Division of refined neutrosophic numbers

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Abstract: Previously, the mathematical operations on refined neutrosophic numbers were studied by researchers, but these studies did not address the division of refined neutrosophic numbers. The aim of this research was how to find the division, in addition to discussing special cases of dividing refined neutrosophic numbers.

Keywords: division; indeterminacy; refined neutrosophic numbers; division conditions of refined neutrosophic numbers.

1. Introduction and Preliminaries

To describe a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, and contradiction, Smarandache suggested the neutrosophic Logic as an alternative to the current logics. Smarandache made refined neutrosophic numbers available in the following form: $(a, b_1I_1, b_2I_2, \ldots, b_nI_n)$ where $a, b_1, b_2, \ldots, b_n \in R$ or C [1]

Agboola introduced the concept of refined neutrosophic algebraic structures [2]. Also, the refined neutrosophic rings I was studied in paper [3], where it assumed that I splits into two indeterminacies I_1 [contradiction (true (T) and false (F))] and I_2 [ignorance (true (T) or false (F))]. It then follows logically that: [3]

$$I_1I_1 = {I_1}^2 = I_1 \qquad (1)$$

$$I_2I_2 = {I_2}^2 = I_2 \qquad (2)$$

$$I_1I_2 = I_2I_1 = I_1 \qquad (3)$$

In addition, there are many papers presenting studies on refined neutrosophic numbers [4-5-6-7-8].

This paper dealt with several topics, in the first part of which introduction and preliminaries were presented, and in the main discussion part the division of refined neutrosophic numbers and the conditions related to them were studied. In the last part, the conclusion was presented.

Main Discussion

Division of refined neutrosophic numbers

Let $\dot{w}_1, \dot{w}_2$ are two refined neutrosophic numbers, where:

$$\dot{w}_1 = \dot{a}_1 + \dot{b}_1 I_1 + \dot{c}_1 I_2 \quad , \quad \dot{w}_2 = \dot{a}_2 + \dot{b}_2 I_1 + \dot{c}_2 I_2$$

To find $\dot{w}_1 \div \dot{w}_2$, we can write:

$$\frac{\dot{a}_1 + \dot{b}_1 I_1 + \dot{c}_1 I_2}{\dot{a}_2 + \dot{b}_2 I_1 + \dot{c}_2 I_2} \equiv x + yI_1 + zI_2$$

where x , y and z are real unknowns.

$$\dot{a}_1 + \dot{b}_1 I_1 + \dot{c}_1 I_2 \equiv (\dot{a}_2 + \dot{b}_2 I_1 + \dot{c}_2 I_2)(x + yI_1 + zI_2)$$

$$\dot{a}_1 + \dot{b}_1 I_1 + \dot{c}_1 I_2 \equiv \dot{a}_2 x + \dot{a}_2 y I_1 + \dot{a}_2 z I_2 + \dot{b}_2 I_1 x + \dot{b}_2 I_1 y I_1 + \dot{b}_2 z I_1 + \dot{c}_2 I_2 x + \dot{c}_2 y I_1 + \dot{c}_2 z I_2$$

$$\dot{a}_1 + \dot{b}_1 I_1 + \dot{c}_1 I_2 \equiv \dot{a}_2 x + [\dot{b}_2 x + (\dot{a}_2 + \dot{b}_2 + \dot{c}_2) y + \dot{b}_2 z] I_1 + [\dot{c}_2 x + (\dot{a}_2 + \dot{c}_2) z] I_2$$

where: $I_1 I_2 = I_2 I_1 = I_1$

by identifying the coefficients, we get:

$$\dot{a}_2 x = \dot{a}_1$$
$$\dot{b}_2 x + (\dot{a}_2 + \dot{b}_2 + \dot{c}_2) y + \dot{b}_2 z = \dot{b}_1$$
$$\dot{c}_2 x + (\dot{a}_2 + \dot{c}_2) z = \dot{c}_1$$

We obtain unique one solution only, provided that:

$$\begin{vmatrix} \dot{a}_2 & 0 & 0 \\ \dot{b}_2 & \dot{a}_2 + \dot{b}_2 + \dot{c}_2 & \dot{b}_2 \\ \dot{c}_2 & 0 & \dot{a}_2 + \dot{c}_2 \end{vmatrix} \neq 0 \quad \Rightarrow \quad \dot{a}_2(\dot{a}_2 + \dot{c}_2)(\dot{a}_2 + \dot{b}_2 + \dot{c}_2) \neq 0$$

