Interval Valued Secondary k-Range Symmetric Neutrosophic Fuzzy Matrices

M.Anandhkumar¹, G.Punithavalli², R.Jegan³, Said Broumi⁴

¹Assistant Professor, Department of Mathematics, IFET College of Engineering (Autonomous), Villupuram, Tamilnadu, India. anandhkumarmm@mail.com
²Assistant Professor, Department of Mathematics Annamalai University (Deputed to Government Arts College, Chidambaram) punithavarman78@gmail.com
³Associate Professor, Department of Mathematics Panimalar Engineering College, Chennai, Tamilnadu, India. rjch12571@gmail.com
⁴Laboratory of Information Processing, Faculty of Science Ben M’Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco broumisaid78@gmail.com

Abstract. The characterization of interval valued (IV) secondary k-range symmetric (RS) Neutrosophic fuzzy matrices have been examined in this study with an example. It is discussed how IV s-k RS, s-RS, IV k-RS, and IV RS matrices relate to one another. We establish the necessary and sufficient criteria for IV s-k RS Neutrosophic fuzzy matrices. The existence of several generalized inverses of a matrix in IV Neutrosophic fuzzy matrices. It is also established what are the equivalent criteria for various g-inverses of an IV s − k RS fuzzy matrix to be an IV s − k RS. The generalized inverses of an IV s − k RS P corresponding to the sets P{1, 2}, P{1, 2, 3} and P{1, 2, 4} are characterized.

Keywords: IV Neutrosophic Fuzzy matrix, IV RS Neutrosophic fuzzy matrix, s-k- RS IV Neutrosophic fuzzy matrix.

1. Introduction

Matrices are crucial in many fields of research in science and engineering. The traditional matrix theory is unable to address problems involving numerous kinds of uncertainties. Zadeh [1] first introduced fuzzy sets (FSs) in 1965. These are traditionally defined by their membership value or grade of membership. Assigning membership values to a fuzzy set can sometimes be challenging. Atanassov [2] introduced intutionistic FSs to solve the problem of assigning non-membership values. Smarandache [3] introduced the concept of neutrosophic sets (NSs) to handle indeterminate information and deal with problems that involve imprecision, uncertainty, and inconsistency.

Fuzzy matrices are used to solve certain kinds of issues. Many researchers have since completed numerous works. Only membership values are addressed by fuzzy matrices. These matrices cannot

1.1 Research Gap

Jayashri [24] presented the concept of range and kernel-symmetry principles to fuzzy matrix. We have applied the range and principles to NFM in this context. We have examined some of the results and extended concepts to NFMs. We first present equivalent characterizations for a RS matrix. We then derive the equivalent conditions that NFMs must meet to show range symmetry. We also find equivalent conditions that allow various generalized inverses to have range symmetric.

Notations:

IVNFM = Interval valued Neutrosophic Fuzzy Matrix,

IV = Interval valued,

RS = Range Symmetric

\[ (P_\mu, P_\lambda, P_v)_L^T = \text{Transpose of the IVNFM } (P_\mu, P_\lambda, P_v)_L, \]

\[ (P_\mu, P_\lambda, P_v)_U^T = \text{Transpose of the IVNFM } (P_\mu, P_\lambda, P_v)_U, \]

\[ (P_\mu, P_\lambda, P_v)_L^+ = \text{Moore-Penrose inverse of IVNFM } (P_\mu, P_\lambda, P_v)_L, \]

\[ (P_\mu, P_\lambda, P_v)_U^+ = \text{Moore-Penrose inverse of IVNFM } (P_\mu, P_\lambda, P_v)_U, \]

\[ R((P_\mu, P_\lambda, P_v)_L) = \text{Row space of } (P_\mu, P_\lambda, P_v)_L \]
Neutrosophic Sets and Systems, Vol. 61, 2023

M.Anandhkumar, G.Punithavalli, R.Jegan, Said Broumi, Interval Valued Secondary k-Range Symmetric Neutrosophic Fuzzy Matrices

\[ R(\{P_\mu, P_\lambda, P_\nu\}) = \text{Row space of } [P_\mu, P_\lambda, P_\nu], \]

\[ C(\{P_\mu, P_\lambda, P_\nu\}) = \text{Column space of } [P_\mu, P_\lambda, P_\nu], \]

\[ C(\{P_\mu, P_\lambda, P_\nu\}) = \text{Column space of } [P_\mu, P_\lambda, P_\nu], \]

2. Preliminaries and Definitions

2.1 Preliminary

If \( \kappa(y) = (y_{k[1]}, y_{k[2]}, y_{k[3]}, \ldots, y_{k[n]}) \in \mathbb{F}^{n \times 1} \) for \( y = y_1, y_2, \ldots, y_n \in \mathbb{F}^{1 \times n} \), where \( K \) is involuntary, The corresponding Permutation matrix is satisfied using the conditions

\[ (P.2.1) \ \ K^K = I_n \]

By the definition of \( V \), and \( R(\chi) = K\chi \)

\[ (P.2.2) \ \ V = V^T, \quad V^TV = I_n \text{ and } V^2 = I \]

\[ (P.2.3) \ \ R(\{P_\mu, P_\lambda, P_\nu\}_L) = R(\{\mu_\mu, P_\lambda, P_\nu\}_L)V, \quad R(\{P_\mu, P_\lambda, P_\nu\}_L) = \]

\[ R(\{P_\mu, P_\lambda, P_\nu\}_L)K \]

\[ R(\{P_\mu, P_\lambda, P_\nu\}_L) = R(\{P_\mu, P_\lambda, P_\nu\}_L)V, \quad R(\{P_\mu, P_\lambda, P_\nu\}_L) = \]

\[ R(\{P_\mu, P_\lambda, P_\nu\}_L)K \]

\[ R(\{P_\mu, P_\lambda, P_\nu\}_L) = R(\{P_\mu, P_\lambda, P_\nu\}_L)V, \quad R(\{P_\mu, P_\lambda, P_\nu\}_L) = \]

\[ R(\{P_\mu, P_\lambda, P_\nu\}_L)K \]

\[ (P.2.4) \ \ R(\{P_\mu, P_\lambda, P_\nu\}_L) = R(\{P_\mu, P_\lambda, P_\nu\}_L)V, \quad R(\{P_\mu, P_\lambda, P_\nu\}_L) = \]

