Neutrosophic Sets and Systems, Vol. 58, 2023

University of New Mexico

SOME RESULTS IN NEUTROSOPHIC SOFT METRIC SPACES

M. Jeyaraman[1],J.Johnsy[2] and R. Pandiselvi[3],

[1]PG and Research Department of Mathematics, Raja Doraisingam Govt. Arts College, Sivagangai, Affiliated to Alagappa University, Karaikudi, Tamil Nadu, India. E-mail: jeya.math@gmail.com. ORCID: orcid.org/0000-0002-0364-1845

[2] Research Scholar, P.G. and Research Department of Mathematics, Raja Doraisingam Govt. Arts College, Sivagangai. Affiliated to Alagappa University, Karaikudi, Tamilnadu, India. E-mail: johnsy.math@gmail.com

[3]PG and Research Department of Mathematics, The Madura College (Autonomous), Madurai, Tamil Nadu, India. E-mail: rpselvi@gmail.com. ORCID: orcid.org/0000-0002-8236-964X

*Correspondence: jeya.math@gmail.com

Abstract. In this research, we define a novel concept termed neutrosophic soft metric space as well as investigate its fundamental characteristics. Additionally, in neutrosophic soft metric spaces, we present various topological features of this newly developed space, such as the soft open sphere and the soft closed sphere. The neutrosophic soft metric space has already been studied in depth, and its topological as well as structural features have been mapped out.

Keywords: Metric spaces, Soft set, Open ball, Closed set, Neutrosophic soft metric space.

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1. **Introduction**

Since its development in 1965, Zadeh's [19] fuzzy set has made a significant impact throughout all of logical thought. A lot of real-world issues can be solved thanks to this notion, however it is not sufficient for other difficulties. For this purpose, Atanassov [1] developed Intuitionistic Fuzzy Sets (IFS). After establishing the IFS, it generalises findings from research on Fuzzy Sets. Smarandache [12] defines a subclass of the crisp set called the Neutrosophic Set (NS). Neutrosophy is a theoretical framework that made its way into print in 1998. Fuzzy set was incorporated into probabilistic metric space to create Fuzzy Metric Space (FMS) [11]. A proof of the Fuzzy Sets version of Baire's Category Theorem in FMS form is presented and several fundamental ideas of Fuzzy Sets are analysed in [6]. Since then, FMS have gained widespread use in fields including medical imaging, data processing, and decision making. Molodtsov [9] first proposed soft set theory as a problem-free mathematical method for dealing with uncertainties. Parameterizing the universal set, we get the soft set, a collection of subsets. Any collection of phrases, natural integers, etc., may be used as the parameter set. Therefore, soft sets theory has appealing uses in a wide variety of contexts.

Neutrosophic soft metric space is a novel concept we developed, and its fundamental characteristics were investigated. Additionally, in neutrosophic soft metric spaces, we present various topological structures of this newly discovered space, such as the soft open ball and the soft closed ball. Neutrosophic soft metric space has been studied for its many topological and structural characteristics.

2. PRELIMINARIES

Definition 2.1. [13] A mapping given by $S : X \to P(U)$, then a pair (S, X) is called a soft set over U. Then a soft set is characterized by a class of subsets of U.

Definition 2.2. A 6-tuple $(\mathfrak{X}, \Lambda, \Omega, \Upsilon, \odot, \oplus)$ is known to be an Neutrosophic Soft Metric Space (shortly NSMS), $\mathfrak{X}$ is an arbitrary non empty set, $\odot$ and $\oplus$, a neutrosophic CTN and CTCN and Λ, Ω and Υ are neutrosophic on $SP(\mathfrak{X}^2) \times (0, \infty)SP \to [0, 1]SP$ satisfying the following conditions: For all $\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \ddot{\iota}_{\alpha_3} \in SP(\mathfrak{X}), \vartheta, \hat{\nu} > 0$.

(i) $0 \leq \Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) \leq 1; 0 \leq \Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) \leq 1; 0 \leq \Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) \leq 1;$

(ii) $\Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) + \Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) + \Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) \leq 3;$

(iii) $\Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) > 0;$

(iv) $\Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = 1$ if and only if $\ddot{\xi}_{\alpha_1} = \ddot{\kappa}_{\alpha_2};$

(v) $\Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = \Lambda(\ddot{\kappa}_{\alpha_2}, \ddot{\xi}_{\alpha_1}, \vartheta);$

(vi) $\Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) \odot \Lambda(\ddot{\kappa}_{\alpha_2}, \ddot{\iota}_{\alpha_3}, \hat{\nu}) \leq \Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\iota}_{\alpha_3}, \vartheta + \hat{\nu}),$ for all$\vartheta, \hat{\nu} > 0;$

