



The indefinite symbolic plithogenic integrals

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Abstract: In order to calculate the indefinite integrals of the symbolic plithogenic field, we used the substitution method, which was provided in this article. We also established a theorem that allowed us to locate the majority of the integrals for the symbolic plithogenic functions, in addition to the condition that must be met for the integration operation to be possible.

Keywords: symbolic plithogenic; division of symbolic plithogenic numbers; indefinite; integrals; substitution.

1. Introduction and Preliminaries

To The genesis, origination, formation, development, and evolution of new entities through dynamics of contradictory and/or neutral and/or noncontradictory multiple old entities is known as plithogenic. Plithogeny advocates for the integration of theories from several fields.

We use numerous "knowledges" from domains like soft sciences, hard sciences, arts and literature theories, etc. as "entities" in this study, this is what Smarandache introduced, as he presented a study on plithogeny, plithogenic set, logic, probability, and statistics [2], in addition to presenting introduction to the symbolic plithogenic algebraic structures (revisited), through which he discussed several ideas, including mathematical operations on plithogenic numbers [1]. Also, an overview of plithogenic set and symbolic plithogenic algebraic structures was discussed by him [3]. It is thought that the symbolic n-plithogenic sets are a good place to start when developing algebraic extensions for other classical structures including rings, vector spaces, modules, and equations [4-5-6-7].

Alhasan also presented several papers on calculus, in which he discussed neutrosophic definite and indefinite integrals. He also presented the most important applications of definite integrals in neutrosophic logic [8-9].

Integration is important in human life, and one of its most important applications is the calculation of area, size and arc length. In our reality we find things that cannot be precisely defined, and that contain an indeterminacy part. This is the reason for studying neutrosophic integration and methods of its integration in this paper.

Smarandache presented the division operation in the symbolic plithogenic field as follows [1]:

Division of Symbolic Plithogenic Components

$$\frac{P_i}{P_j} = \begin{cases} x_0 + x_1P_1 + x_2P_2 + \dots + x_jP_j + P_i & x_0 + x_1 + x_2 + \dots + x_j = 0 & i > j \\ x_0 + x_1P_1 + x_2P_2 + \dots + x_iP_i & x_0 + x_1 + x_2 + \dots + x_i = 1 & i = j \\ \emptyset & & i < j \end{cases}$$

where all coefficients $x_0, x_1, x_2, \dots, x_i, \dots \in SPS$.

Division of Symbolic Plithogenic Numbers

Let consider two symbolic plithogenic numbers as below:

$$PN_r = a_0 + a_1P_1 + a_2P_2 + \dots + a_rP_r$$

$$PN_s = b_0 + b_1P_1 + b_2P_2 + \dots + b_sP_s$$

$$\frac{PN_r}{PN_s} = \begin{cases} \text{none, one many} & r \geq s \\ \emptyset & r < s \end{cases}$$

This paper dealt with several topics, in the first part of which introduction and preliminaries were presented, and in the main discussion part the indefinite symbolic plithogenic integrals. In the last part, a conclusion to the paper is given.

Main Discussion

The indefinite symbolic plithogenic integrals

Definition 1

Let $f: SPS \rightarrow SPS$ to evaluate $\int f(x, PN)dx$

where $PN = d_0 + d_1P_1 + d_2P_2 + \dots + d_nP_n$

put: $x = g(u) \Rightarrow dx = g'(u)du$

by substitution, we get:

$$\int f(x, PN)dx = \int f(u)g'(u)du$$

then we can directly integral it.

Theorem 1

If $\int f(x, PN)dx = \varphi(x, PN)$, then:

$$\int PN_r f(PN_s x + PN_n) dx = \frac{PN_r}{PN_s} \varphi(PN_s x + PN_n) + PC$$

provided that $\frac{PN_r}{PN_s}$ is divisible.

where $PN_r = a_0 + a_1P_1 + a_2P_2 + \dots + a_rP_r$, $PN_s = b_0 + b_1P_1 + b_2P_2 + \dots + b_sP_s$, $PN_n = c_0 + c_1P_1 + c_2P_2 + \dots + c_nP_n$ and $PC = c_0 + c_1P_1 + c_2P_2 + \dots + c_rP_r \in SPS$ is symbolic plithogenic constant.