Definition 2.1 IV Neutrosophic fuzzy matrix (IVNFM): An IV Neutrosophic fuzzy matrix \( P \) of order \( m \times n \) is defined as \( P = \{x_{ij}, p_{ij}\mu, p_{ij}\lambda, p_{ij}\nu\} \) where \( p_{ij}\mu, p_{ij}\lambda, \) and \( p_{ij}\nu \) are the subsets of \([0,1]\) which are denoted by \( p_{ij}\mu = [p_{ij}L, p_{ij}U], p_{ij}\lambda = [p_{ij}L, p_{ij}U] \) and \( p_{ij}\nu = [p_{ij}L, p_{ij}U] \) which maintaining the condition \( 0 \leq p_{ij}U + p_{ij}L \leq 3, 0 \leq p_{ij}U + p_{ij}L \leq 3, 0 \leq p_{ij}L \leq 1, 0 \leq p_{ij}L \leq 1, 0 \leq p_{ij}L \leq 1. \)

Example 2.1 Consider an IV Neutrosophic Fuzzy Matrix

\[
\begin{bmatrix}
< [0,0], [1,1], [1,1] > \\
< [0,0], [0,0], [0,0] >
\end{bmatrix}
\]

\[
\begin{bmatrix}
< [0,0], [1,1], [1,1] > \\
< [0,0], [0,0], [0,0] >
\end{bmatrix}
\]
Neutrosophic Sets and Systems, Vol. 61, 2023

Lower Limit NFM, \([P_\mu, P_\lambda, P_\nu]_L = \begin{bmatrix} <0,1,1> & <0,1,0,2,0,2> \\ <0,1,0,2,0,2> & <0,1,1> \end{bmatrix}\)

Upper Limit NFM, \([P_\mu, P_\lambda, P_\nu]_U = \begin{bmatrix} <0,1,1> & <0,3,0,4,0,5> \\ <0,3,0,4,0,5> & <0,1,1> \end{bmatrix}\)

and \(Q = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0,2,0,4],[0,3,0,5],[1,0,1,0,5]> \\ <[0,2,0,4],[0,3,0,5],[1,0,1,0,5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}\)

Then, \(P + Q = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0,2,0,4],[0,2,0,4],[1,0,1,0,5]> \\ <[0,2,0,4],[0,2,0,4],[1,0,1,0,5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}\)

\(|P| = <[0,0],[1,1],[1,1]> + <[0,0],[1,1],[1,1]> + <[0,1,0,3],[0,2,0,4],[0,2,0,5]> + <[0,1,0,3],[0,2,0,4],[0,2,0,5]> > <[0,1,0,3],[0,2,0,4],[0,2,0,5]>

Definition 2.2. For an IV Neutrosophic fuzzy matrix \(P\) is RS fuzzy matrix iff \(R([P_\mu, P_\lambda, P_\nu]_L) = R([P_\mu, P_\lambda, P_\nu]_U) = R([P_\mu, P_\lambda, P_\nu]_U^T)\).

Lemma 2.1. For a matrix \(A\) belongs to \(F_n\) and a permutation fuzzy matrix \(P, R(A) = R(B)\) iff \(R(PAQ^T) = R(PAQ^T)\).

Lemma 2.2. For interval valued fuzzy matrix \(P=KP\) iff \(KP=(KP)(KP)^T(KP)\), IV fuzzy matrix \(\Leftrightarrow PK=(PK)(PK)^T(KP)\) IV fuzzy matrix.

3. Interval valued Secondary k-KS Neutrosophic fuzzy matrix

Definition 3.1. For a Neutrosophic fuzzy matrix \(P = \langle[P_\mu, P_\lambda, P_\nu]_L, [P_\mu, P_\lambda, P_\nu]_U \rangle \in IVNFM_{nm}\)

is an IV s - symmetric fuzzy matrix iff \([P_\mu, P_\lambda, P_\nu]_L = V([P_\mu, P_\lambda, P_\nu]_L^T)V\) and \([P_\mu, P_\lambda, P_\nu]_U^T = V([P_\mu, P_\lambda, P_\nu]_U^T)V\).

Definition 3.2 For a Neutrosophic fuzzy matrix \(P\) is an IV s - RS fuzzy matrix iff

\(R([P_\mu, P_\lambda, P_\nu]_L) = R(V[P_\mu, P_\lambda, P_\nu]_L^T)V, R([P_\mu, P_\lambda, P_\nu]_U) = R(V[P_\mu, P_\lambda, P_\nu]_U^T)V\).

Definition 3.3. For a NFM \(A = \langle[P_\mu, P_\lambda, P_\nu]_L, [P_\mu, P_\lambda, P_\nu]_U \rangle\) is an IV s-k- RS fuzzy matrix iff
\[ R\left(\left[P_{\mu}, P_{\lambda}, P_{v}\right]_L\right) = R\left(KV[P_{\mu}, P_{\lambda}, P_{v}]_L^TVK\right), \quad R\left(\left[P_{\mu}, P_{\lambda}, P_{v}\right]_U\right) = R\left(KV[P_{\mu}, P_{\lambda}, P_{v}]_U^TVK\right). \]

**Lemma 3.1.** For a Neutrosophic fuzzy matrix \( P \) is an IV s-RS Neutrosophic fuzzy matrix \( \iff \) \( VA =< V[P_{\mu}, P_{\lambda}, P_{v}]_L, \quad V[P_{\mu}, P_{\lambda}, P_{v}]_U > \) IV RS Neutrosophic fuzzy matrix

\( \iff \) \( AV =< [P_{\mu}, P_{\lambda}, P_{v}]_L V, [P_{\mu}, P_{\lambda}, P_{v}]_U V > \) is an IV RS Neutrosophic fuzzy matrix.

