NHSS}}(K) = \left\{ \left(\Pi_{j=1}^{n}\delta_j, < \frac{\varkappa}{\overline{\mu}_{\Pi_{j=1}^{n}\delta_j}(\varkappa), \overline{\eta}_{\Pi_{j=1}^{n}\delta_j}(\varkappa), \overline{\nu}_{\Pi_{j=1}^{n}\delta_j}(\varkappa)} > \right), \forall \varkappa \in U \right\}.$$

where,

$$\underline{\mu}_{\Pi_{j=1}^{n}\delta_j}(\varkappa) = \left\{ \bigwedge \mu_{\Pi_{j=1}^{n}\delta_j}(\varkappa) : \mu_{\Pi_{j=1}^{n}\delta_j}(\varkappa) \in K \cap (\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j); (\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j) \subseteq K, \forall \varkappa \in U \right\}$$
$$\underline{\eta}_{\Pi_{j=1}^{n}\delta_j}(\varkappa) = \left\{ \bigwedge \eta_{\Pi_{j=1}^{n}\delta_j}(\varkappa) : \eta_{\Pi_{j=1}^{n}\delta_j}(\varkappa) \in K \cap (\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j); (\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j) \subseteq K, \forall \varkappa \in U \right\}$$
$$\underline{\nu}_{\Pi_{j=1}^{n}\delta_j}(\varkappa) = \left\{ \bigvee \nu_{\Pi_{j=1}^{n}\delta_j}(\varkappa) : \nu_{\Pi_{j=1}^{n}\delta_j}(\varkappa) \in K \cap (\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j); (\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j) \subseteq K, \forall \varkappa \in U \right\}$$
$$\overline{\mu}_{\Pi_{j=1}^{n}\delta_j}(\varkappa) = \left\{ \bigvee \mu_{\Pi_{j=1}^{n}\delta_j}(\varkappa) : \mu_{\Pi_{j=1}^{n}\delta_j}(\varkappa) \in K \cup (\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j); (\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j) \subseteq K, \forall \varkappa \in U \right\}$$
$$\overline{\eta}_{\Pi_{j=1}^{n}\delta_j}(\varkappa) = \left\{ \bigvee \eta_{\Pi_{j=1}^{n}\delta_j}(\varkappa) : \eta_{\Pi_{j=1}^{n}\delta_j}(\varkappa) \in K \cup (\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j); (\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j) \subseteq K, \forall \varkappa \in U \right\}$$
$$\overline{\nu}_{\Pi_{j=1}^{n}\delta_j}(\varkappa) = \left\{ \bigwedge \nu_{\Pi_{j=1}^{n}\delta_j}(\varkappa) : \nu_{\Pi_{j=1}^{n}\delta_j}(\varkappa) \in K \cup (\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j); (\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j) \subseteq K, \forall \varkappa \in U \right\}$$

where "min" and "max" operators are denoted by "$\bigwedge$" and "$\bigvee$", respectively. It is easy to see that $\underline{apr}_{NHSS}(K)$ and $\overline{apr}_{NHSS}(K)$ are two $\mathcal{NHSS}$s over $P_{NH}(U)$. It is said that K is a neutrosophic hypersoft definable set if $\underline{apr}_{\mathcal{NHSS}}(K) = \overline{apr}_{\mathcal{NHSS}}(K)$, otherwise it is referred to as a Neutrosophic hypersoft rough sets ($\mathcal{NHSRS}s$).

Example 3.2. Let $U = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4\}$. Define the attributes sets by,
$\Delta_1 = \{e_{11}, e_{12}\}, \Delta_2 = \{e_{21}, e_{22}\}, \Delta_3 = \{e_{31}, e_{32}\}$.
Let $\delta_1 = \{e_{11}, e_{12}\}, \delta_2 = \{e_{21}, e_{22}\}, \delta_3 = \{e_{31}\}$ that is $\Pi_{j=1}^{n}\delta_j \subseteq \Pi_{j=1}^{n}\Delta_j$, $j = 1, 2, 3$. Let the $\mathcal{NHSS}$,

$$\begin{aligned}\left(\mathbb{N}_1, \Pi_{j=1}^{3}\delta_i\right) = \Big\{ & ((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_1}{< .5, .2, .3 >}, \frac{\varkappa_2}{< .7, .3, .2 >}\}),\\ & ((e_{13}, e_{23}, e_{33}), \{\frac{x_2}{< .8, .4, .2 >}, \frac{x_3}{< .7, .8, .9 >}, \frac{x_4}{< .6, .1, .4 >}\}),\\ & ((e_{11}, e_{22}, e_{31}), \{\frac{\varkappa_2}{< .3, .2, .5 >}\}),\\ & ((e_{12}, e_{21}, e_{31}), \{\frac{\varkappa_3}{< .8, .4, .1 >}, \frac{\varkappa_4}{< .1, .5, .5 >}\}),\\ & ((e_{12}, e_{22}, e_{31}), \{\frac{\varkappa_1}{< .5, .2, .3 >}, \frac{\varkappa_4}{< .4, .3, .2 >}\}),\\ & ((e_{13}, e_{21}, e_{31}), \{\frac{x_2}{< .8, .9, .2 >}, \frac{x_3}{< .4, .2, .7 >}\})\Big\}\end{aligned}$$

$\alpha_1 = \{e_{11}\}, \alpha_2 = \{e_{21}, e_{22}\}, \alpha_3 = \{e_{31}, e_{32}\}$ that is $\Pi_{j=1}^{n}\alpha_j \subseteq \Pi_{j=1}^{n}\Delta_j$ $j = 1, 2, 3$. Let the $\mathcal{NHSS}$,

$$\begin{aligned}\big(\mathbb{N}_2, \Pi_{i=1}^{3}\alpha_i\big) =\Big\{&((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_2}{<.2, .5, .6>}, \frac{\varkappa_3}{<.8, .6, .1>}\}),\\ &((e_{13}, e_{23}, e_{33}), \{\frac{x_2}{<.6, .3, .8>}, \frac{x_3}{<.2, .7, .3>}\}),\\ &((e_{11}, e_{22}, e_{31}), \{\frac{\varkappa_2}{<.6, .2, .3>}\}),\\ &((e_{11}, e_{21}, e_{32}), \{\frac{\varkappa_1}{<.4, .3, .5>}, \frac{\varkappa_4}{<.7, .3, .2>}\}),\\ &((e_{11}, e_{22}, e_{32}), \{\frac{\varkappa_3}{<.4, .4, .2>}, \frac{\varkappa_4}{<.1, .3, .8>}\}),\\ &((e_{13}, e_{21}, e_{31}), \{\frac{x_2}{<.9, .2, .4>}, \frac{x_3}{<.8, .1, .9>}\})\}\end{aligned}$$

$\beta_1 = \{e_{11}, e_{12}\}, \beta_2 = \{e_{21}\}, \beta_3 = \{e_{31}, e_{32}\}$ that is $\Pi_{j=1}^{n}\beta_j \subseteq \Pi_{j=1}^{n}\Delta_j$ $j = 1, 2, 3$. Let the $\mathcal{NHSS}$,

$$\begin{aligned}\big(\mathbb{N}_3, \Pi_{j=1}^{3}\beta_j\big) =\Big\{&((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_2}{<.7, .4, .5>}, \frac{\varkappa_3}{<.9, .6, .7>}, \frac{\varkappa_4}{<.4, .6, .8>}\}),\\ &((e_{13}, e_{23}, e_{33}), \{\frac{x_2}{<.4, .6, .8>}, \frac{x_3}{<.8, .4, .9>}\}),\\ &((e_{12}, e_{21}, e_{31}), \{\frac{\varkappa_2}{<.5, .6, .3>}, \frac{\varkappa_4}{<.6, .3, .4>}\}),\\ &((e_{11}, e_{21}, e_{32}), \{\frac{\varkappa_1}{<.7, .9, .3>}, \frac{\varkappa_3}{<.6, .7, .8>}, \frac{\varkappa_4}{<.3, .8, .2>}\}),\\ &((e_{12}, e_{21}, e_{32}), \{\frac{\varkappa_1}{<.6, .3, .5>}, \frac{\varkappa_3}{<.4, .5, .9>}\}),\\ &((e_{13}, e_{21}, e_{31}), \{\frac{x_3}{<.8, .4, .1>}\})\}\end{aligned}$$

Let K be a $\mathcal{NHSS}$ defined as

$$\begin{aligned}K =\Big\{&((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_1}{<.6, .4, .9>}, \frac{\varkappa_2}{<.5, .6, .9>}, \frac{\varkappa_3}{<.2, .5, .6>}, \frac{\varkappa_4}{<.8, .6, .6>}\}),\\ &((e_{11}, e_{22}, e_{31}), \{\frac{\varkappa_2}{<.8, .5, .2>}\}),\\ &((e_{12}, e_{21}, e_{31}), \{\frac{\varkappa_2}{<.7, .4, .9>}, \frac{\varkappa_3}{<.5, .7, .8>}, \frac{\varkappa_4}{<.6, .4, .8>}\}),\\ &((e_{12}, e_{22}, e_{31}), \{\frac{\varkappa_1}{<.7, .8, .2>}, \frac{\varkappa_4}{<.5, .6, .3>}\}),\\ &((e_{11}, e_{21}, e_{32}), \{\frac{\varkappa_1}{<.6, .3, .8>}, \frac{\varkappa_3}{<.6, .6, .2>}, \frac{\varkappa_4}{<.7, .5, .8>}\}),\\ &((e_{11}, e_{22}, e_{32}), \{\frac{\varkappa_3}{<.3, .7, .9>}, \frac{\varkappa_4}{<.8, .9, .2>}\}),\\ &((e_{12}, e_{21}, e_{32}), \{\frac{\varkappa_1}{<.6, .3, .8>}, \frac{\varkappa_3}{<.4, .6, .9>}\})\Big\}\end{aligned}$$

Then the $\underline{lower}$ and $\overline{upper}$ $\mathcal{NHSS}$ approximation of K are calculated as

$$\underline{apr}_{\mathcal{NHSS}}(K) =\Big\{((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_2}{<.2, .3, .9>}\})\Big\}$$

$$\overline{apr}_{\mathcal{NHSS}}(K) = \Big\{((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_1}{<.6,.4,.3>}, \frac{\varkappa_2}{<.7,.6,.2>}, \frac{\varkappa_3}{<.9,.6,.1>}, \frac{\varkappa_4}{<.8,.6,.6>}\}),$$
$$((e_{11}, e_{22}, e_{31}), \{\frac{\varkappa_2}{<.8,.5,.2>}\}),$$
$$((e_{12}, e_{21}, e_{31}), \{\frac{\varkappa_2}{<.7,.6,.2>}, \frac{\varkappa_3}{<.8,.7,.1>}, \frac{\varkappa_4}{<.6,.5,.4>}\}),$$
$$((e_{12}, e_{22}, e_{31}), \{\frac{\varkappa_1}{<.7,.8,.2>}, \frac{\varkappa_4}{<.5,.6,.2>}\}),$$
$$((e_{11}, e_{21}, e_{32}), \{\frac{\varkappa_1}{<.7,.9,.3>}, \frac{\varkappa_3}{<.6,.7,.2>}, \frac{\varkappa_4}{<.7,.8,.2>}\}),$$
$$((e_{11}, e_{22}, e_{32}), \{\frac{\varkappa_3}{<.4,.7,.2>}, \frac{\varkappa_4}{<.8,.9,.2>}\}),$$
$$((e_{12}, e_{21}, e_{32}), \{\frac{\varkappa_1}{<.6,.3,.5>}, \frac{\varkappa_3}{<.4,.6,.9>}\})\Big\}.$$

