Neutrosophic Linguistic valued Hypersoft Set with Application: Medical Diagnosis and Treatment

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Abstract: Language is closely connected to the concepts of uncertainty and indeterminacy, as it functions as a fundamental tool for the expression and communication of information. Linguistic formulations possess inherent qualities of ambiguity, imprecision, and vagueness. The comprehension of language frequently hinges upon contextual factors, individual interpretation, and subjective viewpoints, resulting in ambiguities in comprehension. Neutrosophic-linguistic valued hypersoft sets (N-LVHS) play a pivotal role in decision-making by effectively managing linguistic uncertainty, modeling real-world complexity, and accommodating multidimensional information. In the realm of medical diagnosis and treatment, several limitations tied to language and indeterminacy persist. Patients often use vague or imprecise language to describe their symptoms, complicating the accurate identification of ailments. Moreover, diagnostic criteria are subjectively defined, leading to inconsistencies in diagnoses. Disease progression, characterized by its complexity and unpredictability, adds further indeterminacy in treatment planning. The variability in patient responses to treatments introduces uncertainties in outcome prediction. Inconclusive test results and limited clinical data may compound these challenges, underscoring the need for innovative approaches like N-LVHS to address these linguistic and indeterminacy-related limitations and improve the precision and efficacy of medical decision-making and treatment procedures. In constructing an N-LVHS framework for medical diagnosis and treatment, relevant factors, and linguistic terms characterizing medical conditions and treatments are identified. For example, disease severity could be described using terms such as "mild," "moderate," and "severe," while treatment effectiveness may be categorized as "low," "moderate," and "high." Each factor is then assigned neutrosophic values based on their measured impacts. This approach provides a more precise representation of the complex medical diagnostic and treatment landscape. The findings of this study have the potential to assist medical practitioners, researchers, and policymakers in optimizing medical diagnosis and treatment strategies, enhancing patient outcomes, and improving healthcare practices.

Keywords: Indeterminacy, Uncertainty, Neutrosophic set, Linguistic quantifiers; linguistic set; hypersoft set; aggregate operators; multi-criteria decision-making (MCDM).

1. Introduction

Language is closely connected to the concepts of uncertainty and indeterminacy, as it functions as a fundamental tool for the expression and communication of information. Linguistic formulations possess inherent qualities of ambiguity, imprecision, and vagueness. The comprehension of language frequently hinges upon contextual factors, individual interpretation, and subjective viewpoints, resulting in ambiguities in comprehension. The concept of indeterminacy comes because of the inherent intricacy of language, wherein the demarcation between categories can be ambiguous, and numerous interpretations can simultaneously exist. The examination of this relationship necessitates the acknowledgment of the role played by linguistic imprecision and subjectivity in generating uncertainty within the realms of communication and decision-making. The utilization of frameworks such as fuzzy logic or neutrosophic set theory can offer a systematic methodology for handling linguistic uncertainty and indeterminacy. These frameworks provide a range of tools to quantify, model, and effectively navigate the intricate nature of language in diverse applications, such as decision-making and information processing.

Within the realm of medical diagnosis and treatment, there have been notable constraints identified pertaining to language and indeterminacy. These limits have the potential to affect the precision and effectiveness of healthcare treatments. One of the primary difficulties that develops stems from the inherent ambiguity included in the descriptions of symptoms offered by patients. Frequently, individuals seeking medical attention employ inaccurate or ambiguous terminology when articulating their medical concerns, hence posing challenges for healthcare practitioners in accurately comprehending and classifying symptoms [1]. The presence of linguistic indeterminacy has the potential to impede the accuracy of both diagnosis and suggestions for treatment. Furthermore, the subjectivity of diagnosis criteria in many medical disorders adds an additional degree of ambiguity to the procedure. There may be variations in diagnostic criteria across healthcare practitioners, which can result in inconsistencies in the diagnosis and treatment decisions [2]. The presence of subjectivity may be intensified by the intricate and uncertain course of illness advancement, leading to uncertainty in selecting the most appropriate treatment strategy [3].

Moreover, it is worth noting that patients' reactions to medical interventions frequently demonstrate a considerable degree of variability, hence amplifying the inherent uncertainty associated with forecasting the outcomes of treatments. The inclusion of patient-specific characteristics, genetic factors, and variances in physiological responses all contribute to the presence of uncertainty in healthcare decision-making [4]. Additionally, the presence of equivocal test results and a scarcity of comprehensive clinical data, both of which are commonly seen in medical practice, contribute to increased ambiguity and uncertainty, hence posing challenges in the development of precise diagnostic and treatment approaches [5].

Considering the linguistic and indeterminacy-related obstacles, researchers have investigated novel methodologies such as Neutrosophic Linguistic Fuzzy-Valued Hypersoft Sets to improve the accuracy and effectiveness of medical decision-making and treatment protocols. According to Das et al. [6], these frameworks facilitate the ability of healthcare professionals to effectively handle linguistic ambiguity, effectively represent intricate medical data, and effectively integrate several aspects of uncertainty. As a result, these frameworks play a crucial role in enhancing the dependability of diagnoses and the development of personalized treatment plans.
In 1975, Zadeh [7] introduced the concept of linguistic variables and their application in approximate reasoning, particularly in decision-making. These concepts are now widely used in multi-criteria decision-making (MCDM), which aims to enhance decision-making, improve transparency, and facilitate robust solutions aligned with goals and objectives. Delgado, et al. [8] presented linguistic decision-making models, [9] proposed a method based on linguistic aggregation operators, and Wu et al. [10] proposed a multiple criteria decision-making model under linguistic environment.

In 1998, Smarandache introduced a new idea to deal with uncertain, inconsistent, and indeterminate environments, known as neutrosophic sets (NS) [11]. NS incorporates indeterminacy values along with membership and non-membership values (T, I, and F), which are independent of each other. Based on these neutrosophic numbers assigned by decision-makers (DM), NS was expanded to include concepts such as bipolar neutrosophic sets (BPNS) [16], single-valued neutrosophic sets (SVNS) [12], multi-valued neutrosophic sets (MVNS) [13], interval-valued neutrosophic sets (IVNS) [14], and multi-valued interval neutrosophic sets (MVINS) [15]. The application of the neutrosophic linguistic set and application was presented by [16]. These concepts found immediate applications in real-world situations, particularly in multi-criteria decision-making (MCDM) problems. Various strategies have been proposed by scholars to address MCDM, including TOPSIS, AHP, VIKOR, ELECTRE, WSM, WPM, and others [17-22].

The applications of neutrosophic sets and their hybrids in MCDM approaches have been explored by numerous scholars [23–26] and [27]. By employing mathematical methods, real-world problems such as human resource selection, gadget selection, shortest path selection, robot selection, security considerations, medical equipment selection, and environmental safety measures can be addressed. To overcome the limitations and challenges of existing set architectures, Molodstov introduced the concept of a soft set (SS) [28]. The application and the concept of soft topology were described by [29–30]. Maji extended a soft set by combining it with neutrosophic sets, leading to the theory of neutrosophic soft sets (NSS) to address indeterminacy [31]. Deli introduced interval-valued neutrosophic soft sets (IVNSS) along with fundamental concepts, operations, and decision-making techniques [32]. Alkhazaleh introduced the concept of n-valued refined neutrosophic soft sets (nVNRSS) [33], while Alkhazaleh and Hazaymeh presented their operations and applications in MCDM methods [34]. With the development of set structures, operators, and applications, measuring the similarity between sets became crucial. Broumi addressed this by proposing various similarity measures for neutrosophic sets [35]. The application in medical equipment selection and prediction of FIFA 2018 results has been presented by [36–37].

A hypersoft set (HSS), which Smarandache first introduced in 2018 [38], The set is described as a mapping from the desired set of attributes and the power set of the universal set to the cartesian product of attributes, which are further subdivided. Extensions, including fuzzy hypersoft sets (FHSs), intuitionistic hypersoft sets (IHSs), and neutrosophic hypersoft sets (NHSs), have also been proposed to accommodate various levels of truth, uncertainty, and indeterminacy [38]. Neutrosophic hypersoft sets (NHSs), including single-valued neutrosophic hypersoft sets (SVNHSs) [39] and aggregate operators [40], multi-valued neutrosophic hypersoft sets (m-PNHSs), interval-valued neutrosophic hypersoft sets (IVNHSs), and multi-valued interval neutrosophic hypersoft sets (m-PIVNHSs), were defined by [41]. Matrix notations and MCDM algorithms along with case studies were presented by [42]. The distance and similarity measures of NHSs were employed in MCDM techniques, specifically in medicine and nanotechnology [43–47]. The concept of linguistic hypersoft set (LHSs) and fuzzy linguistic hypersoft set (LFHSs) has been proposed by [48-49]. Some more optimization and decision-making approaches [50-53] are used to solve optimization problems. The machine learning tools along with decision-making algorithms has been employed by [54-56] in many real-world examples in which the optimization of the process has been shown [57-58].

The literature review shows that existing approaches cannot resolve the uncertainty or indeterminacy of the further bifurcated attributes of linguistic variables, without considering any standard approach, aggregate operators, and similarity measures for assigning neutrosophic values to decision-making problems. The following lists the distinctive characteristics of our proposed work in comparison to the limitations of previously published methodologies and demonstrates how our contributions stand out as distinctive and potentially superior.

1. So, ultimately, it is the first objective to propose the necessary definition of neutrosophic linguistic- valued hypersoft set (N-LVHS). Aggregate operators, distance, and similarity measures and MCDM algorithms.

2. The implementation of neutrosophic linguistic-valued hypersoft sets in medical diagnostic and treatment protocols presents challenges, including uncertainty and indeterminacy of language and potential computational difficulties. These frameworks present a transformative methodology that allows healthcare professionals to quantitatively analyze, model, and effectively traverse the intricate linguistic aspects of patient symptoms, diagnostic criteria, and treatment alternatives.

3. Furthermore, these approaches provide a systematic method for establishing uniformity in linguistic terminology within the healthcare field, hence mitigating the presence of subjective interpretations and discrepancies in diagnostic criteria. Moreover, the capacity to manage diverse medical data and integrate several aspects of uncertainty enhances the holistic comprehension of intricate medical problems and facilitates the customization of treatment approaches according to the unique requirements of each patient. The utilization of these

novel frameworks makes a substantial contribution to the progress of precision medicine and the enhancement of healthcare quality.