**Proof.** Let Neutrosophic fuzzy matrix \( P \) is s-RS fuzzy matrix

\[ R\left(\left[P_{\mu}, P_{\lambda}, P_{v}\right]_L\right) = R\left(V[P_{\mu}, P_{\lambda}, P_{v}]_L^Tv\right) \quad \quad \text{[Definition 3.2]} \]

\[ R\left(\left[P_{\mu}, P_{\lambda}, P_{v}\right]_L V\right) = R\left(\left[P_{\mu}, P_{\lambda}, P_{v}\right]_L V\right)^T \]

\[ [P_{\mu}, P_{\lambda}, P_{v}]_L V \quad \text{is RS.} \quad \quad \text{[By P.22]} \]

\[ R\left(V[P_{\mu}, P_{\lambda}, P_{v}]_L V V^T\right) = R\left(VV[P_{\mu}, P_{\lambda}, P_{v}]_L^T V\right) \]

\[ R\left(V[P_{\mu}, P_{\lambda}, P_{v}]_L\right) = R\left(V[P_{\mu}, P_{\lambda}, P_{v}]_L^T\right) \]

\( V[P_{\mu}, P_{\lambda}, P_{v}]_L \quad \text{is RS.} \)

Similar manner

\[ R\left(\left[P_{\mu}, P_{\lambda}, P_{v}\right]_U\right) = R\left(V[P_{\mu}, P_{\lambda}, P_{v}]_U^Tv\right) \]

\[ R\left(\left[P_{\mu}, P_{\lambda}, P_{v}\right]_U V\right) = R\left(\left[P_{\mu}, P_{\lambda}, P_{v}\right]_U V\right)^T \]

\[ [P_{\mu}, P_{\lambda}, P_{v}]_U V \quad \text{is RS.} \]

\[ R\left(V[P_{\mu}, P_{\lambda}, P_{v}]_U V V^T\right) = R\left(VV[P_{\mu}, P_{\lambda}, P_{v}]_U^T V\right) \]

\[ R\left(V[P_{\mu}, P_{\lambda}, P_{v}]_U\right) = R\left(V[P_{\mu}, P_{\lambda}, P_{v}]_U^T\right) \]

\( V[P_{\mu}, P_{\lambda}, P_{v}]_U \quad \text{is RS.} \)

Therefore, \( VP =< V[P_{\mu}, P_{\lambda}, P_{v}]_L, V[P_{\mu}, P_{\lambda}, P_{v}]_U > \) is an IV symmetric.

**Example 3.1** Let us consider IV NFM

\[ P = \begin{bmatrix} <0,0>,<1,1>,<1,1> & <0.1,0.3>,<0.2,0.4>,<0.2,0.5> \\ <0.1,0.3>,<0.2,0.4>,<0.2,0.5> & <0,0>,<1,1>,<1,1> \end{bmatrix} \]
Lower Limit NFM, \([P_L, P_A, P_T]\) _L_ = \[
\begin{bmatrix}
<0.1,1> & <0.1,0.2,0.2>\\
<0.1,0.2,0.2> & <0.1,1>
\end{bmatrix}
\]

Upper Limit NFM, \([P_L, P_A, P_T]\) _U_ = \[
\begin{bmatrix}
<0.1,1> & <0.3,0.4,0.5>\\
<0.3,0.4,0.5> & <0.1,1>
\end{bmatrix}
\]

\[V = \begin{bmatrix} <0,0,0> & <1,1,0> \\
<1,1,0> & <0,0,0> \end{bmatrix}, \quad K = \begin{bmatrix} <1,1,0> & <0,0,0> \\
<0,0,0> & <1,1,0> \end{bmatrix}.\]

\[KVP_L^{TVK} = \begin{bmatrix} <1,1,0> & <0,0,0> \\
<0,0,0> & <1,1,0> \\
<1,1,0> & <0,0,0> \\
<0,0,0> & <1,1,0> \end{bmatrix}, \quad KVP_U^{TVK} = \begin{bmatrix} <0,1,0.2> & <0,2,0.2> \\
<0,1,0.2,0.2> & <0,1,0.2> \end{bmatrix}\]

\[KVP_L^{TVK} \neq P_L, \quad KVP_U^{TVK} \neq P_U\]

\[P_L = KP_LK\]

\[KP_LK = \begin{bmatrix} <1,1,0> & <0,0,0> \\
<0,0,0> & <1,1,0> \\
<1,1,0> & <0,0,0> \\
<0,0,0> & <1,1,0> \end{bmatrix}; \quad KVP_L^{TVK} \neq P_L\]

\[P_U = KP_UK\]

\[N(P_U) = N(KVP_L^{TVK}) = <0,0,0>\]

Therefore \(P\) is symmetric NFM, range symmetric NFM, kernel symmetric, but not both \(\kappa\) -symmetric and \(s-\kappa\) -symmetric NFM.

**Example 2.2.** Let us consider IV NFM,
\[
P = \begin{bmatrix}
[0.7, 0.2], [0.3, 0.4], [0.4, 0.6] & [0.5, 0.4], [0.3, 0.3], [0.4, 0.2] \\
[0.5, 0.4], [0.3, 0.3], [0.4, 0.2] & [0.7, 0.2], [0.3, 0.4], [0.4, 0.6]
\end{bmatrix}
\]

\[
= \begin{bmatrix}
<0, 0, 0> & <1, 1, 0> \\
<1, 1, 0> & <0, 0, 0>
\end{bmatrix} ,
K = \begin{bmatrix}
<1, 1, 0> & <0, 0, 0> \\
<0, 0, 0> & <1, 1, 0>
\end{bmatrix} .
\]

Lower Limit NFM, \( P_L = \begin{bmatrix}
<0.7, 0.3, 0.4> & <0.5, 0.3, 0.4> \\
<0.5, 0.3, 0.4> & <0.7, 0.3, 0.4>
\end{bmatrix} .
\]

Upper Limit NFM, \( P_U = \begin{bmatrix}
<0.2, 0.4, 0.6> & <0.4, 0.3, 0.2> \\
<0.4, 0.3, 0.2> & <0.2, 0.4, 0.6>
\end{bmatrix} .
\]

\[
KVP^T L VK = \begin{bmatrix}
<1, 1, 0> & <0, 0, 0> \\
<0, 0, 0> & <1, 1, 0>
\end{bmatrix} \begin{bmatrix}
<0, 0, 0> & <1, 1, 0> \\
<1, 1, 0> & <0, 0, 0>
\end{bmatrix} \begin{bmatrix}
<0.7, 0.3, 0.4> & <0.5, 0.3, 0.4> \\
<0.5, 0.3, 0.4> & <0.7, 0.3, 0.4>
\end{bmatrix}
\]

\[
KVP^T L VK = \begin{bmatrix}
<0.7, 0.3, 0.4> & <0.5, 0.3, 0.4> \\
<0.5, 0.3, 0.4> & <0.7, 0.3, 0.4>
\end{bmatrix} = P_L
\]

\( P \) is symmetric, RS, s-κ-symmetric and hence s-κ- kernel symmetric.

