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Extraction of Knowledge from Uncertain Data Employing Weighted Bipolar and Neutrosophic Soft Sets

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Abstract: The discovery of soft sets is accredited to Molodtsov. This theory can cope with difficult circumstances with a lot of ambiguity, like those where deciding is hard. The bipolar soft set (BSS) and neutrosophic soft set (NSS) are algebraic models that can be viewed as soft set expansions. The BSS theory states that we weigh the pros and cons when deciding and NSS theory can handle belief system ambiguity, contradiction, and lack of knowledge due to its truth and falsity membership values. The concept of BSS and NSS are explained in comprehensive detail in this article. This article examined the weighted bipolar soft set (WBSS) and the weighted neutrosophic soft set (WNSS), as well as how to make accurate decisions under uncertain or inadequate information. A detailed comparison of information extraction approaches using weighted bipolar and neutrosophic soft sets may be lacking in the literature. These strategies may have been studied separately, but there may be little research comparing their performance under different settings and with diverse data. Filling this gap with a thorough and rigorous comparison study would help comprehend these techniques' practical benefits and drawbacks.

Keywords: Decision making problem, Soft set, Neutrosophic soft set, Bipolar soft set, Weighted Neutrosophic Soft Set, Weighted bipolar soft set, Uncertain data.

1. Introduction

This research is motivated by the increasing prevalence of indeterminate, imprecise, and uncertain data in our data-driven society. In disciplines varying from healthcare and finance to environmental science and decision support, traditional approaches to data analysis frequently fall short of handling these complexities effectively. Weighted bipolar and neutrosophic soft sets can explicitly model and extract knowledge from dual viewpoint and indeterminate data, meeting a critical need for advanced tools to empower decision-makers with more comprehensive insights and support interdisciplinary research. The goal of this effort is to close the knowledge gap between theoretical developments and real-world applications, which will eventually improve our capacity to make intelligent decisions and gather insightful information from the ambiguous data environments of the modern world.

The novelty of this work is in its detailed comparison of WBSS and WNSS, both of which are employed for extracting information from uncertain data. This work provides new insights into the relative effectiveness of these two frameworks by systematically evaluating their strengths and weaknesses, allowing researchers and practitioners to choose the best methodology based on data uncertainty. As a result of this study, we now have a better knowledge of how these soft computing methods can be used in real life. This will help people make better decisions based on data in many different areas.

This study is necessary due to the ubiquitous prevalence of uncertainty in modern data-driven decision-making processes across multiple domains. Now more than ever, advanced techniques are needed to model and extract knowledge from uncertain data in the era of big data, when information frequently comprises inaccurate, inconsistent, and incomplete parts. Weighted bipolar soft sets and weighted neutrosophic soft sets offer intriguing paths for addressing this difficulty since they enable the explicit consideration of both positive and negative elements of uncertainty and indeterminacy. This study is vital for expanding our capacity to make informed decisions, manage risks, and extract valuable insights in settings where standard data analysis approaches fail to cope with the complexities of uncertain data. This research is crucial for advancing our ability to make informed decisions, manage risks, and extract valuable insights in context.

Through reading this article, we gained an understanding of the fundamental concepts and algorithms behind WBSS and WNSS, such as how these methodologies contribute to the process of decision-making in the face of ambiguous data and an example of this process. This paper demonstrates how we may obtain an accurate ranking order of items by assigning weights to each parameter in the ranking criteria. Within the scope of this research, a comparison study is carried out between WBSS and WNSS.

The area that needs more exploration is how to evaluate and quantify the uncertainty in the knowledge that has been retrieved. In what ways can we effectively express this uncertainty and what level of confidence can we place in the knowledge that is derived utilizing these soft set models. We might research how these soft set models can be modified for streaming or real-time data environments, in which data is continually incoming. What kinds of methods can be used for online learning.

This research is needed to handle today's data-driven world's growing uncertainty and imprecision. Diverse decision-makers struggle to choose acceptable methods to extract insight from such data. To clarify their benefits and applicability in diverse settings, weighted bipolar soft sets and neutrosophic soft sets must be systematically compared. This study helps decision support, risk assessment, and insights production in complicated, uncertain data by guiding uncertainty management.

The efficiency of WBSS and WNSS sets may rely on data features, hence this study might not be applicable to all uncertain data circumstances. The study might not have looked at all uncertainty modeling techniques, so it might not have included other useful methods for comparison. This research can improve decision-making by systematically comparing two prominent uncertainty modeling techniques, helping practitioners and researchers navigate uncertain data landscapes across domains.

The main objective of this work is to investigate knowledge extraction methodologies using WBSS and WNSS, conduct a comprehensive comparative analysis to assess their performance across diverse datasets and scenarios, identify their strengths and weaknesses in handling uncertainty, ambiguity, and imprecision in data, and evaluate their applicability in real-world decision-making and data analysis tasks.

While individual studies have explored these methodologies separately, there is a notable dearth of systematic and rigorous comparative analyses that assess their performance under varying conditions and with diverse datasets. Such a gap hinders a clear understanding of when and where each method excels, potentially limiting their practical utility. Addressing this gap is essential to provide researchers and practitioners with valuable insights into the relative strengths and weaknesses of these techniques, enabling informed choices for knowledge extraction in scenarios characterized by uncertainty, ambiguity, and imprecision in data.

A review of the literature on WBSS and WNSS is presented in section 2 of this article. The core principle, method, and decision-making example employing WBSS are described in Section 3. In

Section 4, we'll learn about the idea behind WNSS and the algorithm it employs to make decisions. The hypothesis for this investigation is presented in Section 5. In Section 6, the comparative research between the WBSS and the WNSS is discussed. The sensitivity analysis for each method is discussed in detail in Section 7. In Section 8, we present the results discussion, and in Section 9, we provide the summary and final thoughts.

2. Literature Review

Uncertainty management is a challenge for researchers and decision makers across all fields and scientific disciplines, from the fundamental to the managerial, social, and technological. To solve this issue, a great number of different initiatives have been started. Even though each method has its own set of advantages and has demonstrated its usefulness, the theory of soft sets, which was developed by Molodtsov generalizes fuzzy set and rough set techniques [1]. This makes it a significant development in this field. Soft sets have been provided with some procedures in [2]. Newly specified operations on soft sets are discussed in [3], along with some algebraic structures were considered related to these operations. Soft rings were introduced by Bera and Mahapatra [4], soft vector spaces by Faried et al. [5], soft graph representations by Ali et al. [6], soft topological spaces by Asaad et al. [7], soft intersection semigroups by Elavarasan et al. [8], soft lattice ordered sets by Kashif et al. [9], and a novel method to soft sets by Cagman and Eraslan [10]. Maji et al. were the ones who first began applying soft sets in the context of decision making [11]. Numerous writers have since added to the body of literature on the topic, such as extensive work regarding the implementations in the decision-making problem was conducted in [12].

Fuzzy soft set concept was discussed by El-Atik et al. [13]. The object parameter methodology was recommended in this article for use in the process of forecasting unseen data in imprecise fuzzy soft sets [14]. Yiarayong put forward the notion of bipolar-valued fuzzy sets [15]. Alqaraleh et al. discussed the bipolar fuzzy soft sets and use this recognition in a decision-making scenario [16]. Different approaches to introducing BSS were proposed by Deli and Karaaslan in 2020 [17], and subsequent work on bipolar soft groups was done by Karaaslan et al. [18]. You can look at these articles to learn more about the bipolarity in soft sets and related subjects, as well as see some examples of its practical applications [19-21].