4. This contribution has the potential to benefit various fields that rely on language-based decision making, such as natural language processing, sentiment analysis, and artificial intelligence, among others.

5. The use of COVID-19 as a case study demonstrates the complexity of the epidemic, where linguistic ambiguities are crucial. Patients frequently exhibit ambiguous and overlapping symptoms, and the characteristics of the virus may be described inexactly in medical records. N-LVHS is ideally suited to handle this difficulty because of its ability to model and control linguistic ambiguities and indeterminacies. N-LVHS can help with precise symptom assessment, data analysis, and diagnostic judgments by quantifying and structuring linguistic concepts.

The organization of the research paper is structured in the following manner: Section 2 provides an in-depth examination of the fundamental principles that form the basis of linguistic hypersoft sets (N-LVHS). In the subsequent section, we present a comprehensive analysis of N-LVHS, encompassing precise definitions, core concepts, and illustrated instances. Additionally, we explore the fundamental properties and operations associated with N-LVHS. Section 4 serves to introduce the operational laws that govern N-LVHS, so establishing the fundamental principles upon which the future parts are built. In this paper, Sections 5 and 6 provide a detailed exposition of the Neutrosophic Linguistic Valued-Hypersoft Ordered Weighted Geometric Averaging Operator (NLV-HSOWGAO) and the Neutrosophic Linguistic Valued-Hypersoft Weighted Geometric Averaging Operator (NLV-HSWGAO), respectively. In the sixth section, we present a well-defined framework for MCDM that utilizes the "N-LVHS Algorithm to solve MCDM problems." This framework is further illustrated by means of a case study. The findings of the study and their implications are concisely outlined in section 7, culminating in a discussion of possible avenues for further research. The visual representation of the paper's overall layout may be observed in Figure 1, providing a clear point of reference.

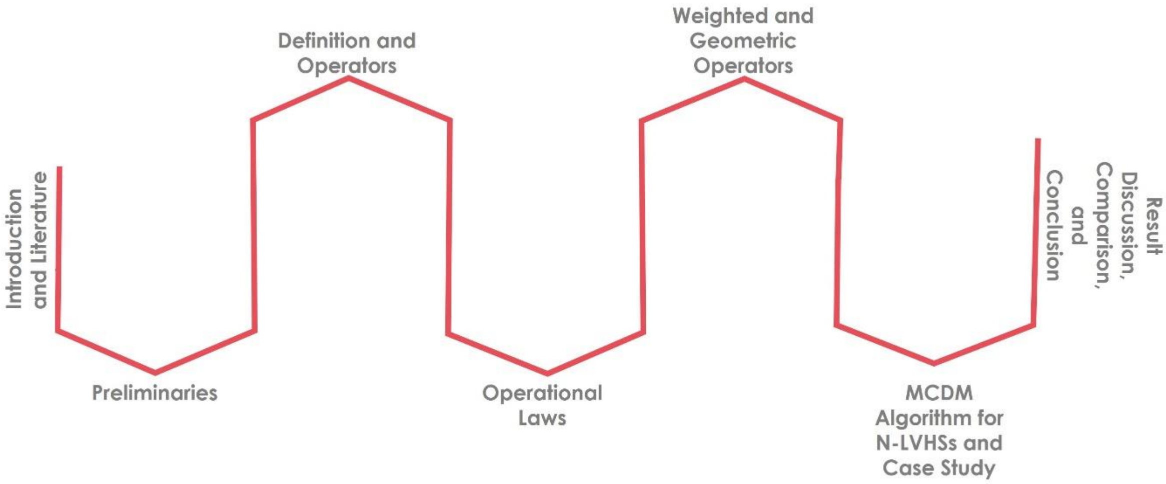

Figure 1. Layout of the paper

2. Preliminary section

In this section, we go through some basic definitions that support the construction of the framework of this paper: linguistic set, linguistic quantifiers, soft set, and hypersoft set (HSS).

Definition 2.1. Linguistic Set [7]

Let $\mathrm{K} = \{\kappa^1, \kappa^2, \kappa^3, \dots, \kappa^t\}$ $where \ \ t = 2n+1: \ \ n \geq 1 \ \ and \ \ n \ \ \in \mathbb{R}^+$, be a finite strictly increasing set. For example, if n = 1 then,

$\mathrm{K} = \{\kappa^1, \kappa^2, \kappa^3\} = \{very \ \ bad, fair, very \ \ good\}$

For Linguistic set, which is under consideration, the relationship to its elements κ^t and the superscript t will be strictly increasing. To define the continuity this set is extended to $\mathrm{K} = \{\kappa^\beta : \beta \in \mathbb{R}\}$ $where \ \ \beta \ \ is \ \ also \ \ stricly \ \ incrasing.$

Definition 2.2. Hypersoft Set [38]

Let, $a^1, a^2, a^3, \dots, a^t$ for $t \geq 1$ be t distinct parameters, whose corresponding parametric values are respectively the sets $\mathcal{L}^1, \mathcal{L}^2, \mathcal{L}^3, \dots, \mathcal{L}^t$ with $\mathcal{L}^i \cap \mathcal{L}^j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \dots, t\}$.

Then the pair $(\mathcal{F}, \mathbb{L})$ where $\mathbb{L} = \{\mathcal{L}^1 \times \mathcal{L}^2 \times \mathcal{L}^3 \times \dots \times \mathcal{L}^t$: t is finite and real valued} is known as Hypersoft set over $\mho$ with mapping $\mathcal{F}: \ \mathbb{L} = \mathcal{L}^1 \times \mathcal{L}^2 \times \mathcal{L}^3 \times \dots \times \mathcal{L}^t \ \longrightarrow P(\mho)$.

Definition 2.3. Linguistic Hypersoft Set [48]

Let, $\alpha^1, \alpha^2, \alpha^3, \dots, \alpha^t$ for $t \geq 1$ be t distinct parameters, whose corresponding parametric values are respectively the sets $\Upsilon^1, \Upsilon^2, \Upsilon^3, \dots, \Upsilon^t$ with $\Upsilon^i \cap \Upsilon^j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \dots, t\}$.

Then the pair (Γ, Λ) where $\Lambda = \{\Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \times \dots \times \Upsilon^t$: t is finite and real valued} is known as hypersoft set over Ω with mapping $\Gamma : \Lambda = \Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \times \dots \times \Upsilon^t \ \longrightarrow P(\Omega)$.

Then the linguistic hypersoft set will be,

$$\Gamma(\{\mathrm{M}(\Omega)(\imath)\}) : M \subseteq \Lambda \quad \& \quad \imath \in \mathrm{K} = \{\kappa^1, \kappa^2, \kappa^3, \dots, \kappa^t\} \ \ where \ \ t = 2n+1: \ \ n \geq 1, \quad n \in \mathbb{R}^+\}$$

3. Neutrosophic-Linguistic Valued Hypersoft Set (N-LVHS)

In this section, we propose N-LVHS with its set structure properties.

Definition 3.1: Neutrosophic Linguistic Valued Hypersoft Set (N-LVHS)

Let, $\alpha^1, \alpha^2, \alpha^3, \dots, \alpha^t$ for $t \geq 1$ be t distinct parameters, whose corresponding parametric values are respectively the sets $\Upsilon^1, \Upsilon^2, \Upsilon^3, \dots, \Upsilon^t$ with $\Upsilon^m \cap \Upsilon^n = \emptyset$, for $m \neq n$, and m, n ∈ {1, 2, …, t}.

Then the pair (Γ, Λ) where $\Lambda = \{\Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \times \dots \times \Upsilon^t\}$ $where$ t is finite and real valued} is known as hypersoft set over Ω with mapping $\Gamma : \Lambda = \Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \times \dots \times \Upsilon^t \ \longrightarrow P(\Omega)$.

Then the neutrosophic-linguistic valued hypersoft set will be,

$$\Gamma(\alpha(k)) = \{M(\alpha(\mathrm{T}, \mathrm{I}, \mathcal{F})) \ \mid \ \mathrm{T}, \mathrm{I}, \mathcal{F} \in \mathrm{k} = \{\kappa^1, \kappa^2, \kappa^3, \dots, \kappa^t\} \ \}$$

$where$ k is the set of lingusitic quantifiers in ascending order i.e. low to high.

Numerical Example 3.1.1:

Let $\Omega = \{\sigma^1, \sigma^2, \sigma^3, \sigma^4\}$ and set $M((\alpha(k)) = \{\sigma^2, \sigma^3\} \subset \Omega$.

Consider the parameters be: $\alpha^1 = nationality, \alpha^2 =$ gender, $\alpha^3 =$ color, and their respective parametric values are:

Nationality = $\Upsilon^1 = \{Pakistani, Chinese, American\}$

Gender = $\Upsilon^2 = \{Male, Female\}$

Color = $\Upsilon^3 = \{Pink, Black, Orange\}$

Then the function $\Gamma : \Lambda = \Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \longrightarrow P(\Omega)$ and assume the hypersoft set,

$\Gamma(\{Pakistani, Male, Orange\}) = \{\sigma^2, \sigma^3\} = M(\alpha(\mathrm{T}, \mathrm{I}, \mathcal{F})))$

The neutrosophic-linguistic valued hypersoft set (N-LVHS), $\Gamma(\sigma^K) = \{M(\alpha(\mathrm{T}, \mathrm{I}, \mathcal{F}))| \ \mathrm{T}, \mathrm{I}, \mathcal{F} \in \mathrm{k}\}$

$\Gamma(\{Pakistani, Male, Orange\}) = \{\sigma^2, \sigma^3\} = \{\sigma^2(v.high, medium, low), \sigma^3(low, v.high, medium)\} = L.$

Similarly,

$\Gamma_1(\{Pakistani, Male, Pink\}) = \{\sigma^2(medium, medium, medium), \sigma^3(low, low, high)\} = L_1$

$\Gamma_2(\{Chinese, Female, Pink\}) = \{\sigma^1(medium, medium, medium), \sigma^4(v.v.low, medium, high)\} = L_2$

$\Gamma_3(\{American, male, black\}) = \{\sigma^1(v.v.high, medium, low), \sigma^3(v.low, high, low)\} = L_3$

Definition 3.2: Let $(\Gamma_1, \Lambda_1) = L_1$ be a N-LVHS, then the subset L_s can be defined as. $\Gamma\Big((\alpha(\mathrm{k})) = \{M(\alpha(\mathrm{T}, \mathrm{I}, \mathcal{F}))| \ \mathrm{T}, \mathrm{I}, \mathcal{F} \in \mathrm{k}\Big)\}$

1. $L_s \subseteq L_1$;
2. $\forall \sigma \in L_s, \Gamma_2(\sigma) \subseteq \Gamma_1(\sigma)$.