(vii) $\Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, .) : (0, \infty) \to (0, 1](E)$ is neutrosophic continuous (NC);

(viii) $\lim_{\vartheta \to \infty} \Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = 1$ for all $\vartheta > 0;$

(ix) $\Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) < 1;$

(x) $\Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = 0$ if and only if $\ddot{\xi}_{\alpha_1} = \ddot{\kappa}_{\alpha_2};$

(xi) $\Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = \Omega(\ddot{\kappa}_{\alpha_2}, \ddot{\xi}_{\alpha_1}, \vartheta);$

(xii) $\Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) \oplus \Omega(\ddot{\kappa}_{\alpha_2}, \ddot{\iota}_{\alpha_3}, \hat{\nu}) \geq \Omega(\ddot{\xi}_{\alpha_1}, \ddot{\iota}_{\alpha_3}\vartheta + \hat{\nu}),$ for all $\vartheta, \hat{\nu} > 0;$

(xiii) $\Omega(\ddot{\xi}, \ddot{\kappa}, .) : (0, \infty) \to (0, 1](E)$ is NC;

(xiv) $\lim_{\vartheta \to \infty} \Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = 0$ for all $\vartheta > 0;$

(xv) $\Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) < 1;$

(xvi) $\Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = 0$ if and only if $\ddot{\xi}_{\alpha_1} = \ddot{\kappa}_{\alpha_2};$

(xvii) $\Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = \Upsilon(\ddot{\kappa}_{\alpha_2}, \ddot{\xi}_{\alpha_1}, \vartheta);$

(xviii) $\Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) \oplus \Upsilon(\ddot{\kappa}_{\alpha_2}, \ddot{\iota}_{\alpha_3}, \hat{\nu}) \geq \Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\iota}_{\alpha_3}, \vartheta + \hat{\nu}),$ for all $\vartheta, \hat{\nu} > 0;$

(xix) $\Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, .) : (0,\infty) \to (0,1](E)$ is NC;
(xx) $\lim_{\vartheta\to\infty} \Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = 0$ for all $\vartheta > 0$;
(xxi) If $\vartheta > 0$ then $\Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = 0; \Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = 1; \Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = 1$.
Then, $(\Lambda, \Omega, \Upsilon)$ is called an NMS on $SP(\mathfrak{X})$. The functions Λ, Υ and Ω denote degree of nearness, inconclusiveness and non-nearness between $\ddot{\xi}_{\alpha_1}$ and $\ddot{\kappa}_{\alpha_2}$ with respect to ϑ respectively.

Example 2.3. Define $S : SP(\mathfrak{X}) \times SP(\mathfrak{X}) \times (0,\infty) \to [0,1]$ by
$\Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = \frac{\vartheta}{\vartheta + d(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2})}; \Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = \frac{d(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2})}{\vartheta + d(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2})}; \Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) = \frac{d(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2})}{\vartheta}$ for all $\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2} \in \mathfrak{X}$ and $\vartheta > 0$ and $\omega \odot \tau = \min\{\omega, \tau\}$ and $\omega \oplus \tau = \max\{\omega, \tau\}$ for all $\omega, \tau \in [0,1](E)$.
Then $(\mathfrak{X}, \Lambda, \Omega, \Upsilon, \odot, \oplus)$ is an NSMS.

3. MAIN RESULTS

Definition 3.1. An NSMS $(\mathfrak{X}, \Lambda, \Omega, \Upsilon, \odot, \oplus)$ and $\varsigma \in (0,1)P, \vartheta > 0$, the set $\breve{B}(\ddot{\xi}_{e_1}, \varsigma, \vartheta) = \{\ddot{\kappa}_{e_2} \in \mathfrak{X} : \Lambda(\ddot{\xi}_{e_1}, \ddot{\kappa}_{e_2}, \vartheta) > 1 - \varsigma, \Omega(\ddot{\xi}_{e_1}, \ddot{\kappa}_{e_2}, \vartheta) < \varsigma, \Upsilon(\ddot{\xi}_{e_1}, \ddot{\kappa}_{e_2}, \vartheta) < \varsigma\}$ is known to be Neutrosophic Soft Open Ball (NSOB).

