Neutrosophic Vision of the Expected Opportunity Loss Criterion (NEOL) Decision Making Under Risk

*1 Maissam Jdid , 2 Florentin Smarandache
1 Faculty of Science, Damascus University, Damascus, Syria
maissam.jdid66@damascusuniversity.edu.sy
2 University of New Mexico ،Mathematics, Physics and Natural Sciences Division
705 Gurley Ave., Gallup, NM 87301, USA
smarand@unm.edu
*Correspondence: jdidmaisam@gmail.com

Abstract:
One of the major challenges facing decision-makers at the present time is obtaining complete information about the issue under study, due to the unstable conditions of the work environment that are beyond the control of decision-makers, which requires them to reach an optimal decision in light of these circumstances and fluctuations and to benefit from the data that is collected. Collected by specialists to determine the appropriate probability distribution corresponding to random cases of nature, here we are faced with the issue of making a decision in the event of risk because the probability distribution is a distribution linked to the data controlled by the conditions of the work environment, which entails a great risk. Decision makers bear the responsibility of choosing the optimal decision that reduces This risk is achieved and the greatest possible profit and the least possible loss are achieved. The issue of decision-making becomes more complex as the number of events increases, and we are in dire need of an ideal study of the issue that takes into account all the circumstances of the work environment. The concept of missed opportunity is very useful in analyzing the decision making under risk, after making the decision and the occurrence of events, the decision makers may regret and wish they had chosen actions different from those they chose at the beginning. To reduce the regret of the decision makers and minimize the expected lost opportunity, researchers in the field of classical operations research presented the criterion of the expected lost opportunity through which the decision can be determined. The ideal with the least percentage of regret. In this research, we present a neutrosophic vision of the expected opportunity loss criterion by taking the data of the issue under study. neutrosophic values are ranges whose lowest limit expresses profit in the worst conditions, and only the highest represents profit in the best conditions.

key words:
Operations research; decision-making theory; decision making under risk; neutrosophic science; neutrosophic decision-making theory; neutrosophic missed opportunity criterion

Introduction:

Decision-making theory depends on the data provided by specialists in collecting data on the issue under study and the type of this data: whether it is confirmed data, uncertain data, or random data repeated according to a certain probability distribution law, through which the methods that must be followed to obtain the optimal decision are determined. The need to know the results of any decision before making it when the data is uncertain or the data is random and the decision maker does not know anything about the state that nature will take or even about the chances of any of it occurring, so the decision maker uses the primary information to obtain additional information that reduces the risk. In classical logic, a group of methods were used that helped the decision maker to make the ideal decision, and since this decision depends on specific classical values that do not take into account the changes that may occur in the work environment, and in light of the changes, fluctuations, and challenges that the decision maker faces in all fields, they increase day by day. Day after day, there is a need for a new study that relies on data that has a margin of freedom and takes into account all circumstances, from the best to the worst. Therefore, many researchers and those interested in studying operations research methods have presented many papers, including [9-1], which is considered a new vision, a neutrosophical vision for this. The methods are based on the concepts and foundations laid down by the founder of this logic, see [10]. Through the indeterminacy of the elements of the profit matrix and used in the decision-making process, an ideal neutrosophic decision is obtained. In this research, we will take the elements of the profit (or loss) matrix and the probabilities are neutrosophic values in the form of ranges, the lowest of which corresponds to the worst states of nature and the highest of which corresponds to the best states of nature, to formulate the expected opportunity loss criterion used to choose the optimal decision under risk. We will apply this criterion to the example that was presented in the research [9], where it was done. Using other criteria to make decisions under risk. By comparing the results provided by each criterion, decision makers can use the appropriate criterion for the issue under study.

