NSS

Neutrosophic Sets and Systems, Vol. 60, 2023

University of New Mexico

Single valued neutrosophic ordered subalgebras of ordered BCI-algebras

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Abstract: In order to apply neutrosophic set theory to ordered BCI-algebra, the notion of single valued neutrosophic ordered subalgebra is introduced and several properties are investigated. The conditions under which single valued neutrosophic level subsets become ordered subalgebras are explored. It is investigated when the T-neutrosophic q-set, I-neutrosophic q-set and F-neutrosophic q-set can be ordered subalgebras. A special set $\mathcal{X}_0^1$ is created and conditions are established in which it becomes an ordered subalgebra.

Keywords: Ordered BCI-algebra, single valued neutrosophic ordered subalgebra, ordered subalgebra, single valued neutrosophic level subset, T- (resp., F- and I-) neutrosophic q-set, T- (resp., F- and I-) neutrosophic $\in\vee q$-set.

2020 Mathematics Subject Classification. 06F35, 03G25, 08A72.

1 Introduction

The neutrosophic set was introduced in the 1990s by Florentin Smarandache as an extension of fuzzy sets and intuitionistic fuzzy sets. This set is a useful device for handling uncertainty, ambiguity and incomplete information as a mathematical framework that extends the concept of classical sets. There is no room for uncertainty in classical set theory. This is because elements only belong to or do not belong to a set in classical set theory. However, in many real-world situations, we encounter imprecise or uncertain information. Neutrosophic sets provide a way to represent and reason with such information. One can use neutrosophic sets in various fields and applications, for example, decision-making, image processing, pattern recognition, expert systems, artificial intelligence, etc. Neutrosophic sets can be applied to algebraic structures to handle uncertainty and indeterminacy in mathematical operations, for example, neutrosophic field, neutrosophic ring, neutrosophic linear algebra, neutrosophic group, etc. The neutrosophic set is also applied to logical algebras (see [1], [2], [5], [6], [7], [8],[9], [10],[11], [12]). These applications show how neutrosophic sets can be incorporated into algebraic structures to accommodate uncertainty and indeterminacy in mathematical operations. By extending classical algebraic structures to neutrosophic sets, it becomes possible to perform algebraic calculations and analyses in scenarios lacking accurate or complete information. K. Iséki [3] first introduced the BCI-algebra. This algebra is a generalized version of the BCK-algebra introduced by K. Iséki and S. Tanaka [4] so as to generalize the set difference in set theory. Recently, Yang et al. [13] attempted to generalize BCI-algebra, and introduced the ordered BCI-algebra.

To apply neutrosophic set theory to the ordered BCI-algebra is the aim of this paper. The notion of single valued neutrosophic ordered subalgebras in ordered BCI-algebras is introduced, and several properties are investigated. We explore The conditions under which single valued neutrosophic level subsets become ordered subalgebras are explored. When the T-neutrosophic q-set, I-neutrosophic q-set and F-neutrosophic q-set can be ordered subalgebras is looked at. A special set $\mathcal{Q}_0^1$ is made and the conditions that it becomes an ordered subalgebra are found.

2 Preliminaries

Definition 2.1 ([13])**.** Suppose that Q is a set with a constant "ϵ", a binary operation " $\rightarrow$ " and a binary relation " $\leq_Q$ ". $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$ is said to be an *ordered BCI-algebra* (for simplicity, OBCI-algebra) if $\mathbf{Q}$ satisfies:

$$(\forall \mathfrak{w}, \mathfrak{z}, \mathfrak{u} \in Q)(\epsilon \leq_Q (\mathfrak{w} \rightarrow \mathfrak{z}) \rightarrow ((\mathfrak{z} \rightarrow \mathfrak{u}) \rightarrow (\mathfrak{w} \rightarrow \mathfrak{u}))), \tag{2.1}$$
$$(\forall \mathfrak{w}, \mathfrak{z} \in Q)(\epsilon \leq_Q \mathfrak{w} \rightarrow ((\mathfrak{w} \rightarrow \mathfrak{z}) \rightarrow \mathfrak{z})), \tag{2.2}$$
$$(\forall \mathfrak{w} \in Q)(\epsilon \leq_Q \mathfrak{w} \rightarrow \mathfrak{w}), \tag{2.3}$$
$$(\forall \mathfrak{w}, \mathfrak{z} \in Q)(\epsilon \leq_Q \mathfrak{w} \rightarrow \mathfrak{z},\ \epsilon \leq_Q \mathfrak{z} \rightarrow \mathfrak{w}\ \Rightarrow\ \mathfrak{w} = \mathfrak{z}), \tag{2.4}$$
$$(\forall \mathfrak{w}, \mathfrak{z} \in Q)(\mathfrak{w} \leq_Q \mathfrak{z}\ \Leftrightarrow\ \epsilon \leq_Q \mathfrak{w} \rightarrow \mathfrak{z}), \tag{2.5}$$
$$(\forall \mathfrak{w}, \mathfrak{z} \in Q)(\epsilon \leq_Q \mathfrak{w},\ \mathfrak{w} \leq_Q \mathfrak{z}\ \Rightarrow\ \epsilon \leq_Q \mathfrak{z}). \tag{2.6}$$

Obviously $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$ with $Q = \{\epsilon\}$ is an OBCI-algebra, which is said to be the *trivial OBCI-algebra*.

Proposition 2.2 ([13])**.** *Let $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$ be an OBCI-algebra. The following hold in $\mathbf{Q}$:*

$$(\forall \mathfrak{w} \in Q)(\epsilon \rightarrow \mathfrak{w} = \mathfrak{w}). \tag{2.7}$$
$$(\forall \mathfrak{w}, \mathfrak{z}, \mathfrak{u} \in Q)(\mathfrak{u} \rightarrow (\mathfrak{z} \rightarrow \mathfrak{w}) = \mathfrak{z} \rightarrow (\mathfrak{u} \rightarrow \mathfrak{w})). \tag{2.8}$$
$$(\forall \mathfrak{w}, \mathfrak{z}, \mathfrak{u} \in Q)(\epsilon \leq_Q \mathfrak{w} \rightarrow \mathfrak{z}\ \Rightarrow\ \epsilon \leq_Q (\mathfrak{z} \rightarrow \mathfrak{u}) \rightarrow (\mathfrak{w} \rightarrow \mathfrak{u})). \tag{2.9}$$
$$(\forall \mathfrak{w}, \mathfrak{z}, \mathfrak{u} \in Q)(\epsilon \leq_Q \mathfrak{w} \rightarrow \mathfrak{z},\ \epsilon \leq_Q \mathfrak{z} \rightarrow \mathfrak{u}\ \Rightarrow\ \epsilon \leq_Q \mathfrak{w} \rightarrow \mathfrak{u}). \tag{2.10}$$
$$(\forall \mathfrak{w}, \mathfrak{z}, \mathfrak{u} \in Q)(\epsilon \leq_Q (\mathfrak{u} \rightarrow (\mathfrak{z} \rightarrow \mathfrak{w})) \rightarrow (\mathfrak{z} \rightarrow (\mathfrak{u} \rightarrow \mathfrak{w}))). \tag{2.11}$$
$$(\forall \mathfrak{w}, \mathfrak{z}, \mathfrak{u} \in Q)(\epsilon \leq_Q \mathfrak{u} \rightarrow (\mathfrak{z} \rightarrow \mathfrak{w})\ \Rightarrow\ \epsilon \leq_Q \mathfrak{z} \rightarrow (\mathfrak{u} \rightarrow \mathfrak{w})). \tag{2.12}$$
$$(\forall \mathfrak{w}, \mathfrak{z} \in Q)(((\mathfrak{w} \rightarrow \mathfrak{z}) \rightarrow \mathfrak{z}) \rightarrow \mathfrak{z} = \mathfrak{w} \rightarrow \mathfrak{z}). \tag{2.13}$$
$$(\forall \mathfrak{w} \in Q)((\mathfrak{w} \rightarrow \mathfrak{w}) \rightarrow \mathfrak{w} = \mathfrak{w}). \tag{2.14}$$
$$(\forall \mathfrak{w}, \mathfrak{z}, \mathfrak{u} \in Q)(\epsilon \leq_Q (\mathfrak{z} \rightarrow \mathfrak{u}) \rightarrow ((\mathfrak{w} \rightarrow \mathfrak{z}) \rightarrow (\mathfrak{w} \rightarrow \mathfrak{u}))). \tag{2.15}$$
$$(\forall \mathfrak{w}, \mathfrak{z}, \mathfrak{u} \in Q)(\epsilon \leq_Q \mathfrak{w} \rightarrow \mathfrak{z}\ \Rightarrow\ \epsilon \leq_Q (\mathfrak{u} \rightarrow \mathfrak{w}) \rightarrow (\mathfrak{u} \rightarrow \mathfrak{z})). \tag{2.16}$$

Definition 2.3 ([13])**.** Let A be a subset of Q. A is said to be

- a *subalgebra* of $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$ if it satisfies:

$$(\forall \mathfrak{w}, \mathfrak{z} \in Q)(\mathfrak{w}, \mathfrak{z} \in A \Rightarrow \mathfrak{w} \rightarrow \mathfrak{z} \in A). \tag{2.17}$$

- an *ordered subalgebra* of $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$ if it satisfies:

$$(\forall \mathfrak{w}, \mathfrak{z} \in Q)(\mathfrak{w}, \mathfrak{z} \in A, \epsilon \leq_Q \mathfrak{w}, \epsilon \leq_Q \mathfrak{z} \Rightarrow \mathfrak{w} \rightarrow \mathfrak{z} \in A). \tag{2.18}$$

We recall that all subalgebras are ordered subalgebras, whereas the converse is not necessarily true (see [13]).