Theorem 3.2. *Let $\breve{B}(\ddot{\xi}_{e_1}, \varsigma, \vartheta)$ be an NSOB, hence it is a Open Set (OS).*

Proof. Consider $\breve{B}(\ddot{\xi}_{e_1}, \varsigma, \vartheta)$ is an OB. Let $\ddot{\kappa}_{e_2} \in \breve{B}(\ddot{\xi}_{e_1}, \varsigma, \vartheta)$.
Then $\Lambda(\ddot{\xi}_{e_1}, \ddot{\kappa}_{e_2}, \vartheta) > 1 - \varsigma, \Omega(\ddot{\xi}_{e_1}, \ddot{\kappa}_{e_2}, \vartheta) < \varsigma, \Upsilon(\ddot{\xi}_{e_1}, \ddot{\kappa}_{e_2}, \vartheta) < \varsigma$.
Since $\Lambda(\ddot{\xi}_{e_1}, \ddot{\kappa}_{e_2}, \vartheta) > 1 - \varsigma$. There exists $\vartheta_0 \in (0, \vartheta)$ such that $\Lambda(\ddot{\xi}_{e_1}, \ddot{\kappa}_{e_2}, \vartheta) > 1 - \varsigma, \Omega(\ddot{\xi}_{e_1}, \ddot{\kappa}_{e_2}, \vartheta) < \varsigma$ and $\Upsilon(\ddot{\xi}_{e_1}, \ddot{\kappa}_{e_2}, \vartheta) < \varsigma$.
If take $\varsigma_0 = \Lambda(\ddot{\xi}_{e_1}, \ddot{\kappa}_{e_2}, \vartheta_0)$ then for $\varsigma_0 > 1 - \varsigma, \rho \in (0,1)$ such that $\varsigma_0 > 1 - \rho > 1 - \varsigma$.
Now for given ς and ρ such that $\varsigma_0 > 1 - \rho$, there exist $\varsigma_1, \varsigma_2 \in (0,1)$ so that $\varsigma_0 \odot \varsigma_1 > 1 - \rho$ and $(1 - \varsigma_0) \oplus (1 - \varsigma_2) \leq \rho$ and $(1 - \varsigma_0) \oplus (1 - \varsigma_3) \leq \rho$.
Choose $\varsigma_4 = \max\{\varsigma_1, \varsigma_2, \varsigma_3\}$. Consider an OB $\breve{B}(\ddot{\kappa}_{e_2}, 1 - \varsigma_4, \vartheta - \vartheta_0)$.
We will show that $\breve{B}(\ddot{\kappa}_{e_2}, 1 - \varsigma_4, \vartheta - \vartheta_0) \subset \breve{B}(\ddot{\xi}_{e_1}, \varsigma, \vartheta)$.
Consider $\ddot{\iota}_{e_3} \in \breve{B}(\ddot{\kappa}_{e_2}, 1 - \varsigma_4, \vartheta - \vartheta_0)$, then $\Lambda(\ddot{\kappa}_{e_2}, \ddot{\iota}_{e_3}, \vartheta - \vartheta_0) > \varsigma_4, \Omega(\ddot{\kappa}_{e_2}, \ddot{\iota}_{e_3}, \vartheta - \vartheta_0) < \varsigma_4$ and $\Upsilon(\ddot{\kappa}_{e_2}, \ddot{\iota}_{e_3}, \vartheta - \vartheta_0) < \varsigma_4$.
Hence,

$$\Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\iota}_{\alpha_3}, \vartheta) \geq \Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta_0) \odot \Lambda(\ddot{\kappa}_{\alpha_2}, \ddot{\iota}_{\alpha_3}, \vartheta - \vartheta_0) \geq \varsigma_0 \odot \varsigma_4 \geq \varsigma_0 \odot \varsigma_1 \geq 1 - \rho > 1 - \varsigma,$$

$$\Omega(\ddot{\xi}_{\alpha_1}, \ddot{\iota}_{\alpha_3}, \vartheta) \leq \Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta_0) \oplus \Omega(\ddot{\kappa}_{\alpha_2}, \ddot{\iota}_{\alpha_3}, \vartheta - \vartheta_0) \leq (1 - \varsigma_0) \oplus (1 - \varsigma_4) \leq (1 - \varsigma_0) \oplus (1 - \varsigma_2) \leq \rho < \varsigma,$$

$$\Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\iota}_{\alpha_3}, \vartheta) \leq \Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta_0) \oplus \Upsilon(\ddot{\kappa}_{\alpha_2}, \ddot{\iota}_{\alpha_3}, \vartheta - \vartheta_0) \leq (1 - \varsigma_0) \oplus (1 - \varsigma_4) \leq (1 - \varsigma_0) \oplus (1 - \varsigma_2) \leq \rho < \varsigma$$

It shows that $\ddot{\iota}_{\alpha_3} \in \breve{B}(\ddot{\xi}_{\alpha_1}, \varsigma, \vartheta)$ and $\breve{B}(\ddot{\kappa}_{\alpha_2}, 1 - \varsigma_4, \vartheta - \vartheta_0) \subset \breve{B}(\ddot{\xi}_{\alpha_1}, \varsigma, \vartheta)$. □

Remark 3.3. Consider an NSMS $(\mathfrak{X}, \Lambda, \Omega, \Upsilon, \odot, \oplus)$. Define $\hat{\nu}(\Lambda, \Omega, \Upsilon) = \{K \subset \mathfrak{X} :$ for each $\ddot{\xi}_{\alpha_1} \in K,$ there exists$\vartheta > 0\}$ and $\varsigma \in (0,1)$ such that $\breve{B}(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) \subset K\}$. Hence, $\hat{\nu}(\Lambda, \Omega, \Upsilon)$ is a topology on $\mathfrak{X}$.

