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Neutrosophic Soft Generalized b-Closed Sets in Neutrosophic Soft Topological Spaces

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Abstract: The notion of neutrosophic soft generalized sets is introduced as a general mathematical tool that incorporates useful properties of both neutrosophic generalized sets and soft sets. In this study, we acquaint the notion of neutrosophic soft generalized b-closed (open) (gb, in short) sets in neutrosophic soft topological spaces. In addition, the relations of this set with other neutrosophic soft closed and open sets and various properties are examined. Furthermore, the properties of the gb-closure and gb-interior operator in neutrosophic soft sets and gb-neighborhood in neutrosophic soft topological spaces are investigated.

Keywords: Neutrosophic soft sets; Neutrosophic soft topology; Neutrosophic soft generalized sets; Neutrosophic soft generalized b-closed (open) sets; Neutrosophic soft generalized b-closure (interior); Neutrosophic soft point; Neutrosophic soft generalized b-neighborhood

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1. Introduction

Model uncertainties in the solution of problems in many different fields such as science, social science, engineering, and medicine, to cope with the structural difficulties of the classical sets, to evaluate the problems in terms of uncertain situations, to model set structures that bring a new approach different from the classical set structure have been defined. One of these set structures is the fuzzy set theory defined by Zadeh [1]. Later, the fuzzy set is generalized as the concept of the intuitionistic fuzzy set by Atanassov [2] in 1986. The fuzzy set is defined by the membership function. However, since it is not practical to create a membership function for each state, Molodotsov [3] defined soft set theory in 1999. This theory is more functional than compared to other structures in practice for decision-making and solving problems involving uncertainties. In the fuzzy set and intuitionistic fuzzy set theories, the values of an element such as being a member and not being a member are emphasized, while the uncertain values are not emphasized. To meet this need, Smarandache

[4] defined the neutrosophic set theory for solving problems involving imprecise, ambiguous, and inconsistent data. Later, many researchers have done successful studies on different combinations of theories such as soft sets, intuitionistic sets, fuzzy sets, and neutrosophic sets [5-12]. One of these combinations is the neutrosophic soft set theory, first described by Maji [13] and later edited by Deli and Broumi [14]. On the other hand, Salama and Alblowi [15] used the concept of a generalized set [16] in neutrosophic sets and described the generalized neutrosophic set and neutrosophic topological space. Broumi [17] defined the generalized neutrosophic soft set by combining the notion of the generalized neutrosophic set proposed by Salama and Alblowi in [15], and the notion of the soft set proposed by Molodtsov in [3].

Applications of the topology of neutrosophic depend on the neutrosophic internal and closing properties of the neutrosophic open and closed sets. For this reason, topologists have identified new set structures in neutrosophic soft sets [18,19] and generalized sets [20-23] using the properties of neutrosophic open and closed sets. In recent years, studies on the properties of these set structures and the properties of their different combinations, which are defined in neutrosophic soft and various topographical spaces, have diversified, and become important research [24-32] topic. In addition, some research [33-40] has been done on how to use neutrosophic and other topological spaces and different sets in fields such as image processing, medical diagnosis, decision-making, information systems, data analysis, industry, graphic structures, applied mathematics, and computer coding.

The b-sets identified by Dimitrije [41] in 1996 are one of these defined structures and have a stronger relationship with other set structures. Akdag and Ozkan [42] defined and studied the concept of the soft b-closed set in 2014. In addition, Ebenanjar [18] et al. studied the notion of the neutrosophic soft b-open set. Das and Pramanik [43] studied generalized neutrosophic b-open sets. In recent years, many studies on b-sets in neutrosophic spaces have been carried out by many researchers. Soft b-separation axioms were studied by Khattak et al. [44], generalized b-closed sets in fuzzy neutrosophic bitopological spaces were studied by Mohammed and Raheem [30], neutrosophic b-generalized sets and their continuity were studied by Maheswari and Chandrasekar [31], and neutrosophic soft ∗b open sets studied by Mehmood [45]. Later, studies using b-sets diversified [46-51].

For modern topology, which is heavily dependent on set theory ideas, in this work, we present a set, so-called "neutrosophic soft generalized b-closed (open)", and its basic properties. We then present the basics of the properties of the gb-closure and gb-interior operator in neutrosophic soft sets and gb-neighborhood in neutrosophic soft topological spaces. This set can be applied in different neutrosophic topological spaces and different set types in the future, and be considered as the starting point for the expansion of concepts such as continuity, compactness, connectedness, and separation axioms through these sets.

2. Preliminaries

In this section, some descriptions and properties that will be used in the article will be given.

Definition 2.1 [3]. Let's assume that $\mathcal{U}$ and E are the initial universe and the set of parameters, respectively. Let the symbol $P(\mathcal{U})$ denote the power set of $\mathcal{U}$. In this case, the pair (F, A) defined over $\mathcal{U}$, where A is a subset of E parameters, is called a soft set. Where $F: A \to P(\mathcal{U})$ is a mapping.

Definition 2.2 [4]. A neutrosophic set (NS, in short) A on the universe of discourse $\mathcal{U}$ is defined as $A = \{< u, T_A(u), I_A(u), F_A(u) >: u \in \mathcal{U}\}$, where T_A, I_A, F_A: $\mathcal{U} \to]^{-}0,1[^{+}$ and

$$^{-}0 \leq T_A(u) + I_A(u) + F_A(u) \leq 3^{+}.$$

Where $T_A(u)$, $I_A(u)$, and $F_A(u)$ which represent the degree of membership function (or Truth), the degree of indeterminacy, and the degree of non-membership (or Falsehood) respectively of each element $u \in \mathcal{U}$ to the set A.

We take the neutrosophic set in the subset of [0,1] since it is not feasible to use a neutrosophic set with values from $]^{-}0,1[^{+}$ in real-life applications such as scientific and engineering calculations.

Definition 2.3 [14]. Let's assume that $\mathcal{U}$ and E are the initial universe and the set of parameters, respectively. Let the symbol $P(\mathcal{U})$ denote the set of all neutrosophic sets of $\mathcal{U}$. Then $(\tilde{F}, E)$ is called a neutrosophic soft set (NSS, in short) over $\mathcal{U}$, where $\tilde{F}: E \to P(\mathcal{U})$ is a mapping.

It can be defined as a parametrized family of some elements of the set $P(\mathcal{U})$ and written as a set of ordered pairs,

$$(\tilde{F}, E) = \left\{\left(e < u, T_{\tilde{F}(e)}(u), I_{\tilde{F}(e)}(u), F_{\tilde{F}(e)}(u) >: u \in \mathcal{U}\right): e \in E\right\}$$

where $T_{\tilde{F}(e)}(u), I_{\tilde{F}(e)}(u), F_{\tilde{F}(e)}(u) \in [0,1]$, respectively called the truth-membership, indeterminacy-membership, and the falsity-membership function of $(\tilde{F}, E)$. The inequality

$$0 \leq T_{\tilde{F}(e)}(u) + I_{\tilde{F}(e)}(u) + F_{\tilde{F}(e)}(u) \leq 3$$

is satisfied.

Throughout this paper, the symbol $NSS(\mathcal{U}_E)$ will indicate the class of all neutrosophic soft sets on $\mathcal{U}$, and $\tilde{F}_E$ will be replaced instead of $(\tilde{F}, E)$.

Definition 2.4 [52]. Let $\tilde{F}_E$, $\tilde{G}_E \in NSS(\mathcal{U}_E)$. Then

(i) $\tilde{F}_E$ is said to be a *null* set if $T_{\tilde{F}(e)}(u) = 0$, $I_{\tilde{F}(e)}(u) = 0$, $F_{\tilde{F}(e)}(u) = 1$; for all $e \in E$, for all $u \in \mathcal{U}$.

It is denoted by $\emptyset_E$. Obviously $(\emptyset_E)^c = 1_E$.

(ii) $\tilde{F}_E$ is said to be an *absolute* set if $T_{\tilde{F}(e)}(u) = 1$, $I_{\tilde{F}(e)}(u) = 1$, $F_{\tilde{F}(e)}(u) = 0$; for all $e \in E$, for all $u \in \mathcal{U}$.

It is denoted by 1_E. Obviously $(1_E)^c = \emptyset_E$.

(iii) the neutrosophic soft union of $\tilde{F}_E$ and $\tilde{G}_E$, denoted $\tilde{F}_E \cup \tilde{G}_E = \tilde{H}_E$, is defined as $\tilde{H}_E = \{(e < u, T_{\tilde{H}(e)}(u), I_{\tilde{H}(e)}(u), F_{\tilde{H}(e)}(u) >: u \in \mathcal{U}): e \in E\}$ where,

$$T_{\tilde{H}(e)}(u) = max\{T_{\tilde{F}(e)}(u), T_{\tilde{G}(e)}(u)\},$$
$$I_{\tilde{H}(e)}(u) = max\{I_{\tilde{F}(e)}(u), I_{\tilde{G}(e)}(u)\},$$
$$F_{\tilde{H}(e)}(u) = min\{F_{\tilde{F}(e)}(u), F_{\tilde{G}(e)}(u)\}.$$

(iv) the neutrosophic soft intersection of $\tilde{F}_E$ and $\tilde{G}_E$, denoted $\tilde{F}_E \cap \tilde{G}_E = \tilde{H}_E$, is defined as $\tilde{H}_E = \{(e < u, T_{\tilde{H}(e)}(u), I_{\tilde{H}(e)}(n), F_{\tilde{H}(e)}(u) >: u \in \mathcal{U}): e \in E\}$ where,

$$T_{\tilde{H}(e)}(u) = min\{T_{\tilde{F}(e)}(u), T_{\tilde{G}(e)}(u)\},$$
$$I_{\tilde{H}(e)}(u) = min\{I_{\tilde{F}(e)}(u), I_{\tilde{G}(e)}(u)\},$$
$$F_{\tilde{H}(e)}(u) = max\{F_{\tilde{F}(e)}(u), F_{\tilde{G}(e)}(u)\}.$$

(v) $\tilde{F}_E$ is a subset of $\tilde{G}_E$, denoted by $\tilde{F}_E \subseteq \tilde{G}_E$. If for all $e \in E$, for all $u \in \mathcal{U}$; $T_{\tilde{F}(e)}(u) \leq T_{\tilde{G}(e)}(u)$, $I_{\tilde{F}(e)}(u) \leq I_{\tilde{G}(e)}(u)$, $F_{\tilde{F}(e)}(u) \geq F_{\tilde{G}(e)}(u)$.

(vi) the neutrosophic soft complement of $\tilde{F}_E$, denoted $(\tilde{F}_E)^c$, is defined as

$$(\tilde{F}_E)^c = \{(e < u, F_{\tilde{F}(e)}(u), 1 - I_{\tilde{F}(e)}(u), T_{\tilde{F}(e)}(u) >: u \in \mathcal{U}): e \in E\}$$

Obvious that $((\tilde{F}_E)^c)^c = \tilde{F}_E$.

(vii) the neutrosophic soft difference of $\tilde{F}_E$ and $\tilde{G}_E$, denoted $\tilde{F}_E \setminus \tilde{G}_E = \tilde{H}_E$, is defined as $\tilde{H}_E = \{(e < u, T_{\tilde{H}(e)}(u), I_{\tilde{H}(e)}(u), F_{\tilde{H}(e)}(u) >: u \in \mathcal{U}): e \in E\}$ where,

$$T_{\tilde{H}(e)}(u) = min\{T_{\tilde{F}(e)}(u), T_{\tilde{G}(e)}(u)\},$$
$$I_{\tilde{H}(e)}(u) = min\{I_{\tilde{F}(e)}(u), 1 - I_{\tilde{G}(e)}(u)\},$$
$$F_{\tilde{H}(e)}(u) = max\{F_{\tilde{F}(e)}(u), F_{\tilde{G}(e)}(u)\}.$$

Definition 2.5 [52, 53]. Let $\tilde{\tau}_{NSS} \subset NSS(\mathcal{U}_E)$. Then $\tilde{\tau}_{NSS}$ is said to be a neutrosophic soft topology (NST, in short) on $\mathcal{U}$ if

(i) $\emptyset_E$ and 1_E belong to $\tilde{\tau}_{NSS}$,

(ii) $\cup_{i \in I} (\tilde{F}_E)_i \in \tilde{\tau}_{NSS}$ for each $(\tilde{F}_E)_i \in \tilde{\tau}_{NSS}$,

(iii) $\tilde{F}_E \cap \tilde{G}_E \in \tilde{\tau}_{NSS}$ for any $\tilde{F}_E, \tilde{G}_E \in \tilde{\tau}_{NSS}$.