Let Q be a non-empty set. A *single valued neutrosophic set* in Q is a structure of the form:

$$\mathcal{C}_\sim := \{\langle \mathfrak{y}; \widetilde{\mathcal{C}}_T(\mathfrak{y}), \widetilde{\mathcal{C}}_I(\mathfrak{y}), \widetilde{\mathcal{C}}_F(\mathfrak{y})\rangle \mid \mathfrak{y} \in Q\}$$

where $\widetilde{\mathcal{C}}_F : Q \rightarrow [0,1]$ is a false membership function, $\widetilde{\mathcal{C}}_I : Q \rightarrow [0,1]$ is an indeterminate membership function, and $\widetilde{\mathcal{C}}_T : Q \rightarrow [0,1]$ is a truth membership function. For brevity, the symbol $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is used for the single valued neutrosophic set

$$\mathcal{C}_\sim := \{\langle \mathfrak{y}; \widetilde{\mathcal{C}}_T(\mathfrak{y}), \widetilde{\mathcal{C}}_I(\mathfrak{y}), \widetilde{\mathcal{C}}_F(\mathfrak{y})\rangle \mid \mathfrak{y} \in Q\}.$$

Given a single valued neutrosophic set $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ in Q, we consider the following sets.

$$\begin{aligned}
\mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho) &:= \{\mathfrak{y} \in Q \mid \widetilde{\mathcal{C}}_T(\mathfrak{y}) \geq \varrho\}, \\
\mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma) &:= \{\mathfrak{y} \in Q \mid \widetilde{\mathcal{C}}_I(\mathfrak{y}) \geq \sigma\}, \\
\mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta) &:= \{\mathfrak{y} \in Q \mid \widetilde{\mathcal{C}}_F(\mathfrak{y}) \leq \delta\},
\end{aligned}$$

which are called *single valued neutrosophic level subsets* of Q where $\varrho, \sigma, \delta \in [0,1]$.

3 Single valued neutrosophic ordered subalgebras

Here the notion of single valued neutrosophic (ordered) subalgebras is introduced and several properties are investigated. Unless otherwise specified, we henceforth denote an OBCI-algebra by $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$.

Definition 3.1. Let $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ be a single valued neutrosophic set in Q. $\mathcal{C}_\sim$ is said to be a *single valued neutrosophic subalgebra* of $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$ if $\mathcal{C}_\sim$ satisfies:

$$(\forall \mathfrak{y}, \mathfrak{k} \in Q) \begin{pmatrix} \widetilde{\mathcal{C}}_T(\mathfrak{y} \rightarrow \mathfrak{k}) \geq \min\{\widetilde{\mathcal{C}}_T(\mathfrak{y}), \widetilde{\mathcal{C}}_T(\mathfrak{k})\} \\ \widetilde{\mathcal{C}}_I(\mathfrak{y} \rightarrow \mathfrak{k}) \geq \min\{\widetilde{\mathcal{C}}_I(\mathfrak{y}), \widetilde{\mathcal{C}}_I(\mathfrak{k})\} \\ \widetilde{\mathcal{C}}_F(\mathfrak{y} \rightarrow \mathfrak{k}) \leq \max\{\widetilde{\mathcal{C}}_F(\mathfrak{y}), \widetilde{\mathcal{C}}_F(\mathfrak{k})\} \end{pmatrix}. \tag{3.1}$$

Definition 3.2. Let $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ be a single valued neutrosophic set in Q. $\mathcal{C}_\sim$ is said to be a *single* *valued neutrosophic ordered subalgebra* of $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$ if $\mathcal{C}_\sim$ satisfies:

$$(\forall \mathfrak{y}, \mathfrak{k} \in Q) \left(\epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{k} \ \Rightarrow \begin{cases} \widetilde{\mathcal{C}}_T(\mathfrak{y} \rightarrow \mathfrak{k}) \geq \min\{\widetilde{\mathcal{C}}_T(\mathfrak{y}), \widetilde{\mathcal{C}}_T(\mathfrak{k})\} \\ \widetilde{\mathcal{C}}_I(\mathfrak{y} \rightarrow \mathfrak{k}) \geq \min\{\widetilde{\mathcal{C}}_I(\mathfrak{y}), \widetilde{\mathcal{C}}_I(\mathfrak{k})\} \\ \widetilde{\mathcal{C}}_F(\mathfrak{y} \rightarrow \mathfrak{k}) \leq \max\{\widetilde{\mathcal{C}}_F(\mathfrak{y}), \widetilde{\mathcal{C}}_F(\mathfrak{k})\} \end{cases} \right). \tag{3.2}$$

Example 3.3. Let $Q = \{0, \epsilon, j, 1\}$ be a set, where 0 and 1 are the least element and the greatest element of Q, respectively. A binary operation " $\rightarrow$ " on Q is provided by the table below:

$\rightarrow$	1	ϵ	j	0
1	1	0	0	0
ϵ	1	ϵ	j	0
j	1	j	ϵ	0
0	1	1	1	1

Let $\leq_Q := \{(1,1), (j,1)\}, (\epsilon, 1), (j, j), (0, j), (\epsilon, \epsilon), (0, \epsilon), (0, 0)\}$. Then $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$ is an OBCI-algebra (see [13]).

(i) Let $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ be a single valued neutrosophic set in Q provided by the table below:

Q	$\widetilde{\mathcal{C}}_T(\mathfrak{y})$	$\widetilde{\mathcal{C}}_I(\mathfrak{y})$	$\widetilde{\mathcal{C}}_F(\mathfrak{y})$
1	0.68	0.73	0.31
ϵ	0.68	0.73	0.31
j	0.24	0.49	0.59
0	0.68	0.73	0.31

Clearly $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic subalgebra of $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$.

(ii) Let $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ be a single valued neutrosophic set in Q provided by the table below:

Q	$\widetilde{\mathcal{C}}_T(\mathfrak{y})$	$\widetilde{\mathcal{C}}_I(\mathfrak{y})$	$\widetilde{\mathcal{C}}_F(\mathfrak{y})$
1	0.38	0.25	0.63
ϵ	0.64	0.76	0.29
j	0.38	0.25	0.63
0	0.64	0.76	0.29

Clearly $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic ordered subalgebra of $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$.

It is certain that every single valued neutrosophic subalgebra is a single valued neutrosophic ordered subalgebra. However, as the following example shows, the converse is not necessarily true. From this point of view, we can say that the single valued neutrosophic ordered subalgebra is a generalization of the single valued neutrosophic subalgebra.

$\rightarrow$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0
1	1	0	0	0	0
$\frac{3}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0
$\frac{1}{2}$	1	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	0
$\frac{1}{4}$	1	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	0
0	1	1	1	1	1

Example 3.4. Suppose that $Q = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$ is a set with a binary operation " $\rightarrow$ " provided by the table: and that $\leq_Q$ is the natural order in Q. Certainly $\mathbf{Q} := (Q, \rightarrow, \frac{3}{4}, \leq_Q)$ is an OBCI-algebra (see [13]). Suppose that $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic set in Q provided by the table:

Q	$\widetilde{\mathcal{C}}_T(\mathfrak{y})$	$\widetilde{\mathcal{C}}_I(\mathfrak{y})$	$\widetilde{\mathcal{C}}_F(\mathfrak{y})$
1	0.37	0.25	0.63
$\frac{3}{4}$	0.68	0.76	0.29
$\frac{1}{2}$	0.37	0.25	0.63
$\frac{1}{4}$	0.37	0.25	0.63
0	0.68	0.76	0.29

It is routine to check that $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic ordered subalgebra of $\mathbf{Q} := (Q, \rightarrow, \frac{3}{4}, \leq_Q)$. However, $\mathcal{C}_\sim$ is not a single valued neutrosophic subalgebra of $\mathbf{Q} := (Q, \rightarrow, \frac{3}{4}, \leq_Q)$ since

$$\widetilde{\mathcal{C}}_T(0 \rightarrow \tfrac{3}{4}) = \widetilde{\mathcal{C}}_T(1) = 0.37 \not\geq 0.68 = \min\{\widetilde{\mathcal{C}}_T(0), \widetilde{\mathcal{C}}_T(\tfrac{3}{4})\}$$

and/or

$$\widetilde{\mathcal{C}}_F(0 \rightarrow \tfrac{3}{4}) = \widetilde{\mathcal{C}}_F(1) = 0.63 \not\leq 0.29 = \max\{\widetilde{\mathcal{C}}_F(0), \widetilde{\mathcal{C}}_F(\tfrac{3}{4})\}.$$

Theorem 3.5. *A single valued neutrosophic set $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ in Q is a single valued neutrosophic ordered subalgebra of $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$ if and only if $\mathcal{C}_\sim$ satisfies:*

$$(\forall \mathfrak{y}, \mathfrak{k} \in Q) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\, \epsilon \leq_Q \mathfrak{k},\, \mathfrak{y} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho_{\mathfrak{y}}),\, \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho_{\mathfrak{k}}) \\ \Rightarrow \mathfrak{y} \rightarrow \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \min\{\varrho_{\mathfrak{y}}, \varrho_{\mathfrak{k}}\}) \end{array} \right), \tag{3.3}$$

$$(\forall \mathfrak{w}, \mathfrak{z} \in Q) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{w},\, \epsilon \leq_Q \mathfrak{z},\, \mathfrak{w} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma_{\mathfrak{w}}),\, \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma_{\mathfrak{z}}) \\ \Rightarrow \mathfrak{w} \rightarrow \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \min\{\sigma_{\mathfrak{w}}, \sigma_{\mathfrak{z}}\}) \end{array} \right), \tag{3.4}$$

$$(\forall \mathfrak{y}, \mathfrak{z} \in Q) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\, \epsilon \leq_Q \mathfrak{z},\, \mathfrak{y} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta_{\mathfrak{y}}),\, \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta_{\mathfrak{z}}) \\ \Rightarrow \mathfrak{y} \rightarrow \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \max\{\delta_{\mathfrak{y}}, \delta_{\mathfrak{z}}\}) \end{array} \right). \tag{3.5}$$

