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The neutrosophic numerical integration and MATLAB

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Abstract: In this paper, the idea of nitro has been introduced to numerical integrals, where we studied neutrosophic numerical integrals by submitting the neutrosophic trapezoidal method and estimation of error of the neutrosophic trapezoidal method, in addition to supporting examples for that and verified using MATLAB.

Keywords: neutrosophic numerical integrals; estimation of error; the neutrosophic trapezoidal method.

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1. Introduction

In contrast to the current logics, Smarandache suggested the Neutrosophic Logic to describe a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, and contradiction. Smarandache introduced the concept of neutrosophy as a new school of philosophy [4]. He presented the definition of the standard form of neutrosophic real number, the Neutrosophic statistics [3-5-6], and professor Smarandache entered the concept of preliminary calculus of the differential and integral calculus, where he introduced for the first time the notions of neutrosophic mereo-limit, mereo-continuity, mereoderivative, and mereo-integral [1]. A number of studies in the area of integration and differentiation were given by Y. Alhasan [10-11-16], also he presented the definition of the concept of neutrosophic complex numbers and its properties [2-9]. The AH isometry was used to study many structures such as conic sections, real analysis concepts, and geometrical surfaces [13-14-15].

The calculation of area, size, and arc length is one of integration's most crucial uses in daily life. We encounter things in our world that are ill-defined and partially indeterminate.

There are four sections of paper. first section, which also includes a study of neutrosophic science, serves as an introduction. In the second section, a few definitions and theories of the neutrosophic integrals are offered. The 3th section frames neutrosophic numerical integrals and

MATLAB, in which the neutrosophic trapezoidal method and estimation of error of the neutrosophic trapezoidal method were studied. In 4th section, a conclusion to the paper is given.

2. Preliminaries

2.1. Neutrosophic integration by substitution method [16]

Definition2.1.1

$Let \quad f: D_f \subseteq R \to R_f \cup \{I\}$, to evaluate $\int f(x)dx$

Put: $x = g(u) \quad \Rightarrow \quad dx = \acute{g}(u)du$

By substitution, we get:

$$\int f(x)dx = \int f(u)\acute{g}(u)du$$

then we can directly integral it.

Theorme2.1.1:

If $\int f(x, I)dx = \varphi(x, I) \quad then,$

$$\int f\big((a+bI)x + c + dI\big)\, dx = \left(\frac{1}{a} - \frac{b}{a(a+b)} I\right) \varphi\big((a+bI)x + c + dI\big) + C$$

where C is an indeterminate real constant, $a \neq 0$, $a \neq -b$ and b, c, d are real numbers, while I = indeterminacy.

3. Methods of the neutrosophic numerical integration

3.1 The neutrosophic trapezoidal method

Let:

$$\int_{a+bI}^{c+dI} f(x, I)\, dx$$

Where $f(x, I)$ is neutrosophic function and a, b, c, d are real numbers, while $I =$ indeterminacy ($I \in [0,1]$).

we divide the interval $[a + bI, c + dI]$ into n equal parts:

$$a + bI = x_0 + I\acute{x}_0 \ , x_1 + I\acute{x}_1 \ , x_2 + I\acute{x}_2, \dots, x_{n-1} + I\acute{x}_{n-1}, x_n + I\acute{x}_n = c + dI$$

such that the length of each sub interval is:

$$h_I = \frac{c + dI - (a + bI)}{n}$$

$$= \frac{c - a + (d - b)I}{n} = \Delta x_I$$

Let $y_0 + I\acute{y}_0 \ , y_1 + I\acute{y}_1 \ , y_2 + I\acute{y}_2, \dots, y_{n-1} + I\acute{y}_{n-1}, y_n + I\acute{y}_n$ be y –coordinate, whereas:

$$y_0 + I\acute{y}_0 = f(x_0 + I\acute{x}_0), y_1 + I\acute{y}_1 = f(\ x_1 + I\acute{x}_1), y_2 + I\acute{y}_2 = f(x_2 + I\acute{x}_2), \dots, y_{n-1} + I\acute{y}_{n-1}$$
$$= f(\ x_{n-1} + I\acute{x}_{n-1}), y_n + I\acute{y}_n = f(x_n + I\acute{x}_n)$$