Theorem 3.4. *Every NSMS is Hausdorff.*

Proof. Let $(\mathfrak{X}, \Lambda, \Omega, \Upsilon, \odot, \oplus)$ be a NSMS. Choose $\ddot{\xi}_{\alpha_1}$ and $\ddot{\kappa}_{\alpha_2}$ as two distinct points in $\mathfrak{X}$. Hence, $0 < \Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) < 1, 0 < \Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) < 1, 0 < \Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) < 1$.
Take $\varsigma_1 = \Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta), \varsigma_2 = \Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta), \varsigma_3 = \Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta)$ and $\varsigma = \max\{\varsigma_1, 1-\varsigma_2, 1-\varsigma_3\}$. If we take $\varsigma_0 \in (\varsigma, 1)$, then there exist $\varsigma_4, \varsigma_5, \varsigma_6$ such that $\varsigma_4 \odot \varsigma_4 \geq \varsigma_0, (1-\varsigma_5) \oplus (1-\varsigma_5) \leq 1-\varsigma_0$ and $(1-\varsigma_6) \oplus (1-\varsigma_6) \leq 1-\varsigma_0$.
Let $\varsigma_7 = \max\{\varsigma_4, \varsigma_5, \varsigma_6\}$. If we consider the $\breve{B}\left(\ddot{\xi}_{\alpha_1}, 1-\varsigma_7, \frac{\vartheta}{2}\right)$ and $\breve{B}\left(\ddot{\kappa}_{\alpha_2}, 1-\varsigma_7, \frac{\vartheta}{2}\right)$, then clearly $\breve{B}\left(\ddot{\xi}_{\alpha_1}, 1-\varsigma_7, \frac{\vartheta}{2}\right) \bigcap \breve{B}\left(\ddot{\kappa}_{\alpha_2}, 1-\varsigma_7, \frac{\vartheta}{2}\right) = \emptyset$. From here, if we choose $\ddot{\iota}_{\alpha_3} \in \breve{B}\left(\ddot{\xi}_{\alpha_1}, 1-\varsigma_7, \frac{\vartheta}{2}\right) \bigcap \breve{B}\left(\ddot{\kappa}_{\alpha_2}, 1-\varsigma_7, \frac{\vartheta}{2}\right)$ then

$$\begin{aligned}
\varsigma_1 = \Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) &\geq \Lambda\left(\ddot{\xi}_{\alpha_1}, \ddot{\iota}_{\alpha_3}, \frac{\vartheta}{2}\right) \odot \Lambda\left(\ddot{\iota}_{\alpha_3}, \ddot{\kappa}_{\alpha_2}, \frac{\vartheta}{2}\right) \\
&\geq \varsigma_7 \odot \varsigma_7 \geq \varsigma_4 \odot \varsigma_4 \geq \varsigma_0 > \varsigma_1, \\
\varsigma_2 = \Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) &\leq \Omega\left(\ddot{\xi}_{\alpha_1}, \ddot{\iota}_{\alpha_3}, \frac{\vartheta}{2}\right) \oplus \Omega\left(\ddot{\iota}_{\alpha_3}, \ddot{\kappa}_{\alpha_2}, \frac{\vartheta}{2}\right) \\
&\leq (1-\varsigma_7) \oplus (1-\varsigma_7) \leq (1-\varsigma_5) \oplus (1-\varsigma_5) \leq (1-\varsigma_0) < \varsigma_2, \\
\varsigma_3 = \Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) &\leq \Upsilon\left(\ddot{\xi}_{\alpha_1}, \ddot{\iota}_{\alpha_3}, \frac{\vartheta}{2}\right) \oplus \Upsilon\left(\ddot{\iota}_{\alpha_3}, \ddot{\kappa}_{\alpha_2}, \frac{\vartheta}{2}\right) \\
&\leq (1-\varsigma_7) \oplus (1-\varsigma_7) \leq (1-\varsigma_6) \oplus (1-\varsigma_6) \leq (1-\varsigma_0) < \varsigma_3.
\end{aligned}$$

which is a contradiction. Therefore, we say that NSMS is Hausdorff. □

Definition 3.5. Let $(\mathfrak{X}, \Lambda, \Omega, \Upsilon, \odot, \oplus)$ be a NSMS. A subset A of $\mathfrak{X}$ is called Neutrosophic Bounded (NB), if there exist $\vartheta > 0$ and $\varsigma \in (0,1)$ such that $\Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) > 1-\varsigma, \Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) < \varsigma$ and $\Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta) < \varsigma$, for all $\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2} \in A$.

