



## The Problem of Planning Neutrosophic Hydropower Systems (Converting Some Nonlinear Neutrosophic Models into Linear Neutrosophic Models)

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### Abstract:

The core of operations research activity focuses on creating and using models. These models may be linear models, non-linear models, dynamic models, and others. Linear models are considered one of the most important and most widely used operations research models due to the availability of appropriate algorithms through which we can obtain the optimal solution, which prompts us to benefit from the nature of the topic under study and the information available to us about the variables in it to transform it into linear models. In classical logic, many nonlinear programming problems have been processed and transformed into linear programming problems. In this research, we present a study of the issue of planning hydroelectric systems, where the general policy for operating this system specifies two prices for selling the produced electricity, which makes this issue a non-linear programming issue. We will turn it into a linear programming problem using linear programming concepts, and then we will use Boolean concepts. Neutrosophic studies of linear programming and non-linear programming are presented to provide a neutrosophic formulation of the issue of planning hydroelectric systems, through which we obtain a neutrosophic linear model whose optimal solution fits all the conditions that the system's operating environment may experience during the operating period. During the two operating periods.

### key words:

Operations research; linear models; nonlinear models; neutrosophic logic; neutrosophic linear models; neutrosophic nonlinear models; converting neutrosophic nonlinear models into linear neutrosophic models; the issue of planning neutrosophic hydroelectric systems; converting nonlinear neutrosophic models into linear neutrosophic models.

### 1. Introduction:

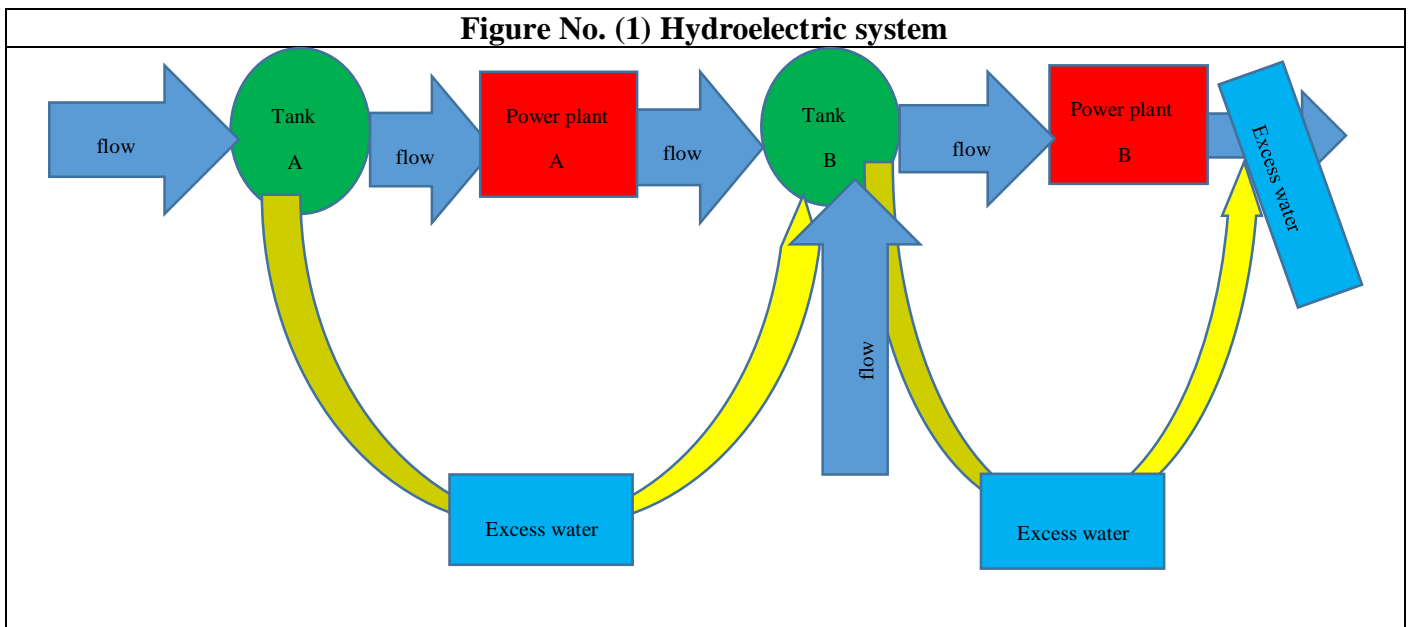
Mathematical programming problems are generally concerned with allocating scarce resources of labor, machinery, and capital and using them in the best possible way, such that costs are reduced to their minimum or profits are maximized, by choosing the optimal solution from a set of possible solutions, and in doing so it depends on transforming the problem under study into a model. Mathematical and appropriate techniques are used for this type of model to reach the optimal solution. Linear programming problems are among the most widely used problems in most fields, and the reason for this is the availability of many appropriate techniques to find the optimal solution. Therefore, we find that students and researchers in the field of operations research always seek to Converting realistic problems into linear models if the data of the problem allow it. In this research, we present a study of one of the important issues, which is the issue of generating electrical power through the flow of water to power plants, where we described a hydroelectric system that operates in two periods and produces an amount of electricity that is sold at two different prices. This depends on the quantity produced, and because of this difference in price for the quantities produced, we find that the mathematical model that we will obtain is a non-linear model, based on the information contained in reference [1]. In this research, we will present the issue using classical logic in a detailed manner through which we explain how to address some Nonlinear programming issues using linear programming concepts, and in

order to obtain accurate results that take into account all conditions and factors that can affect the amount of water used in the process of generating electrical power, we will use the concepts of neutrosophic logic, the logic whose studies and research, whose concepts have been used in most fields, have provided more results. Accuracy of the results that we were obtaining using the concepts of classical logic see [2-12], using previous studies that we presented using the concepts of neutrosophic logic for the topics of linear programming and nonlinear programming [13-24], and the result of this study will be to transform a neutrosophic nonlinear programming problem into A neutrosophic linear programming problem. The studies presented in the papers [17-20] can be used to find the optimal solution.

## 2. Discussion:

**Based on the study mentioned in Reference [1], we present the following example through which we present the classic formulation of the issue of planning hydroelectric energy systems:**

The administration controls the operation of a system consisting of two water tanks, each equipped with two electrical power plants, as shown in the following figure:



This system works by sending a quantity of water to power plants to generate electrical power. The experts provided the following information:

### ✚ Information about the power plants during the two periods:

- 1- A volume of water of 1 Kilo-Acre-Foot (*KAF*), can generate 400 Megawatt-hour (*MWh*) of electricity in power plant *A* and 200 *MWh* in power plant *B*.
- 2- The maximum power that can be generated in power plant *A* is 60000 *MWh*, and in power plant *B* the maximum power that can be generated is 35000 *MWh*.
- 3- An amount of electricity amounting to 50000 *MWh*, can be sold at a price of 20\$ *MWh*, while the excess quantity is sold at a price of 14\$ *MWh*.

### ✚ Information about the tanks during the two periods:

- 1- The maximum capacity of reservoir *A* is 2000 *KAF* and the maximum capacity of reservoir *B* is 1500 *KAF*.
- 2- The level of reservoir *A* at the beginning of the first period is 1900 *KAF* and the level of reservoir *B* is 850 *KAF*.
- 3- The minimum level allowed in tank *A* is 1200 *KAF* and in tank *B* is 800 *KAF*.

- 4- The flow to tank *A* during the first period is 200 *KAF* and to tank *B* is 40 *KAF*.
- 5- The flow to tank *A* during the second period is 130 *KAF* and to tank *B* is 15 *KAF*.

**we should be noted here:**

- Power plant *A* is supplied from tank *A* and power plant *B* from tank *B*.
- When the tanks are completely full, some of the water is drained through the drainage channels so that flooding does not occur.
- For tank *B*, water is supplied from the following sources:
  - The aforementioned outflow.
  - From power plant *A*, the water that is supplied to the power plant after its use goes out to tank *B*.
  - Water drained from tank *A* so that flooding does not occur.

