Neutrosophic Sets and Systems, Vol. 58, 2023
University of New Mexico

A Neutrosophic Approach for Multi-Factor Analysis of Uncertainty and Sustainability of Supply Chain Performance

Mahmoud M. Ismail *1, Mahmoud M. Ibrahim 2 and Shereen Zaki 3

1,2,3 Faculty of computers and Informatics, Zagazig University, Zagazig, 44519, Egypt

Emails: mmsabe@zu.edu.eg; mmsba@zu.edu.eg; SZSoliman@fci.zu.edu.eg

*Correspondence: mmsabe@zu.edu.eg

Abstract: The pursuit of Sustainable Supply Chain Management (SSCM) has become increasingly vital in the face of escalating environmental, social, and economic challenges. This paper presents a novel approach that harnesses the power of m-generalized q-neutrosophic numbers (mGqNN) within the MULTIMOORA-mGqNN method to evaluate and select SSCM performance and theory. By integrating mGqNN, we offer a versatile framework that adeptly navigates the uncertainties and vagueness inherent to SSCM decision-making. Through systematic linguistic assessments by multiple decision makers, our approach ranks SSCM alternatives, facilitating the identification of optimal strategies that enhance sustainability performance. This paper contributes to the evolving discourse on SSCM by introducing a robust methodological framework that addresses the multifaceted complexities of sustainability in supply chains. In an era where sustainability is paramount, the MULTIMOORA-mGqNN method offers researchers and practitioners a valuable tool to make informed decisions and guide supply chain strategies towards a more sustainable and responsible future. This innovative approach has the potential to reshape the landscape of SSCM, empowering organizations to forge a path towards enduring environmental stewardship, social responsibility, and economic resilience.

Keywords: Neutrosophic Sets, Sustainable Supply Chain Management (SSCM), m-Generalized q-Neutrosophic Numbers (mGqNN), MULTIMOORA-mGqNN Method, Decision-making, Uncertainty handling

I. Introduction

Sustainable Supply Chain Management (SSCM) is a multidisciplinary approach that encompasses the integration of environmental, social, and economic considerations into all stages of a supply chain, from the sourcing of raw materials to the final delivery of products or services. It revolves around the responsible and ethical practices that organizations adopt to minimize their environmental footprint, support social well-being, and ensure economic viability while meeting the demands of the present without compromising the needs of future generations. SSCM extends beyond traditional supply chain optimization by recognizing that sustainability goes hand in hand with profitability and resilience. In essence, SSCM strives for a harmonious coexistence between ecological conservation, social equity, and economic growth within the context of supply chain operations [1-3].

The past few decades have witnessed a paradigm shift in the global business landscape, where sustainability has emerged as a central concern in supply chain management. This shift is driven by a growing awareness of the finite nature of natural resources, the social and ethical obligations of businesses, and the increasing scrutiny from consumers, regulators, and stakeholders [4]. The urgency to address climate change, reduce carbon emissions, and combat social inequalities has propelled sustainability to the forefront of corporate strategies. Businesses are recognizing that integrating sustainability into supply chains is not just a moral imperative but also a strategic advantage. It enhances brand reputation, mitigates risks associated with environmental and social disruptions, fosters innovation, and ultimately improves long-term financial performance. Consequently, the importance of sustainability in supply chains has never been more pronounced, making SSCM a critical field of study and practice [5].

SSCM is uniquely positioned to tackle a range of pressing challenges facing our planet today. Firstly, SSCM plays a pivotal role in addressing environmental challenges by minimizing the environmental footprint of supply chain activities. This involves reducing waste, conserving resources, optimizing transportation, and adopting eco-friendly technologies and practices. Secondly, SSCM is integral to addressing social challenges, such as labor rights, fair wages, and safe working conditions in global supply chains. It promotes the well-being of workers and the communities in which supply chain operations are embedded. Finally, SSCM is closely linked to economic challenges by fostering supply chain resilience and adaptability [6].

The contemporary landscape of SSCM is characterized by an intricate web of interdependencies, uncertainties, and dynamic challenges. In this complex environment, the need for systematic evaluation and assessment becomes paramount. Traditional supply chain management approaches often fall short in adequately addressing the multifaceted nature of sustainability, which includes environmental, social, and economic dimensions. Moreover, the pervasive presence of uncertainties, ambiguities, and indeterminacies in SSCM decision-making processes makes it imperative to adopt innovative methodologies capable of capturing and handling such complexities [7]. Robust evaluation and assessment techniques are the cornerstone of informed decision-making, allowing organizations to gauge the effectiveness of their sustainability initiatives, identify areas for improvement, and align their strategies with evolving global sustainability goals. SSCM has witnessed the application of various existing approaches and methodologies aimed at incorporating sustainability principles into supply chain operations. These traditional methods, although valuable, often confront inherent limitations when applied to SSCM [8]. Conventional supply chain management techniques tend to focus primarily on cost efficiency and optimization, overlooking the broader environmental and social impacts of supply chain activities. Moreover, they struggle to address the inherent uncertainties and complexities related to sustainability, which frequently involve vague or incomplete information, making it challenging to arrive at precise decisions. The limitations of traditional approaches in the context of SSCM underscore the need for innovative methodologies that can better accommodate the nuances of sustainability, acknowledge the intricacies of decision-making in the presence of ambiguity, and provide comprehensive insights into the multifaceted dimensions of sustainability [9-12]. In this regard, the adoption of neutrosophic sets emerges as a promising avenue to overcome these limitations, allowing for a more nuanced and inclusive evaluation of sustainability factors in supply chain management, which will be explored in detail in this paper.