Proof:

put: $PN_s x + PN_n = u \Rightarrow PN_s dx = du$

$$\Rightarrow dx = \frac{1}{PN_s} du$$

$$\begin{aligned} \int PN_r f(PN_s x + PN_n) dx &= \int PN_r f(u) \frac{1}{PN_s} du \\ &= \int \frac{PN_r}{PN_s} f(u) du \\ &= \frac{PN_r}{PN_s} \varphi(u) + PC \end{aligned}$$

back to the variable x , we get:

$$\int PN_r f(PN_s x + PN_n) dx = \frac{PN_r}{PN_s} \varphi(PN_s x + PN_n) + PC$$

Using the previous theorem, we get on:

$$1) \int PN_r (PN_s x + PN_n)^n dx = \frac{PN_r (PN_s x + PN_n)^{n+1}}{PN_s (n+1)} + PC$$

$$2) \int \frac{PN_r}{PN_s x + PN_n} dx = \frac{PN_r}{PN_s} \ln|PN_s x + PN_n| + PC$$

$$3) \int PN_r e^{PN_s x + PN_n} dx = \frac{PN_r}{PN_s} e^{PN_s x + PN_n} + PC$$

$$4) \int \frac{PN_r}{\sqrt{PN_s x + PN_n}} dx = 2 \frac{PN_r}{PN_s} \sqrt{PN_s x + PN_n} + PC$$

$$5) \int PN_r \cos(PN_s x + PN_n) dx = \frac{PN_r}{PN_s} \sin(PN_s x + PN_n) + PC$$

$$6) \int PN_r \sin(PN_s x + PN_n) dx = \frac{PN_r}{PN_s} \cos(PN_s x + PN_n) + PC$$

$$7) \int PN_r \sec^2(PN_s x + PN_n) dx = \frac{PN_r}{PN_s} \tan(PN_s x + PN_n) + PC$$

$$8) \int PN_r \csc^2(PN_s x + PN_n) dx = \frac{PN_r}{PN_s} \cot(PN_s x + PN_n) + PC$$

$$9) \int PN_r \sec(PN_s x + PN_n) \tan(PN_s x + PN_n) dx = \frac{PN_r}{PN_s} \sec(PN_s x + PN_n) + PC$$

$$10) \int PN_r \csc(PN_s x + PN_n) \cot(PN_s x + PN_n) dx = \frac{PN_r}{PN_s} \csc(PN_s x + PN_n) + PC$$

Example 1

$$1) \int P_2 (P_1 x + 5 - 3P_1 + 4P_2)^5 dx = \frac{P_2 (P_1 x + 5 - 3P_1 + 4P_2)^6}{P_1 \cdot 6} + PC$$

$$= (x_0 + x_1 P_1 + P_2) \frac{(P_1 x + 5 - 3P_1 + 4P_2)^6}{6} + PC$$

where:

$$\begin{aligned} \frac{P_2}{P_1} = x_0 + x_1 P_1 + x_2 P_2 &\Rightarrow P_2 = x_0 P_1 + x_1 P_1 + x_2 P_2 \\ &\Rightarrow P_2 = (x_0 + x_1) P_1 + x_2 P_2, \text{ then:} \end{aligned}$$

$$x_0 + x_1 = 0 \text{ and } x_2 = 1$$

hence: $\frac{P_2}{P_1} = x_0 + x_1 P_1 + P_2$, where: $x_0 + x_1 = 0$

let's check the answer:

$$\begin{aligned} \frac{d}{dx} \left[(x_0 + x_1 P_1 + P_2) \frac{(P_1 x + 5 - 3P_1 + 4P_2)^6}{6} + PC \right] &= 6P_1 (x_0 + x_1 P_1 + P_2) \frac{(P_1 x + 5 - 3P_1 + 4P_2)^5}{6} \\ &= (x_0 P_1 + x_1 P_1 + P_2) (P_1 x + 5 - 3P_1 + 4P_2)^5 \\ &= ((x_0 + x_1) P_1 + P_2) (P_1 x + 5 - 3P_1 + 4P_2)^5 \end{aligned}$$

but we have: $x_0 + x_1 = 0$, then:

$$\frac{d}{dx} \left[(x_0 + x_1 P_1 + P_2) \frac{(P_1 x + 5 - 3P_1 + 4P_2)^6}{6} + PC \right] = P_2 (P_1 x + 5 - 3P_1 + 4P_2)^5$$