**What is required is to build a mathematical model of the functioning of this system so that it achieves the maximum profit from the process of selling electricity. We know that to formulate a mathematical programming program, we follow the following three steps:**

**First step:** We identify the unknowns in the problem (decision variables) and express them in algebraic symbols.

**Second step:** We define all constraints and express them with equations or inequalities that are mathematical functions of the unknown variables.

**Step Three:** We define the objective function and represent it as a linear function of the unknown variables. It should be made as large or as small as possible.

According to the text of the issue, in order to obtain the required mathematical model, the issue must be studied according to the data for each period and the necessary relationships determined. Here we find that a study must be presented for the first and second steps, specific to each period separately.

**✚ The study for the first period:**

**based on the previous information, we organize the following tables:**

**Information about the power plants of the first period:**

<b>Table No. (1) Information about power plants A and B during the first period</b>		
power plants	power plant A	power plant B
Information		
Water supplied to the power plant during the first period	unknown	unknown
Maximum generating capacity during the first period	60000	35000
The power it can generate for a volume of water of 1 <i>KAF</i>	400	200

**Information about tanks during the first period:**

<b>Table No. (2) Information about reservoirs A and B during the first period</b>		
Tanks	Tank A	Tank B
Information		
Maximum capacity	2000	1500
Minimum permissible level	1200	800
The ratio at the beginning of the first period	1900	850
Excess water drained from the tank to prevent flooding	unknown	unknown
Flow during the first period	200	40

Reservoir level at the end of the first period	unknown	unknown
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**The first step regarding this issue during the first period of work, we find that the unknowns**

- 1- The amount of power produced is divided into two parts: a quantity that is sold for 20\$ *MWh*, and a quantity that is sold for 14\$ *MWh*.
- 2- The amount of water that must be supplied to power plant *A* will be denote  $x_1$ .
- 3- The amount of water supplied to power plant *B* will be denoted  $x_2$ .
- 4- The amount of water that must be drained from tank *A* so that flooding does not occur. We will denote it  $x_3$ .
- 5- The amount of water that must be drained from tank *B* so that flooding does not occur. We will denote it  $x_4$ .
- 6- Tank *A* level at the end of the first period. We will denote it  $x_5$ .
- 7- Tank *B* level at the end of the first period. We will denote it  $x_6$ .

**The second step regarding this issue during the first period of work we find the following restrictions:**

- 1- We assume that the amount of power sold during the first period at a price of 20\$ *MWh*, is  $X_1$  and the amount of power sold at a price of 14\$ *MWh*, is  $X_2$ . The amount of power produced during the first period is:

$$X = X_1 + X_2$$

Electrical power constraint: Since every 1 *KAF* generates electrical power of 400 *MWh*, in power plant *A* when supplying this plant with the quantity  $x_1$ *KAF*, the power produced during the first period of this plant will be  $400x_1$ . The same situation applies to power plant *B*, the amount of power produced is  $200x_2$ , and therefore the amount of power produced during the first period of the two plants is:

$$400x_1 + 200x_2$$

The amount of power produced during the first period must equal the amount of power sold during this period, meaning we get the following restriction:

$$400x_1 + 200x_2 = X_1 + X_2 \quad (1)$$

- 2- Constraint on the amount of water that must be supplied to Power Plant *A*: Since the maximum power that this plant can produce during the first period is 60000 *MWh*, and every 1 *KAF* of water in this plant produces 400 *MWh*, we get the following constraint:

$$\frac{60000}{400} = 150 \Rightarrow x_1 \leq 150 \quad (2)$$

Restricting the amount of water that must be supplied to power plant *B*: Since the maximum power that this plant can produce during the first period is 35000 *MWh*, and every 1 *KAF* of water in this plant produces 200 *MWh*, we get the following restriction:

$$\frac{35000}{200} = 175 \Rightarrow x_2 \leq 175 \quad (3)$$

- 3- Maintaining the amount of water in tank *A*:  
The level of tank *A* at the beginning of the first period, plus the amount of flow into this tank, must equal the amount of water supplied to power plant *A* from tank *A* + the amount of water that must be drained so that a flood does not occur + the tank level at the end of the first period, i.e.:

$$1900 + 200 = x_1 + x_3 + x_5 \Rightarrow$$

$$x_1 + x_3 + x_5 = 2100 \quad (4)$$

Since the minimum level allowed in this tank is 1200 and the maximum capacity of this tank is 2000, the level of tank A at the end of the first period must be limited between the values 1200 and 2000 and thus we obtain the following double restriction:

$$1200 \leq x_5 \leq 2000 \quad (5)$$

4- Maintaining the amount of water in tank B:

The water supplied to power plant B + the water drained from tank B so that there is no flooding + the level of tank B at the end of the first period must equal the amount of water supplied to tank B from power plant A (it is the same amount of water that was supplied to power plant A from the tank A) + the amount of water that must be drained from tank A so that a flood does not occur + the level of tank B at the beginning of the first period + the amount of flow into tank B in the first period, i.e.:

$$x_2 + x_4 + x_6 = x_1 + x_3 + 850 + 40 \Rightarrow$$

$$x_2 + x_4 + x_6 - x_1 - x_3 = 890 \quad (6)$$

Since the minimum level allowed in this tank is 800 and the maximum capacity of this tank is 1500, the level of tank B at the end of the first period must be limited between the values 800 and 1500, and thus we obtain the following double restriction:

$$800 \leq x_6 \leq 1500 \quad (7)$$

**✚ The study for the second period:**

**Information about the power plants of the second period:**

<b>Table No. (1) Information about power plants A and B during the second period</b>		
Information \ power plants	power plant A	power plant B
Water supplied to the power plant during the second period	Unknown	Unknown
Maximum generating capacity during the second period	60000	35000
The power it can generate for a volume of water of 1 KAF	400	200

**Information about tanks during the first period:**

<b>Table No. (2) Information about reservoirs A and B during the second period</b>		
Information \ Tanks	Tank A	Tank B
Maximum capacity	2000	1500
Minimum permissible level	1200	800
The ratio at the beginning of the second period is the same as the ratio at the end of the first period	Unknown	Unknown
Excess water drained from the tank to prevent flooding	Unknown	Unknown
Flow during the second period	130	15
Reservoir level at the end of the second period	Unknown	Unknown

**The first step regarding this issue during the second period of work, we find that the unknowns are:**

- 1- The amount of power produced is divided into two parts: a quantity that is sold for 20\$ *MWh*, and a quantity that is sold for 14\$ *MWh*.
- 2- The amount of water that must be supplied to power plant *A* will be denoted  $y_1$ .
- 3- The amount of water supplied to power plant *B* will be denoted  $y_2$ .
- 4- The amount of water that must be drained from tank *A* so that flooding does not occur. We will denote it  $y_3$ .
- 5- The amount of water that must be drained from tank *B* so that flooding does not occur. We will denote it  $y_4$ .
- 6- Tank *A* level at the end of the second period. We will denote it  $y_5$ .
- 7- Tank *B* level at the end of the second period. We will denote it  $y_6$ .