Philosophically, Smarandache introduced the concept of a neutrosophic set (NS) for the first time [22]. A NS can be defined in terms of its truth-membership degree, indeterminacy-membership degree, or falsity-membership degree. This broadens the applicability of concepts like fuzzy set and interval-valued fuzzy set. The NS and the set theoretic operators need to be described to satisfy the requirements of a scientific or engineering investigation. Otherwise, it will be challenging to implement in the situations that occur. Thus, Smarandache proposed the SVNS concept. The set-theoretic operators and many different features of SVNS have been discussed in [23]. In a SVNS setting, these papers suggested a multi-attribute decision making (MADM) approach based on the correlation coefficient [24, 25]. By utilizing SVNS similarity measures, authors refined and expanded upon previous clustering and decision-making techniques [26, 27]. A novel SVNS similarity measure has been introduced and used to aid in decision-making [28].

TOPSIS technique to solve decision making problems on multi-attribute SVNS was expanded here [29]. To evaluate its subsethood [30], this paper developed a measure that was applied to MADM. Its relations were proposed by Latreche et al. [31], and their properties were explored. For the purposes of cluster analysis and MADM, Luo et al. devised a novel distance measure of SVNSs [32]. SVNS aggregation operators based on t-conorm, and t-norm were proposed by Rong et al. and applied in MADM [33]. Simplified neutrosophic sets and a cross-entropy aggregation algorithm were suggested in [34]. Broumi et al. offer single valued neutrosophic graphs in [35], while suggest bipolar single valued neutrosophic graphs in [36].

SVNS are suggested by [37], which combines the benefits of NS with those of soft sets. Based on SVNS, a few novel operators and a soft matrix have been specified by Broumi et al. [38]. Evaluation of Q-Neutrosophic soft expert set has been defined by Al-Hijjawi et al. [39]. Neutrosophic vague soft expert set theory was described in [40]. Currently, researchers are concentrating on developing and presenting theories for coping with ambiguity [41-42], elaborating those theories with relevant examples. Numerous researchers today are hard at work debating the veracity of Neutrosophy in decision issues, as the TOPSIS method and NSS are commonly used in finding solutions in the decision-making problems [43–44].

3. Weighted Bipolar Soft Set Theory

3.1. Soft Set Theory

Let Ġ represents the initial universe set and X represents the parameters that have been defined. Power set of Ġ is denoted by Ṗ(Ġ). A pair (L, X) is called a soft set over Ġ, where L is a mapping given by [1],

L: X → Ṗ (Ġ).

Here, L(û)(ϑ) = Ø if ϑ ∉ Ġ. As ϑ(û) is approximate function of the soft set (L, X) and the value is a set called ϑ-element of the soft set for all ϑ ∈ Ġ.

3.2. Bipolar Soft Set Theory

3.2.1. Definition

Let R_1 and R_2 are two nonempty subsets of R, as $R_1 \cup R_2 = R$ and $R_1 \cap R_2 = \emptyset$. Then, (Ƴ, Ɲ, R) is BSS over Ġ, where Ƴ and Ɲ are set valued mappings, where Ƴ: $R_1 \longrightarrow$ P(Ġ), Ɲ: $R_2 \longrightarrow$ P(Ġ) and Ƴ(û) ∩ Ɲ(Ƴ(û)) = ∅, where Ƴ: $R_1 \longrightarrow R_2$ is a bijective function [15].

3.2.2. Properties

1) Let ($Ƴ_1$, $Ɲ_1$, R) and ($Ƴ_2$, $Ɲ_2$, Ķ) are two BSS. ($Ƴ_1$, $Ɲ_1$, R) is a bipolar soft subset of ($Ƴ_2$, $Ɲ_2$, Ķ) if, R ⊆ Ķ, along with ∀û ∈ ẓ, $Ƴ_1$(û) ⊆ $Ƴ_2$(û) and $Ɲ_2$(¬û) ⊆ $Ɲ_1$(¬û). We can write it as, ($Ƴ_1$, $Ɲ_1$, R) ⊆ ($Ƴ_2$, $Ɲ_2$, Ķ) [16].
2) ($Ƴ_1$, $Ɲ_1$, R) and ($Ƴ_2$, $Ɲ_2$, Ķ) are said to be equal if and only if ($Ƴ_1$, $Ɲ_1$, R) ⊆ ($Ƴ_2$, $Ɲ_2$, Ķ) and ($Ƴ_2$, $Ɲ_2$, Ķ) ⊆ ($Ƴ_1$, $Ɲ_1$, R). We can write it as, ($Ƴ_1$, $Ɲ_1$, R) = ($Ƴ_2$, $Ɲ_2$, Ķ)
3) Let (Ƴ, Ɲ, R) is a BSS. Then, (Ƴ, Ɲ, R)^c = ($Ƴ^c$, $Ɲ^c$, R) = {⟨ û, $Ƴ^c$(û) = X− Ƴ(û), $Ɲ^c$(û) = Ƴ− Ɲ(û)⟩}.
4) (Ƴ, Ɲ, R) is null, if ∀û ∈ ẓ, Ƴ(û) = ∅ and Ɲ(û) = Ġ. Defined as {⟨∅, Ġ, R⟩}.
5) (Ƴ, Ɲ, R) is absolute, if ∀û ∈ ẓ, Ƴ(û) = Ġ and Ɲ(û) = ∅. Defined as {⟨Ġ, ∅, R⟩}.

3.2.3. Tabular Representation of BSS

Let, Ġ = Universal set = {ϑ_1, ϑ_2, ϑ_3, ϑ_4, ϑ_5}

W = Set of parameters = {$û_1$, $û_2$, $û_3$, $û_4$}

Then, ¬W = {¬$û_1$, ¬$û_2$, ¬$û_3$, ¬$û_4$}

Ƴ($û_1$) = {ϑ_1, ϑ_5} Ɲ(¬$û_1$) = {ϑ_2, ϑ_3, ϑ_4}

Ƴ($û_2$) = {ϑ_2, ϑ_4} Ɲ(¬$û_2$) = {ϑ_1, ϑ_3, ϑ_5}

Ƴ($û_3$) = {ϑ_3, ϑ_4, ϑ_5} Ɲ(¬$û_3$) = {ϑ_1, ϑ_2}

Ƴ($û_4$) = {ϑ_5} Ɲ(¬$û_4$) = {ϑ_1, ϑ_2, ϑ_3, ϑ_4}

Here, BSS (Ƴ, Ɲ, R) represented by this table 1.

Table 1. BSS (Ɏ, Ɲ, Ɍ)

(Ɏ, Ɲ, Ɍ)	û1	û2	û3	û4
ϑ_1	1	-1	-1	-1
ϑ_2	-1	1	-1	-1
ϑ_3	-1	-1	1	-1
ϑ_4	-1	1	1	-1
ϑ_5	1	-1	1	1

Here, Table [1] represents BSS using equation (1). Where, $\xi_{\delta\tau}$ is the δ-th entry of the τ-th column of the table.

$$\xi_{\delta\tau} = \begin{cases} 1 & \text{if } \vartheta_\delta \in Ɏ(\hat{u}_\tau) \\ 0 & \text{if } \vartheta_\delta \in \dot{G} - \{Ɏ(\hat{u}_\tau) \cup Ɲ(\neg\hat{u}_\tau)\} \\ -1 & \text{if } \vartheta_\delta \in Ɲ(\neg\hat{u}_\tau) \end{cases} \quad (1)$$

3.2.4. Algorithm

1) The BSS (Ɏ, Ɲ, W).
2) Enter the parameters that have been chosen. Ɍ ⊆ W.
3) Decision parameter $Ɗ_\delta$ calculated considering all the selected parameters for each row.

$$Ɗ_\delta = \sum_\tau \xi_{\delta\tau} \quad (2)$$

4) Find out φ, where; $Ɗ_\varphi = \max(Ɗ_\delta)$.
5) The best option available is the item denoted by ϑ_φ, if φ might take on more than one value, then the value of φ that is selected can be any one of them.