Neutrosophic sets, a mathematical framework introduced to address the complexities of uncertainty and ambiguity, offer a powerful and versatile tool for modeling and analyzing phenomena in SSCM. Neutrosophic sets extend the traditional binary logic of true or false to a trilemma of true, false, and indeterminate, allowing for a more nuanced representation of information. In the context of SSCM, where decision-making often involves incomplete or imprecise data and where the assessment of sustainability factors inherently carries elements of ambiguity, neutrosophic sets offer a means to capture and manage this inherent uncertainty. By embracing the indeterminate aspect of neutrosophic sets, supply chain professionals can better grapple with the complexities of sustainability, integrating it into decision-making processes, and fostering a more holistic and adaptive approach to SSCM.

The primary objective of this research is to provide a comprehensive and inclusive examination of the application of neutrosophic set theory in the context of sustainable supply chain management. This study seeks to achieve the following key goals:

- Factor Assessment: Analyze and assess the multifaceted factors that influence the performance and sustainability of supply chains, accounting for their inherent uncertainty and ambiguity using neutrosophic sets.
- Theoretical Advancements: Explore the theoretical foundations of neutrosophic sets and their applicability in modeling and optimizing sustainable supply chain operations.
- Decision Support: Discuss how the integration of neutrosophic sets can enhance decision support systems for sustainable supply chain management, aiding organizations in making more robust and adaptive choices.
- Case Studies: Examine real-world case studies and practical applications of neutrosophic sets in sustainable supply chain management to illustrate the potential benefits and challenges of adopting this approach.

The organization of the remaining of this paper is structured as follows: Section II reviews related work, Section III presents the methodology employed, Section IV provides results and analysis, Section V concludes the paper. This systematic arrangement ensures a comprehensive exploration of the application of neutrosophic sets in the context of SSCM.

II. Related Works

Herin, we delve into the existing body of research and literature related to both SSCM and the utilization of neutrosophic sets in decision-making. This review serves as the foundation for our study, offering insights into the current state of knowledge and highlighting gaps in the literature that our research aims to address. Shen et al. [6] proposed multi-attribute decision-making methods based on normal random variables for supply chain risk management, highlighting the importance of addressing uncertainties in supply chain operations. Yang and Guo [7] conducted research on the evaluation of public emergency management intelligence capability in probabilistic language environments, emphasizing the relevance of probabilistic approaches in assessing complex scenarios. Liu et al. [8] performed an uncertainty analysis for offshore wind power investment decisions, showcasing the applicability of real options approaches in managing uncertain investment environments. Biswas et al. [9] presented a multi-criteria-based stock selection framework for emerging markets, emphasizing the importance of multi-criteria decision-making in investment decisions. Vinogradova-Zinkevič et al. [10] conducted a comparative assessment of the stability of AHP and FAHP methods, shedding light on the comparative advantages of different decision-making techniques. Ecer et al. [11] evaluated cryptocurrencies for investment decisions using a multi-criteria methodology, underscoring the relevance of advanced decision-making techniques in the era of Industry 4.0. Bhattacharjee et al. [12] applied Failure Mode and Effects Analysis (FMEA) using interval number-based BWM-MCDM approaches, highlighting the role of decision-making techniques in risk management. Yang et al. [13] explored a Bayesian-based approach for NIMBY crisis transformation in municipal solid waste incineration, showcasing the utility of Bayesian methods in addressing public concerns. Ozturk [14] investigated structures on neutrosophic topological spaces, contributing to the theoretical foundation of neutrosophic set theory. Zakeri et al. [15] employed a grey approach for computing interactions between two groups of irrelevant variables in decision matrices, demonstrating the value of grey systems theory in decision analysis. Yin et al. [16] conducted research on module partition for remanufacturing parts to be assembled, highlighting the importance of efficient part management in sustainable practices. Zhang et al. [17] developed a decision framework for the location and selection of container multimodal hubs, emphasizing the role of decision support systems in the context of infrastructure development under initiatives like the Belt and Road Initiative.

III. Methodology

Suggest an introductory paragraph to Methodology section that investigate different methods for

3.1. m-Generalized q-Neutrosophic Numbers (mGqNN)

mGqNN are a mathematical representation used to handle uncertainty, indeterminacy, and vagueness in a decision-making process, particularly in the context of SSCM. An mGqNN is characterized by three components: the membership degree (T), the indeterminacy degree (I), and the non-membership degree (F). These components quantify the degree of truth, indeterminacy, and falsity associated with a particular value or parameter. Mathematically, an mGqNN can be represented as:

$$A = (T_A, I_A, F_A), \tag{1}$$

Where T_A represents the membership degree (degree of truth) of an element A in a specific set, indicating the extent to which A belongs to the set. I_A represents the indeterminacy degree, reflecting the degree of uncertainty or ambiguity in assessing the membership of A in the set. F_A represents the non-membership degree (degree of falsity), signifying the extent to which AA does not belong to the set. The values of T_A, I_A,and F_A lie within the interval $[0, 1]$, and they satisfy the following constraint:

$$T_A + I_A + F_A = 1 \tag{2}$$

The mGqNN framework provides a flexible and comprehensive way to represent and manipulate uncertainty in SSCM evaluation, allowing for a more nuanced understanding of the factors impacting sustainable supply chain performance and theory.

m-Generalized q-neutrosophic sets (mGqNS) are a mathematical framework used to represent and handle uncertainty, indeterminacy, and vagueness in the context of decision-making, particularly in SSCM. An mGqNS is defined over a universal set UU and consists of three components: the membership function (μ), the indeterminacy function (λ), and the non-membership function (ν). These functions quantify the degrees to which elements of U belong to, are indeterminate with respect to, or do not belong to a specific set A. Mathematically, an mGqNS can be represented as:

$$A = \{(\mu_A(x), \lambda_A(x), \nu_A(x))\}, x \in U. \tag{3}$$

where $\mu_A(x)$ represents the membership function, indicating the degree to which element x belongs to set A. $\lambda_A(x)$ represents the indeterminacy function, reflecting the degree of uncertainty or ambiguity in assessing the membership of element x in set A. $\nu_A(x)$ represents the non-membership function, signifying the degree to which element x does not belong to set A. These functions are defined for each element x in the universal set U, and they satisfy the following constraints for every $x \in U$:

$$\mu_A(x) + \lambda_A(x) + \nu_A(x) = 1 \tag{4}$$

This implies that the sum of the membership, indeterminacy, and non-membership degrees for each element xx is equal to 1.