**The second step regarding this issue during the second period of work we find the following restrictions:**

- 1- We assume that the amount of power sold during the first period at a price of 20\$ per megawatt-hour is  $Y_1$  and the amount of power sold at a price of 14\$ per megawatt-hour is  $Y_2$ . The amount of power produced during the first period is:

$$Y = Y_1 + Y_2$$

Electrical power constraint: Since every 1 *KAF* generates electrical power of 400 *MWh*, in power plant *A*, when supplying this plant with the quantity  $y_1$  *KAF*, the power produced during the second period of this plant will be  $400 y_1$ . The same situation is for power plant *B*, the amount of power produced is  $200y_2$ , and therefore the amount of power produced during the second period of the two plants is:

$$400y_1 + 200y_2$$

The amount of power produced during the second period must equal the amount of power sold during this period, meaning we get the restriction:

$$400y_1 + 200y_2 = Y_1 + Y_2 \quad (8)$$

- 2- Constraint on the amount of water that must be supplied to power Plant *A*: Since the maximum power that this plant can produce during the second period is 60000 *MWh*, and every 1 *KAF* of water in this plant produces 400 *MWh*, we get the following constraint:

$$\frac{60000}{400} = 150 \Rightarrow y_1 \leq 150 \quad (9)$$

Restricting the amount of water that must be supplied to power plant *B*: Since the maximum power that this plant can produce during the second period is 35000 *MWh*, and every 1 *KAF* of water in this plant produces 200 *MWh*, we get the following restriction:

$$\frac{35000}{200} = 175 \Rightarrow y_2 \leq 175 \quad (10)$$

- 3- Maintaining the amount of water in tank *A*:  
The level of tank *A* at the beginning of the second period, plus the amount flowing into this tank, must equal the amount of water supplied to it Power factor *A* of tank *A* + the amount of water that must be drained so that there is no flooding + the tank level at the end the first period, i.e.:

$$x_5 + 130 = y_1 + y_3 + y_5 \Rightarrow y_1 + y_3 + y_5 - x_5 = 130 \quad (11)$$

Since the minimum level allowed in this tank is 1200 and the maximum capacity of this tank is 2000, the level of tank *A* at the end of the first period must be

limited between the values 1200 and 2000 and thus we obtain the following double restriction:

$$1200 \leq y_5 \leq 2000 \quad (12)$$

4- Maintaining the amount of water in tank  $B$ :

The water supplied to power plant  $B$  + the water drained from tank  $B$  so that there is no flooding + the level of tank  $B$  at the end of the second period must equal the amount of water supplied to tank  $B$  from power plant  $A$  (it is the same amount of water that was supplied to power plant  $A$  from the tank  $A$ ) + the amount of water that must be drained from tank  $A$  so that a flood does not occur + the level of tank  $B$  at the beginning of the second period + the amount of flow into tank  $B$  in the first period, i.e.:

$$\begin{aligned} y_2 + y_4 + y_6 &= x_1 + y_3 + x_6 + 15 \Rightarrow \\ y_2 + y_4 + y_6 - x_1 - y_3 - x_6 &= 15 \quad (13) \end{aligned}$$

Since the minimum level allowed in this tank is 800 and the maximum capacity of this tank is 1500, the level tank  $B$  at the end of the second period must be confined between the values 800 and 1500, and thus we obtain the double entry.

$$800 \leq y_6 \leq 1500 \quad (14)$$

**The third step: In the problem, determine the objective function relation:**

From the data of the issue, we found that the department responsible for the workflow set two prices for selling the electrical power produced during the two periods, according to the quantity sold, and then they are:

The amount of power produced during the first period is given by the following relation:

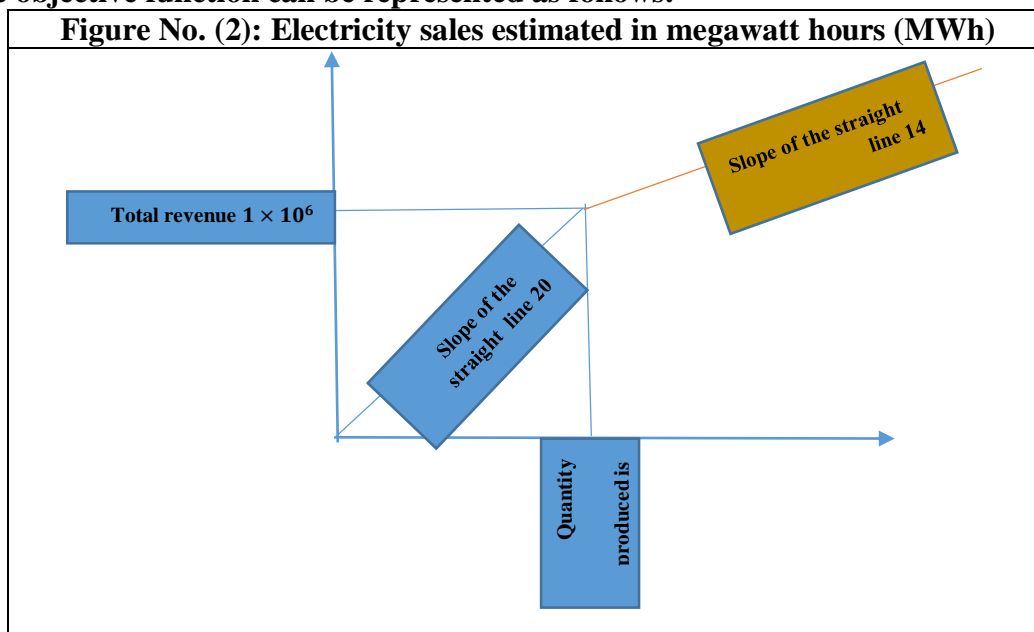
$$X = X_1 + X_2$$

Where  $X_1$  is the amount of power sold during this period at a price of 20\$  $MWh$ , and  $X_2$  is the amount of power sold during this period is at a price of 14\$  $MWh$ , the amount of power produced during the second period is given by the following relation:

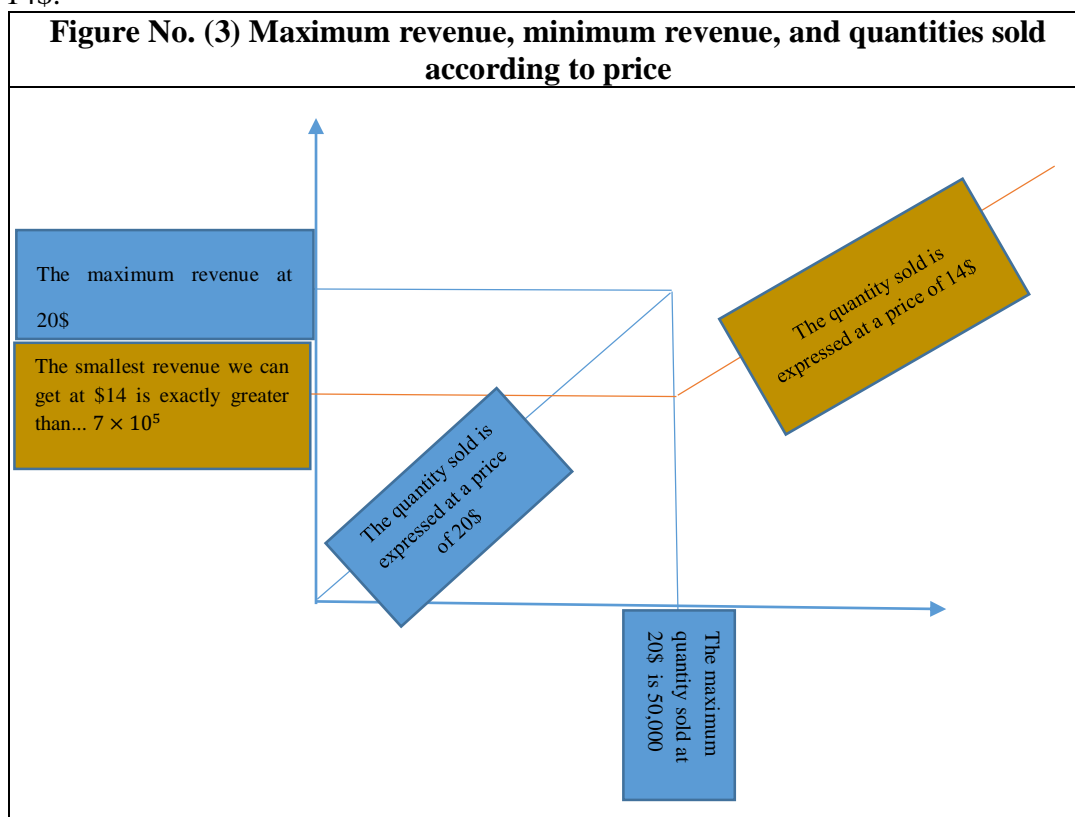
$$Y = Y_1 + Y_2$$

Where  $Y_1$  is the amount of power sold during this period at a price of 20\$  $MWh$ , and  $Y_2$  is the amount of power sold during this period is at a price of 14\$  $MWh$ , in the following figure we show the maximum revenue and the quantities sold of power corresponding to the price of 20\$  $MWh$ , and the smallest revenue that we can obtain from power and the quantities sold of power corresponding to the price of 14\$  $MWh$ .