Then the area between the two lines $a + bI, c + dI$, the curve of $f(x, I)$ and the x-axis equal the sum of upright trapezoidal which are bounded from up by arc of the neutrosophic function, Where the area of the first neutrosophic trapezoidal is:

$$\left(\frac{y_0 + I\acute{y}_0 + y_1 + I\acute{y}_1}{2}\right) h_I$$

and the area of the second neutrosophic trapezoidal is:

$$\left(\frac{y_1 + I\acute{y}_1 + y_2 + I\acute{y}_2}{2}\right) h_I$$

And the area of the last neutrosophic trapezoidal is:

$$\left(\frac{y_{n-1} + I\acute{y}_{n-1} + y_n + I\acute{y}_n}{2}\right) h_I$$

Hence:

$$\int_{a+bI}^{c+dI} f(x,I)\,dx = \left(\frac{y_0 + I\acute{y}_0 + y_1 + I\acute{y}_1}{2}\right) h_I + \left(\frac{y_1 + I\acute{y}_1 + y_2 + I\acute{y}_2}{2}\right) h_I + .. + \left(\frac{y_{n-1} + I\acute{y}_{n-1} + y_n + I\acute{y}_n}{2}\right) h_I$$

$$= \frac{h_I}{2}[y_0 + I\acute{y}_0 + 2y_1 + 2I\acute{y}_1 + \cdots + 2y_{n-1} + 2I\acute{y}_{n-1} + y_n + I\acute{y}_n]$$

$$= \frac{h_I}{2}[y_0 + I\acute{y}_0 + 2(y_1 + I\acute{y}_1 + \cdots + y_{n-1} + I\acute{y}_{n-1}) + y_n + I\acute{y}_n]$$

3.1.1 Estimation of error

$$E_I \le \frac{1}{12} {h_I}^3 Max\left|\acute{\acute{f}}(x,I)\right| \quad ; \quad a + bI \le x \le c + dI$$

Then the error of the step (subinterval) is:

$$E_I \le \frac{n}{12} {h_I}^3 Max\left|\acute{\acute{f}}(x,I)\right| \quad ; \quad a + bI \le x \le c + dI$$

We have:

$$h_I = \frac{c + dI - (a + bI)}{n} \qquad \Rightarrow \quad n = \frac{c + dI - (a + bI)}{h_I}$$

Then the estimation of error of the neutrosophic trapezoidal method is:

$$E \le \frac{c + dI - (a + bI)}{12} {h_I}^2 Max\left|\acute{\acute{f}}(x,I)\right| \quad ; \quad a + bI \le x \le c + dI$$

Example 3.1

Evaluate

$$\int_{0+0I}^{1+I} \frac{dx}{1 + x^2}$$

by trapezoidal where $n = 4$, then calculate the occurred error.

Solution:

$$h_I = \frac{c - a + (d - b)I}{n} = \frac{1 + I}{4} = 0.25 + 0.25I$$

Then we will divide the interval $[0, 1+I]$ into four subintervals of length: $h_I = 0.25 + 0.25I$

(x, I)	0	$0.25(1+I)$	$0.5(1+I)$	$0.75(1+I)$	$1+I$
$f(x, I)$	1	$0.9412 - 0.1412I$	$0.8 - 0.3I$	$0.64 - 0.3323I$	$0.5 - 0.3I$

where:

