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Interval Valued Secondary k-Range Symmetric Neutrosophic Fuzzy Matrices

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Abstract. The characterization of interval valued (IV) secondary k- range symmetric (RS) Neutrosophic fuzzy matrices have been examined in this study with an example. It is discussed how IV s-k RS, s- RS, IV k- RS, and IV RS matrices relate to one another. We establish the necessary and sufficient criteria for IV s-k RS Neutrosophic fuzzy matrices. The existence of several generalized inverses of a matrix in IV Neutrosophic fuzzy matrices. It is also established what are the equivalent criteria for various g-inverses of an IV $s-\kappa$ RS fuzzy matrix to be an IV $s-\kappa$ RS. The generalized inverses of an IV $s-\kappa$ RS P corresponding to the sets P{1, 2}, P{1, 2, 3} and P{1, 2, 4} are characterized.

Keywords: IV Neutrosophic Fuzzy matrix, IV RS Neutrosophic fuzzy matrix, s-k- RS IV Neutrosophic fuzzy matrix.

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1. Introduction

Matrices are crucial in many fields of research in science and engineering. The traditional matrix theory is unable to address problems involving numerous kinds of uncertainties. Zadeh [1] first introduced fuzzy sets (FSs) in 1965. These are traditionally defined by their membership value or grade of membership. Assigning membership values to a fuzzy set can sometimes be challenging. Atanassov [2] introduced intuitionistic FSs to solve the problem of assigning non-membership values. Smarandache [3] introduced the concept of neutrosophic sets (NSs) to handle indeterminate information and deal with problems that involve imprecision, uncertainty, and inconsistency.

Fuzzy matrices are used to solve certain kinds of issues. Many researchers have since completed numerous works. Only membership values are addressed by fuzzy matrices. These matrices cannot handle values that are not membership. Khan, Shyamal, and Pal [4] have studied intuitionistic fuzzy matrices (IFMs) for the first time. Atanassov [5,6] has discussed IFS and Operations over IV IFS. Hashimoto [7] has studied Canonical form of a transitive matrix. Kim and Roush [8] have studied generalized fuzzy matrices. Lee [9] has studied Secondary Skew Symmetric, Secondary Orthogonal Matrices. Hill and Waters [10] have analyzed On k-Real and k-Hermitian matrices. Meenakshi [11] has focussed Fuzzy Matrix: Theory and Applications. Meenakshi and Jaya Shree [12] have studied On k-kernel symmetric matrices. Meenakshi and Krishanmoorthy [13] have characterized On Secondary k-Hermitian matrices. Meenakshi and Jaya Shree [14] have studied On K -range symmetric matrices. Jaya shree [15] has studied Secondary κ-Kernel Symmetric Fuzzy Matrices. Shyamal and Pal [16] Interval valued Fuzzy matrices. Meenakshi and Kalliraja [17] have studied Regular Interval valued Fuzzy matrices. But, practically it is difficult to measure the membership or non-membership value as a point. Anandhkumar [18,19] has studied Pseudo Similarity of NFM and On various Inverse of NFM. Anandhkumar,et.al [20] have studied Generalized Symmetric Neutrosophic Fuzzy Matrices. Anandhkumar,et.al [21] have discussed Reverse Sharp and Left-T Right-T Partial Ordering on Neutrosophic Fuzzy Matrices. Pal and Susanta Kha [22] have studied IV Intuitionistic Fuzzy Matrices. Vidhya and Irene Hepzibah [23] have discussed on Interval Valued Neutrosophic Fuzzy Matrices.

1.1 Research Gap

Jayashri [24] presented the concept of range and kernel-symmetry principles to fuzzy matrix. We have applied the range and principles to NFM in this context. We have examined some of the results and extended concepts to NFMs. We first present equivalent characterizations for a RS matrix. We then derive the equivalent conditions that NFMs must meet to show range symmetry. We also find equivalent conditions that allow various generalized inverses to have range symmetric.

Notations:

IVNFM = Interval valued Neutrosophic Fuzzy Matrix,

IV =Interval valued,

RS = Range Symmetric

$[P_\mu, P_\lambda, P_v]_L^T$ = Transpose of the IVNFM $[P_\mu, P_\lambda, P_v]_L$,

$[P_\mu, P_\lambda, P_v]_U^T$ = Transpose of the IVNFM $[P_\mu, P_\lambda, P_v]_U$,

$[P_\mu, P_\lambda, P_v]_L^+$ = Moore-Penrose inverse of IVNFM $[P_\mu, P_\lambda, P_v]_L$,

$[P_\mu, P_\lambda, P_v]_U^+$ = Moore-Penrose inverse of IVNFM $[P_\mu, P_\lambda, P_v]_U$,

$R\left([P_\mu, P_\lambda, P_v]_L\right)$ = Row space of $[P_\mu, P_\lambda, P_v]_L$

$R\left([P_\mu, P_\lambda, P_v]_U\right)$ = Row space of $[P_\mu, P_\lambda, P_v]_U$,