The objective function can be represented as follows:



From the previous figure, we notice that the objective function is a discrete linear function. It is linear in the fields  $[0, 50000]$  and  $]50000, \infty[$ . Therefore, the quantity of electricity sold can be divided into two parts: the part sold at a price of 20\$/ MWh, and the part sold at a price: 14\$/ MWh, and the following figure shows the quantities produced and the maximum revenue for power sold at a price of 20\$ and the quantities produced and the smallest revenue we can get from power sold at a price of 14\$.





From the above, we can represent the objective function with a linear function as follows:

$$Z = 20(X_1 + Y_1) + 14(X_2 + Y_2) \quad (15)$$

**Mathematical model:**

**Find:**

$$Z = 20(X_1 + Y_1) + 14(X_2 + Y_2) \rightarrow Max$$

**Within the conditions:**

$$400x_1 + 200x_2 = X_1 + X_2$$

$$x_1 \leq 150$$

$$x_2 \leq 175$$

$$x_1 + x_3 + x_5 = 2100$$

$$x_5 \leq 2000$$

$$x_5 \geq 1200$$

$$x_2 + x_4 + x_6 - x_1 - x_3 = 890$$

$$x_6 \leq 1500$$

$$x_6 \geq 800$$

$$400y_1 + 200y_2 = Y_1 + Y_2$$

$$y_1 \leq 150$$

$$y_2 \leq 175$$

$$y_1 + y_3 + y_5 - x_5 = 130$$

$$y_5 \leq 2000$$

$$y_5 \geq 1200$$

$$y_2 + y_4 + y_6 - x_1 - y_3 - x_6 = 15$$

$$y_6 \leq 1500$$

$$y_6 \geq 800$$

$$x_j \geq 0, y_j \geq 0 ; j = 1, 2, \dots, 6 \text{ and } (X_1, X_2, Y_1, Y_2) \geq 0$$

It is a linear model in which the direct Simplex algorithm and its modifications can be used to find the optimal solution through which we achieve the maximum profit, but this solution will be a classic value, a specific value, appropriate to the data that was used, and any change in the conditions of the work environment will affect this data, and therefore the solution we obtain will be inappropriate. It may cause the organization to suffer significant losses, so we suggest that this issue be studied using the concepts of neutrosophic logic by taking data that is subject to change in neutrosophic values, as in the following generalized study:

❖ **The general neutrosophical formulation of the issue of planning hydroelectric power systems:**

In this section, we present a general neutrosophic formulation of the issue of planning hydroelectric systems, because, as we know, some data in the text of the issue are subject to change during the course of work in the system due to many natural and other factors. In order to obtain a more accurate and appropriate study for all circumstances, we will take these data as neutrosophic values.

Relying on the information contained in references [3,14,21], we present the following neutrosophic study:

**Neutrosophic numbers [2]:**

The neutrosophic number is given by the following formula:  $a \pm bI$ , where  $a$  and  $b$  are real or complex coefficients, and  $I$  is the indeterminacy. It can be any domain, set, or any neighborhood of real values. Therefore, in order to obtain the neutrosophic mathematical model for the problem of planning hydroelectric systems, we will take the data that Affected by the factors and conditions surrounding the system's operating

environment, neutrosophic numbers are any of the form  $Nb_i$  and  $Na_{ij}$ , indefinite values. Completely determined, they can be any neighborhood of the real numbers  $a_{ij}$  and  $b_i$  written in one of the forms:

$$Na_{ij} = a_{ij} + \varepsilon_{ij} \text{ and } Nb_i = b_i + \mu_i \text{ where } \varepsilon_{ij} \in [\lambda_{1ij}, \lambda_{2ij}] \text{ or } \varepsilon_{ij} \in \{\lambda_{1ij}, \lambda_{2ij}\}.$$

**Neutrosophic Mathematical Model [21]:**

In case of examples in which the goal and constraints are in the form of neutrosophic mathematical functions, then the neutrosophic mathematical model is written in the following form:

$$Nf = Nf(x_1, x_2, \dots, x_n) \rightarrow (Max) \text{ or } (Min)$$

Subject to the following restrictions:

$$Ng_i(x_1, x_2, \dots, x_n) \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} Nb_i ; i = 1, 2, \dots, m$$

$$x_1, x_2, \dots, x_n \geq 0$$

**The general form of the neutrosophic linear model [14]:**

The general neutrosophic form of the linear mathematical model is given in short form as follows:

$$Nf = \sum_{j=1}^n (c_j \pm \varepsilon_j)x_j \rightarrow (Max) \text{ or } (Min)$$

Within the restrictions:

$$\sum_{j=1}^n a_{ij}x_j \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} b_i \pm \delta_i ; i = 1, 2, \dots, m$$

$$x_j \geq 0$$

**Then we get the following neutrosophic formulation:**

A department controls the operation of a system consisting of two water tanks, each equipped with a plant to generate electrical power. This system works by sending a quantity of water to power plants to generate electrical power. The experts provided the following information:

**Information about the power plants during the two periods:**

- 1- A volume of water of 1 *KAF*, can generate  $p_A$  *MWh*, of electricity in power plant *A* and  $p_B$  *MWh*, in power plant *B*.
- 2- The maximum power that can be generated in power plant *A* is  $NP_A$  *MWh*, and in power plant *B* the maximum power that can be generated is  $NP_B$  *MWh*.
- 3- An amount of electricity amounting to  $K$  *MWh*, can be sold at a price of  $C_{N1}$  \$ *MWh*, while the excess quantity is sold at a price of  $C_{N2}$  \$ *MWh*.

**Information about the tanks during the two periods:**

- 1- The maximum capacity of reservoir *A* is  $S_A$  *KAF* and the maximum capacity of reservoir *B* is  $S_B$  *KAF*.
- 2- The level of reservoir *A* at the beginning of the first period is  $NM_{A1}$  *KAF* and the level of reservoir *B* is  $NM_{B1}$  *KAF*.
- 3- The minimum level allowed in tank *A* is  $NL_A$  *KAF* and in tank *B* is  $NL_B$  *KAF*.
- 4- The flow to tank *A* during the first period is  $T_{AN1}$  *KAF* and to tank *B* is  $T_{BN1}$  *KAF*.
- 5- The flow to tank *A* during the second period is  $T_{AN2}$  *KAF* and to tank *B* is  $T_{BN2}$  *KAF*.

**We should be noted here:**

- Power plant *A* is supplied from tank *A* and power plant *B* from tank *B*.
- When the tanks are completely full, some of the water is drained through the drainage channels so that flooding does not occur.

- For tank *B*, water is supplied from the following sources:
  - The aforementioned outflow.
  - From power plant *A*, the water that is supplied to the power plant after its use goes out to tank *B*.
  - Water drained from tank *A* so that flooding does not occur.

**What is required is to build a mathematical model of the functioning of this system so that it achieves the maximum profit from the process of selling electricity. We know that to formulate a mathematical programming program, we follow the following three steps:**

**First step:** We identify the unknowns in the problem (decision variables) and express them in algebraic symbols.

**Second step:** We define all constraints and express them with equations or inequalities that are mathematical functions of the unknown variables.