= (The same integral function)

$$2) \int \frac{2P_1 + 3}{P_1 x + 1 + 2P_1 - 7P_2 + 4P_5} dx = \text{does not exist}$$

because:

$$\frac{2P_1 + 3}{P_1} = x_0 + x_1 P_1$$

$$2P_1 + 3 = x_0 P_1 + x_1 P_1$$

$$2P_1 + 3 = (x_0 + x_1) P_1$$

then: $x_0 + x_1 = 2$, but we are not able to catch the free coefficient 1 from the left-hand side so:

$$\frac{2P_1 + 3}{P_1} = (\text{does not exist})$$

$$3) \int e^{P_3 x - 3 + P_2} dx = \text{does not exist}$$

because: $\frac{1}{P_3} = (\text{does not exist})$

$$4) \int 6P_4 \cos((P_4 - 3P_2)x + 2 - P_3 + P_4) dx = \frac{6P_4}{P_4 - 3P_2} \sin((P_4 - 3P_2)x + 2 - P_3 + P_4) + PC$$

$$= (x_0 + x_1 P_1 + x_2 P_2 - 3P_4) \sin((P_4 - 3P_2)x + 2 - P_3 + P_4) + PC$$

where:

$$\frac{6P_4}{P_4 - 3P_2} = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4$$

$$6P_4 = (P_4 - 3P_2)(x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4)$$

$$6P_4 = x_0P_4 + x_1P_4 + x_2P_4 + x_3P_4 + x_4P_4 - 3x_0P_2 - 3x_1P_2 - 3x_2P_2 - 3x_3P_3 - 3x_4P_4$$

$$6P_4 = -3(x_0 + x_1 + x_2)P_2 - 3x_3P_3 + (x_0 + x_1 + x_2 + x_3 - 2x_4)P_4, \text{ then:}$$

$$x_0 + x_1 + x_2 = 0, x_3 = 0 \text{ and } x_4 = -3$$

$$\text{hence: } \frac{6P_4}{P_4 - 3P_2} = x_0 + x_1P_1 + x_2P_2 - 3P_4, \text{ where: } x_0 + x_1 + x_2 = 0$$

$$\begin{aligned} 5) \int (P_3 + 5P_2 - 5P_1 + 6) \sec^2((P_3 + 5)x - 8 + 4P_1 - 5P_2 + 3P_3) dx \\ = \frac{P_3 + 5P_2 - 5P_1 + 6}{P_3 + 5} \tan((P_3 + 5)x - 8 + 4P_1 - 5P_2 + 3P_3) + PC \\ = \left(\frac{6}{5} + P_1 + P_2 - \frac{1}{30}P_3 \right) \tan((P_3 + 5)x - 8 + 4P_1 - 5P_2 + 3P_3) + PC \end{aligned}$$

where:

$$\frac{P_3 + 5P_2 - 5P_1 + 6}{P_3 + 5} = x_0 + x_1P_1 + x_2P_2 + x_3P_3$$

$$P_3 + 5P_2 - 5P_1 + 6 = (P_3 + 5)(x_0 + x_1P_1 + x_2P_2 + x_3P_3)$$

$$P_3 + 5P_2 - 5P_1 + 6 = x_0P_3 + x_1P_3 + x_2P_3 + x_3P_3 + 5x_0 + 5x_1P_1 + 5x_2P_2 + 5x_3P_3$$

$$P_3 + 5P_2 - 5P_1 + 6 = 5x_0 + 5x_1P_1 + 5x_2P_2 + (x_0 + x_1 + x_2 + 6x_3)P_3, \text{ then:}$$

$$x_0 = \frac{6}{5}, x_1 = 1, x_2 = -1 \text{ and } x_3 = -\frac{1}{30}$$

$$\text{hence: } \frac{P_3 + 5P_2 - 5P_1 + 6}{P_3 + 5} = \frac{6}{5} + P_1 + P_2 - \frac{1}{30}P_3$$