3.2.5. Example-1

Let's say a new client interested in purchasing a car from a selection of available cars. It's possible that he would choose the car that suits his requirements the best based on a set of criteria. Let, $\dot{G}$= {ϑ_1, ϑ_2, ϑ_3, ϑ_4, ϑ_5, ϑ_6} a set of cars.

W= {$\hat{u}_1$, $\hat{u}_2$, $\hat{u}_3$, $\hat{u}_4$, $\hat{u}_5$, $\hat{u}_6$, $\hat{u}_7$, $\hat{u}_8$} set of parameters.

As, $\hat{u}_1$= automated
$\hat{u}_2$= petrol car
$\hat{u}_3$= cheap
$\hat{u}_4$= comfortable seat
$\hat{u}_5$= air conditioning
$\hat{u}_6$= power windows
$\hat{u}_7$= remote start
$\hat{u}_8$= air bag

¬W= {$\neg\hat{u}_1$, $\neg\hat{u}_2$, $\neg\hat{u}_3$, $\neg\hat{u}_4$, $\neg\hat{u}_5$, $\neg\hat{u}_6$, $\neg\hat{u}_7$, $\neg\hat{u}_8$} = {Not automated, Not a patrol car, Not cheap, No comfortable seat, No air conditioning, No power windows, No remote start, No air bag}.

Let, Ɏ($\hat{u}_1$) = {ϑ_1, ϑ_2, ϑ_3} Ɲ($\neg\hat{u}_1$) = {ϑ_4, ϑ_5}
Ɏ($\hat{u}_2$) = {ϑ_3, ϑ_4, ϑ_5} Ɲ($\neg\hat{u}_2$) = {ϑ_1}
Ɏ($\hat{u}_3$) = {ϑ_1, ϑ_5} Ɲ($\neg\hat{u}_3$) = {ϑ_2, ϑ_3}

Ɏ(û₄) = {ϑ₁, ϑ₃, ϑ₅} Ɲ(¬û₄) = {ϑ₂, ϑ₆}

Ɏ(û₅) = {ϑ₂, ϑ₄, ϑ₅} Ɲ(¬û₅) = {ϑ₃}

Ɏ(û₆) = {ϑ₃, ϑ₅, ϑ₆} Ɲ(¬û₆) = {ϑ₁}

Ɏ(û₇) = {ϑ₂, ϑ₃} Ɲ(¬û₇) = {ϑ₅, ϑ₆}

Ɏ(û₈) = {ϑ₄, ϑ₅, ϑ₆} Ɲ(¬û₈) = {ϑ₃}

1) Data entry for the BSS (Ɏ, Ɲ, W) should follow the table 2.
2) Assume, set of selected parameters by the client; R= {$\hat{u}_1$, $\hat{u}_2$, $\hat{u}_4$, $\hat{u}_5$, $\hat{u}_6$, $\hat{u}_8$}.
3) After determining the parameters to use, we can determine the value of the decision parameter Ɗ and then describe the BSS using those parameters in the manner given in table 3.
4) The value of Ɗ; $Ɗ_5$ = max $Ɗ_\delta$ = 4 and hence φ = 5.
5) According to the criteria that the client had chosen, the ϑ_5 or fifth car is the ideal one to recommend to the customer. If ϑ_5 is not accessible, then the client has the option of selecting either ϑ_3 or ϑ_4 as their replacement. The customer can choose any one of these two cars between the third and fourth car. In the situation that ϑ_3 and ϑ_4 are not available, then the choice will be made between ϑ_2 and ϑ_6.

Table 2. BSS (Ɏ, Ɲ, W)

(Ɏ, Ɲ, W)	û₁	û₂	û₃	û₄	û₅	û₆	û₇	û₈
ϑ₁	1	-1	1	1	0	-1	0	0
ϑ₂	1	0	-1	-1	1	0	1	0
ϑ₃	1	1	-1	1	-1	1	1	-1
ϑ₄	-1	1	0	0	1	0	0	1
ϑ₅	-1	1	1	1	1	1	-1	1
ϑ₆	0	0	0	-1	0	1	-1	1

Table 3. BSS (Ɏ, Ɲ, R)

(Ɏ, Ɲ, W)	û₁	û₂	û₄	û₅	û₆	û₈	Ɗ
ϑ₁	1	-1	1	0	-1	0	0
ϑ₂	1	0	-1	1	0	0	1
ϑ₃	1	1	1	-1	1	-1	2
ϑ₄	-1	1	0	1	0	1	2
ϑ₅	-1	1	1	1	1	1	4
ϑ₆	0	0	-1	0	1	1	1

This table reveals that some objects have the same decision value, making it impossible to rank them based on expert's values given to each parameter. ϑ_5 received the highest decision value, resulting in first position. ϑ_3 and ϑ_4 both had the same decision value of 2, making it impossible to decide which object is best. Similarly, ϑ_2 and ϑ_6 also had the same decision value of 1, making it impossible to determine which object is better. Here the ranking order of object is, $\vartheta_5 > \vartheta_3 = \vartheta_4 > \vartheta_2 = \vartheta_6 > \vartheta_1$.

3.3. Weighted Bipolar Soft Set Theory

3.3.1. Definition

The idea of WBSS is a hybridization of soft sets and weighted parameters of BSS. In the WBSS, certain weightages are assigned to parameters that are required for the decision-making process or that are selected for it. Because some of the features are more significant than others, it is necessary to provide higher priority to those characteristics while giving lower importance to the other criteria. When applied to a decision-making challenge, this strategy yields more precise results. These weights are assigned by the people who make decisions and vary from person to person. As a result, the decision that is made by each decision maker will be unique because not everyone's priorities are the same. For WBSS, the entries are determined by.

$$\Pi_{\delta\tau} = \begin{cases} \xi_{\delta\tau} \times \eta_{\tau} & \text{if } \xi_{\delta\tau} = 1 \\ 0 & \text{if } \xi_{\delta\tau} = 0 \\ \xi_{\delta\tau} \times (1 - \eta_{\tau}) & \text{if } \xi_{\delta\tau} = -1 \end{cases} \quad (3)$$

Where, $\xi_{\delta\tau}$ = entries in BSS (Ƴ, Ɲ, Ʀ).

The formula that is used to determine an object's weighted decision value is as follows:

$$Ɗ_{\delta} = \sum_{\tau} \Pi_{\delta\tau} \quad (4)$$

3.3.2. Algorithm

1) Enter Weighted Bipolar Soft Set (Ƴ, Ɲ, W).
2) Enter the parameters that have been chosen. Ʀ ⊆ W.
3) Based on the selected parameters, construct the WBSS (Ƴ, Ɲ, Ʀ) weighted table.
4) Weighted Decision parameter $Ɗ_{\delta}$, has been calculated considering all the selected parameters for each row.
5) Find out φ, where; $Ɗ_{\varphi} = \max(Ɗ_{\delta})$.
6) The best option available is the item denoted by ϑ_{φ}, if φ might take on more than one value, then the value of φ that is selected can be any one of them.

3.3.3. Flowchart of WBSS

Figure 1 shows the flowchart diagram of Weighted Bipolar Soft Set.