- $f(0) = 1$
- $f\big(0.25(1+I)\big) = \frac{1}{1+\big(0.25(1+I)\big)^2} = \frac{1}{1.0625+0.1875I} = 0.9412 - 0.1412I$
- $f\big(0.5(1+I)\big) = \frac{1}{1+\big(0.5(1+I)\big)^2} = \frac{1}{1.25+0.75I} = 0.8 - 0.3I$
- $f\big(0.75(1+I)\big) = \frac{1}{1+\big(0.75(1+I)\big)^2} = \frac{1}{1.5625+1.6875I} = 0.64 - 0.3323I$
- $f(1+I) = \frac{1}{1+(1+I)^2} = \frac{1}{2+3I} = 0.5 - 0.3I$

then:

$$\int_{0+0I}^{1+I} \frac{dx}{1+x^2} = \frac{h_I}{2}[y_0 + I\acute{y}_0 + 2(y_1 + I\acute{y}_1 + y_2 + I\acute{y}_2 + y_3 + I\acute{y}_3) + y_4 + I\acute{y}_4]$$

$$= \frac{0.25 + 0.25I}{2}[1 + 2(0.9412 - 0.1412I + 0.8 - 0.3I + 0.64 - 0.3323I) + 0.5 - 0.3I]$$

$$= \frac{0.25 + 0.25I}{2}[6.2624 - 1.847I]$$

$$= (0.25 + 0.25I)[3.1312 - 0.9235I] = 0.7828 + 0.32105I$$

To finding the occurred error:

$$f(x, I) = \frac{1}{1+x^2} \quad \Rightarrow \quad \acute{f}(x, I) = \frac{-2x}{(1+x^2)^2}$$

$$\Rightarrow \acute{\acute{f}}(x, I) = \frac{-2(1+x^2)^2 + 6x^2(1+x^2)}{(1+x^2)^4} = \frac{-2 - 2x^2 + 8x^2}{(1+x^2)^3} = \frac{6x^2 - 2}{(1+x^2)^3}$$

$$\acute{\acute{f}}(0) = -2$$

$$\acute{\acute{f}}(1+I) = \frac{22I}{8 + 117I} = \frac{22}{125}I$$

$$\left|\acute{\acute{f}}(0)\right| > \left|\acute{\acute{f}}(1+I)\right| \quad on \quad I = [0,1]$$

$$\Rightarrow \qquad Max\left|\acute{\acute{f}}(x, I)\right| = \left|\acute{\acute{f}}(0)\right| = |-2| = 2$$

then:

$$E_I \leq \frac{c+dI-(a+bI)}{12} {h_I}^2 \quad Max\left|\acute{\acute{f}}(x,I)\right| \quad ; \quad a+bI \leq x \leq c+dI$$

$$E_I \leq \frac{1+I}{12}(0.25+0.25I)^2 \quad |-2| \quad ; \quad 0 \leq x \leq 1+I$$

$$E_I \leq \frac{0.0625+0.4375I}{12}(2) \approx 0.01042+0.07292I \qquad (1)$$

Note:
If we put $I=0$ in (1), we get: $E_I \leq 0.01042$ (the same result without indeterminacy).

Example 3.2
Evaluate

$$\int_{0.5+0.5I}^{1+I} xe^x\, dx$$

by trapezoidal where $h_I = 0.1+0.1I$, then calculate the occurred error.

Solution:

$$h_I = 0.1+0.1I$$

(x,I)	$0.5+0.5I$	$0.6+0.6I$	$0.7+0.7I$	$0.8+0.8I$	$0.9+0.9I$	$1+I$
$f(x,I)$	$0.8244 + 1.894I$	$1.09326 + 2.89086I$	$1.40966 + 2.89086I$	$1.7804 + 6.1444I$	$2.21364 + 8.67564I$	$2.7183 + 12.0599I$

where:

- $f(0.5+0.5I) = (0.5+0.5I)e^{0.5+0.5I}$

$$= (0.5+0.5I)\left(\sqrt{e}+I\left[e-\sqrt{e}\right]\right)$$

$$= (0.5+0.5I)(1.6487+1.0696I)$$

$$\approx 0.8244+1.894I$$

- $f(0.6+0.6I) = (0.6+0.6I)e^{(0.6+0.6I)}$

$$= (0.6+0.6I)(e^{0.6}+I[e^{1.2}-e^{0.6}])$$

$$= (0.6+0.6I)(1.8221+1.498I)$$

$$\approx 1.09326+2.89086I$$

- $f(0.7+0.7I) = (0.7+0.7I)e^{(0.7+0.7I)}$

$$= (0.7+0.7I)(e^{0.7}+I[e^{1.4}-e^{0.7}])$$

$$= (0.7+0.7I)(2.0138+2.0414I)$$

$$\approx 1.40966 + 2.89086I$$

- $f(0.8 + 0.8I) = (0.8 + 0.8I)e^{(0.8+0.8I)}$

$$= (0.8 + 0.8I)(e^{0.8} + I[e^{1.6} - e^{0.8}])$$

$$= (0.8 + 0.8I)(2.2255 + 2.7275I)$$

$$\approx 1.7804 + 6.1444I$$

- $f(0.9 + 0.9I) = (0.9 + 0.9I)e^{(0.9+0.9I)}$

$$= (0.9 + 0.9I)(e^{0.9} + I[e^{1.8} - e^{0.9}])$$

$$= (0.9 + 0.9I)(2.4596 + 3.5900I)$$

$$\approx 2.21364 + 8.67564I$$

- $f(1 + I) = (1 + I)e^{(1+I)}$

$$= (1 + I)(e + I[e^2 - e])$$

$$= (1 + I)(2.7183 + 4.6708I)$$

$$\approx 2.7183 + 12.0599I$$

then:

$$\int_{0.5+0.5I}^{1+I} xe^x\,dx = \frac{h_I}{2}[y_0 + I\acute{y}_0 + 2(y_1 + I\acute{y}_1 + y_2 + I\acute{y}_2 + y_3 + I\acute{y}_3 + y_4 + I\acute{y}_4) + y_5 + I\acute{y}_5]$$

$$= \frac{0.1 + 0.1I}{2}[0.8244 + 1.894I + 2(1.09326 + 2.89086I + 1.40966 + 4.26762I + 1.7804 + 6.1444I + 2.21364 + 8.67564I) + 2.7183 + 12.0599I]$$

$$= \frac{0.1 + 0.1I}{2}[16.53662 + 57.91094I]$$

$$= 0.826831 + 0.826831I + 2.895547I + 2.895547I$$

$$= 0.826831 + 6.6179251I \qquad (1)$$

to finding the occurred error:

$$f(x, I) = xe^x \quad \Longrightarrow \quad \acute{f}(x, I) = (1 + x)e^x$$

$$\Longrightarrow \quad \acute{\acute{f}}(x, I) = (2 + x)e^x$$

$$\acute{\acute{f}}(0.5 + 0.5I) = (2.5 + 0.5I)e^{0.5+0.5I} = (2.5 + 0.5I)(e^{0.5} + I[e - e^{0.5}])$$

$$= (2.5 + 0.5I)(1.64872 + 1.06956I)$$

$$= 4.1218 + 4.1218I + 0.53478I + 0.53478I$$

$$= 4.1218 + 5.19136I$$

$$\acute{\acute{f}}(1 + I) = (3 + I)e^{1+I} = (3 + I)(e + I[e^2 - e])$$

$$= (3 + I)(2.7183 + 4.6708I)$$

$$= 8.1549 + 8.1549I + 4.6708I + 4.6708I$$

$$= 8.1549 + 17.4965I$$

$$\left|\acute{\acute{f}}(1 + I)\right| > \left|\acute{\acute{f}}(0.5 + 0.5I)\right| \quad on \quad I = [0,1]$$

$$\Longrightarrow \qquad Max\left|\acute{\acute{f}}(x, I)\right| = \left|\acute{\acute{f}}(1 + I)\right| = |8.1549 + 17.4965I|$$