Proof. Assume that $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic ordered subalgebra of $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$. Let $\mathfrak{y}, \mathfrak{k} \in Q$ be such that $\epsilon \leq_Q \mathfrak{y}$, $\epsilon \leq_Q \mathfrak{k}$, $\mathfrak{y} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho_{\mathfrak{y}})$ and $\mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho_{\mathfrak{k}})$. Then $\widetilde{\mathcal{C}}_T(\mathfrak{y}) \geq \varrho_{\mathfrak{y}}$ and $\widetilde{\mathcal{C}}_T(\mathfrak{k}) \geq \varrho_{\mathfrak{k}}$, which imply that $\widetilde{\mathcal{C}}_T(\mathfrak{y} \rightarrow \mathfrak{k}) \geq \min\{\widetilde{\mathcal{C}}_T(\mathfrak{y}), \widetilde{\mathcal{C}}_T(\mathfrak{k})\} \geq \min\{\varrho_{\mathfrak{y}}, \varrho_{\mathfrak{k}}\}$. Hence $\mathfrak{y} \rightarrow \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \min\{\varrho_{\mathfrak{y}}, \varrho_{\mathfrak{k}}\})$. Similarly, one is capable of verifying that $\mathfrak{w} \rightarrow \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \min\{\sigma_{\mathfrak{w}}, \sigma_{\mathfrak{z}}\})$ for all $\mathfrak{w} \in$

$\mathcal{Q}(\widetilde{\mathcal{C}}_I;\sigma_{\mathfrak{w}})$ and $\mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I;\sigma_{\mathfrak{z}})$ with $\epsilon \leq_Q \mathfrak{w}$ and $\epsilon \leq_Q \mathfrak{z}$. Now, let $\mathfrak{y}, \mathfrak{z} \in Q$ be such that $\epsilon \leq_Q \mathfrak{y}$, $\epsilon \leq_Q \mathfrak{z}$, $\mathfrak{y} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F;\delta_{\mathfrak{y}})$ and $\mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F;\delta_{\mathfrak{z}})$. Then $\widetilde{\mathcal{C}}_F(\mathfrak{y}) \leq \delta_{\mathfrak{y}}$ and $\widetilde{\mathcal{C}}_F(\mathfrak{z}) \leq \delta_{\mathfrak{z}}$. Thus

$$\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{z}) \leq \max\{\widetilde{\mathcal{C}}_F(\mathfrak{y}), \widetilde{\mathcal{C}}_F(\mathfrak{z})\} \leq \max\{\delta_{\mathfrak{y}}, \delta_{\mathfrak{z}}\},$$

and so $\mathfrak{y} \to \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \max\{\delta_{\mathfrak{y}}, \delta_{\mathfrak{z}}\})$.

Conversely, suppose that $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic set in Q that satisfies the conditions (3.3), (3.4) and (3.5). If $\widetilde{\mathcal{C}}_T(\mathfrak{w} \to \mathfrak{z}) < \min\{\widetilde{\mathcal{C}}_T(\mathfrak{w}), \widetilde{\mathcal{C}}_T(\mathfrak{z})\}$ for some $\mathfrak{w}, \mathfrak{z} \in Q$ with $\epsilon \leq_Q \mathfrak{w}$ and $\epsilon \leq_Q \mathfrak{z}$, then

$$\widetilde{\mathcal{C}}_T(\mathfrak{w} \to \mathfrak{z}) < \varrho_{\mathfrak{o}} \leq \min\{\widetilde{\mathcal{C}}_T(\mathfrak{w}), \widetilde{\mathcal{C}}_T(\mathfrak{z})\}$$

for some $\varrho_{\mathfrak{o}} \in (0, 1]$. Hence $\mathfrak{w}, \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho_{\mathfrak{o}})$ and $\mathfrak{w} \to \mathfrak{z} \notin \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho_{\mathfrak{o}})$, a contradiction, and thus

$$\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) \geq \min\{\widetilde{\mathcal{C}}_T(\mathfrak{y}), \widetilde{\mathcal{C}}_T(\mathfrak{k})\}$$

for all $\mathfrak{y}, \mathfrak{k} \in Q$ with $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$. Similarly, we can obtain $\widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}) \geq \min\{\widetilde{\mathcal{C}}_I(\mathfrak{y}), \widetilde{\mathcal{C}}_I(\mathfrak{k})\}$ for all $\mathfrak{y}, \mathfrak{k} \in Q$ with $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$. Let there be $\mathfrak{z}, \mathfrak{u} \in Q$ so that $\epsilon \leq_Q \mathfrak{z}$, $\epsilon \leq_Q \mathfrak{u}$ and $\widetilde{\mathcal{C}}_F(\mathfrak{z} \to \mathfrak{u}) > \max\{\widetilde{\mathcal{C}}_F(\mathfrak{z}), \widetilde{\mathcal{C}}_F(\mathfrak{u})\}$. If we take $\delta := \max\{\widetilde{\mathcal{C}}_F(\mathfrak{z}), \widetilde{\mathcal{C}}_F(\mathfrak{u})\}$, then $\mathfrak{z}, \mathfrak{u} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta)$ and $\mathfrak{z} \to \mathfrak{u} \notin \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta)$, a contradiction. Hence $\widetilde{\mathcal{C}}_F(\mathfrak{k} \to \mathfrak{o}) \leq \max\{\widetilde{\mathcal{C}}_F(\mathfrak{k}), \widetilde{\mathcal{C}}_F(\mathfrak{o})\}$ for all $\mathfrak{k}, \mathfrak{o} \in Q$ with $\epsilon \leq_Q \mathfrak{k}$ and $\epsilon \leq_Q \mathfrak{o}$. Consequently, $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$. □

Theorem 3.6. *Given a single valued neutrosophic set $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ in Q, its nonempty single valued neutrosophic level subsets $\mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho)$, $\mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma)$ and $\mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta)$ are ordered subalgebras of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$ for all $\varrho, \sigma \in (0.5, 1]$ and $\delta \in [0, 0.5)$ if and only if the following fact is established.*

$$(\forall \mathfrak{y}, \mathfrak{k} \in Q) \left(\epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{k} \ \Rightarrow \begin{cases} \max\{\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}), 0.5\} \geq \min\{\widetilde{\mathcal{C}}_T(\mathfrak{y}), \widetilde{\mathcal{C}}_T(\mathfrak{k})\} \\ \max\{\widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}), 0.5\} \geq \min\{\widetilde{\mathcal{C}}_I(\mathfrak{y}), \widetilde{\mathcal{C}}_I(\mathfrak{k})\} \\ \min\{\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}), 0.5\} \leq \max\{\widetilde{\mathcal{C}}_F(\mathfrak{y}), \widetilde{\mathcal{C}}_F(\mathfrak{k})\} \end{cases} \right). \tag{3.6}$$

Proof. Assume that the nonempty single valued neutrosophic level subsets $\mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho)$, $\mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma)$ and $\mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta)$ are ordered subalgebras of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$ for all $\varrho, \sigma \in (0.5, 1]$ and $\delta \in [0, 0.5)$. Let there be $\mathfrak{w}, \mathfrak{z} \in Q$ that satisfies $\epsilon \leq_Q \mathfrak{w}$, $\epsilon \leq_Q \mathfrak{z}$ and $\max\{\widetilde{\mathcal{C}}_I(\mathfrak{w} \to \mathfrak{z}), 0.5\} < \min\{\widetilde{\mathcal{C}}_I(\mathfrak{w}), \widetilde{\mathcal{C}}_I(\mathfrak{z})\} := \sigma$. We get $\sigma \in (0.5, 1]$ and $\mathfrak{w}, \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma)$. Since $\mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma)$ is an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$, it follows that $\mathfrak{w} \to \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma)$. Hence $\widetilde{\mathcal{C}}_I(\mathfrak{w} \to \mathfrak{z}) \geq \sigma = \min\{\widetilde{\mathcal{C}}_I(\mathfrak{w}), \widetilde{\mathcal{C}}_I(\mathfrak{z})\}$, a contradiction. Thus

$$\max\{\widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}), 0.5\} \geq \min\{\widetilde{\mathcal{C}}_I(\mathfrak{y}), \widetilde{\mathcal{C}}_I(\mathfrak{k})\}$$

for all $\mathfrak{y}, \mathfrak{k} \in Q$ with $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$. In a similar way, we have

$$\max\{\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}), 0.5\} \geq \min\{\widetilde{\mathcal{C}}_T(\mathfrak{y}), \widetilde{\mathcal{C}}_T(\mathfrak{k})\}$$

for all $\mathfrak{y}, \mathfrak{k} \in Q$ with $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$. If we say

$$\min\{\widetilde{\mathcal{C}}_F(\mathfrak{w} \to \mathfrak{z}), 0.5\} > \max\{\widetilde{\mathcal{C}}_F(\mathfrak{w}), \widetilde{\mathcal{C}}_F(\mathfrak{z})\} := \delta$$

for some $\mathfrak{w}, \mathfrak{z} \in Q$ satisfying $\epsilon \leq_Q \mathfrak{w}$ and $\epsilon \leq_Q \mathfrak{z}$, then $\delta \in [0, 0.5)$ and $\mathfrak{w}, \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta)$. But $\mathfrak{w} \rightarrow \mathfrak{z} \notin \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta)$, a contradiction. Thus $\min\{\widetilde{\mathcal{C}}_F(\mathfrak{y} \rightarrow \mathfrak{k}), 0.5\} \leq \max\{\widetilde{\mathcal{C}}_F(\mathfrak{y}), \widetilde{\mathcal{C}}_F(\mathfrak{k})\}$ for all $\mathfrak{y}, \mathfrak{k} \in Q$ with $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$.