$C\left([P_\mu, P_\lambda, P_v]_L\right)$ = Column space of $[P_\mu, P_\lambda, P_v]_L$,

$C\left([P_\mu, P_\lambda, P_v]_U\right)$ = Column space of $[P_\mu, P_\lambda, P_v]_U$

2. Preliminaries and Definitions

2.1 Preliminary

If $\kappa(y)=(y_{k[1]}, y_{k[2]}, y_{k[3]},\ldots, y_{k[n]}) \in F_{n\times 1}$ for $y = y_1, y_2,..,y_n \in F_{[1\times n]}$, where K is involuntary, The corresponding Permutation matrix is satisfied using the conditions

(P.2.1) $KK^T = K^T K = I_n$, $K = K^T$, $K^2 = I$

By the definition of V, and $R(x) = Kx$

(P.2.2) $V = V^T$, $VV^T = V^TV = I_n$ and $V^2 = I$

(P.2.3) $R\left([P_\mu, P_\lambda, P_v]_L\right) = R\left([P_\mu, P_\lambda, P_v]_L\right)V$, $R\left([P_\mu, P_\lambda, P_v]_L\right) =$

$R\left([P_\mu, P_\lambda, P_v]_L\right)K$

$R\left([P_\mu, P_\lambda, P_v]_U\right) = R\left([P_\mu, P_\lambda, P_v]_U\right)V$, $R\left([P_\mu, P_\lambda, P_v]_U\right) =$

$R\left([P_\mu, P_\lambda, P_v]_U\right)K$

(P.2.4) $R\left([P_\mu, P_\lambda, P_v]_L V\right)^T = R\left(V[P_\mu, P_\lambda, P_v]_L{}^T\right)$, $R\left(V[P_\mu, P_\lambda, P_v]_L\right)^T =$

$R\left([P_\mu, P_\lambda, P_v]_L{}^T V\right)$

$R\left([P_\mu, P_\lambda, P_v]_U V\right)^T = R\left(V[P_\mu, P_\lambda, P_v]_U{}^T\right)$, $R\left(V[P_\mu, P_\lambda, P_v]_U\right)^T =$

$R\left([P_\mu, P_\lambda, P_v]_U{}^T V\right)$

Definition:2.1 IV Neutrosophic fuzzy matrix (IVNFM): An IV Neutrosophic fuzzy matrix P of order m×n is defined as $P = [x_{ij}, < p_{ij\mu}, p_{ij\lambda}\ p_{ijv} >]_{m\times n}$ where $p_{ij\mu}$, $p_{ij\lambda}$ and p_{ijv} are the subsets of [0,1] which are denoted by $p_{ij\mu} = [p_{ij\mu L}, p_{ij\mu U}]$, $p_{ij\lambda} = [p_{ij\lambda L}, p_{ij\lambda U}]$ and $p_{ijv} = [p_{ijvL}, p_{ijvU}]$ which maintaining the condition $0 \le p_{ij\mu U} + p_{ij\lambda U} + p_{ijvU} \le 3$, $0 \le p_{ij\mu L} + p_{ij\lambda L} + p_{ijvL} \le 3$, $0 \le p_{\mu L} \le p_{\mu U} \le 1$, $0 \le p_{\lambda L} \le p_{\lambda U} \le 1$, $0 \le p_{vL} \le p_{vU} \le 1$.

Example2.1 Consider an IV Neutrosophic Fuzzy Matrix

$$P = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0.1,0.3],[0.2,0.4],[0.2,0.5]> \\ <[0.1,0.3],[0.2,0.4],[0.2,0.5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}$$

Lower Limit NFM, $[P_\mu, P_\lambda, P_v]_L = \begin{bmatrix} <0,1,1> & <0.1, 0.2, 0,2> \\ <0.1, 0.2, 0.2> & <0,1,1> \end{bmatrix}$

Upper Limit NFM, $[P_\mu, P_\lambda, P_v]_U = \begin{bmatrix} <0,1,1> & <0.3, 0.4, 0.5> \\ <0.3, 0.4, 0.5> & <0,1,1> \end{bmatrix}$

and $Q = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0.2,0.4],[0.3,0.5],[0.1,0.5]> \\ <[0.2,0.4],[0.3,0.5],[0.1,0.5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}$

Then, $P + Q = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0.2,0.4],[0.2,0.4],[0.1,0.5]> \\ <[0.2,0.4],[0.2,0.4],[0.1,0.5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}$

$Q = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0.1,0.3],[0.3,0.5],[0.2,0.5]> \\ <[0.1,0.3],[0.3,0.5],[0.2,0.5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}$

$|P| = <[0,0],[1,1],[1,1]><[0,0],[1,1],[1,1]> + <[0.1,0.3],[0.2,0.4],[0.2,0.5]><[0.1,0.3],[0.2,0.4],[0.2,0.5]>$

$|P| = <[0,0],[1,1],[1,1]> + <[0.1,0.3],[0.2,0.4],[0.2,0.5]> = <[0.1,0.3],[0.2,0.4],[0.2,0.5]>$

Definition 2.2. For IV Neutrosophic fuzzy matrix P is RS fuzzy matrix iff $R\left([P_\mu, P_\lambda, P_v]_L\right) =$

$R\left([P_\mu, P_\lambda, P_v]_L{}^T\right)$ and $R\left([P_\mu, P_\lambda, P_v]_U\right) = R\left([P_\mu, P_\lambda, P_v]_U{}^T\right)$.