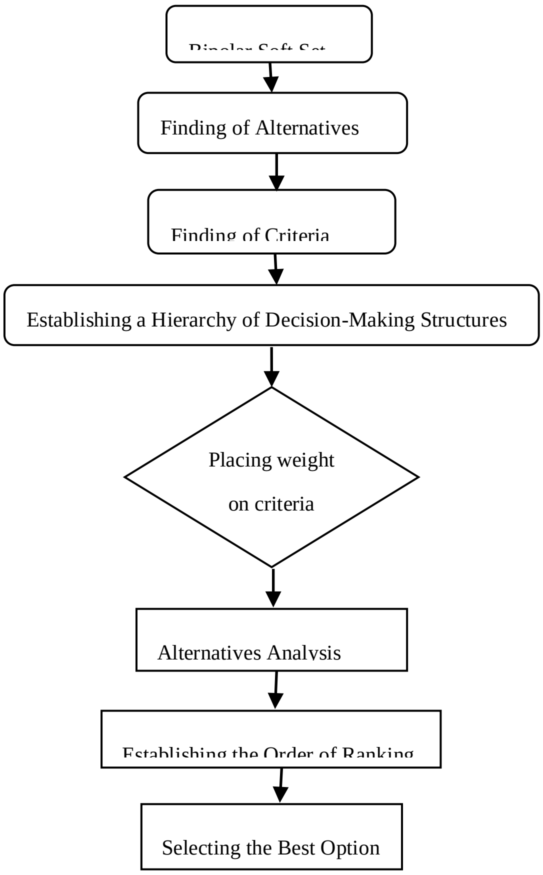


Figure 1. Flowchart of WBSS

3.3.4. Example

Let us assume example 1 to explain this algorithm for WBSS. Now employ this revised strategy to address the initial issue. Start the updated algorithm's third step after giving the parameters weights based on priority.

R = {$\hat{u}_1$, $\hat{u}_2$, $\hat{u}_4$, $\hat{u}_5$, $\hat{u}_6$, $\hat{u}_8$}

Weight of $\hat{u}_1$: $\eta_1 = 0.9$

Weight of $\hat{u}_2$: $\eta_2 = 0.7$

Weight of $\hat{u}_4$: $\eta_4 = 0.8$

Weight of $\hat{u}_5$: $\eta_5 = 0.7$

Weight of $\hat{u}_6$: $\eta_6 = 0.5$

Weight of $\hat{u}_8$: $\eta_8 = 0.9$

Table 4. WBSS (Ƴ, Ɲ, R)

(Ƴ, Ɲ, R)	$\hat{u}_1$	$\hat{u}_2$	$\hat{u}_4$	$\hat{u}_5$	$\hat{u}_6$	$\hat{u}_8$	**Ɗ**
ϑ_1	0.9	0.3	0.8	0	0.5	0	2.5
ϑ_2	0.9	0	0.2	0.7	0	0	1.8
ϑ_3	0.9	0.7	0.8	0.3	0.5	0.1	3.3
ϑ_4	0.1	0.7	0	0.7	0	0.9	2.4
ϑ_5	0.1	0.7	0.8	0.7	0.5	0.9	3.7
ϑ_6	0	0	0.2	0	0.5	0.9	1.6

Table 4 represents WBSS (Y, Ɲ, Ɍ) including weightage of each parameter and calculated the decision parameter $Ɗ_δ$. Max($Ɗ_δ$) = $Ɗ_5$ = 3.7 and hence φ = 5. From the table, ϑ_5 or the fifth car is the greatest possible selection object, that car is the best option for the consumer according to his priorities. In the event, if the fifth vehicle is not accessible, then the third one ϑ_3 will be selected as the alternative. If option 3 is unavailable, the customer will select ϑ_1 followed by ϑ_4. The ranking order of object is, $\vartheta_5 > \vartheta_3 > \vartheta_1 > \vartheta_4 > \vartheta_2 > \vartheta_6$.

After considering these two options side by side, the fifth car is the best one to buy for that client. According to BSS, if there is no fifth car available, the customer has the option of selecting either the third or the fourth car. However, according to WBSS, if the fifth one is not available, the customer should purchase the third one instead. From the WBSS table, we were able to determine the ranking order of items based on the values that experts had assigned to each parameter, and now we can choose which one is the most suitable.

4. Weighted Neutrosophic Soft Set Theory

4.1. Neutrosophic Soft Set Theory

4.1.1. Definition

Neutrosophic soft set (NSS) (L, X) over Ġ is defined by a mapping [23], L: X → P(Ġ);
Here, L = Approximate function of the NSS(L, X).

$(L, X) = \{\hat{u}, \langle \vartheta, T_L(\vartheta), I_L(\vartheta), F_L(\vartheta)\rangle: \vartheta \in \dot{G} \text{ and } \hat{u} \in X\}$

And, Power set of Ġ is denoted by Ṕ(Ġ).

$T_L(\vartheta)$, $I_L(\vartheta)$, $F_L(\vartheta) \in [0, 1]$, are the truth-membership, indeterminacy-membership, and falsity-membership function respectively. Supremum of each T, I, F is 1 so, $0 \le T_L(\vartheta) + I_L(\vartheta) + F_L(\vartheta) \le 3$. A statement or a neutrosophic term describes each of the parameters.

4.1.2. Properties

1) Let (L, X) and (V, N) be two NSS. (L, X) is neutrosophic soft subset of (V, N) if
   (i) $X \subset N$
   (ii) $T_{L(\hat{u})}(\vartheta) \le T_{V(\hat{u})}(\vartheta)$, $I_{L(\hat{u})}(\vartheta) \le I_{V(\hat{u})}(\vartheta)$, $F_{L(\hat{u})}(\vartheta) \ge F_{V(\hat{u})}(\vartheta)$, $\forall \hat{u} \in X$, $\vartheta \in \dot{G}$.
   Symbolized $(L, X) \subset (V, N)$.

   (L, X) is neutrosophic soft super set of (V, N) if (V, N) is neutrosophic soft subset of (L, X). Denoted $(L, X) \supset (V, N)$ [24].
2) Equality of two NSSs can be written as, (L, X) = (V, N). If $(L, X) \subseteq (V, N)$ and $(L, X) \supseteq (V, N)$.
3) Let W = {$\hat{u}_1, \hat{u}_2, \hat{u}_3, \hat{u}_4$} set of parameters. The NOT set of W = ˥W = {$˥\hat{u}_1, ˥\hat{u}_2, \cdots ˥\hat{u}_\pi$}, where $˥\hat{u}_\tau$ = not $\hat{u}_\tau$, ∀τ.
4) Complement of NSS = $(L, X)^c = (L^c, ˥X)$, Where $L^c : ˥X \to P(\dot{G})$, with $T^c_{L(\vartheta)} = F^c_{L(\vartheta)}$, $I^c_{L(\vartheta)} = I^c_{L(\vartheta)}$, $F^c_{L(\vartheta)} = T^c_{L(\vartheta)}$.
5) A neutrosophic soft set (L, X) defined as empty or null, If $T_{L(\hat{u})}(\vartheta) = 0$, $F_{L(\hat{u})}(\vartheta) = 0$ and $I_{L(\hat{u})}(\vartheta) = 0$, $\forall \vartheta \in \dot{G}$, $\forall \hat{u} \in X$ [32].

4.1.3. Comparison Table

It is a table whose rows are objects $\vartheta_1, \vartheta_2, \ldots, \vartheta_\omega$ and columns are parameters $\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_\pi$. The entries $\varrho_{\delta\tau}$ are calculated by, $\varrho_{\delta\tau} = m + q - b$. Where; m = Count of instances where $T_{\vartheta(\delta)}(\hat{u}_\tau)$ is greater than or equivalent to $T_{\vartheta(\varphi)}(\hat{u}_\tau)$, for $\vartheta_\delta \neq \vartheta_\varphi$, $\forall \vartheta_\varphi \in \dot{G}$, q = Count of instances where $I_{\vartheta(\delta)}(\hat{u}_\tau)$ is greater than or equivalent to $I_{\vartheta(\varphi)}(\hat{u}_\tau)$, for $\vartheta_\delta \neq \vartheta_\varphi$, $\forall \vartheta_\varphi \in \dot{G}$, b = Count of instances where $F_{\vartheta(\delta)}(\hat{u}_\tau)$ is greater than or equivalent to $F_{\vartheta(\varphi)}(\hat{u}_\tau)$, for $\vartheta_\delta \neq \vartheta_\varphi$, $\forall \vartheta_\varphi \in \dot{G}$ [45].