In this case, the triplet $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is called a neutrosophic soft topological space (NSTS, in short) over $\mathcal{U}$. The members of $\tilde{\tau}_{NSS}$ are said to be a neutrosophic soft open set (NSOS, in short) and their complements are said to be a neutrosophic soft closed set (NSCS, in short).

Definition 2.6 [53]. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{F}_E \in NSS(\mathcal{U}_E)$. Then,

(i) the interior of $\tilde{F}_E$, denoted $int(\tilde{F}_E)$, is described as

$$int(\tilde{F}_E) = \cup \{\tilde{G}_E : \tilde{G}_E \text{ is an NSOS in } \mathcal{U} \text{ and } \tilde{G}_E \subseteq \tilde{F}_E\}.$$

(ii) the closure of $\tilde{F}_E$, denoted $cl(\tilde{F}_E)$, is defined as

$$cl(\tilde{F}_E) = \cap \{\tilde{K}_E : \tilde{K}_E \text{ is an NSCS in } \mathcal{U} \text{ and } \tilde{K}_E \supseteq \tilde{F}_E\}.$$

Definition 2.7 [54]. The NSS $u^e_{(\alpha,\beta,\gamma)}$ is said to be a neutrosophic soft point, for every $u \in \mathcal{U}$, $0 < \alpha, \beta, \gamma \leq 1$, $e \in E$, and is described as

$$u^e_{(\alpha,\beta,\gamma)}(e')(y) = \begin{cases} (\alpha,\beta,\gamma) & if \quad e' = e \quad and \quad y = u \\ (0,0,1) & if \quad e' \neq e \quad or \quad y \neq u. \end{cases}$$

Definition 2.8 [54]. Let $\tilde{F}_E$ be an NSS over $\mathcal{U}$. We say that $u^e_{(\alpha,\beta,\gamma)} \in \tilde{F}_E$ read as belonging to the NSS $\tilde{F}_E$ whenever

$$\alpha \leq T_{\tilde{F}(e)}(u), \beta \leq I_{\tilde{F}(e)}(u) \text{ and } \gamma \geq F_{\tilde{F}(e)}(u).$$

Definition 2.9 [54]. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{F}_E \in NSS(\mathcal{U}_E)$. If there exists an NSOS $\tilde{G}_E$ such that $u^e_{(\alpha,\beta,\gamma)} \in \tilde{G}_E \subset \tilde{F}_E$, then $\tilde{F}_E$ is called a neutrosophic soft neighborhood of the neutrosophic soft point $u^e_{(\alpha,\beta,\gamma)} \in \tilde{F}_E$.

3. Neutrosophic Soft b-Closed Sets

In this section, we introduce the elementary descriptions and outcomes of the netrosophic soft closed and netrosophic soft-b-closed set theories that will be required in the future chapter.

Definition 3.1 [18, 19]. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{F}_E \in NSS(\mathcal{U}_E)$. Then

(i) $\tilde{F}_E$ is called a neutrosophic soft regular closed (open) set (NS-rCS (NS-rOS), in short) if $\tilde{F}_E = cl\left(int(\tilde{F}_E)\right)$ ($\tilde{F}_E = int\left(cl(\tilde{F}_E)\right)$).

(ii) $\tilde{F}_E$ is called a neutrosophic soft pre-closed (open) set (NS-pCS (NS-pOS), in short) if $cl\left(int(\tilde{F}_E)\right) \subseteq \tilde{F}_E$ ($\tilde{F}_E \subseteq int\left(cl(\tilde{F}_E)\right)$).

(iii) $\tilde{F}_E$ is called a neutrosophic soft semi-closed (open) set (NS-sCS (NS-sOS), in short) if $int\left(cl(\tilde{F}_E)\right) \subseteq \tilde{F}_E$ ($\tilde{F}_E \subseteq cl\left(int(\tilde{F}_E)\right)$).

(iv) $\tilde{F}_E$ is called a neutrosophic soft α-closed (open) set (NS-αCS (NS-αOS), in short) if $cl\left(int\left(cl(\tilde{F}_E)\right)\right) \subseteq \tilde{F}_E$ ($\tilde{F}_E \subseteq int\left(cl\left(int(\tilde{F}_E)\right)\right)$).

Definition 3.2. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{F}_E \in NSS(\mathcal{U}_E)$. Then,

(i) the regular closure of $\tilde{F}_E$, denoted $cl_r(\tilde{F}_E)$, is defined as
$$cl_r(\tilde{F}_E) = \cap \{\tilde{K}_E : \tilde{K}_E \text{ is an NS-rCS in } \mathcal{U} \text{ and } \tilde{K}_E \supseteq \tilde{F}_E\}.$$

(ii) the regular interior of $\tilde{F}_E$, denoted $int_r(\tilde{F}_E)$, is defined as
$$int_r(\tilde{F}_E) = \cup \{\tilde{G}_E : \tilde{G}_E \text{ is an NS-rOS in } \mathcal{U} \text{ and } \tilde{G}_E \subseteq \tilde{F}_E\}.$$

(iii) the pre-closure of $\tilde{F}_E$, denoted $cl_p(\tilde{F}_E)$, is defined as
$$cl_p(\tilde{F}_E) = \cap \{\tilde{K}_E : \tilde{K}_E \text{ is an NS-pCS in } \mathcal{U} \text{ and } \tilde{K}_E \supseteq \tilde{F}_E\}.$$

(iv) the pre-interior of $\tilde{F}_E$, denoted $int_p(\tilde{F}_E)$, is defined as
$$int_p(\tilde{F}_E) = \cup \{\tilde{G}_E : \tilde{G}_E \text{ is an NS-pOS in } \mathcal{U} \text{ and } \tilde{G}_E \subseteq \tilde{F}_E\}.$$

(v) the semi-closure of $\tilde{F}_E$, denoted $cl_s(\tilde{F}_E)$, is defined as

$$cl_s(\tilde{F}_E) = \cap \{\tilde{K}_E: \ \tilde{K}_E \text{ is an NS-sCS in } \mathcal{U} \text{ and } \tilde{K}_E \supseteq \tilde{F}_E\}.$$

(vi) the semi-interior of $\tilde{F}_E$, denoted $int_s(\tilde{F}_E)$, is defined as

$$int_s(\tilde{F}_E) = \cup \{\tilde{G}_E: \tilde{G}_E \text{ is an NS-sOS in } \mathcal{U} \text{ and } \tilde{G}_E \subseteq \tilde{F}_E\}.$$

(vii) the α-interior of $\tilde{F}_E$, denoted $int_\alpha(\tilde{F}_E)$, is defined as

$$int_\alpha(\tilde{F}_E) = \cup \{\tilde{G}_E: \tilde{G}_E \text{ is an NS-}\alpha\text{OS in } \mathcal{U} \text{ and } \tilde{G}_E \subseteq \tilde{F}_E\}.$$

(viii) the α-closure of $\tilde{F}_E$, denoted $cl_\alpha(\tilde{F}_E)$, is defined as

$$cl_\alpha(\tilde{F}_E) = \cap \{\tilde{K}_E: \ \tilde{K}_E \text{ is an NS-}\alpha\text{CS in } \mathcal{U} \text{ and } \tilde{K}_E \supseteq \tilde{F}_E\}.$$

Theorem 3.1. [18] Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{F}_E \in NSS(\mathcal{U}_E)$. Then,

(i) $cl_p(\tilde{F}_E) = \tilde{F}_E \cup cl\left(int(\tilde{F}_E)\right)$, $int_p(\tilde{F}_E) = \tilde{F}_E \cap int\left(cl(\tilde{F}_E)\right)$,

(ii) $cl_s(\tilde{F}_E) = \tilde{F}_E \cup int\left(cl(\tilde{F}_E)\right)$, $int_s(\tilde{F}_E) = \tilde{F}_E \cap cl\left(int(\tilde{F}_E)\right)$,

(iii) $cl_\alpha(\tilde{F}_E) = \tilde{F}_E \cup cl\left(int\left(cl(\tilde{F}_E)\right)\right)$, $int_\alpha(\tilde{F}_E) = \tilde{F}_E \cap int\left(cl\left(int(\tilde{F}_E)\right)\right)$.

Definition 3.3 [18]. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be a NSTS over $\mathcal{U}$ and $\tilde{F}_E \in NSS(\mathcal{U}_E)$. Then

(i) $\tilde{F}_E$ is called a neutrosophic soft b-closed set (NS-bCS, in short) if $int\left(cl(\tilde{F}_E)\right) \cap cl\left(int(\tilde{F}_E)\right) \subseteq \tilde{F}_E$.

(ii) $\tilde{F}_E$ is called a neutrosophic soft b-open set (NS-bOS, in short) if $\tilde{F}_E \subseteq int\left(cl(\tilde{F}_E)\right) \cup cl\left(int(\tilde{F}_E)\right)$.

Example 3.1. Let $\mathcal{U} = \{u_1, u_2\}$, $\mathrm{E} = \{\mathrm{e}_1, \mathrm{e}_2\}$ and $\tau_{NSS} = \{\emptyset_E, 1_E, (\tilde{F}_E)_1\}$ where $(\tilde{F}_E)_1$ is an NSS over $\mathcal{U}$, defined as

$$(\tilde{F}_E)_1 = \begin{Bmatrix} e_1 = \{< u_1, 0.6,0.4,0.7 >, < u_2, 0.2,0.5,0.5 >\} \\ e_2 = \{< u_1, 0.6,0.5,0.8 >, < u_2, 0.4,0.3,0.5 >\} \end{Bmatrix}.$$

Then τ_{NSS} defines an NST on $\mathcal{U}$, and so $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is an NSTS over $\mathcal{U}$. An NSS $\tilde{G}_E$ in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is defined as

$$\tilde{G}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.7,0.8,0.9 >, < u_2, 0.8,0.2,0.1 >\} \\ e_2 = \{< u_1, 0.3,0.3,0.3 >, < u_2, 0.7,0.1,0.1 >\} \end{Bmatrix}.$$

Then, $\tilde{G}_E$ is an NS-bCS in $\mathcal{U}$ since $int\left(cl(\tilde{G}_E)\right) \cap cl\left(int(\tilde{G}_E)\right) = int(1_E) \cap cl(\emptyset_E) = \emptyset_E \subseteq \tilde{G}_E$. Also, an NSS $\tilde{K}_E$ in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is defined as

$$\tilde{K}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.3,0.2,0.7 >, < u_2, 0.1,0.4,0.5 >\} \\ e_2 = \{< u_1, 0.2,0.3,0.9 >, < u_2, 0.2,0.2,0.8 >\} \end{Bmatrix}.$$

Then, $\tilde{K}_E$ is an NS-bOS in $\mathcal{U}$ because $\tilde{K}_E \subseteq int\left(cl(\tilde{K}_E)\right) \cup cl\left(int(\tilde{K}_E)\right) = int\left((\tilde{F}_E)_1{}^c\right) \cup cl(\emptyset_E) = (\tilde{F}_E)_1$.