Lemma 2.1. For a matrix A belongs to F_n and a permutation fuzzy matrix P, R(A) = R(B) iff $R(PAQ^T) = R(PAQ^T)$.

Lemma2.2. For interval valued fuzzy matrix P=KP^{T}K iff KP=(KP)(KP)^T(KP), IV fuzzy matrix ⇔ PK=(PK)(PK)^T(PK) IV fuzzy matrix.

3. Interval valued Secondary k-KS Neutrosophic fuzzy matrix

Definition 3.1. For a Neutrosophic fuzzy matrix $P = <[P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U> \in IVNFM_{nn}$

is an IV s - symmetric fuzzy matrix iff $[P_\mu, P_\lambda, P_v]_L = V\left([P_\mu, P_\lambda, P_v]_L{}^T\right)V$ and $[P_\mu, P_\lambda, P_v]_U$

$= V\left([P_\mu, P_\lambda, P_v]_U{}^T\right)V$.

Definition 3.2 For a Neutrosophic fuzzy matrix P is an IV s- RS fuzzy matrix iff

$$R\left([P_\mu, P_\lambda, P_v]_L\right) = R\left(V[P_\mu, P_\lambda, P_v]_L{}^T V\right), \quad R\left([P_\mu, P_\lambda, P_v]_U\right) = R\left(V[P_\mu, P_\lambda, P_v]_U{}^T V\right).$$

Definition 3.3. For a NFM $A = <[P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U>$ is an IV s-k- RS fuzzy matrix iff

$R\left([P_\mu, P_\lambda, P_v]_L\right) = R\left(KV[P_\mu, P_\lambda, P_v]_L^T VK\right)$, $R\left([P_\mu, P_\lambda, P_v]_U\right) = R\left(KV[P_\mu, P_\lambda, P_v]_U^T VK\right)$.

Lemma 3.1. For a Neutrosophic fuzzy matrix P is an IV s- RS Neutrosophic fuzzy matrix $\Leftrightarrow$

$VA = < V[P_\mu, P_\lambda, P_v]_L, \quad V[P_\mu, P_\lambda, P_v]_U >$ IV RS Neutrosophic fuzzy matrix

$\Leftrightarrow$ $AV = < [P_\mu, P_\lambda, P_v]_L V, [P_\mu, P_\lambda, P_v]_U V >$ is an IV RS Neutrosophic fuzzy matrix.

Proof. Let Neutrosophic fuzzy matrix P is s-RS fuzzy matrix

$\Leftrightarrow R\left([P_\mu, P_\lambda, P_v]_L\right) = R\left(V[P_\mu, P_\lambda, P_v]_L^T V\right)$ [Definition 3.2]

$\Leftrightarrow R\left([P_\mu, P_\lambda, P_v]_L V\right) = R\left([P_\mu, P_\lambda, P_v]_L V\right)^T$

$\Leftrightarrow [P_\mu, P_\lambda, P_v]_L V$ is RS. [By P.2.2]

$\Leftrightarrow R\left(V[P_\mu, P_\lambda, P_v]_L VV^T\right) = R\left(VV[P_\mu, P_\lambda, P_v]_L^T V\right)$

$\Leftrightarrow R\left(V[P_\mu, P_\lambda, P_v]_L\right) = R\left(V[P_\mu, P_\lambda, P_v]_L\right)^T$

$\Leftrightarrow V[P_\mu, P_\lambda, P_v]_L$ is RS.

Similar manner

$\Leftrightarrow R\left([P_\mu, P_\lambda, P_v]_U\right) = R\left(V[P_\mu, P_\lambda, P_v]_U^T V\right)$

$\Leftrightarrow R\left([P_\mu, P_\lambda, P_v]_U V\right) = R\left([P_\mu, P_\lambda, P_v]_U V\right)^T$

$\Leftrightarrow [P_\mu, P_\lambda, P_v]_U V$ is RS.

$\Leftrightarrow R\left(V[P_\mu, P_\lambda, P_v]_U VV^T\right) = R\left(VV[P_\mu, P_\lambda, P_v]_U^T V\right)$

$\Leftrightarrow R\left(V[P_\mu, P_\lambda, P_v]_U\right) = R\left(V[P_\mu, P_\lambda, P_v]_U\right)^T$

$\Leftrightarrow V[P_\mu, P_\lambda, P_v]_U$ is RS.

Therefore, $VP = < V[P_\mu, P_\lambda, P_v]_L, V[P_\mu, P_\lambda, P_v]_U >$ is an IV symmetric.