**Example 2.3.** Let us consider IV NFM

Lower Limit NFM, \( [P_L, P_L, P_L] = \begin{bmatrix}
<0, 0, 0> & <0, 0, 0> & <1, 1, 0> \\
<1, 1, 0> & <0, 0, 0> & <1, 1, 0> \\
<0, 0, 0> & <1, 1, 0> & <1, 1, 0>
\end{bmatrix} \)

\( K = \begin{bmatrix}
<0, 0, 0> & <1, 1, 0> & <0, 0, 0> \\
<1, 1, 0> & <0, 0, 0> & <1, 1, 0> \\
<0, 0, 0> & <1, 1, 0> & <1, 1, 0>
\end{bmatrix} , V = \begin{bmatrix}
<0, 0, 0> & <0, 0, 0> & <1, 1, 0> \\
<0, 0, 0> & <1, 1, 0> & <0, 0, 0> \\
<1, 1, 0> & <0, 0, 0> & <0, 0, 0>
\end{bmatrix} .
\]

\( KV = \begin{bmatrix}
<0, 0, 0> & <1, 1, 0> & <0, 0, 0> \\
<1, 1, 0> & <0, 0, 0> & <1, 1, 0> \\
<0, 0, 0> & <1, 1, 0> & <1, 1, 0>
\end{bmatrix} \)

\( KV = \begin{bmatrix}
<0, 1, 0> & <0, 1, 0> & <0, 1, 0> \\
<0, 1, 0> & <0, 1, 0> & <0, 1, 0> \\
<0, 1, 0> & <0, 1, 0> & <0, 1, 0>
\end{bmatrix} \)
\[ VK = \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix} \]

\[ VK = \begin{bmatrix} <0,1,0> & <0,1,0> & <1,1,0> \\ <1,1,0> & <0,1,0> & <0,1,0> \end{bmatrix} \]

\[ P_L^T VK = \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \end{bmatrix} \]

\[ KVP_L^T VK = \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \end{bmatrix} \]

\[ P_L \neq KVP_L^T VK \]

Hence \( P \) is not \( s \)-k-symmetric and not RS. But \( s \)-k-kernel symmetric.

i.e) \( N(P_L) = N(KVP_L^T VK) = <0,0,0> \)

**Theorem 3.1.** The following conditions are equivalent for \( P \in \text{IVNFM}_n \)

(i) \( P = <[P, P, P]_L, [P, P, P]_U, [P, P, P]_{LV} > \in \text{IVNFM}_{uv} \) is an IV \( s \)-\( k \)-RS.

(ii) \( KVP = <KV[P, P, P]_L, KV[P, P, P]_U > \) is an IV RS.

(iii) \( PKV = <[P, P, P]_L, KV[P, P, P]_U, KV > \) is an IV RS.

(iv) \( VP = <V[P, P, P]_L, V[P, P, P]_U, V > \) is an IV \( k \)-RS.

(v) \( PK = <[P, P, P]_L, K, [P, P, P]_U, K > \) is an IV \( s \)-RS.

(vi) \( P^T \) is an IV \( s \)-\( k \)-RS.


(ix) \( C(K\{P, \mu \}, P, v, L) = C(K\{P, \mu \}, P, v, L)^T, C(K\{P, \mu \}, P, v, U) \)
\[ = C(K\{P, \mu \}, P, v, L)^T \]

\[ = VK[P, \mu \}, P, v, L]^T VKH \text{ for } H \in IVNFM \]

\[ = H, VK[P, \mu \}, P, v, L]^T VK \text{ for } H \in IVNFM \]

\[ = KV[P, \mu \}, P, v, L]^T VKH \text{ for } H \in IVNFM \]

\[ = H, KV[P, \mu \}, P, v, L] KV \text{ for } H \in IVNFM \]

**Proof:** (i) iff (ii) iff (iv)

Let P is an IV \( s-\kappa RS \)

Let \([P, \mu \}, P, v, L] is a \( s-\kappa RS \).

\[ \text{ (By Definition 3.3)} \]

\[ \text{ By (P.23)} \]

\[ \iff KVP = VK[P, \mu \}, P, v, L], KV[P, \mu \}, P, v, L] > \text{ is an IV RS} \]

\[ \iff VP = V[P, \mu \}, P, v, L], V[P, \mu \}, P, v, L] > \text{ is an IV } \kappa- RS \]

As a conclusion (i) iff (ii) iff (iv) is true

(i) iff (ii) iff (vi)

Let P is an IV \( s-\kappa RS \)
\( \Leftrightarrow R(KV[P_\mu, P_\alpha, P_v,L]) = R(KV[P_\mu, P_\alpha, P_v,U])^T, R(KV[P_\mu, P_\alpha, P_v,L]) = R(KV[P_\mu, P_\alpha, P_v,U])^T, \)

\( \Leftrightarrow R(VK(KV[P_\mu, P_\alpha, P_v,L])) = R((VK)(P_\mu, P_\alpha, P_v,L)^TK(KV)^T) \)

\( R(VK(KV[P_\mu, P_\alpha, P_v,L])) = R((VK)(P_\mu, P_\alpha, P_v,L)^TK(KV)^T) \)

\( \Rightarrow AKV = \left[ [P_\mu, P_\alpha, P_v,L], KV, [P_\mu, P_\alpha, P_v,U], KV \right] \text{is an IV RS} \)

\( \Rightarrow AK = \left[ [P_\mu, P_\alpha, P_v,L], K, [P_\mu, P_\alpha, P_v,U], K \right] \text{is an IV s-RS} \)

As a conclusion (i) \( \Leftrightarrow \) (iii) \( \Leftrightarrow \) (v) is true. (ii) \( \Leftrightarrow \) (ix)

\( KVA = \left[ KV[P_\mu, P_\alpha, P_v,L], KV[P_\mu, P_\alpha, P_v,U] \right] \text{is an IV RS} \)