From this, we get on the conditions for the division of two refined neutrosophic numbers to exist:

$$\dot{a}_2 \neq 0 \quad , \ \dot{a}_2 \neq -\dot{c}_2 \quad \text{and} \ \dot{a}_2 \neq -\dot{b}_2 - \dot{c}_2$$

then:

$$x = \frac{\dot{a}_1}{\dot{a}_2}$$

$$z = \frac{\dot{a}_2 \dot{c}_1 - \dot{a}_1 \dot{c}_2}{\dot{a}_2(\dot{a}_2 + \dot{c}_2)}$$

$$\frac{\dot{a}_1 \dot{b}_2}{\dot{a}_2} + (\dot{a}_2 + \dot{b}_2 + \dot{c}_2) y + \frac{\dot{a}_2 \dot{b}_2 \dot{c}_1 - \dot{a}_1 \dot{b}_2 \dot{c}_2}{\dot{a}_2(\dot{a}_2 + \dot{c}_2)} = \dot{b}_1$$

$$(\dot{a}_2 + \dot{b}_2 + \dot{c}_2) y = \dot{b}_1 - \frac{\dot{a}_1 \dot{b}_2}{\dot{a}_2} - \left(\frac{\dot{a}_2 \dot{b}_2 \dot{c}_1 - \dot{a}_1 \dot{b}_2 \dot{c}_2}{\dot{a}_2(\dot{a}_2 + \dot{c}_2)}\right)$$

$$(\dot{a}_2+\dot{b}_2+\dot{c}_2)y=\frac{\dot{b}_1\dot{a}_2(\dot{a}_2+\dot{c}_2)-\dot{a}_1\dot{b}_2(\dot{a}_2+\dot{c}_2)-\dot{a}_2\dot{b}_2\dot{c}_1+\dot{a}_1\dot{b}_2\dot{c}_2}{\dot{a}_2(\dot{a}_2+\dot{c}_2)}$$

$$(\dot{a}_2+\dot{b}_2+\dot{c}_2)y=\frac{\dot{a}_2{}^2\dot{b}_1+\dot{a}_2\dot{b}_1\dot{c}_2-\dot{a}_1\dot{a}_2\dot{b}_2-\dot{a}_1\dot{b}_2\dot{c}_2-\dot{a}_2\dot{b}_2\dot{c}_1+\dot{a}_1\dot{b}_2\dot{c}_2}{\dot{a}_2(\dot{a}_2+\dot{c}_2)}$$

$$y=\frac{\dot{a}_2{}^2\dot{b}_1+\dot{a}_2\dot{b}_1\dot{c}_2-\dot{a}_1\dot{a}_2\dot{b}_2-\dot{a}_2\dot{b}_2\dot{c}_1}{\dot{a}_2(\dot{a}_2+\dot{c}_2)(\dot{a}_2+\dot{b}_2+\dot{c}_2)}$$

hence:

$$\frac{\dot{a}_1+\dot{b}_1I_1+\dot{c}_1I_2}{\dot{a}_2+\dot{b}_2I_1+\dot{c}_2I_2}\equiv\frac{\dot{a}_1}{\dot{a}_2}+\left[\frac{\dot{a}_2{}^2\dot{b}_1+\dot{a}_2\dot{b}_1\dot{c}_2-\dot{a}_1\dot{a}_2\dot{b}_2-\dot{a}_2\dot{b}_2\dot{c}_1}{\dot{a}_2(\dot{a}_2+\dot{c}_2)(\dot{a}_2+\dot{b}_2+\dot{c}_2)}\right]I_1+\left[\frac{\dot{a}_2\dot{c}_1-\dot{a}_1\dot{c}_2}{\dot{a}_2(\dot{a}_2+\dot{c}_2)}\right]I_2$$

Example1:

$$\frac{4+I_1+I_2}{1+2I_1+3I_2}=4-\frac{1}{4}I_1-\frac{11}{4}I_2$$

Let's check the answer:

$$(1+2I_1+3I_2)\left(4-\tfrac{1}{4}I_1-\tfrac{11}{4}I_2\right)=4+I_1+I_2 \quad \text{(True)}$$

As consequences, we have:

1) $\dfrac{\dot{a}_1+\dot{b}_1I_1+\dot{c}_1I_2}{k(\dot{a}_1+\dot{b}_1I_1+\dot{c}_1I_2)}=\dfrac{1}{k}$

$where\ \ k\neq 0\ \ ,\dot{a}_1\neq 0\quad ,\ \dot{a}_1\neq -\dot{b}_1$ and $\dot{a}_1\neq -\dot{b}_1-\dot{c}_1$

2) $\dfrac{I_1}{\dot{a}_2+\dot{b}_2I_1+\dot{c}_2I_2}=\dfrac{\dot{a}_2\dot{b}_1+\dot{b}_1\dot{c}_2}{(\dot{a}_2+\dot{c}_2)(\dot{a}_2+\dot{b}_2+\dot{c}_2)}I_1$