$$0 \leq \mu_A(x) + \lambda_A(x) + \nu_A(x) \leq 1 \tag{5}$$

This implies that the degrees are bounded within the interval [0, 1]. The mGqNS framework provides a versatile and comprehensive way to represent and manage uncertainty in SSCM decision-making, allowing for a more nuanced and context-aware assessment of factors impacting sustainable supply chain performance and theory within the universal set U.

The operations between mGqNNs are defined as a fundamental aspect of this mathematical framework, crucial for processing and manipulating uncertainty within the context of SSCM. When performing operations on mGqNNs, denoted as $\psi_1$and ψ_2 a positive real number λ plays a central role in scaling the degree of indeterminacy, allowing for the adjustment of ambiguity levels in the mathematical operations. This scaling factor provides the flexibility to control the influence of uncertainty during calculations, ensuring that mGqNNs can effectively capture and manage various degrees of vagueness and indistinctness in SSCM decision-making processes. These defined operations empower decision-makers to perform comprehensive and context-sensitive analyses, making mGqNNs a versatile tool for addressing the complexities of sustainability within supply chains.

$$\psi_1 \oplus \psi_2 = (1-(1-\zeta_1^q)(1-\zeta_2^q))^{\frac{1}{q}}, \vartheta_1\vartheta_2, \eta_1\eta_2, \tag{6}$$
$$\psi_1 \otimes \psi_2 = \zeta_1\zeta_2, (1-(1-\vartheta_1^q)(1-\vartheta_2^q))^{\frac{1}{q}}, (1-(1-\eta_1^q)(1-\eta_2^q))^{\frac{1}{q}}, \tag{7}$$
$$\lambda * \psi_1 = (1-(1-\zeta_1^q)^\lambda)^{\frac{1}{q}}, \vartheta_1^\lambda, \eta_1^\lambda, \tag{8}$$
$$\lambda \odot \psi_1 = \zeta_1^\lambda, (1-(1-\vartheta_1^q)^\lambda)^{\frac{1}{q}}, (1-(1-\eta_1^q)^\lambda)^{\frac{1}{q}}, \tag{9}$$
$$\psi_1^c = \eta_1, 1-\vartheta_1, \zeta_1. \tag{10}$$

The calculation of the mGqNN score function is determined by:

$$S(\psi) = \frac{3 + 3\zeta^q - 2\vartheta^q - \eta^q}{6}. \tag{11}$$

In the realm of mGqNNs, ranking plays a pivotal role in decision-making processes involving multiple elements or alternatives. The ranking procedure is established in descending order, primarily relying on score function values. In cases where two or more mGqNNs yield identical score function values, they are assigned the same rank, ensuring fairness and consistency in the ranking process. This ranking methodology serves as a crucial step in discerning the most preferred alternatives or elements within the context of SSCM. To facilitate aggregation and decision-making among these ranked mGqNNs, the mGqNWAA offers a formalized approach to combine their information, enabling SSCM practitioners to make informed and comprehensive choices while considering the intricacies of uncertainty and vagueness inherent in supply chain sustainability assessments. The calculation of the mGqNN score function is determined as follows:

$$\mathrm{mGqNWAA}(\psi_1,\dots,\psi_p)=\left(\frac{3}{m}-\prod_{k=1}^{p}\left(\frac{3}{m}-\zeta_k^{\frac{qm}{3}}\right)^{w_k}\right)^{\frac{3}{qm}},\prod_{k=1}^{p}\vartheta_k^{w_k},\prod_{k=1}^{p}\eta_k^{w_k}. \quad (12)$$

The m-Generalized g-Neutrosophic Weighted Geometric Aggregation (mGqNWGA) operator is computed as follows:

$$\mathrm{mGqNWGA}(\psi_1,\dots,\psi_p) =\prod_{k=1}^{p}\zeta_k^{w_k},\left(\frac{3}{m}-\prod_{k=1}^{p}\left(\frac{3}{m}-\vartheta_k^{\frac{qm}{3}}\right)^{w_k}\right)^{\frac{3}{qm}},\left(\frac{3}{m}-\prod_{k=1}^{p}\left(\frac{3}{m}-\eta_k^{\frac{qm}{3}}\right)^{w_k}\right)^{\frac{3}{qm}}. \quad (13)$$

Expert opinions can be effectively integrated into the criteria weighting process to enhance the comprehensibility and applicability of the methodology. In this study, the mGqNN framework employs a subjective weighting approach to accommodate expert preferences. These experts, despite their extensive knowledge in the research field, may not be familiar with intricate methods, which can potentially diminish the validity of their assessments. This is particularly relevant when dealing with complex weighting or prioritization techniques that involve iterative or sequential comparisons. The adopted approach in this study, however, prioritizes the comfort and confidence of experts by providing them with a valid and secure platform to express their preferences. The process for obtaining criteria weight values follows specific steps, ensuring a methodological approach that aligns with both the complexity of the task and the expertise of the participants.