**Step Three:** We define the objective function and represent it as a linear function of the unknown variables. It should be made as large or as small as possible. According to the text of the issue, in order to obtain the required mathematical model, the issue must be studied according to the data for each period and the necessary relationships determined. Here we find that a study must be presented for the first and second steps, specific to each period separately.

**✚ The study for the first period:**

**Based on the previous information, we organize the following tables:**

**Information about the power plants of the first period:**

<b>Table No. (1) Information about power plants A and B during the first period</b>		
power plants	power plant A	power plant B
Information		
Water supplied to the power plant during the first period	Unknown	Unknown
Maximum generating capacity during the first period	$NP_A$	$NP_B$
The power it can generate for a volume of water of 1 KAF	$p_A$	$p_B$

**Information about tanks during the first period:**

<b>Table No. (2) Information about reservoirs A and B during the first period</b>		
Tanks	Tank A	Tank B
Information		
Maximum capacity	$S_A$	$S_B$
Minimum permissible level	$NL_A$	$NL_B$
The ratio at the beginning of the first period	$NM_{A1}$	$NM_{B1}$
Excess water drained from the tank to prevent flooding	Unknown	Unknown
Flow during the first period	$T_{AN1}$	$T_{BN1}$
Reservoir level at the end of the first period	Unknown	Unknown

**The first step regarding this issue during the first period of work, we find that the unknowns:**

- 1- The amount of power produced is divided into two parts: a quantity that is sold for  $C_{N1}$  \$ MWh, and a quantity that is sold for  $C_{N2}$  \$ MWh.
- 2- The amount of water that must be supplied to power plant A will be denoted  $x_1$ .
- 3- The amount of water supplied to power plant B will be denoted  $x_2$ .
- 4- The amount of water that must be drained from tank A so that flooding does not occur. We will denote it  $x_3$ .
- 5- The amount of water that must be drained from tank B so that flooding does not occur. We will denote it  $x_4$ .
- 6- Tank A level at the end of the first period. We will denote it  $x_5$ .
- 7- Tank B level at the end of the first period. We will denote it  $x_6$ .

**The second step regarding this issue during the first period of work we find the following restrictions:**

- 1- We assume that the amount of power sold during the first period at a price of  $C_{N1}$  \$ MWh, is  $X_1$  and the amount of power sold at a price of  $C_{N2}$  \$ MWh, is  $X_2$ . The amount of power produced during the first period is:

$$X = X_1 + X_2$$

Electrical power constraint: Since every 1 KAF generates electrical power of 400 MWh, in power plant A when supplying this plant with the quantity  $x_1$  KAF, the power produced during the first period of this plant will be  $p_{A1} x_1$ . The same situation applies to power plant B, the amount of power produced is  $p_B x_2$ , and therefore the amount of power produced during the first period of the two plants is:

$$p_A x_1 + p_B x_2$$

The amount of power produced during the first period must equal the amount of power sold during this period, meaning we get the following restriction:

$$p_A x_1 + p_B x_2 = X_1 + X_2 \quad (1)$$

- 2- Constraint on the amount of water that must be supplied to power plant A: Since the maximum power that this plant can produce during the first period is  $NP_A$  MWh, and every 1 KAF of water in this plant produces  $p_A$  MWh, we get the following constraint:

$$\frac{NP_A}{p_A} \Rightarrow x_1 \leq \frac{NP_A}{p_A} \quad (2)$$

Restricting the amount of water that must be supplied to power plant B: Since the maximum power that this plant can produce during the first period is  $NP_B$  megawatt-hours, and every 1 KAF of water in this plant produces  $p_B$  MWh, we get the following restriction:

$$\frac{NP_B}{p_B} \Rightarrow x_2 \leq \frac{NP_B}{p_B} \quad (3)$$

- 3- Maintaining the amount of water in tank A:

The level of tank A at the beginning of the first period, plus the amount of flow into this tank, must equal the amount of water supplied to power plant A from tank A + the amount of water that must be drained so that a flood does not occur + the tank level at the end of the first period, i.e.:

$$NM_{A1} + T_{AN1} = x_1 + x_3 + x_5 \Rightarrow x_1 + x_3 + x_5 = NM_{A1} + T_{AN1} \quad (4)$$

Since the minimum level allowed in this tank is  $S_A$  and the maximum capacity of this tank is  $NL_A$ , the level of tank A at the end of the first period must be limited between the values  $NL_A$  and  $S_A$  thus we obtain the following double restriction:

$$NL_A \leq x_5 \leq S_A \quad (5)$$

- 4- Maintaining the amount of water in tank B:

The water supplied to power plant  $B$  + the water drained from tank  $B$  so that there is no flooding + the level of tank  $B$  at the end of the first period must equal the amount of water supplied to tank  $B$  from power plant  $A$  (it is the same amount of water that was supplied to power plant  $A$  from the tank  $A$ ) + the amount of water that must be drained from tank  $A$  so that a flood does not occur + the level of tank  $B$  at the beginning of the first period + the amount of flow into tank  $B$  in the first period, i.e.:

$$x_2 + x_4 + x_6 = x_1 + x_3 + NM_{B1} + T_{BN1} \Rightarrow$$

$$x_2 + x_4 + x_6 - x_1 - x_3 = NM_{B1} + T_{BN1} \quad (6)$$

Since the minimum level allowed in this tank is  $NL_B$  and the maximum capacity of this tank is  $S_B$ , the level of tank  $B$  at the end of the first period must be limited between the values  $NL_B$  and  $S_B$ , thus we obtain the following double restriction:

$$NL_B \leq x_6 \leq S_B \quad (7)$$

**✚ The study for the second period:**

**Information about the power plants of the second period:**

<b>Table No. (1) Information about power plants A and B during the second period</b>		
power plants	power plant A	power plant B
Information		
Water supplied to the power plant during the second period	Unknown	Unknown
Maximum generating capacity during the second period	$NP_A$	$NP_B$
The power it can generate for a volume of water of 1 KAF	$p_A$	$p_B$

**Information about tanks during the first period:**

<b>Table No. (2) Information about reservoirs A and B during the second period</b>		
Tanks	Tank A	Tank B
Information		
Maximum capacity	$S_A$	$S_B$
Minimum permissible level	$NL_A$	$NL_B$
The ratio at the beginning of the second period is the same as the ratio at the end of the first period	Unknown	Unknown
Excess water drained from the tank to prevent flooding	Unknown	Unknown
Flow during the second period	$T_{AN2}$	$T_{BN2}$
Reservoir level at the end of the second period	Unknown	Unknown

**The first step regarding this issue during the second period of work, we find that the unknowns are:**

- 1- The amount of power produced is divided into two parts: a quantity that is sold for  $C_{N1}$  \$ MWh, and a quantity that is sold for  $C_{N2}$  \$ MWh.
- 2- The amount of water that must be supplied to power plant  $A$  will be denoted  $y_1$ .
- 3- The amount of water supplied to power plant  $B$  will be denoted  $y_2$ .
- 4- The amount of water that must be drained from tank  $A$  so that flooding does not occur. We will denote it  $y_3$ .

- 5- The amount of water that must be drained from tank  $B$  so that flooding does not occur. We will denote it  $y_4$ .
- 6- Tank  $A$  level at the end of the second period. We will denote it  $y_5$ .
- 7- Tank  $B$  level at the end of the second period. We will denote it  $y_6$ .