Discussion:

The risk lies in the view of decision making when the movement of nature is random and subject to a probability distribution that may not be completely known, but rather a distribution assumed by experts, or by the decision maker himself, which reflects its effects on the decision itself. To reduce the risk, the probabilities are calculated or estimated. Through practical facts or a statistical study taken from previous experiments and studies, classical operations research methods presented a study of decision-making theory in its three cases: the case of confirmed data, uncertain data, and random data, where appropriate criteria were set for each case so that decision makers can make decisions that limit losses. But these decisions are appropriate for work conditions similar to the work conditions in which the data was collected, and any change may cause a large and unexpected loss. Therefore, in previous research [8] we presented a neutrosophical vision of some standards for decision-making in the case of uncertain data, and in another research [9]. We presented some criteria for decision-making under risk, as a complement to what we presented from the neutrosophical study of decision-making under risk. In this research, we present a neutrosophical vision for the expected opportunity loss criterion.

1- The classic general formulation of the decision problem:

The decision maker has alternatives $A(a_1, a_2, \dots, a_m)$where m is the number of alternatives available to the decision maker, and the states that nature can take in the future $\theta(\theta_1, \theta_2, \dots, \theta_n)$ where n is the number of states that nature can take at Its movement, and the amount of profit or loss that the decision maker will achieve is $X(a_i, \theta_j)$, or by short code X_{ij} .Then the profit matrix is given by the following table:

States of nature / Alternatives	θ_1	θ_2	...	θ_n
a_1	X_{11}	X_{12}	...	X_{1n}
a_2	X_{21}	X_{22}	...	X_{2n}
...	...	...	...	...
a_m	X_{m1}	X_{m2}	...	X_{mn}
$P(\theta_j)$	$P(\theta_1)$	$P(\theta_2)$	...	$P(\theta_n)$

Table No. (1) Classic general data for the decision-making issue (profit matrix)

2- The general neutrosophic formulation of the decision-making problem:

The decision maker has the alternatives $A(a_1, a_2, \dots, a_m)$where m is the number of alternatives available to the decision maker, and the states that nature can take in the future are $\theta(\theta_1, \theta_2, \dots, \theta_n)$where n is the number of states that nature can take when they move (they are independent of each other), We symbolize the amount of profit or loss that the decision maker will achieve $NX = X(a_i, \theta_j) \pm \varepsilon_{ij}$, or by short code NX_{ij} These are neutrosophic values, and ε_{ij} ,it is indeterminacy, it can be$\varepsilon_{ij} \in [\lambda_1, \lambda_2]$or $\varepsilon_{ij} \in \{\lambda_1, \lambda_2\}$. $^-0$

Also, the law of probability distribution to which the possible states of nature are subject, we take it as a neutrosophic number series or a neutrosophic mathematical function that corresponds to each state of nature with the probability of its occurrence:

$$NP(\theta_j) = P(\theta_j) + \mu_j \text{ . Where } {}^-0 \le \sum_{j=1}^{n} NP(\theta_j) \le 3^+ \text{ and } 0 \le P(\theta_j) \le 1.$$

μ_j it is indeterminacy that can by $\mu_j \in [\delta_1, \delta_2]$ or $\mu_j \in \{\delta_1, \delta_2\}$. Based on the previous data, the goal is to choose the optimal alternative according to the available states of nature in order to obtain the greatest possible profit or the least possible loss. We organize the previous information in the following table:

States of nature / Alternatives	θ_1	θ_2	…	θ_n
a_1	$X_{11} \pm \varepsilon_{11}$	$X_{12} \pm \varepsilon_{12}$	…	$X_{1n} \pm \varepsilon_{1n}$
a_2	$X_{21} \pm \varepsilon_{21}$	$X_{22} \pm \varepsilon_{22}$	…	$X_{2n} \pm \varepsilon_{2n}$
…	…	…	…	…
a_m	$X_{m1} \pm \varepsilon_{m1}$	$X_{m2} \pm \varepsilon_{m2}$	…	$X_{mn} \pm \varepsilon_{mn}$
$P(\theta_j)$	$P(\theta_1)$	$P(\theta_2)$	…	$P(\theta_n)$

Table No. (2) Neutrosophic general data for the decision-making issue (profit matrix)

3- **In a previous study [9], we presented a neutrosophical vision of three criteria used to determine the optimal decision under risk:**

a. Neutrosophic aspiration level criterion.
b. Neutrosophic most likely criterion.
c. Neutrosophic largest expected values criterion.