This holds only when linguistic variables σ^k satisfy the property i.e., each σ^k of $(\Gamma_s, \Lambda_s) \leq \sigma^k$ of (Γ_1, Λ_1).

Example 3.2.1: Recall Example 1. The function $\Gamma_2 : \Lambda_s = \Upsilon^1 \times \Upsilon^2 \longrightarrow P(\Omega)$ and assume the hypersoft set, $\Gamma_2(\{Pakistani, Male\}) = \{\sigma^2(medium, medium, medium)\} = L_s$. Where $\Lambda_s \subseteq \Lambda$ and $L_s \subseteq L_1$.

Definition 3.3: Empty neutrosophic-linguistic valued hypersoft set (EN-LVHS) can be defined as.

$\Gamma_1 : \Lambda_E = \Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \times \ldots \times \Upsilon^n \longrightarrow P(\Omega)$

such that each Υ^{i} $(i \leq n)$ *is empty.* $\Gamma_1(\{L_E(\Omega)\})$

1. $(\Gamma_1, \Lambda_E)^{\phi} = L_E$ **if** $\forall \Gamma_1(\sigma^k) = \phi : \forall \sigma^{\mathrm{k}} \in \Lambda_E$.

Example 3.3.1: Recall Example 1. The function $\Gamma_1 : \Lambda_E = \Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \longrightarrow P(\Omega)$ and assume the Hypersoft set, $\Gamma_1(\emptyset) = \emptyset = L_E$. Where $\Lambda_E \subseteq \Lambda$.

Definition 3.4: The AND operation on two $(\Gamma_1, \Lambda_1) = L_1$ and $(\Gamma_2, \Lambda_2) = L_2$ neutrosophic-linguistic valued hypersoft set (N-LVHS) can be defined by;

1. $L_1 \wedge L_2 = (\Gamma_3, \Lambda_3) = L_3$; max of (σ^{k})
2. $(\sigma_i, \sigma_j) = \sigma_k = L_3$ where $\sigma_i \in \sigma_1$ *and* $\sigma_j \in L_2$ with $i \neq j$;
3. $\Gamma_3(\sigma_i, \sigma_j) = \Gamma_1(\sigma_i) \cup \Gamma_2(\sigma_j)$

Definition 3.5: The OR operation on two $(\Gamma_1, \Lambda_1) = L_1$ and $(\Gamma_2, \Lambda_2) = L_2$ neutrosophic-linguistic valued hypersoft set (N-LVHS) be defined by.

1. $L_1 \vee L_2 = (\Gamma_3, \Lambda_3) = L_3$;
2. $(\sigma_i, \sigma_j) = \sigma_k = L_3$ where $\sigma_i \in L_1$ *and* $\sigma_j \in L_2$ with $i \neq j$;
3. $\Gamma_3(\sigma_i, \sigma_j) = \Gamma_1(\sigma_i) \cap \Gamma_2(\sigma_j)$

Definition 3.6: The NOT operation on (Γ,Λ) neutrosophic-linguistic valued hypersoft set (N-LVHS) can be defined by.

1. $\sim L = \sim(\Gamma,\Lambda) =\sim \Upsilon^1 \times\sim \Upsilon^2 \times\sim \Upsilon^3 \times \ldots \times\sim \Upsilon^n$;
2. $\sim L =\sim \prod \sigma_i : i = 1,2,3,\ldots,n$
3. $|\sim L| = n - Tuple$

Definition 3.7: The Complement on $(\Gamma,\Lambda) = L$ neutrosophic-linguistic valued hypersoft set (N-LVHS) can be defined by.

1. $(\Gamma,\Lambda)^{\sim} = (\Gamma^{\sim},\sim L)$; $\Gamma^{\sim}:\sim L \longrightarrow P(\Omega)$.
2. $\Gamma^{\sim}(\sim\sigma) = \Omega\backslash\ \Gamma(\sigma);\ \ \forall\sigma \in L$

Proposition 3.8: Let $(\Gamma,\Lambda) = L$, $(\Gamma_1,\Lambda_1) = L_1$, $(\Gamma_2,\Lambda_2) = L_2$ and $(\Gamma_3,\Lambda_3) = L_3$ be neutrosophic-linguistic valued hypersoft set (N-LVHS) then following holds.

1. $(\Gamma_1,\Lambda_1) \subseteq (\Gamma_1,\Lambda_1)$
2. $(\Gamma_1,\Lambda_E)^{\Phi} \subseteq (\Gamma_1,\Lambda_1)$
3. $\sim(\sim L) = L$
4. $\sim(\Gamma_1,\Lambda_E)^{\Phi} = \Omega$
5. If $(\Gamma_1,\Lambda_1) \subseteq (\Gamma_2,\Lambda_2)$ and $(\Gamma_2,\Lambda_2) \subseteq (\Gamma_2,\Lambda_2)$ **then** $(\Gamma_1,\Lambda_1) = (\Gamma_2,\Lambda_2)$
   Iff each $\sigma^k$of $(\Gamma_1,\Lambda_1) = \sigma^k$ of (Γ_2,Λ_2).

This property holds only when linguistic variables satisfy the property i.e., each $\sigma^k$of $(\Gamma_1,\Lambda_1) = \sigma^k$of (Γ_2,Λ_2).

6. If $(\Gamma_1,\Lambda_1) \subseteq (\Gamma_2,\Lambda_2)$ and $(\Gamma_2,\Lambda_2) \subseteq (\Gamma_3,\Lambda_3)$ then $(\Gamma_1,\Lambda_1) \subseteq (\Gamma_3,\Lambda_3)$.

This property holds only when linguistic variables satisfy the property i.e., each σ^k of $(\Gamma_1,\Lambda_1) = \sigma^k$ of $(\Gamma_2,\Lambda_2) = \sigma^k$of (Γ_3,Λ_3).

Proof: Recall L, L_1, L_2 and L_3 from example 3.3.1.

1. $\Gamma_1(\{Pakistani, Male, Pink\}) = \{\sigma^2,\sigma^3\} = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, low)\} = L_1$ $\because \sigma^2(perfect, medium, low) \in L_1$ $also$ $\sigma^3(low, medium, low) \in L_1$ $\Rightarrow \sigma^2,\sigma^3 \in L_1$

Thus $(\Gamma_1,\Lambda_1) \subseteq L_1 = (\Gamma_1,\Lambda_1)$.

2. Consider $L_1 = (\Gamma_1,\Lambda_1)$
   $\because$ $\phi \in L_1$ $\Rightarrow$ $(\Gamma_1,\Lambda_E)^{\Phi} \in L_1$

Thus $(\Gamma_1,\Lambda_E)^{\Phi} \subseteq L_1 = (\Gamma_1,\Lambda_1)$ $(\Gamma_1,\Lambda_E)^{\Phi} \subseteq (\Gamma_1,\Lambda_1)$.

3. Consider $L = \{\sigma^2(perfect, medium, low), \sigma^3(0)\}$, apply definition 6, $we\ get, (\sim L) = \{\sigma^1(none, none, none), \sigma^4(perfect, medium, low)\}$ again apply definition 6, we get; $\sim(\sim L) = \{\sigma^2(perfect, medium, low), \sigma^3(none, none, none)\} = L$
4. Consider $(\Gamma_1,\Lambda_E)^{\Phi} = \phi \Rightarrow \phi \in L_E$ taking complement, $\sim(L_E) = \Omega\backslash\ \Gamma_1(\sigma^k\ ) = \phi$;

$\Rightarrow\ \sim(L_E) = \Omega$

$hence$ $\sim(\Gamma_1,\Lambda_E)^{\Phi} = \Omega$.

5. Consider, $(\Gamma_1,\Lambda_1) = \{\sigma^1(high, medium, low), \sigma^3(low, medium, v.low)\}$
   $(\Gamma_2,\Lambda_2) = \{\sigma^1(high, medium, low), \sigma^3(low, low, v.low)\}$

Each linguistic variable K^i of $(\Gamma_1,\Lambda_1) =$ linguistic variable K^i of (Γ_2,Λ_2) then this implies that $(\Gamma_1,\Lambda_1) \subseteq (\Gamma_2,\Lambda_2)$ $also$ $(\Gamma_2,\Lambda_2) \subseteq (\Gamma_1,\Lambda_1)$

thus $(\Gamma_2,\Lambda_2) = (\Gamma_1,\Lambda_1)$.

Counter Example:

Consider,

$(\Gamma_1, \Lambda_1) = \{\sigma^2(high, medium, low), \sigma^3(v.low, low, low)\}$

and

$$(\Gamma_2, \Lambda_2) = \{\sigma^2(perfect, low, low), \sigma^3(low, medium, v.low)\}$$

Each linguistic variable K^i of $(\Gamma_1, \Lambda_1) <$ linguistic variable K^i of (Γ_2, Λ_2) then this implies that $(\Gamma_1, \Lambda_1) \subseteq (\Gamma_2, \Lambda_2)$ *But* $(\Gamma_2, \Lambda_2) \nsubseteq (\Gamma_1, \Lambda_1)$ since linguistic variable of $(\Gamma_2, \Lambda_2) >$ linguistic variable of (Γ_1, Λ_1).

$$(\Gamma_2, \Lambda_2) \neq (\Gamma_1, \Lambda_1)$$

6. Same as 5.

4. Operational Laws on LHSS

In this section, we discuss the importance of operational laws and theorems and propose for N-LVHS. Let $(\Gamma_1, \Lambda_1) = L_1$ and $(\Gamma_2, \Lambda_2) = L_2$ be two N-LVHS, where $\Lambda_1 = \{\Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \times \ldots \times \Upsilon^n$: n is finite and real valued} over Ω with mapping $\Gamma : \Lambda_1 = \Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \times \ldots \times \Upsilon^n \longrightarrow P(\Omega)$ and $\Lambda_2 = \{\Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \times \ldots \times \Upsilon^m$: m is finite and real valued} over Ω with mapping $\Gamma_2 : \Lambda_2 = \Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \times \ldots \times \Upsilon^m \longrightarrow P(\Omega)$

such that.

$$\Gamma(\alpha(k)) = \{M(\alpha(\mathrm{T}, \mathrm{I}, \mathcal{F})) \mid \mathrm{T}, \mathrm{I}, \mathcal{F} \in \mathrm{k} = \{\kappa^1, \kappa^2, \kappa^3, \ldots, \kappa^t\}\}$$

where k is the set of lingusitic quantifiers in ascending order i.e. low to high.