\( \Rightarrow R\left( KV[P_\mu, P_\alpha, P_v,L] \right) = R\left( \left( KV[P_\mu, P_\alpha, P_v,L] \right)^T \right), R\left( KV[P_\mu, P_\alpha, P_v,U] \right) \)

\( = R\left( \left( KV[P_\mu, P_\alpha, P_v,U] \right)^T \right) \)

(ii) \( \Leftrightarrow \) (ix) is true. (ii) \( \Leftrightarrow \) (vii)

\( KVP = \left[ KV[P_\mu, P_\alpha, P_v,L], KV[P_\mu, P_\alpha, P_v,U] \right] \text{is an IV RS} \)

\( \Rightarrow R\left( KV[P_\mu, P_\alpha, P_v,L] \right) = R\left( \left( KV[P_\mu, P_\alpha, P_v,L] \right)^T \right), R\left( KV[P_\mu, P_\alpha, P_v,U] \right) \)

\( = R\left( \left( KV[P_\mu, P_\alpha, P_v,U] \right)^T \right) \)

\( \Rightarrow R\left( [P_\mu, P_\alpha, P_v,L] \right) = R\left( [P_\mu, P_\alpha, P_v,L]^TK^T \right), R\left( [P_\mu, P_\alpha, P_v,U] \right) = R\left( [P_\mu, P_\alpha, P_v,U]^TK^T \right) \)

As a conclusion (ii) \( \Leftrightarrow \) (vii) is true. (iii) \( \Leftrightarrow \) (viii)

\( PVK = \left[ [P_\mu, P_\alpha, P_v,L], VK, [P_\mu, P_\alpha, P_v,U], VK \right] \text{is an IV RS} \)

\( \Rightarrow R\left( [P_\mu, P_\alpha, P_v,L] \right) = R\left( \left( [P_\mu, P_\alpha, P_v,L] \right)^T \right), R\left( [P_\mu, P_\alpha, P_v,U] \right) \)

\( = R\left( \left( [P_\mu, P_\alpha, P_v,U] \right)^T \right) \)

\( \Rightarrow R\left( [P_\mu, P_\alpha, P_v,L] \right) = R\left( [P_\mu, P_\alpha, P_v,L]^TK^T \right), R\left( [P_\mu, P_\alpha, P_v,U] \right) = R\left( [P_\mu, P_\alpha, P_v,U]^TK^T \right) \)

As a conclusion (iii) \( \Leftrightarrow \) (viii) is true. (i) \( \Leftrightarrow \) (vi)

Let \( A = \left[ [P_\mu, P_\alpha, P_v,L], [P_\mu, P_\alpha, P_v,U] \right] \in \text{IVNFM}_m \) is an IV s-\( \kappa \) RS

\( \Leftrightarrow R([P_\mu, P_\alpha, P_v,L]) = R(KV[P_\mu, P_\alpha, P_v,L]^TK^T), R([P_\mu, P_\alpha, P_v,U]) = R(KV[P_\mu, P_\alpha, P_v,U]^TK^T), \)

(By Definition 3.3)
\[ (KVA)^T = (KV[P_\mu, P_\alpha, P_v]_L, KV[P_\mu, P_\alpha, P_v]_L) \] is an IV RS

\[ A^T VK = ([P_\mu, P_\alpha, P_v]_L, [P_\mu, P_\alpha, P_v]_L, VK) \] is an IV RS

\[ P^T = ([P_\mu, P_\alpha, P_v]_L, [P_\mu, P_\alpha, P_v]_L) \] is an IV s-\( \kappa \) RS

As a conclusion (i) \( \iff \) (vi) is true

(i) \( \iff \) (xii) \( \iff \) (xi)

Let \( P \) is an IV s-\( \kappa \) RS

Consider \([P_\mu, P_\alpha, P_v]_L\) is a s-\( \kappa \) RS

\[ C([P_\mu, P_\alpha, P_v]_L) = C(KV[P_\mu, P_\alpha, P_v, L], C([P_\mu, P_\alpha, P_v, L]_L) = C(KV[P_\mu, P_\alpha, P_v, L, VK]) \]

By (P.2.3)

\[ [P_\mu, P_\alpha, P_v]_L = H_1 KV[P_\mu, P_\alpha, P_v, L]_L VK, [P_\mu, P_\alpha, P_v, L]_L = H_1 KV[P_\mu, P_\alpha, P_v, L, VK] \]

for \( H_1 \in IVNFM \). As a result (i) \( \iff \) (xii) \( \iff \) (xi) true.

(ii) \( \iff \) (xiii) \( \iff \) (x)

\[ AVK = [P_\mu, P_\alpha, P_v, L] VK, [P_\mu, P_\alpha, P_v, L] VK \] is an IV RS

\[ AV = [P_\mu, P_\alpha, P_v, L] VK, [P_\mu, P_\alpha, P_v, L] VK \] is an IV \( \kappa \)-RS

\[ R(V[P_\mu, P_\alpha, P_v, L]) = R(K(V[P_\mu, P_\alpha, P_v, L])^T K), \]

\[ R(V[P_\mu, P_\alpha, P_v, L]) = R(K(V[P_\mu, P_\alpha, P_v, L])^T K), \]

[By Definition 3.3]

\[ R([P_\mu, P_\alpha, P_v, L]) = R((P_\mu, P_\alpha, P_v, L)^T VK), R([P_\mu, P_\alpha, P_v, L]) = R((P_\mu, P_\alpha, P_v, L)^T VK), \]

\[ R([P_\mu, P_\alpha, P_v, L]) = R((KV[P_\mu, P_\alpha, P_v, L])^T K), R([P_\mu, P_\alpha, P_v, L]) = R((KV[P_\mu, P_\alpha, P_v, L])^T K) \]

\[ [P_\mu, P_\alpha, P_v, L] = VK[P_\mu, P_\alpha, P_v, L], VK[H_1], [P_\mu, P_\alpha, P_v, L] = VK[P_\mu, P_\alpha, P_v, L] \]

As a conclusion (ii) \( \iff \) (xiii) \( \iff \) (x) is true

The above statement can be reduced to the equivalent requirement that a matrix be an IV s- RS for \( K = I \) in particular.