Example 3.1 Let us consider IV NFM

$$P = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0.1,0.3],[0.2,0.4],[0.2,0.5]> \\ <[0.1,0.3],[0.2,0.4],[0.2,0.5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}$$

Lower Limit NFM, $[P_\mu, P_\lambda, P_v]_L = \begin{bmatrix} <0,1,1> & <0.1,0.2,0,2> \\ <0.1,0.2,0.2> & <0,1,1> \end{bmatrix}$,

Upper Limit NFM, $[P_\mu, P_\lambda, P_v]_U = \begin{bmatrix} <0,1,1> & <0.3,0.4,0.5> \\ <0.3,0.4,0.5> & <0,1,1> \end{bmatrix}$

$V = \begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix}$, $K = \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix}$,

$$KVP_L^TVK = \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix} \begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix} \begin{bmatrix} <0,1,1> & <0.1,0.2,0,2> \\ <0.1,0.2,0.2> & <0,1,1> \end{bmatrix} \begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix} \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix}$$

$$KVP_L^TVK = \begin{bmatrix} <0,1,0.2> & <0,0.2,0.2> \\ <0.1,0.2,0.2> & <0,1,0.2> \end{bmatrix}$$

$$KVP_L^TVK \neq P_L$$

Similarly, $KVP_U^TVK \neq P_U$

$$P_L = KP_LK$$

$$KP_LK = \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix} \begin{bmatrix} <0,1,1> & <0.1,0.2,0,2> \\ <0.1,0.2,0.2> & <0,1,1> \end{bmatrix} \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix}$$

$$KP_LK = \begin{bmatrix} <0,1,0.2> & <0.1,0.2,0.2> \\ <0.1,0.2,0.2> & <0.1,1,0.2> \end{bmatrix} \neq P_L$$

Similarly, $P_U \neq KP_UK$

N (PL) = $N(KVP_L^TVK) = <0,0,0>$

Therefore P is symmetric NFM, range symmetric NFM, kernel symmetric, but not both κ –symmetric and s- κ - symmetric NFM.

Example 2.2. Let us consider IV NFM,

$$P = \begin{bmatrix} <[0.7,0.2],[0.3,0.4],[0.4,0.6]> & <[0.5,0.4],[0.3,0.3],[0.4,0.2]> \\ <[0.5,0.4],[0.3,0.3],[0.4,0.2]> & <[0.7,0.2],[0.3,0.4],[0.4,0.6]> \end{bmatrix} V$$

$$= \begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix}, K = \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix},$$

Lower Limit NFM, $P_L = \begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.4> \end{bmatrix},$

Upper Limit NFM, $P_U = \begin{bmatrix} <0.2,0.4,0.6> & <0.4,0.3,0.2> \\ <0.4,0.3,0.2> & <0.2,0.4,0.6> \end{bmatrix}$

$$KVP_L^T VK = \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix} \begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix} \begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.4> \end{bmatrix} \begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix} \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix}$$

$$KVP_L^T VK = \begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.4> \end{bmatrix} = P_L$$

P is symmetric, RS, s-κ-symmetric and hence s- k- kernel symmetric.

Example 2.3. Let us consider IV NFM

Lower limit NFM, $[P_\mu, P_\lambda, P_v]_L = \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0.5,0.3,0.4> & <1,1,0> & <0,0,0> \\ <0.4,0.2,0.6> & <0.5,0.3,0.4> & <0,0,0> \end{bmatrix}$

$$K = \begin{bmatrix} <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \\ <0,0,0> & <0,0,0> & <1,1,0> \end{bmatrix}, V = \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix}$$

$$KV = \begin{bmatrix} <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \\ <0,0,0> & <0,0,0> & <1,1,0> \end{bmatrix} \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix}$$

$$KV = \begin{bmatrix} <0,1,0> & <0,1,0> & <0,1,0> \\ <0,1,0> & <0,1,0> & <1,1,0> \\ <1,1,0> & <0,1,0> & <0,1,0> \end{bmatrix}$$

$$VK=\begin{bmatrix}<0,0,0> & <0,0,0> & <1,1,0>\\ <0,0,0> & <1,1,0> & <0,0,0>\\ <1,1,0> & <0,0,0> & <0,0,0>\end{bmatrix}\begin{bmatrix}<0,0,0> & <1,1,0> & <0,0,0>\\ <1,1,0> & <0,0,0> & <0,0,0>\\ <0,0,0> & <0,0,0> & <1,1,0>\end{bmatrix}$$

$$VK=\begin{bmatrix}<0,1,0> & <0,1,0> & <1,1,0>\\ <1,1,0> & <0,1,0> & <0,1,0>\\ <0,1,0> & <1,1,0> & <0,1,0>\end{bmatrix}$$

$$P_L^{T}VK=\begin{bmatrix}<0.5,0.8,0.4> & <0.4,0.8,0.6> & <0,0,0.4>\\ <0,0.7,0> & <0.5,0.7,0> & <0,0.7,0>\\ <0,0,0> & <0,0,0> & <1,0,0>\end{bmatrix}$$