We chose the appropriate alternative for the following question:

Example 1:

We have the following table of alternatives and states of nature:

States of nature / Alternatives	θ_1	θ_2	θ_3
a_1	$[300,350]$	$[100,150]$	$[400,450]$
a_2	$[-220,-170]$	$[170,220]$	$[500,550]$
a_3	$[-400,-350]$	$[200,250]$	$[300,350]$
a_4	$[160,210]$	$[300,350]$	$[200,250]$
$P(\theta_j)$	$[0.3,0.45]$	$[0.1,0.25]$	$[0.6,0.75]$

Table No. (3) Neutrosophic profit matrix table

It is required to determine the appropriate alternative using:

Neutrosophic aspiration level criterion:

According to the following data, the level of ambition of the decision maker:

The profit belongs to the range $M \in [300,350]$.

The loss belongs to the range $N \in [200,250]$.

The appropriate alternative that achieves the level of ambition of the decision maker in profit and loss is alternative a_1.

Neutrosophic most likely criterion:

From the table we notice that the most likely case is case θ_3, then the issue will lead to a decision in case of confirmation according to the following table:

Most likely case / Alternative	θ_3
a_1	$[400,450]$
a_2	$[500,550]$
a_3	$[300,350]$
a_4	$[200,250]$
$P(\theta_3)$	$[0.6,0.75]$

Table No. (4): Table of the most likely neutrosophic states

We choose the largest value in the condition column θ_3, which is $[500,550]$ corresponding to the alternative a_2, and a_2 is the appropriate alternative.

Largest expected values criterion:

States of nature / Alternatives	θ_1	θ_2	θ_3	$E(a_i)$
a_1	$[300,350]$	$[100,150]$	$[400,450]$	$[340,532.5]$
a_2	$[-220,-170]$	$[170,220]$	$[500,550]$	$[251,391]$
a_3	$[-400,-350]$	$[200,250]$	$[300,350]$	$[80,167.5]$
a_4	$[160,210]$	$[300,350]$	$[200,250]$	$[198,369.5]$
$P(\theta_j)$	$[0.3,0.45]$	$[0.1,0.25]$	$[0.6,0.75]$	

Table No. (5): Table of expected neutrosophic values

By comparing the elements of column $E(a_i)$, we notice that the largest expected values are $[340,532.5]$ corresponding to alternative a_1. Alternative a_1 is the appropriate alternative according to this criterion.

4- **In this research, we present a neutrosophical vision of another of the criteria used to choose the appropriate alternative decision making under risk:**

Expected opportunity loss criterion (EOL):

Based on the information contained in references [11-13], we find that the minimum expected lost opportunity criterion depends on choosing the decision that guarantees us the least regret, i.e., the decision with the lost opportunity, and it is calculated according to the following steps:

From the profit matrix:

we choose the largest profit value corresponding to each state of nature, θ_j, and let M_j be:

$$M_j = \underbrace{Max}_{i} X_{ij}$$

We form the Regret Matrix from the following relation:

$$X'_{ij} = M_j - X_{ij}$$

We obtain the following regret matrix:

States of nature / Alternatives	θ_1	θ_2	…	θ_n
a_1	X'_{11}	X'_{12}	…	X'_{1n}
a_2	X'_{21}	X'_{22}	…	X'_{2n}
…	…	…	…	…
a_m	X'_{m1}	X'_{m2}	…	X'_{mn}
$P(\theta_j)$	$P(\theta_1)$	$P(\theta_2)$	…	$P(\theta_n)$

Table No. (6) Classic regret matrix

c. We calculate the expected value corresponding to each alternative:

$$E(a_i) = \sum_{j=1}^{n} P(\theta_j) \, . X'_{ij} \quad ; i = 1,2, \dots, m$$

We symbolize the appropriate alternative, through which we will determine the optimal decision, with the symbol E_K, and it is calculated from the following relation:

$$E_K = \underbrace{Min}_{i} [E(a_i)] \quad ; i = 1,2, \dots, m$$

Neutrosophic Vision of the Expected Opportunity Loss Criterion (NEOL):