Corollary:3.1 The following statements are equivalent for \( P \in IVNFM_{nm} \)

(i) \( P = [P_\mu, P_\alpha, P_v]_L, [P_\mu, P_\alpha, P_v]_L > \in IVNFM_{nm} \) is an IV s-RS.
(ii) $VP = \langle [P_{\mu}, P_{\lambda}, P_{\nu}], P_{\mu}, P_{\lambda}, P_{\nu} \rangle_L >$ is an IV RS.

(iii) $PV = \langle [P_{\mu}, P_{\lambda}, P_{\nu}], V, [P_{\mu}, P_{\lambda}, P_{\nu}] \rangle_U >$ is an IV RS.

(iv) $P^T = \langle [P_{\mu}, P_{\lambda}, P_{\nu}]_L, [P_{\mu}, P_{\lambda}, P_{\nu}]_U \rangle_T >$ is an IV $s$ - RS.

(v) $R \left( [P_{\mu}, P_{\lambda}, P_{\nu}]_L \right) = R \left( [P_{\mu}, P_{\lambda}, P_{\nu}]_L V \right), R \left( [P_{\mu}, P_{\lambda}, P_{\nu}]_U \right) = R \left( [P_{\mu}, P_{\lambda}, P_{\nu}]_U V \right)$

(vi) $R \left( [P_{\mu}, P_{\lambda}, P_{\nu}]_L \right) = R \left( [P_{\mu}, P_{\lambda}, P_{\nu}]_L V \right), R \left( [P_{\mu}, P_{\lambda}, P_{\nu}]_U \right) = R \left( [P_{\mu}, P_{\lambda}, P_{\nu}]_U V \right)$

(vii) $C \left( KV[P_{\mu}, P_{\lambda}, P_{\nu}]_L \right) = C \left( V[P_{\mu}, P_{\lambda}, P_{\nu}]_L \right)^T, C \left( KV[P_{\mu}, P_{\lambda}, P_{\nu}]_U \right) = C \left( V[P_{\mu}, P_{\lambda}, P_{\nu}]_U \right)^T$

(viii) $[P_{\mu}, P_{\lambda}, P_{\nu}]_L = V[P_{\mu}, P_{\lambda}, P_{\nu}]_L VH_1, [P_{\mu}, P_{\lambda}, P_{\nu}]_U$

$= V[P_{\mu}, P_{\lambda}, P_{\nu}]_U VH_1$ for $H_1 \in IVNFM$

(ix) $[P_{\mu}, P_{\lambda}, P_{\nu}]_L = H_1 V[P_{\mu}, P_{\lambda}, P_{\nu}]_L V, [P_{\mu}, P_{\lambda}, P_{\nu}]_U$

$= H_1 V[P_{\mu}, P_{\lambda}, P_{\nu}]_U V$ for $H_1 \in IVNFM$

(x) $[P_{\mu}, P_{\lambda}, P_{\nu}]_L = V[P_{\mu}, P_{\lambda}, P_{\nu}]_L VH_1, [P_{\mu}, P_{\lambda}, P_{\nu}]_U$

$= V[P_{\mu}, P_{\lambda}, P_{\nu}]_U VH_1$ for $H_1 \in IVNFM$

(xi) $[P_{\mu}, P_{\lambda}, P_{\nu}]_L = H_1 V[P_{\mu}, P_{\lambda}, P_{\nu}]_L V, [P_{\mu}, P_{\lambda}, P_{\nu}]_U$

$= H_1 V[P_{\mu}, P_{\lambda}, P_{\nu}]_U V$ for $H_1 \in IVNFM$

**Theorem 3.2.** For $P = \langle [P_{\mu}, P_{\lambda}, P_{\nu}], [P_{\mu}, P_{\lambda}, P_{\nu}]_L > \in IVNFM_{\omega}$ then any two of the conditions below imply the other

(i) $P$ is an IV $\kappa$- RS.

(ii) $P$ is an IV $s$-$\kappa$- RS.

(iii) $R \left( [P_{\mu}, P_{\lambda}, P_{\nu}]_L \right)^T = R \left( VK[P_{\mu}, P_{\lambda}, P_{\nu}]_L \right)^T, R \left( [P_{\mu}, P_{\lambda}, P_{\nu}]_U \right)^T = R \left( VK[P_{\mu}, P_{\lambda}, P_{\nu}]_U \right)^T$

**Proof:** (i) and (ii) implies (iii)

Let $P$ is an IV $s$ - $\kappa$ RS
\[ R((P_{\mu},P_{\hat{\alpha}},P_{v},L)_{P}) = R((P_{\mu},P_{\hat{\alpha}},P_{v},U)_{P}) = R((P_{\mu},P_{\hat{\alpha}},P_{v},V)_{P}) \]

[By Theorem 3.1]

\[ R(K[P_{\mu},P_{\hat{\alpha}},P_{v},L]_{P}) = R(K[P_{\mu},P_{\hat{\alpha}},P_{v},U]_{P}) = R(K[P_{\mu},P_{\hat{\alpha}},P_{v},V]_{P}) \]

\[ = R(K[P_{\mu},P_{\hat{\alpha}},P_{v},U]_{P})^{T}K \]  

[By Lemma 2.2]

\[ R[(P_{\mu},P_{\hat{\alpha}},P_{v})^{T}] = R\left((VK[P_{\mu},P_{\hat{\alpha}},P_{v},L]_{P})^{T}\right), R\left((P_{\mu},P_{\hat{\alpha}},P_{v},U)_{P})^{T}\right) = R\left((VK[P_{\mu},P_{\hat{\alpha}},P_{v},U])^{T}\right) \]

(i) & (ii) implies (iii) is true

(i)& (iii) implies (ii)

\[ P \text{ is an IV s-k-RS} \]

\[ R(K[P_{\mu},P_{\hat{\alpha}},P_{v},L]_{P}) = R\left([P_{\mu},P_{\hat{\alpha}},P_{v},L]_{P})^{T}\right), R\left(K[P_{\mu},P_{\hat{\alpha}},P_{v},U]_{P}) = R\left(K[P_{\mu},P_{\hat{\alpha}},P_{v},U])^{T}\right) \]