Then the operational laws on N-LVHS can be defined with some necessary conditions.

Definition 4.1 Union of N-LVHS

Case 1: $L_1 \cup L_2 = \{\prod \alpha^i(K^i) \times \prod \alpha^j(K^j) \in \prod_{i=1}^n \Upsilon^i \times \prod_{j=1}^n \Upsilon^j\}$

Where, $\alpha^i(k^i) \in \prod_{i=1}^n \Upsilon^i$, *and* $\alpha^j(k^j) \in \prod_{j=1}^n \Upsilon^j$ *should be distinct* with $\Upsilon^i \cap \Upsilon^j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, t\}$ and $\mathrm{k} = \{\kappa^1, \kappa^2, \kappa^3, \ldots, \kappa^t\}$.

Case 2: $L_1 \cup L_2 = \{\alpha^i(k^i) \in \prod_{i=1}^n \Upsilon^i \times \prod_{j=1}^n \Upsilon^j\}$

with $i = j$ *, and linguistic* variable k^i *of* σ^i *should be same.*

Example: Consider 3.1.1,

Case 1:

$\Gamma_1(\{Pakistani, male, black\}) = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low)\} = L_1$

$\Gamma_2(\{American, Female, Pink\}) = \{\sigma^1(high, medium, low), \sigma^4(low, medium, v.low)\} = L_2$

$\because \Upsilon^i \cap \Upsilon^j = \emptyset$

$L_1 \cup L_2$

$= \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low), \sigma^1(high, medium, low), \sigma^4(low, medium, v.low)\}$.

Case 2:

$\Gamma_1(\{Pakistani, male, black\}) = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low)\} = L_1$

$\Gamma_2(\{Pakistani, female, pink\}) = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low)\} = L_2$

$$\because \Upsilon^i \cap \Upsilon^j \neq \emptyset \text{ with } i = j$$

$L_1 \cup L_2 = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low)\}$.

Case 3; (Counter example) \Restriction:

$\Gamma_1(\{Pakistani, male, black\}) = \{\sigma^2(high, medium, low), \sigma^3(v.low, medium, v.low)\} = L_1$

$\Gamma_2(\{Pakistani, female, pink\}) = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low)\} = L_2$

$$\because \ \Upsilon^i \ \cap \ \Upsilon^j \ \neq \ \emptyset \text{ with } i = j$$

Each linguistic value k^i of L_1 *is less then* linguistic value k^i of L_2 then this implies $L_1 \cup L_2$ can be defined with some restriction i.e., consider highest linguistic value k^i of each attribute.

Example:

$L_1 = \{\sigma^2(high, medium, low), \sigma^3(low, medium, v.low)\}$

$L_2 = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low)\}$

As, $\sigma^2(low, medium, v.low) < \sigma^2(perfect, medium, low), and$

$$\sigma^3(v.low, medium, v.low) < \sigma^3(low, medium, v.low)$$

Then $L_1 \cup L_2 = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low)\}$.

Definition 4.2 Intersection of N-LVHS

Let $(\Gamma_1, \Lambda_1) = L_1$ and $(\Gamma_2, \Lambda_2) = L_2$ be two N-LVHS and $\mu \geq 0$, then the intersection can be defined as;

$$L_1 \cap L_2 = \left\{ \prod \alpha^i(k^i) \times \prod \alpha^j(k^j) \in \prod_{i=1}^{n} \Upsilon^i \times \prod_{j=1}^{n} \Upsilon^j \right\} = \emptyset$$

Where, $\alpha^i(k^i) \in \prod_{i=1}^{n} \Upsilon^i, and \quad \alpha^j(k^j) \in \prod_{j=1}^{n} \Upsilon^j$ *should be distinct* with $\Upsilon^i \cap \Upsilon^j = \emptyset$,

for $i = j$, and $i, j \in \{1, 2, \ldots, t\}$ and $\{M(\alpha(T, I, \mathcal{F})) \mid T, I, \mathcal{F} \in k = \{\kappa^1, \kappa^2, \kappa^3, \ldots, \kappa^t\}$.

Case 2: $L_1 \cap L_2 = \{\alpha^i(k^i) \in \prod_{i=1}^{n} \Upsilon^i \times \prod_{j=1}^{n} \Upsilon^j\}$

with $i = j$ *, and fuzzy value* k^i *of* σ^i Then $L_1 \cap L_2 = L_1$ *or* L_2

Example: Consider,

Case 1:

$\Gamma_1(\{Pakistani, male, black\}) = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low)\} = L_1$

$\Gamma_2(\{American, Female, Pink\}) = \{\sigma^1(high, medium, low), \sigma^4(low, medium, v.low)\} = L_2$

$$\because \ \Upsilon^i \ \cap \ \Upsilon^j \ = \ \emptyset \quad L_1 \cap L_2 = \{\emptyset\}$$

Case 2:

$\Gamma_1(\{Pakistani, male, black\}) = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low)\} = L_1$

$\Gamma_2(\{Pakistani, female, pink\}) = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low)\} = L_2$

$$\because \ \Upsilon^i \ \cap \ \Upsilon^j \ \neq \ \emptyset \text{ with } i = j$$

$$L_1 \cap L_2 = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low)\}.$$

Case 3: (Counter example) \ Restriction

$\Gamma_1(\{Pakistani, male, black\}) = \{\sigma^2(high, medium, low), \sigma^3(v.low, low, v.v.low)\} = L_1$

$\Gamma_2(\{Pakistani, female, pink\}) = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low)\} = L_2$

$$\because \ \Upsilon^i \ \cap \ \Upsilon^j \ \neq \ \emptyset \text{ with } i = j$$

Each linguistic value k^i of L_1 *is less then* linguistic value k^i of L_2 then this implies $L_1 \cup L_2$ can be defined with some restriction i.e., consider highest linguistic value k^i of each attribute.

Example:

$L_1 = \{\sigma^2(high, medium, low), \sigma^3(v.low, low, v.v.low)\}$

$L_2 = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low)\}$

As,

$\sigma^2(high, medium, low) < \sigma^2(perfect, medium, low)$,

and

$\sigma^3(v.low, low, v.v.low) < \sigma^3(low, medium, v.low)$

Then $L_1 \cap L_2 = \emptyset$.

Theorem 4.3: If L_1, L_2 and L_3 be three N-LVHS then the following holds:

i. $L_1 \cup L_1 = L_1$
ii. $L_1 \cup \emptyset = L_1$
iii. $L_1 \cap L_1 = L_1$
iv. $L_1 \cap \emptyset = \emptyset$
v. $L_1 \cup L_2 = L_2 \cup L_1$
vi. $L_1 \cap L_2 = L_2 \cap L_1$
vii. $L_1 \cup (L_2 \cup L_3) = (L_1 \cup L_2) \cup L_3)$
viii. If $L_1 \subset L_2$ and $L_2 \subset L_1$ the $L_1 = L_2$.
ix. $\mu(L_1) = \mu L_1$; $\mu \geq 0$.
x. $\mu(L_1 \cup L_2) = \mu(L_2 \cup L_1)$

The proofs are straight forward. ■

Theorem 4.4

If L_1, L_2 be two N-LVHS then the operations are given as follows:

1. $\mu \times L_1 = L_{\mu\times 1}$; μ (Linguistic variable);
2. $L_1 \oplus L_2 = L_{1\oplus 2}$;
3. $L_1 \otimes L_2 = L_{1\otimes 2}$;
4. $(L_1)^{\mu} = L_{1^{\mu}}$.

Proof:

1. Consider, $\Gamma_1(\{Pakistani, male, black\}) = \{\sigma^2(perfect, medium, low), \sigma^3(low, medium, v.low)\} = L_1$ and $\mu = 0.4$,

The proofs are straight forward. ■

5. Some Aggregation Operators

Aggregate operators are essential in decision-making processes, combining and aggregating linguistic quantifiers or numerical values to assess factors. They enable informed analysis and evaluation of complex information, handling multiple criteria simultaneously, such as language, quality, reliability, and customer satisfaction, allowing for comprehensive evaluation and comparison.

Definition 5.1 NLV-HSWGAO

Consider, $\alpha^1, \alpha^2, \alpha^3, \ldots, \alpha^t$ for $t \geq 1$ be t distinct parameters, whose corresponding parametric values are respectively the sets $\Upsilon^1, \Upsilon^2, \Upsilon^3, \ldots, \Upsilon^t$ with $\Upsilon^i \cap \Upsilon^j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, t\}$.