Definition 3.6. If $A \subseteq \bigcup_{U \in C_N} U$, a collection C_N of OSs is said to be an Open Cover(OC) of A. A subspace A of a NSMS is compact, if every OC of A has a finite subcover. If every sequence in A has a convergent subsequence to a point in A, then it is called sequential compact.

Theorem 3.7. *If A is a compact member of an NSMS* $(\mathfrak{X}, \Lambda, \Omega, \Upsilon, \odot, \oplus)$ *then it is NSMS bounded.*

Proof. Consider A is a compact member of an NSMS $\mathfrak{X}$. Let $\vartheta > 0$ and $0 < \varsigma < 1$. Let $\{\breve{B}(\ddot{\xi}_{\alpha_1}, \varsigma, \vartheta) : \ddot{\xi}_{\alpha_1} \in A\}$ be an open cover of A. Since A is compact, there exists $\ddot{\xi}_{\alpha_1}, \ddot{\xi}_{\alpha_2}, \ddot{\xi}_{\alpha_3} \ldots \ddot{\xi}_{\alpha_n} \in A$ such that $A \subseteq \bigcup_{i=1}^{n} \breve{B}(\ddot{\xi}_{\alpha_i}, \varsigma, \vartheta)$. Let $\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2} \in A$. Then $\ddot{\xi}_{\alpha_1} \in \breve{B}(\ddot{\xi}_{\alpha_i}, \varsigma, \vartheta)$ and $\ddot{\kappa}_{\alpha_2} \in \breve{B}(\ddot{\xi}_{\alpha_j}, \varsigma, \vartheta)$ for some i, j.
Then $\Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\xi}_{\alpha_i}, \vartheta) > 1 - \varsigma, \Omega(\ddot{\xi}_{\alpha_1}, \ddot{\xi}_{\alpha_i}, \vartheta) < \varsigma, \Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\xi}_{\alpha_i}, \vartheta) < \varsigma$ and $\Lambda(\ddot{\kappa}_{\alpha_1}, \ddot{\xi}_{\alpha_j}, \vartheta) > 1 - \varsigma, \Omega(\ddot{\kappa}_{\alpha_1}, \ddot{\xi}_{\alpha_j}, \vartheta) < \varsigma, \Upsilon(\ddot{\kappa}_{\alpha_1}, \ddot{\xi}_{\alpha_j}, \vartheta) < \varsigma$
Now, let $\alpha = \min\{\Lambda(\ddot{\xi}_{\alpha_i}, \ddot{\xi}_{\alpha_j}, \vartheta) : 1 \leq i, j \leq n\}, \beta = \max\{\Omega(\ddot{\xi}_{\alpha_i}, \ddot{\xi}_{\alpha_j}, \vartheta) : 1 \leq i, j \leq n\}$ and $\gamma = \max\{\Upsilon(\ddot{\xi}_{\alpha_i}, \ddot{\xi}_{\alpha_j}, \vartheta) : 1 \leq i, j \leq n\}$.
Then $\alpha, \beta, \gamma > 0$, from here, for $0 < \ddot{\xi}_{\alpha_1}, \ddot{\xi}_{\alpha_2}, \ddot{\xi}_{\alpha_3} < 1$.
Next, we have

$$\begin{aligned}
\Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, 3\vartheta) &\geq \Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\xi}_{\alpha_i}, \vartheta) \odot \Lambda(\ddot{\xi}_{\alpha_i}, \ddot{\xi}_{\alpha_j}, \vartheta) \odot \Lambda(\ddot{\xi}_{\alpha_i}, \ddot{\kappa}_{\alpha_1}, \vartheta) \\
&\geq (1-\varsigma) \odot (1-\varsigma) \odot \mu \geq 1 - \varsigma_1, \text{for some} \quad 0 < \varsigma_1 < 1 \\
\Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, 3\vartheta) &\leq \Omega(\ddot{\xi}_{\alpha_1}, \ddot{\xi}_{\alpha_i}, \vartheta) \oplus \Omega(\ddot{\xi}_{\alpha_i}, \ddot{\xi}_{\alpha_j}, \vartheta) \oplus \Omega(\ddot{\xi}_{\alpha_i}, \ddot{\kappa}_{\alpha_1}, \vartheta) \\
&\leq \varsigma \oplus \varsigma \oplus \beta \leq \varsigma_2, \text{ for some} \quad 0 < \varsigma_2 < 1 \\
\Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, 3\vartheta) &\leq \Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\xi}_{\alpha_i}, \vartheta) \oplus \Upsilon(\ddot{\xi}_{\alpha_i}, \ddot{\xi}_{\alpha_j}, \vartheta) \oplus \Upsilon(\ddot{\xi}_{\alpha_i}, \ddot{\kappa}_{\alpha_1}, \vartheta) \\
&\leq \varsigma \oplus \varsigma \oplus \gamma \leq \varsigma_3, \text{for some} \quad 0 < \varsigma_3 < 1.
\end{aligned}$$