Theorem 3.3. *Let $\left(U,\ \mathbb{N},\ \Pi_{j=1}^{n}\delta_j\right)$ be a neutrosophic hypersoft approximation space, and $K, L \in P_{NH}(U)$, then the following properties hold.*

i. $\underline{apr}_{\mathcal{NHSS}}(K) \subseteq K \subseteq \overline{apr}_{\mathcal{NHSS}}(K)$
ii. $\underline{apr}_{\mathcal{NHSS}}(0_{(U,\ \Pi_{j=1}^{n}\delta_j)}) = 0_{(U,\ \Pi_{j=1}^{n}\delta_j)}$, $\overline{apr}_{\mathcal{NHSS}}(1_{(U,\ \Pi_{j=1}^{n}\delta_j)}) = 1_{(U,\ \Pi_{j=1}^{n}\delta_j)}$
iii. *If $K \subseteq L$, then $\underline{apr}_{\mathcal{NHSS}}(K) \subseteq \underline{apr}_{\mathcal{NHSS}}(L)$*
iv. *If $K \subseteq L$, then $\overline{apr}_{\mathcal{NHSS}}(K) \subseteq \overline{apr}_{\mathcal{NHSS}}(L)$*
v. $\underline{apr}_{\mathcal{NHSS}}(K \cap L) \subseteq \underline{apr}_{\mathcal{NHSS}}(K) \cap \underline{apr}_{\mathcal{NHSS}}(L)$
vi. $\underline{apr}_{\mathcal{NHSS}}(K \cup L) \subseteq \underline{apr}_{\mathcal{NHSS}}(K) \cup \underline{apr}_{\mathcal{NHSS}}(L)$
vii. $\overline{apr}_{\mathcal{NHSS}}(K \cap L) \subseteq \overline{apr}_{\mathcal{NHSS}}(K) \cap \overline{apr}_{\mathcal{NHSS}}(L)$
viii. $\overline{apr}_{\mathcal{NHSS}}(K \cup L) \subseteq \overline{apr}_{\mathcal{NHSS}}(K) \cup \overline{apr}_{\mathcal{NHSS}}(L)$

Proof. (i) From the Definition 3.1, we can conclude that $\underline{apr}_{\mathcal{NHSS}}(K) \subseteq K$.
In addition, from the definition of neutrosophic hypersoft upper approximation,
$\forall \left(\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j\right) \cap K \neq 0_{(U,\ \Pi_{j=1}^{n}\delta_j)}$, $\overline{\mu}, \overline{\nu}, \overline{\eta} \in K \cup \left(\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j\right)$.
Hence , $K \subseteq \overline{apr}_{\mathcal{NHSS}}(K)$.
Thus, $\underline{apr}_{\mathcal{NHSS}}(K) \subseteq K \subseteq \overline{apr}_{\mathcal{NHSS}}(K)$.
(ii) From Definition 3.1, the proof of (ii) naturally follows.
(iii) Let $K \subseteq L$ and $\left(\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j\right) \subseteq K, j = (1, 2, ..., n)$.
Then $\underline{apr}_{\mathcal{NHSS}}(K) = K \cap \left(\bigcap_{j=1}^{n}\left(\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j\right)\right)$.
Also, we have $(\left(\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j\right) \subseteq K$ then $\left(\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j\right) \subset L$.
Hence $\underline{apr}_{\mathcal{NHSS}}(L) = L \cap \left(\bigcap_{j=1}^{n}\left(\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j\right)\right)$.
This implies $\underline{apr}_{\mathcal{NHSS}}(K) \subseteq \underline{apr}_{\mathcal{NHSS}}(L)$.
(iv) Let $K \subseteq L$ and $\left(\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j\right) \cap K \neq \emptyset, j = 1, 2, ..., n$.
Then $\overline{apr}_{\mathcal{NHSS}}(K) = K \cup \left(\bigcup_{j=1}^{n}\left(\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j\right)\right)$. For $K \subseteq L$,
then $\left(\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j\right) \cap L \neq \emptyset$ and $\overline{apr}_{\mathcal{NHSS}}(L) = L \cup \left(\bigcup_{j=1}^{n}\left(\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j\right)\right)$.

This implies $\overline{apr}_{\mathcal{NHSS}}(K) \subseteq \overline{apr}_{\mathcal{NHSS}}(L)$.
(v) Let $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \underline{apr}_{\mathcal{NHSS}}(K \cap L)$.
There exist $\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)$ such that $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \subseteq \underline{apr}_{\mathcal{NHSS}}(K \cap L)$, $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \subseteq K$ and $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \subseteq L$.
Therefore $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \underline{apr}_{\mathcal{NHSS}}(K)$ and
$\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \underline{apr}_{\mathcal{NHSS}}(L)$, implying $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \underline{apr}_{\mathcal{NHSS}}(K) \cap \underline{apr}_{\mathcal{NHSS}}(L)$.
Thus $\underline{apr}_{\mathcal{NHSS}}(K \cap L) \subseteq \underline{apr}_{\mathcal{NHSS}}(K) \cap \underline{apr}_{\mathcal{NHSS}}(L)$.
(vi) Let $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \notin \underline{apr}_{\mathcal{NHSS}}(K \cup L)$.
There exist $\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)$ such that $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \nsubseteq \underline{apr}_{\mathcal{NHSS}}(K \cap L)$, hence it follows that $\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \nsubseteq K$ and $\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \nsubseteq L$.
Therefore $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \notin \underline{apr}_{\mathcal{NHSS}}(K)$ and $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \notin \underline{apr}_{\mathcal{NHSS}}(L)$,
implying $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \notin \underline{apr}_{\mathcal{NHSS}}(K) \cup \underline{apr}_{\mathcal{NHSS}}(L)$.
Thus $\underline{apr}_{\mathcal{NHSS}}(K \cup L) \subseteq \underline{apr}_{\mathcal{NHSS}}(K) \cup \underline{apr}_{\mathcal{NHSS}}(L)$.
(vii) Let $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \overline{apr}_{\mathcal{NHSS}}(K \cap L)$.
There exist $\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)$ such that $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \cap (K \cap L) \neq 0_{(U,\ \Pi_{j=1}^{n}\delta_j)}$, $\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \cap (K) \neq 0_{(U,\ \Pi_{j=1}^{n}\delta_j)}$ and $\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \cap (L) \neq 0_{(U,\ \Pi_{j=1}^{n}\delta_j)}$.
Therefore $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \overline{apr}_{\mathcal{NHSS}}(K)$ and
$\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \overline{apr}_{\mathcal{NHSS}}(L)$, implying $\varkappa_{\mu,\nu,\eta}^{\Pi_{i=1}^{n}A_i} \in \overline{apr}_{\mathcal{NHSS}}(K) \cap \overline{apr}_{\mathcal{NHSS}}(L)$.
(viii) Let $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \overline{apr}_{\mathcal{NHSS}}(K \cup L)$.
There exist $\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)$ such that $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \cap (K \cup L) \neq 0_{(U,\ \Pi_{j=1}^{n}\delta_j)}$, it follows that
$\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \cap (K) \neq 0_{(U,\ \Pi_{j=1}^{n}\delta_j)}$ or $\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \cap (L) \neq 0_{(U,\ \Pi_{j=1}^{n}\delta_j)}$.
Therefore $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \overline{apr}_{\mathcal{NHSS}}(K)$ or $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \overline{apr}_{\mathcal{NHSS}}(L)$.
Hence $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \overline{apr}_{\mathcal{NHSS}}(K) \cup \overline{apr}_{\mathcal{NHSS}}(L)$.
Thus $\overline{apr}_{\mathcal{NHSS}}(K \cup L) \subseteq \overline{apr}_{\mathcal{NHSS}}(K) \cup \overline{apr}_{\mathcal{NHSS}}(L)$ □

The converse of properties (i), (iii), (iv), (v), (vi), (vii) and (viii) in Theorem 3.3 does not hold.

Theorem 3.4. *Let $\left(U,\ \mathbb{N},\ \Pi_{j=1}^{n}\delta_j\right)$ be a neutrosophic hypersoft approximation space, and $K, L \in P_{NH}(U)$, then the following properties hold.*

i. $\underline{apr}_{\mathcal{NHSS}}\left(\underline{apr}_{\mathcal{NHSS}}(K)\right) = \underline{apr}_{\mathcal{NHSS}}(K)$
ii. $\overline{apr}_{\mathcal{NHSS}}\left(\overline{apr}_{\mathcal{NHSS}}(K)\right) \supseteq \overline{apr}_{\mathcal{NHSS}}(K)$
iii. $\overline{apr}_{\mathcal{NHSS}}(K) \subseteq \overline{apr}_{\mathcal{NHSS}}\left(\underline{apr}_{\mathcal{NHSS}}(K)\right)$.
iv. $\overline{apr}_{\mathcal{NHSS}}\left(\underline{apr}_{\mathcal{NHSS}}(K)\right) \supseteq \underline{apr}_{\mathcal{NHSS}}(K)$.

Proof. (i) Let $\varkappa^{\Pi_{j=1}^{n}\delta_j}_{(\mu,\eta,\nu)} \in \underline{apr}_{\mathcal{NHSS}}(K)$.
Then, we have $\varkappa^{\Pi_{j=1}^{n}\delta_j}_{(\mu,\eta,\nu)} \in \left(\mathbb{N}_j, \Pi_{j=1}^{n}\delta_j\right) \subseteq \underline{apr}_{\mathcal{NHSS}}(K)$.
So $\varkappa^{\Pi_{j=1}^{n}\delta_j}_{(\mu,\eta,\nu)} \in \underline{apr}_{\mathcal{NHSS}}\left(\underline{apr}_{\mathcal{NHSS}}(K)\right)$.
Therefore $\underline{apr}_{\mathcal{NHSS}}(K) \subseteq \underline{apr}_{\mathcal{NHSS}}\left(\underline{apr}_{\mathcal{NHSS}}(K)\right)$.
From the Theorem 3.3 $\underline{apr}_{\mathcal{NHSS}}(K) \subseteq K$ using (iii) of Theorem 3.3 we obtain $\underline{apr}_{\mathcal{NHSS}}\left(\underline{apr}_{\mathcal{NHSS}}(K)\right) \subseteq \underline{apr}_{\mathcal{NHSS}}(K)$.
Hence $\underline{apr}_{\mathcal{NHSS}}\left(\underline{apr}_{\mathcal{NHSS}}(K)\right) = \underline{apr}_{\mathcal{NHSS}}(K)$.
(ii) Let $P = \overline{apr}_{\mathcal{NHSS}}(K)$. Using property (i) of Theorem 3.3,
we get $P \subseteq \overline{apr}_{\mathcal{NHSS}}(K)$.
Hence $\overline{apr}_{\mathcal{NHSS}}\left(\overline{apr}_{\mathcal{NHSS}}(K)\right) \supseteq \overline{apr}_{\mathcal{NHSS}}(K)$.
(iii) Let $P = \overline{apr}_{\mathcal{NHSS}}(K)$.
Using property (i) of Theorem 3.3, we got $\underline{apr}_{\mathcal{NHSS}}(K) \subseteq P$.
Hence $\underline{apr}_{\mathcal{NHSS}}\left(\overline{apr}_{\mathcal{NHSS}}(K)\right) \subseteq \overline{apr}_{\mathcal{NHSS}}(K)$.
(iv) Let $Q = \underline{apr}_{\mathcal{NHSS}}(K)$.
Using property (i) of Theorem 3.3, we got $\overline{apr}_{\mathcal{NHSS}}(K) \supseteq Q$.
Hence $\underline{apr}_{\mathcal{NHSS}} \subseteq \overline{apr}_{\mathcal{NHSS}}(K)\left(\underline{apr}_{\mathcal{NHSS}}(K)\right)$. □

The converse of properties (ii), (iii) and (iv) in Theorem 3.4 does not hold.