Definition 3.4 [18]. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{F}_E \in NSS(\mathcal{U}_E)$. Then,

(i) the b-interior of $\tilde{F}_E$, denoted $int_b(\tilde{F}_E)$, is defined as

$$int_b(\tilde{F}_E) = \cup \{\tilde{G}_E : \tilde{G}_E \text{ is an NS-bOS in } \mathcal{U} \text{ and } \tilde{G}_E \subseteq \tilde{F}_E\}.$$

(ii) the b-closure of $\tilde{F}_E$, denoted $cl_b(\tilde{F}_E)$, is defined as

$$cl_b(\tilde{F}_E) = \cap \{\tilde{K}_E : \tilde{K}_E \text{ is an NS-bCS in } \mathcal{U} \text{ and } \tilde{K}_E \supseteq \tilde{F}_E\}.$$

Example 3.2. Let $\mathcal{U} = \{u_1, u_2\}$, $\mathrm{E} = \{\mathrm{e}_1, \mathrm{e}_2\}$ and $\tau_{\mathrm{NSS}} = \{\emptyset_\mathrm{E}, 1_\mathrm{E}, (\tilde{\mathrm{F}}_\mathrm{E})_1, (\tilde{\mathrm{F}}_\mathrm{E})_2\}$ where $(\tilde{\mathrm{F}}_\mathrm{E})_1$ and $(\tilde{\mathrm{F}}_\mathrm{E})_2$ are NSSs over $\mathcal{U}$, defined as

$$(\tilde{\mathrm{F}}_\mathrm{E})_1 = \begin{Bmatrix} \mathrm{e}_1 = \{< u_1, 0.7,0.7,0.5 >, < u_2, 0.7,0.5,0.5 >\} \\ \mathrm{e}_2 = \{< u_1, 0.5,0.6,0.4 >, < u_2, 0.6,0.4,0.6 >\} \end{Bmatrix},$$

$$(\tilde{\mathrm{F}}_\mathrm{E})_2 = \begin{Bmatrix} \mathrm{e}_1 = \{< u_1, 0.4,0.2,0.8 >, < u_2, 0.3,0.5,0.8 >\} \\ \mathrm{e}_2 = \{< u_1, 0.4,0.3,0.6 >, < u_2, 0.6,0.3,0.6 >\} \end{Bmatrix}.$$

Then τ_{NSS} defines an NST on $\mathcal{U}$, and thus $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is an NSTS over $\mathcal{U}$. An NSS $\tilde{\mathrm{F}}_\mathrm{E}$ in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is defined as

$$\tilde{\mathrm{F}}_\mathrm{E} = \begin{Bmatrix} \mathrm{e}_1 = \{< u_1, 0.2,0.1,0.8 >, < u_2, 0.2,0.3,0.7 >\} \\ \mathrm{e}_2 = \{< u_1, 0.1,0.2,0.9 >, < u_2, 0.2,0.4,0.6 >\} \end{Bmatrix}.$$

By Definition 3.3 (i), $\emptyset_\mathrm{E}, 1_\mathrm{E}, (\tilde{\mathrm{F}}_\mathrm{E})_1, (\tilde{\mathrm{F}}_\mathrm{E})_2$,and $\tilde{\mathrm{F}}_\mathrm{E}$ are NS-bCSs in $\mathcal{U}$ and by Definition 3.4 (ii) $(\tilde{\mathrm{F}}_\mathrm{E})_1 \supseteq \tilde{F}_E$, $1_\mathrm{E} \supseteq \tilde{F}_E$, and $\tilde{\mathrm{F}}_\mathrm{E} \supseteq \tilde{F}_E$. Hence, $cl_b(\tilde{F}_E) = (\tilde{\mathrm{F}}_\mathrm{E})_1 \cap 1_\mathrm{E} \cap \tilde{\mathrm{F}}_\mathrm{E} = \tilde{\mathrm{F}}_\mathrm{E}$.

Example 3.3. Let $\mathcal{U} = \{u_1, u_2, u_3\}$, $\mathrm{E} = \{\mathrm{e}_1, \mathrm{e}_2\}$ and $\tau_{\mathrm{NSS}} = \{\emptyset_\mathrm{E}, 1_\mathrm{E}, (\tilde{\mathrm{F}}_\mathrm{E})_1\}$ where $(\tilde{\mathrm{F}}_\mathrm{E})_1$ is NSS over $\mathcal{U}$, defined as

$$(\tilde{\mathrm{F}}_\mathrm{E})_1 = \begin{Bmatrix} \mathrm{e}_1 = \{< u_1, 0.5,0.6,0.3 >, < u_2, 0.6,0.5,0.2 >, < u_3, 0.5,0.3,0.4 >\} \\ \mathrm{e}_2 = \{< u_1, 0.7,0.8,0.2 >, < u_2, 0.7,0.4,0.3 >, < u_3, 0.2,0.4,0.1 >\} \end{Bmatrix}.$$

Then τ_{NSS} defines an NST on $\mathcal{U}$, and hence $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is an NSTS over $\mathcal{U}$. An NSS $\tilde{\mathrm{F}}_\mathrm{E}$ in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is defined as

$$\tilde{\mathrm{F}}_\mathrm{E} = \begin{Bmatrix} \mathrm{e}_1 = \{< u_1, 0.3,0.4,0.6 >, < u_2, 0.1,0.5,0.7 >, < u_3, 0.4,0.6,0.5 >\} \\ \mathrm{e}_2 = \{< u_1, 0.3,0.1,0.8 >, < u_2, 0.2,0.5,0.7 >, < u_3, 0.1,0.2,0.3 >\} \end{Bmatrix}.$$

By Definition 3.3 (ii), $\emptyset_\mathrm{E}$, 1_E, $(\tilde{\mathrm{F}}_\mathrm{E})_1$, and $\tilde{\mathrm{F}}_\mathrm{E}$ are NS-bOSs in $\mathcal{U}$ and by Definition 3.4 (i) $\emptyset_\mathrm{E} \subseteq \tilde{F}_E$, $\tilde{\mathrm{F}}_\mathrm{E} \subseteq \tilde{F}_E$. Thus, $int_b(\tilde{F}_E) = \emptyset_\mathrm{E} \cup \tilde{\mathrm{F}}_\mathrm{E} = \tilde{\mathrm{F}}_\mathrm{E}$.

Theorem 3.2 [18]. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{F}_E \in NSS(\mathcal{U}_E)$. Then

(i) $\tilde{F}_E$ is a neutrosophic soft b-closed set iff $\tilde{F}_E = cl_b(\tilde{F}_E)$,

(ii) $\tilde{F}_E$ is a neutrosophic soft b-open set iff $\tilde{F}_E = int_b(\tilde{F}_E)$,

(iii) $cl_b(\emptyset_E) = \emptyset_E, cl_b(1_E) = 1_E$,

(iv) $int_b(\emptyset_E) = \emptyset_E, int_b(1_E) = 1_E$,

(v) $\tilde{F}_E$ is a neutrosophic soft b-closed set iff $cl_b\left(cl_b(\tilde{F}_E)\right) = cl_b(\tilde{F}_E)$,

(vi) $\left(int_b(\tilde{F}_E)\right)^c = cl_b(\tilde{F}_E)^c$,

(vii) $\left(cl_b(\tilde{F}_E)\right)^c = int_b(\tilde{F}_E)^c$.

Example 3.4. Let us take into account the topology, NS-bCS, and NS-bOS that are given in Example 3.1. By Definition 3.3 (i), $\emptyset_E$, 1_E, $(\tilde{F}_E)_1$, and $\tilde{G}_E$ are NS-bCSs in $\mathcal{U}$ and by Definition 3.4 (ii) $1_E \supseteq \tilde{G}_E$ and $\tilde{G}_E \supseteq \tilde{G}_E$. Then, $cl_b(\tilde{G}_E) = 1_E \cap \tilde{G}_E = \tilde{G}_E$. Hence, $cl_b(\tilde{G}_E) = \tilde{G}_E$.

By Definition 3.3 (ii), $\emptyset_E$, 1_E, $(\tilde{F}_E)_1$, and $\widetilde{N}_E$ are NS-bOSs in $\mathcal{U}$ and by Definition 3.4 (i), $\emptyset_E \subseteq \widetilde{N}_E$, $\widetilde{N}_E \subseteq \widetilde{N}_E$. Then, $int_b(\widetilde{N}_E) = \emptyset_E \cup \widetilde{N}_E = \widetilde{N}_E$. Thus, $int_b(\widetilde{N}_E) = \widetilde{N}_E$.
By Definition 3.4 (ii), $1_E \supseteq 1_E$, $cl_b(1_E) = 1_E$. Also, By Definition 3.3 (i), $\emptyset_E, 1_E$, and $(\tilde{F}_E)_1$ are NS-bCSs in $\mathcal{U}$ and by Definition 3.4 (ii), $\emptyset_E \supseteq \emptyset_E$, $1_E \supseteq \emptyset_E$, and $(\tilde{F}_E)_1 \supseteq \emptyset_E$. Then, $cl_b(\emptyset_E) = 1_E \cap \emptyset_E \cap (\tilde{F}_E)_1 = \emptyset_E$. Similarly, by taking the complement, Theorem 3.2 (iv) also provides. By Theorem 3.2 (i), $cl_b(\tilde{G}_E) = \tilde{G}_E$. So, $cl_b\left(cl_b(\tilde{G}_E)\right) = cl_b(\tilde{G}_E)$ is obtained. By Theorem 3.2 (ii), $int_b(\widetilde{N}_E) = \widetilde{N}_E$. Therefore, $\left(int_b(\widetilde{N}_E)\right)^c = (\widetilde{N}_E)^c$ is obtained. Also, By Definition 3.3 (i), $\emptyset_E$, 1_E, $(\tilde{F}_E)_1$, $(\widetilde{N}_E)^c$ are NS-bCSs in $\mathcal{U}$ and by Definition 3.4 (ii) $1_E \supseteq (\widetilde{N}_E)^c$ and $(\widetilde{N}_E)^c \supseteq (\widetilde{N}_E)^c$. Thus, $cl_b\left((\widetilde{N}_E)^c\right) = 1_E \cap (\widetilde{N}_E)^c = (\widetilde{N}_E)^c$. Hence, $cl_b\left((\widetilde{N}_E)^c\right) = (\widetilde{N}_E)^c$. Therefore, $\left(int_b(\widetilde{N}_E)\right)^c = cl_b\left((\widetilde{N}_E)^c\right)$ is obtained. Similarly, by taking the complement, Theorem 3.2 (vii) also provides.

Theorem 3.3 [18]. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{G}_E, \widetilde{K}_E \in NSS(\mathcal{U}_E)$. Then,
- (i) $int_b(\tilde{G}_E) \subseteq int_b(\widetilde{K}_E)$ if $\tilde{G}_E \subseteq \widetilde{K}_E$,
- (ii) $cl_b(\tilde{G}_E) \subseteq cl_b(\widetilde{K}_E)$ if $\tilde{G}_E \subseteq \widetilde{K}_E$,
- (iii) $cl_b(\tilde{G}_E \cup \widetilde{K}_E) \supseteq cl_b(\tilde{G}_E) \cup cl_b(\widetilde{K}_E)$.
- (iv) $cl_b(\tilde{G}_E \cap \widetilde{K}_E) \subseteq cl_b(\tilde{G}_E) \cap cl_b(\widetilde{K}_E)$.
- (v) $int_b(\tilde{G}_E \cup \widetilde{K}_E) \supseteq int_b(\tilde{G}_E) \cup int_b(\widetilde{K}_E)$.
- (vi) $int_b(\tilde{G}_E \cap \widetilde{K}_E) \subseteq int_b(\tilde{G}_E) \cap int_b(\widetilde{K}_E)$.

Theorem 3.4 [18]. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{F}_E \in NSS(\mathcal{U}_E)$. Then,
- (i) $cl_b(\tilde{F}_E) = \tilde{F}_E \cup [int(cl(\tilde{F}_E)) \cap cl(int(\tilde{F}_E))]$,
- (ii) $int_b(\tilde{F}_E) = \tilde{F}_E \cap [int(cl(\tilde{F}_E)) \cup cl(int(\tilde{F}_E))]$.

4. Neutrosophic Soft Generalized b-Closed Sets

In this section, we present and examine the description of the neutrosophic soft generalized b-closed set in neutrosophic soft topological spaces and its related properties. In addition, we give generalized definitions of the neutrosophic soft regular, pre, semi, and α sets and their relations with the neutrosophic soft generalized b-closed set.
Definition 4.1. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{F}_E \in NSS(\mathcal{U}_E)$. Then
- (i) $\tilde{F}_E$ is called a neutrosophic soft generalized closed set (NS-gCS, in short) if $cl(\tilde{F}_E) \subseteq \tilde{G}_E$ whenever $\tilde{F}_E \subseteq \tilde{G}_E$ and $\tilde{G}_E$ is an NSOS in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$.
- (ii) $\tilde{F}_E$ is called a neutrosophic soft generalized b-closed set (NS-gbCS, in short) if $cl_b(\tilde{F}_E) \subseteq \tilde{G}_E$ whenever $\tilde{F}_E \subseteq \tilde{G}_E$ and $\tilde{G}_E$ is an NSOS in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$.

Theorem 4.1. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{F}_E \in NSS(\mathcal{U}_E)$. Every NS-gCS is an NS-gbCS.