Let $\mathfrak{A}: \Lambda = \Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \times \ldots \times \Upsilon^t \longrightarrow P(\Omega) = \Gamma(\sigma^K) = \{M(\alpha(\mathrm{T}, \mathrm{I}, \mathcal{F}))| \ \mathrm{T}, \mathrm{I}, \mathcal{F} \in \mathrm{k} = \{\kappa^1, \kappa^2, \kappa^3, \ldots, \kappa^t\}\}$ (1)

if $\mathfrak{A}^\omega \ (\alpha^1, \alpha^2, \alpha^3, \ldots, \alpha^t) = \prod_{t=1}^{n} (\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F}))^{(\omega^t)}$

Such that

$\mathfrak{A}^\omega \ (\alpha^1, \alpha^2, \alpha^3, \ldots, \alpha^t) =$

${\alpha^1}_i^{\omega^1} \otimes {\alpha^2}_i^{\omega^2} \otimes {\alpha^3}_i^{\omega^3} \otimes \ldots \otimes {\alpha^t}_i^{\omega^t} = \sigma_i(\mathrm{T}, \mathrm{I}, \mathcal{F})$

Where $\omega = (\omega^1, \omega^2, \omega^3, \ldots, \omega^t)^T$ is the exponential weighting vector of the $\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F}) \in \{M(\alpha(\mathrm{T}, \mathrm{I}, \mathcal{F}))\}$ and $\omega^t \in [0, 1]$ with $\sum_{t=1}^{n} \omega^t = 1$, and $\mathrm{k} = \{\kappa^1, \kappa^2, \kappa^3, \ldots, \kappa^t\}$. Then $\mathfrak{A}$ is called neutrosophic linguistic valued- hypersoft weighted geometric averaging operator (NLV-HSWGAO).

Example: Assume $\omega = (0.4, 0.3, 0.3)^T$ then NLV-HSWGAO $\{\sigma^2(Pakistani, Male, Orange), \sigma^3(Pakistani, Male, Orange)\} =$

$$\sigma^2 \begin{pmatrix} Pakistani(low, medium, v.low), Male(medium, medium, v.low), \\ Orange(high, low, v.low) \end{pmatrix}$$

$$\because \mathfrak{A}^\omega \ (\alpha^1, \alpha^2, \alpha^3, \ldots, \alpha^t) = \prod_{t=1}^{n} (\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F}))^{(\omega^t)}$$

$= {\alpha^1}_i^{\omega^1} \otimes {\alpha^2}_i^{\omega^2} \otimes {\alpha^3}_i^{\omega^3} \otimes \ldots \otimes {\alpha^t}_i^{\omega^t} = \sigma_i(\mathrm{T}, \mathrm{I}, \mathcal{F})$

$= \{\mathrm{Pakistani}(low, medium, v.low)^{0.4}, \mathrm{Male}(medium, medium, v.low)^{0.3}, \ \mathrm{Orange}(high, low, v.low)^{0.3}\}$

$= \sigma^2\{(low, medium, v.low)^{0.4} + (medium, medium, v.low)^{0.3} + (high, low, v.low)^{0.3}\}$

$\sigma^2(v.v.low, v.low, low)$

Similarly, $\sigma^3(none, none, none)$.

Definition 5.2 NLV-HSOWGAO

Consider, $\alpha^1, \alpha^2, \alpha^3, \ldots, \alpha^t$ for $t \geq 1$ be t distinct parameters, whose corresponding parametric values are respectively the sets $\Upsilon^1, \Upsilon^2, \Upsilon^3, \ldots, \Upsilon^t$ with $\Upsilon^i \cap \Upsilon^j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, t\}$.

Let

$\mathfrak{O}: \Lambda = \Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \times \ldots \times \Upsilon^t \longrightarrow P(\Omega)$

$\Gamma(\alpha(\mathrm{T}, \mathrm{I}, \mathcal{F})) = \{M(\alpha(\mathrm{T}, \mathrm{I}, \mathcal{F})) \ | \ \mathrm{T}, \mathrm{I}, \mathcal{F} \in \mathrm{k} =$

$\{\kappa^1, \kappa^2, \kappa^3, \ldots, \kappa^t\}\}$ (2)

If $\mathfrak{O}^\omega \ (\alpha^1, \alpha^2, \alpha^3, \ldots, \alpha^t) = \prod_{t=1}^{n} (\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F}))^{(\omega^t)}$

Such that $\mathfrak{O}^\omega \ (\alpha^1, \alpha^2, \alpha^3, \ldots, \alpha^t) = {\alpha^1}_i^{\omega^1} \otimes {\alpha^2}_i^{\omega^2} \otimes {\alpha^3}_i^{\omega^3} \otimes \ldots \otimes {\alpha^t}_i^{\omega^t} = \sigma_i(\mathrm{T}, \mathrm{I}, \mathcal{F})$

Subject to the condition, the linguistic values of α_i should be in ascending order. Where $\omega = (\omega^1, \omega^2, \omega^3, \ldots, \omega^t)^T$ is the exponential weighting vector of the $\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F}) \in \{\{M(\alpha(\mathrm{T}, \mathrm{I}, \mathcal{F})) \ | \ \mathrm{T}, \mathrm{I}, \mathcal{F} \in [0,1]\}\}$ and $\omega^t \in [0, 1]$ with $\sum_{t=1}^{n} \omega^t = 1$, and $\mathrm{T}, \mathrm{I}, \mathcal{F} \in [0,1]$ then $\mathfrak{O}$ is called neutrosophic linguistic valued-hypersoft ordered weighted geometric averaging operator (NLV-HSOWGAO).

Example: Assume $\omega = (0.4, 0.3, 0.3)^T$ then LHSOWGAO $\{\sigma^2(Pakistani, Male, Orange),$

$\sigma^3(Pakistani, Male, Orange) =$

$$\sigma^2\begin{pmatrix} Pakistani(low, medium, v.low), Male(medium, medium, v.low), \\ Orange(high, low, v.low) \end{pmatrix}$$

$$\because \mathfrak{D}^\omega\ (\alpha^1, \alpha^2, \alpha^3, \dots, \alpha^t) = \prod_{t=1}^{n} (\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F}))^{(\omega^t)}$$

$= {\alpha^1}_i^{\omega^1} \otimes {\alpha^2}_i^{\omega^2} \otimes {\alpha^3}_i^{\omega^3} \otimes \dots \otimes {\alpha^t}_i^{\omega^t} = \sigma_i(\mathrm{T}, \mathrm{I}, \mathcal{F})$

= {Pakistani$(low, medium, v.low)^{0.4}$, Male$(medium, medium, v.low)^{0.3}$, Orange$(high, low, v.low)^{0.3}$}

= $\sigma^2(v.v.low,\ v.low\ ,low\ )$

Similarly,

$\sigma^3(none, none, none)$

Theorem 5.1:

1. $\min_i(\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F})) \le \mathfrak{A}^\omega\ (\alpha^1, \alpha^2, \dots, \alpha^t) \le \max_i(\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F}))$

2. $\min_i(\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F})) \le \mathfrak{D}^\omega\ (\alpha^1, \alpha^2, \dots, \alpha^t) \le \max_i(\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F}))$

Proof: The proofs are straight forward. ∎

Theorem 5.2:

1. $\mathfrak{D}^\omega\ (\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F})) = \mathfrak{D}^\omega\ (\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F}))$

Where $(\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F}))$ is any permutation of $(\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F}))$

2. If $\forall(\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F})) = (\alpha(\mathrm{T}, \mathrm{I}, \mathcal{F}))$ for all t, then $\mathfrak{D}^\omega\ (\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F})) = \sigma_i(\mathrm{T}, \mathrm{I}, \mathcal{F})$

3. If $(\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F})) \le (\hat{\alpha}^t(\mathrm{T}, \mathrm{I}, \mathcal{F}))$ for all t, then $\mathfrak{D}^\omega\ (\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F})) \le \mathfrak{D}^\omega\ (\hat{\alpha}^t(\mathrm{T}, \mathrm{I}, \mathcal{F}))$

Proof: The proofs are straight forward. ∎

6. N-LVHS Algorithm to solve MCDM Problem

A decision-making technique based on neutrosophic linguistic valued-hypersoft weighted geometric averaging operator (NLV-HSWGAO) has been used to construct an algorithm known as neutrosophic linguistic valued hypersoft set based multi-criteria group decision-making (N-LVHS) algorithm. The graphical representation of the proposed N-LVHS algorithm is presented in Figure 2.

Step1: *Consider,* $\alpha^1, \alpha^2, \alpha^3, \dots, \alpha^t$ *for* $t \ge 1$ *be* t *distinct parameters, whose corresponding parametric values are respectively the sets* $\Upsilon^1, \Upsilon^2, \Upsilon^3, \dots, \Upsilon^t$ *with* $\Upsilon^i \cap \Upsilon^j = \emptyset$, *for* $i \ne j$, *and* $i, j \in \{1, 2, \dots, t\}$. *Let* $\omega = (\omega^1, \omega^2, \omega^3, \dots, \omega^t)^T$ *be the exponential weighting vector. Where* $\omega^t \ge 0$ *, and* $\sum_{t=1}^{n} \omega^t = 1$.

Let $\mathfrak{A}$: $\Lambda = \Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \times \dots \times \Upsilon^t \longrightarrow P(\Omega)$

$$\mathfrak{A}(\alpha(k)) = \{M(\alpha(\mathrm{T}, \mathrm{I}, \mathcal{F}))\ \mid\ \mathrm{T}, \mathrm{I}, \mathcal{F} \in \mathrm{k} = \{\kappa^1, \kappa^2, \kappa^3, \dots, \kappa^t\}\ \}$$

The decision-maker $\mathcal{D}$ *assign the values with the linguistic quantifiers and assign linguistic variable to each alternative as* $H_i = \{(\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F})\ \mid\ i = 1, 2, \dots, t$ *and* $\mathrm{k} \in \{\kappa^1, \kappa^2, \kappa^3, \dots, \kappa^t\}\}$ *, and construct a neutrosophic linguistic preference table for* $(\alpha^t(\mathrm{T}, \mathrm{I}, \mathcal{F}))^{(\omega^t)}$.

Step2: *Construct a matrix* $[\alpha_{ij}]_{\ i\ \times j}$ *for* $\mathcal{D}$ *using neutrosophic linguistic valued hypersoft weighted geometric averaging operator (NLV-HSWGAO),*

$$\sigma_{\mathrm{i}}^{\mathrm{t}}(\mathrm{T},\mathrm{I},\mathcal{F}) = {\alpha^{1}}_{i}^{\omega^{1}} \otimes {\alpha^{2}}_{i}^{\omega^{2}} \otimes {\alpha^{3}}_{i}^{\omega^{3}} \otimes \ldots \otimes {\alpha^{t}}_{i}^{\omega^{t}}$$

Step3: List *the aggregated values of all the alternatives* $\sigma_{\mathrm{i}}^{\mathrm{t}}(\mathrm{T},\mathrm{I},\mathcal{F})$.