$$KVP_L^{T}VK=\begin{bmatrix}<0,1,0> & <0,1,0> & <0,1,0>\\ <0,1,0> & <0,1,0> & <1,1,0>\\ <1,1,0> & <0,1,0> & <0,1,0>\end{bmatrix}\begin{bmatrix}<0.5,0.8,0.4> & <0.4,0.8,0.6> & <0,0,0.4>\\ <0,0.7,0> & <0.5,0.7,0> & <0,0.7,0>\\ <0,0,0> & <0,0,0> & <1,0,0>\end{bmatrix}$$

$$KVP_L^{T}VK=\begin{bmatrix}<0,0,0> & <0,0.2,0> & <0,0,0>\\ <0,0,0> & <0,0,0> & <1,0,0>\\ <0.5,0,0> & <0.4,0,0> & <0,0,0>\end{bmatrix}\neq P_L$$

$$P_L \neq KVP_L^{T}VK$$

Hence P is not s- k-symmetric and not RS. But s- k- kernel symmetric.

i.e) $N(P_L) = N(KVP_L^TVK) = <0,0,0>$

Theorem 3.1. The following conditions are equivalent for $P \in \text{IVNFM}_n$

(i) $P=<[P_\mu,P_\lambda,P_v]_L,[P_\mu,P_\lambda,P_v]_U>\in \text{IVNFM}_{nn}$ is an IV $s-\kappa$ RS.

(ii) $KVP=<KV[P_\mu,P_\lambda,P_v]_L, KV[P_\mu,P_\lambda,P_v]_U>$ is an IV RS.

(iii) $PKV=<[P_\mu,P_\lambda,P_v]_L KV,[P_\mu,P_\lambda,P_v]_U KV>$ is an IV RS.

(iv) $VP=<V[P_\mu,P_\lambda,P_v]_L, V[P_\mu,P_\lambda,P_v]_U>$ is an IV k- RS.

(v) $PK=<[P_\mu,P_\lambda,P_v]_L K,[P_\mu,P_\lambda,P_v]_U K>$ is an IV s- RS.

(vi) P^T is an IV s-k RS.

(vii) $R([P_\mu,P_\lambda,P_v]_L)=\left(R[P_\mu,P_\lambda,P_v]_L^{T}\,VK\right), R([P_\mu,P_\lambda,P_v]_U)=\left(R[P_\mu,P_\lambda,P_v]_U^{T}\,VK\right)$

(viii) $R([P_\mu,P_\lambda,P_v]_L^{T})=\left(R[P_\mu,P_\lambda,P_v]_L\,VK\right), R([P_\mu,P_\lambda,P_v]_U^{T})=\left(R[P_\mu,P_\lambda,P_v]_U\,VK\right)$

(ix) $C(KV[P_{\mu},P_{\lambda},P_{v}]_{L}) = C\left(KV[P_{\mu},P_{\lambda},P_{v}]_{L}{}^{T}\right)^{T}, C(KV[P_{\mu},P_{\lambda},P_{v}]_{U})$

$= C\left(KV[P_{\mu},P_{\lambda},P_{v}]_{U}{}^{T}\right)^{T}$

(x) $[P_{\mu},P_{\lambda},P_{v}]_{L} = VK[P_{\mu},P_{\lambda},P_{v}]_{L}{}^{T}\, VKH_{1}, [P_{\mu},P_{\lambda},P_{v}]_{U}$

$= VK[P_{\mu},P_{\lambda},P_{v}]_{U}{}^{T}\, VKH_{1}$ for $H_{1} \in IVNFM$

(xi) $[P_{\mu},P_{\lambda},P_{v}]_{L} = H_{1}KV[P_{\mu},P_{\lambda},P_{v}]_{L}{}^{T}\, KV, [P_{\mu},P_{\lambda},P_{v}]_{U}$

$= H_{1}KV[P_{\mu},P_{\lambda},P_{v}]_{U}{}^{T}\, VK$ for $H_{1} \in IVNFM$

(xii) $[P_{\mu},P_{\lambda},P_{v}]_{L}{}^{T} = KV[P_{\mu},P_{\lambda},P_{v}]_{L}\, VKH_{1}, [P_{\mu},P_{\lambda},P_{v}]_{U}{}^{T}$

$= KV[P_{\mu},P_{\lambda},P_{v}]_{U}\, VKH_{1}$ for $H_{1} \in IVNFM$

(xiii) $[P_{\mu},P_{\lambda},P_{v}]_{L}{}^{T} = H_{1}KV[P_{\mu},P_{\lambda},P_{v}]_{L}\, KV, [P_{\mu},P_{\lambda},P_{v}]_{U}$

$= H_{1}KV[P_{\mu},P_{\lambda},P_{v}]_{U}\, VK$ for $H_{1} \in IVNFM$

Proof: (i) iff (ii) iff (iv)

Let P is an IV s – κ RS

Let $[P_{\mu},P_{\lambda},P_{v}]_{L}$ is a s – κ RS.