Proof: Let $\tilde{F}_E \subseteq \tilde{G}_E$ and $\tilde{G}_E$ be an NSOS in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$. Then, since $\tilde{F}_E$ is an NS-gCS, $cl(\tilde{F}_E) \subseteq \tilde{G}_E$. Therefore, $cl_b(\tilde{F}_E) \subseteq cl(\tilde{F}_E)$ and $cl(\tilde{F}_E) \subseteq \tilde{G}_E$. Thus, $\tilde{F}_E$ is an NS-gbCS in $\mathcal{U}$.

Remark 4.1. Example 4.1 shows that every NS-gCS is an NS-gbCS but the converse is not always true. Moreover, nor can we say that every non-NS-gbCS must be an NS-gCS.

Example 4.1. Let $\mathcal{U} = \{u_1, u_2\}$, $\mathrm{E} = \{\mathrm{e}_1, \mathrm{e}_2\}$ and $\tau_{\mathrm{NSS}} = \{\emptyset_{\mathrm{E}}, 1_{\mathrm{E}}, (\widetilde{\mathrm{N}}_{\mathrm{E}})_1, (\widetilde{\mathrm{N}}_{\mathrm{E}})_2\}$ where $(\widetilde{\mathrm{N}}_{\mathrm{E}})_1$ and $(\widetilde{\mathrm{N}}_{\mathrm{E}})_2$ are NSSs over $\mathcal{U}$, defined as

$$(\widetilde{\mathrm{N}}_{\mathrm{E}})_1 = \begin{Bmatrix} \mathrm{e}_1 = \{< u_1, 0.8,0.8,0.3 >, < u_2, 0.6,0.5,0.4 >\} \\ \mathrm{e}_2 = \{< u_1, 0.7,0.9,0.2 >, < u_2, 0.7,0.5,0.6 >\} \end{Bmatrix},$$

$$(\widetilde{\mathrm{N}}_{\mathrm{E}})_2 = \begin{Bmatrix} \mathrm{e}_1 = \{< u_1, 0.7,0.7,0.4 >, < u_2, 0.6,0.4,0.5 >\} \\ \mathrm{e}_2 = \{< u_1, 0.5,0.6,0.3 >, < u_2, 0.7,0.3,0.8 >\} \end{Bmatrix}.$$

Then τ_{NSS} defines an NST on $\mathcal{U}$, and therefore $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is an NSTS over $\mathcal{U}$. An NSS $\tilde{\mathrm{F}}_{\mathrm{E}}$ in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is defined as

$$\tilde{F}_E = \begin{Bmatrix} \mathrm{e}_1 = \{< u_1, 0.1,0.1,0.9 >, < u_2, 0.3,0.2,0.7 >\} \\ \mathrm{e}_2 = \{< u_1, 0.1,0.1,0.8 >, < u_2, 0.2,0.4,0.6 >\} \end{Bmatrix}.$$

Then, for the NSOS $(\widetilde{\mathrm{N}}_{\mathrm{E}})_1$, we have $\tilde{F}_E \subseteq (\widetilde{\mathrm{N}}_{\mathrm{E}})_1$. By Theorem 3.4. (i) $cl_b(\tilde{F}_E) = \tilde{F}_E \cup \left[int\left(cl(\tilde{F}_E)\right) \cap cl\left(int(\tilde{F}_E)\right)\right] = \tilde{F}_E \cup [int(1_{\mathrm{E}}) \cap cl(\emptyset_{\mathrm{E}})] = \tilde{F}_E \subseteq (\widetilde{\mathrm{N}}_{\mathrm{E}})_1$. $cl(\tilde{F}_E) = 1_{\mathrm{E}} \nsubseteq (\widetilde{\mathrm{N}}_{\mathrm{E}})_1$ is obtained according to Definition 4.1 (i). So, $\tilde{F}_E$ is an NS-gbCS in $\mathcal{U}$ but not NS-gCS. Intercalarily, an NSS $\widetilde{\mathrm{K}}_{\mathrm{E}}$ in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is defined as

$$\widetilde{\mathrm{K}}_{\mathrm{E}} = \begin{Bmatrix} \mathrm{e}_1 = \{< u_1, 0.7,0.8,0.4 >, < u_2, 0.6,0.5,0.5 >\} \\ \mathrm{e}_2 = \{< u_1, ,0.5,0.7,0.2 >, < u_2, 0.7,0.4,0.7 >\} \end{Bmatrix}.$$

Now, we have $\tilde{K}_E \subseteq (\tilde{N}_{\mathrm{E}})_1$. Because $cl_b(\widetilde{\mathrm{K}}_{\mathrm{E}}) = \tilde{K}_E \cup \left[int\left(cl(\tilde{K}_E)\right) \cap cl\left(int(\tilde{K}_E)\right)\right] = \tilde{K}_E \cup \left[int(1_{\mathrm{E}}) \cap cl\left((\widetilde{\mathrm{N}}_{\mathrm{E}})_2\right)\right] = 1_{\mathrm{E}}$, we have $cl_b(\widetilde{\mathrm{K}}_{\mathrm{E}}) \nsubseteq (\widetilde{\mathrm{N}}_{\mathrm{E}})_1$. Hence, $\tilde{K}_E$ is not an NS-gbCS in $\mathcal{U}$. Then since $cl(\tilde{K}_E) = 1_{\mathrm{E}}$, we have $cl(\tilde{K}_E) \nsubseteq (\widetilde{\mathrm{N}}_{\mathrm{E}})_1$. Thus, $\tilde{K}_E$ is not an NS-gCS in $\mathcal{U}$.

Definition 4.2. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{F}_E \in NSS(\mathcal{U}_E)$. Let $\tilde{G}_E$ be an NSOS in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$. Then

(i) $\tilde{F}_E$ is called a neutrosophic soft generalized regular closed set (NS-grCS, in short) if $cl_r(\tilde{F}_E) \subseteq \tilde{G}_E$ whenever $\tilde{F}_E \subseteq \tilde{G}_E$.

(ii) $\tilde{F}_E$ is called a neutrosophic soft generalized pre-closed set (NS-gpCS, in short) if $cl_p(\tilde{F}_E) \subseteq \tilde{G}_E$ whenever $\tilde{F}_E \subseteq \tilde{G}_E$.

(iii) $\tilde{F}_E$ is called a neutrosophic soft generalized semi-closed set (NS-gsCS, in short) if $cl_s(\tilde{F}_E) \subseteq \tilde{G}_E$ whenever $\tilde{F}_E \subseteq \tilde{G}_E$.

(iv) $\tilde{F}_E$ is called a neutrosophic soft α-generalized closed set (NS-αgCS, in short) if $cl_\alpha(\tilde{F}_E) \subseteq \tilde{G}_E$ whenever $\tilde{F}_E \subseteq \tilde{G}_E$.

Theorem 4.2. In a NSTS $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$

(i) Every NS-CS is an NS-gbCS.

(ii) Every NS-rCS is an NS-gbCS.

(iii) Every NS-αCS is an NS-gbCS.
(iv) Every NS-αgCS is an NS-gbCS.
(v) Every NS-pCS is an NS-gbCS.
(vi) Every NS-gpCS is an NS-gbCS.
(vii) Every NS-bCS is an NS-gbCS.
(viii) Every NS-sCS is an NS-gbCS.
(ix) Every NS-gsCS is an NS-gbCS.

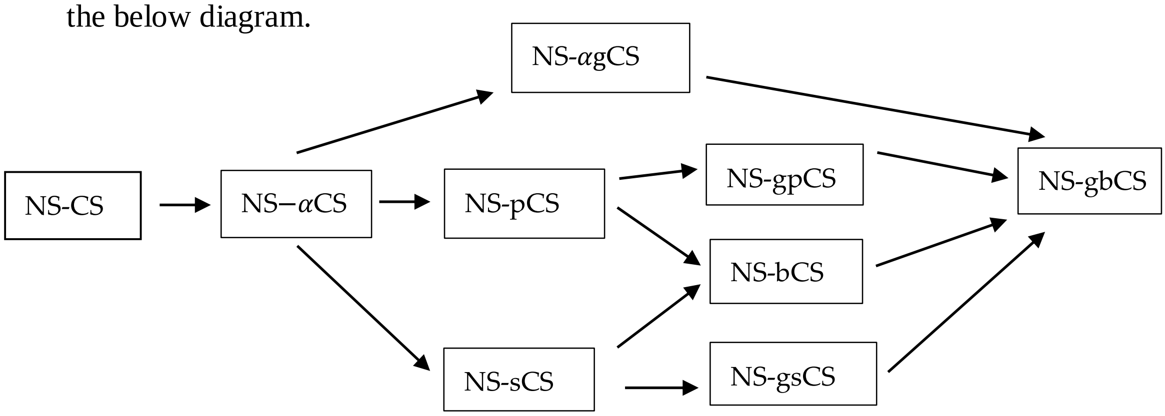
Proof: Let $\tilde{F}_E \subseteq \tilde{G}_E$ and $\tilde{G}_E$ be an NSOS in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$. Then,
(i) since $\tilde{F}_E$ is an NS-CS and $cl_b(\tilde{F}_E) \subseteq cl(\tilde{F}_E)$, $cl_b(\tilde{F}_E) \subseteq cl(\tilde{F}_E) = \tilde{F}_E \subseteq \tilde{G}_E$. Thus, $\tilde{F}_E$ is an NS-gbCS in $\mathcal{U}$.

(ii) since $\tilde{F}_E$ is an NS-rCS, $cl\left(int(\tilde{F}_E)\right) = \tilde{F}_E$ which implies $cl\left(int(\tilde{F}_E)\right) = cl(\tilde{F}_E)$. Therefore, $cl(\tilde{F}_E) = \tilde{F}_E$. Hence, $\tilde{F}_E$ is an NS-CS in $\mathcal{U}$. By Theorem 4.2 (i), $\tilde{F}_E$ is an NS-gbCS in $\mathcal{U}$.
(iii) since $\tilde{F}_E$ is an NS-αCS, $cl_\alpha(\tilde{F}_E) = \tilde{F}_E$. So, $cl_b(\tilde{F}_E) \subseteq cl_\alpha(\tilde{F}_E) = \tilde{F}_E \subseteq \tilde{G}_E$. Thus, $\tilde{F}_E$ is an NS-gbCS in $\mathcal{U}$.
(iv) since $\tilde{F}_E$ is an NS-αgCS, $cl_\alpha(\tilde{F}_E) \subseteq \tilde{G}_E$. Therefore, $cl_b(\tilde{F}_E) \subseteq cl_\alpha(\tilde{F}_E)$, $cl_b(\tilde{F}_E) \subseteq \tilde{G}_E$. Thus, $\tilde{F}_E$ is an NS-gbCS in $\mathcal{U}$.
(v) since $\tilde{F}_E$ is an NS-pCS, by Definition 3.1 (ii) $cl(int(\tilde{F}_E)) \subseteq \tilde{F}_E$ which implies $int\left(cl(\tilde{F}_E)\right) \cap cl\left(int(\tilde{F}_E)\right) \subseteq cl(\tilde{F}_E) \cap cl\left(int(\tilde{F}_E)\right) \subseteq \tilde{F}_E$. Therefore, $cl_b(\tilde{F}_E) \subseteq \tilde{G}_E$. Hence, $\tilde{F}_E$ is an NS-gbCS in $\mathcal{U}$.

(vi) since $\tilde{F}_E$ is an NS-gpCS, $cl_p(\tilde{F}_E) \subseteq \tilde{G}_E$. Thus, $cl_b(\tilde{F}_E) \subseteq cl_p(\tilde{F}_E)$, $cl_b(\tilde{F}_E) \subseteq \tilde{G}_E$. So, $\tilde{F}_E$ is an NS-gbCS in $\mathcal{U}$.
(vii) since $\tilde{F}_E$ is an NS-bCS, by Definition 3.3 (i) $int(cl(\tilde{F}_E)) \cap cl(int(\tilde{F}_E)) \subseteq \tilde{F}_E$. Therefore, $cl_b(\tilde{F}_E) = \tilde{F}_E \cup \left(int(cl(\tilde{F}_E)) \cap cl(int(\tilde{F}_E))\right) \subseteq \tilde{F}_E$. So, $cl_b(\tilde{F}_E) \subseteq \tilde{G}_E$. Thus, $\tilde{F}_E$ is an NS-gbCS in $\mathcal{U}$.
(viii) since $\tilde{F}_E$ is an NS-sCS, Definition 3.1 (iii) $int(cl(\tilde{F}_E)) \subseteq \tilde{F}_E$ which implies $int(cl(\tilde{F}_E)) \cap cl(int(\tilde{F}_E)) \subseteq \tilde{F}_E$. Therefore, $\tilde{F}_E$ is an NS-bCS in $\mathcal{U}$. By Theorem 4.2 (vii), $\tilde{F}_E$ is an NS-gbCS in $\mathcal{U}$.
(ix) since $\tilde{F}_E$ is an NS-gsCS, $cl_s(\tilde{F}_E) \subseteq \tilde{G}_E$. Hence, $cl_b(\tilde{F}_E) \subseteq cl_s(\tilde{F}_E)$, $cl_b(\tilde{F}_E) \subseteq \tilde{G}_E$. Thus, $\tilde{F}_E$ is an NS-gbCS in $\mathcal{U}$.