Remark 3.5. Let $\left(U,\ \mathbb{N},\ \Pi_{j=1}^{n}\delta_j\right)$ be a neutrosophic hypersoft approximation space, and $K, L \in P_{NH}(U)$, then the following properties hold.

i. $\underline{apr}_{\mathcal{NHSS}}(K^c) \neq [\overline{apr}_{\mathcal{NHSS}}(K)]^c$
ii. $\overline{apr}_{\mathcal{NHSS}}(K^c) \neq [\underline{apr}_{\mathcal{NHSS}}(K)]^c$

Definition 3.6. Let $\left(\mathbb{N}_1, \Pi_{j=1}^{n}\delta j\right)$ and $\left(\mathbb{N}_2, \Pi_{j=1}^{n}\delta j\right)$ be two $\mathcal{NHSS}$s over the same universe set U. Then "$\left(\mathbb{N}_1, \Pi_{j=1}^{n}\delta j\right)$ difference $\left(\mathbb{N}_2, \Pi_{j=1}^{n}\delta j\right)$" operation on them is denoted by $\left((\mathbb{N}_1 \setminus \mathbb{N}_2), \Pi_{j=1}^{n}\delta j\right)$ and is defined by
$\left((\mathbb{N}_1 \setminus \mathbb{N}_2), \Pi_{j=1}^{n}\delta j\right) = \left(\mathbb{N}_1, \Pi_{j=1}^{n}\delta j\right) \cap \left(\mathbb{N}_2, \Pi_{j=1}^{n}\delta j\right)^c$
$= \{\Pi_{j=1}^{n}\delta j, < \varkappa, (1-\mu)_{\mathbb{N}_j,\Pi_{j=1}^{n}\delta j}(\varkappa), (1-\eta)_{\mathbb{N}_j,\Pi_{j=1}^{n}\delta j}(\varkappa), (1-\nu)_{\mathbb{N}_j,\Pi_{j=1}^{n}\delta j}(\varkappa) >: \varkappa \in U : \Pi_{j=1}^{n}\delta j \subseteq \Pi_{j=1}^{n}\Delta_j\}$.
where
$\mu_{(\mathbb{N}_1\setminus\mathbb{N}_2),\Pi_{j=1}^{n}\delta j}(\varkappa) = min\{\mu_{\mathbb{N}_1,\Pi_{j=1}^{n}\delta j}(\varkappa), \mu_{\mathbb{N}_2,\Pi_{j=1}^{n}\delta j}(\varkappa)\}$
$\eta_{(\mathbb{N}_1\setminus\mathbb{N}_2),\Pi_{j=1}^{n}\delta j}(\varkappa) = min\{\eta_{\mathbb{N}_1,\Pi_{j=1}^{n}\delta j}(\varkappa), \eta_{\mathbb{N}_2,\Pi_{j=1}^{n}\delta j}(\varkappa)\}$
$\nu_{(\mathbb{N}_1\setminus\mathbb{N}_2),\Pi_{j=1}^{n}\delta j}(\varkappa) = max\{\nu_{\mathbb{N}_1,\Pi_{j=1}^{n}\delta j}(\varkappa), \nu_{\mathbb{N}_2,\Pi_{j=1}^{n}\delta j}(\varkappa)\}$.

Definition 3.7. Let $\underline{apr}_{\mathcal{NHSS}}(K)$ and $\overline{apr}_{\mathcal{NHSS}}(K)$ be neutrosophic hypersoft $\underline{lower}$ and $\overline{upper}$ approximations of $K \in P_{NH}(U)$ with respect to the neutrosophic hypersoft approximation space K, respectively. Then

$pos_{\mathcal{NHSS}}(K) = \underline{apr}_{\mathcal{NHSS}}(K)$

$neg_{\mathcal{NHSS}}(K) = (\overline{apr}_{\mathcal{NHSS}}(K))^c$

$bnd_{\mathcal{NHSS}}(K) = \overline{apr}_{\mathcal{NHSS}}(K) \setminus \underline{apr}_{\mathcal{NHSS}}(K)$

are called the neutrosophic hypersoft positive region, neutrosophic hypersoft negative region and neutrosophic hypersoft boundary region of K, respectively.

Example 3.8. Consider Example 3.2. The neutrosophic hypersoft rough positive, negative, and boundary region can then be computed as follows:

$$\begin{aligned} pos_{\mathcal{NHSS}}(K) =& \underline{apr}_{\mathcal{NHSS}}(K) \\ =& \Big\{((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_2}{< .2, .3, .9 >}\})\Big\} \end{aligned}$$

$$\begin{aligned} neg_{\mathcal{NHSS}}(K) =& (\overline{apr}_{\mathcal{NHSS}}(K))^c \\ =& \Big\{((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_1}{< .4, .6, .7 >}, \frac{\varkappa_2}{< .3, .4, .8 >}, \frac{\varkappa_3}{< .1, .4, .9 >}, \frac{\varkappa_4}{< .2, .4, .4 >}\}), \\ & ((e_{11}, e_{22}, e_{31}), \{\frac{\varkappa_2}{< .2, .5, .8 >}\}), \\ & ((e_{12}, e_{21}, e_{31}), \{\frac{\varkappa_2}{< .3, .4, .8 >}, \frac{\varkappa_3}{< .2, .3, .9 >}, \frac{\varkappa_4}{< .4, .5, .6 >}\}), \\ & ((e_{12}, e_{22}, e_{31}), \{\frac{\varkappa_1}{< .3, .2, .8 >}, \frac{\varkappa_4}{< .5, .4, .8 >}\}), \\ & ((e_{11}, e_{21}, e_{32}), \{\frac{\varkappa_1}{< .3, .1, .7 >}, \frac{\varkappa_3}{< .4, .3, .8 >}, \frac{\varkappa_4}{< .3, .2, .8 >}\}), \\ & ((e_{11}, e_{22}, e_{32}), \{\frac{\varkappa_3}{< .6, .3, .8 >}, \frac{\varkappa_4}{< .2, .1, .8 >}\}), \\ & ((e_{12}, e_{21}, e_{32}), \{\frac{\varkappa_1}{< .4, .7, .5 >}, \frac{\varkappa_3}{< .6, .4, .1 >}\})\Big\} \end{aligned}$$

$$(\underline{apr}_{\mathcal{NHSS}}(K))^c = \Big\{((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_2}{< .8, .7, .1 >}\})\Big\}$$

$$\begin{aligned} Bnd_{\mathcal{NHSS}}(K) =& \overline{apr}_{\mathcal{NHSS}}(K) \setminus \underline{apr}_{\mathcal{NHSS}}(K) \\ =& \overline{apr}_{\mathcal{NHSS}}(K) \cap (\underline{apr}_{\mathcal{NHSS}}(K))^c \\ =& \Big\{((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_2}{< .7, .6, .2 >}\})\Big\} \end{aligned}$$

Obviously, $\underline{apr}_{\mathcal{NHSS}}(K) \neq \overline{apr}_{\mathcal{NHSS}}(K)$. So K is $\mathcal{NHSS}$ in the approximation space $\Big(U, \mathbb{N}, \Pi_{j=1}^{n}\delta_j\Big)$.

Theorem 3.9. *Let $\Big(U,\ \mathbb{N},\ \Pi_{j=1}^{n}\delta_j\Big)$ be a neutrosophic hypersoft approximation space, and $K, L \in P_{NH}(U)$, then $\underline{apr}_{\mathcal{NHSS}}(K \setminus L) \subseteq \underline{apr}_{\mathcal{NHSS}}(K) \setminus \underline{apr}_{\mathcal{NHSS}}(L)$.*

Proof. Let $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \underline{apr}_{\mathcal{NHSS}}(K \setminus L)$.
There exist $\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)$ such that $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \subseteq \underline{apr}_{\mathcal{NHSS}}(K \setminus L)$, $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \subseteq K$ and $\varkappa_{1-\mu,1-\nu,1-\eta}^{\Pi_{j=1}^{n}\delta_j} \in \left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) \subseteq L^c$.
Thus $\underline{apr}_{\mathcal{NHSS}}(K \setminus L) \subseteq \underline{apr}_{\mathcal{NHSS}}(K) \setminus \underline{apr}_{\mathcal{NHSS}}(L)$.
Therefore $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \underline{apr}_{\mathcal{NHSS}}(K)$ and
$\varkappa_{1-\mu,1-\nu,1-\eta}^{\Pi_{j=1}^{n}\delta_j} \in \underline{apr}_{\mathcal{NHSS}}(L^c)$, implying $\varkappa_{\mu,\nu,\eta}^{\Pi_{j=1}^{n}\delta_j} \in \underline{apr}_{\mathcal{NHSS}}(K) \setminus \underline{apr}_{\mathcal{NHSS}}(L)$.
Thus $\underline{apr}_{\mathcal{NHSS}}(K \cap L) \subseteq \underline{apr}_{\mathcal{NHSS}}(K \setminus \underline{apr}_{\mathcal{NHSS}}(L)$. □

The converse of Theorem 3.9 does not hold.

Remark 3.10. Let $\left(U,\ \mathbb{N},\ \Pi_{j=1}^{n}\delta_j\right)$ be a neutrosophic hypersoft approximation space, and $K, L \in P_{NH}(U)$, then

$$\overline{apr}_{\mathcal{NHSS}}(K \setminus L) \neq \overline{apr}_{\mathcal{NHSS}}(K) \setminus \overline{apr}_{\mathcal{NHSS}}(L).$$

Example 3.11. Let $\left(U,\ \mathbb{N},\ \Pi_{j=1}^{n}\delta_j\right)$ be a neutrosophic hypersoft approximation space, and $K, L \in P_{NH}(U)$, based on Example 3.2 defined as,

$$\begin{aligned} K = \Big\{ & \big((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_1}{<.6,.4,.9>}, \frac{\varkappa_2}{<.5,.6,.9>}, \frac{x_3}{<.2,.5,.6>}, \frac{\varkappa_4}{<.8,.6,.6>}\}\big), \\ & \big((e_{13}, e_{23}, e_{33}), \{\frac{\varkappa_2}{<.8,.6,.9>}, \frac{\varkappa_3}{<.4,.6,.8>}, \frac{\varkappa_4}{<.8,.4,.9>}\}\big), \\ & \big((e_{11}, e_{22}, e_{31}), \{\frac{\varkappa_2}{<.8,.5,.2>}\}\big), \\ & \big((e_{12}, e_{21}, e_{31}), \{\frac{\varkappa_2}{<.7,.4,.9>}, \frac{\varkappa_3}{<.5,.7,.8>}, \frac{\varkappa_4}{<.6,.4,.8>}\}\big), \\ & \big((e_{12}, e_{22}, e_{31}), \{\frac{\varkappa_1}{<.7,.8,.2>}, \frac{\varkappa_4}{<.5,.6,.3>}\}\big), \\ & \big((e_{11}, e_{21}, e_{32}), \{\frac{\varkappa_1}{<.6,.3,.8>}, \frac{\varkappa_3}{<.6,.6,.2>}, \frac{\varkappa_4}{<.7,.5,.8>}\}\big), \\ & \big((e_{11}, e_{22}, e_{32}), \{\frac{\varkappa_3}{<.3,.7,.9>}, \frac{\varkappa_4}{<.8,.9,.2>}\}\big), \\ & \big((e_{12}, e_{21}, e_{32}), \{\frac{\varkappa_1}{<.6,.3,.8>}, \frac{\varkappa_3}{<.4,.6,.9>}\}\big) \Big\} \end{aligned}$$