**The second step regarding this issue during the second period of work we find the following restrictions:**

- 1- We assume that the amount of power sold during the first period at a price of  $C_{N1}$  \$  $MWh$ , is  $Y_1$  and the amount of power sold at a price of  $C_{N2}$  \$  $MWh$ , is  $Y_2$ . The amount of power produced during the first period is:

$$Y = Y_1 + Y_2$$

Electrical power constraint: Since every 1  $KAF$  generates electrical power of  $p_A$   $MWh$ , in power plant  $A$ , when supplying this plant with the quantity  $y_1$   $KAF$ , the power produced during the second period of this plant will be  $p_A y_1$ . The same situation is for power plant  $B$ , the amount of power produced is  $p_B y_2$ , and therefore the amount of power produced during the second period of the two plants is:

$$p_A y_1 + p_B y_2$$

The amount of power produced during the second period must equal the amount of power sold during this period, meaning we get the restriction:

$$p_A y_1 + p_B y_2 = Y_1 + Y_2 \quad (8)$$

- 2- Constraint on the amount of water that must be supplied to power plant  $A$ : Since the maximum power that this plant can produce during the second period is  $NP_A$   $MWh$ , and every 1  $KAF$  of water in this plant produces  $p_A$   $MWh$ , we get the following constraint:

$$\frac{NP_A}{p_A} \Rightarrow y_1 \leq \frac{NP_A}{p_A} \quad (9)$$

Restricting the amount of water that must be supplied to power plant  $B$ : Since the maximum power that this plant can produce during the second period is  $NP_B$   $MWh$ , and every 1  $KAF$  of water in this plant produces  $p_B$   $MWh$ , we get the following restriction:

$$\frac{NP_B}{p_B} \Rightarrow y_2 \leq \frac{NP_B}{p_B} \quad (10)$$

- 3- Maintaining the amount of water in tank  $A$ :

The level of tank  $A$  at the beginning of the second period, plus the amount flowing into this tank, must equal the amount of water supplied to it Power factor  $A$  of tank  $A$  + the amount of water that must be drained so that there is no flooding + the tank level at the end the first period, i.e.:

$$\begin{aligned} x_5 + T_{A2} &= y_1 + y_3 + y_5 \Rightarrow \\ y_1 + y_3 + y_5 - x_5 &= T_{A2} \quad (11) \end{aligned}$$

Since the minimum level allowed in this tank is  $NL_A$  and the maximum capacity of this tank is  $S_A$ , the level of tank  $A$  at the end of the first period must be limited between the values  $NL_A$  and  $S_A$  thus we obtain the following double restriction:

$$NL_A \leq y_5 \leq S_A \quad (12)$$

- 4- Maintaining the amount of water in tank  $B$ :

The water supplied to power plant  $B$  + the water drained from tank  $B$  so that there is no flooding + the level of tank  $B$  at the end of the second period must equal the amount of water supplied to tank  $B$  from power plant  $A$  (it is the same amount of water that was supplied to power plant  $A$  from the tank  $A$ ) + the amount of water that must be drained from tank  $A$  so that a flood does not occur + the level of tank  $B$

at the beginning of the second period + the amount of flow into tank  $B$  in the first period, i.e.:

$$\begin{aligned} y_2 + y_4 + y_6 &= x_1 + y_3 + x_6 + T_{BN2} \implies \\ y_2 + y_4 + y_6 - x_1 - y_3 - x_6 &= T_{BN2} \quad (13) \end{aligned}$$

Since the minimum level allowed in this tank is  $NL_B$  and the maximum capacity of this tank is  $S_B$ , the level tank  $B$  at the end of the second period must be confined between the values  $NL_B$  and  $S_B$ , and thus we obtain the double entry.

$$NL_B \leq y_6 \leq S_B \quad (14)$$

**The third step: In the problem, determine the objective function relation:**

From the data of the issue, we found that the department responsible for the workflow set two prices for selling the electrical power produced during the two periods, according to the quantity sold, and then they are:

The amount of power produced during the first period is given by the following relation:

$$X = X_1 + X_2$$

Where  $X_1$  is the amount of power sold during this period at a price of  $C_{N1}$  \$  $MWh$ , and  $X_2$  is the amount of power sold during this period is at a price of  $C_{N2}$  \$  $MWh$ , the amount of power produced during the second period is given by the following relation:

$$Y = Y_1 + Y_2$$

Where  $Y_1$  is the amount of power sold during this period at a price of  $C_{N1}$  \$  $MWh$ , and  $Y_2$  is the amount of power sold during this period is at a price of  $C_{N2}$  \$  $MWh$ , in the following figure we show the maximum revenue and the quantities sold of power corresponding to the price of  $C_{N1}$  \$  $MWh$ , and the smallest revenue that we can obtain from power and the quantities sold of power corresponding to the price of  $C_{N2}$  \$  $MWh$ .

From the previous figure, we notice that the objective function is a discrete linear function. It is linear in the fields  $[0,50000]$  and  $]50000, \infty[$ . Therefore, the quantity of electricity sold can be divided into two parts: the part sold at a price of  $C_{N1}$  \$/ $MWh$ , and the part sold at a price:  $C_{N2}$  \$/ $MWh$ , and the following figure shows the quantities produced and the maximum revenue for power sold at a price of  $C_{N1}$  \$ and the quantities produced and the smallest revenue we can get from power sold at a price of  $C_{N2}$  \$  $MWh$ .

From the above, we can represent the objective function with a linear function as follows:

$$Z = C_{N1}(X_1 + Y_1) + C_{N2}(X_2 + Y_2) \quad (15)$$

**Mathematical model:**

**Find:**

$$Z = C_{N1}(X_1 + Y_1) + C_{N2}(X_2 + Y_2) \rightarrow Max$$

**Within the conditions:**

$$\begin{aligned} p_A x_1 + p_B x_2 &= X_1 + X_2 \\ x_1 &\leq \frac{NP_A}{p_A} \\ x_2 &\leq \frac{NP_B}{p_B} \\ x_1 + x_3 + x_5 &= NM_{A1} + T_{AN1} \\ y_5 &\geq NL_A \\ y_5 &\leq S_A \\ x_2 + x_4 + x_6 - x_1 - x_3 &= NM_{B1} + T_{BN1} \end{aligned}$$

$$\begin{aligned}
x_6 &\leq S_B \\
x_6 &\geq NL_B \\
p_A y_1 + p_B y_2 &= Y_1 + Y_2 \\
y_1 &\leq \frac{NP_A}{p_A} \\
y_2 &\leq \frac{NP_B}{p_B} \\
y_1 + y_3 + y_5 - x_5 &= T_{AN2} \\
y_5 &\leq S_A \\
y_5 &\geq NL_A \\
y_2 + y_4 + y_6 - x_1 - y_3 - x_6 &= T_{BN2} \\
y_6 &\leq S_B \\
y_6 &\geq NL_B \\
x_j \geq 0, y_j \geq 0; j &= 1, 2, \dots, 6 \text{ and } (X_1, X_2, Y_1, Y_2) \geq 0
\end{aligned}$$

❖ **Example text using neutrosophic values:**

A department controls the operation of a system consisting of two water tanks, each equipped with a plant to generate electrical energy. This system works by sending a quantity of water to power plants to generate electrical energy. The experts provided the following information:

**Information about the power plants during the two periods:**

- 1- A volume of water of 1 *KAF*, can generate 400 *MWh*, of electricity in power plant *A* and 200 *MWh*, in power plant *B*.
- 2- The maximum power that can be generated in power plant *A* is [50000,70000] *MWh*, and in power plant *B* the maximum power that can be generated is [30000,40000] *MWh*.
- 3- An amount of electricity amounting to 50000 *MWh*, can be sold at a price of {15,20,25}\$ *MWh*, while the excess quantity is sold at a price of {10,14,15}\$ *MWh*.

**Information about the tanks during the two periods:**

- 1- The maximum capacity of reservoir *A* is 2000 *KAF* and the maximum capacity of reservoir *B* is 1500 *KAF*.
- 2- The level of reservoir *A* at the beginning of the first period is [1700,1900] *KAF* and the level of reservoir *B* is [650,850] *KAF*.
- 3- The minimum level allowed in tank *A* is [1000,1200] *KAF* and in tank *B* is [600,800] *KAF*.
- 4- The flow to tank *A* during the first period is [150,250] *KAF* and to tank *B* is [30,50] *KAF*.
- 5- The flow to tank *A* during the second period is [80,180] *KAF* and to tank *B* is [10,20] *KAF*.