$$6) \int P_2 \csc(P_4x) \cot(P_4x) dx = \text{does not exist}$$

$$\text{because: } \frac{P_2}{P_4} = (\text{does not exist})$$

$$\begin{aligned} 7) \int \frac{P_2}{\sqrt{P_2x + 3 - P_1}} dx \\ = \frac{2P_2}{P_2} \sqrt{P_2x + 3 - P_1} + PC \\ = 2(x_0 + x_1P_1 + x_2P_2) \sqrt{P_2x + 3 - P_1} + PC \end{aligned}$$

where:

$$\frac{2P_2}{P_2} = x_0 + x_1P_1 + x_2P_2$$

$$2P_2 = x_0P_2 + x_1P_2 + x_2P_2$$

$$2P_2 = (x_0 + x_1 + x_2)P_2$$

then: $x_0 + x_1 + x_2 = 2$

hence: $\frac{2P_2}{P_2} = x_0 + x_1P_1 + x_2P_2$, where: $x_0 + x_1 + x_2 = 2$

Theorem 2

Let $f: SPS \rightarrow SPS$, then:

$$\int \frac{\hat{f}(x, PN)}{f(x, PN)} dx = \ln|f(x, PN)| + PC$$

Proof:

$$\begin{aligned} \text{put: } f(x, PN) = u & \Rightarrow \hat{f}(x, PN) dx = du \\ & \Rightarrow dx = \frac{1}{\hat{f}(x, PN)} du \\ & \Rightarrow dx = \frac{1}{\hat{u}} du \end{aligned}$$

$$\int \frac{\hat{f}(x, PN)}{f(x, PN)} dx = \int \frac{\hat{u} 1}{u \hat{u}} du = \int \frac{1}{u} du = \ln|u| + PC$$

back to the $f(x, PN)$, we get:

$$\int \frac{\hat{f}(x, PN)}{f(x, PN)} dx = \ln|f(x, PN)| + PC$$

Example 2

$$1) \int \frac{(3 + 2P_1 - 7P_2 + P_3 - 5P_4)x^7}{(3 + 2P_1 - 7P_2 + P_3 - 5P_4)x^8 + 8 - P_1} dx = \frac{1}{8} \ln|(3 + 2P_1 - 7P_2 + P_3 - 5P_4)x^8 + 8 - P_1| + PC$$

$$2) \int \frac{(1 + 4P_1 - P_2)e^{(1+4P_1-P_2)x+2P_3}}{e^{(1+4P_1-P_2)x+2P_3} + 5P_4} dx = \ln|e^{(1+4P_1-P_2)x+2P_3} + 5P_4| + PC$$

$$3) \int (P_5 + 3) \tan(P_4 + 1)x dx = (P_5 + 3) \int \frac{\sin(P_5 + 1)x}{\cos(P_5 + 1)x} dx = \frac{P_5 + 3}{P_4 + 1} \ln|(P_4 + 1)x| + PC$$

$$= (3 - P_5) \ln|\cos(P_4 + 1)x| + PC$$

where:

$$\frac{P_5 + 3}{P_4 + 1} = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5$$

$$P_5 + 3 = (P_4 + 1)(x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5)$$

$$P_5 + 3 = x_0P_4 + x_1P_4 + x_2P_4 + x_3P_4 + x_4P_4 + x_5P_5 + x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5$$

$$P_5 + 3 = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + (x_0 + x_1 + x_2 + x_3 + 2x_4)P_4 + (2x_5)P_5, \text{ then:}$$

$$x_0 = 3, x_1 = 0, x_2 = 0, x_3 = 0, x_4 = \frac{-3}{2}, x_5 = \frac{1}{2}$$

hence: $\frac{P_5+3}{P_4+1} = 3 - \frac{3}{2}P_4 + \frac{1}{2}P_5$

$$\begin{aligned}
 4) \int \frac{-7P_4}{1 + \tan(P_3x)} dx &= \int \frac{-7P_4}{1 + \frac{\sin(P_3x)}{\cos(P_3x)}} dx \\
 &= \frac{1}{2} \int \frac{-14P_4 \cos(P_3x)}{\cos(P_3x) + \sin(P_3x)} dx \\
 &= \frac{-7P_4}{2P_3} \int \frac{\cos(P_3x) + \sin(P_3x) + \cos(P_3x) - \sin(P_3x)}{\cos(P_3x) + \sin(P_3x)} dx \\
 &= \frac{-7P_4}{2P_3} \int dx + \frac{-7P_4}{2P_3} \int \frac{\cos(P_3x) - \sin(P_3x)}{\cos(P_3x) + \sin(P_3x)} dx \\
 &= \left(x_0 + x_1P_1 + x_2P_2 + x_3P_3 - \frac{7}{2}P_4\right)x + \left(x_0 + x_1P_1 + x_2P_2 + x_3P_3 - \frac{7}{2}P_4\right) \ln|\cos(P_3x) + \sin(P_3x)| + PC
 \end{aligned}$$