Step4: *Finally, list the alternatives with highest truthiness* (T) *value. The maximum truthiness* (T)*, will represent the positive ideal alternative.*

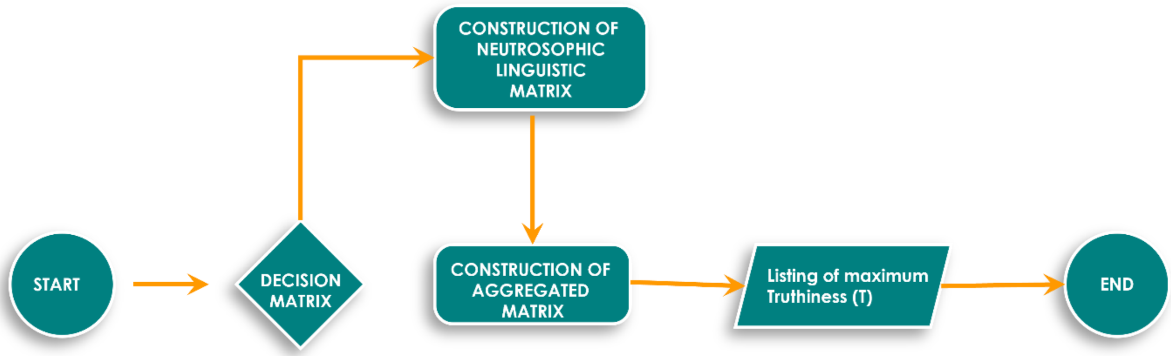

Figure 2. Graphical representation of proposed N-LVHS algorithm

6.1 Illustrative example

The use of COVID-19 as a case study demonstrates the complexity of the epidemic, where linguistic ambiguities are crucial. Patients frequently exhibit ambiguous and overlapping symptoms, and the characteristics of the virus may be described inexactly in medical records. N-LVHS is ideally suited to handle this difficulty because of its ability to model and control linguistic ambiguities and indeterminacies. N-LVHS can help with precise symptom assessment, data analysis, and diagnostic judgments by quantifying and structuring linguistic concepts. The latest COVID-19 statistics on WHO website are shown in Figure 3. (Data retrieved on 14 Oct 2023, https://covid19.who.int/table.)

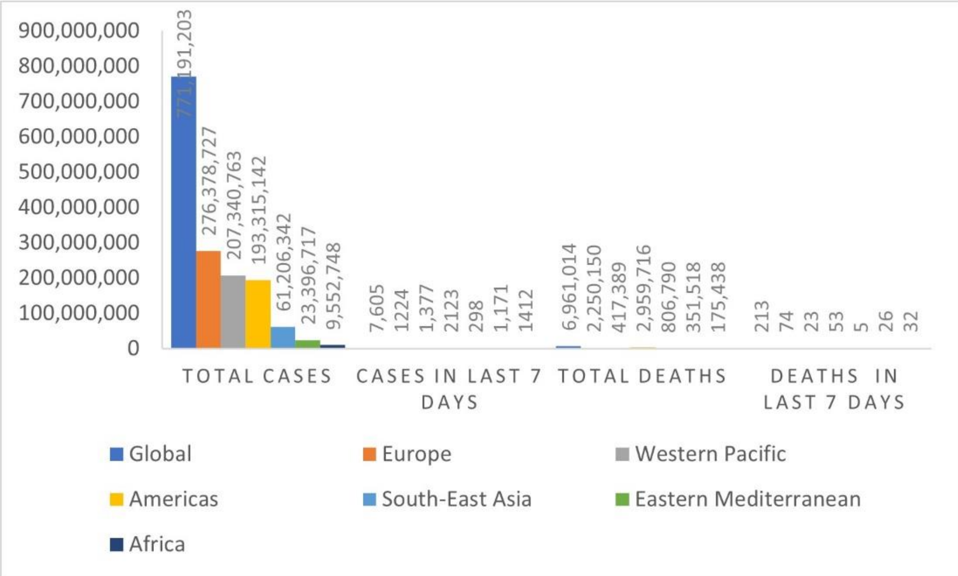

Figure 3. COVID-19 current statistics.

6.2 Demonstration of proposed example

A patient often presents a set of symptoms to a doctor during a medical checkup. However, it can frequently be difficult for clinicians to make a specific diagnosis due to the ambiguity and overlap in symptoms. Let's choose a fictitious example to show how N-LVHS would perform in it. Ten patients visit their doctor's office with a cough, a fever, lethargy, and shortness of breath. These symptoms lack specificity, making the diagnosis questionable even if they are symptomatic of several medical diseases, including COVID-19. To evaluate their symptoms more precisely, the doctor uses N-LVHS, and data presented in table 1.

Consider $P = \{P^1, P^2, \dots, P^{10}\}$ *be* ten patients as alternatives, and doctor want to diagnose. The goal should be to identify COVID-19 positive patients, while minimizing any unintended negative consequences**.** Consider the parameters be: α^1 = fever, α^2 = cough, α^3 = lethargy, and α^4 = shortness of breath.

Then the function $\Gamma : \Lambda = \Upsilon^1 \times \Upsilon^2 \times \Upsilon^3 \times \Upsilon^4 \longrightarrow P(\Omega)$ and assume the hypersoft set $P = \{P^1, P^2, \dots, P^{10}\} \subset \Omega$ where $\Omega = \{P^1, P^2, \dots, P^{10}\}$ be the universal set.

Step1: Construction of neutrosophic linguistic preference table for alternatives

Patient No. / Symptoms	**Fever**	**Cough**	**Lethargy**	**Shortness of breath**
P01	$(high, low, v.low)$	$(v.v.high, v.v.low, none)$	$(high, medium, low)$	$(v.high, low, medium)$
P02	$(low, v.low, high)$	$(high, medium, low)$	$(v.high, low, medium)$	(low, low, low)
P03	$(perfect, none, none)$	$(none, v.low, none)$	$(low, low, high)$	$(high, v.low, high)$
P04	$(v.high, none, v.low)$	$(low, v.low, high)$	$(medium, medium, low)$	$(medium, low, low)$
P05	$(low, high, medium)$	$(v.low, high, medium)$	$(v.low, low, high)$	$(low, low, high)$

P06	$(high, medium, high)$	$(medium, high, low)$	$(none, high, v.high)$	$(medium, high, high)$
P07	$(medium, low, none)$	$(high, high, low)$	$(low, none, low)$	$(v.v.high, low, low)$
P08	$(v.v.high, none, high)$	$(medium, low, none)$	$(high, medium, none)$	$(v.v.v.low, low, none)$
P09	$(high, low, low)$	$(high, none, none)$	$(none, low, low)$	$(high, low, high)$
P10	$(v.v.high, low, v.low)$	$(v.high, medium, none)$	$(medium, high, low)$	$(medium, low, medium)$

Table 1: Doctor patient interaction and information gathering in neutrosophic linguistic form.

Step2: Construction of neutrosophic linguistic valued hypersoft weighted geometric averaging operator (NLV-HSWGAO) based matrix.

$$\begin{array}{c} patients \\ P^1 \\ P^2 \\ P^3 \\ P^4 \\ P^5 \\ P^6 \\ P^7 \\ P^8 \\ P^9 \\ P^{10} \end{array} = \begin{bmatrix} NLV-HSWGAO\ \ values \\ (v.v.high, low, v.low) \\ (medium, low, none) \\ (high, medium, none) \\ (medium, low, medium) \\ (medium, low, low) \\ (high, high, low) \\ (high, medium, high) \\ (perfect, none, none) \\ (v.high, low, medium) \\ (low, v.low, high) \end{bmatrix}$$

Step3: List *the aggregated values of all the alternatives* $\sigma_i^t(\mathrm{T}, \mathrm{I}, \mathcal{F})$.

$$\begin{array}{c} patients \\ P^1 \\ P^2 \\ P^3 \\ P^4 \\ P^5 \\ P^6 \\ P^7 \\ P^8 \\ P^9 \\ P^{10} \end{array} = \begin{bmatrix} aggregated\ \ values \\ v.v.high \\ medium \\ high \\ medium \\ medium \\ high \\ high \\ perfect \\ v.high \\ low \end{bmatrix}$$

Step4: *Finally, list the alternatives with highest truthiness* (T) *value. The maximum truthiness* (T)*, will represent the positive ideal alternative.*

Alternative	**Result**
P01	*Positive*
P02	*Negative*
P03	*Positive*
P04	*Negative*
P05	*Negative*
P06	*Positive*
P07	*Positive*
P08	*Positive*
P09	*Positive*
P10	*Negative*

In this imaginary case study, we saw the difficulty that doctors frequently encounter when patients present with symptoms that are vague and common to several different illnesses. The symptoms that the 10 patients with cough, fever, lethargy, and shortness of breath experienced were symptomatic of several disorders, including the frequently occurring COVID-19. The doctor used the N-LVHS algorithm to solve this diagnostic since it uses cutting-edge language models to analysis and interpret patient information. The N-LVHS delivered a more accurate and data-driven assessment, greatly reducing diagnostic ambiguity, by carefully examining the patients' symptoms and comparing them with a wide pool of medical data. The relation between the symptoms and diagnostic has been presented in Figure 4.