Then the $\underline{lower}$ and $\overline{upper}$ $\mathcal{NHSS}$ approximation of K are calculated as

$$\begin{aligned} \underline{apr}_{\mathcal{NHSS}}(K) = \Big\{ & \big((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_2}{<.2,.3,.9>}\}\big), \\ & \big((e_{13}, e_{23}, e_{33}), \{\frac{\varkappa_2}{<.4,.3,.9>}, \frac{\varkappa_3}{<.2,.4,.9>}\}\big) \Big\} \end{aligned}$$

$$\overline{apr}_{\mathcal{NHSS}}(K) = \Big\{((e_{11},e_{21},e_{31}),\{\frac{\varkappa_1}{<.6,.4,.3>},\frac{\varkappa_2}{<.7,.6,.2>},\frac{\varkappa_3}{<.9,.6,.1>},\frac{\varkappa_4}{<.8,.6,.6>}\}),$$
$$((e_{13},e_{23},e_{33}),\{\frac{\varkappa_2}{<.8,.6,.2>},\frac{\varkappa_3}{<.8,.8,.3>},\frac{\varkappa_4}{<.8,.4,.4>}\}),$$
$$((e_{11},e_{22},e_{31}),\{\frac{\varkappa_2}{<.8,.5,.2>}\}),$$
$$((e_{12},e_{21},e_{31}),\{\frac{\varkappa_2}{<.7,.6,.2>},\frac{\varkappa_3}{<.8,.7,.1>},\frac{\varkappa_4}{<.6,.5,.4>}\}),$$
$$((e_{12},e_{22},e_{31}),\{\frac{\varkappa_1}{<.7,.8,.2>},\frac{\varkappa_4}{<.5,.6,.2>}\}),$$
$$((e_{11},e_{21},e_{32}),\{\frac{\varkappa_1}{<.7,.9,.3>},\frac{\varkappa_3}{<.6,.7,.2>},\frac{\varkappa_4}{<.7,.8,.2>}\}),$$
$$((e_{11},e_{22},e_{32}),\{\frac{\varkappa_3}{<.4,.7,.2>},\frac{\varkappa_4}{<.8,.9,.2>}\}),$$
$$((e_{12},e_{21},e_{32}),\{\frac{\varkappa_1}{<.6,.3,.5>},\frac{\varkappa_3}{<.4,.6,.9>}\})\Big\}$$

$$Let\ L = \Big\{((e_{11},e_{21},e_{31}),\{\frac{\varkappa_1}{<.7,.5,.4>},\frac{\varkappa_2}{<.6,.7,.8>},\frac{\varkappa_3}{<.3,.6,.2>},\frac{\varkappa_4}{<.9,.6,.3>}\}),$$
$$((e_{11},e_{22},e_{31}),\{\frac{\varkappa_2}{<.9,.5,.1>}\}),$$
$$((e_{12},e_{21},e_{31}),\{\frac{\varkappa_2}{<.9,.8,.1>},\frac{\varkappa_3}{<.6,.8,.5>},\frac{\varkappa_4}{<.7,.5,.4>}\}),$$
$$((e_{11},e_{21},e_{32}),\{\frac{\varkappa_1}{<.8,.5,.6>},\frac{\varkappa_3}{<.8,.6,.1>},\frac{\varkappa_4}{<.8,.6,.3>}\}),$$
$$((e_{12},e_{21},e_{32}),\{\frac{\varkappa_1}{<.8,.5,.3>},\frac{\varkappa_3}{<.6,.8,.7>}\}),$$
$$((e_{13},e_{21},e_{31}),\{\frac{\varkappa_2}{<.7,.1,.3>},\frac{\varkappa_3}{<.9,.3,.8>}\})\Big\}$$

Then the $\underline{lower}$ and $\overline{upper}$ $\mathcal{NHSS}$ approximation of L are calculated as

$$\underline{apr}_{\mathcal{NHSS}}(L) = \Big\{((e_{11},e_{21},e_{31}),\{\frac{\varkappa_2}{<.2,.3,.8>}\}),$$
$$((e_{13},e_{21},e_{31}),\frac{\varkappa_3}{<.4,.1,.9>}\})\Big\}$$

$$\overline{apr}_{\mathcal{NHSS}}(L) = \Big\{((e_{11},e_{21},e_{31}),\{\frac{\varkappa_1}{<.7,.5,.3>},\frac{\varkappa_2}{<.7,.7,.2>},\frac{\varkappa_3}{<.9,.6,.1>},\frac{\varkappa_4}{<.9,.6,.3>}\}),$$
$$((e_{11},e_{22},e_{31}),\{\frac{\varkappa_2}{<.9,.5,.1>}\}),$$
$$((e_{12},e_{21},e_{31}),\{\frac{\varkappa_2}{<.9,.8,.1>},\frac{\varkappa_3}{<.8,.8,.1>},\frac{\varkappa_4}{<.7,.5,.4>}\}),$$
$$((e_{11},e_{21},e_{32}),\{\frac{\varkappa_1}{<.8,.9,.1>},\frac{\varkappa_3}{<.6,.7,.8>},\frac{\varkappa_4}{<.8,.8,.2>}\}),$$
$$((e_{12},e_{21},e_{32}),\{\frac{\varkappa_1}{<.8,.5,.3>},\frac{\varkappa_3}{<.6,.8,.7>}\}),$$
$$((e_{13},e_{21},e_{31}),\{\frac{\varkappa_2}{<.9,.9,.2>},\frac{\varkappa_3}{<.9,.4,.1>}\})\Big\}$$

$$\underline{apr}_{\mathcal{NHSS}}(K)/\underline{apr}_{\mathcal{NHSS}}(L) = \Big\{((e_{11},e_{21},e_{31}),\{\frac{\varkappa_2}{<.2,.3,.9>}\})\Big\}$$

$$\overline{apr}_{\mathcal{NHSS}}(K)/\overline{apr}_{\mathcal{NHSS}}(L) = \Big\{((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_1}{< .3, .4, .7 >}, \frac{\varkappa_2}{< .3, .3, .8 >}, \frac{\varkappa_3}{< .1, .4, .9 >}, \frac{\varkappa_4}{< .1, .4, .7 >}\}),$$
$$((e_{11}, e_{22}, e_{31}), \{\frac{\varkappa_2}{< .1, .5, .9 >}\}),$$
$$((e_{12}, e_{21}, e_{31}), \{\frac{\varkappa_2}{< .1, .2, .9 >}, \frac{\varkappa_3}{< .2, .2, .9 >}, \frac{\varkappa_4}{< .3, .5, .6 >}\}),$$
$$((e_{11}, e_{21}, e_{32}), \{\frac{\varkappa_1}{< .2, .1, .9 >}, \frac{\varkappa_3}{< .4, .3, .2 >}, \frac{\varkappa_4}{< .2, .2, .8 >}\}),$$
$$((e_{12}, e_{21}, e_{32}), \{\frac{\varkappa_1}{< .2, .5, .7 >}, \frac{\varkappa_3}{< .4, .2, .3 >}\})\Big\}$$

$$L^c = \Big\{((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_1}{< .3, .5, .6 >}, \frac{\varkappa_2}{< .4, .3, .2 >}, \frac{\varkappa_3}{< .7, .4, .8 >}, \frac{\varkappa_4}{< .1, .4, .7 >}\}),$$
$$((e_{11}, e_{22}, e_{31}), \{\frac{\varkappa_2}{< .1, .5, .9 >}\}),$$
$$((e_{12}, e_{21}, e_{31}), \{\frac{\varkappa_2}{< .1, .2, .9 >}, \frac{\varkappa_3}{< .4, .2, .5 >}, \frac{\varkappa_4}{< .2, .4, .7 >}\}),$$
$$((e_{11}, e_{21}, e_{32}), \{\frac{\varkappa_1}{< .6, .3, .8 >}, \frac{\varkappa_3}{< .2, .5, .7 >}, \frac{\varkappa_4}{< .4, .2, .3 >}\}),$$
$$((e_{12}, e_{21}, e_{32}), \{\frac{\varkappa_1}{< .3, .9, .7 >}, \frac{\varkappa_3}{< .1, .7, .2 >}\})\Big\}$$

$$K \setminus L = \Big\{((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_1}{< .3, .4, .9 >}, \frac{\varkappa_2}{< .4, .3, .9 >}, \frac{\varkappa_3}{< .2, .5, .8 >}, \frac{\varkappa_4}{< .1, .4, .7 >}\}),$$
$$((e_{11}, e_{22}, e_{31}), \{\frac{\varkappa_2}{< .1, .5, .9 >}\}),$$
$$((e_{12}, e_{21}, e_{31}), \{\frac{\varkappa_2}{< .1, .2, .9 >}, \frac{\varkappa_3}{< .4, .2, .8 >}, \frac{\varkappa_4}{< .2, .4, .8 >}\}),$$
$$((e_{11}, e_{21}, e_{32}), \{\frac{\varkappa_1}{< .6, .3, .8 >}, \frac{\varkappa_3}{< .2, .5, .7 >}, \frac{\varkappa_4}{< .4, .2, .8 >}\}),$$
$$((e_{12}, e_{21}, e_{32}), \{\frac{\varkappa_1}{< .3, .3, .8 >}, \frac{\varkappa_3}{< .1, .6, .9 >}\})\Big\}$$

$$\underline{apr}_{\mathcal{NHSS}}(K \setminus L) = \Big\{((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_2}{< .2, .3, .9 >}\})\Big\}$$

$$\overline{apr}_{\mathcal{NHSS}}(K \setminus L) = \Big\{((e_{11}, e_{21}, e_{31}), \{\frac{\varkappa_1}{< .5, .4, .3 >}, \frac{\varkappa_2}{< .7, .5, .2 >}, \frac{\varkappa_3}{< .9, .6, .1 >}, \frac{\varkappa_4}{< .4, .6, .8 >}\}),$$
$$((e_{11}, e_{22}, e_{31}), \{\frac{\varkappa_2}{< .6, .5, .3 >}\}),$$
$$((e_{12}, e_{21}, e_{31}), \{\frac{\varkappa_2}{< .5, .6, .3 >}, \frac{\varkappa_3}{< .8, .4, .1 >}, \frac{\varkappa_4}{< .6, .5, .4 >}\}),$$
$$((e_{11}, e_{21}, e_{32}), \{\frac{\varkappa_1}{< .7, .9, .3 >}, \frac{\varkappa_3}{< .6, .7, .7 >}, \frac{\varkappa_4}{< .7, .8, .2 >}\}),$$
$$((e_{12}, e_{21}, e_{32}), \{\frac{\varkappa_1}{< .6, .3, .5 >}, \frac{\varkappa_3}{< .4, .6, .9 >}\})\Big\}$$

Hence,

$$\underline{apr}_{\mathcal{NHSS}}(K \setminus L) \subseteq \underline{apr}_{\mathcal{NHSS}}(K) \setminus \underline{apr}_{\mathcal{NHSS}}(L)$$
$$\overline{apr}_{\mathcal{NHSS}}(K \setminus L) \neq \overline{apr}_{\mathcal{NHSS}}(K) \setminus \overline{apr}_{\mathcal{NHSS}}(L).$$

4. Equality Properties on Neutrosophic Hypersoft Rough Sets

In this section, we define equity between neutrosophic hypersoft rough sets.

Definition 4.1. Let $\left(U,\ \mathbb{N},\ \Pi_{j=1}^{n}\delta_j\right)$ be a neutrosophic hypersoft approximation space, then $\forall\ K, L \in P_{NH}(U)$, we define the following binary relations:

(i). Sets K and L are in lower $\mathcal{NHSS}$ equal related iff

$K \preccurlyeq_{\mathcal{NHSS}}\ L \Longleftrightarrow \underline{apr}_{\mathcal{NHSS}}(K) = \underline{apr}_{\mathcal{NHSS}}(L)$.

(ii). Sets K and L are in upper $\mathcal{NHSS}$ equal related iff

$K \curlyeqprec_{\mathcal{NHSS}}\ L \Longleftrightarrow \overline{apr}_{\mathcal{NHSS}}(K) = \overline{apr}_{\mathcal{NHSS}}(L)$.