$$\Leftrightarrow R([P_{\mu},P_{\lambda},P_{v}]_{L}) = R(KV[P_{\mu},P_{\lambda},P_{v}]_{L}{}^{T}VK), R([P_{\mu},P_{\lambda},P_{v}]_{U}) = R(KV[P_{\mu},P_{\lambda},P_{v}]_{U}{}^{T}VK),$$

(By Definition 3.3)

$$\Leftrightarrow R(KV[P_{\mu},P_{\lambda},P_{v}]_{L}) = R(KV[P_{\mu},P_{\lambda},P_{v}]_{L})^{T}, R([P_{\mu},P_{\lambda},P_{v}]_{U}) = R(KV[P_{\mu},P_{\lambda},P_{v}]_{U})^{T}$$

By (P.2.3)

$\Leftrightarrow$ $KVP = < KV[P_{\mu},P_{\lambda},P_{v}]_{L}, KV[P_{\mu},P_{\lambda},P_{v}]_{U} >$ is an IV RS

$\Leftrightarrow$ $VP = < V[P_{\mu},P_{\lambda},P_{v}]_{L}, V[P_{\mu},P_{\lambda},P_{v}]_{U} >$ is an IV κ- RS

As a conclusion (i) iff (ii) iff (iv) is true

(i) iff (ii) iff (v)

Let P is an IV s – κ RS

$\Leftrightarrow R(KV[P_\mu,P_\lambda,P_v]_L) = R(KV[P_\mu,P_\lambda,P_v]_L)^T, R(KV[P_\mu,P_\lambda,P_v]_U) = R(KV[P_\mu,P_\lambda,P_v]_U)^T,$

$\Leftrightarrow R(VK(KV[P_\mu,P_\lambda,P_v]_L)) = R((VK)[P_\mu,P_\lambda,P_v]_L{}^T VK(VK)^T)$,

$R(VK(KV[P_\mu,P_\lambda,P_v]_U)) = R((VK)[P_\mu,P_\lambda,P_v]_U{}^T VK(VK)^T)$

$\Leftrightarrow AKV = \left[[P_\mu,P_\lambda,P_v]_L KV, [P_\mu,P_\lambda,P_v]_U KV\right]$ is an IV RS

$\Leftrightarrow AK = \left[[P_\mu,P_\lambda,P_v]_L K, [P_\mu,P_\lambda,P_v]_U K\right]$ is an IV s- RS

As a conclusion (i) $\Leftrightarrow$ (iii) $\Leftrightarrow$ (v) is true. (ii) $\Leftrightarrow$ (ix)

$KVA = \left[KV[P_\mu,P_\lambda,P_v]_L, KV[P_\mu,P_\lambda,P_v]_U\right]$ is an IV RS

$\Leftrightarrow R\left(KV[P_\mu,P_\lambda,P_v]_L\right) = R\left(\left(KV[P_\mu,P_\lambda,P_v]_L\right)^T\right), R\left(KV[P_\mu,P_\lambda,P_v]_U\right)$

$= R\left(\left(KV[P_\mu,P_\lambda,P_v]_U\right)^T\right)$

(ii) $\Leftrightarrow$ (ix) is true. (ii) $\Leftrightarrow$ (vii)

$KVP = \left[KV[P_\mu,P_\lambda,P_v]_L, KV[P_\mu,P_\lambda,P_v]_U\right]$ is an IV RS.

$\Leftrightarrow R\left(KV[P_\mu,P_\lambda,P_v]_L\right) = R\left(\left(KV[P_\mu,P_\lambda,P_v]_L\right)^T\right), R\left(KV[P_\mu,P_\lambda,P_v]_U\right)$

$= R\left(\left(KV[P_\mu,P_\lambda,P_v]_U\right)^T\right)$

$\Leftrightarrow R\left([P_\mu,P_\lambda,P_v]_L\right) = R\left([P_\mu,P_\lambda,P_v]_L{}^T VK\right), R\left([P_\mu,P_\lambda,P_v]_U\right) = R\left([P_\mu,P_\lambda,P_v]_U{}^T VK\right)$

As a conclusion (ii) $\Leftrightarrow$ (vii) is true. (iii) $\Leftrightarrow$ (viii)

$PVK = \left[[P_\mu,P_\lambda,P_v]_L VK, [P_\mu,P_\lambda,P_v]_U VK\right]$

$\Leftrightarrow R\left([P_\mu,P_\lambda,P_v]_L VK\right) = R\left(\left([P_\mu,P_\lambda,P_v]_L VK\right)^T\right), R\left([P_\mu,P_\lambda,P_v]_U VK\right)$

$= R\left(\left([P_\mu,P_\lambda,P_v]_U VK\right)^T\right)$

$\Leftrightarrow R\left([P_\mu,P_\lambda,P_v]_L VK\right) = R\left([P_\mu,P_\lambda,P_v]_L\right)^T, R\left([P_\mu,P_\lambda,P_v]_U VK\right) = R\left([P_\mu,P_\lambda,P_v]_U\right)^T$

As a conclusion (iii) $\Leftrightarrow$ (viii) is true. (i) $\Leftrightarrow$ (vi)