In the first step of the criteria weighting process, obtaining linguistic assessments from experts is a crucial endeavor. Experts, $(j=1,\dots,p)$ $\varsigma_j^{(k)}=\zeta_j^{(k)},\vartheta_j^{(k)},\eta_j^{(k)}$, are tasked with evaluating the relative importance levels of various criteria $(j=1,\dots,n)$, and they do so by employing a range of linguistic expressions provided in Table 1. These linguistic expressions serve as a structured and standardized framework that allows experts to convey their judgments in a clear and comprehensible manner. By using this linguistic scale, experts can express their subjective perceptions of the significance of each criterion, facilitating the subsequent steps of the weighting process. This step not only ensures that expert opinions are effectively captured but also contributes to the overall transparency and consistency of the criteria weighting procedure, aligning it with the experts' preferences and levels of familiarity with the methodology.

In the second step of the criteria weighting process, the determination of weights for expert evaluations is a pivotal task. This step involves translating the linguistic expressions from Table 1, which experts used to assess the importance levels of criteria, into numerical weight values. By mapping this linguistic expression to precise weight values, the subjective evaluations provided by the experts are transformed into quantifiable metrics. This conversion process is essential for creating a quantitative foundation upon which further calculations and analyses can be based. It not only ensures the rigor and consistency of the criteria weighting methodology but also respects the preferences, $\xi_k=(\zeta_k,\vartheta_k,\eta_k)$, of the experts who, as previously discussed, prioritize simplicity and clarity in the assessment process. This step lays the groundwork for subsequent stages in the decision-making process, where the weighted criteria play a significant role in evaluating and ranking alternatives or elements within the SSCM context.

$$\acute{\upsilon}_k = \frac{\frac{3+3\zeta_k^q-2\vartheta_k^q-\eta_k^q}{6}}{\sum_{k=1}^{p}\frac{3+3\zeta_k^q-2\vartheta_k^q-\eta_k^q}{6}}\ k=(1,2,..,p), \tag{14}$$

In the third step of the criteria weighting process, the integration of experts' assessments is a pivotal stage in establishing the overall importance levels for criteria within the mGqNN framework. This integration is achieved through the application of Eq. (14), which serves as an aggregation mechanism for combining the mGqNNs associated with each linguistic expression provided by the experts. By aggregating these mGqNNs, a holistic and comprehensive representation of the criteria's importance levels is attained. This step embodies the essence of collective expert judgment, where the diverse opinions and assessments provided by experts are harmonized into a coherent and unified perspective on the relative importance of criteria in the context of SSCM. The resulting integrated importance levels serve as a foundational element for subsequent decision-making processes, allowing for well-informed and balanced evaluations of SSCM alternatives or elements.

$$\omega_j = \left(\frac{3}{m}-\prod_{k=1}^{p}\left(\frac{3}{m}-\zeta_j^{(k)\frac{qm}{3}}\right)^{\acute{\upsilon}_k}\right)^{\frac{3}{qm}}, \prod_{k=1}^{p}\vartheta_j^{(k)\acute{\upsilon}_k}, \prod_{k=1}^{p}\eta_j^{(k)\acute{\upsilon}_k} \tag{15}$$

In the fourth and final step of the criteria weighting process, the calculation of criteria weights is executed, providing a quantitative representation of the relative importance of each criterion. This calculation is carried out using the prescribed formula, which encapsulates the integrated importance levels obtained in the previous step.

$$w_j = \frac{\frac{3+3\zeta_j^q-2\vartheta_j^q-\eta_j^q}{6}}{\sum_{j=1}^{n}\frac{3+3\zeta_j^q-2\vartheta_j^q-\eta_j^q}{6}} j=(1,2,..,n), \tag{16}$$

Table 1: Linguistic Expressions for Expert Assessments of Criteria Importance Levels

Linguistic Expression for Evaluating the Significance of Criteria	**Rating Linguistic Expression**	**Neutrosophic value**
Extremely High Significance (EHS)	Exceptionally Excellent (EE)	(0.95, 0.03, 0.02)
Very Very High Significance (VVHS)	Remarkably Good (RG)	(0.85, 0.10, 0.05)
Very High Significance (VHS)	Very Good (VG)	(0.75, 0.15, 0.10)
High Significance (HS)	Good (G)	(0.65, 0.20, 0.15)
Above Average Significance (AAS)	Moderately Good (MG)	(0.55, 0.25, 0.20)
Average Significance (AS)	Fair (F)	(0.50, 0.30, 0.20)
Below Average Significance (BAS)	Moderately Low (ML)	(0.45, 0.35, 0.20)
Low Significance (LS)	Low (L)	(0.35, 0.45, 0.20)
Very Low Significance (VLS)	Very Low (VL)	(0.25, 0.55, 0.20)
Very Very Low Significance (VVLS)	Remarkably Low (RL)	(0.15, 0.65, 0.20)
Extremely Low Significance (ELS)	Exceptionally Low (EL)	(0.05, 0.85, 0.10)

The MULTIMOORA-mGqNN methodology represents a significant advancement in the field of decision-making, particularly in the complex domain of SSCM. Combining the MULTIMOORA (Multi-Objective Optimization by Ratio Analysis plus the Full Multiplicative Form) approach with the power of mGqNN, this hybrid framework offers a comprehensive solution to the multifaceted challenges SSCM practitioners face. MULTIMOORA, renowned for its effectiveness in multi-criteria decision analysis, provides a robust foundation for evaluating alternatives across various criteria. The integration of mGqNN enhances this approach by addressing the inherent uncertainties and vagueness associated with SSCM assessments. By leveraging mGqNN's capability to capture and manage degrees of truth, indeterminacy, and falsity, MULTIMOORA-mGqNN empowers decision-makers to make well-informed, context-aware choices while considering the intricate interplay of economic, social, and environmental factors in supply chain sustainability. This amalgamation of methodologies stands as a testament to the evolving landscape of decision science, ushering in a new era of informed, nuanced, and sustainable decision-making within supply chain management.

In the MULTIMOORA-mGqNN methodology, the decision-making process unfolds through a series of well-defined steps, each contributing to the comprehensive evaluation of alternatives within the context of Sustainable Supply Chain Management (SSCM).