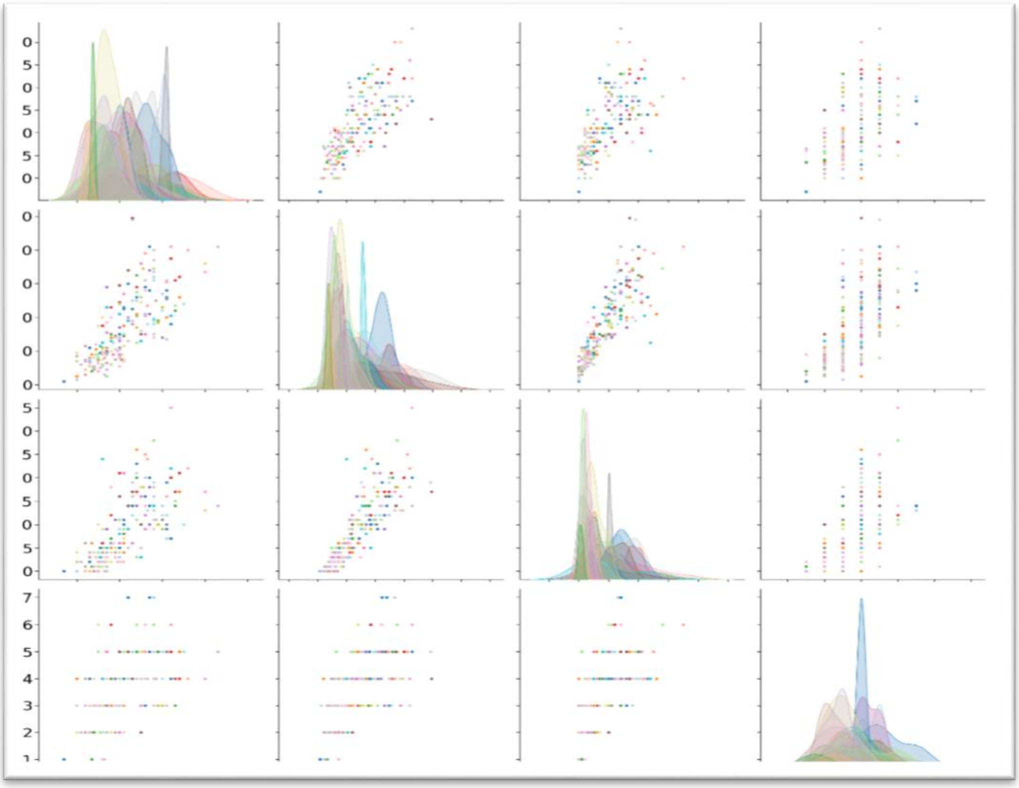

Figure 4. Symptoms relation with diagnosis and information.

6.3 Result discussion comparison and future directions

Certainly, comparing the outcomes of the N-LVHS algorithm with those of current diagnostic techniques offers important insights into the prospective advantages of this cutting-edge instrument. Traditional diagnostic techniques frequently depend on clinical judgement and medical expertise, which can be difficult in situations with confusing symptoms like those seen in our study. It is interesting that, in contrast to our suggested method, the existing approaches use a completely different methodology to calculate the results of alternatives table 2, presents the comparison with existing approaches.

N-LVHS, in comparison, uses cutting-edge language models and medical data to analyze symptoms in a more thorough and data-driven way. N-LVHS has a significant edge in terms of diagnosis accuracy because it can consider a wide range of medical data, new research, and real-time data. Additionally, it excels at managing risk and adjusting to new medical information, which is particularly important in situations like the COVID-19 pandemic.

Method	*Positive*	*Negative*
LHSs (Saqlain et al. [48])	*P01, P03, P06, P07, P08, P09, P10*	*P02, P04, P05*
FLHSs (Saqlain et al. [49])	*P01, P03, P06, P07, P08, P09*	*P02, P04, P05, P10*
N-LVHS (Proposed)	*P01, P03, P06, P07, P08, P09*	*P02, P04, P05, P10*

Table 2. Result comparison with existing studies.

The potential of N-LVHS to improve healthcare outcomes and supplement conventional diagnostic techniques is highlighted by this comparison. While it's important to recognize that AI-driven technologies cannot take the place of a healthcare professional's knowledge and experience, their integration can greatly improve diagnostic accuracy, especially in cases when symptoms are complex and difficult to identify. A new age of more precise, effective, and patient-centered healthcare is promised by further research and collaboration efforts between AI technology and the medical sector.

7. Conclusion

In conclusion, this study emphasizes the importance of language and the difficulties it presents when it comes to medical diagnosis and treatment. This study provides a possible method for enhancing healthcare decision-making by introducing Neutrosophic-Linguistic Valued Hypersoft Sets (N-LVHS), a potent tool that successfully regulates linguistic uncertainty and indeterminacy.

The necessary definitions, notions, aggregate operators, and algorithms has been proposed in this paper. The N-LVHS can be used as a crucial tool to solve the complexities of medical practice in a constantly changing healthcare environment where language-driven ambiguity and uncertainty prevail. This study contributes to the ongoing effort to provide healthcare that is more effective and patient-centered by addressing linguistic imprecision and indeterminacy. It emphasizes the significance of integrating cutting-edge linguistic and computational tools to improve healthcare practices in a complex and uncertain world. N-LVHS will need to be expanded to include a wider range of medical conditions in the future, and data scientists and healthcare professionals will need to work together to improve N-LVHS algorithms. The proposed study has the potential for a wide range of case study applications in numerous fields. It can be used in market research to understand customer attitude, environmental impact assessments to balance intricate ecological, social, and economic issues, and disaster preparedness to determine resource allocation and reaction plans.

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References

[1] Kuhn T, Basch P, Barr M, Yackel T. (2015). Medical Informatics Committee of the American College of Physicians. Clinical documentation in the 21st century: executive summary of a policy position paper from the American College of Physicians. Annals of Internal Medicine, 162(4), 301-303. doi: 10.7326/M14-2128.

[2] Verhoog, S., Eijgermans, D.G.M., Fang, Y. (2022). Contextual determinants associated with children's and adolescents' mental health care utilization: a systematic review. European Child & Adolescent Psychiatry, (2022). https://doi.org/10.1007/s00787-022-02077-5

[3] Pienta, K.J., Hammarlund EU, Axelrod R, Brown JS, Amend SR. (2020). Poly-aneuploid cancer cells promote evolvability, generating lethal cancer. Evolutionary Applications, 13(7), 1626-1634. doi: 10.1111/eva.12929.

[4] Relling MV, Evans WE. (2015). Pharmacogenomics in the clinic. Nature. 526, 343-350. doi: 10.1038/nature15817.

[5] Kho AN, Cashy JP, Jackson KL, Pah AR, Goel S, Boehnke J, Humphries JE, Kominers SD, Hota BN, Sims SA, Malin BA, French DD, Walunas TL, Meltzer DO, Kaleba EO, Jones RC, Galanter WL. (2015). Design and implementation of a privacy preserving electronic health record linkage tool in Chicago. Journal of the American Medical Informatics Association, 22(5), 1072-1080. doi: 10.1093/jamia/ocv038.

[6] Das R, Mukherjee A, Tripathy BC. (2022). Application of Neutrosophic Similarity Measures in Covid-19. Annals of Data Science. 9(1), 55–70. doi: 10.1007/s40745-021-00363-8.

[7] Zadeh, L. A. (1975). The Concept of a Linguistic Variable and its Application to Approximate Reasoning-II. Information Sciences, 8, 199-249.

[8] Delgado, M., Verdegay, L. J., Vila, M. A. (1992). Linguistic decision-making models. International Journal of Intelligent, 7, 479-492. Doi: https://doi.org/10.1002/int.4550070507

[9] Jiang, L., Liu, H., Cui, N. (2021). A semantic model for computing with flexible linguistic expressions and the application in MCGDM. Computers & Industrial Engineering, 158, 107409. Doi: https://doi.org/10.1016/j.cie.2021.10740

[10] Wu, X., Liao, H., Benjamin, L., Zavadskas, E. K. (2023). A Multiple Criteria Decision-Making Method with Heterogeneous Linguistic Expressions. IEEE Transactions on Engineering Management, 70(5), 1857-1870. Doi: https://doi.org/10.1109/TEM.2021.3072590

[11] Smarandache, F. (1998). Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis and synthetic analysis, (October).

[12] Wang, H., Y. Zhang, and R. Sunderraman. (2010). Single-Valued Neutrosophic Sets, Multispace and Multistructure, Neutrosophic Transdisciplinarity, 100 Collected Papers of Sciences, Vol. IV, 410-419. North-European Scientific Publishers Hanko, Finland 2010. ISBN: 978-952-67349-2-7

[13] Aiwu, Z., Jianguo, Du., and Hongjun, G. (2015). Interval Valued Neutrosophic Sets and Multi-attribute Decision-making Based on Generalized Weighted Aggregation Operator. Journal of Intelligent and Fuzzy Systems, 29(6), 2697-2706. doi: 10.13195/j.kzyjc.2014.0467.

[14] Wang, H., Smarandache, F., Zhang, Y., and Sunderraman, R. (2005). Interval Neutrosophic Sets and Logic: Theory and Applications in Computing. ArXiv. /abs/cs/0505014.

[15] Zhang, X. and Xu, Z. (2014). Extension of TOPSIS to Multiple Criteria Decision Making with Pythagorean Fuzzy Sets. International Journal of Intelligent Systems, 29, 1061-1078. https://doi.org/10.1002/int.21676

[16] Amalini. L. N., Kamal. M., Abdullah. L. and Saqlain. M. (2020) Multi-Valued Interval Neutrosophic Linguistic Soft Set Theory and Its Application in Knowledge Management, CAAI Transactions on Intelligence Technology, 5, 200-208. https://doi.org/10.1049/trit.2020.0036

[17] Deli, I., Ali, M., and Smarandache, F. (2015). Bipolar neutrosophic sets and their application based on multicriteria decision-making problems, 2015 International Conference on Advanced Mechatronic Systems (ICAMechS), Beijing, China, 249-254. doi: 10.1109/ICAMechS.2015.7287068.

[18] Adriyendi. (2015). Multi-attribute decision-making using simple additive weighting and weighted product in food choice. International Journal of Information Engineering and Electronic Business, 6, 8-14. Doi: 10.5815/ijieeb.2015.06.0

[19] Delgado, M., Verdegay, J.L., and Vila, M.A. (1992). Linguistic decisionˆamaking models. International Journal of Intelligent Systems, 7, 479-492. DOI:10.1002/int.4550070507

[20] Afshari, A. R., and Mojahed, M. (2010). Simple additive weighting approach to personnel selection problem. International Journal of Innovation, Management and Technology, 1(5), 511-515.