(iii). Sets K and L are $\mathcal{NHSS}$ equal related iff

$K \approx_{\mathcal{NHSS}} L \Longleftrightarrow \underline{apr}_{\mathcal{NHSS}}(K) = \underline{apr}_{\mathcal{NHSS}}(L)\ \&\ \overline{apr}_{\mathcal{NHSS}}(K) = \overline{apr}_{\mathcal{NHSS}}(L)$.

Theorem 4.2. *Let* $\left(U,\ \mathbb{N},\ \Pi_{j=1}^{n}\delta_j\right)$ *be a neutrosophic hypersoft approximation space, then* $\forall$ $K, L, M, N \in P_{NH}(U)$*. Then the following hold:*

(i). If $K \subseteq L$ *and* $L \curlyeqprec_{\mathcal{NHSS}} 0_{(U,\Pi_{j=1}^{n}\delta_j)}$*, then* $K \curlyeqprec_{\mathcal{NHSS}} 0_{(U,\Pi_{j=1}^{n}\delta_j)}$.

(ii). If $K \subseteq L$ *and* $K \curlyeqprec_{\mathcal{NHSS}} 1_{(U,\Pi_{j=1}^{n}\delta_j)}$*, then* $L \curlyeqprec_{\mathcal{NHSS}} 1_{(U,\Pi_{j=1}^{n}\delta_j)}$.

(iii) $K \curlyeqprec_{\mathcal{NHSS}} L \Longleftrightarrow K \curlyeqprec_{\mathcal{NHSS}} (K \cup L) \curlyeqprec_{\mathcal{NHSS}} L$.

(iv) $K \curlyeqprec_{\mathcal{NHSS}} L, M \curlyeqprec_{\mathcal{NHSS}} N \Longrightarrow (K \cup M) \curlyeqprec_{\mathcal{NHSS}} (L \cup N)$.

Proof. (i). Given $K \subseteq L$ and $L \curlyeqprec_{\mathcal{NHSS}} 0_{(U,\Pi_{j=1}^{n}\delta_j)}$,
so that $\overline{apr}_{\mathcal{NHSS}}(K) \subseteq \overline{apr}_{\mathcal{NHSS}}(L)$ and $\overline{apr}_{\mathcal{NHSS}}(L) = 0_{(U,\Pi_{j=1}^{n}\delta_j)}$.
Hence, $\overline{apr}_{\mathcal{NHSS}}(K) = 0_{(U,\Pi_{j=1}^{n}\delta_j)} = \overline{apr}_{\mathcal{NHSS}}(L)$.
(ii). Given, $K \curlyeqprec_{\mathcal{NHSS}} 1_{(U,\Pi_{j=1}^{n}\delta_j)}$ and $K \subseteq L$,
then $\overline{apr}_{\mathcal{NHSS}}(K) = \overline{apr}_{\mathcal{NHSS}}(1_{(U,\Pi_{j=1}^{n}\delta_j)})$ and $\overline{apr}_{\mathcal{NHSS}}(K) \subseteq \overline{apr}_{\mathcal{NHSS}}(L)$.
But we know that $\overline{apr}_{\mathcal{NHSS}}(K) \subseteq \overline{apr}_{\mathcal{NHSS}}(1_{(U,\Pi_{j=1}^{n}\delta_j)})$,
hence $\overline{apr}_{\mathcal{NHSS}}(K) = \overline{apr}_{\mathcal{NHSS}}(1_{(U,\Pi_{j=1}^{n}\delta_j)})$.

We note here that $K \curlyeqprec_{\mathcal{NHSS}} L$ iff $K \cap L \curlyeqprec_{\mathcal{NHSS}} K$ and $K \cap L \curlyeqprec_{\mathcal{NHSS}} L$ is not true in general.

(iii) Assume that $K \curlyeqprec_{\mathcal{NHSS}} L$. By definition 4.1(ii), we have
$\overline{apr}_{\mathcal{NHSS}}(K) = \overline{apr}_{\mathcal{NHSS}}(L)$.
From Theorem 3.3, it is known that $\overline{apr}_{\mathcal{NHSS}}(K \cup L) = \overline{apr}_{\mathcal{NHSS}}(K) \cup \overline{apr}_{\mathcal{NHSS}}(L)$. There by we obtain $\overline{apr}_{\mathcal{NHSS}}(K \cup L) = \overline{apr}_{\mathcal{NHSS}}(K) = \overline{apr}_{\mathcal{NHSS}}(L)$.
Consequently, $K \curlyeqprec_{\mathcal{NHSS}} L \Rightarrow K \curlyeqprec_{\mathcal{NHSS}} (K \cup L) \curlyeqprec_{\mathcal{NHSS}} L$.
Conversely, if $K \curlyeqprec_{\mathcal{NHSS}} (K \cup L) \curlyeqprec_{\mathcal{NHSS}} L$ then it is obvious that $K \curlyeqprec_{\mathcal{NHSS}} L$ from the transitivity of $\curlyeqprec_{\mathcal{NHSS}}$.
(iv) Suppose that $K \curlyeqprec_{\mathcal{NHSS}} L$ and $M \curlyeqprec_{\mathcal{NHSS}} N$. By Definition 4.1(ii),
we can write $\overline{apr}_{\mathcal{NHSS}}(K) = \overline{apr}_{\mathcal{NHSS}}(L)$ and $\overline{apr}_{\mathcal{NHSS}}(M) = \overline{apr}_{\mathcal{NHSS}}(N)$. By considering Theorem 3.3, we have $\overline{apr}_{\mathcal{NHSS}}(K \cup L) = \overline{apr}_{\mathcal{NHSS}}(K) \cup \overline{apr}_{\mathcal{NHSS}}(L)$ and $\overline{apr}_{\mathcal{NHSS}}(M \cup N) =$

$\overline{apr}_{\mathcal{NHSS}}(M) \cup \overline{apr}_{\mathcal{NHSS}}(N)$.
Thereby, we conclude that $\overline{apr}_{\mathcal{NHSS}}(K \cup M) = \overline{apr}_{\mathcal{NHSS}}(L \cup N)$ and so $\overline{apr}_{\mathcal{NHSS}}(K \cup M) \preccurlyeq_{\mathcal{NHSS}} \overline{apr}_{\mathcal{NHSS}}(L \cup N)$. □

The converse of property (iv) in Theorem 4.2 does not hold.

Example 4.3. Let $\left(U,\ \mathbb{N},\ \Pi_{j=1}^{n}\delta_j\right)$ be a neutrosophic hypersoft approximation space, then $\forall\ K, L \in P_{NH}(U)$ based on Example 3.2, $K \preccurlyeq_{\mathcal{NHSS}}\ L \Longleftrightarrow \overline{apr}_{\mathcal{NHSS}}(K) = \overline{apr}_{\mathcal{NHSS}}(L)$,

$$K = \{((e_{11}, e_{21}, e_{31}), \{\frac{x_1}{< .6, .4, .9 >}, \frac{x_2}{< .5, .6, .9 >}, \frac{x_3}{< .2, .5, .6 >}, \frac{x_4}{< .8, .6, .6 >}\}),$$
$$((e_{11}, e_{22}, e_{31}), \{\frac{x_2}{< .8, .5, .2 >}\}),$$
$$((e_{12}, e_{21}, e_{31}), \{\frac{x_2}{< .7, .4, .9 >}, \frac{x_3}{< .5, .7, .8 >}, \frac{x_4}{< .6, .4, .8 >}\}),$$
$$((e_{11}, e_{21}, e_{32}), \{\frac{x_1}{< .6, .3, .8 >}, \frac{x_3}{< .6, .6, .2 >}, \frac{x_4}{< .7, .5, .8 >}\}),$$
$$((e_{12}, e_{21}, e_{32}), \{\frac{x_1}{< .6, .3, .8 >}, \frac{x_3}{< .4, .6, .9 >}\})\}$$

The lower and upper $\mathcal{NHSS}$ approximation of K are calculated as

$$\underline{apr}_{NHSS}(K) = \{((e_{11}, e_{21}, e_{31}), \{\frac{x_2}{< .2, .3, .9 >}\})\}$$

$$\overline{apr}_{NHSS}(K) = \{((e_{11}, e_{21}, e_{31}), \{\frac{x_1}{< .6, .4, .3 >}, \frac{x_2}{< .7, .6, .2 >}, \frac{x_3}{< .9, .6, .1 >}, \frac{x_4}{< .8, .6, .6 >}\}),$$
$$((e_{11}, e_{22}, e_{31}), \{\frac{x_2}{< .8, .5, .2 >}\}),$$
$$((e_{12}, e_{21}, e_{31}), \{\frac{x_2}{< .7, .6, .2 >}, \frac{x_3}{< .8, .7, .1 >}, \frac{x_4}{< .6, .5, .4 >}\}),$$
$$((e_{11}, e_{21}, e_{32}), \{\frac{x_1}{< .7, .9, .3 >}, \frac{x_3}{< .6, .7, .2 >}, \frac{x_4}{< .7, .8, .2 >}\}),$$
$$((e_{12}, e_{21}, e_{32}), \{\frac{x_1}{< .6, .3, .5 >}, \frac{x_3}{< .4, .6, .9 >}\})\}$$

$$L = \{((e_{11}, e_{21}, e_{31}), \{\frac{x_1}{< .6, .4, .5 >}, \frac{x_2}{< .6, .6, .8 >}, \frac{x_3}{< .8, .6, .4 >}, \frac{x_4}{< .8, .6, .6 >}\}),$$
$$((e_{11}, e_{22}, e_{31}), \{\frac{x_2}{< .8, .5, .2 >}\}),$$
$$((e_{12}, e_{21}, e_{31}), \{\frac{x_2}{< .7, .5, .6 >}, \frac{x_3}{< .6, .7, .4 >}, \frac{x_4}{< .6, .4, .8 >}\}),$$
$$((e_{11}, e_{21}, e_{32}), \{\frac{x_1}{< .7, .4, .6 >}, \frac{x_3}{< .6, .7, .2 >}, \frac{x_4}{< .7, .6, .5 >}\}),$$
$$((e_{12}, e_{21}, e_{32}), \{\frac{x_1}{< .6, .3, .7 >}, \frac{x_3}{< .4, .6, .9 >}\})\}$$

The lower and upper $\mathcal{NHSS}$ approximation of L are calculated as

$$\underline{apr}_{NHSS}(L) = \{((e_{11}, e_{21}, e_{31}), \{\frac{x_2}{< .2, .3, .8 >}\})\}$$

$$\overline{apr}_{NHSS}(L) = \{((e_{11}, e_{21}, e_{31}), \{\frac{x_1}{<.6, .4, .3>}, \frac{x_2}{<.7, .6, .2>}, \frac{x_3}{<.9, .6, .1>}, \frac{x_4}{<.8, .6, .6>}\}),$$
$$((e_{11}, e_{22}, e_{31}), \{\frac{x_2}{<.8, .5, .2>}\}),$$
$$((e_{12}, e_{21}, e_{31}), \{\frac{x_2}{<.7, .6, .2>}, \frac{x_3}{<.8, .7, .1>}, \frac{x_4}{<.6, .5, .4>}\}),$$
$$((e_{11}, e_{21}, e_{32}), \{\frac{x_1}{<.7, .9, .3>}, \frac{x_3}{<.6, .7, .2>}, \frac{x_4}{<.7, .8, .2>}\}),$$
$$((e_{12}, e_{21}, e_{32}), \{\frac{x_1}{<.6, .3, .5>}, \frac{x_3}{<.4, .6, .9>}\})\}$$

Theorem 4.4. *Let* $\left(U,\ \mathbb{N},\ \Pi_{j=1}^{n}\delta_j\right)$ *be a neutrosophic hypersoft approximation space, then* $\forall$ $K, L, M, N \in P_{NH}(U)$. *Then the following hold:*

(i). If $K \subseteq L$ *and* $L \preccurlyeq_{\mathcal{NHSS}} 0_{(U, \Pi_{j=1}^{n}\delta_j)}$, *then* $K \preccurlyeq_{\mathcal{NHSS}} 0_{(U, \Pi_{j=1}^{n}\delta_j)}$.