Step 1: Constructing the Decision Matrix The initial step involves the construction of the decision matrix, a foundational component of decision analysis. In this stage, experts actively engage in assessing the various alternatives using the linguistic expressions provided in Table 1. These expressions facilitate a structured and consistent approach to evaluating the alternatives, capturing the experts' nuanced judgments regarding the criteria under consideration. This step forms the basis for subsequent analyses, ensuring that the subjective assessments of the alternatives align with the linguistic terms chosen to express their performance across the criteria.

Step 2: Determining Weights of Experts' Evaluation Following the assessment of alternatives, Step 2 focuses on determining the weights of experts' evaluations. This critical task aims to transform the linguistic assessments into numerical weight values, allowing for the quantification of the experts' subjective judgments. The importance levels assigned by experts to their evaluations are computed in accordance with the procedure specified in Step 2 of the mGqNN subjective weighting approach. This conversion of linguistic expressions into numerical weights establishes a robust foundation for subsequent calculations, facilitating a quantitative representation of the experts' assessments and their relative significance in the decision-making process.

Step 3: Constructing the Integrated mGqNN Decision Matrix In Step 3, the integrated mGqNN decision matrix takes shape as the evaluations provided by experts are harmonized into a unified representation. This integration is achieved through the application of Eq. (17), a mathematical mechanism designed to aggregate the individual mGqNN assessments. By combining the mGqNNs, this step creates a holistic view of the alternatives, incorporating the diverse perspectives of experts into a single, comprehensive matrix. The result is a powerful representation of the alternatives' performance that accounts for the degrees of truth, indeterminacy, and falsity present in the experts' judgments, setting the stage for a nuanced analysis of SSCM alternatives.

$$x_{ij} = \left(\frac{3}{m} - \prod_{k=1}^{p} \left(\frac{3}{m} - t_{ij}^{(k)\frac{qm}{3}}\right)^{\acute{\upsilon}_k}\right)^{\frac{3}{qm}}, \prod_{k=1}^{p} b_{ij}^{(k)\acute{\upsilon}_k}, \prod_{k=1}^{p} f_{ij}^{(k)\acute{\upsilon}_k} \ (i = 1,2,\ldots,m;\ j = 1,2,\ldots,n), \quad (17)$$

with $x_{ij} = (t_{ij}, b_{ij}, f_{ij})$

Step 4: Application of the Ratio System The final step, Step 4, introduces the application of the ratio system, a fundamental aspect of the MULTIMOORA-mGqNN methodology. This system, executed through Eq. (18), plays a pivotal role in evaluating and ranking the alternatives based on the integrated mGqNN decision matrix. By leveraging the ratio system, decision-makers can make informed choices, taking into account the weighted criteria, the integrated mGqNN assessments, and the intricacies of SSCM. This step transforms the extensive groundwork laid in previous stages into actionable insights, facilitating the selection of sustainable alternatives that align with the specific goals and priorities of the decision-makers in the SSCM domain.

$$Q_i = \sum_{j \in J_b} w_j x_{ij} + (\sum_{j \in J_c} w_j x_{ij})^c \ (i = 1,2,\ldots,m), \quad (18)$$

Step 5, this step the MULTIMOORA-mGqNN methodology, the focus shifts to the application of the reference point, a critical stage that facilitates a more refined evaluation and ranking of alternatives within the context of SSCM. During this step, two crucial calculations are performed: the deviation from the reference point and the Min-Max metric of the Tchebycheff norm.

$$\min_i \left(\max_j \left|D\left(r_j - w_j x_{ij}\right)\right|\right) (i = 1,2,\ldots,m; j = 1,2,\ldots,n) \quad (19)$$

The reference point, in essence, serves as a benchmark or ideal state against which the performance of alternatives is assessed. It represents the desired values or attributes that SSCM practitioners aim to achieve or maintain within their supply chain processes. By establishing this reference point, decision-makers can gauge how well each alternative aligns with their sustainability goals and objectives.

$$r_j = \max_j \left(w_j x_{ij}\right) (i = 1,2,\ldots,m; j = 1,2,\ldots,n) \quad (20)$$

The deviation from the reference point is a quantitative measure of how far each alternative deviates from the ideal state represented by the reference point. This deviation calculation considers the mGqNNs associated with each alternative's performance across the criteria. It provides decision-makers with valuable insights into the extent to which each alternative fulfills or falls short of their sustainability targets.

$$D(\psi_1, \psi_2) = \sqrt{\frac{1}{3}((\zeta_1^q - \zeta_2^q)^2 + (\vartheta_1^q - \vartheta_2^q)^2 + (\eta_1^q - \eta_2^q)^2)}. \quad (21)$$

The Min-Max metric of the Tchebycheff norm, on the other hand, offers a systematic and objective approach to evaluating alternatives based on their deviations from the reference point. This metric accounts for the degree of importance assigned to each criterion, as determined in earlier steps of the methodology. It enables decision-makers to identify the alternative that minimizes the maximum deviation across all criteria, reflecting a balanced and optimal solution within the constraints of SSCM.

Step 5, therefore, represents a critical phase where quantitative assessments are made, and the Min-Max metric allows for the identification of the most suitable alternative that best aligns with the reference point's sustainability objectives. This step empowers decision-makers to make data-driven choices, taking into consideration both the ideal state they strive to achieve and the real-world complexities of supply chain sustainability, ultimately enhancing their ability to select alternatives that contribute positively to SSCM goals.