Example2:

$$\frac{I_1}{2-4I_1+3I_2}=I_1$$

Let's check the answer:

$$(2-4I_1+3I_2)(I_1)=I_1 \quad \text{(True)}$$

3) $\dfrac{I_2}{\dot{a}_2+\dot{b}_2I_1+\dot{c}_2I_2}=\left[\dfrac{-\dot{b}_2\dot{c}_1}{(\dot{a}_2+\dot{c}_2)(\dot{a}_2+\dot{b}_2+\dot{c}_2)}\right]I_1+\left[\dfrac{\dot{c}_1}{\dot{a}_2+\dot{c}_2}\right]I_2$

Example3:

$$\frac{I_2}{1+3I_1-5I_2}=-\frac{3}{4}I_1-\frac{1}{4}I_2$$

Let's check the answer:

$$(1+3I_1-5I_2)\left(-\tfrac{3}{4}I_1-\tfrac{1}{4}I_2\right)=I_2 \quad \text{(True)}$$

4) $\frac{I_1+I_2}{\dot{a}_2+\dot{b}_2 I_1+\dot{c}_2 I_2}=\left[\frac{\dot{a}_2\dot{b}_1+\dot{b}_1\dot{c}_2-\dot{b}_2\dot{c}_1}{(\dot{a}_2+\dot{c}_2)(\dot{a}_2+\dot{b}_2+\dot{c}_2)}\right]I_1+\left[\frac{\dot{c}_1}{\dot{a}_2+\dot{c}_2}\right]I_2$

Example4:

$$\frac{I_1+I_2}{2+I_1+2I_2}=\frac{3}{20}I_1+\frac{1}{4}I_2$$

Let's check the answer:

$$(2+I_1+2I_2)\left(\frac{3}{20}I_1+\frac{1}{4}I_2\right)=I_1+I_2 \qquad \text{(True)}$$

5) $\frac{\dot{a}_1+\dot{b}_1 I_1+\dot{c}_1 I_2}{k(I_1+I_2)}=undefined$

$where\ \ k,\dot{a}_1,\dot{b}_1$ and $\dot{c}_1$ any real number.

In particular:

$i)\ \frac{\dot{a}_1+\dot{b}_1 I_1+\dot{c}_1 I_2}{I_1+I_2}=undefined$

$ii)\ \frac{\dot{a}_1+\dot{b}_1 I_1+\dot{c}_1 I_2}{I_1}=undefined$

$iii)\ \frac{\dot{a}_1+\dot{b}_1 I_1+\dot{c}_1 I_2}{I_2}=undefined$

6) $\frac{\dot{a}_1+\dot{b}_1 I_1+\dot{c}_1 I_2}{k}=\ \frac{\dot{a}_1}{k}+\frac{\dot{a}_1}{k}I_1+\frac{\dot{c}_1}{k}I_2\ \ ;k\neq 0$

7) $\frac{k}{\dot{a}_2+\dot{b}_2 I_1+\dot{c}_2 I_2}\equiv\frac{k}{\dot{a}_2}+k\left[\frac{-\dot{b}_2}{(\dot{a}_2+\dot{c}_2)(\dot{a}_2+\dot{b}_2+\dot{c}_2)}\right]I_1-\left[\frac{k\dot{c}_2}{\dot{a}_2(\dot{a}_2+\dot{c}_2)}\right]I_2$

Where $\dot{a}_2\neq 0$, $\dot{a}_2\neq-\dot{c}_2$ and $\dot{a}_2\neq-\dot{b}_2-\dot{c}_2$

8) $\frac{k(I_1+I_2)}{\dot{a}_2+\dot{b}_2 I_1+\dot{c}_2 I_2}=k\ \left[\frac{\dot{a}_2\dot{b}_1+\dot{b}_1\dot{c}_2-\dot{b}_2\dot{c}_1}{(\dot{a}_2+\dot{c}_2)(\dot{a}_2+\dot{b}_2+\dot{c}_2)}\right]I_1+\left[\frac{k\dot{c}_1}{\dot{a}_2+\dot{c}_2}\right]I_2$

Example5:

$$\frac{4I_1+4I_2}{2+I_1+2I_2}=\frac{3}{5}I_1+I_2$$

Let's check the answer:

$$(2+I_1+2I_2)\left(\frac{3}{5}I_1+I_2\right)=4(I_1+I_2) \qquad \text{(True)}$$

Conclusions

In this work, we conclusion formula to evaluate division of refined neutrosophic numbers, also, we get on the conditions for the division of two refined neutrosophic numbers to exist. In addition to providing direct special cases for finding the result of the division.

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