[21] Adriyendi. (2015). Multi-attribute decision-making using simple additive weighting and weighted product in food choice. International Journal of Information Engineering and Electronic Business, 6, 8-14. DOI: 10.5815/ijieeb.2015.06.0

[22] Abdullah, L., and Adawiyah, C.W. (2014). Simple Additive Weighting Methods of Multi-Criteria Decision Making and Applications: A Decade Review. International Journal of Information Processing and Management, 5, 39-49.

[23] Abdel-Baset, M. Chang, V., and Gamal, A. (2019). Evaluation of the green supply chain management practices: A novel neutrosophic approach. Computers in Industry, 108, 210-220. https://doi.org/10.1016/j.compind.2019.02.013

[24] Abdel-Basset, M., Saleh, M., Gamal, A., and Smarandache, F. (2019). An approach of TOPSIS technique for developing supplier selection with group decision making under type- 2 neutrosophic number. Applied Soft Computing, 77, 438-452. https://doi.org/10.1016/j.asoc.2019.01.035

[25] Abdel-Basset, M., Manogaran, G., Gamal, A., and Smarandache, F. (2019). A group decision-making framework based on neutrosophic TOPSIS approach for smart medical device selection. Journal of Medical Systems, 43(2), 38. DOI: 10.1007/s10916-019-1156-1

[26] Abdel-Basset, M., Mohamed, M., Elhoseny, M., Son, L. H., Chiclana, F., and Zaied, A. E. H. (2019). Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases. Artificial intelligence in medicine, 101, 101735. https://doi.org/10.1016/j.artmed.2019.101735

[27] Abdel-Basset, M., A. Gamal, G. Manogaran, L. H. Son, and H. V. Long. (2020). A novel group decision making model based on neutrosophic sets for heart disease diagnosis, Multimedia Tools and Applications, 79(15-16), 9977-10002. doi: 10.1007/s11042-019-07742-7.

[28] Molodtsov, D. (1999). Soft set theory - First results, Computers and mathematics with applications, 37(4-5), 19-31. doi: 10.1016/30898-1221(99)00056-5

[29] Riaz, M. Smarandache F., Firdous A., Fakhar, A. (2019). On Soft Rough Topology with Multi-Attribute Group Decision Making, Mathematics, 7(1), 1-18.

[30] Riaz, M., Davvaz, B., Firdous A., Fakhar, A. (2019). Novel Concepts of Soft Rough Set Topology with Applications, Journal of Intelligent and Fuzzy Systems, 36(4), 3579-3590. DOI: 10.3233/JIFS-181648.

[31] Maji, K P. (2013). Neutrosophic soft set. Annals of Fuzzy Mathematics and Informatics, 5(1), 157-168.

[32] Deli, I. (2017). Interval-valued neutrosophic soft sets and its decision-making. International Journal of Machine Learning and Cybernetics. 8, 665–676. https://doi.org/10.1007/s13042-015-0461-3.

[33] Alkhazaleh, S. (2016). n-valued refined neutrosophic soft set theory, 2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Vancouver, BC, Canada, 2016, pp. 2475-2480. doi: 10.1109/FUZZIEEE.2016.7738004.

[34] Alkhazaleh, S. and Hazaymeh,A. (2018). n-Valued Refined Neutrosophic Soft Sets and their Applications in Decision-Making Problems and Medical Diagnosis. Journal of Artificial Intelligence and Soft Computing Research, 8(1), 79-86. https://doi.org/10.1515/jaiscr-2018-0005

[35] Broumi, S., and Smarandache, F. (2012). Several Similarity Measures of Neutrosophic Sets. Neutrosophic Sets and Systems, 1, 54-62.

[36] Saeed, M., Saqlain, M., Mehmood, A., K.A., N., and Yaqoob, S. (2020). Multi-Polar Neutrosophic Soft Sets with Application in Medical Diagnosis and Decision-Making. Neutrosophic Sets and Systems, 33, 183-207. doi: 10.5281/ zenodo. 3782895

[37] Saeed M., Saqlain M. and Riaz M. (2019). Application of Generalized Fuzzy TOPSIS in Decision Making for Neutrosophic Soft set to Predict the Champion of FIFA 2018: A Mathematical Analysis, Punjab University Journal of Mathematics, 51(8), 111-126.

[38] Smarandache F. (2018). Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set, Neutrosophic Sets and Systems, 22, 168-170.

[39] Saqlain M., S. Moin, M. N. Jafar, M. Saeed, and F. Smarandache. (2020). Aggregate Operators of Neutrosophic Hypersoft Set, Neutrosophic Sets and Systems, 32,294-306. doi: 10.5281/zenodo. 3723155.

[40] Saqlain M, Sana M, Jafar N, Saeed. M, Said. B, (2020). Single and Multi-valued Neutrosophic Hypersoft set and Tangent Similarity Measure of Single valued Neutrosophic Hypersoft Sets, Neutrosophic Sets and Systems (NSS), 32, 317-329. doi: 10.5281 zenodo. 3723165.

[41] Saqlain M., and X. L. Xin. (2020). Interval Valued, m-Polar and m-Polar Interval Valued Neutrosophic Hypersoft Sets," Neutrosophic Sets and Systems,36,389-399. doi: 10.5281/ zenodo. 4065475.

[42] Saqlain M., Saeed, M., Zulqarnain M R, Sana M. (2021). Neutrosophic Hypersoft Matrix Theory: Its Definition, Operators, and Application in Decision-Making of Personnel Selection Problem, Neutrosophic Operational Research, Springer, 10.1007/978-3-03057197-9.

[43] Saqlain M., M. Riaz, M. A. Saleem, and M. S. Yang. (2021). Distance and Similarity Measures for Neutrosophic HyperSoft Set (NHSS) with Construction of NHSSTOPSIS and Applications, IEEE Access, 9, 30803-30816. 2021. doi: 10.1109/ACCESS. 2021.3059712.

[44] Jafar N. M., Saeed, M., Saqlain, M., and Yang, M. S. (2021). Trigonometric Similarity Measures for Neutrosophic Hypersoft Sets with Application to Renewable Energy Source Selection, IEEE Access,9,129178 129187. doi: 10.1109/ ACCESS. 2021.3112721.

[45] Ihsan, M., Saeed, M., & Rahman, A. U. (2023). An intelligent fuzzy parameterized MADM-approach to optimal selection of electronic appliances based on neutrosophic hypersoft expert set. Neutrosophic Sets and Systems 53, 459-481. https://doi.org/10.5281/zenodo.7536071

[46] Rahman, A. U., Saeed, M., Mohammed, M. A., Al-Waisy, A. S., Kadry, S., & Kim, J. (2022). An innovative fuzzy parameterized MADM approach to site selection for dam construction based on sv-complex neutrosophic hypersoft set. AIMS Mathematics, 8(2), 4907–4929. https://doi.org/10.3934/math.2023245

[47] Saeed, M., Ahsan, M., Ur Rahman, A., Saeed, M. H., & Mehmood, (2021). A. An application of neutrosophic hypersoft mapping to diagnose brain tumor and propose appropriate treatment. Journal of Intelligent & Fuzzy Systems, 41(1), 1677-1699. DOI: http://doi.org/10.3233/JIFS-210482

[48] Saqlain, M., Garg, H., Kumam, P., Kumam, W. (2023). Uncertainty and decision-making with multi-polar interval-valued neutrosophic hypersoft set: A distance, similarity measure, and machine learning approach, Alexandria Engineering Journal, 84, 323-332. https://doi.org/10.1016/j.aej.2023.11.001

[49] Saqlain, M., Kumam, P., Kumam, W. and Phiangsungnoen, S. (2023). Proportional Distribution Based Pythagorean Fuzzy Fairly Aggregation Operators with Multi-Criteria Decision-Making, IEEE Access, 11, 72209-72226. https://doi.org/10.1109/ACCESS.2023.3292273

[50] Hicham, N., Nassera, H., & Karim, S. (2023). Strategic Framework for Leveraging Artificial Intelligence in Future Marketing Decision-Making. Journal of Intelligent and Management Decision, 2(3), 139-150. https://doi.org/10.56578/jimd020304

[51] Jana, C. & Pal, M. (2023). Interval-Valued Picture Fuzzy Uncertain Linguistic Dombi Operators and Their Application in Industrial Fund Selection. Journal of Industrial Intelligence, 1(2), 110-124. https://doi.org/10.56578/jii010204

[52] Saqlain, M. (2023). Sustainable Hydrogen Production: A Decision-Making Approach Using VIKOR and Intuitionistic Hypersoft Sets. Journal of Intelligent Management Decision, 2(3), 130-138. https://doi.org/10.56578/jimd020303

[53] Haq, H. B. U. & Saqlain, M. (2023). Iris Detection for Attendance Monitoring in Educational Institutes Amidst a Pandemic: A Machine Learning Approach. Journal of Industrial Intelligence, 1(3), 136-147. https://doi.org/10.56578/jii010301

[54] Smarandache F. (2023). New Types of Soft Sets "HyperSoft Set, IndetermSoft Set, Indeterm HyperSoft Set, and TreeSoft Set": An Improved Version. Neutrosophic Systems with Applications, 8, 35–41. https://doi.org/10.61356/j.nswa.2023.41

[55] Ramya G, & Francina Shalini A. (2023). Trigonometric Similarity Measures of Pythagorean Neutrosophic Hypersoft Sets. Neutrosophic Systems with Applications, 9, 91–100. https://doi.org/10.61356/j.nswa.2023.53

[56] Haque, T. S., Alam, S., & Chakraborty, A. (2022). Selection of most effective COVID-19 virus protector using a novel MCGDM technique under linguistic generalized spherical fuzzy environment. Computational and Applied Mathematics, 41(2), 84.

[57] Tešić, D., Božanić, D., Radovanović, M., & Petrovski, A. (2023). Optimising Assault Boat Selection for Military Operations: An Application of the DIBR II-BM-CoCoSo MCDM Model. Journal of Intelligent and Management Decision, 2(4), 160-171. https://doi.org/10.56578/jimd020401

[58] Stević, Ž., Mujaković, N., Goli, A., & Moslem, S. (2023). Selection of Logistics Distribution Channels for Final Product Delivery: FUCOM-MARCOS Model, Journal of Intelligent and Management Decision, 2(4), 172-178. https://doi.org/10.56578/jimd020402

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