Conversely, a single valued neutrosophic set $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ in Q satisfies the condition (3.6). Assume that $\mathfrak{y}, \mathfrak{k} \in Q$ are such that $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$. If $\mathfrak{y}, \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho) \cap \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma)$ for all $\varrho, \sigma \in (0.5, 1]$, then $\max\{\widetilde{\mathcal{C}}_T(\mathfrak{y} \rightarrow \mathfrak{k}), 0.5\} \geq \min\{\widetilde{\mathcal{C}}_T(\mathfrak{y}), \widetilde{\mathcal{C}}_T(\mathfrak{k})\} \geq \varrho > 0.5$ and

$$\max\{\widetilde{\mathcal{C}}_I(\mathfrak{y} \rightarrow \mathfrak{k}), 0.5\} \geq \min\{\widetilde{\mathcal{C}}_I(\mathfrak{y}), \widetilde{\mathcal{C}}_I(\mathfrak{k})\} \geq \sigma > 0.5.$$

Hence $\widetilde{\mathcal{C}}_T(\mathfrak{y} \rightarrow \mathfrak{k}) \geq \varrho$ and $\widetilde{\mathcal{C}}_I(\mathfrak{y} \rightarrow \mathfrak{k}) \geq \sigma$, that is, $\mathfrak{y} \rightarrow \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho) \cap \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma)$. Now, if $\mathfrak{y}, \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta)$ for all $\delta \in [0, 0.5)$, then $\min\{\widetilde{\mathcal{C}}_F(\mathfrak{y} \rightarrow \mathfrak{k}), 0.5\} \leq \max\{\widetilde{\mathcal{C}}_F(\mathfrak{y}), \widetilde{\mathcal{C}}_F(\mathfrak{k})\} \leq \delta < 0.5$ and so $\widetilde{\mathcal{C}}_F(\mathfrak{y} \rightarrow \mathfrak{k}) \leq \delta$, i.e., $\mathfrak{y} \rightarrow \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta)$. Therefore $\mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho)$, $\mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma)$ and $\mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta)$ are ordered subalgebras of $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$ for all $\varrho, \sigma \in (0.5, 1]$ and $\delta \in [0, 0.5)$. □

Given a single valued neutrosophic set $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ in Q and $\varrho, \sigma, \delta \in [0, 1]$, we consider the sets:

$$\begin{aligned} T_q(\mathcal{C}_\sim, \varrho) &:= \{\mathfrak{y} \in Q \mid \widetilde{\mathcal{C}}_T(\mathfrak{y}) + \varrho > 1\}, \\ I_q(\mathcal{C}_\sim, \sigma) &:= \{\mathfrak{y} \in Q \mid \widetilde{\mathcal{C}}_I(\mathfrak{y}) + \sigma > 1\}, \\ F_q(\mathcal{C}_\sim, \delta) &:= \{\mathfrak{y} \in Q \mid \widetilde{\mathcal{C}}_F(\mathfrak{y}) + \delta < 1\}, \end{aligned}$$

which are called the *T-neutrosophic q-set*, *I-neutrosophic q-set* and *F-neutrosophic q-set*, respectively, of $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$. Also, we consider the sets:

$$\begin{aligned} T_{\in\vee q}(\mathcal{C}_\sim, \varrho) &= \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho) \cup T_q(\mathcal{C}_\sim, \varrho), \\ I_{\in\vee q}(\mathcal{C}_\sim, \sigma) &= \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma) \cup I_q(\mathcal{C}_\sim, \sigma), \\ F_{\in\vee q}(\mathcal{C}_\sim, \delta) &= \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta) \cup F_q(\mathcal{C}_\sim, \delta), \end{aligned}$$

which are called the *T-neutrosophic $\in\vee q$-set*, *I-neutrosophic $\in\vee q$-set* and *F-neutrosophic $\in\vee q$-set*, respectively, of $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$.

Theorem 3.7. *If $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic ordered subalgebra of $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$, then its nonempty T- (resp., I- and F-) neutrosophic q-set $T_q(\mathcal{C}_\sim, \varrho)$ (resp., $I_q(\mathcal{C}_\sim, \sigma)$ and $F_q(\mathcal{C}_\sim, \delta)$) is an ordered subalgebra of $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$ for all $\varrho \in (0, 1]$ (resp., $\sigma \in (0, 1]$ and $\delta \in [0, 1)$).*

Proof. Suppose that $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic ordered subalgebra of $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$, and that $\mathfrak{y}, \mathfrak{k} \in Q$ are so that $\epsilon \leq_Q \mathfrak{y}$, $\epsilon \leq_Q \mathfrak{k}$ and $\mathfrak{y}, \mathfrak{k} \in T_q(\mathcal{C}_\sim, \varrho)$ for $\varrho \in (0, 1]$. Then $\widetilde{\mathcal{C}}_T(\mathfrak{y}) + \varrho > 1$ and $\widetilde{\mathcal{C}}_T(\mathfrak{k}) + \varrho > 1$. It follows that

$$\widetilde{\mathcal{C}}_T(\mathfrak{y} \rightarrow \mathfrak{k}) + \varrho \geq \min\{\widetilde{\mathcal{C}}_T(\mathfrak{y}), \widetilde{\mathcal{C}}_T(\mathfrak{k})\} + \varrho = \min\{\widetilde{\mathcal{C}}_T(\mathfrak{y}) + \varrho, \widetilde{\mathcal{C}}_T(\mathfrak{k}) + \varrho\} > 1.$$

Hence $\mathfrak{y} \rightarrow \mathfrak{k} \in T_q(\mathcal{C}_\sim, \varrho)$, and therefore $T_q(\mathcal{C}_\sim, \varrho)$ is an ordered subalgebra of $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$ for all $\varrho \in (0, 1]$. Similarly, we can verify that $I_q(\mathcal{C}_\sim, \sigma)$ is an ordered subalgebra of $\mathbf{Q} := (Q, \rightarrow, \epsilon, \leq_Q)$ for all $\sigma \in (0, 1]$. Now, let $\mathfrak{y}, \mathfrak{k} \in F_q(\mathcal{C}_\sim, \delta)$ for all $\delta \in [0, 1)$ and $\mathfrak{y}, \mathfrak{k} \in Q$ with $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$. Then $\widetilde{\mathcal{C}}_F(\mathfrak{y}) + \delta < 1$

and $\widetilde{\mathcal{C}}_F(\mathfrak{k}) + \delta < 1$, which imply that

$$\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}) + \delta \leq \max\{\widetilde{\mathcal{C}}_F(\mathfrak{y}), \widetilde{\mathcal{C}}_F(\mathfrak{y})\} + \delta = \max\{\widetilde{\mathcal{C}}_F(\mathfrak{y}) + \delta, \widetilde{\mathcal{C}}_F(\mathfrak{y}) + \delta\} < 1.$$

Hence $\mathfrak{y} \to \mathfrak{k} \in F_q(\mathcal{C}_\sim, \delta)$ for all $\delta \in [0, 1)$ and $\mathfrak{y}, \mathfrak{k} \in Q$ with $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$, and therefore $F_q(\mathcal{C}_\sim, \delta)$ is an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$ for all $\delta \in [0, 1)$. □

Theorem 3.8. *Suppose that a single valued neutrosophic set $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ in Q satisfies:*

$$(\forall \mathfrak{y}, \mathfrak{k} \in Q) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{k},\ \mathfrak{y} \in T_q(\mathcal{C}_\sim, \varrho_\mathfrak{y}),\ \mathfrak{k} \in T_q(\mathcal{C}_\sim, \varrho_\mathfrak{k}) \\ \Rightarrow \mathfrak{y} \to \mathfrak{k} \in T_{\in\vee q}(\mathcal{C}_\sim, \min\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\}) \end{array} \right), \tag{3.7}$$

$$(\forall \mathfrak{w}, \mathfrak{z} \in Q) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{w},\ \epsilon \leq_Q \mathfrak{z},\ \mathfrak{w} \in I_q(\mathcal{C}_\sim, \sigma_\mathfrak{w}),\ \mathfrak{k} \in I_q(\mathcal{C}_\sim, \sigma_\mathfrak{z}) \\ \Rightarrow \mathfrak{w} \to \mathfrak{z} \in I_{\in\vee q}(\mathcal{C}_\sim, \min\{\sigma_\mathfrak{w}, \sigma_\mathfrak{z}\}) \end{array} \right), \tag{3.8}$$

$$(\forall \mathfrak{y}, \mathfrak{z} \in Q) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{z},\ \mathfrak{y} \in F_q(\mathcal{C}_\sim, \delta_\mathfrak{y}),\ \mathfrak{z} \in F_q(\mathcal{C}_\sim, \delta_\mathfrak{z}) \\ \Rightarrow \mathfrak{y} \to \mathfrak{z} \in F_{\in\vee q}(\mathcal{C}_\sim, \max\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\}) \end{array} \right), \tag{3.9}$$

The nonempty T-neutrosophic q-set $T_q(\mathcal{C}_\sim, \varrho)$, I-neutrosophic q-set $I_q(\mathcal{C}_\sim, \sigma)$ and F-neutrosophic q-set $F_q(\mathcal{C}_\sim, \delta)$ are ordered subalgebras of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$ for all $\varrho, \sigma \in (0.5, 1]$ and $\delta \in [0, 0.5)$).