(ii). If $K \subseteq L$ *and* $L \preccurlyeq_{\mathcal{NHSS}} 1_{(U, \Pi_{j=1}^{n}\delta_j)}$, *then* $K \preccurlyeq_{\mathcal{NHSS}} 1_{(U, \Pi_{j=1}^{n}\delta_j)}$.

(iii) $K \preccurlyeq_{\mathcal{NHSS}} L \Longleftrightarrow K \preccurlyeq_{\mathcal{NHSS}} (K \cup L) \preccurlyeq_{NHSS} L$.

(iv) $K \preccurlyeq_{\mathcal{NHSS}} L, M \preccurlyeq_{\mathcal{NHSS}} N \Longrightarrow (K \cup M) \preccurlyeq_{\mathcal{NHSS}} (L \cup N)$.

Proof. By considering Definition 4.1(i), and Theorem 3.3, it can be proved similar to the proof of Theorem 4.2. □

The converse of property (iv) in Theorem 4.4. does not hold.

Example 4.5. Let $\left(U,\ \mathbb{N},\ \Pi_{j=1}^{n}\delta_j\right)$ be a neutrosophic hypersoft approximation space, then $\forall\ K, L \in P_{NH}(U)$ based on Example 3.2, $K \preccurlyeq_{\mathcal{NHSS}}\ L \Longleftrightarrow \underline{apr}_{\mathcal{NHSS}}(K) = \underline{apr}_{\mathcal{NHSS}}(L)$,

$$K = \{((e_{11}, e_{21}, e_{31}), \{\frac{x_1}{<.6, .4, .9>}, \frac{x_2}{<.5, .6, .9>}, \frac{x_3}{<.2, .5, .6>}, \frac{x_4}{<.8, .6, .6>}\}),$$
$$((e_{11}, e_{22}, e_{31}), \{\frac{x_2}{<.8, .5, .2>}\}),$$
$$((e_{12}, e_{21}, e_{31}), \{\frac{x_2}{<.7, .4, .9>}, \frac{x_3}{<.5, .7, .8>}, \frac{x_4}{<.6, .4, .8>}\}),$$
$$((e_{11}, e_{21}, e_{32}), \{\frac{x_1}{<.6, .3, .8>}, \frac{x_3}{<.6, .6, .2>}, \frac{x_4}{<.7, .5, .8>}\}),$$
$$((e_{12}, e_{21}, e_{32}), \{\frac{x_1}{<.6, .3, .8>}, \frac{x_3}{<.4, .6, .9>}\})\}$$

The lower and upper $\mathcal{NHSS}$ approximation of K are calculated as

$$\underline{apr}_{NHSS}(K) = \{((e_{11}, e_{21}, e_{31}), \{\frac{x_2}{<.2, .3, .9>}\})\}$$

$$\overline{apr}_{NHSS}(K) = \{((e_{11}, e_{21}, e_{31}), \{\frac{x_1}{< .6, .4, .3 >}, \frac{x_2}{< .7, .6, .2 >}, \frac{x_3}{< .9, .6, .1 >}, \frac{x_4}{< .8, .6, .6 >}\}),$$
$$((e_{11}, e_{22}, e_{31}), \{\frac{x_2}{< .8, .5, .2 >}\}),$$
$$((e_{12}, e_{21}, e_{31}), \{\frac{x_2}{< .7, .6, .2 >}, \frac{x_3}{< .8, .7, .1 >}, \frac{x_4}{< .6, .5, .4 >}\}),$$
$$((e_{11}, e_{21}, e_{32}), \{\frac{x_1}{< .7, .9, .3 >}, \frac{x_3}{< .6, .7, .2 >}, \frac{x_4}{< .7, .8, .2 >}\}),$$
$$((e_{12}, e_{21}, e_{32}), \{\frac{x_1}{< .6, .3, .5 >}, \frac{x_3}{< .4, .6, .9 >}\})\}$$

$$L = \{((e_{11}, e_{21}, e_{31}), \{\frac{x_1}{< .7, .5, .4 >}, \frac{x_2}{< .6, .7, .9 >}, \frac{x_3}{< .3, .6, .2 >}, \frac{x_4}{< .9, .6, .3 >}\}),$$
$$((e_{11}, e_{22}, e_{31}), \{\frac{x_2}{< .9, .5, .1 >}\}),$$
$$((e_{12}, e_{21}, e_{31}), \{\frac{x_2}{< .9, .8, .1 >}, \frac{x_3}{< .6, .8, .5 >}, \frac{x_4}{< .7, .5, .4 >}\}),$$
$$((e_{11}, e_{21}, e_{32}), \{\frac{x_1}{< .8, .5, .6 >}, \frac{x_3}{< .8, .6, .1 >}, \frac{x_4}{< .8, .6, .3 >}\}),$$
$$((e_{12}, e_{21}, e_{32}), \{\frac{x_1}{< .8, .5, .3 >}, \frac{x_3}{< .6, .8, .7 >}\})\}$$

The lower and upper $\mathcal{NHSS}$ approximation of L are calculated as

$$\underline{apr}_{NHSS}(L) = \{((e_{11}, e_{21}, e_{31}), \{\frac{x_2}{< .2, .3, .9 >}\})\}$$

$$\overline{apr}_{NHSS}(L) = \{((e_{11}, e_{21}, e_{31}), \{\frac{x_1}{< .7, .5, .3 >}, \frac{x_2}{< .7, .7, .2 >}, \frac{x_3}{< .9, .6, .1 >}, \frac{x_4}{< .9, .6, .3 >}\}),$$
$$((e_{11}, e_{22}, e_{31}), \{\frac{x_2}{< .9, .5, .1 >}\}),$$
$$((e_{12}, e_{21}, e_{31}), \{\frac{x_2}{< .9, .8, .1 >}, \frac{x_3}{< .8, .8, .1 >}, \frac{x_4}{< .7, .5, .4 >}\}),$$
$$((e_{11}, e_{21}, e_{32}), \{\frac{x_1}{< .8, .9, .1 >}, \frac{x_3}{< .6, .7, .8 >}, \frac{x_4}{< .8, .8, .2 >}\}),$$
$$((e_{12}, e_{21}, e_{32}), \{\frac{x_1}{< .8, .5, .3 >}, \frac{x_3}{< .6, .8, .7 >}\})\}$$

5. Reduct and Core of Neutrosophic Hypersoft Rough Sets

In this section, we discuss reduct, core, dispensable, and indispensable neutrosophic hypersoft rough sets.

Definition 5.1. Let $(\acute{\acute{R}}_N, \Pi_{j=1}^n \acute{X}_j), (\acute{\acute{S}}_N, \Pi_{j=1}^n \acute{Y}_j)$ & $\acute{\acute{T}}_N, \Pi_{j=1}^n \acute{Z}_j)$ be $\mathcal{NHSS}$s on U, where $(\Pi_{j=1}^n \acute{X}_j, \Pi_{j=1}^n \acute{Y}_j, \Pi_{j=1}^n \acute{Z}_j) \subseteq \left(\mathbb{N}, \Pi_{j=1}^n \delta_j\right)$, and $\acute{\acute{R}}_N : \Pi_{j=1}^n \acute{X}_j \to P_N(U)$, $\acute{\acute{S}}_N : \Pi_{j=1}^n \acute{Y}_j \to P_N(U)$, $\acute{\acute{T}}_N : \Pi_{i=1}^n \acute{Z}_i \to P_N(U)$ are mappings.
Let $\left(\mathbb{N}, \Pi_{j=1}^n \delta_j\right) = \{(\acute{\acute{R}}_N : \Pi_{j=1}^n \acute{X}_j), (\acute{\acute{S}}_N : \Pi_{j=1}^n \acute{Y}_j) \& (\acute{\acute{T}}_N : \Pi_{j=1}^n \acute{Z}_j)\}$.
We define approximate neutrosophic hypersoft set, which is denoted by APP.
$APP\left(\mathbb{N}, \Pi_{j=1}^n \delta j\right)$
$= APP\left\{(\acute{\acute{R}}_N, \Pi_{j=1}^n \acute{X}_i), (\acute{\acute{S}}_N, \Pi_{j=1}^n \acute{Y}_j) \& (\acute{\acute{T}}_N, \Pi_{j=1}^n \acute{Z}_j)\right\}$

$= \Big\{(r_{\Pi_{j=1}^{n}\alpha_j}, s_{\Pi_{i=1}^{n}\beta_j}, t_{\Pi_{j=1}^{n}\gamma_j}), \bigcap\{\acute{\acute{R}}_N, (r_{\Pi_{j=1}^{n}\alpha_j}), \acute{\acute{S}}_N, (s_{\Pi_{j=1}^{n}\beta_j}), \acute{\acute{T}}_N, (t_{\Pi_{j=1}^{n}\gamma_j})\}$
$/1 \leq \alpha_j, \beta_j, \gamma_j \leq n\Big\}$
$= \Big\{(\Pi_{j=1}^{n} e_j,\ \mathbb{N}, \Pi_{j=1}^{n}(e_j)) \setminus \Pi_{j=1}^{n} e_j \in \Big(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\Big)\Big\},$
where $r_{\Pi_{j=1}^{n}\alpha_j} \in \acute{\acute{R}}_N,\ s_{\Pi_{j=1}^{n}\beta_j} \in \acute{\acute{S}}_N,\ t_{\Pi_{j=1}^{n}\gamma_j} \in \acute{\acute{T}}_N$ and
$\Pi_{j=1}^{n} e_j \in \Big(\mathbb{N}, \Pi_{j=1}^{n}\delta_i\Big) \subseteq \acute{\acute{R}}_N \times \acute{\acute{S}}_N \times \acute{\acute{T}}_N,$
$(\mathbb{N}, \Pi_{j=1}^{n}(e_j)) = \bigcap\Big\{\acute{\acute{R}}_N, (r_{\Pi_{j=1}^{n}\alpha_j}), \acute{\acute{S}}_N, (s_{\Pi_{j=1}^{n}\beta_j}), \acute{\acute{T}}_N, (t_{\Pi_{j=1}^{n}\gamma_j})\Big\}.$
Also we write the difference in approximate neutrosophic hypersoft sets as
$APP\Big(\Big(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\Big) - (\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j)\Big) = APP\Big((\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j), (\acute{\acute{T}}_N, \Pi_{j=1}^{n}\acute{Z}_j)\Big).$

Definition 5.2. Two approximate neutrosophic hypersoft sets $APP(\mathbb{G}, \Pi_{j=1}^{n}\acute{\acute{B}}_j)$ and $APP(\mathbb{H}, \Pi_{k=1}^{n}\acute{\acute{C}}_k)$ are said to be equal, that is $APP(\mathbb{G}, \Pi_{j=1}^{n}\acute{\acute{B}}_j) = APP(\mathbb{H}, \Pi_{k=1}^{n}\acute{\acute{C}}_k)$ if for every $\Pi_{j=1}^{n} b_j \in \Pi_{j=1}^{n}\acute{\acute{B}}_j$ there exists one $\Pi_{k=1}^{n} c_k \in \Pi_{k=1}^{n}\acute{\acute{C}}_k$ such that $\mathbb{G}, (\Pi_{j=1}^{n} b_j) = \mathbb{H}, (\Pi_{k=1}^{n} c_k)$ for some $1 \leq j, k \leq n$ and for every $\Pi_{k=1}^{n} c_k \in \Pi_{k=1}^{n}\acute{\acute{C}}_k$ there exists one $\Pi_{j=1}^{n} b_j \in \Pi_{j=1}^{n}\acute{\acute{B}}_j$ such that $\mathbb{H}, (\Pi_{k=1}^{n} c_k) = \mathbb{H}, (\Pi_{k=1}^{n} c_k)$ for some $1 \leq j, k \leq n$, where $\Pi_{j=1}^{n}\acute{\acute{B}}_j, \Pi_{k=1}^{n}\acute{\acute{C}}_k \subseteq \Pi_{j,k=1}^{n}\Delta_{jk}$ and $G : \Pi_{j=1}^{n}\acute{\acute{B}}_j \to P_N(U), H : \Pi_{k=1}^{n}\acute{\acute{C}}_k \to P_N(U)$.