Decision value of an Object ϑ_δ, δ = {1, 2, …, ω} is $Ɗ_\delta$, where; $Ɗ_\delta = \sum_\tau \rho_{\delta\tau}$

4.1.4. Algorithm

1) The Neutrosophic Soft Set (L, X) should be entered.
2) Using the NSS (L, X), calculate the comparative matrix.
3) Analyze the value of $Ɗ_\delta$, ∀δ.
4) Calculate φ, where $Ɗ_\varphi = \max(Ɗ_\delta)$.
5) If φ has more than one value, then any one of ϑ_δ could be the preferable choice.

4.1.5. Example-2

Suppose there were five applicants for the teaching position who walked in for an interview. There are certain requirements or characteristics that must be fulfilled for a candidate to be considered for the position of teacher. The person responsible for making the decision or conducting the interview assigned a score to each criterion based on the candidate's performance. The top applicant was selected for the teaching position based on their score from the interview. In order to address the challenge of making decisions regarding NSS, the one above has been taken into consideration.

Let $\dot{G}$ is the universal set of candidates for teacher, $\dot{G} = \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5\}$ and W is the set of parameters, W = $\{\hat{u}_1, \hat{u}_2, \hat{u}_3, \hat{u}_4, \hat{u}_5\}$

Where, $\hat{u}_1$ = Experience
$\hat{u}_2$ = Technical Skill
$\hat{u}_3$ = Behaviour
$\hat{u}_4$ = Communication skill
$\hat{u}_5$ = Punctuality

And, NSS(L, X) = {Experience = {<ϑ_1, 0.9, 0.3, 0.2>, <ϑ_2, 0.7, 0.5, 0.1>, <ϑ_3, 0.4, 0.1, 0.8>, <ϑ_4, 0.7, 0.5, 0.9>, <ϑ_5, 0.5, 0.4, 0.3>}, Technical Skill = {<ϑ_1, 0.8, 0.5, 0.3>, <ϑ_2, 0.5, 0.4, 0.7>, <ϑ_3, 0.9, 0.6, 0.3>, <ϑ_4, 0.4, 0.3, 0.4>, <ϑ_5, 0.6, 0.8, 0.2>}, Behavior = {<ϑ_1, 0.5, 0.7, 0.1>, <ϑ_2, 0.8, 0.6, 0.4>, <ϑ_3, 0.3, 0.1, 0.8>, <ϑ_4, 0.7, 0.5, 0.6>, <ϑ_5, 0.4, 0.3, 0.4>}, Communication skill = {<ϑ_1, 0.7, 0.3, 0.2>, <ϑ_2, 0.6, 0.8, 0.3>, <ϑ_3, 0.8, 0.4, 0.5>, <ϑ_4, 0.5, 0.3, 0.6>, <ϑ_5, 0.7, 0.4, 0.3>}, Punctuality = {<ϑ_1, 0.6, 0.4, 0.2>, <ϑ_2, 0.4, 0.5, 0.3>, <ϑ_3, 0.7, 0.4, 0.1>, <ϑ_4, 0.8, 0.7, 0.2>, <ϑ_5, 0.5, 0.6, 0.4>}}.

The tabular representation of NSS (L, X) is given in Table 5.

Table 5. NSS (L, X)

$\dot{G}$	$\hat{u}_1$	$\hat{u}_2$	$\hat{u}_3$	$\hat{u}_4$	$\hat{u}_5$
ϑ_1	(0.9, 0.3, 0.2)	(0.8, 0.5, 0.3)	(0.5, 0.7, 0.1)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.2)
ϑ_2	(0.7, 0.5, 0.1)	(0.5, 0.4, 0.7)	(0.8, 0.6, 0.4)	(0.6, 0.8, 0.3)	(0.4, 0.5, 0.3)
ϑ_3	(0.4, 0.1, 0.8)	(0.9, 0.6, 0.3)	(0.3, 0.1, 0.8)	(0.8, 0.4, 0.5)	(0.7, 0.4, 0.1)
ϑ_4	(0.7, 0.5, 0.9)	(0.4, 0.3, 0.4)	(0.7, 0.5, 0.6)	(0.5, 0.3, 0.6)	(0.8, 0.7, 0.2)
ϑ_5	(0.5, 0.4, 0.3)	(0.6, 0.8, 0.2)	(0.4, 0.3, 0.4)	(0.7, 0.4, 0.3)	(0.5, 0.6, 0.4)

Table 6 shows the comparative table for the above NSS (L, X), after calculating comparative value and decision value for each object.

Table 6: Comparative table of the NSS (L, X)

Ġ	û₁	û₂	û₃	û₄	û₅	Decision value
ϑ_1	4	3	6	4	1	18
ϑ_2	7	-2	5	3	-1	12
ϑ_3	-3	5	-4	4	4	6
ϑ_4	4	-3	2	-3	6	6
ϑ_5	1	6	0	4	0	11

It is visible from the above table that the first applicant ϑ_1 received the highest decision value or score, which is 18. This is the primary reason why the first applicant is the most qualified individual to be appointed as a teacher. If applicant ϑ_1 is not present, the position will be given to candidate ϑ_2, who received the second highest score in the interview. Similarly, if the second applicant is absent, the fifth option, ϑ_5, will be selected.

According to the NSS table, some objects share the same decision value, hence a ranking based on the values assigned by experts to each attribute is impossible. Due to the limitations of the NSS table, we are unable to determine the ranking order of each object. The ranking order of object is, $\vartheta_1 > \vartheta_2 > \vartheta_5 > \vartheta_3 = \vartheta_4$.

4.2. Weighted Neutrosophic Soft Set Theory

4.2.1. Definition

The idea of WNSS is a hybridization of soft sets and weighted parameters of NSS. If a weight, which is a real positive integer greater than 1, is applied on the parameter of a NSS, then the set is referred to as being WNSS. The entries of WNSS [45];

$$Å_{\eth\tau} = \eta_{\eth\tau} \times \varrho_{\eth\tau} \,;$$

Where, $\eta_{\eth\tau}$ = Weight of each parameter.

$\varrho_{\eth\tau}$ = ðτ-th entry in the table of NSS.

We refer to (L, X^{η}) as the WNSS for the NSS (L, X) with weights η associated with the parameter û.

4.2.3. Algorithm

1) Enter Weighted Neutrosophic Soft Set (L, X^{η}).
2) Using the WNSS (L, X^{η}), calculate the comparative matrix.
3) Decision parameter $Ɗ_{\eth}$, has been calculated considering all the parameters for each row.
4) Find out φ, where; $Ɗ_{\varphi} = \max(Ɗ_{\eth})$.
5) The best option available is the item denoted by $Ɗ_{\varphi}$, if φ might take on more than one value, then the value of ð that is selected can be any one of them.

4.2.4. Flowchart of WNSS

Figure 2 shows the flowchart diagram of Weighted Neutrosophic Soft Set.