Taking $\varsigma = \max\{\varsigma_1, \varsigma_2, \varsigma_3\}$ and $\vartheta_0 = 3\vartheta$, we have $\Lambda(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta_0) > 1 - \varsigma, \Omega(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta_0) < \varsigma$ and $\Upsilon(\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2}, \vartheta_0) < \varsigma$, for all $\ddot{\xi}_{\alpha_1}, \ddot{\kappa}_{\alpha_2} \in A$. Hence A is NSMS is bounded. □

Theorem 3.8. *If $(\mathfrak{X}, \Lambda, \Omega, \Upsilon, \odot, \oplus)$ is an NSMS and $\tau_{(\Lambda,\Omega,\Upsilon)}$ is a topology on $\mathfrak{X}$. Then $\ddot{\xi}_{\alpha_n} \to \ddot{\xi}_{\alpha_1}$ iff $\Lambda(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) \to 1, \Omega(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) \to 0$ and $\Upsilon(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) \to 0$ as $n \to \infty$ for $\{\ddot{\xi}_{\alpha_n}\}$ in $\mathfrak{X}$.*

Proof. Let $\vartheta > 0$. Consider $\ddot{\xi}_{\alpha_n} \to \ddot{\xi}_{\alpha_1}$. There exist $n_0 \in \mathbb{N}$ such that $\ddot{\xi}_{\alpha_n} \in \breve{B}(\ddot{\xi}_{\alpha_1}, \varsigma, \vartheta)$ for all $n \geq n_0, \varsigma \in (0, 1)$.
Then $1 - \Lambda(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) < \varsigma, \Omega(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) < \varsigma$ and $\Upsilon(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) < \varsigma$.
Hence $\Lambda(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) \to 1, \Omega(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) \to 0$ and $\Upsilon(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) \to 0$ as $n \to \infty$.
Conversely, $\Lambda(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) \to 1, \Omega(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) \to 0$ and $\Upsilon(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) \to 0$, as $n \to \infty$,
Then for $\varsigma \in (0, 1)$, there exists $n_0 \in \mathbb{N}$ such that $1 - \Lambda(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) < \varsigma, \Omega(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) < \varsigma$ and $\Upsilon(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) < \varsigma$, for each $n \geq n_0$.
It follows that $\Lambda(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) > 1-\varsigma, \Omega(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) < \varsigma$ and $\Upsilon(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) < \varsigma$, for each $n \geq n_0$.
Thus $\ddot{\xi}_{\alpha_n} \in \breve{B}(\ddot{\xi}_{\alpha_1}, \varsigma, \vartheta)$ for each $n \geq n_0$. Hence $\ddot{\xi}_{\alpha_n} \to \ddot{\xi}_{\alpha_1}$. □

Theorem 3.9. *If $(\mathfrak{X}, \Lambda, \Omega, \Upsilon, \odot, \oplus)$ is an NSMS and Cauchy sequence in $\mathfrak{X}$ has a convergent sequence. Then $(\mathfrak{X}, \Lambda, \Omega, \Upsilon, \odot, \oplus)$ is a complete NSMS.*