Definition 5.3. The neutrosophic hypersoft set $(\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j))$ is dispensable in
$\Big\{(\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_i), (\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j), (\acute{\acute{T}}_N, \Pi_{j=1}^{n}\acute{Z}_j))\Big\}$
if $APP\Big(\Big(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\Big)\Big) = APP\Big(\Big(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\Big) - (\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j)\Big).$
Suppose if $APP\Big(\Big(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\Big) \neq APP\Big(\Big(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\Big) - (\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j)\Big),$
then $(\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j)$ is indispensable in $\Big(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\Big)$.

Definition 5.4. The neutrosophic hypersoft set $APP(\Big(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\Big))$ is independent if each $(\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j) \subseteq (\mathbb{N}, \Pi_{j=1}^{n}\delta_j)$ is indispensable in $(\mathbb{N}, \Pi_{j=1}^{n}\delta_j)$. Otherwise neutrosophic hypersoft set $(\mathbb{N}, \Pi_{j=1}^{n}\delta_j)$ is dependent.

Definition 5.5. The set of all indispensable neutrosophic hypersoft sets in $APP\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)$ is called the core of $APP\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)$ and is denoted $CORE\Big(APP\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)\Big)$.

Theorem 5.6. $CORE\Big(APP\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)\Big) = \bigcap RED\Big(APP\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)\Big),$
where $RED\Big(APP\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)\Big)$ *is the family of all reducts of* $APP\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)$.

Proof. If $(\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j)$ is a reduct of $\Big(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\Big)$ and
$(\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j) \in \Big(\mathbb{N}, \Pi_{j=1}^{n}\delta Y_j\Big) - (\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j),$
then $APP\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) = APP(\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j),$
$(\acute{\acute{R}}_N,\ \acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j) \subseteq \Big(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\Big) - \Big\{(\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{Y}_j)\Big\} \subseteq \Big(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\Big).$

Note that, if $\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right), (\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j), (\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j)$,
then $APP(\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j) = APP(\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j)$.
Assuming that $(\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j) = \left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) - \left\{(\acute{\acute{S}}_N, \Pi_{j=1}^{n}\delta Y_j)\right\}$ we conclude that $(\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j)$ is superfluous, i.e. $(\acute{\acute{S}}_N, \Pi_{j=1}^{n}\delta Y_j) \notin CORE\left(\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)\right)$ and
$CORE\left(\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)\right) \subseteq \bigcap\left\{(\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j) : (\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j) \in RED\left(\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)\right)\right\}$.
Suppose $\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j) \notin CORE\left(\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)\right)$, i.e. $(\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j)$ is superfluous in $\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)$. That means $APP(\left(\mathbb{N},\ prod_{j=1}^{n}\delta_j\right)) = APP\left(\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) - \left\{(\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j)\right\}\right)$, which implies that there exists an independent subset $\acute{\acute{T}}_N, \Pi_{j=1}^{n}\acute{Z}_j \subseteq \left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right) - \{\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j)\}$, such that $APP(\acute{\acute{T}}_N, \Pi_{j=1}^{n}\acute{z}_j) = APP(\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right))$. Obviously $\acute{\acute{T}}_N$ is reduct of $\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)$ and $\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j) \in \acute{\acute{T}}_N, \Pi_{j=1}^{n}\acute{Z}_j$. This shows that
$CORE\left(\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)\right) \supseteq \bigcap\left\{(\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j) : (\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j) \in RED\left(\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)\right)\right\}$ □

Example 5.7. Now we consider an apartment evaluation problem. Suppose $U = \{\varkappa_1, \varkappa_2, ..., \varkappa_9\}$ be a set of nine apartments, $E = \{\Delta_1, \Delta_2, \Delta_3, \Delta_4\}$ be the set of attributes, where $\{\Delta_1 - rate; \Delta_2 - condition; \Delta_3 - infra - structure; \Delta_4 - environs\}$. The values of rate are $\{e_{11} - high; e_{12} - normal; e_{13} - low\}$, the values of condition are $\{e_{21} - worth; e_{22} - not\ worth\}$, the values of infra-structure are $\{e_{31} - super; e_{32} - ok; e_{33} - worst\}$, and the values of environs are $\{e_{41} - quiet; e_{42} - a\ little\ noisy; e_{43} - noisy; e_{44} - quite\ noisy\}$. The evaluation results are listed below. The reduct and core are evaluated as follows.
Here, $\Pi_{j=1}^{4}\delta_j = \left\{\Pi_{j=1}^{3}\acute{W}_j, \Pi_{j=1}^{2}\acute{X}_j, \Pi_{j=1}^{3}\acute{Y}_j, \Pi_{j=1}^{4}\acute{Z}_j\right\}$
For attribute rate
$\acute{W}_j$: e_{11}-high $= \{\varkappa_1, \varkappa_4, \varkappa_5, \varkappa_7\}$, e_{12}-normal $= \{\varkappa_2, \varkappa_8\}$, e_{13}-low $= \{\varkappa_3, \varkappa_6, \varkappa_9\}$;
For attribute condition:
$\acute{X}_j$: e_{21}-worth $= \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_6\}$, e_{22}-not worth $= \{\varkappa_4, \varkappa_5, \varkappa_7, \varkappa_8, \varkappa_9\}$;
For attribute infrastructure:
$\acute{Y}_j$: e_{31}-super $= \{\varkappa_1, \varkappa_2, \varkappa_3\}$, e_{32}-ok $= \{\varkappa_4, \varkappa_5, \varkappa_6, \varkappa_7, \varkappa_8\}$, e_{33}-worst $= \{x9\}$;
For attribute environs:
$\acute{Z}_j$: e_{41}-quiet $= \{\varkappa_1, \varkappa_2\}$, e_{42}-a little noisy $= \{\varkappa_3, \varkappa_6\}$, e_{43}-noisy $= \{\varkappa_4, \varkappa_5, \varkappa_7\}$, e_{44}-quite noisy $= \{\varkappa_8, \varkappa_9\}$;
Let $\acute{\acute{Q}}_N, \Pi_{j=1}^{n}\acute{W}_j = \{\Delta_1 - rate\}$ $\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j = \{\Delta_2 - condition\}$, $\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j = \{\Delta_3 - infrastructure\}$, $\acute{\acute{T}}_N, \Pi_{j=1}^{n}\acute{Z}_j = \{\Delta_4 - environs\} \subseteq \Pi_{j=1}^{4}\Delta_j$.

$$(\acute{\acute{Q}}_N,\Pi_{j=1}^{n}\acute{W}_j)=\Big\{((e_{11},e_{12},e_{13}),\acute{\acute{Q}}_N(e_{11},e_{12},e_{13}))\Big\}$$
$$=\Big\{((e_{11},e_{12},e_{13}),\{\frac{\varkappa_1}{<.3,.4,.9>},\frac{\varkappa_4}{<.5,.6,.6>},\frac{\varkappa_5}{<.2,.8,.6>},\frac{\varkappa_7}{<.8,.9,.6>}\},$$
$$\{\frac{\varkappa_2}{<.8,.2,.7>},\frac{\varkappa_8}{<.7,.9,.4>}\},$$
$$\{\frac{\varkappa_3}{<.6,.4,.8>},\frac{\varkappa_6}{<.6,.3,.2>},\frac{\varkappa_9}{<.7,.8,.8>}\})\Big\}$$

$$(\acute{\acute{R}}_N,\Pi_{j=1}^{n}\acute{X}_j)=\Big\{((e_{21},e_{22}),\acute{\acute{R}}_N(e_{21},e_{22}))\Big\}$$
$$=\Big\{((e_{21},e_{22}),\{\frac{\varkappa_1}{<.8,.5,.3>},\frac{\varkappa_2}{<.5,.9,.4>},\frac{\varkappa_3}{<.9,.3,.4>},\frac{\varkappa_6}{<.4,.8,.6>}\},$$
$$\{\frac{\varkappa_4}{<.8,.3,.2>},\frac{\varkappa_5}{<.6,.6,.2>},\frac{\varkappa_7}{<.6,.4,.9>}\frac{\varkappa_8}{<.5,.3,.1>},\frac{\varkappa_9}{<.9,.9,.2>}\})\Big\}$$

$$(\acute{\acute{S}}_N,\Pi_{j=1}^{n}\acute{Y}_j)=\Big\{((e_{31},e_{32},e_{33}),\acute{\acute{S}}_N(e_{31},e_{22},e_{33}))\Big\}$$
$$=\Big\{((e_{31},e_{32},e_{33}),\{\frac{\varkappa_1}{<.6,.8,.7>},\frac{\varkappa_2}{<.2,.9,.2>},\frac{\varkappa_3}{<.3,.6,.1>}\},$$
$$\{\frac{\varkappa_4}{<.7,.5,.3>},\frac{\varkappa_5}{<.9,.4,.6>},\frac{\varkappa_6}{<.2,.3,.8>},\frac{\varkappa_7}{<.6,.3,.9>},\frac{\varkappa_8}{<.6,.8,.3>}\},$$
$$\{\frac{\varkappa_9}{<.6,.3,.8>}\})\Big\}$$

$$(\acute{\acute{T}}_N,\Pi_{i=1}^{n}\acute{Z}_j)=\Big\{((e_{41},e_{42},e_{43},e_{44}),\acute{\acute{T}}_N(e_{41},e_{42},e_{43},e_{44}))\Big\}$$
$$=\Big\{((e_{41},e_{42},e_{43},e_{44}),\{\frac{\varkappa_1}{<.6,.4,.9>},\frac{\varkappa_2}{<.5,.6,.9>}\},$$
$$\{\frac{\varkappa_3}{<.8,.5,.2>},\frac{\varkappa_6}{<.7,.3,.4>}\},$$
$$\{\frac{\varkappa_4}{<.7,.1,.9>},\frac{\varkappa_5}{<.8,.4,.1>},\frac{\varkappa_7}{<.5,.3,.8>}\},$$
$$\{\frac{\varkappa_8}{<.7,.8,.2>},\frac{\varkappa_9}{<.6,.3,.8>}\})\Big\}$$

$$APP\Big((\mathbb{N},\Pi_{j=1}^{n}\delta_j)\Big)=APP\Big((\acute{\acute{Q}}_N,\Pi_{j=1}^{n}\acute{W}_j),(\acute{\acute{R}}_N,\Pi_{j=1}^{n}\acute{X}_j),(\acute{\acute{S}}_N,\Pi_{j=1}^{n}\acute{Y}_j),(\acute{\acute{T}}_N,\Pi_{j=1}^{n}\acute{Z}_j)\Big)$$
$$=\Big\{((e_{11},e_{12},e_{13}),(e_{21},e_{22}),(e_{31},e_{32},e_{33}),(e_{41},e_{42},e_{43},e_{44}),\{\frac{\varkappa_1}{<.3,.4,.9>}\},$$
$$\{\frac{\varkappa_4}{<.5,.1,.9>},\frac{\varkappa_5}{<.2,.4,.6>},\frac{\varkappa_7}{<.5,.3,.9>}\},$$
$$\{\frac{\varkappa_2}{<.2,.2,.9>}\},$$
$$\{\frac{\varkappa_8}{<.5,.3,.4>}\},$$
$$\{\frac{\varkappa_3}{<.3,.3,.8>}\},$$
$$\{\frac{\varkappa_6}{<.2,.3,.8>}\},$$
$$\{\frac{\varkappa_9}{<.6,.3,.8>}\})\Big\}$$

$$APP\Big((\mathbb{N},\Pi_{j=1}^{n}\delta_j)-(\acute{\acute{Q}}_N,\Pi_{j=1}^{n}\acute{W}_i)\Big)=APP\Big((\acute{\acute{R}}_N,\Pi_{j=1}^{n}\acute{X}_i),(\acute{\acute{S}}_N,\Pi_{j=1}^{n}\acute{Y}_i),(\acute{\acute{T}}_N,\Pi_{j=1}^{n}\acute{Z}_j)\Big)$$
$$=\Big\{((e_{21},e_{22}),(e_{31},e_{32},e_{33}),(e_{41},e_{42},e_{43},e_{44}),\{\frac{\varkappa_1}{<.6,.4,.9>},\frac{\varkappa_2}{<.2,.6,.9>}\},$$
$$\{\frac{\varkappa_4}{<.7,.1,.9>},\frac{\varkappa_5}{<.6,.4,.6>},\frac{\varkappa_7}{<.5,.3,.9>}\},$$
$$\{\frac{\varkappa_3}{<.3,.3,.4>}\},$$
$$\{\frac{\varkappa_6}{<.2,.3,.8>}\},$$
$$\{\frac{\varkappa_8}{<.5,.3,.3>}\},$$
$$\{\frac{\varkappa_9}{<.6,.3,.8>}\})\Big\}$$
$$\neq APP\Big((\mathbb{N},\Pi_{j=1}^{n}\delta_j)\Big)$$