There fore,(i) & (iii) implies (ii)

\[ R\left([P_{\mu},P_{\hat{\alpha}},P_{v},L]_{P}) = R\left(KV[P_{\mu},P_{\hat{\alpha}},P_{v},L]_{P})^{T}\right), R\left([P_{\mu},P_{\hat{\alpha}},P_{v},U]) = R\left(KV[P_{\mu},P_{\hat{\alpha}},P_{v},U])^{T}\right) \]

\[ P \text{ is an IV s-k-RS} \]  

[By Theorem 3.1]

\[ \Rightarrow \text{(ii) is true} \]

(ii) & (iii) implies (i)

\[ P \text{ is an IV s- } \kappa \text{-RS} \]

\[ R(K[P_{\mu},P_{\hat{\alpha}},P_{v},L]_{P}) = R\left(K[P_{\mu},P_{\hat{\alpha}},P_{v},U]_{P}) = R\left(K[P_{\mu},P_{\hat{\alpha}},P_{v},V]_{P}) \]  

\[ \Rightarrow \text{(i) & (ii) implies (iii) is true} \]

\[ P = <[P_{\mu},P_{\hat{\alpha}},P_{v},L], [P_{\mu},P_{\hat{\alpha}},P_{v},U]_{P} > \epsilon \text{ IVNFM}_{\kappa} \]  

is an IV s- } \kappa \text{-RS}.

Therefore, (i) is true, Hence the theorem.

4. IV s - \kappa \text{ RS regular Neutrosophic fuzzy matrices}

In this section, it was discovered that there are various generalized inverses of matrices in IVNFM. The comparable standards for different g-inverses of an IV s-k RS Neutrosophic fuzzy matrix to be IV s-k RS are also established. The generalized inverses of an IV s - \kappa RS P corresponding to the sets P{1, 2}, P{1, 2, 3} and P{1, 2, 4} are characterized.

**Theorem 4.1:** Let \( P \in \text{IVNFM}_{\kappa}, Z \) belongs to P{1,2} and PW, ZW are an IV s- \kappa RS. Then P is an IV s- \kappa - RS iff \( W = <[W_{\mu},W_{\hat{\alpha}},W_{v},L], [W_{\mu},W_{\hat{\alpha}},W_{v},U]_{P} > \) is an IV s- \kappa - RS.
Proof: Let $P = \langle [P_\mu, P_\lambda, P_v], [P_\mu, P_\lambda, P_v] \rangle \in \text{IVNFM}_m$

$$R\left( KV[P_\mu, P_\lambda, P_v] \right) = R\left( \left( KV[P_\mu, P_\lambda, P_v] W[P_{\mu L}, P_{\lambda L}, P_v] \right) \right) \subseteq R\left( W[P_\mu, P_\lambda, P_v] \right)$$

$$= R\left( WVV[P_\mu, P_\lambda, P_v] \right) \subseteq R\left( WVKKV[P_\mu, P_\lambda, P_v] \right) \subseteq R\left( KV[P_\mu, P_\lambda, P_v] \right)$$

Hence, $R\left( KV[P_\mu, P_\lambda, P_v] \right) = R\left( W[P_\mu, P_\lambda, P_v] \right)$

$$= R\left( KV \left( W[P_\mu, P_\lambda, P_v] \right)^T \right) \quad \text{[WP is IV s- } \kappa\text{-RS]}$$

$$= R\left( \left( W[P_\mu, P_\lambda, P_v] \right)^T \right) \quad \text{[WP is IV s- } \kappa\text{-RS]}$$

$$\implies R\left( KV[W_\mu, W_\lambda, W_v] \right) = R\left( \left( KV[W_\mu, W_\lambda, W_v] \right)^T \right) \quad \text{[KVW is an IV RS]}$$

$$\iff R\left( KV[W_\mu, W_\lambda, W_v] \right) = R\left( \left( KV[W_\mu, W_\lambda, W_v] \right)^T \right)$$

$$\iff R\left( KV[W_\mu, W_\lambda, W_v] \right)$$

$$\iff KVX = \left[ KV[W_\mu, W_\lambda, W_v], KV[W_\mu, W_\lambda, W_v] \right]$$

is an IV RS

$W = \langle [W_\mu, W_\lambda, W_v], [W_\mu, W_\lambda, W_v] \rangle \in \text{IVNFM}_m$ is an IV RS.

Theorem 4.2: Let $P \in \text{IVNFM}_m$ $W = \langle [W_\mu, W_\lambda, W_v], [W_\mu, W_\lambda, W_v] \rangle \in \text{P}[1,2,3]$, then $R(KV[P_\mu, P_\lambda, P_v]) =$

$$R(KV[X_{\mu L}, X_{\lambda L}, X_{v L}])^T, \text{ and } R(KV[P_\mu, P_\lambda, P_v]) = R(KV[Z_{\mu L}, Z_{\lambda L}, Z_{v L}])^T$$

Then $P = \langle [P_\mu, P_\lambda, P_v], [P_\mu, P_\lambda, P_v] \rangle \in \text{IVNFM}_m$ is IV s- $\kappa$-RS $\iff W = \langle [W_\mu, W_\lambda, W_v], [W_\mu, W_\lambda, W_v] \rangle$ is IV s- $\kappa$-RS.

Proof: Given $P[1,2,3]$, Hence,
If $KVA$ is an IV RS

$$
or [P, P, P]_{[\mu, \mu, \mu]} = [P, P, P]_{[\mu, \mu, \mu], L},$$

$$[W, W, W]_{[\mu, \mu, \mu], L}[P, P, P]_{[\mu, \mu, \mu], L} = [W, W, W]_{[\mu, \mu, \mu], L},$$

$$([P, P, P]_{[\mu, \mu, \mu], L}[W, W, W]_{[\mu, \mu, \mu], L}]^T = [P, P, P]_{[\mu, \mu, \mu], L}[W, W, W]_{[\mu, \mu, \mu], L}$$

Consider, $R\left((K[V[P, P, P]_{[\mu, \mu, \mu], L}]^T = R\left([W, W, W]_{[\mu, \mu, \mu], L}^T[P, P, P]_{[\mu, \mu, \mu], L}^T VK\right)\left[\text{By using } P = PW\right]

= R\left(KV\left([P, P, P]_{[\mu, \mu, \mu], L}[W, W, W]_{[\mu, \mu, \mu], L}\right)^T\right)