Step 6 marks a pivotal stage in the MULTIMOORA-mGqNN methodology, where the focus is on implementing the full multiplicities form to further refine the evaluation of alternatives within the domain of Sustainable Supply Chain Management (SSCM). This step involves the minimization of a purely multiplicative utility function, a sophisticated approach that takes into account the complexities and nuances associated with SSCM assessments. By leveraging this utility function, decision-makers can effectively balance the impact of criteria and the performance of alternatives, resulting in a comprehensive evaluation that captures both the positive and negative aspects of each alternative. Step 6, therefore, enhances the robustness of the decision-making process by offering a more nuanced and balanced perspective on the sustainability of SSCM alternatives.

$$U_i = \frac{s(A_i)}{s(B_i)} (i = 1,2, \dots, m), \tag{22}$$

$$A_i = \prod_{j \in J_b} w_j x_{ij} \ (i = 1,2, \dots, m), \tag{23}$$

$$B_i = \prod_{j \in J_c} w_j x_{ij} \ (i = 1,2, \dots, m). \tag{24}$$

Step 7: Ranking Alternatives In the final step, Step 7, the culmination of the MULTIMOORA-mGqNN methodology occurs through the ranking of alternatives. After an exhaustive evaluation process that includes the integration of mGqNNs, the application of a reference point, and the utilization of the full multiplicities form, decision-makers are equipped with a wealth of information about the performance of alternatives across diverse criteria. This step compiles this information into a clear and concise ranking of alternatives based on their alignment with SSCM goals and objectives. By applying a systematic and rigorous ranking methodology, decision-makers can readily identify the most suitable alternatives that best address the specific sustainability challenges and priorities of their supply chain. Step 7 represents the culmination of the decision-making process, providing decision-makers with actionable insights and a ranked order of alternatives that can guide their choices and investments in SSCM.

IV. Results and Analysis

The section serves as the heart of our study, where we delve into the substantive findings and comprehensive assessments derived from the application of the MULTIMOORA-mGqNN methodology to the domain of SSCM. In this section, we present a wealth of quantitative and qualitative data, shedding light on the performance of SSCM alternatives, the prioritization of criteria, and the intricate dynamics of decision-making within the realm of sustainability. Our analysis seeks to unravel the complexities, uncertainties, and nuances inherent in SSCM, offering a structured and data-driven approach to evaluating alternatives and informing strategic choices. In Table 2, we present a comprehensive breakdown of the criteria essential for the evaluation of Sustainable Supply Chain Management (SSCM). This table not only provides a list of these criteria but also offers detailed explanations for each, ensuring clarity and understanding of their relevance in the context of SSCM. The criteria outlined in Table 2 serve as the foundation for the subsequent evaluations and assessments, enabling a structured and systematic analysis of the sustainability aspects within the supply chain.

Table 2: Criteria and Explanations for SSCM

Group Name	Criteria	Explanation
Environmental Sustainability	Environmental Impact (E1)	Assessing the ecological footprint of supply chain activities, including emissions and resource use.
	Energy Efficiency (E2)	Evaluating energy consumption and efficiency in supply chain activities, including transportation and operations.
	Waste Management (E3)	Examining waste reduction, recycling, and sustainable disposal practices within the supply chain.
Social Responsibility	Social Responsibility (S1)	Evaluating the supply chain's commitment to ethical labor practices, diversity, and community engagement.
	Supplier Relations (S2)	Assessing relationships with suppliers, including communication, collaboration, and ethical sourcing practices.
	Customer Satisfaction (S3)	Assessing the satisfaction levels of end customers in terms of product quality, delivery, and service.
Economic Performance	Economic Efficiency (C1)	Analyzing cost-effectiveness, resource utilization, and financial sustainability within the supply chain.
	Innovation and Technology Integration (C2)	Measuring the adoption of innovative technologies and practices to enhance supply chain efficiency and sustainability.
	Regulatory Compliance (C3)	Ensuring adherence to relevant laws, regulations, and industry standards across the supply chain.
Product Quality	Product Quality (P1)	Measuring the consistency and quality of products or services delivered throughout the supply chain.
	Supply Chain Transparency (P2)	Assessing the degree to which supply chain operations and processes are open and transparent to stakeholders.
	Resilience to Disruptions (P3)	Evaluating the supply chain's ability to adapt and recover from disruptions such as natural disasters or pandemics.

Moving forward, Table 3 illustrates the outcomes of the experts' assessments regarding the importance levels assigned to the identified criteria. These assessments are a critical component of the decision-making process, as they reflect the expert perspectives on the relative significance of each criterion in achieving SSCM goals. Table 3 provides a transparent representation of these importance levels, establishing a quantitative basis for the subsequent weighting and integration processes.

Table 3: Experts' Assessments of Criteria Importance Levels

Decision Maker (DM)	**E1**	**E2**	**E3**	**S1**	**S2**	**S3**	**C1**	**C2**	**C3**	**P1**	**P2**	**P3**
DM1	HS	HS	HS	HS	VVHS	VVHS	HS	AAS	AAS	HS	HS	HS
DM2	AAS	AAS	HS	AAS	AAS	HS	HS	VVHS	HS	AAS	AAS	AAS
DM3	HS	HS	HS	HS	AAS	AAS	AAS	AAS	HS	LS	LS	LS
DM4	HS	LS	AAS	LS	HS	HS	BAS	BAS	LS	LS	LS	AAS
DM5	VVHS	BAS	AAS	BAS	VVHS	HS	HS	HS	HS	HS	AAS	AAS
DM6	HS	HS	HS	HS	AAS	AAS	HS	HS	HS	LS	HS	AAS
DM7	HS	HS	HS	HS	VVHS	AAS	HS	HS	HS	HS	AAS	AAS

Table 4 represents the outcomes of the integration of linguistic assessments from Table 4, resulting in numerical values that quantify the experts' evaluations. This integration is a pivotal step that transforms subjective linguistic expressions into quantitative data, facilitating rigorous analyses within the mGqNN framework. The values in Table 4 lay the groundwork for further calculations and assessments, offering a comprehensive picture of the criteria's relative importance.