Hence, $(\acute{\acute{Q}}_N,\Pi_{j=1}^{n}\acute{W}_j)$ is indispensable in$(\mathbb{N},\Pi_{j=1}^{n}\delta_j)$.

$$APP\Big((\mathbb{N},\Pi_{j=1}^{n}\delta_j)-(\acute{\acute{R}}_N,\Pi_{j=1}^{n}\acute{X}_j)\Big)=APP\Big((\acute{\acute{Q}}_N,\Pi_{j=1}^{n}\acute{W}_j,\acute{\acute{S}}_N,\Pi_{j=1}^{n}\acute{Y}_j,(\acute{\acute{T}}_N,\Pi_{j=1}^{n}\acute{Z}_j)\Big)$$

$$=\Big\{((e_{11},e_{12},e_{13}),(e_{31},e_{32},e_{33}),(e_{41},e_{42},e_{43},e_{44}),\{\frac{\varkappa_1}{<.3,.4,.9>}\},$$
$$\{\frac{\varkappa_4}{<.5,.1,.9>},\frac{\varkappa_5}{<.2,.4,.6>},\frac{\varkappa_7}{<.5,.3,.9>}\},$$
$$\{\frac{\varkappa_2}{<.2,.2,.9>}\},$$
$$\{\frac{\varkappa_8}{<.6,.8,.4>}\},$$
$$\{\frac{\varkappa_3}{<.3,.4,.8>}\},$$
$$\{\frac{\varkappa_6}{<.2,.3,.8>}\},$$
$$\{\frac{\varkappa_9}{<.6,.3,.8>}\})\Big\}$$
$$=APP\Big((\mathbb{N},\Pi_{i=1}^{n}\delta_j)\Big)$$

Hence, $(\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j)$ is dispensable in$(\mathbb{N}, \Pi_{j=1}^{n}\delta_j)$.

$$APP\Big((\mathbb{N}, \Pi_{j=1}^{n}\delta_j) - (\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j)\Big) = APP\Big((\acute{\acute{Q}}_N, \Pi_{j=1}^{n}\acute{W}_j, \acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j, (\acute{\acute{T}}_N, \Pi_{j=1}^{n}\acute{Z}_j)\Big)$$

$$=\Big\{((e_{11}, e_{12}, e_{13}), (e_{21}, e_{22}), (e_{41}, e_{42}, e_{43}, e_{44}), \{\frac{\varkappa_1}{< .3, .4, .9 >}\},$$
$$\{\frac{\varkappa_4}{< .5, .1, .9 >}, \frac{\varkappa_5}{< .2, .4, .6 >}, \frac{\varkappa_7}{< .5, .3, .9 >}\},$$
$$\{\frac{\varkappa_2}{< .5, .2, .9 >}\},$$
$$\{\frac{\varkappa_8}{< .5, .3, .4 >}\},$$
$$\{\frac{\varkappa_3}{< .6, .3, .8 >}, \frac{\varkappa_6}{< .4, .3, .6 >}\},$$
$$\{\frac{\varkappa_9}{< .6, .3, .8 >}\})\Big\}$$
$$\neq APP\Big((\mathbb{N}, \Pi_{j=1}^{n}\delta_j)\Big)$$

Hence, $(\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j)$ is indispensable in$(\mathbb{N}, \Pi_{j=1}^{n}\delta_j)$.

$$APP\Big((\mathbb{N}, \Pi_{j=1}^{n}\delta_j) - (\acute{\acute{T}}_N, \Pi_{j=1}^{n}\acute{Z}_j)\Big) = APP\Big((\acute{\acute{Q}}_N, \Pi_{j=1}^{n}\acute{W}_j, \acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j, (\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j)\Big)$$

$$=\Big\{((e_{11}, e_{12}, e_{13}), (e_{21}, e_{22}), (e_{31}, e_{32}, e_{33}), \{\frac{\varkappa_1}{< .3, .4, .9 >}\},$$
$$\{\frac{\varkappa_4}{< .5, .3, .6 >}, \frac{\varkappa_5}{< .2, .4, .6 >}, \frac{\varkappa_7}{< .6, .3, .9 >}\},$$
$$\{\frac{\varkappa_2}{< .2, .2, .7 >}\},$$
$$\{\frac{\varkappa_8}{< .5, .3, .4 >}\},$$
$$\{\frac{\varkappa_3}{< .3, .3, .8 >}\},$$
$$\{\frac{\varkappa_6}{< .2, .3, .8 >}\},$$
$$\{\frac{\varkappa_9}{< .6, .3, .8 >}\})\Big\}$$
$$= APP\Big((\mathbb{N}, \Pi_{j=1}^{n}\delta_j)\Big)$$

Hence, $(\acute{\acute{T}}_N, \Pi_{j=1}^{n}\acute{Z}_j)$ is dispensable in$(\mathbb{N}, \Pi_{j=1}^{n}\delta_j)$.

The set of four approximate neutrosophic hypersoft sets $APP\Big((\acute{\acute{Q}}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j), (\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j), (\acute{\acute{T}}_N, \Pi_{j=1}^{n}\acute{Z}_j)\Big)$ is the same as the $APP\Big((\acute{\acute{Q}}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j), (\acute{\acute{T}}_N, \Pi_{j=1}^{n}\acute{Z}_j)\Big)$ and $APP\Big((\acute{\acute{Q}}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j), (\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j)\Big)$. In order to find reducts of the approximate neutrosophic hypersoft sets $APP\Big((\mathbb{N}, \Pi_{j=1}^{n}\delta_j)\Big) = APP\Big((\acute{\acute{Q}}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{\acute{R}}_N, \Pi_{j=1}^{n}\acute{X}_j), (\acute{\acute{S}}_N, \Pi_{j=1}^{n}\acute{Y}_j)\Big)$ we have to check

whether $APP\Big((\acute{Q}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{R}_N, \Pi_{j=1}^{n}\acute{X}_j), (\acute{S}_N, \Pi_{j=1}^{n}\acute{Y}_j)\Big)$ are independent or not.
Because $APP\Big((\acute{Q}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{S}_N, \Pi_{j=1}^{n}\acute{Y}_j), (\acute{T}_N, \Pi_{j=1}^{n}\acute{Z}_j)\Big) \neq APP((\acute{Q}_N, \Pi_{j=1}^{n}\acute{W}_j))$
$APP\Big((\acute{Q}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{S}_N, \Pi_{j=1}^{n}\acute{Y}_j), (\acute{T}_N, \Pi_{j=1}^{n}\acute{Z}_j)\Big) \neq APP((\acute{S}_N, \Pi_{j=1}^{n}\acute{Y}_j))$
$APP\Big((\acute{Q}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{S}_N, \Pi_{j=1}^{n}\acute{Y}_j), (\acute{T}_N, \Pi_{j=1}^{n}\acute{Z}_j)\Big) \neq APP((\acute{T}_N, \Pi_{j=1}^{n}\acute{Z}_j))$, hence the approximate neutrosophic hypersoft sets are independent and consequently
$APP\Big((\acute{Q}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{S}_N, \Pi_{j=1}^{n}\acute{Y}_j), (\acute{T}_N, \Pi_{j=1}^{n}\acute{Z}_j)\Big)$ is reduct of $APP\Big((\mathbb{N}, \Pi_{j=1}^{n}\delta_j)\Big)$. Proceeding in the same way we find that $APP\Big((\acute{Q}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{R}_N, \Pi_{j=1}^{n}\acute{X}_j), (\acute{S}_N, \Pi_{j=1}^{n}\acute{Y}_j)\Big)$ is also a reduct of $APP\Big((\mathbb{N}, \Pi_{j=1}^{n}\delta_j)\Big)$.

Thus there are two reducts of the approximate neutrosophic hypersoft sets $APP\Big((\mathbb{N}, \Pi_{j=1}^{n}\delta_j)\Big)$,
$APP\Big((\acute{Q}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{S}_N, \Pi_{j=1}^{n}\acute{Y}_j), (\acute{T}_N, \Pi_{j=1}^{n}\acute{Z}_j)\Big)$
and $APP\Big((\acute{Q}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{R}_N, \Pi_{j=1}^{n}\acute{X}_j), (\acute{S}_N, \Pi_{j=1}^{n}\acute{Y}_j)\Big)$
From Theorem 5.6,

$$CORE\Big(APP\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)\Big) = \bigcap RED\Big(APP\left(\mathbb{N}, \Pi_{j=1}^{n}\delta_j\right)\Big)$$
$$= \{APP\Big((\acute{Q}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{S}_N, \Pi_{j=1}^{n}\acute{Y}_j), (\acute{T}_N, \Pi_{j=1}^{n}\acute{Z}_j)\Big)\}$$
$$\bigcap$$
$$\{APP\Big((\acute{Q}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{R}_N, \Pi_{j=1}^{n}\acute{X}_j), (\acute{S}_N, \Pi_{j=1}^{n}\acute{Y}_j)\Big)\}$$
$$= APP\Big((\acute{Q}_N, \Pi_{j=1}^{n}\acute{W}_j), (\acute{S}_N, \Pi_{j=1}^{n}\acute{Y}_j)\Big)$$

is the core of $APP\Big((\mathbb{N}, \Pi_{j=1}^{n}\delta_i)\Big)$ and also $APP\Big((\acute{Q}_N, \Pi_{j=1}^{n}\acute{W}_j), ((\acute{S}_N, \Pi_{j=1}^{n}\acute{Y}_j)\Big)$ is indispensable in $APP\Big((\mathbb{N}, \Pi_{j=1}^{n}\delta_j)\Big)$. If these nine houses are the training samples, then we have two different kinds of evaluation references for other input samples $\{rate; infrastructure; environs\}$, $\{rate; condition; infrastructure\}$ and it is clear that the attributes $\{rate; infrastructure\}$ is the key attributes for the evaluation of apartments.

6. Conclusion

This article defines the neutrosophic hypersoft rough set by combining the notions three sets: neutrosophic set, hypersoft set, and rough set. The study of fundamental properties such as union, intersection, and complement are illustrated using examples. The lower and upper rough neutrosophic hypersoft approximations are then specified and validated. The relationship between the core and reduct on the neutrosophic hypersoft rough set is illustrated with examples. The ideas of reduct and core can be used in data reduction and identification of vital set of attributes in decision making problems. We propose to work on multi-attribute, multi-criteria decision making problems using the theoretical properties of reduct, core and equity defined in this work as our future research direction.

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