Remark 4.2. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be a NSTS over $\mathcal{U}$ and $\tilde{F}_E \in NSS(\mathcal{U}_E)$. Then, every neutrosophic soft regular, closed, α, pre, semi, g, αg, gs, gp, and the b-closed set is NS-gbCS.

- ❖ We can also see the relationships between NS-gbCS and NS-CS sets with the help of the below diagram.



Remark 4.3. Example 4.2 and 4.3 show that the inverse of the applications in the diagram above is not always true.

Example 4.2. Let $\mathcal{U} = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2\}$ and $\tau_{NSS} = \{\emptyset_E, 1_E, (\widetilde{N}_E)_1, (\widetilde{N}_E)_2\}$ where $(\widetilde{N}_E)_1$ and $(\widetilde{N}_E)_2$ are NSSs over $\mathcal{U}$, defined as

$$(\widetilde{N}_E)_1 = \begin{Bmatrix} e_1 = \{< u_1, 0.5,0.4,0.4 >, < u_2, 0.3,0.6,0.4 >, < u_3, 0.2,0.4,0.4 >\} \\ e_2 = \{< u_1, 0.7,0.6,0.2 >, < u_2, 0.2,0.3,0.6 >, < u_3, 0.2,0.5,0.2 >\} \end{Bmatrix},$$

$$(\widetilde{N}_E)_2 = \begin{Bmatrix} e_1 = \{< u_1, 0.2,0.1,0.8 >, < u_2, 0.3,0.3,0.5 >, < u_3, 0.2,0.3,0.5 >\} \\ e_2 = \{< u_1, 0.2,0.2,0.9 >, < u_2, 0.1,0.2,0.7 >, < u_3, 0.1,0.5,0.4 >\} \end{Bmatrix}.$$

Then τ_{NSS} defines an NST on $\mathcal{U}$, and therefore $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is an NSTS over $\mathcal{U}$. An NSS $\widetilde{K}_E$ in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is defined as

$$\widetilde{K}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.9,0.8,0.2 >, < u_2, 0.4,0.6,0.3 >, < u_3, 0.7,0.8,0.1 >\} \\ e_2 = \{< u_1, 0.7,0.6,0.1 >, < u_2, 0.5,0.7,0.2 >, < u_3, 0.3,0.6,0.2 >\} \end{Bmatrix}.$$

Then, $\widetilde{K}_E$ is an NS-gbCS in $\mathcal{U}$ but not an NS-bCS in $\mathcal{U}$ since $int(cl(\widetilde{K}_E)) \cap cl\left(int(\widetilde{K}_E)\right) = \left((\widetilde{N}_E)_2\right)^c \nsubseteq \widetilde{K}_E$. Also, by Definition 3.1 $\widetilde{K}_E$ is not an NS-sCS, NS-pCS, NS-αCS, respectively.

Example 4.3. Let $\mathcal{U} = \{u_1, u_2\}$, $E = \{e_1, e_2\}$ and $\tau_{NSS} = \{\emptyset_E, 1_E, (\widetilde{N}_E)_1, (\widetilde{N}_E)_2, (\widetilde{N}_E)_3, (\widetilde{N}_E)_4, (\widetilde{N}_E)_5\}$ where $(\widetilde{N}_E)_1, (\widetilde{N}_E)_2, (\widetilde{N}_E)_3, (\widetilde{N}_E)_4$, and $(\widetilde{N}_E)_5$ are NSSs over $\mathcal{U}$, defined as

$$(\widetilde{N}_E)_1 = \begin{Bmatrix} e_1 = \{< u_1, 0.3,0.2,0.6 >, < u_2, 0.2,0.1,0.4 >\} \\ e_2 = \{< u_1, 0.1,0.2,0.7 >, < u_2, 0.4,0.2,0.6 >\} \end{Bmatrix},$$

$$(\widetilde{N}_E)_2 = \begin{Bmatrix} e_1 = \{< u_1, 0.4,0.2,0.5 >, < u_2, 0.6,0.4,0.3 >\} \\ e_2 = \{< u_1, 0.1,0.3,0.6 >, < u_2, 0.4,0.2,0.5 >\} \end{Bmatrix},$$

$$(\widetilde{N}_E)_3 = \begin{Bmatrix} e_1 = \{< u_1, 0.5,0.6,0.2 >, < u_2, 0.7,0.5,0.3 >\} \\ e_2 = \{< u_1, 0.4,0.5,0.3 >, < u_2, 0.5,0.5,0.5 >\} \end{Bmatrix},$$

$$(\widetilde{N}_E)_4 = \begin{Bmatrix} e_1 = \{< u_1, 0.5,0.8,0.2 >, < u_2, 0.7,0.6,0.3 >\} \\ e_2 = \{< u_1, 0.4,0.7,0.3 >, < u_2, 0.6,0.7,0.2 >\} \end{Bmatrix},$$

$$(\widetilde{N}_E)_5 = \begin{Bmatrix} e_1 = \{< u_1, 0.5,0.3,0.5 >, < u_2, 0.6,0.4,0.3 >\} \\ e_2 = \{< u_1, 0.2,0.4,0.6 >, < u_2, 0.5,0.3,0.5 >\} \end{Bmatrix}.$$

Then τ_{NSS} defines an NST on $\mathcal{U}$, and therefore $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is an NSTS over $\mathcal{U}$. Then, for the NSOS $(\widetilde{N}_E)_1$, we have $(\widetilde{N}_E)_1 \subseteq (\widetilde{N}_E)_1$. By Theorem 3.4. (i) $cl_b\left((\widetilde{N}_E)_1\right) = (\widetilde{N}_E)_1 \cup$

$\left[int\left(cl((\widetilde{N}_E)_1)\right) \cap cl\left(int((\widetilde{N}_E)_1)\right)\right] = (\widetilde{N}_E)_1 \cup \left[int\left({(\widetilde{N}_E)_1}^c\right) \cap cl((\widetilde{N}_E)_1)\right] = (\widetilde{N}_E)_1 \subseteq$ $(\widetilde{N}_E)_1$. Thus, by Definition 4.1 (ii) $(\widetilde{N}_E)_1$ is an NS-gbCS in $\mathcal{U}$. But since $cl\left(int(cl(\widetilde{N}_E)_1)\right) = {(\widetilde{N}_E)_1}^c \nsubseteq (\widetilde{N}_E)_1$, hence $(\widetilde{N}_E)_1$ is not an NS-αCS in $\mathcal{U}$. Intercalarily, by Definition 3.1. and Definition 4.2, $(\widetilde{N}_E)_1$ is not an NS-pCS, NS-rCS, NS-gpCS, NS-αgCS, respectively.

Remark 4.4. Example 4.4 shows that the union and intersection of any two NS-gbCSs need not be NS-gbCS.

Example 4.4. Let $\mathcal{U} = \{u_1, u_2\}$, $E = \{e_1, e_2\}$ and $\tau_{NSS} = \{\emptyset_E, 1_E, (\tilde{F}_E)_1\}$ where $(\tilde{F}_E)_1$ is NSS over $\mathcal{U}$, defined as

$$(\tilde{F}_E)_1 = \begin{Bmatrix} e_1 = \{< u_1, 0.8,0.8,0.1 >, < u_2, 0.6,0.5,0.4 >\} \\ e_2 = \{< u_1, 0.7,0.9,0.2 >, < u_2, 0.4,0.5,0.4 >\} \end{Bmatrix}.$$

Then τ_{NSS} defines an NST on $\mathcal{U}$, and so $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is an NSTS over $\mathcal{U}$. Two NSSs $\widetilde{G}_E$ and $\widetilde{K}_E$ in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ are defined as

$$\widetilde{G}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.1,0.2,0.9 >, < u_2, 0.2,0.3,0.8 >\} \\ e_2 = \{< u_1, 0.1,0.3,0.7 >, < u_2, 0.4,0.3,0.5 >\} \end{Bmatrix},$$

$$\widetilde{K}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.8,0.8,0.1 >, < u_2, 0.6,0.5,0.4 >\} \\ e_2 = \{< u_1, 0.7,0.9,0.2 >, < u_2, 0.3,0.5,0.4 >\} \end{Bmatrix}.$$

By Definition 4.1 (ii), NSSs $\widetilde{G}_E$ and $\widetilde{K}_E$ are NS-gbCSs in $\mathcal{U}$, but since $\widetilde{G}_E \cup \widetilde{K}_E \subseteq (\tilde{F}_E)_1$ and $cl_b(\widetilde{G}_E \cup \widetilde{K}_E) = 1_E \nsubseteq (\tilde{F}_E)_1$, and thus $\widetilde{G}_E \cup \widetilde{K}_E$ is not an NS-gbCS in $\mathcal{U}$.

Now, two NSSs $\widetilde{M}_E$ and $\widetilde{N}_E$ in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ are defined as

$$\widetilde{M}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.9,0.9,0.1 >, < u_2, 0.6,0.7,0.3 >\} \\ e_2 = \{< u_1, 0.7,1.0,0.1 >, < u_2, 0.5,0.5,0.4 >\} \end{Bmatrix},$$

$$\widetilde{N}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.8,0.8,0.1 >, < u_2, 0.7,0.5,0.4 >\} \\ e_2 = \{< u_1, 0.8,0.9,0.2 >, < u_2, 0.4,0.6,0.4 >\} \end{Bmatrix}.$$

By Definition 4.1 (ii), NSSs $\widetilde{M}_E$ and $\widetilde{N}_E$ are NS-gbCSs in $\mathcal{U}$, but since $\widetilde{M}_E \cap \widetilde{N}_E \subseteq (\tilde{F}_E)_1$ and $cl_b(\widetilde{M}_E \cap \widetilde{N}_E) = 1_E \nsubseteq (\tilde{F}_E)_1$, and hence $\widetilde{M}_E \cap \widetilde{N}_E$ is not an NS-gbCS in $\mathcal{U}$.

Theorem 4.3. Let $\tilde{F}_E$ be an NS-gbCS in an NSTS $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$. If $\tilde{F}_E \subseteq \tilde{G}_E \subseteq cl_b(\tilde{F}_E)$, then $\tilde{G}_E$ is also an NS-gbCS in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$.

Proof. Let $\widetilde{H}_E$ be an NSOS in $\mathcal{U}$ such that $\tilde{G}_E \subseteq \widetilde{H}_E$, then $\tilde{F}_E \subseteq \widetilde{H}_E$. Since $\tilde{F}_E$ is an NS-gbCS set in $\mathcal{U}$, it follows $cl_b(\tilde{F}_E) \subseteq \widetilde{H}_E$. Now, $\tilde{G}_E \subseteq cl_b(\tilde{F}_E)$ implies $cl_b(\tilde{G}_E) \subseteq cl_b(cl_b(\tilde{F}_E)) = cl_b(\tilde{F}_E)$. Thus, $cl_b(\tilde{G}_E) \subseteq \widetilde{H}_E$. Hence, $\tilde{G}_E$ is an NS-gbCS in $\mathcal{U}$.

Definition 4.3. An NSS $\tilde{F}_E$ of an NSTS $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is called a neutrosophic soft gb-open set (NS-gbOS, in short) if $(\tilde{F}_E)^c$ is an NS-gbCS in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$.