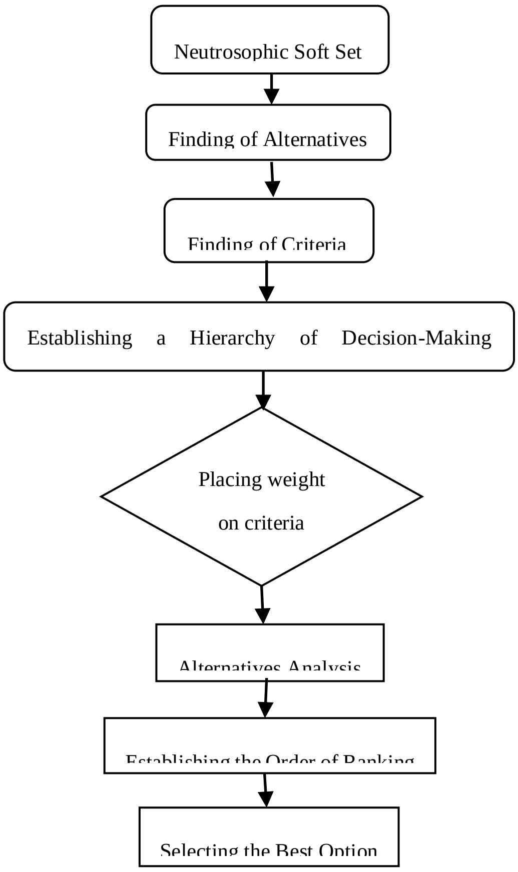


Figure 2. Flowchart of WNSS

4.2.5. Example

Let us consider example 2. Putting the weights on the parameters Experience, Technical Skill, Behavior, Communication skill, Punctuality the WNSS corresponding to the NSS (L, X) denoted by (L, X^{η}) and is given in the following table 7.

According to the decision maker or interviewer, each criterion or parameter was assigned a weight $(\eta)_j$; weight of parameters, for j= {1, 2, 3, 4, 5}.

Where, $(\eta)_1$ = Weight of Experience = 0.7

$(\eta)_2$ = Weight of Technical Skill = 0.9

$(\eta)_3$ = Weight of Behavior = 0.4

$(\eta)_4$ = Weight of Communication skill = 0.6

$(\eta)_5$ = Weight of Punctuality = 0.5

Table 7. WNSS (L, X^{η})

Ġ	**û₁**	**û₂**	**û₃**	**û₄**	**û₅**
ϑ_1	(0.63, 0.21, 0.14)	(0.72, 0.45, 0.27)	(0.20, 0.28, 0.04)	(0.42, 0.18, 0.12)	(0.30, 0.20, 0.10)
ϑ_2	(0.49, 0.35, 0.07)	(0.45, 0.36, 0.63)	(0.32, 0.24, 0.16)	(0.36, 0.48, 0.18)	(0.20, 0.25, 0.15)
ϑ_3	(0.28, 0.07, 0.56)	(0.81, 0.54, 0.27)	(0.12, 0.04, 0.32)	(0.48, 0.24, 0.30)	(0.35, 0.20, 0.05)
ϑ_4	(0.49, 0.35, 0.63)	(0.36, 0.27, 0.36)	(0.28, 0.20, 0.24)	(0.30, 0.18, 0.36)	(0.40, 0.35, 0.10)
ϑ_5	(0.35, 0.28, 0.21)	(0.54, 0.72, 0.18)	(0.16, 0.12, 0.16)	(0.42, 0.24, 0.18)	(0.25, 0.30, 0.20)

Table 8 shows the comparative table for the above WNSS.

Table 8: Comparative table of WNSS (L, X^{η})

Ġ	**$\hat{u}_1$**	**$\hat{u}_2$**	**$\hat{u}_3$**	**$\hat{u}_4$**	**$\hat{u}_5$**	**Decision value**
ϑ_1	4	3	6	4	1	18
ϑ_2	7	-2	5	3	-1	12
ϑ_3	-3	5	-4	4	4	6
ϑ_4	3	-3	2	-3	6	5
ϑ_5	1	6	0	4	0	11

It is clear from the data presented in the chart that the first candidate, ϑ_1, was given the maximum possible score of 18, representing the best decision value. The position will be offered to candidate ϑ_2, who obtained the second highest score in the interview if applicant ϑ_1 is not present for the selection process. In a similar fashion, the fifth choice, which is designated by the letter ϑ_5, will be chosen if the second candidate is not present. In the NSS, we do not know the candidate ϑ_3 and ϑ_4's position or the number that the interviewer gave them. However, with the help of this WNSS, we were able to obtain precise information regarding the applicants ϑ_3 and ϑ_4 and their respective rankings. Therefore, if candidate ϑ_5 is not available, applicant ϑ_3 can be selected, and then ϑ_4 comes next.

Based on the values and weightage supplied to each parameter by the experts, we were able to establish the ranking order of items in the WNSS table and select the best option. The ranking order of object is, $\vartheta 1 > \vartheta 2 > \vartheta 5 > \vartheta 3 > \vartheta 4$.

5. Hypothesis

The incorporation of weighted attributes in bipolar soft sets enhances the accuracy and flexibility of knowledge representation and extraction in uncertain and imprecise data environments, leading to improved decision-making outcomes when compared to traditional bipolar soft sets that do not consider attribute weighting.

The introduction of attribute weighting in neutrosophic soft sets enhances the adaptability and effectiveness of knowledge extraction in contexts characterized by uncertainty and indeterminacy, resulting in superior decision support capabilities compared to traditional neutrosophic soft sets without attribute weighting.

6. Comparison study

6.1. Comparison of WBSS and WNSS

This comparative research presents an overview of the most important aspects, strengths, and problems of WBSS and WNSS. Table 9 shows the comparative analysis between WBSS and WNSS.

Table 9: Comparison analysis between WBSS and WNSS

Aspect	WBSS	WNSS
Definition	In WBSS, each element is associated with both a positive and a negative membership degree, along with a weight that indicates the strength or significance of that element.	In WNSS, each element is characterized by a degree of membership, non-membership, and indeterminacy, along with a weight that signifies the importance of that element.
Membership Interpretation	The positive and negative membership degrees represent the levels of acceptance and rejection of an element with respect to a certain property or concept. The weights provide a measure of the element's influence in the decision-making process.	The membership, non- membership and indeterminacy degrees capture the ambiguity and uncertainty in an element's classification into a particular category. The weight reflects the relative importance of the element's attributes.
Handling Uncertainty	This framework is effective in capturing uncertainty when there are conflicting opinions about an element's affiliation with a particular property. It accounts for both favorable and unfavorable viewpoints.	This framework is suitable for handling uncertainty in a scenario where the information about an element's membership, non-membership, or indeterminacy is incomplete or vague.
Decision-Making	The use of positive and negative membership degrees, along with weights, enables a comprehensive evaluation of elements considering both supportive and opposing characteristics. Elements with higher weights might have a stronger impact on decision outcomes.	The incorporation of weights can allow certain elements to carry more significance in decision-making processes. This prioritization can be based on the relative importance of elements in a specific context.

6.2. Comparison of BSS and WBSS

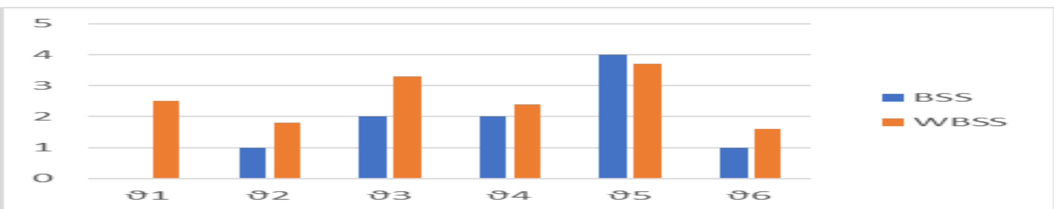


Figure 3. Ranking of objects' orders from our example using BSS and WBSS

Figure 3 shows a graph, that compares the results of BSS and WBSS approach in our example to rank the same set of items and demonstrate how their rankings change. Here, the x-axis represents the objects and the y-axis represents the decision values, with the graph displaying the decision values for each object. As can be seen, ϑ_5 is the superior option for both strategies, earning it number 1 in our rankings. However, ϑ_3 and ϑ_4 are ranked the same as rank 2. If for some reason ϑ_5 is not available, then we will have to settle with either ϑ_3 or ϑ_4 as our alternative. In a similar manner, ϑ_2 and ϑ_6 are placed in the same order inside the ranking. We are unable to conclude which option is preferable. However, with the help of WBSS, we were able to determine that ϑ_3 has higher priority than ϑ_4, and that ϑ_2 has higher priority than ϑ_6 by giving weightage to each parameter.