where:

$$\begin{aligned}
 \frac{-7P_4}{2P_3} &= x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 \\
 -7P_4 &= (2P_3)(x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4) \\
 -7P_4 &= 2x_0P_3 + 2x_1P_3 + 2x_2P_3 + 2x_3P_3 + 2x_4P_4 \\
 -7P_4 &= 2(x_0 + x_1 + x_2 + x_3)P_3 + 2x_4P_4, \text{ then:} \\
 x_0 + x_1 + x_2 + x_3 &= 0 \text{ and } x_4 = -\frac{7}{2}
 \end{aligned}$$

hence: $\frac{6P_4}{P_4-3P_2} = x_0 + x_1P_1 + x_2P_2 + x_3P_3 - \frac{7}{2}P_4$, where: $x_0 + x_1 + x_2 + x_3 = 0$

Theorem 3

Let $f: SPS \rightarrow SPS$, then:

$$\int \frac{\hat{f}(x, PN)}{\sqrt{f(x, PN)}} dx = 2\sqrt{f(x, PN)} + PC$$

Proof:

$$\begin{aligned}
 \text{put: } f(x, PN) = u &\Rightarrow \hat{f}(x, PN)dx = du \\
 &\Rightarrow dx = \frac{1}{\hat{f}(x, PN)} du \\
 &\Rightarrow dx = \frac{1}{\hat{u}} du
 \end{aligned}$$

$$\int \frac{\hat{f}(x, PN)}{\sqrt{f(x, PN)}} dx = \int \frac{\hat{u}}{\sqrt{u}} \frac{1}{\hat{u}} du = \int \frac{1}{\sqrt{u}} du = 2\sqrt{u} + PC$$

back to $f(x, PN)$, we get:

$$\int \frac{\hat{f}(x, PN)}{\sqrt{f(x, PN)}} dx = 2\sqrt{f(x, PN)} + PC$$

Example 3

$$1) \int \frac{(5 + P_1 - 4P_2 + 2P_3)x - 7P_2}{\sqrt{(10 + 2P_1 - 8P_2 + 4P_3)x^2 - 28P_2x}} dx = \frac{-1}{2} \sqrt{(10 + 2P_1 - 8P_2 + 4P_3)x^2 - 28P_2x} + PC$$

$$2) \int \frac{(1 + P_1)x^9}{\sqrt{(1 + P_1)x^{10} - P_1 + 9P_2}} dx = \frac{2}{9} \sqrt{(1 + P_1)x^{10} - P_1 + 9P_2} + PC$$

Theorem 4

$f: SPS \rightarrow SPS$, then:

$$\int [f(x, PN)]^n \hat{f}(x, PN) dx = \frac{[f(x, PN)]^{n+1}}{n+1} + PC$$

Proof:

$$\begin{aligned} \text{put: } f(x, PN) = u & \Rightarrow \hat{f}(x, PN) dx = du \\ & \Rightarrow dx = \frac{1}{\hat{f}(x, PN)} du \end{aligned}$$

$$\Rightarrow dx = \frac{1}{\hat{u}} du$$

$$\int [f(x, PN)]^n \hat{f}(x, PN) dx = \int u^n \hat{u} \frac{1}{\hat{u}} du = \int u^n du = \frac{u^{n+1}}{n+1} + PC$$

back to $f(x, PN)$, we get:

$$\int [f(x, PN)]^n \hat{f}(x, PN) dx = \frac{[f(x, PN)]^{n+1}}{n+1} + PC$$

Example 5

$$\begin{aligned} 1) \int P_3 x^3 [(P_2 + 1)x^3]^4 dx &= \frac{1}{4} \int 4P_3 x^3 [(P_2 + 1)x^3]^4 dx \\ &= \frac{P_3}{P_2 + 1} \frac{[(3 + 2I_1 + 2I_2)x^3]^5}{5} + PC \end{aligned}$$