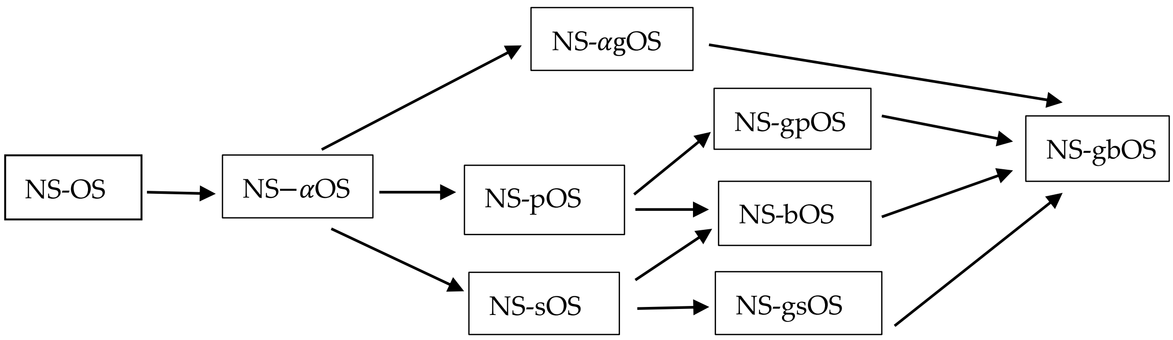
Theorem 4.4. An NSS $\widetilde{N}_E$ of an NSTS $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is an NS-gbOS if and only if $\widetilde{M}_E \subseteq int_b(\widetilde{N}_E)$ whenever $\widetilde{M}_E \subseteq \widetilde{N}_E$ and $\widetilde{M}_E$ is a neutrosophic soft closed set in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$.

Proof. Suppose $\widetilde{N}_E$ is an NS-gbOS in $\mathcal{U}$. Then $(\widetilde{N}_E)^c$ is an NS-gbCS in $\mathcal{U}$. Let $\widetilde{M}_E$ be a neutrosophic soft b-closed set in $\mathcal{U}$ such that $\widetilde{M}_E \subseteq \widetilde{N}_E$. Then $(\widetilde{N}_E)^c \subseteq (\widetilde{M}_E)^c$, $(\widetilde{M}_E)^c$ is a neutrosophic soft b-open set in $\mathcal{U}$. Since $(\widetilde{N}_E)^c$ is an NS-gbCS, $cl_b(\widetilde{N}_E)^c \subseteq (\widetilde{M}_E)^c$, which implies $(int_b(\widetilde{N}_E))^c \subseteq (\widetilde{M}_E)^c$. Thus, $\widetilde{M}_E \subseteq int_b(\widetilde{N}_E)$.

Conversely, assume that $\widetilde{M}_E \subseteq int_b(\widetilde{N}_E)$, whenever $\widetilde{M}_E \subseteq \widetilde{N}_E$ and $\widetilde{M}_E$ be a neutrosophic soft b-closed set in $\mathcal{U}$. Then $(int_b(\widetilde{N}_E))^c \subseteq (\widetilde{M}_E)^c \subseteq \widetilde{H}_E$, where $\widetilde{H}_E$ is a neutrosophic soft b-open set in $\mathcal{U}$. Hence, $cl_b(\widetilde{N}_E)^c \subseteq \widetilde{H}_E$, which implies $(\widetilde{N}_E)^c$ is an NS-gbCS. Therefore, $\widetilde{N}_E$ is an NS-gbOS**.**

Remark 4.5. An NSS $\widetilde{F}_E$ is called NS-gCS, NS-grCS NS-gpCS, NS-gsCS, NS-αgCS if the complement of $(\widetilde{F}_E)^c$ is a neutrosophic soft generalized open set, neutrosophic soft generalized regular open set, neutrosophic soft generalized pre-open set, neutrosophic soft generalized semi-open set and neutrosophic soft α-generalized open set (NS-gOS, NS-grOS, NS-gpOS, NS-gsOS, NS-αgOS, in short resp.), respectively.

- In the diagram, we have shown the relationship between NS-gbOS and NS-OSs.



Remark 4.6. Example 4.5 and 4.6 show that the inverse of the applications in the diagram above is not always true.

Example 4.5. Let $\mathcal{U} = \{u_1, u_2\}$, $E = \{e_1, e_2\}$ and $\tau_{NSS} = \{\emptyset_E, 1_E, (\tilde{F}_E)_1\}$ where $(\tilde{F}_E)_1$ is NSS over $\mathcal{U}$, defined as

$$(\widetilde{F}_E)_1 = \begin{Bmatrix} e_1 = \{< u_1, 0.4,0.5,0.6 >, < u_2, 0.5,0.4,0.7 >\} \\ e_2 = \{< u_1, 0.1,0.2,0.6 >, < u_2, 0.5,0.5,0.6 >\} \end{Bmatrix}.$$

Then τ_{NSS} defines an NST on $\mathcal{U}$, and hence $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is an NSTS over $\mathcal{U}$. An NSS $\widetilde{K}_E$ in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is defined as

$$\widetilde{K}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.3,0.4,0.4 >, < u_2, 0.7,0.4,0.5 >\} \\ e_2 = \{< u_1, 0.3,0.8,0.1 >, < u_2, 0.4,0.4,0.6 >\} \end{Bmatrix}.$$

By Theorem 3.4 (ii) and Theorem 4.4, $\widetilde{G}_E = \emptyset_E \subseteq int_b(\widetilde{K}_E)$ and $\widetilde{G}_E = \emptyset_E \subseteq \widetilde{K}_E$, so $\widetilde{K}_E$ is an NS-gbOS in $\mathcal{U}$, but not NS-bOS in $\mathcal{U}$ since $\widetilde{K}_E \not\subseteq int\left(cl(\widetilde{K}_E)\right) \cup cl\left(int(\widetilde{K}_E)\right) = (\widetilde{F}_E)_1$. Although NSS $\widetilde{K}_E$ is an NS-gbOS in $\mathcal{U}$, it is not NS-αOS, NS-OS, respectively.

Example 4.6. Let $\mathcal{U} = \{u_1, u_2\}$, $E = \{e_1, e_2\}$ and $\tau_{NSS} = \{\emptyset_E, 1_E, (\tilde{F}_E)_1\}$ where $(\tilde{F}_E)_1$ is NSS over $\mathcal{U}$, defined as

$$(\widetilde{F}_E)_1 = \begin{Bmatrix} e_1 = \{< u_1, 0.5,0.5,0.6 >, < u_2, 0.4,0.4,0.7 >\} \\ e_2 = \{< u_1, 0.1,0.3,0.4 >, < u_2, 0.6,0.5,0.6 >\} \end{Bmatrix}.$$

Then τ_{NSS} defines an NST on $\mathcal{U}$, and hence $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is an NSTS over $\mathcal{U}$. An NSS $\tilde{K}_E$ in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is defined as

$$\tilde{K}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.6,0.6,0.3 >, < u_2, 0.8,0.7,0.1 >\} \\ e_2 = \{< u_1, 0.6,0.9,0.1 >, < u_2, 0.7,0.5,0.4 >\} \end{Bmatrix}.$$

By Theorem 4.4 and Definition 4.2, ${(\tilde{F}_E)_1}^c \subseteq \tilde{K}_E$ and ${(\tilde{F}_E)_1}^c \subseteq int_b(\tilde{K}_E) = \tilde{K}_E \cap \left[int\left(cl(\tilde{K}_E)\right) \cup cl\left(int(\tilde{K}_E)\right)\right] = \tilde{K}_E \cap [int(1_E) \cup cl((\tilde{F}_E)_1)] = \tilde{K}_E$, thus $\tilde{K}_E$ is an NS-gbOS in $\mathcal{U}$, but not NS-αgOS in $\mathcal{U}$ since ${(\tilde{F}_E)_1}^c \subseteq \tilde{K}_E$ and ${(\tilde{F}_E)_1}^c \nsubseteq int_\alpha(\tilde{K}_E) = \tilde{K}_E \cap \left[int\left(cl\left(int(\tilde{K}_E)\right)\right)\right] = (\tilde{F}_E)_1$. Further, although NSS $\tilde{K}_E$ is an NS-gbOS in $\mathcal{U}$, it is not NS-sOS in $\mathcal{U}$.

Theorem 4.5. Let $\tilde{M}_E$ be an NS-gbOS in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$. If $int_b(\tilde{M}_E) \subseteq \tilde{N}_E \subseteq \tilde{M}_E$, then $\tilde{N}_E$ is an NS-gbOS in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$.

Proof. Let $\tilde{M}_E$ be NS-gbOS and $\tilde{N}_E$ be any NSS in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ such that $int_b(\tilde{M}_E) \subseteq \tilde{N}_E \subseteq \tilde{M}_E$. Thus, $(\tilde{M}_E)^c$ is an NS-gbCS, and $(\tilde{M}_E)^c \subseteq (\tilde{N}_E)^c \subseteq cl_b(\tilde{M}_E)^c$. So, $(\tilde{M}_E)^c$ is an NS-gbCS. Hence, $\tilde{N}_E$ is an NS-gbOS of $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$.

Remark 4.7. Example 4.7 shows that the union and intersection of any two NS-gbOSs need not be NS-gbOSs.

Example 4.7. Let $\mathcal{U} = \{u_1, u_2\}$, $E = \{e_1, e_2\}$ and $\tau_{NSS} = \{\emptyset_E, 1_E, (\tilde{F}_E)_1\}$ where $(\tilde{F}_E)_1$ is NSS over $\mathcal{U}$, defined as follows

$$(\tilde{F}_E)_1 = \begin{Bmatrix} e_1 = \{< u_1, 0.7,0.8,0.2 >, < u_2, 0.6,0.7,0.5 >\} \\ e_2 = \{< u_1, 0.8,0.6,0.2 >, < u_2, 0.4,0.5,0.4 >\} \end{Bmatrix}.$$

Then τ_{NSS} defines an NST on $\mathcal{U}$, and hence $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is an NSTS over $\mathcal{U}$. Two NSS $\tilde{G}_E$ and $\tilde{K}_E$ in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ are defined as

$$\tilde{G}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.1,0.2,0.8 >, < u_2, 0.4,0.3,0.7 >\} \\ e_2 = \{< u_1, 0.1,0.3,0.8 >, < u_2, 0.4,0.5,0.4 >\} \end{Bmatrix},$$

$$\tilde{K}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.2,0.2,0.7 >, < u_2, 0.5,0.1,0.6 >\} \\ e_2 = \{< u_1, 0.2,0.4,1.0 >, < u_2, 0.3,0.4,0.7 >\} \end{Bmatrix}.$$

It can be easily seen that both $\tilde{G}_E$ and $\tilde{K}_E$ are NS-gbOSs in $\mathcal{U}$, but $\tilde{G}_E \cup \tilde{K}_E$ is not NS-gbOS in $\mathcal{U}$ because ${(\tilde{F}_E)_1}^c \nsubseteq int_b(\tilde{G}_E \cup \tilde{K}_E) = \emptyset_E$.

Now, two NSSs $\tilde{M}_E$ and $\tilde{N}_E$ in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ are defined as

$$\tilde{M}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.8,0.8,0.1 >, < u_2, 0.7,0.7,0.3 >\} \\ e_2 = \{< u_1, 0.9,0.7,0.5 >, < u_2, 0.4,0.6,0.4 >\} \end{Bmatrix},$$

$$\tilde{N}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.2,0.2,0.7 >, < u_2, 0.5,0.3,0.6 >\} \\ e_2 = \{< u_1, 0.2,0.4,0.8 >, < u_2, 0.5,0.5,0.3 >\} \end{Bmatrix}.$$

It can be easily seen that both $\tilde{M}_E$ and $\tilde{N}_E$ are NS-gbOSs in $\mathcal{U}$, but $\tilde{M}_E \cap \tilde{N}_E$ is not NS-gbOS in $\mathcal{U}$ since ${(\tilde{F}_E)_1}^c \nsubseteq int_b(\tilde{M}_E \cap \tilde{N}_E) = \emptyset_E$.

Definition 4.4. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{F}_E \in NSS(\mathcal{U}_E)$. Then,

(i) the neutrosophic soft generalized b- interior of $\tilde{F}_E$, denoted $int_{gb}(\tilde{F}_E)$, is defined as $int_{gb}(\tilde{F}_E) = \cup \{\tilde{G}_E : \tilde{G}_E$ is an NS-gbCS in $\mathcal{U}$ and $\tilde{G}_E \subseteq \tilde{F}_E\}$.