Let $A = \langle [P_\mu,P_\lambda,P_v]_L, [P_\mu,P_\lambda,P_v]_U \rangle \in IVNFM_{nn}$ is an IV s – κ RS

$\Leftrightarrow R([P_\mu,P_\lambda,P_v]_L) = R(KV[P_\mu,P_\lambda,P_v]_L{}^T VK), R([P_\mu,P_\lambda,P_v]_U) = R(KV[P_\mu,P_\lambda,P_v]_U{}^T VK),$

(By Definition 3.3)

$\Leftrightarrow\ (KVA)^T = (KV[P_\mu, P_\lambda, P_v]_L, KV[P_\mu, P_\lambda, P_v]_U)^T$ is an IV RS

$\Leftrightarrow\ A^T VK = ([P_\mu, P_\lambda, P_v]_L VK, [P_\mu, P_\lambda, P_v]_U VK)$ is an IV RS

$\Leftrightarrow\ P^T = \left([P_\mu, P_\lambda, P_v]_L{}^T, [P_\mu, P_\lambda, P_v]_U{}^T\right)$ is an IV s – κ RS

As a conclusion (i) $\Leftrightarrow$ (vi) is true

(i) $\Leftrightarrow$ (xii) $\Leftrightarrow$ (xi)

Let P is an IV s – κ RS

Consider $[P_\mu, P_\lambda, P_v]_L$ is a s – κ RS

$$\Leftrightarrow C([P_\mu, P_\lambda, P_v]_L{}^T) = C(\mathrm{KV}[P_\mu, P_\lambda, P_v]_L VK), \mathrm{C}([P_\mu, P_\lambda, P_v]_U{}^T) = C(\mathrm{KV}[P_\mu, P_\lambda, P_v]_U VK)$$

By (P.2.3)

$$\Leftrightarrow [P_\mu, P_\lambda, P_v]_L = H_1 \mathrm{KV}[P_\mu, P_\lambda, P_v]_L{}^T VK, [P_\mu, P_\lambda, P_v]_U{}^T = H_1 \mathrm{KV}[P_\mu, P_\lambda, P_v]_U VK$$

for $\mathrm{H}_1 \in IVNFM$. As a result (i) $\Leftrightarrow$ (xii) $\Leftrightarrow$ (xi) true.

(ii) $\Leftrightarrow$ (xiii) $\Leftrightarrow$ (x)

$\Leftrightarrow AVK = \left[[P_\mu, P_\lambda, P_v]_L VK, [P_\mu, P_\lambda, P_v]_U VK\right]$ is an IV RS

$\Leftrightarrow AV = \left[[P_\mu, P_\lambda, P_v]_L V, [P_\mu, P_\lambda, P_v]_U V\right]$ is an IV κ- RS

$$\Leftrightarrow R(\mathrm{V}[P_\mu, P_\lambda, P_v]_L) = R(\mathrm{K}\left(\mathrm{V}[P_\mu, P_\lambda, P_v]_L\right)^T K),$$

$$\mathrm{R}(\mathrm{V}[P_\mu, P_\lambda, P_v]_U) = R(K\left(V[P_\mu, P_\lambda, P_v]_U\right)^T K),$$ [By Definition 3.3]

$$\Leftrightarrow R([P_\mu, P_\lambda, P_v]_L) = R\left([P_\mu, P_\lambda, P_v]_L\right)^T VK),\ \mathrm{R}([P_\mu, P_\lambda, P_v]_U) = R\left([P_\mu, P_\lambda, P_v]_U\right)^T VK),$$

$$\Leftrightarrow R([P_\mu, P_\lambda, P_v]_L{}^T) = R\left(\mathrm{KV}[P_\mu, P_\lambda, P_v]_L\right)^T K),\ R([P_\mu, P_\lambda, P_v]_U{}^T) = R\left(KV[P_\mu, P_\lambda, P_v]_U\right)$$

$$[P_\mu, P_\lambda, P_v]_L = VK[P_\mu, P_\lambda, P_v]_L{}^T\ \mathrm{VK}\, H_1, [P_\mu, P_\lambda, P_v]_U = VK[P_\mu, P_\lambda, P_v]_U\ \mathrm{VK}\ for\ H_1 \in IVNFM$$

As a conclusion (ii) $\Leftrightarrow$ (xiii) $\Leftrightarrow$ (x) is true

The above statement can be reduced to the equivalent requirement that a matrix be an IV s- RS for K = I in particular.

Corollary:3.1 The following statements are equivalent for $P \in \mathrm{IVNFM}_{nn}$

(i) $P = < [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U > \in \mathrm{IVNFM}_{nn}$ is an IV s-RS.

(ii) $VP = < V[P_\mu, P_\lambda, P_v]_L, V[P_\mu, P_\lambda, P_v]_U >$ is an IV RS.

(iii) $PV = < [P_\mu, P_\lambda, P_v]_L V, [P_\mu, P_\lambda, P_v]_U V >$ is an IV RS.

(iv) $P^T = < [P_\mu, P_\lambda, P_v]_L{}^T, [P_\mu, P_\lambda, P_v]_U{}^T >$ is an IV s – RS.