Using the data in the general neutrosophic formulation of the risk decision problem we find: we choose the largest profit value corresponding to each state of nature, θ_j, and let NM_j be:

$$NM_j = \underbrace{Max}_{i} NX_{ij}$$

We form the Regret Matrix from the following relationship:

$$NX'_{ij} = NM_j - NX_{ij}$$

We obtain the following neutrosophic regret matrix:

States of nature / Alternatives	θ_1	θ_2	…	θ_n
a_1	NX'_{11}	NX'_{12}	…	NX'_{1n}
a_2	NX'_{21}	NX'_{22}	…	NX'_{2n}
…	…	…	…	…
a_m	NX'_{m1}	NX'_{m2}	…	NX'_{mn}
$NP(\theta_j)$	$NP(\theta_1)$	$NP(\theta_2)$	…	$NP(\theta_n)$

Table No. (7) Neutrosophic Regret Matrix

We calculate the expected neutrosophic value corresponding to each alternative in the regret matrix:

$$NE(a_i) = \sum_{j=1}^{n} NP(\theta_j) \, . NX'_{ij} \quad ; i = 1,2, \dots , m$$

We symbolize the optimal expected neutrosophic minimum value, through which we will determine the appropriate alternative, with the symbol NE_K, and it is calculated from the following relation:

$$NE_K = \underbrace{Min}_{i} [NE(a_i)] \quad ; i = 1,2, \dots , m$$

Example 2:

We apply the neutrosophic expected value criterion to the following data in Example No. (1):

States of nature / Alternatives	θ_1	θ_2	θ_3
a_1	$[300,350]$	$[100,150]$	$[400,450]$
a_2	$[-220,-170]$	$[170,220]$	$[500,550]$
a_3	$[-400,-350]$	$[200,250]$	$[300,350]$
a_4	$[160,210]$	$[300,350]$	$[200,250]$
$P(\theta_j)$	$[0.3,0.45]$	$[0.1,0.25]$	$[0.6,0.75]$

Table No. (8) Neutrosophic profit matrix table for the problem

What is required is to determine the appropriate alternative using the expected lost opportunity criterion:

We form the neutrosophic regret matrix using the following relation:

$$NX'_{ij} = NM_j - NX_{ij}$$

We obtain the following matrix:

States of nature / Alternatives	θ_1	θ_2	θ_3	$E(a_i)$
a_1	0	200	100	$[80,125]$
a_2	520	130	0	$[169,266.5]$
a_3	700	100	200	$[340,490]$
a_4	140	0	300	$[222,288]$
$P(\theta_j)$	$[0.3,0.45]$	$[0.1,0.25]$	$[0.6,0.75]$	

Table No. (9) Regret matrix table and expected value of the neutrosophic minimum

In our example, we find that the alternative a_1 corresponding to the minimum expected value of neutrosophic $[80,125]$is the appropriate alternative that achieves the lowest value of regret and achieves a profit whose expected value is $[340,532.5]$.

Conclusion and results:

Through the previous study, we presented a neutrosophic vision of one of the important criteria used for decision-making under risk, which can be used in many life issues to reduce the regret resulting from making a decision on a specific issue by using neutrosophic value data, and to help decision makers in choosing an example decision in the case of risk, we present the following comparison is between the results of some of the criteria used to make a decision that suits all circumstances in the event of risk. We leave them to choose the criterion that can be relied upon and is appropriate for the issue under study through the following table.

Alternative and profit / criterion	Alternative	profit
Neutrosophic aspiration level criterion	a_1	[400,450]
Neutrosophic most likely criterion	a_2	[500,550]
Neutrosophic largest expected values criterion.	a_1	[340,532.5]
Neutrosophic expected opportunity loss criterion	a_1	[340,532.5]

Table No. (10): Comparison table

We note that alternative a_1 is the appropriate alternative according to three criteria. The criterion of the most likely neutrosophic state determines alternative a_2, best alternative.

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