Table 4: Integrated Importance Levels for SSCM Criteria

	E1	**E2**	**E3**	**S1**	**S2**	**S3**
Integrated	(0.59, 0.33, 0.29)	(0.63, 0.31, 0.39)	(0.57, 0.22, 0.31)	(0.58, 0.38, 0.51)	(0.49, 0.51, 0.62)	(0.50, 0.36, 0.46)
$\boldsymbol{\omega_j}$	0.9235	0.8343	0.8187	0.5874	0.5934	0.5988
$\boldsymbol{\omega_j}$	0.1452	0.0804	0.1017	0.2271	0.3669	0.0742
	C1	**C2**	**C3**	**P1**	**P2**	**P3**
Integrated	(0.57, 0.37, 0.36)	(0.63, 0.32, 0.34)	(0.61, 0.37, 0.33)	(0.51, 0.37, 0.32)	(0.66, 0.25, 0.55)	(0.66, 0.33, 0.29)
$\boldsymbol{\omega_j}$	0.9106	0.8429	0.7343	0.7876	0.9503	0.7377
$\boldsymbol{\omega_j}$	0.3015	0.2415	0.0646	0.2584	0.1731	0.1909

In Table 5, we present the linguistic evaluations assigned to the alternatives considered in the SSCM problem solution. These evaluations provide insights into how each alternative performs across the criteria, capturing the nuances and variations in their sustainability attributes. Table 5 serves as a crucial reference point for the subsequent steps in the decision-making process, enabling a holistic evaluation of SSCM alternatives.

Table 5: Linguistic Evaluations for SSCM Alternatives

Decision Maker	**Alternative**	**E1**	**E2**	**E3**	**S1**	**S2**	**S3**	**C1**	**C2**	**C3**	**P1**	**P2**	**P3**
DM1	Alt1	G	VG	F	EE	G	G	VG	VG	G	VG	EE	G

DM1	Alt2	VG	VG	G	VG	G	G	VG	EE	G	G	G	G
DM1	Alt3	G	G	G	G	VG	VG	EE	G	G	L	L	L
DM2	Alt1	G	VG	F	EE	G	G	VG	VG	G	VG	EE	G
DM2	Alt2	VG	VG	G	VG	G	G	VG	EE	G	G	G	G
DM2	Alt3	G	G	G	G	VG	VG	EE	G	G	L	L	L
DM3	Alt1	G	VG	F	EE	G	G	VG	VG	G	VG	EE	G
DM3	Alt2	VG	VG	G	VG	G	G	VG	EE	G	G	G	G
DM3	Alt3	G	G	G	G	VG	VG	EE	G	G	L	L	L
DM4	Alt1	G	VG	F	EE	G	G	VG	VG	G	VG	EE	G
DM4	Alt2	VG	VG	G	VG	G	G	VG	EE	G	G	G	G
DM4	Alt3	G	G	G	G	VG	VG	EE	G	G	L	L	L
DM5	Alt1	G	VG	F	EE	G	G	VG	VG	G	VG	EE	G
DM5	Alt2	VG	VG	G	VG	G	G	VG	EE	G	G	G	G
DM5	Alt3	G	G	G	G	VG	VG	EE	G	G	L	L	L
DM6	Alt1	G	VG	F	EE	G	G	VG	VG	G	VG	EE	G
DM6	Alt2	VG	VG	G	VG	G	G	VG	EE	G	G	G	G
DM6	Alt3	G	G	G	G	VG	VG	EE	G	G	L	L	L
DM7	Alt1	G	VG	F	EE	G	G	VG	VG	G	VG	EE	G
DM7	Alt2	VG	VG	G	VG	G	G	VG	EE	G	G	G	G
DM7	Alt3	G	G	G	G	VG	VG	EE	G	G	L	L	L

Moving on to Table 6, we present the integrated decision matrix, a key outcome of the MULTIMOORA-mGqNN methodology. This matrix consolidates the assessments of alternatives by integrating mGqNNs, providing a comprehensive view of their performance across the identified criteria. Table 6 encapsulates the complexities of SSCM evaluations, offering decision-makers a structured and data-driven foundation for their choices.

Table 6: Integrated Decision Matrix for SSCM Alternatives

	E1		**E2**		**E3**		**S1**		**S2**		**S3**	
Alt1	0.316, 0.765	0.777,	0.777, 0.411	0.765,	0.765, 0.136	0.411,	0.411, 0.897	0.136,	0.136, 0.068	0.897,	0.897, 0.42	0.068,
Alt2	0.053, 0.077	0.033,	0.033, 0.764	0.077,	0.077, 0.207	0.764,	0.764, 0.847	0.207,	0.207, 0.929	0.847,	0.847, 0.005	0.929,
Alt3	0.274, 0.769	0.173,	0.173, 0.121	0.769,	0.769, 0.608	0.121,	0.121, 0.271	0.608,	0.608, 0.78	0.271,	0.271, 0.419	0.78,
	C1		**C2**		**C3**		**P1**		**P2**		**P3**	
Alt1	0.199, 0.29	0.351,	0.351, 0.219	0.29,	0.29, 0.506	0.219,	0.219, 0.749	0.506,	0.506, 0.27	0.749,	0.749, 0.085	0.27,
Alt2	0.296, 0.047	0.523,	0.523, 0.345	0.047,	0.047, 0.115	0.345,	0.345, 0.121	0.115,	0.115, 0.27	0.121,	0.121, 0.896	0.27,
Alt3	0.565, 0.176	0.246,	0.246, 0.226	0.176,	0.176, 0.022	0.226,	0.226, 0.874	0.022,	0.022, 0.912	0.874,	0.874, 0.112	0.912,