Proof. Suppose that a single valued neutrosophic set $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ satisfies (3.7), (3.8) and (3.9). Let $\mathfrak{y}, \mathfrak{k} \in Q$ be such that $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$. If $\mathfrak{y}, \mathfrak{k} \in T_q(\mathcal{C}_\sim, \varrho) \cap I_q(\mathcal{C}_\sim, \sigma)$ for all $\varrho, \sigma \in (0.5, 1]$, then $\mathfrak{y} \to \mathfrak{k} \in T_{\in\vee q}(\mathcal{C}_\sim, \varrho) \cap I_{\in\vee q}(\mathcal{C}_\sim, \sigma)$ by (3.7) and (3.8). Hence $\mathfrak{y} \to \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho)$ or $\mathfrak{y} \to \mathfrak{k} \in T_q(\mathcal{C}_\sim, \varrho)$; and $\mathfrak{y} \to \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma)$ or $\mathfrak{y} \to \mathfrak{k} \in I_q(\mathcal{C}_\sim, \sigma)$. If $\mathfrak{y} \to \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho) \cap \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma)$, then $\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) \geq \varrho > 1 - \varrho$ and $\widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}) \geq \sigma > 1 - \sigma$ since $\varrho, \sigma \in (0.5, 1]$. Hence $\mathfrak{y} \to \mathfrak{k} \in T_q(\mathcal{C}_\sim, \varrho) \cap I_q(\mathcal{C}_\sim, \sigma)$, and so $T_q(\mathcal{C}_\sim, \varrho)$ and $I_q(\mathcal{C}_\sim, \sigma)$ are ordered subalgebras of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$. If $\mathfrak{y}, \mathfrak{k} \in F_q(\mathcal{C}_\sim, \delta)$, then $\mathfrak{y} \to \mathfrak{k} \in F_{\in\vee q}(\mathcal{C}_\sim, \delta)$ by (3.9). Hence $\mathfrak{y} \to \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta)$ or $\mathfrak{y} \to \mathfrak{k} \in F_q(\mathcal{C}_\sim, \delta)$ If $\mathfrak{y} \to \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta)$, then $\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}) \leq \delta < 1 - \delta$ since $\delta < 0.5$. Thus $\mathfrak{y} \to \mathfrak{k} \in F_q(\mathcal{C}_\sim, \delta)$, and therefore $F_q(\mathcal{C}_\sim, \delta)$ is an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$. □

Proposition 3.9. *Given a single valued neutrosophic set $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ in Q, if the nonempty T-neutrosophic q-set $T_q(\mathcal{C}_\sim, \varrho)$, I-neutrosophic q-set $I_q(\mathcal{C}_\sim, \sigma)$ and F-neutrosophic q-set $F_q(\mathcal{C}_\sim, \delta)$ are ordered subalgebras of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$ for all $\varrho, \sigma \in (0, 0.5]$ and $\delta \in [0.5, 1)$, then the following assertion is valid.*

$$(\forall \mathfrak{y}, \mathfrak{k} \in Q)(\forall \varrho_\mathfrak{y}, \varrho_\mathfrak{k} \in (0, 0.5]) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{k},\ \mathfrak{y} \in T_q(\mathcal{C}_\sim, \varrho_\mathfrak{y}),\ \mathfrak{k} \in T_q(\mathcal{C}_\sim, \varrho_\mathfrak{k}) \\ \Rightarrow \mathfrak{y} \to \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \max\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\}) \end{array} \right), \tag{3.10}$$

$$(\forall \mathfrak{w}, \mathfrak{z} \in Q)(\forall \sigma_\mathfrak{w}, \sigma_\mathfrak{z} \in (0, 0.5]) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{w},\ \epsilon \leq_Q \mathfrak{z},\ \mathfrak{w} \in I_q(\mathcal{C}_\sim, \sigma_\mathfrak{w}),\ \mathfrak{z} \in I_q(\mathcal{C}_\sim, \sigma_\mathfrak{z}) \\ \Rightarrow \mathfrak{w} \to \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \max\{\sigma_\mathfrak{w}, \sigma_\mathfrak{z}\}) \end{array} \right), \tag{3.11}$$

$$(\forall \mathfrak{y}, \mathfrak{z} \in Q)(\forall \delta_\mathfrak{y}, \delta_\mathfrak{z} \in [0.5, 1)) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{z},\ \mathfrak{y} \in F_q(\mathcal{C}_\sim, \delta_\mathfrak{y}),\ \mathfrak{z} \in F_q(\mathcal{C}_\sim, \delta_\mathfrak{z}) \\ \Rightarrow \mathfrak{y} \to \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\}) \end{array} \right), \tag{3.12}$$

Proof. Suppose that $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic set in Q and that the nonempty T-neutrosophic q-set $T_q(\mathcal{C}_\sim, \varrho)$, I-neutrosophic q-set $I_q(\mathcal{C}_\sim, \sigma)$ and F-neutrosophic q-set $F_q(\mathcal{C}_\sim, \delta)$ are ordered subalgebras of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$ for all $\varrho, \sigma \in (0, 0.5]$ and $\delta \in [0.5, 1)$. For every $\mathfrak{y}, \mathfrak{k} \in Q$ with $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$, let $\varrho_\mathfrak{y}, \varrho_\mathfrak{k} \in (0, 0.5]$ be such that $\mathfrak{y} \in T_q(\mathcal{C}_\sim, \varrho_\mathfrak{y})$ and $\mathfrak{k} \in T_q(\mathcal{C}_\sim, \varrho_\mathfrak{k})$. Then

$\mathfrak{y}, \mathfrak{k} \in T_q(\mathcal{C}_\sim, \max\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\})$ and $\max\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\} \in (0, 0.5]$, and so $\mathfrak{y} \to \mathfrak{k} \in T_q(\mathcal{C}_\sim, \max\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\})$. It follows that $\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) > 1 - \max\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\} \geq \max\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\}$. Hence $\mathfrak{y} \to \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \max\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\})$, i.e., (3.10) is valid. By the similar way, we can get the result (3.11). For every $\mathfrak{y}, \mathfrak{z} \in Q$ with $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{z}$, let $\mathfrak{y} \in F_q(\mathcal{C}_\sim, \delta_\mathfrak{y})$ and $\mathfrak{z} \in F_q(\mathcal{C}_\sim, \delta_\mathfrak{z})$ for all $\delta_\mathfrak{y}, \delta_\mathfrak{z} \in [0.5, 1)$. Then $\mathfrak{y}, \mathfrak{z} \in F_q(\mathcal{C}_\sim, \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\})$ and $\min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\} \in [0.5, 1)$. Hence $\mathfrak{y} \to \mathfrak{z} \in F_q(\mathcal{C}_\sim, \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\})$, which implies that $\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{z}) + \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\} < 1$. It follows that $\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{z}) < 1 - \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\} \leq \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\}$. Therefore $\mathfrak{y} \to \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\})$, which proves (3.12). □

Proposition 3.10. *Given a single valued neutrosophic set* $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ *in* Q*, if the nonempty* T*-neutrosophic* q*-set* $T_q(\mathcal{C}_\sim, \varrho)$*,* I*-neutrosophic* q*-set* $I_q(\mathcal{C}_\sim, \sigma)$ *and* F*-neutrosophic* q*-set* $F_q(\mathcal{C}_\sim, \delta)$ *are ordered subalgebras of* $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$ *for all* $\varrho, \sigma \in (0.5, 1]$ *and* $\delta \in [0, 0.5)$*, then the following hold true.*

$$(\forall \mathfrak{y}, \mathfrak{k} \in Q)(\forall \varrho_\mathfrak{y}, \varrho_\mathfrak{k} \in (0.5, 1]) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{k},\ \mathfrak{y} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho_\mathfrak{y}),\ \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho_\mathfrak{k}) \\ \Rightarrow \mathfrak{y} \to \mathfrak{k} \in T_q(\mathcal{C}_\sim, \max\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\}) \end{array} \right), \tag{3.13}$$

$$(\forall \mathfrak{w}, \mathfrak{z} \in Q)(\forall \sigma_\mathfrak{w}, \sigma_\mathfrak{z} \in (0.5, 1]) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{w},\ \epsilon \leq_Q \mathfrak{z},\ \mathfrak{w} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma_\mathfrak{w}),\ \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma_\mathfrak{z}) \\ \Rightarrow \mathfrak{w} \to \mathfrak{z} \in I_q(\mathcal{C}_\sim, \max\{\sigma_\mathfrak{w}, \sigma_\mathfrak{z}\}) \end{array} \right), \tag{3.14}$$

$$(\forall \mathfrak{y}, \mathfrak{z} \in Q)(\forall \delta_\mathfrak{y}, \delta_\mathfrak{z} \in [0, 0.5)) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{z},\ \mathfrak{y} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta_\mathfrak{y}),\ \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta_\mathfrak{z}) \\ \Rightarrow \mathfrak{y} \to \mathfrak{z} \in F_q(\mathcal{C}_\sim, \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\}) \end{array} \right), \tag{3.15}$$