Proof. Consider $\{\ddot{\xi}_{\alpha_n}\}$ is a Cauchy sequence and $\{\ddot{\xi}_{\alpha_{n_i}}\}$ is a member of $\{\ddot{\xi}_{\alpha_n}\}$ that converge to $\ddot{\xi}_{\alpha_1}$. We have to prove $\{\ddot{\xi}_{\alpha_n}\} \to \ddot{\xi}_{\alpha_1}$. Let $\vartheta > 0$ and $\hat{\nu} \in (0,1)$. Consider $\varsigma \in (0,1)$ such that $(1-\varsigma) \odot (1-\varsigma) \geq 1-\hat{\nu}$ and $\varsigma \oplus \varsigma \leq \hat{\nu}$. Since $\{\ddot{\xi}_{\alpha_n}\}$ is Cauchy sequence, there is $\alpha_{n_0} \in \mathbb{N}$ such that $\Lambda\left(\ddot{\xi}_{\alpha_m}, \ddot{\xi}_{\alpha_n}, \frac{\vartheta}{2}\right) > 1-\varsigma, \Omega\left(\ddot{\xi}_{\alpha_m}, \ddot{\xi}_{\alpha_n}, \frac{\vartheta}{2}\right) < \varsigma$ and $\Upsilon\left(\ddot{\xi}_{\alpha_m}, \ddot{\xi}_{\alpha_n}, \frac{\vartheta}{2}\right) < \varsigma$, for all $\alpha_m, \alpha_n \geq \alpha_{n_0}$. Since $\ddot{\xi}_{\alpha_{n_i}} \to \ddot{\xi}_{\alpha_1}$, there is positive integer α_{i_p} such that $\alpha_{i_p} > \alpha_{n_0}, \Lambda\left(\ddot{\xi}_{\alpha_{i_p}}), \ddot{\xi}_{\alpha_1}, \frac{\vartheta}{2}\right) > 1-\varsigma, \Omega\left(\ddot{\xi}_{\alpha_{i_p}}), \ddot{\xi}_{\alpha_1}, \frac{\vartheta}{2}\right) < \varsigma$ and $\Upsilon\left(\ddot{\xi}_{\alpha_{i_p}}, \ddot{\xi}_{\alpha_1}, \frac{\vartheta}{2}\right) < \varsigma$.
Thus if $\alpha_n \geq \alpha_{n_0}, \Lambda(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) \geq \Lambda\left(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_{i_p}}, \frac{\vartheta}{2}\right) \odot \Lambda\left(\ddot{\xi}_{\alpha_{i_p}}, \ddot{\xi}_{\alpha_1}, \frac{\vartheta}{2}\right) > (1-\varsigma) \odot (1-\varsigma) \geq 1-\hat{\nu}, \Omega(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) \leq \Omega\left(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_{i_p}}, \frac{\vartheta}{2}\right) \oplus \Omega\left(\ddot{\xi}_{\alpha_{i_p}}, \ddot{\xi}_{\alpha_1}, \frac{\vartheta}{2}\right) < \varsigma \oplus \varsigma \leq \hat{\nu}$ and $\Upsilon(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_1}, \vartheta) \leq \Upsilon\left(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_{i_p}}, \frac{\vartheta}{2}\right) \oplus \Upsilon\left(\ddot{\xi}_{\alpha_{i_p}}, \ddot{\xi}_{\alpha_1}, \frac{\vartheta}{2}\right) < \varsigma \oplus \varsigma \leq \hat{\nu}$
Thus $\ddot{\xi}_{\alpha_{i_p}}) \to \ddot{\xi}_{\alpha_1}$ and hence $(\mathfrak{X}, \Lambda, \Omega, \Upsilon, \odot, \oplus)$ is complete. □

Theorem 3.10. *If a sequence $\{\Upsilon n : n \in \mathbb{N}\}$ is dense open members of a complete NSMS $(\mathfrak{X}, \Lambda, \Omega, \Upsilon, \odot, \oplus)$. Then $\bigcap_{n\in\mathbb{N}} \upsilon_n$ is dense in $\mathfrak{X}$.*