6.3. Comparison of NSS and WNSS

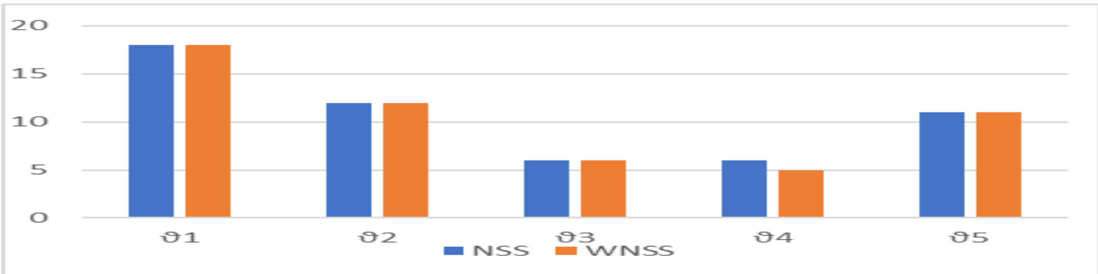


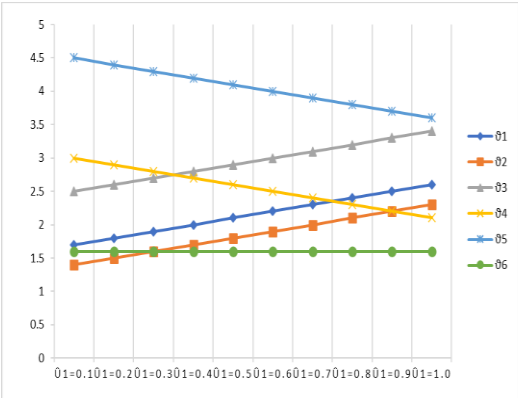
Figure 4. Ranking of objects' orders from our example using NSS and WNSS

Figure 4 shows the differences in ranking order of objects that we got from our example by applying NSS and WNSS approaches. The graph displays the choice values for each object, with the x-axis representing the objects and the y-axis representing the decision values. As we can see that, for both the approaches ϑ_1 is the best choice and got rank 1. Then ϑ_2 got the rank 2 and ϑ_5 got the rank 3. If in any situation ϑ_1 is not available, then we can go for ϑ_2 followed by ϑ_5. But ϑ_3 and ϑ_4 are in same ranking as rank 4. We can't find out which one is best out of ϑ_3 and ϑ_4. By using WNSS, we got that in between ϑ_3 and ϑ_4, ϑ_3 has more priority.

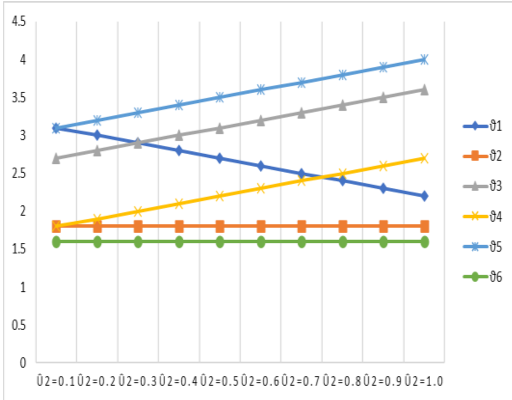
7. Sensitivity Analysis

The results were subjected to sensitivity analysis in order to verify their dependability and validity as well as to look at how they changed when certain inputs and parameters were changed. Approaches to decision-making sometimes involve defining certain criteria in a manner that is open to interpretation and is dependent on the decision-makers' perceptions of the situation as well as the degree to which environmental hazards are present. So, these factors change depending on the situation where the system for making decisions is being modeled. Here, WBSS and WNSS have undergone sensitivity analysis from the standpoint of parameter modifications. The sensitivity analysis that is going to be performed on WBSS is going to assess the effect that a change in parameters $\hat{u}_1$, $\hat{u}_2$, $\hat{u}_4$, $\hat{u}_5$, $\hat{u}_6$, and $\hat{u}_8$ will have on the evaluation of ranking orders of objects. The impact of a modification in parameters $\hat{u}_1$, $\hat{u}_2$, $\hat{u}_3$, $\hat{u}_4$, $\hat{u}_5$ on the assessment of object ranking orders will be examined through sensitivity analysis on WNSS. In this case, each variable is set according to the preferences of the experts. Therefore, multiple experiments were conducted with different values for these factors to demonstrate their significant impact on the final ranking order using WBSS and WNSS approaches.

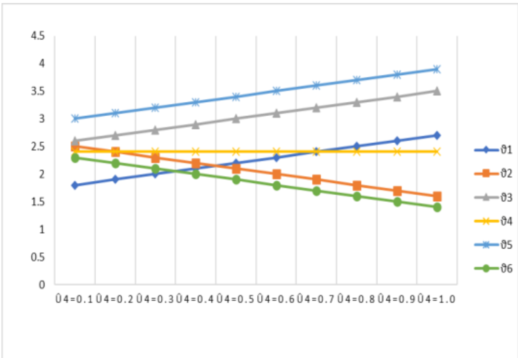
7.1. Sensitivity Analysis of WBSS



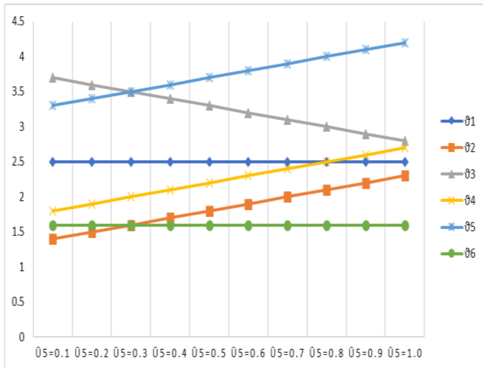
(a)



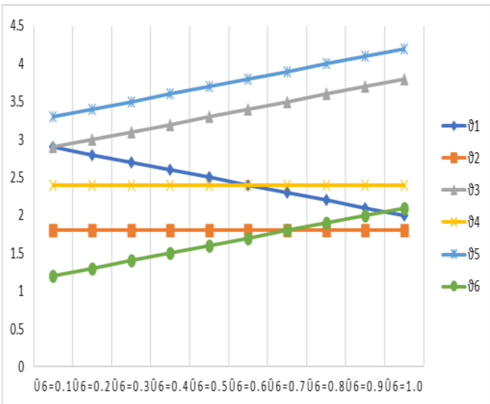
(b)



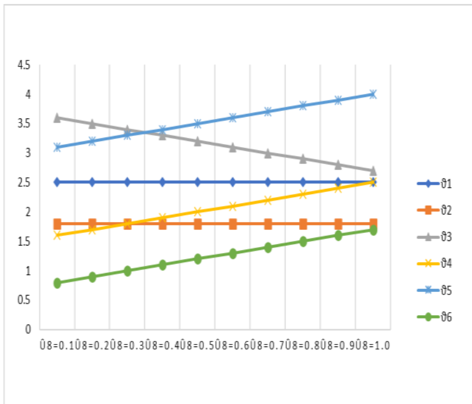
(c)



(d)



(e)



(f)

Figure 5. This figure shows the evaluation of ranking orders of object by changing the values of parameters during WBSS approach. Effect of each parameter on decision making result has been shown here; (a) Influence of parameter $\hat{u}_1$ on ranking order evaluation; (b) Influence of parameter $\hat{u}_2$ on ranking order evaluation; (c) Influence of parameter $\hat{u}_4$ on ranking order evaluation; (d) Influence of parameter $\hat{u}_5$ on ranking order evaluation; (e) Influence of parameter $\hat{u}_6$ on ranking order evaluation; (f) Influence of parameter $\hat{u}_8$ on ranking order evaluation.