Proof. Suppose that $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic set in Q and that the nonempty T-neutrosophic q-set $T_q(\mathcal{C}_\sim, \varrho)$, I-neutrosophic q-set $I_q(\mathcal{C}_\sim, \sigma)$ and F-neutrosophic q-set $F_q(\mathcal{C}_\sim, \delta)$ are ordered subalgebras of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$ for all $\varrho, \sigma \in (0.5, 1]$ and $\delta \in [0, 0.5)$. For every $\mathfrak{y}, \mathfrak{k} \in Q$ with $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$, let $\sigma_\mathfrak{w}, \sigma_\mathfrak{z} \in (0.5, 1]$ be such that $\mathfrak{w} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma_\mathfrak{w})$ and $\mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma_\mathfrak{z})$. Then $\widetilde{\mathcal{C}}_I(\mathfrak{w}) \geq \sigma_\mathfrak{w} > 1 - \sigma_\mathfrak{w} \geq 1 - \max\{\sigma_\mathfrak{w}, \sigma_\mathfrak{z}\}$ and $\widetilde{\mathcal{C}}_I(\mathfrak{z}) \geq \sigma_\mathfrak{z} > 1 - \sigma_\mathfrak{z} \geq 1 - \max\{\sigma_\mathfrak{w}, \sigma_\mathfrak{z}\}$, which induce $\mathfrak{w}, \mathfrak{z} \in I_q(\mathcal{C}_\sim, \max\{\sigma_\mathfrak{w}, \sigma_\mathfrak{z}\})$ and $\max\{\sigma_\mathfrak{w}, \sigma_\mathfrak{z}\} \in (0.5, 1]$. Since $I_q(\mathcal{C}_\sim, \max\{\sigma_\mathfrak{w}, \sigma_\mathfrak{z}\})$ is an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$, we have $\mathfrak{w} \to \mathfrak{z} \in I_q(\mathcal{C}_\sim, \max\{\sigma_\mathfrak{w}, \sigma_\mathfrak{z}\})$. In a similar way, one obtains get $\mathfrak{y} \to \mathfrak{k} \in T_q(\mathcal{C}_\sim, \max\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\})$ for all $\varrho_\mathfrak{y}, \varrho_\mathfrak{k} \in (0.5, 1]$ and $\mathfrak{y}, \mathfrak{k} \in Q$ with $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$. For every $\mathfrak{y}, \mathfrak{z} \in Q$ with $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{z}$, let $\delta_\mathfrak{y}, \delta_\mathfrak{z} \in [0, 0.5)$ be such that $\mathfrak{y} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta_\mathfrak{y})$ and $\mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta_\mathfrak{z})$. Then $\widetilde{\mathcal{C}}_F(\mathfrak{y}) \leq \delta_\mathfrak{y} < 1 - \delta_\mathfrak{y} \leq 1 - \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\}$ and $\widetilde{\mathcal{C}}_F(\mathfrak{z}) \leq \delta_\mathfrak{z} < 1 - \delta_\mathfrak{z} \leq 1 - \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\}$. Hence $\mathfrak{y}, \mathfrak{z} \in F_q(\mathcal{C}_\sim, \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\})$, and so $\mathfrak{y} \to \mathfrak{z} \in F_q(\mathcal{C}_\sim, \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\})$ since $\min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\} \in [0, 0.5)$ and $F_q(\mathcal{C}_\sim, \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\})$ is an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$. □

Proposition 3.11. *Given a single valued neutrosophic set* $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ *in* Q*, let the nonempty* T*-neutrosophic* $\in\vee q$*-set* $T_{\in\vee q}(\mathcal{C}_\sim, \varrho)$*,* I*-neutrosophic* $\in\vee q$*-set* $I_{\in\vee q}(\mathcal{C}_\sim, \sigma)$ *and* F*-neutrosophic* $\in\vee q$*-set* $F_{\in\vee q}(\mathcal{C}_\sim, \delta)$ *be ordered subalgebras of* $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$ *for all* $\varrho, \sigma \in (0, 1]$ *and* $\delta \in [0, 1)$*. The following assertions are established.*

$$(\forall \mathfrak{y}, \mathfrak{k} \in Q)(\forall \varrho_\mathfrak{y}, \varrho_\mathfrak{k} \in (0, 1]) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{k},\ \mathfrak{y} \in T_q(\mathcal{C}_\sim, \varrho_\mathfrak{y}),\ \mathfrak{k} \in T_q(\mathcal{C}_\sim, \varrho_\mathfrak{k}) \\ \Rightarrow \mathfrak{y} \to \mathfrak{k} \in T_{\in\vee q}(\mathcal{C}_\sim, \max\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\}) \end{array} \right), \tag{3.16}$$

$$(\forall \mathfrak{w}, \mathfrak{z} \in Q)(\forall \sigma_\mathfrak{w}, \sigma_\mathfrak{z} \in (0, 1]) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{w},\ \epsilon \leq_Q \mathfrak{z},\ \mathfrak{w} \in I_q(\mathcal{C}_\sim, \sigma_\mathfrak{w}),\ \mathfrak{k} \in I_q(\mathcal{C}_\sim, \sigma_\mathfrak{z}) \\ \Rightarrow \mathfrak{w} \to \mathfrak{z} \in I_{\in\vee q}(\mathcal{C}_\sim, \max\{\sigma_\mathfrak{w}, \sigma_\mathfrak{z}\}) \end{array} \right), \tag{3.17}$$

$$(\forall \mathfrak{y}, \mathfrak{z} \in Q)(\forall \delta_\mathfrak{y}, \delta_\mathfrak{z} \in [0, 1)) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{z},\ \mathfrak{y} \in F_q(\mathcal{C}_\sim, \delta_\mathfrak{y}),\ \mathfrak{z} \in F_q(\mathcal{C}_\sim, \delta_\mathfrak{z}) \\ \Rightarrow \mathfrak{y} \to \mathfrak{z} \in F_{\in\vee q}(\mathcal{C}_\sim, \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\}) \end{array} \right). \tag{3.18}$$

Proof. Given a single valued neutrosophic set $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ in Q, suppose that the nonempty T-neutrosophic $\in\vee q$-set $T_{\in\vee q}(\mathcal{C}_\sim, \varrho)$, I-neutrosophic $\in\vee q$-set $I_{\in\vee q}(\mathcal{C}_\sim, \sigma)$ and F-neutrosophic $\in\vee q$-set $F_{\in\vee q}(\mathcal{C}_\sim, \delta)$

are ordered subalgebras of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$ for all $\varrho, \sigma \in (0,1]$ and $\delta \in [0,1)$. Let $\mathfrak{y}, \mathfrak{k} \in Q$ be so that $\epsilon \leq_Q \mathfrak{y}$, $\epsilon \leq_Q \mathfrak{k}$, $\mathfrak{y} \in T_q(\mathcal{C}_\sim, \varrho_\mathfrak{y})$ and $\mathfrak{k} \in T_q(\mathcal{C}_\sim, \varrho_\mathfrak{k})$ for all $\varrho_\mathfrak{y}, \varrho_\mathfrak{k} \in (0,1]$. Then $\mathfrak{y} \in T_{\in\vee q}(\mathcal{C}_\sim, \varrho_\mathfrak{y}) \subseteq T_{\in\vee q}(\mathcal{C}_\sim, \max\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\})$ and $\mathfrak{k} \in T_{\in\vee q}(\mathcal{C}_\sim, \varrho_\mathfrak{k}) \subseteq T_{\in\vee q}(\mathcal{C}_\sim, \max\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\})$, which imply from the hypothesis that $\mathfrak{y} \to \mathfrak{k} \in T_{\in\vee q}(\mathcal{C}_\sim, \max\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\})$. By the similarly way, $\mathfrak{w} \to \mathfrak{z} \in I_{\in\vee q}(\mathcal{C}_\sim, \max\{\sigma_\mathfrak{w}, \sigma_\mathfrak{z}\})$ is established for all $\sigma_\mathfrak{w}, \sigma_\mathfrak{z} \in (0,1]$ and $\mathfrak{w}, \mathfrak{z} \in Q$ satisfying $\epsilon \leq_Q \mathfrak{w}$, $\epsilon \leq_Q \mathfrak{z}$, $\mathfrak{w} \in I_q(\mathcal{C}_\sim, \sigma_\mathfrak{w})$ and $\mathfrak{z} \in I_q(\mathcal{C}_\sim, \sigma_\mathfrak{z})$. For every $\mathfrak{y}, \mathfrak{z} \in Q$ satisfying $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{z}$, let $\mathfrak{y} \in F_q(\mathcal{C}_\sim, \delta_\mathfrak{y})$ and $\mathfrak{z} \in F_q(\mathcal{C}_\sim, \delta_\mathfrak{z})$ for all $\delta_\mathfrak{y}, \delta_\mathfrak{z} \in [0,1)$. Then $\mathfrak{y} \in F_q(\mathcal{C}_\sim, \delta_\mathfrak{y}) \subseteq F_q(\mathcal{C}_\sim, \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\})$ and $\mathfrak{z} \in F_q(\mathcal{C}_\sim, \delta_\mathfrak{z}) \subseteq F_q(\mathcal{C}_\sim, \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\})$. Since $F_q(\mathcal{C}_\sim, \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\})$ is an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$, we obtain $\mathfrak{y} \to \mathfrak{z} \in F_{\in\vee q}(\mathcal{C}_\sim, \min\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\})$, as required. □

Given a single valued neutrosophic set $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ in Q, consider the set:

$$\mathcal{Q}_0^1 := \{\mathfrak{y} \in Q \mid \widetilde{\mathcal{C}}_T(\mathfrak{y}) > 0,\ \widetilde{\mathcal{C}}_I(\mathfrak{y}) > 0,\ \widetilde{\mathcal{C}}_F(\mathfrak{y}) < 1\}. \tag{3.19}$$

We find conditions for the set $\mathcal{Q}_0^1$ in (3.19) to be an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$.