(ii) the neutrosophic soft generalized b-closure of $\tilde{F}_E$, denoted $cl_{gb}(\tilde{F}_E)$, is defined as $cl_{gb}(\tilde{F}_E) = \cap \{\tilde{K}_E : \tilde{K}_E$ is an NS-gbCS in $\mathcal{U}$ and $\tilde{K}_E \supseteq \tilde{F}_E\}$.

Theorem 4.6. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{N}_E \in NSS(\mathcal{U}_E)$. Then,

(i) $\tilde{N}_E$ is an NS-gbCS iff $\tilde{N}_E = cl_{gb}(\tilde{N}_E)$,

(ii) $\tilde{N}_E$ is an NS-gbOS iff $\tilde{N}_E = int_{gb}(\tilde{N}_E)$,

(iii) $int_{gb}(\emptyset_E) = \emptyset_E$, $int_{gb}(1_E) = 1_E$,

(iv) $cl_{gb}(\emptyset_E) = \emptyset_E$, $cl_{gb}(1_E) = 1_E$,

(v) $\left(int_{gb}(\tilde{N}_E)\right)^c = cl_{gb}\left((\tilde{N}_E)^c\right)$,

(vi) $\left(cl_{gb}(\tilde{N}_E)\right)^c = int_{gb}\left((\tilde{N}_E)^c\right)$,

(vii) $\left(int_{gb}(\tilde{N}_E)^c\right)^c = cl_{gb}(N_E)$,

(viii) $\left(cl_{gb}(\tilde{N}_E)^c\right)^c = int_{gb}(\tilde{N}_E)$.

Proof.

(i) Suppose $\tilde{N}_E = cl_{gb}(\tilde{N}_E) = \cap \{\tilde{K}_E : \tilde{K}_E$ is an NS-gbCS in $\mathcal{U}$ and $\tilde{K}_E \supseteq \tilde{N}_E\}$ then $\tilde{N}_E \in \cap \{\tilde{K}_E : \tilde{K}_E$ is an NS-gbCS in $\mathcal{U}$ and $\tilde{K}_E \supseteq \tilde{N}_E\}$ which implies $\tilde{N}_E$ is an NS-gbCS.

Conversely, suppose $\tilde{N}_E$ is an NS-gbCS in $\mathcal{U}$. We take $\tilde{N}_E \subseteq \tilde{N}_E$ and $\tilde{N}_E$ is an NS-gbCS. $\tilde{N}_E \in \cap \{\tilde{K}_E : \tilde{K}_E$ is an NS-gbCS in $\mathcal{U}$ and $\tilde{K}_E \supseteq \tilde{N}_E\}$. $\tilde{N}_E \subseteq \tilde{K}_E$ implies $N_E \subseteq \cap \{\tilde{K}_E : \tilde{K}_E$ is an NS-gbCS in $\mathcal{U}$ and $\tilde{K}_E \supseteq \tilde{N}_E\} = cl_{gb}(\tilde{N}_E)$. This proves (i).

(ii) Proved by taking complement in (i).

(iii) Since the sets 1_E and $\emptyset_E$ are NS-gbOSs, the largest NS-gbOS neutrosophic subset of $\mathcal{U}$ is the set $int_{gb}(1_E)$, and the largest NS-gbOS neutrosophic subset of $\emptyset_E$ is the set $int_{gb}(\emptyset_E)$. Thus, $int_{gb}(\emptyset_E) = \emptyset_E$ and $int_{gb}(1_E) = 1_E$.

(iv) As $cl_{gb}(\emptyset_E)$ is the smallest NS-gbCS on $\mathcal{U}$ containing $\tilde{\emptyset}_E$ and $cl_{gb}(1_E)$ is the smallest NS-gbCS on $\mathcal{U}$ containing 1_E, we have $cl_{gb}(\emptyset_E) = \emptyset_E$, $cl_{gb}(1_E) = 1_E$.

(v) Since $int_{gb}(\tilde{N}_E) = \cup \{\tilde{M}_E : \tilde{M}_E$ is an NS-gbOS in $\mathcal{U}$ and $\tilde{M}_E \subseteq \tilde{N}_E\}$ then $\left(int_{gb}(\tilde{N}_E)\right)^c = \cap \{(\tilde{M}_E)^c : (\tilde{M}_E)^c$ is an NS-gbCS in $\mathcal{U}$ and $(\tilde{M}_E)^c \supseteq (\tilde{N}_E)^c\}$. Replacing $(\tilde{M}_E)^c$ by $\tilde{K}_E$, we get $\left(int_{gb}(\tilde{N}_E)\right)^c = \cap \{\tilde{K}_E : \tilde{K}_E$ is an NS-gbCS in $\mathcal{U}$ and $\tilde{K}_E \supseteq (\tilde{N}_E)^c\}$.

Therefore, $\left(int_{gb}(\tilde{N}_E)\right)^c = cl_{gb}\left((\tilde{N}_E)^c\right)$. This proves (v).

(vi) Proof is similar to the above part (v).

(vii) Since $int_{gb}(\widetilde{N}_E) = \cup\{\widetilde{M}_E : \widetilde{M}_E$ is an NS-gbOS in $\mathcal{U}$ and $\widetilde{M}_E \subseteq \widetilde{N}_E\}$ then $\left(int_{gb}(\widetilde{N}_E)\right)^c = \cap\{(\widetilde{M}_E)^c : (\widetilde{M}_E)^c$ is an NS-gbCS in $\mathcal{U}$ and $(\widetilde{N}_E)^c \subseteq (\widetilde{M}_E)^c\} = cl_{gb}((\widetilde{N}_E)^c)$. Then replacing $\widetilde{N}_E$ by $(\widetilde{N}_E)^c$, we get $\left(int_{gb}(\widetilde{N}_E)^c\right)^c = \cap\{(\widetilde{M}_E)^c : (\widetilde{M}_E)^c$ is an NS-gbCS in $\mathcal{U}$ and $\left((\widetilde{N}_E)^c\right)^c \subseteq (\widetilde{M}_E)^c\} = cl_{gb}(((\widetilde{N}_E)^c)^c) = cl_{gb}(\widetilde{N}_E)$.

(viii) Proof is similar to the above part (vii).

Example 4.8. Let $\mathcal{U} = \{u_1, u_2\}$, $\mathrm{E} = \{\mathrm{e}_1, \mathrm{e}_2\}$ and $\tau_{\mathrm{NSS}} = \{\emptyset_{\mathrm{E}}, 1_{\mathrm{E}}, (\tilde{\mathrm{F}}_{\mathrm{E}})_1, (\tilde{\mathrm{F}}_{\mathrm{E}})_2\}$ where $(\tilde{\mathrm{F}}_{\mathrm{E}})_1$ and $(\tilde{\mathrm{F}}_{\mathrm{E}})_2$ are NSSs over $\mathcal{U}$, defined as

$$(\tilde{\mathrm{F}}_{\mathrm{E}})_1 = \begin{Bmatrix} \mathrm{e}_1 = \{< u_1, 0.4, 0.6, 0.6 >, < u_2, 0.5, 0.6, 0.4 >\} \\ \mathrm{e}_2 = \{< u_1, 0.2, 0.5, 0.3 >, < u_2, 0.3, 0.6, 0.5 >\} \end{Bmatrix},$$

$$(\tilde{\mathrm{F}}_{\mathrm{E}})_2 = \begin{Bmatrix} \mathrm{e}_1 = \{< u_1, 0.4, 0.5, 0.7 >, < u_2, 0.3, 0.4, 0.5 >\} \\ \mathrm{e}_2 = \{< u_1, 0.1, 0.4, 0.5 >, < u_2, 0.2, 0.4, 0.6 >\} \end{Bmatrix}.$$

Then τ_{NSS} defines an NST on $\mathcal{U}$, and hence $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is an NSTS over $\mathcal{U}$. An NSS $\widetilde{N}_{\mathrm{E}}$ in $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is defined as

$$\widetilde{\mathrm{N}}_{\mathrm{E}} = \begin{Bmatrix} \mathrm{e}_1 = \{< u_1, 0.5, 0.5, 0.7 >, < u_2, 0.6, 0.6, 0.2 >\} \\ \mathrm{e}_2 = \{< u_1, 0.6, 0.4, 0.2 >, < u_2, 0.4, 0.5, 0.4 >\} \end{Bmatrix}.$$

By Definition 4.1 (ii), $\emptyset_{\mathrm{E}}, 1_{\mathrm{E}}, (\tilde{\mathrm{F}}_{\mathrm{E}})_1, (\tilde{\mathrm{F}}_{\mathrm{E}})_2$, and $\widetilde{\mathrm{N}}_{\mathrm{E}}$ are NS-gbCSs in $\mathcal{U}$ and $1_{\mathrm{E}} \supseteq \widetilde{N}_E$, $\widetilde{\mathrm{N}}_{\mathrm{E}} \supseteq \widetilde{N}_E$. So, $cl_{gb}(\widetilde{N}_E) = 1_{\mathrm{E}} \cap \widetilde{\mathrm{N}}_{\mathrm{E}} = \widetilde{\mathrm{N}}_{\mathrm{E}}$. Hence, $cl_{gb}(\widetilde{N}_E) = \widetilde{N}_E$. Similarly, by taking the complement, Theorem 4.6 (ii) also provides.

By Theorem 4.4, $\emptyset_{\mathrm{E}}$, 1_{E}, $(\tilde{\mathrm{F}}_{\mathrm{E}})_1$, $(\tilde{\mathrm{F}}_{\mathrm{E}})_2$, and $(\widetilde{N}_E)^c$ are NS-gbOSs in $\mathcal{U}$ and $\emptyset_{\mathrm{E}}, (\widetilde{N}_E)^c \subseteq (\widetilde{N}_E)^c$. So, $int_{gb}((\widetilde{N}_E)^c) = \emptyset_{\mathrm{E}} \cup (\widetilde{N}_E)^c = (\widetilde{N}_E)^c$. Also, by Theorem 4.6 (i), $\left(cl_{gb}(\widetilde{N}_E)\right)^c = (\widetilde{N}_E)^c$. Hence, $\left(cl_{gb}(\widetilde{N}_E)\right)^c = int_{gb}((\widetilde{N}_E)^c)$ is obtained. Similarly, using the complement, Theorem 4.6 (v) is obtained. In addition, the conditions of Theorem 4.6 (vii) and Theorem 4.6 (viii) are satisfied by replacing $\widetilde{N}_E$ with $(\widetilde{N}_E)^c$.

Definition 4.5. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$. An NSS $\tilde{F}_E$ over $\mathcal{U}$ is said to be a neutrosophic soft gb-neighborhood of the neutrosophic soft point $\mathrm{e}_{\tilde{\mathrm{F}}} \in \tilde{F}_E$ if there exits an NS-gbOS $\tilde{G}_E$ such that $\mathrm{e}_{\tilde{\mathrm{F}}} \in \tilde{G}_E \subseteq \tilde{F}_E$.

All neutrosophic soft gb-neighborhoods of neutrosophic soft point $\mathrm{e}_{\tilde{\mathrm{F}}}$ are called its neutrosophic soft gb-neighborhood system and are denoted by $gb - N_\tau(e_{\tilde{F}})$.

Definition 4.6. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$. An NSS $\tilde{F}_E$ over $\mathcal{U}$ is called neutrosophic soft gb-neighborhood of an NSS $\tilde{G}_E$ if there exists an NS-gbOS $\widetilde{H}_E$ such that $\tilde{G}_E \subseteq \widetilde{H}_E \subseteq \tilde{F}_E$.