**It should be noted here:**

- Power plant *A* is supplied from tank *A* and power plant *B* from tank *B*.
- When the tanks are completely full, some of the water is drained through the drainage channels so that flooding does not occur.
- For tank *B*, water is supplied from the following sources:
  - The aforementioned outflow.
  - From power plant *A*, the water that is supplied to the power plant after its use goes out to tank *B*.
  - Water drained from tank *A* so that flooding does not occur.



**What is required is to build a mathematical model of the functioning of this system so that it achieves the maximum profit from the process of selling electricity, we know that to formulate a mathematical programming program, we follow the following three steps:**

**First step:** We identify the unknowns in the problem (decision variables) and express them in algebraic symbols.

**Second step:** We define all constraints and express them with equations or inequalities that are mathematical functions of the unknown variables.

**Step Three:** We define the objective function and represent it as a linear function of the unknown variables. It should be made as large or as small as possible.

**According to the text of the issue:**

In order to obtain the required mathematical model, the issue must be studied according to the data for each period and the necessary relationships determined.

Here we find that a study must be presented for the first and second steps, specific to each period separately.

**✚ The study for the first period:**

**Based on the previous information, we organize the following tables:**

**Information about the power plants of the first period:**

<b>Table No. (1) Information about power plants A and B during the first period</b>		
Information \ power plants	power plant A	power plant B
Water supplied to the power plant during the first period	Unknown	Unknown
Maximum generating capacity during the first period	[50000,70000]	[30000,40000]
The power it can generate for a volume of water of 1 KAF	400	200

**Information about tanks during the first period:**

<b>Table No. (2) Information about reservoirs A and B during the first period</b>		
Information \ Tanks	Tank A	Tank B
Maximum capacity	2000	1500
Minimum permissible level	[1000,1200]	[600,800]
The ratio at the beginning of the first period	[1700,1900]	[650,850]
Excess water drained from the tank to prevent flooding	Unknown	Unknown
Flow during the first period	[150,250]	[30,50]
Reservoir level at the end of the first period	Unknown	Unknown

**The first step regarding this issue during the first period of work, we find that the unknowns:**

- 1- The amount of power produced is divided into two parts: a quantity that is sold for {15,20,25}\$ MWh, and a quantity that is sold for {10,14,15}\$ MWh.
- 2- The amount of water that must be supplied to power plant A will be denoted  $x_1$ .
- 3- The amount of water supplied to power plant B will be denoted  $x_2$ .

- 4- The amount of water that must be drained from tank *A* so that flooding does not occur. We will denote it  $x_3$ .
- 5- The amount of water that must be drained from tank *B* so that flooding does not occur. We will denote it  $x_4$ .
- 6- Tank *A* level at the end of the first period. We will denote it  $x_5$ .
- 7- Tank *B* level at the end of the first period. We will denote it  $x_6$ .

**The second step regarding this issue during the first period of work we find the following restrictions:**

- 1- We assume that the amount of power sold during the first period at a price of  $\{15,20,25\}$  \$ *MWh*, is  $X_1$  and the amount of power sold at a price of  $\{10,14,15\}$  \$ *MWh*, is  $X_2$ . The amount of power produced during the first period is:

$$X = X_1 + X_2$$

Electrical power constraint: Since every 1 *KAF* generates electrical power of 400 *MWh*, in power plant *A* when supplying this plant with the quantity  $x_1$  *KAF*, the power produced during the first period of this plant will be  $400x_1$ . The same situation applies to power plant *B*, the amount of power produced is  $200x_2$ , and therefore the amount of power produced during the first period of the two plants is:

$$400x_1 + 200x_2$$

The amount of power produced during the first period must equal the amount of power sold during this period, meaning we get the following restriction:

$$400x_1 + 200x_2 = X_1 + X_2 \quad (1)$$

- 2- Constraint on the amount of water that must be supplied to Power Plant *A*: Since the maximum power that this plant can produce during the first period is  $[50000,70000]$  *MWh*, and every 1 *KAF* of water in this plant produces 400 *MWh*, we get the following constraint:

$$\frac{[50000,70000]}{400} \Rightarrow x_1 \leq [125,175] \quad (2)$$

Restricting the amount of water that must be supplied to power plant *B*: Since the maximum power that this plant can produce during the first period is  $[30000,40000]$  *MWh*, and every 1 *KAF* of water in this plant produces 200 *MWh*, we get the following restriction:

$$\frac{[30000,40000]}{200} \Rightarrow x_2 \leq [150,200] \quad (3)$$

- 3- Maintaining the amount of water in tank *A*:

The level of tank *A* at the beginning of the first period, plus the amount of flow into this tank, must equal the amount of water supplied to power plant *A* from tank *A* + the amount of water that must be drained so that a flood does not occur + the tank level at the end of the first period, i.e.:

$$[50000,70000] + [150,250] = x_1 + x_3 + x_5 \Rightarrow x_1 + x_3 + x_5 = [50150,70250] \quad (4)$$

Since the minimum level allowed in this tank is  $[1000,1200]$  and the maximum capacity of this tank is 2000, the level of tank *A* at the end of the first period must be limited between the values  $[1000,1200]$  and 2000 and thus we obtain the following double restriction:

$$[1000,1200] \leq x_5 \leq 2000 \quad (5)$$

- 4- Maintaining the amount of water in tank *B*:

The water supplied to power plant *B* + the water drained from tank *B* so that there is no flooding + the level of tank *B* at the end of the first period must equal the

amount of water supplied to tank *B* from power plant *A* (it is the same amount of water that was supplied to power plant *A* from the tank *A*) + the amount of water that must be drained from tank *A* so that a flood does not occur + the level of tank *B* at the beginning of the first period + the amount of flow into tank *B* in the first period, i.e.:

$$x_2 + x_4 + x_6 = x_1 + x_3 + [650,850] + [30,50] \Rightarrow$$

$$x_2 + x_4 + x_6 - x_1 - x_3 = [680,900] \quad (6)$$

Since the minimum level allowed in this tank is [600,800] and the maximum capacity of this tank is 1500, the level of tank *B* at the end of the first period must be limited between the values [600,800] and 1500, and thus we obtain the following double restriction:

$$[600,800] \leq x_6 \leq 1500 \quad (7)$$

**✚ The study for the second period:**

**Information about the power plants of the second period:**

Table No. (1) Information about power plants A and B during the second period		
Information \ power plants	power plant A	power plant B
Water supplied to the power plant during the second period	Unknown	Unknown
Maximum generating capacity during the second period	[50000,70000]	[30000,40000]
The power it can generate for a volume of water of 1 KAF	400	200

**Information about tanks during the first period:**

Table No. (2) Information about reservoirs A and B during the second period		
Information \ Tanks	Tank A	Tank B
Maximum capacity	2000	1500
Minimum permissible level	[1000,1200]	[600,800]
The ratio at the beginning of the second period is the same as the ratio at the end of the first period	Unknown	Unknown
Excess water drained from the tank to prevent flooding	Unknown	Unknown
Flow during the second period	[80,180]	[10,20]
Reservoir level at the end of the second period	Unknown	Unknown

**The first step regarding this issue during the second period of work, we find that the unknowns:**

- 1- The amount of power produced is divided into two parts: a quantity that is sold for {15,20,25}\$ *MWh*, and a quantity that is sold for {10,14,15}\$ *MWh*.
- 2- The amount of water that must be supplied to power plant *A* will be denoted  $y_1$ .
- 3- The amount of water supplied to power plant *B* will be denoted  $y_2$ .
- 4- The amount of water that must be drained from tank *A* so that flooding does not occur. We will denote it  $y_3$ .

- 5- The amount of water that must be drained from tank  $B$  so that flooding does not occur. We will denote it  $y_4$ .
- 6- Tank  $A$  level at the end of the second period. We will denote it  $y_5$ .
- 7- Tank  $B$  level at the end of the second period. We will denote it  $y_6$ .