Theorem 3.12. *If $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$, then the set $\mathcal{Q}_0^1$ in (3.19) is an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$.*

Proof. Suppose that $\mathfrak{y}, \mathfrak{k} \in Q$ are such that $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$ and $\mathfrak{y}, \mathfrak{k} \in \mathcal{Q}_0^1$. Then $\widetilde{\mathcal{C}}_T(\mathfrak{y}) > 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{y}) > 0$, $\widetilde{\mathcal{C}}_F(\mathfrak{y}) < 1$, $\widetilde{\mathcal{C}}_T(\mathfrak{k}) > 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{k}) > 0$, and $\widetilde{\mathcal{C}}_F(\mathfrak{k}) < 1$. Suppose also that $\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) = 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}) = 0$ and $\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}) = 1$, respectively. Using (3.2) derives to $0 = \widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) \geq \min\{\widetilde{\mathcal{C}}_T(\mathfrak{y}), \widetilde{\mathcal{C}}_T(\mathfrak{k})\} > 0$,

$$0 = \widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}) \geq \min\{\widetilde{\mathcal{C}}_I(\mathfrak{y}), \widetilde{\mathcal{C}}_I(\mathfrak{k})\} > 0, \text{ and } 1 = \widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}) \leq \min\{\widetilde{\mathcal{C}}_F(\mathfrak{y}), \widetilde{\mathcal{C}}_F(\mathfrak{k})\} < 1,$$

respectively. This is a contradiction, and thus $\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) > 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}) > 0$ and $\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}) < 1$, respectively. Hence $\mathfrak{y} \to \mathfrak{k} \in \mathcal{Q}_0^1$, and therefore $\mathcal{Q}_0^1$ is an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$. □

Theorem 3.13. *If a single valued neutrosophic set $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ in Q satisfies:*

$$(\forall \mathfrak{y}, \mathfrak{k} \in Q)(\forall \varrho_\mathfrak{y}, \varrho_\mathfrak{k} \in (0,1]) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{k},\ \mathfrak{y} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho_\mathfrak{y}),\ \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \varrho_\mathfrak{k}) \\ \Rightarrow \mathfrak{y} \to \mathfrak{k} \in T_q(\mathcal{C}_\sim, \min\{\varrho_\mathfrak{y}, \varrho_\mathfrak{k}\}) \end{array} \right), \tag{3.20}$$

$$(\forall \mathfrak{w}, \mathfrak{z} \in Q)(\forall \sigma_\mathfrak{w}, \sigma_\mathfrak{z} \in (0,1]) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{w},\ \epsilon \leq_Q \mathfrak{z},\ \mathfrak{w} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma_\mathfrak{w}),\ \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \sigma_\mathfrak{z}) \\ \Rightarrow \mathfrak{w} \to \mathfrak{z} \in I_q(\mathcal{C}_\sim, \min\{\sigma_\mathfrak{w}, \sigma_\mathfrak{z}\}) \end{array} \right), \tag{3.21}$$

$$(\forall \mathfrak{y}, \mathfrak{z} \in Q)(\forall \delta_\mathfrak{y}, \delta_\mathfrak{z} \in [0,1)) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{z},\ \mathfrak{y} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta_\mathfrak{y}),\ \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \delta_\mathfrak{z}) \\ \Rightarrow \mathfrak{y} \to \mathfrak{z} \in F_q(\mathcal{C}_\sim, \max\{\delta_\mathfrak{y}, \delta_\mathfrak{z}\}) \end{array} \right), \tag{3.22}$$

then the set $\mathcal{Q}_0^1$ in (3.19) is an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$.

Proof. Suppose that $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic set in Q that satisfies the conditions (3.20), (3.21) and (3.22). For every $\mathfrak{y}, \mathfrak{k} \in Q$ with $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$, let $\mathfrak{y}, \mathfrak{k} \in \mathcal{Q}_0^1$. Then $\widetilde{\mathcal{C}}_T(\mathfrak{y}) > 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{y}) > 0$, $\widetilde{\mathcal{C}}_F(\mathfrak{y}) < 1$, $\widetilde{\mathcal{C}}_T(\mathfrak{k}) > 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{k}) > 0$, and $\widetilde{\mathcal{C}}_F(\mathfrak{k}) < 1$. It is clear that $\mathfrak{y} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \widetilde{\mathcal{C}}_T(\mathfrak{y})) \cap \mathcal{Q}(\widetilde{\mathcal{C}}_I; \widetilde{\mathcal{C}}_I(\mathfrak{y})) \cap \mathcal{Q}(\widetilde{\mathcal{C}}_F; \widetilde{\mathcal{C}}_F(\mathfrak{y}))$ and $\mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \widetilde{\mathcal{C}}_T(\mathfrak{k})) \cap \mathcal{Q}(\widetilde{\mathcal{C}}_I; \widetilde{\mathcal{C}}_I(\mathfrak{k})) \cap \mathcal{Q}(\widetilde{\mathcal{C}}_F; \widetilde{\mathcal{C}}_F(\mathfrak{k}))$. Suppose

$$\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) = 0, \widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}) = 0 \text{ and } \widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}) = 1,$$

respectively. We get $\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) + \min\{\widetilde{\mathcal{C}}_T(\mathfrak{y}), \widetilde{\mathcal{C}}_T(\mathfrak{k})\} = \min\{\widetilde{\mathcal{C}}_T(\mathfrak{y}), \widetilde{\mathcal{C}}_T(\mathfrak{k})\} \leq 1$,

$$\widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}) + \min\{\widetilde{\mathcal{C}}_I(\mathfrak{y}), \widetilde{\mathcal{C}}_I(\mathfrak{k})\} = \min\{\widetilde{\mathcal{C}}_I(\mathfrak{y}), \widetilde{\mathcal{C}}_I(\mathfrak{k})\} \leq 1,$$

and $\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}) + \max\{\widetilde{\mathcal{C}}_F(\mathfrak{y}), \widetilde{\mathcal{C}}_F(\mathfrak{k})\} \geq 1$, respectively. Then $\mathfrak{y} \to \mathfrak{k} \notin T_q(\mathcal{C}_\sim, \min\{\widetilde{\mathcal{C}}_T(\mathfrak{y}), \widetilde{\mathcal{C}}_T(\mathfrak{k})\})$,

$$\mathfrak{y} \to \mathfrak{k} \notin I_q(\mathcal{C}_\sim, \min\{\widetilde{\mathcal{C}}_I(\mathfrak{y}), \widetilde{\mathcal{C}}_I(\mathfrak{k})\}),$$

and $\mathfrak{y} \to \mathfrak{k} \notin F_q(\mathcal{C}_\sim, \max\{\widetilde{\mathcal{C}}_F(\mathfrak{y}), \widetilde{\mathcal{C}}_F(\mathfrak{k})\})$, respectively. It contradicts conditions (3.20), (3.21) and (3.22), respectively. Hence $\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) > 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}) > 0$ and $\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}) < 1$, that is, $\mathfrak{y} \to \mathfrak{k} \in \mathcal{Q}_0^1$. Therefore $\mathcal{Q}_0^1$ is an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$. □

Theorem 3.14. *Suppose that a single valued neutrosophic set $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ in Q satisfies:*

$$(\forall \mathfrak{y}, \mathfrak{k} \in Q)(\forall \varrho_{\mathfrak{y}}, \varrho_{\mathfrak{k}} \in (0,1]) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{k},\ \mathfrak{y} \in T_q(\mathcal{C}_\sim, \varrho_{\mathfrak{y}}),\ \mathfrak{k} \in T_q(\mathcal{C}_\sim, \varrho_{\mathfrak{k}}) \\ \Rightarrow \mathfrak{y} \to \mathfrak{k} \in \mathcal{Q}(\widetilde{\mathcal{C}}_T; \min\{\varrho_{\mathfrak{y}}, \varrho_{\mathfrak{k}}\}) \end{array} \right), \tag{3.23}$$

$$(\forall \mathfrak{w}, \mathfrak{z} \in Q)(\forall \sigma_{\mathfrak{w}}, \sigma_{\mathfrak{z}} \in (0,1]) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{w},\ \epsilon \leq_Q \mathfrak{z},\ \mathfrak{w} \in I_q(\mathcal{C}_\sim, \sigma_{\mathfrak{w}}),\ \mathfrak{z} \in I_q(\mathcal{C}_\sim, \sigma_{\mathfrak{z}}) \\ \Rightarrow \mathfrak{w} \to \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_I; \min\{\sigma_{\mathfrak{w}}, \sigma_{\mathfrak{z}}\}) \end{array} \right), \tag{3.24}$$

$$(\forall \mathfrak{y}, \mathfrak{z} \in Q)(\forall \delta_{\mathfrak{y}}, \delta_{\mathfrak{z}} \in [0,1)) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{z},\ \mathfrak{y} \in F_q(\mathcal{C}_\sim, \delta_{\mathfrak{y}}),\ \mathfrak{z} \in F_q(\mathcal{C}_\sim, \delta_{\mathfrak{z}}) \\ \Rightarrow \mathfrak{y} \to \mathfrak{z} \in \mathcal{Q}(\widetilde{\mathcal{C}}_F; \max\{\delta_{\mathfrak{y}}, \delta_{\mathfrak{z}}\}) \end{array} \right). \tag{3.25}$$

The set $\mathcal{Q}_0^1$ in (3.19) is an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$.