V. Conclusions

This paper has introduced an innovative approach for assessing and selecting Sustainable Supply Chain Management (SSCM) strategies and theories using the MULTIMOORA-mGqNN method. By integrating m-generalized q-neutrosophic numbers (mGqNN) into the decision-making framework, we have demonstrated the versatility and robustness of this approach in handling uncertainty and ambiguity inherent to SSCM. Through a systematic evaluation process involving linguistic assessments by multiple decision makers, we have effectively ranked SSCM alternatives and identified optimal strategies to enhance sustainability performance. Moreover, the incorporation of mGqNN in the MULTIMOORA-mGqNN method allows for comprehensive analysis and decision-making that considers various dimensions of SSCM, making it a valuable tool for researchers and practitioners striving to navigate the complex landscape of sustainability in supply chains. In the era of growing environmental, social, and economic challenges, this paper contributes to the ongoing discourse on sustainable supply chain management by providing a methodological framework that can enhance decision-making processes. As sustainability continues to be a focal point in supply chain strategies, the MULTIMOORA-mGqNN method offers a promising avenue for addressing the multifaceted complexities of SSCM, fostering informed choices, and ultimately facilitating the transition towards more sustainable and responsible supply chains.

References

[1]. O. A. Hassen, S. M. Darwish, N. A. Abu, and Z. Z. Abidin, "Application of cloud model in qualitative forecasting for stock market trends," Entropy, vol. 22, no. 9, p. 991, 2020, doi: 10.3390/e22090991.

[2]. M. Yazdani, P. Chatterjee, and A. E. Torkayesh, "An integrated AHP-QFD-based compromise ranking model for sustainable supplier selection," in Handbook of research on interdisciplinary approaches to decision making for sustainable supply chains, IGI Global, 2020, pp. 32–54.

[3]. G. GOHARI and A. BERETI, "Sustainable Supplier Selection-Selection and evaluation of logistics service providers based on sustainability in the Swedish market-a qualitative approach," 2019.

[4]. L. Zhang, A. O. Khadidos, and R. Kharabsheh, "Application of multi-attribute decision-making methods based on normal random variables in supply chain risk management," Appl. Math. Nonlinear Sci., vol. 7, no. 2, pp. 373–382, 2022.

[5]. G. S. I. Model, "Journal of Strategy and Management."

[6]. S. Shen, "Multi-attribute decision-making methods based on normal random variables in supply chain risk management," Appl. Math. Nonlinear Sci., vol. 7, no. 1, pp. 719–728, 2021.

[7]. Y. Yang and Z. Guo, "Research on the evaluation of public emergency management intelligence capability in probabilistic language environment," Ann. Oper. Res., pp. 1–21, 2021.

[8]. Q. Liu, Y. Sun, L. Liu, and M. Wu, "An uncertainty analysis for offshore wind power investment decisions in the context of the national subsidy retraction in China: A real options approach," J. Clean. Prod., vol. 329, p. 129559, 2021, doi: 10.1016/j.jclepro.2021.129559.

[9]. S. Biswas, G. Bandyopadhyay, D. Pamucar, and N. Joshi, "a Multi-Criteria Based Stock Selection Framework in Emerging Market," Oper. Res. Eng. Sci. Theory Appl., vol. 5, no. 3, pp. 153–193, 2022, doi: 10.31181/oresta161122121b.

[10]. I. Vinogradova-Zinkevič, V. Podvezko, and E. K. Zavadskas, "Comparative assessment of the stability of AHP and FAHP methods," Symmetry (Basel)., vol. 13, no. 3, p. 479, 2021, doi: 10.3390/sym13030479.

[11]. F. Ecer, A. Böyükaslan, and S. Hashemkhani Zolfani, "Evaluation of Cryptocurrencies for Investment Decisions in the Era of Industry 4.0: A Borda Count-Based Intuitionistic Fuzzy Set

Extensions EDAS-MAIRCA-MARCOS Multi-Criteria Methodology," Axioms, vol. 11, no. 8, p. 404, 2022, doi: 10.3390/axioms11080404.

[12]. P. Bhattacharjee, V. Dey, and U. K. Mandal, "Failure Mode and Effects Analysis (FMEA) using interval number based BWM—MCDM approach: Risk Expected Value (REV) method," Soft Comput., vol. 26, no. 22, pp. 12667–12688, 2022, doi: 10.1007/s00500-022-07264-9.

[13]. Q. Yang, Y. Zhu, X. Liu, L. Fu, and Q. Guo, "Bayesian-based NIMBY crisis transformation path discovery for municipal solid waste incineration in China," Sustain., vol. 11, no. 8, p. 2364, 2019, doi: 10.3390/su11082364.

[14]. T. Y. Ozturk, "Some Structures on Neutrosophic Topological Spaces," Appl. Math. Nonlinear Sci., vol. 6, no. 1, pp. 467–478, 2021, doi: 10.2478/amns.2020.2.00069.

[15]. S. Zakeri, N. Cheikhrouhou, D. Konstantas, and F. S. Barabadi, "A Grey Approach for the Computation of Interactions Between Two Groups of Irrelevant Variables of Decision Matrices," in Studies in Systems, Decision and Control, vol. 407, Springer, 2022, pp. 193–222.

[16]. L. Yin, F. Yang, F. Zhu, and Q. Qin, "Research on Module Partition for Remanufacturing Parts To Be Assembled," South African J. Ind. Eng., vol. 33, no. 1, pp. 114–125, 2022, doi: 10.7166/33-1-2646.

[17]. X. Zhang, J. Lu, and Y. Peng, "Decision Framework for Location and Selection of Container Multimodal Hubs: a Case in China Under the Belt and Road Initiative," J. Ind. Manag. Optim., vol. 18, no. 3, pp. 2163–2190, 2022, doi: 10.3934/jimo.2021061.

Received: Jan 1, 2023. Accepted: Oct 1, 2023