By altering the values of parameters in our example, the effect on the evaluation of ranking orders of items through WBSS approach is illustrated in this graph. Specifically, the effect shows how changing these values in WBSS approach affects the ranking of the objects.

As can be seen in Figure 5(a), the significance of the parameter $\hat{u}_1$'s influence on the ranking evaluation was investigated by experimenting with a variety of different values for it, ranging from $\hat{u}_1 = 0.1$ to $\hat{u}_1 = 1.0$. The order of the ranking has not been affected in any way, even though the value of the û1 parameter has been modified multiple times. Throughout the entirety of the sensitivity study and parameter value change û1, option ϑ_5 has remained the most favorable choice, followed by option ϑ_3. ϑ_6, on the other hand, maintains its position as the lowest in the order despite the modification in the value of the parameter $\hat{u}_1$.

In a similar manner, the impact on the ranking orders of objects has been illustrated in figures 5(b), 5(c), 5(d), 5(e) and 5(f) by modifying the values of the parameters $\hat{u}_1$, $\hat{u}_2$, $\hat{u}_4$, $\hat{u}_5$, $\hat{u}_6$ and $\hat{u}_8$ accordingly to highlight the sensitivity analysis of the parameters on the ranking orders. As can be seen, ϑ_5 is the greatest option to select out of all the other possible things to go with, and ϑ_3 comes in second place. By modifying the parameter values of $\hat{u}_5$ and $\hat{u}_8$, we can observe that ϑ_3 is greater than ϑ_5 on occasion, but ϑ_5 is usually greater. In a similar vein, if we examine the least one, then we find that ϑ_6 is the one that has less decision worth in every circumstance. The parameter values used in WBSS's sensitivity study had no influence on the final rankings.

7.2. Sensitivity Analysis of WNSS

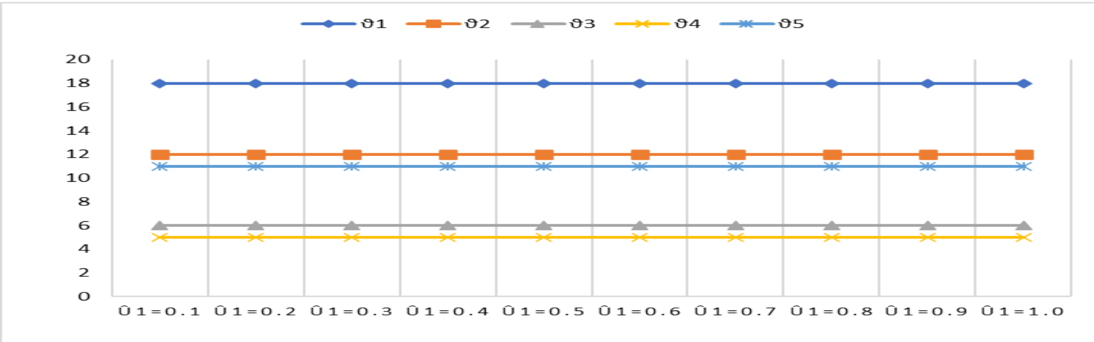


Figure 6. Evaluation of ranking orders of object by changing the values of parameters during WNSS approach.

This graph illustrates the effect that changing the values of the parameters in our example has on the evaluation of ranking orders of items using the WNSS technique. More specifically, the result shows how changing these values in the WNSS method changes how the items are ranked.

The relevance of the parameter $\hat{u}_1$'s impact on the ranking evaluation was studied by testing with a number of various values for it, ranging from $\hat{u}_1 = 0.1$ to $\hat{u}_1 = 1.0$. This was done using Figure 6, which displays the results of the experiment. In spite of the fact that the value of the $\hat{u}_1$ parameter has been altered on several occasions, the sequence in which the rankings are presented has not been altered in any manner at all. Option ϑ_1 has been determined to be the optimal selection throughout the whole of the sensitivity analysis and parameter value change $\hat{u}_1$, with option ϑ_2 coming in a close second. ϑ_4, on the other hand, remains in the position of having the lowest value in the order despite the fact that the value of the parameter $\hat{u}_1$ has been changed.

Changing the values of each parameter in the WNSS model from 0.1 to 1.0, as we did in the previous example, maintains the same order of ranking for the items in the model. The order of the rankings has not been altered in any way as a result of changing the values of any parameters. According to the results of the sensitivity study performed on WNSS, changing the values of the parameters does not affect the ranking orders in any way.

8. Discussion

We observed in BSS database that some items have the same decision value, making it difficult to rank them by expert parameter values. ϑ_5 placed top due to its highest decision value. It is impossible to choose between objects 3 and 4 because they both have a decision value of 2, making it impossible to choose which is the better option. It was also impossible to tell which item is better because ϑ_2 and ϑ_6 both had the same decision value of 1. WBSS recommends buying the third one if the fifth is unavailable. The WBSS table showed the ranking order of things based on experts' parameter values, so we may choose the best one.

The NSS table shows that applicant ϑ_1 had the highest decision value. In the absence of application ϑ_1, the position will be awarded to candidate ϑ_2, who scored second in the interview. The fifth option, ϑ_5, will be chosen if the second candidate is absent. NSS table restrictions prevent us from rating objects. The interviewer's number and position of candidates ϑ_3 and ϑ_4 are unknown in the NSS. Through WNSS, we were able to gather accurate information on candidates ϑ_3 and ϑ_4 and their rankings. Thus, if ϑ_5 is unavailable, ϑ_3 can be chosen, followed by ϑ_4. Based on the experts' parameter values and weightages, we ranked the WNSS table elements and chose the optimal choice.

In WBSS, the combination of positive and negative membership degrees with weights permits a full evaluation of items that takes into account both supportive and opposing qualities. It's possible that factors with higher weights will have a greater bearing on how the decision turns out. Because the WNSS incorporates weights, certain components of the decision-making process may be given the ability to have a greater bearing on the final outcome. This ranking might be done on the basis of the relative value of the components within a particular context.

9. Conclusion

In conclusion, our study has revealed that both weighted bipolar soft sets and weighted neutrosophic soft sets exhibit strengths and applicability in knowledge extraction from uncertain data, with their comparative performance contingent on specific data characteristics and task objectives. While weighted bipolar soft sets excel in scenarios necessitating strict consideration of positive and negative attributes, weighted neutrosophic soft sets offer flexibility to handle inherent data uncertainty. These findings provide valuable insights for decision support, pattern recognition, and data mining applications. Our research contributes to the field of soft computing by illuminating the strengths and weaknesses of these techniques, paving the way for future research on hybrid approaches and domain-specific refinements. Overall, these methodologies serve as versatile tools for navigating the intricacies of uncertain information, offering practitioners informed choices for knowledge extraction in uncertain environments.

In the future, researchers will investigate how deep learning and neural network models can be combined with WBSS and WNSS approaches to improve the ability to retrieve information. In complex datasets, detailed patterns may be easier to discern with the assistance of deep learning. Researchers may use this to tackle scalability problems and use knowledge extraction techniques in situations with massive datasets. They will also be able to develop strategies that efficiently manage enormous amounts of uncertain data. The Human Computer Interaction (HCI) field may be utilized in the future of research to create practitioner-friendly interfaces.

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