$$= |8.1549| + I[|25.6514| - |8.1549|]$$

$$= 8.1549 + 17.4965I$$

then:

$$E_I \leq \frac{c + dI - (a + bI)}{12} {h_I}^2 \quad Max\left|\acute{\acute{f}}(x, I)\right| \quad ; \quad a + bI \leq x \leq c + dI$$

$$E_I \leq \frac{1 + I - (0.5 + 0.5I)}{12}(0.1 + 0I)^2 \quad (8.1549 + 17.4965I) \quad ; \quad 0.5 + 0.5I \leq x \leq 1 + I$$

$$E_I \leq \frac{0.5 + 0.5I}{12}(0.081549 + 0.174965I) \approx 0.00398 + 0.01856I \qquad (*)$$

3.1.2 The neutrosophic numerical integration by using MATLAB

If we go back to the previous example 3.1 and evaluate

$$\int_{0+0I}^{1+I} \frac{dx}{1 + x^2} = 0.7828 + 0.32105I$$

by trapezoidal, where $h_I = 0.1 + 0.1I$, by using MATLAB

solution:

- for $I = 1$ by substitution in (2), we find:

$$\int_{0+0I}^{1+I} \frac{dx}{1 + x^2} = 0.7828 + 0.32105(1) = \boxed{1.10385}$$

Let's now use MATLAB (for $I = 1$):

```
a=0;
>> b=2;
>> h=0.5;
```

```
>> f=0;
>> for x=0.5:(h):1.5;
f=(f+(1/(1+x^2)));
end
>> Am=(h/2)*((1/(1+a^2))+(2*f)+(1/(1+b^2)))

Am =

1.1038
```

- for $I = 0$ by substitution in (2), we find:

$$\int_{0+0I}^{1+I} \frac{dx}{1+x^2} = 0.7828 + 0.32105(0) = \boxed{0.7828}$$

Let's now use MATLAB (for $I = 0$):

```
a=0;
b=1;
h=0.25;
f=0;
for x=0.25:(h):0.75;
f=(f+(1/(1+x^2)));
end
Am=(h/2)*((1/(1+a^2))+(2*f)+(1/(1+b^2)))

Am =

0.7828
```

If we go back to the previous example 3.2 and evaluate

$$\int_{0.5+0.5I}^{1+I} xe^x \, dx$$

by trapezoidal, where $h_I = 0.1 + 0.1I$, by using MATLAB

solution:

- for $I = 1$ by substitution in (1), we find:

$$\int_{1}^{2} xe^x \, dx = 0.826831 + 6.6179251(1) = \boxed{7.4447561}$$

Let's now use MATLAB (for $I = 1$):

```
>> a=1;
b=2;
h=0.2;
f=0;
for x=1.2:(h):1.8;
f=(f+(x*(exp(x))));
```

```
end
>> Am=(h/2)*((a*(exp(a))+2*f+(b*(exp(b)))))

Am =

7.4448
```

➢ for $I = 0$ by substitution in (1), we find:

$$\int_{0.5}^{1} xe^{x}\,dx = 0.826831 + 6.6179251(0) = \boxed{0.826831}$$

Let's now use MATLAB (for $I = 0$):

```
a=0.5;
b=1;
h=0.1;
f=0;
for x=0.6:(h):0.9;
f=(f+(x*(exp(x))));
end
>> Am=(h/2)*((a*(exp(a))+2*f+(b*(exp(b)))))

Am =

0.8268
```

We note that we got the same results, noting that the numerical integration is approximate and the error estimate is studied.

4. Conclusions

This essay expands on the writings we previously wrote on neutrosophic integrals. Integrals play a significant role in daily life since they make several mathematical operations possible in the actual world, this is why we decided to explore the neutrosophic numerical integration, where we introduced the concept of neutrosophic numerical integration through the trapezoidal method, and the estimation of error study. Some examples of this were also solved and the correctness of the results was confirmed using MATLAB. In addition, this study is also regarded as being significant for neutrosophic integral applications.

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