Proof. Consider ϱ is a nonempty open set in $\mathfrak{X}$. Then we have $\varrho \cap \upsilon_1 \neq \emptyset$. Let $\ddot{\xi}_{\alpha_1} \in \varrho \cap \upsilon_1$ (Since υ_1 is dense in $\mathfrak{X}$)
Then $\breve{B}(\ddot{\xi}_{\alpha_1}, \varsigma_1, \vartheta_1) \subset \varrho \cap \upsilon_1$ (Since $\varrho \cap \upsilon_1$ is open) for $\varsigma_1 \in (0,1)$ and $\vartheta_1 > 0$,
Choose $\varsigma_1^{'} < \varsigma_1$ and $\vartheta_1^{'} = \min\{\vartheta_1, 1\}$ such that $\breve{B}(\ddot{\xi}_{\alpha_1}, \varsigma_1^{'}, \vartheta_1^{'}) \subset \varrho \cap \upsilon_1$
Since υ_2 is dense in $\mathfrak{X}, \breve{B}(\ddot{\xi}_{\alpha_1}, \varsigma_1^{'}, \vartheta_1) \subset \varrho \cap \upsilon_2 \neq \emptyset$. Let $\ddot{\xi}_{\alpha_2} \in \breve{B}(\ddot{\xi}_{\alpha_1}, \varsigma_1^{'}, \vartheta_1^{'}) \cap \upsilon_2$.
Then there exist $\varsigma_2 \in (0, \frac{1}{2})$ and $\vartheta_2 > 0$ such that
$\breve{B}(\ddot{\xi}_{\alpha_2}, \varsigma_2, \vartheta_2) \subset \breve{B}(\ddot{\xi}_{\alpha_1}, \varsigma_1^{'}, \vartheta_1^{'}) \cap \upsilon_2$ (Since $\breve{B}(\ddot{\xi}_{\alpha_1}, \varsigma_1^{'}, \vartheta_1^{'}) \cap \upsilon_2$ is open).
Choose $\varsigma_2^{'} < \varsigma_2$ and $\vartheta_2^{'} = \min\left(\vartheta_2, \frac{1}{2}\right)$ such that $\breve{B}(\ddot{\xi}_{\alpha_2}, \varsigma_2^{'}, \vartheta_2^{'}) \subset \breve{B}(\ddot{\xi}_{\alpha_1}, \varsigma_1^{'}, \vartheta_1^{'}) \cap \upsilon_2$
By repeating this procedure, we obtain a sequence $\{\ddot{\xi}_{\alpha_n}\}$ in $\mathfrak{X}$ and a sequence $\{\vartheta_n^{'}\}$ such that $0 < \vartheta_n^{'} < \frac{1}{n}$ and $\breve{B}(\ddot{\xi}_{\alpha_n}, \varsigma_n^{'}, \vartheta_n^{'}) \subset \breve{B}(\ddot{\xi}_{\alpha_{n-1}}), \varsigma_{n-1}^{'}, \vartheta_{n-1}^{'}) \cap \upsilon_n$.
Now, we have to prove $\{\ddot{\xi}_{\alpha_n}\}$ is a Cauchy sequence.
Consider $\alpha_{n_0} \in \mathbb{N}$ such that $\frac{1}{\alpha_{n_0}} < \vartheta$ and $\frac{1}{\alpha_{n_0}} < \hat{\nu}$ for $\vartheta > 0$ and $\hat{\nu} > 0$.
Then $\Lambda(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_m}, \vartheta) \geq \Lambda\left(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_m}, \frac{1}{\alpha_n}\right) \geq 1 - \frac{1}{\alpha_n} > 1-\hat{\nu}$ for $\alpha_n \geq \alpha_{n_0}$ and $\alpha_m \geq \alpha_n \Omega(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_m}, \vartheta) \leq \Omega\left(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_m}, \frac{1}{\alpha_n}\right) \leq \frac{1}{\alpha_n} < \hat{\nu}, \Upsilon(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_m}, t) \leq \Upsilon\left(\ddot{\xi}_{\alpha_n}, \ddot{\xi}_{\alpha_m}, \frac{1}{\alpha_n}\right) \leq \frac{1}{\alpha_n} < \hat{\nu}$.
There fore $\{\ddot{\xi}_{\alpha_n}\}$ is a Cauchy sequence.
Then there exist $\ddot{\xi}_{\alpha_1} \in \mathfrak{X}$ such that $\ddot{\xi}_{\alpha_n} \to \ddot{\xi}_{\alpha_1}$ (since $\mathfrak{X}$ is complete).
Since $\ddot{\xi}_{\alpha_k} \in \breve{B}(\ddot{\xi}_{\alpha_n}, \varsigma_n^{'}, \vartheta_n^{'})$ for $k \geq n$, we obtain $\ddot{\xi}_{\alpha_1} \in \breve{B}(\ddot{\xi}_{\alpha_n}, \varsigma_n^{'}, \vartheta_n^{'})$.
Hence $\breve{B}(\ddot{\xi}_{\alpha_n}, \varsigma_n^{'}, \vartheta_n^{'}) \subset \breve{B}(\ddot{\xi}_{\alpha_{n-1}}), \varsigma_{n-1}^{'}, \vartheta_{n-1}^{'}) \cap \upsilon_n$ for all n.
Therefore $\varrho \cap (\cap_{n\in\mathbb{N}} \upsilon_n) \neq \emptyset$. Thus $\cap_{n\in\mathbb{N}} \upsilon_n$ is dense in $\mathfrak{X}$. □

4. CONCLUSION

Neutrosophic soft metric space was introduced in this study, and it is distinct from the fuzzy soft metric spaces defined in [3]. In this novel setting, we analysed a number of topological configurations. To define an NSMS and investigate its characteristics is the focus of this research. Open ball, open set, Hausdorffness, compactness, completeness, and nowhere dense in NSMS are some of the structural characteristic features that have been identified.

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Received: June 3, 2023. Accepted: Sep 30, 2023