= R\left(([P, P, P]_{[\mu, \mu, \mu], L}[W, W, W]_{[\mu, \mu, \mu], L}]^T \right) \quad \text{[By } P_{23}\text{]}

= R\left((W, W, W]_{[\mu, \mu, \mu], L}ight) \quad \text{[By using } W = PW\]

Similarly, we can consider, $R\left((K[V[P, P, P]_{[\mu, \mu, \mu], U}]^T = R\left([W, W, W]_{[\mu, \mu, \mu], U}^T[P, P, P]_{[\mu, \mu, \mu], U}^T VK\right)\right)$

= R\left(KV\left([P, P, P]_{[\mu, \mu, \mu], U}[W, W, W]_{[\mu, \mu, \mu], U}\right)^T\right)

= R\left(([P, P, P]_{[\mu, \mu, \mu], U}[W, W, W]_{[\mu, \mu, \mu], U}]^T \right) \quad \text{[By } P_{23}\text{]}

= R\left((W, W, W]_{[\mu, \mu, \mu], U}\right) \quad \text{[By using } W = PW\]

= R\left(KV[W, W, W]_{[\mu, \mu, \mu], U}\right) \quad \text{[By } P_{23}\text{]}

If $KVA$ is an IV RS

$$\iff R\left(KV[P, P, P]_{[\mu, \mu, \mu], L}\right) = R\left(KV[P, P, P]_{[\mu, \mu, \mu], L}\right)^T,$$

$$R\left(KV[P, P, P]_{[\mu, \mu, \mu], U}\right) = R\left(KV[P, P, P]_{[\mu, \mu, \mu], U}\right)^T$$

$$\iff R\left(KV[W, W, W]_{[\mu, \mu, \mu], L}\right) = R\left(KV[W, W, W]_{[\mu, \mu, \mu], L}\right)^T,$$

$$KVA = [K[V[W, W, W]_{[\mu, \mu, \mu], L}, KV[W, W, W]_{[\mu, \mu, \mu], U}] \text{ is an IV RS.}$$
Theorem 4.3: Let \( P \in \text{IVNFM}_{mn} \), \( Z \in \{1, 2, 4\} \), \( R(KV[P, P, P])^T = R(KV[W, W, W]) \). Then KVP is an IV s- \( \kappa \)-RS iff

\[
\]

Proof: Given, \( P \in \{1, 2, 4\} \), hence \( [P, P, P] \). Hence \( [P, P, P] \). Therefore,

\[
\]

Consider, \( R(KV[P, P, P])^T = R(KV[W, W, W])^T VK \) \( \text{By using } P = PWP \)

\[
\]

\[
\]

\[
\]

\[
\]

\[
\]

\[
\]

\[
= N((X, X, X, X, X, X)) = R(KV[W, W, W]) \text{ [By P2.3]}
\]

If KVP is an IV RS

\[
\Leftrightarrow R(KV[P, P, P]) = R(KV[P, P, P])^T,
\]

\[
R(KV[P, P, P]) = R(KV[P, P, P])^T
\]

\[
\Leftrightarrow R(KV[W, W, W]) = R(KV[W, W, W])^T
\]
\[ KVX = [KV[W_{\mu}, W_{\lambda}, W_{v}, L}, KV[W_{\mu}, W_{\lambda}, W_{v}, U], \] \] is an IV RS.

\[ W = [W_{\mu}, W_{\lambda}, W_{v}, L], [W_{\mu}, W_{\lambda}, W_{v}, U] > \] is an IV s-k RS.

The aforementioned Theorems reduce to comparable criteria, in particular for \( K = I \), for different \( g \)-inverses of interval valued s- RS to be IV secondary RS.

**Corollary 4.1:** For \( P \in IVNFM_{m}, Z \in P \{1, 2, 3\} \) and \( PW = [P_{\mu}, P_{\lambda}, P_{v}, L], [P_{\mu}, P_{\lambda}, P_{v}, U] > \), \( [P_{\mu}, P_{\lambda}, P_{v}, L], [P_{\mu}, P_{\lambda}, P_{v}, U] > \) are an IV s- RS. Then \( P \) is an IV s- RS iff \( W = [W_{\mu}, W_{\lambda}, W_{v}, L], [W_{\mu}, W_{\lambda}, W_{v}, U] > \) is an IV s- RS.

**Corollary 4.2:** For \( P \in IVNFM_{m}, W \in P \{1, 2, 3\}, R(KV[P_{\mu}, P_{\lambda}, P_{v}, L]), R(W_{\mu}, W_{\lambda}, W_{v}, L]) = R(V[W_{\mu}, W_{\lambda}, W_{v}, L])^T \), \( \), \( R(KV[P_{\mu}, P_{\lambda}, P_{v}, L]) = R(V[W_{\mu}, W_{\lambda}, W_{v}, L])^T \). Then \( P \) is an IV s RS iff \( W = [W_{\mu}, W_{\lambda}, W_{v}, L], [W_{\mu}, W_{\lambda}, W_{v}, U] > \) is an IV s RS.

**Corollary 4.3:** For \( P \in IVNFM_{m}, W \in P, R(V[P_{\mu}, P_{\lambda}, P_{v}, L])^T = R(V[W_{\mu}, W_{\lambda}, W_{v}, L])^T, R(V[P_{\mu}, P_{\lambda}, P_{v}, L])^T, \) \( = R(V[W_{\mu}, W_{\lambda}, W_{v}, L])^T \). Then \( P \) is an IV s RS iff \( W \) is an IV s-RS.

5. Conclusion:
We present equivalent characterizations of an IV k- RS, IV RS, IV s- RS, IV s-k RS NFM. Also, we give the example of s-k-symmetric fuzzy matrix is s-k- RS Neutrosophic fuzzy matrix the opposite isn't always true. We discuss various g-inverse associated with a regular matrices and obtain characterization of set of all inverses. Equivalent conditions for various g-inverses of an Interval Valued s-k-range Symmetric and s-range Symmetric NFMs are determined. In future, we shall prove some related properties of Interval Valued Secondary k-range Symmetric Neutrosophic Fuzzy Matrices.
REFERENCES


Received: Aug 9, 2023. Accepted: Dec. 17, 2023