Example 4.9. Let $\mathcal{U} = \{u_1, u_2\}$, $\mathrm{E} = \{\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3\}$ and $\tau_{\mathrm{NSS}} = \{\emptyset_{\mathrm{E}}, 1_{\mathrm{E}}, (\widetilde{\mathrm{N}}_{\mathrm{E}})_1, (\widetilde{\mathrm{N}}_{\mathrm{E}})_2, (\widetilde{\mathrm{N}}_{\mathrm{E}})_3\}$ where $(\widetilde{N}_{\mathrm{E}})_1$, $(\widetilde{\mathrm{N}}_{\mathrm{E}})_2$, and $(\widetilde{\mathrm{N}}_{\mathrm{E}})_3$ are NSSs over $\mathcal{U}$, defined as

$$(\tilde{N}_E)_1 = \begin{Bmatrix} e_1 = \{< u_1, 0.8,0.8,0.1 >, < u_2, 0.6,0.5,0.4 >\} \\ e_2 = \{< u_1, 0.7,0.9,0.2 >, < u_2, 0.4,0.5,0.4 >\} \end{Bmatrix},$$
$$(\tilde{N}_E)_2 = \begin{Bmatrix} e_1 = \{< u_1, 0.1,0.1,0.9 >, < u_2, 0.4,0.3,0.7 >\} \\ e_2 = \{< u_1, 0.1,0.3,0.7 >, < u_2, 0.3,0.5,0.6 >\} \end{Bmatrix},$$
$$(\tilde{N}_E)_3 = \begin{Bmatrix} e_1 = \{< u_1, 0.1,0.1,0.9 >, < u_2, 0.3,0.2,0.8 >\} \\ e_2 = \{< u_1, 0.1,0.1,0.8 >, < u_2, 0.2,0.4,0.6 >\} \end{Bmatrix}.$$

Then τ_{NSS} defines an NST on $\mathcal{U}$, and hence $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is an NSTS over $\mathcal{U}$. Since $(\tilde{N}_E)_3 \subseteq (\tilde{N}_E)_2 \subseteq (\tilde{N}_E)_1$, thus $(\tilde{N}_E)_1$ is a neutrosophic soft gb-neighborhood of $(\tilde{N}_E)_3$.

Theorem 4.7. If $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$. $\tilde{N}_E$ is an NS-gbOS. Then $\tilde{N}_E$ is a neutrosophic soft gb-neighborhood of each of its neutrosophic soft points.

Proof. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $\tilde{N}_E$ be an NS-gbOS over $\mathcal{U}$. Suppose that $\tilde{N}_E$ is an NS-gbOS. Then for every neutrosophic soft gb-point $e_{\tilde{F}} \in \tilde{N}_E$, we have $e_{\tilde{F}} \in \tilde{N}_E \subseteq \tilde{N}_E$, and so $\tilde{N}_E$ is a neighborhood of $e_{\tilde{F}}$. Thus, N_E is a neighborhood of each of its neutrosophic soft points.

Next, suppose that $\tilde{N}_E$ is a neutrosophic soft gb-neighborhood of its neutrosophic soft points. If $\tilde{N}_E = \tilde{\emptyset}_E$ then $\tilde{N}_E$ is a neutrosophic soft open as $\tilde{\emptyset}_E \in \tilde{\tau}_{NSS}$. But if $\tilde{N}_E \neq \tilde{\emptyset}$ then for each $e_{\tilde{F}} \in \tilde{N}_E$ there exists a neutrosophic soft point $\tilde{M}_E$ such that $e_{\tilde{F}} \in (\tilde{M}_E)_{e_{\tilde{F}}} \subseteq \tilde{N}_E$. $\tilde{N}_E = \cup (\tilde{M}_E)_{e_{\tilde{F}}}$ and so $\tilde{N}_E$ is an NS-gbOS in $\mathcal{U}$, being of the union of NS-gbOSs in $\mathcal{U}$. Hence, it's proved.

Theorem 4.8. Every neutrosophic soft neighborhood of a neutrosophic soft point $e_{\tilde{F}}$ of an NSTS $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ over $\mathcal{U}$ is a neutrosophic soft gb-neighborhood of $e_{\tilde{F}}$.

Proof. Let $\tilde{F}_E$ be a neutrosophic soft gb-neighborhood of $e_{\tilde{F}} \in \tilde{F}_E$. Then there exists an NSOS $\tilde{G}_E$ such that $e_{\tilde{F}} \in \tilde{G}_E \subseteq \tilde{F}_E$. Since we know that every NSOS set is a neutrosophic soft gb-open set, $\tilde{G}_E$ is an NS-gbOS and hence $\tilde{F}_E$ is a neutrosophic soft gb-neighborhood of $e_{\tilde{F}}$. Thus, it's proved.

Remark 4.8. Example 4.10 shows that the converse of Theorem 4.8 is not always true.

Example 4.10. Let $\mathcal{U} = \{u_1, u_2\}$, $E = \{e_1, e_2\}$ and $\tau_{NSS} = \{\emptyset_E, 1_E, (\tilde{N}_E)_1, (\tilde{N}_E)_2, (\tilde{N}_E)_3, (\tilde{N}_E)_4, (\tilde{N}_E)_5\}$ where $(\tilde{N}_E)_1$, $(\tilde{N}_E)_2$, $(\tilde{N}_E)_3$, $(N_E)_4$, and $(\tilde{N}_E)_5$ are NSSs over $\mathcal{U}$, defined as

$$(\tilde{N}_E)_1 = \begin{Bmatrix} e_1 = \{< u_1, 0.1,0.2,0.7 >, < u_2, 0.1,0.2,0.5 >\} \\ e_2 = \{< u_1, 0.3,0.3,0.6 >, < u_2, 0.4,0.5,0.6 >\} \end{Bmatrix},$$
$$(\tilde{N}_E)_2 = \begin{Bmatrix} e_1 = \{< u_1, 0.7,0.4,0.5 >, < u_2, 0.6,0.5,0.3 >\} \\ e_2 = \{< u_1, 0.5,0.5,0.3 >, < u_2, 0.5,0.5,0.5 >\} \end{Bmatrix},$$
$$(\tilde{N}_E)_3 = \begin{Bmatrix} e_1 = \{< u_1, 0.8,0.5,0.3 >, < u_2, 0.7,0.6,0.3 >\} \\ e_2 = \{< u_1, 0.6,0.6,0.3 >, < u_2, 0.5,0.5,0.5 >\} \end{Bmatrix},$$
$$(\tilde{N}_E)_4 = \begin{Bmatrix} e_1 = \{< u_1, 0.8, 0.9,0.1 >, < u_2, 0.8,0.7,0.3 >\} \\ e_2 = \{< u_1, 0.6,0.6,0.3 >, < u_2, 0.6,0.7,0.2 >\} \end{Bmatrix},$$
$$(\tilde{N}_E)_5 = \begin{Bmatrix} e_1 = \{< u_1, 0.6,0.4,0.5 >, < u_2, 0.6,0.2,0.3 >\} \\ e_2 = \{< u_1, 0.4,0.5,0.6 >, < u_2, 0.5,0.5,0.5 >\} \end{Bmatrix}.$$

Then τ_{NSS} defines an NST on $\mathcal{U}$, and hence $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ is an NSTS over $\mathcal{U}$. Here $e_{2_F} = \{< u_1, 0.7, 0.5, 0.4 >, < u_2, 0.4, 0.5, 0.4 >\}$ is a neutrosophic soft point over $\mathcal{U}$. Obviouslyerse $e_{2_F} \in \tilde{G}_E$, where

$$\tilde{G}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.7, 0.8, 0.2 >, < u_2, 0.3, 0.3, 0.4 >\} \\ e_2 = \{< u_1, 0.6, 0.7, 0.3 >, < u_2, 0.5, 0.5, 0.5 >\} \end{Bmatrix}$$

is a neutrosophic soft gb-neighborhood of e_{2_F}, since $e_{2_F} \in \tilde{K}_E \subseteq \tilde{G}_E$, where

$$\tilde{K}_E = \begin{Bmatrix} e_1 = \{< u_1, 0.7, 0.6, 0.3 >, < u_2, 0.2, 0.3, 0.2 >\} \\ e_2 = \{< u_1, 0.6, 0.7, 0.3 >, < u_2, 0.6, 0.5, 0.4 >\} \end{Bmatrix}$$

is an NS-gbOS. But $\tilde{G}_E$ is not a neutrosophic soft neighborhood of e_{2_F}.

Theorem 4.9. Let $(\mathcal{U}, \tilde{\tau}_{NSS}, E)$ be an NSTS over $\mathcal{U}$ and $e_{\tilde{F}}$ be a neutrosophic soft point. Then, $gb - N_\tau(e_{\tilde{F}})$ has the following properties.

(i) If $\tilde{G}_E \in gb - N_\tau(e_{\tilde{F}})$, then $e_{\tilde{F}} \in \tilde{G}_E$,

(ii) If $\tilde{G}_E \in gb - N_\tau(e_{\tilde{F}})$, $\tilde{G}_E \subseteq \tilde{H}_E$, then $\tilde{H}_E \in gb - N_\tau(e_{\tilde{F}})$,

(iii) If $\tilde{G}_E, \tilde{H}_E \in gb - N_\tau(e_{\tilde{F}})$, then $\tilde{G}_E \cup \tilde{H}_E \in gb - N_\tau(e_{\tilde{F}})$,

(iv) If $\tilde{G}_E \in gb - N_\tau(e_{\tilde{F}})$ then there exists $\tilde{M}_E \in gb - N_\tau(e_{\tilde{F}})$, such that $\tilde{M}_E \subseteq \tilde{G}_E$ and $\tilde{M}_E \in gb - N_\tau(e_{\tilde{G}})$ for every $e_{\tilde{G}} \in \tilde{M}_E$.

Proof. (i) Let $\tilde{G}_E \in gb - N_\tau(e_{\tilde{F}})$ then $\tilde{G}_E$is a neutrosophic soft gb-neighborhood of $e_{\tilde{F}}$ which implies $e_{\tilde{F}} \in \tilde{G}_E$.

(ii) Assume $\tilde{G}_E \in gb - N_\tau(e_{\tilde{F}})$ then $\tilde{G}_E$ is a neutrosophic soft gb-neighborhood of $e_{\tilde{F}}$ which implies there exists an NS-gbOS $\tilde{K}_E$, such that $e_{\tilde{F}} \in \tilde{K}_E \subseteq \tilde{G}_E \subseteq \tilde{H}_E$. Hence, $\tilde{H}_E$ is a neutrosophic soft gb-neighborhood of $e_{\tilde{F}}$ and so, $\tilde{H}_E \in gb - N_\tau(e_{\tilde{F}})$.

(iii) If $\tilde{G}_E, \tilde{H}_E \in gb - N_\tau(e_{\tilde{F}})$, then there exists an NS-gbOSs $\tilde{F}_E$ and $\tilde{K}_E$ such that $e_{\tilde{F}} \in \tilde{F}_E \subseteq \tilde{G}_E$ and $e_{\tilde{F}} \in \tilde{K}_E \subseteq \tilde{H}_E$. Obviously, $\tilde{F}_E$ and $\tilde{K}_E$ are contained in $\tilde{H}_E \cup \tilde{G}_E$ which implies $\tilde{G}_E \cup \tilde{H}_E \in gb - N_\tau(e_{\tilde{F}})$.

(iv) If $\tilde{G}_E \in gb - N_\tau(e_{\tilde{F}})$, then there exists an NS-gbOS $\tilde{M}_E$ such that $e_{\tilde{F}} \in \tilde{M}_E \subseteq \tilde{G}_E$. Since $\tilde{M}_E$ is an NS-gbOS and $e_{\tilde{F}} \in \tilde{M}_E \subseteq \tilde{M}_E$, then $\tilde{M}_E \in N_\tau(e_{\tilde{F}})$. Thus, $\tilde{M}_E \in gb - N_\tau(e_{\tilde{F}})$ and $\tilde{M}_E \subseteq \tilde{G}_E$.Again since $\tilde{M}_E$ is an NS-gbOS, so $\tilde{M}_E$ is a neighborhood of each of its neutrosophic soft points. Therefore, $\tilde{M}_E \in N_\tau(e_{\tilde{F}})$ for all $e_{\tilde{F}} \in \tilde{M}_E$. Hence, its proved (iv).

5. Conclusion

In this study, we have introduced the concepts of neutrosophic soft generalized b-closed (open) sets and neutrosophic soft generalized b-interior, neutrosophic soft generalized b-closure, neutrosophic soft generalized b-neighborhood, and explored some of their properties. We have also proved some theorems about neutrosophic soft generalized b-closed sets in neutrosophic soft topological space and analyzed them with appropriate examples. This study is based on theoretical operations of neutrosophic soft generalized b-sets. These sets may be the starting point for new theoretical and applied studies. Therefore, we believe that many new studies can be done in neutrosophic soft generalized b-sets and neutrosophic topological spaces. In addition, this study can be extended to analyze neutrosophic properties such as continuity, compactness, connectedness, and separation axioms by using neutrosophic soft generalized b-sets and other neutrosophic soft generalized sets.

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