Proof. Suppose that $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic set in Q that satisfies the conditions (3.23), (3.24) and (3.25). For every $\mathfrak{y}, \mathfrak{k} \in Q$ with $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$, let $\mathfrak{y}, \mathfrak{k} \in \mathcal{Q}_0^1$. We have $\widetilde{\mathcal{C}}_T(\mathfrak{y}) > 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{y}) > 0$, $\widetilde{\mathcal{C}}_F(\mathfrak{y}) < 1$, $\widetilde{\mathcal{C}}_T(\mathfrak{k}) > 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{k}) > 0$, and $\widetilde{\mathcal{C}}_F(\mathfrak{k}) < 1$. Then $\widetilde{\mathcal{C}}_T(\mathfrak{y}) + 1 > 1$, $\widetilde{\mathcal{C}}_I(\mathfrak{y}) + 1 > 1$, $\widetilde{\mathcal{C}}_F(\mathfrak{y}) + 0 < 1$, $\widetilde{\mathcal{C}}_T(\mathfrak{k}) + 1 > 1$, $\widetilde{\mathcal{C}}_I(\mathfrak{k}) + 1 > 1$, and $\widetilde{\mathcal{C}}_F(\mathfrak{k}) + 0 < 1$. Hence $\mathfrak{y}, \mathfrak{k} \in T_q(\mathcal{C}_\sim, 1) \cap I_q(\mathcal{C}_\sim, 1) \cap F_q(\mathcal{C}_\sim, 0)$. If $\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) = 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}) = 0$ and $\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}) = 1$, respectively, then $\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) < 1 = \min\{1, 1\}$, $\widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}) < 1 = \min\{1, 1\}$, and $\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}) > 0 = \max\{0, 0\}$, respectively. Thus $\mathfrak{y} \to \mathfrak{k} \notin \mathcal{Q}(\widetilde{\mathcal{C}}_T; \min\{1, 1\}) \cap \mathcal{Q}(\widetilde{\mathcal{C}}_I; \min\{1, 1\}) \cap \mathcal{Q}(\widetilde{\mathcal{C}}_F; \max\{0, 0\})$, a contradiction. Hence $\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) > 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}) > 0$ and $\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}) < 1$, that is, $\mathfrak{y} \to \mathfrak{k} \in \mathcal{Q}_0^1$. Therefore $\mathcal{Q}_0^1$ is an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$. □

Theorem 3.15. *If a single valued neutrosophic set $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ in Q satisfies:*

$$(\forall \mathfrak{y}, \mathfrak{k} \in Q)(\forall \varrho_{\mathfrak{y}}, \varrho_{\mathfrak{k}} \in (0,1]) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{k},\ \mathfrak{y} \in T_q(\mathcal{C}_\sim, \varrho_{\mathfrak{y}}),\ \mathfrak{k} \in T_q(\mathcal{C}_\sim, \varrho_{\mathfrak{k}}) \\ \Rightarrow \mathfrak{y} \to \mathfrak{k} \in T_q(\mathcal{C}_\sim, \min\{\varrho_{\mathfrak{y}}, \varrho_{\mathfrak{k}}\}) \end{array} \right), \tag{3.26}$$

$$(\forall \mathfrak{w}, \mathfrak{z} \in Q)(\forall \sigma_{\mathfrak{w}}, \sigma_{\mathfrak{z}} \in (0,1]) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{w},\ \epsilon \leq_Q \mathfrak{z},\ \mathfrak{w} \in I_q(\mathcal{C}_\sim, \sigma_{\mathfrak{w}}),\ \mathfrak{z} \in I_q(\mathcal{C}_\sim, \sigma_{\mathfrak{z}}) \\ \Rightarrow \mathfrak{w} \to \mathfrak{z} \in I_q(\mathcal{C}_\sim, \min\{\sigma_{\mathfrak{w}}, \sigma_{\mathfrak{z}}\}) \end{array} \right), \tag{3.27}$$

$$(\forall \mathfrak{y}, \mathfrak{z} \in Q)(\forall \delta_{\mathfrak{y}}, \delta_{\mathfrak{z}} \in [0,1)) \left(\begin{array}{c} \epsilon \leq_Q \mathfrak{y},\ \epsilon \leq_Q \mathfrak{z},\ \mathfrak{y} \in F_q(\mathcal{C}_\sim, \delta_{\mathfrak{y}}),\ \mathfrak{z} \in F_q(\mathcal{C}_\sim, \delta_{\mathfrak{z}}) \\ \Rightarrow \mathfrak{y} \to \mathfrak{z} \in F_q(\mathcal{C}_\sim, \max\{\delta_{\mathfrak{y}}, \delta_{\mathfrak{z}}\}) \end{array} \right), \tag{3.28}$$

then the set $\mathcal{Q}_0^1$ in (3.19) is an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$.

Proof. Suppose that $\mathcal{C}_\sim := (\widetilde{\mathcal{C}}_T, \widetilde{\mathcal{C}}_I, \widetilde{\mathcal{C}}_F)$ is a single valued neutrosophic set in Q satisfying the conditions (3.26), (3.27) and (3.28), and $\mathfrak{y}, \mathfrak{k} \in Q$ be such that $\epsilon \leq_Q \mathfrak{y}$ and $\epsilon \leq_Q \mathfrak{k}$. If $\mathfrak{y}, \mathfrak{k} \in \mathcal{Q}_0^1$, then $\widetilde{\mathcal{C}}_T(\mathfrak{y}) > 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{y}) > 0$,

$\widetilde{\mathcal{C}}_F(\mathfrak{y}) < 1$, $\widetilde{\mathcal{C}}_T(\mathfrak{k}) > 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{k}) > 0$, and $\widetilde{\mathcal{C}}_F(\mathfrak{k}) < 1$. Hence $\widetilde{\mathcal{C}}_T(\mathfrak{y}) + 1 > 1$, $\widetilde{\mathcal{C}}_I(\mathfrak{y}) + 1 > 1$, $\widetilde{\mathcal{C}}_F(\mathfrak{y}) + 0 < 1$, $\widetilde{\mathcal{C}}_T(\mathfrak{k}) + 1 > 1$, $\widetilde{\mathcal{C}}_I(\mathfrak{k}) + 1 > 1$, and $\widetilde{\mathcal{C}}_F(\mathfrak{k}) + 0 < 1$. Thus $\mathfrak{y}, \mathfrak{k} \in T_q(\mathcal{C}_\sim, 1) \cap I_q(\mathcal{C}_\sim, 1) \cap F_q(\mathcal{C}_\sim, 0)$. If $\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) = 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}) = 0$ and $\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}) = 1$, respectively, then $\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) + 1 = 1$, $\widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}) + 1 = 1$ and $\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}) + 0 = 1$, respectively. It follows that $\mathfrak{y} \to \mathfrak{k} \notin T_q(\mathcal{C}_\sim, \min\{1, 1\}) \cap I_q(\mathcal{C}_\sim, \min\{1, 1\}) \cap F_q(\mathcal{C}_\sim, \max\{0, 0\})$, a contradiction. Hence $\widetilde{\mathcal{C}}_T(\mathfrak{y} \to \mathfrak{k}) > 0$, $\widetilde{\mathcal{C}}_I(\mathfrak{y} \to \mathfrak{k}) > 0$ and $\widetilde{\mathcal{C}}_F(\mathfrak{y} \to \mathfrak{k}) < 1$, i.e., $\mathfrak{y} \to \mathfrak{k} \in \mathcal{Q}_0^1$. Consequently, $\mathcal{Q}_0^1$ is an ordered subalgebra of $\mathbf{Q} := (Q, \to, \epsilon, \leq_Q)$. □

4 Conclusion

Smarandache proposed a single-valued neutrosophic set as a part of neutrosophic theory. This set is an extension of classical set theory that allows for the representation of inconsistent, indeterminate and uncertain information in a more comprehensive manner. Single-valued neutrosophic sets have been applied in various fields, for example, image processing, decision making, medical diagnosis, natural language processing, etc., due to their ability to handle uncertainty and imprecision. Of course, it is well known that research on single-valued neutrosophic sets applied to algebraic structures is also actively underway. To apply the single-valued neutrosophic set to ordered BCI-algebras is the aim of this paper. We introduced the notion of single valued neutrosophic ordered subalgebras in ordered BCI-algebras, and investigated several related properties. We explored the conditions under which single valued neutrosophic level subsets become ordered subalgebras, and when the T-neutrosophic q-set, I-neutrosophic q-set and F-neutrosophic q-set could become ordered subalgebras. We created a special set $\mathcal{Q}_0^1$ and found the conditions that it becomes an ordered subalgebra. Based on the ideas and results of this paper, in the future we will investigate a neutrosophic set version for several types of filters in ordered BCI-algebras.

Acknowledgment

The authors would like to thank the anonymous reviewers for their valuable comments and suggestions.

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Received: July 10, 2023. Accepted: Nov 19, 2023