$$= \frac{1}{2} P_3 \frac{[(3 + 2I_1 + 2I_2)x^3]^5}{5} + PC$$

$$= \frac{1}{10} P_3 [(3 + 2I_1 + 2I_2)x^3]^5 + PC$$

where:

$$\frac{P_3}{P_2 + 1} = x_0 + x_1 P_1 + x_2 P_2 + x_3 P_3$$

$$P_3 = (P_2 + 1)(x_0 + x_1 P_1 + x_2 P_2 + x_3 P_3)$$

$$P_3 = x_0P_3 + x_1P_3 + x_2P_3 + x_3P_3 + x_0 + x_1P_1 + x_2P_2 + x_3P_3$$

$$P_3 = x_0 + x_1P_1 + x_2P_2 + (x_0 + x_1 + x_2 + 2x_3)P_3, \text{ then:}$$

$$x_0 = 0, x_1 = 0, x_2 = 0, x_3 = \frac{1}{2}$$

hence: $\frac{P_3}{P_2+1} = \frac{1}{2}P_3$

$$\begin{aligned} 2) \int \frac{P_2}{\sqrt{P_1x - 5P_1 + 8P_2}} (\sqrt{P_1x - 5P_1 + 8P_2})^{11} dx \\ = \frac{P_2}{P_1} \frac{(\sqrt{(2 + I_1 + I_2)x - I_1 + 2I_2})^{12}}{12} + PC \\ = (x_0 + x_1P_1 + P_2) \frac{(\sqrt{(2 + I_1 + I_2)x - I_1 + 2I_2})^{12}}{12} + PC \end{aligned}$$

where:

$$\begin{aligned} \frac{P_2}{P_1} = x_0 + x_1P_1 + x_2P_2 \quad \Rightarrow \quad P_2 = x_0P_1 + x_1P_1 + x_2P_2 \\ \Rightarrow \quad P_2 = (x_0 + x_1)P_1 + x_2P_2, \text{ then:} \end{aligned}$$

$$x_0 + x_1 = 0 \text{ and } P_2 = 1$$

hence: $\frac{P_2}{P_1} = x_0 + x_1P_1 + P_2$, where: $x_0 + x_1 = 0$

5. Conclusions

In this paper, we discussed integrations in the symbolic plithogenic field, where we presented direct methods for solving most integrations of symbolic plithogenic functions, and we arrived at the condition that must be met in order for integration to be possible.

Acknowledgments " This study is supported via funding from Prince sattam bin Abdulaziz University project number (PSAU/2023/R/1445)".

References

[1] Smarandache, F., " Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)". Neutrosophic Sets and Systems, Volume 53, PP: 653-665, 2023

[2] Smarandache, F., "Plithogeny, Plithogenic Set, Logic, Probability, and Statistics", 2017.

[3] Smarandache, F., "An Overview of Plithogenic Set and Symbolic Plithogenic Algebraic Structures", Journal of Fuzzy Extension and Applications, Volume 4, pp. 48-55, 2023.

- [4] Merkepci, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.
- [5] Rawashdeh, A., "An Introduction To The Symbolic 3-plithogenic Number Theory", Neoma Journal Of Mathematics and Computer Science, 2023.
- [6] Abobala, M., and Allouf, A., " On A Novel Security Scheme for The Encryption and Decryption Of 2×2 Fuzzy Matrices with Rational Entries Based on The Algebra of Neutrosophic Integers and El-Gamal Crypto-System", Neutrosophic Sets and Systems, vol.54, 2023.
- [7] M. B. Zeina, N. Altounji, M. Abobala, and Y. Karmouta, "Introduction to Symbolic 2-Plithogenic Probability Theory," *Galoitica: Journal of Mathematical Structures and Applications*, vol. 7, no. 1, 2023.
- [8] Alhasan, Y., "The neutrosophic integrals and integration methods", Neutrosophic Sets and Systems, Volume 43, pp. 290-301, 2021.
- [9] Alhasan, Y., "The definite neutrosophic integrals and its applications", Neutrosophic Sets and Systems, Volume 49, pp. 277-293, 2022.

Received: July 12, 2023. Accepted: Nov 21, 2023