**The second step regarding this issue during the second period of work we find the following restrictions:**

- 1- We assume that the amount of power sold during the first period at a price of  $\{15,20,25\}$  \$  $MWh$ , is  $Y_1$  and the amount of power sold at a price of  $\{10,14,15\}$  \$  $MWh$ , is  $Y_2$ . The amount of power produced during the first period is:

$$Y = Y_1 + Y_2$$

Electrical power constraint: Since every 1  $KAF$ , generates electrical power of 400  $MWh$ , in power plant  $A$ , when supplying this plant with the quantity  $y_1$   $KAF$ , the power produced during the second period of this plant will be  $400 y_1$ . The same situation is for power plant  $B$ , the amount of power produced is  $200y_2$ , and therefore the amount of power produced during the second period of the two plants is:

$$400y_1 + 200y_2$$

The amount of power produced during the second period must equal the amount of power sold during this period, meaning we get the restriction:

$$400y_1 + 200y_2 = Y_1 + Y_2 \quad (8)$$

- 2- Constraint on the amount of water that must be supplied to power Plant  $A$ : Since the maximum power that this plant can produce during the second period is  $[50000,70000]$   $MWh$ , and every 1  $KAF$  of water in this plant produces 400  $MWh$ , we get the following constraint:

$$\frac{[50000,70000]}{400} \Rightarrow y_1 \leq [125,175] \quad (9)$$

Restricting the amount of water that must be supplied to power plant  $B$ : Since the maximum power that this plant can produce during the second period is  $[30000,40000]$   $MWh$ , and every 1  $KAF$  of water in this plant produces 200  $MWh$ , we get the following restriction:

$$\frac{[30000,40000]}{200} \Rightarrow y_2 \leq [150,200] \quad (10)$$

- 3- Maintaining the amount of water in tank  $A$ :

The level of tank  $A$  at the beginning of the second period, plus the amount flowing into this tank, must equal the amount of water supplied to it Power factor  $A$  of tank  $A$  + the amount of water that must be drained so that there is no flooding + the tank level at the end the first period, i.e.:

$$x_5 + [150,250] = y_1 + y_3 + y_5 \Rightarrow y_1 + y_3 + y_5 - x_5 = [150,250] \quad (11)$$

Since the minimum level allowed in this tank is  $[1000,1200]$  and the maximum capacity of this tank is 2000, the level of tank  $A$  at the end of the first period must be limited between the values  $[1000,1200]$  and 2000 and thus we obtain the following double restriction:

$$[1000,1200] \leq y_5 \leq 2000 \quad (12)$$

- 4- Maintaining the amount of water in tank  $B$ :

The water supplied to power plant  $B$  + the water drained from tank  $B$  so that there is no flooding + the level of tank  $B$  at the end of the second period must

equal the amount of water supplied to tank  $B$  from power plant  $A$  (it is the same amount of water that was supplied to power plant  $A$  from the tank  $A$ ) + the amount of water that must be drained from tank  $A$  so that a flood does not occur + the level of tank  $B$  at the beginning of the second period + the amount of flow into tank  $B$  in the first period, i.e.:

$$y_2 + y_4 + y_6 = x_1 + y_3 + x_6 + [30,50] \Rightarrow$$

$$y_2 + y_4 + y_6 - x_1 - y_3 - x_6 = [30,50] \quad (13)$$

Since the minimum level allowed in this tank is  $[600,800]$  and the maximum capacity of this tank is 1500, the level tank  $B$  at the end of the second period must be confined between the values  $[600,800]$  and 1500, and thus we obtain the double entry.

$$[600,800] \leq y_6 \leq 1500 \quad (14)$$

**The third step: In the problem, determine the objective function relation:**

From the data of the issue, we found that the department responsible for the workflow set two prices for selling the electrical power produced during the two periods, according to the quantity sold, and then they are:

The amount of power produced during the first period is given by the following relation:

$$X = X_1 + X_2$$

Where  $X_1$  is the amount of power sold during this period at a price of  $\{15,20,25\}$  \$  $MWh$ , and  $X_2$  is the amount of power sold during this period is at a price of  $\{10,14,15\}$  \$  $MWh$ , the amount of power produced during the second period is given by the following relation:

$$Y = Y_1 + Y_2$$

Where  $Y_1$  is the amount of power sold during this period at a price of  $\{15,20,25\}$  \$  $MWh$ , and  $Y_2$  is the amount of power sold during this period is at a price of  $\{10,14,15\}$  \$  $MWh$ , in the following figure we show the maximum revenue and the quantities sold of power corresponding to the price of  $\{15,20,25\}$  \$  $MWh$ , and the smallest revenue that we can obtain from power and the quantities sold of power corresponding to the price of  $\{10,14,15\}$  \$  $MWh$ .

From the previous figure, we notice that the objective function is a discrete linear function. It is linear in the fields  $[0,50000]$  and  $]50000, \infty[$ . Therefore, the quantity of electricity sold can be divided into two parts: the part sold at a price of  $\{15,20,25\}$  \$ /  $MWh$ , and the part sold at a price:  $\{10,14,15\}$  \$ /  $MWh$ , and the following figure shows the quantities produced and the maximum revenue for power sold at a price of  $\{15,20,25\}$  \$ and the quantities produced and the smallest revenue we can get from power sold at a price of  $\{10,14,15\}$  \$.

From the above, we can represent the objective function with a linear function as follows:

$$Z = \{15,20,25\}(X_1 + Y_1) + \{10,14,15\}(X_2 + Y_2) \quad (15)$$

**Mathematical model:**

**Find:**

$$Z = \{15,20,25\}(X_1 + Y_1) + \{10,14,15\}(X_2 + Y_2) \rightarrow Max$$

**Within the conditions:**

$$400x_1 + 200x_2 = X_1 + X_2$$

$$x_1 \leq [125,175]$$

$$x_2 \leq [150,200]$$

$$x_1 + x_3 + x_5 = [50150,70250]$$

$$y_5 \leq 2000$$

$$\begin{aligned}
& y_5 \geq [1000,1200] \\
& x_2 + x_4 + x_6 - x_1 - x_3 = [680,900] \\
& x_6 \leq 1500 \\
& x_6 \geq [600,800] \\
& 200y_1 + 400y_2 = Y_1 + Y_2 \\
& y_1 \leq [125,175] \\
& y_2 \leq [150,200] \\
& y_1 + y_3 + y_5 - x_5 = [150,250] \\
& y_5 \leq 2000 \\
& y_5 \geq [1000,1200] \\
& y_2 + y_4 + y_6 - x_1 - y_3 - x_6 = [30,50] \\
& y_6 \leq 1500 \\
& y_6 \geq [600,800] \\
& x_j \geq 0, y_j \geq 0 ; j = 1,2, \dots,6 \text{ and } (X_1, X_2, Y_1, Y_2) \geq 0
\end{aligned}$$

It is a linear neutrosophic model. The direct simplex neutrosophic algorithm and its modifications can be used to find the optimal solution through which we achieve the maximum profit, and it takes into account all the conditions that the system's working environment may experience, through the indeterminacy present in the neutrosophic values that were taken for data that is subject to change due to factors and conditions that can occur. To go through the work environment.

### Conclusion and results:

In this research, we presented a study of the issue of planning hydroelectric systems. From the information contained in reference [1], we reformulated this issue in an expanded way using classical values. Since some of the data in this issue are affected by natural or other factors, we found that the solution we can get from While solving the linear model, there may be an inaccurate solution that does not fit with the conditions that the system's operating environment may experience. Therefore, we presented a complex formula for the problem of planning hydroelectric systems using the concepts of neutrosophic logic, and we obtained a neutrosophic linear model. Special neutrosophic algorithms can be used to solve the linear models that were presented. In previous research to obtain the optimal neutrosophic solution suitable for all conditions

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