(v) $R\left([P_\mu, P_\lambda, P_v]_L\right) = R\left([P_\mu, P_\lambda, P_v]_L{}^T V\right), R\left([P_\mu, P_\lambda, P_v]_U\right) = R\left([P_\mu, P_\lambda, P_v]_U{}^T V\right)$

(vi) $R\left([P_\mu, P_\lambda, P_v]_L{}^T\right) = R\left([P_\mu, P_\lambda, P_v]_L V\right), R\left([P_\mu, P_\lambda, P_v]_U{}^T\right) = R\left([P_\mu, P_\lambda, P_v]_U V\right)$

(vii) $C\left(KV[P_\mu, P_\lambda, P_v]_L\right) = C\left(V[P_\mu, P_\lambda, P_v]_L\right)^T, C\left(KV[P_\mu, P_\lambda, P_v]_U\right) = C\left(V[P_\mu, P_\lambda, P_v]_U\right)^T$

(viii) $[P_\mu, P_\lambda, P_v]_L = V[P_\mu, P_\lambda, P_v]_L{}^T VH_1, [P_\mu, P_\lambda, P_v]_U$

$= V[P_\mu, P_\lambda, P_v]_U{}^T VH_1$ for $H_1 \in IVNFM$

(ix) $[P_\mu, P_\lambda, P_v]_L = H_1 V[P_\mu, P_\lambda, P_v]_L{}^T V, [P_\mu, P_\lambda, P_v]_U$

$= H_1 V[P_\mu, P_\lambda, P_v]_U{}^T V$ for $H_1 \in IVNFM$

(x) $[P_\mu, P_\lambda, P_v]_L{}^T = V[P_\mu, P_\lambda, P_v]_L VH_1, [P_\mu, P_\lambda, P_v]_U{}^T$

$= V[P_\mu, P_\lambda, P_v]_U VH_1$ for $H_1 \in IVNFM$

(xi) $[P_\mu, P_\lambda, P_v]_L{}^T = H_1 V[P_\mu, P_\lambda, P_v]_L V, [P_\mu, P_\lambda, P_v]_U$

$= H_1 V[P_\mu, P_\lambda, P_v]_U V$ for $H_1 \in IVNFM$

Theorem 3.2. For $P = < [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U > \in IVNFM_{nn}$ then any two of the conditions below imply the other

(i) P is an IV κ- RS.

(ii) P is an IV s-κ- RS.

(iii) $R\left([P_\mu, P_\lambda, P_v]_L\right)^T = R\left(VK[P_\mu, P_\lambda, P_v]_L\right)^T, R\left([P_\mu, P_\lambda, P_v]_U\right)^T = R\left(VK[P_\mu, P_\lambda, P_v]_U{}^T\right)$

Proof: (i) and (ii) implies (iii)

Let P is an IV s – κ RS

$$\Rightarrow R\left([P_\mu,P_\lambda,P_v]_L\right)=R\left([P_\mu,P_\lambda,P_v]_L{}^T VK\right),R\left([P_\mu,P_\lambda,P_v]_U\right)=R\left([P_\mu,P_\lambda,P_v]_U{}^T VK\right)$$

[By Theorem 3.1]

$$\Rightarrow R\left(K[P_\mu,P_\lambda,P_v]_L K\right)=R\left(K[P_\mu,P_\lambda,P_v]_L{}^T K\right),R\left(K[P_\mu,P_\lambda,P_v]_U K\right)$$

$$=R\left(K[P_\mu,P_\lambda,P_v]_U{}^T K\right)$$ [By Lemma 2.2]

$$\Rightarrow R[P_\mu,P_\lambda,P_v]_L{}^T=R\left(\left(VK[P_\mu,P_\lambda,P_v]_L\right)^T\right),R\left([P_\mu,P_\lambda,P_v]_U\right)^T=R\left(\left(VK[P_\mu,P_\lambda,P_v]_U\right)^T\right)$$

(i) & (ii) implies (iii) is true

(i)& (iii) implies (ii)

P is an IV κ- RS

$$\Rightarrow R\left(K[P_\mu,P_\lambda,P_v]_L K\right)=R\left(\left([P_\mu,P_\lambda,P_v]_L\right)^T\right),R\left(K[P_\mu,P_\lambda,P_v]_U K\right)=R\left(\left([P_\mu,P_\lambda,P_v]_U\right)^T\right)$$

Therefore, (i) & (iii)

$$\Rightarrow R\left([P_\mu,P_\lambda,P_v]_L\right)=R\left([P_\mu,P_\lambda,P_v]_L{}^T VK\right),R\left([P_\mu,P_\lambda,P_v]_U\right)=R\left([P_\mu,P_\lambda,P_v]_U{}^T VK\right)$$

$$\Rightarrow R\left([P_\mu,P_\lambda,P_v]_L\right)=R\left(\left(KV[P_\mu,P_\lambda,P_v]_L\right)^T\right),R\left([P_\mu,P_\lambda,P_v]_U\right)=R\left(\left(KV[P_\mu,P_\lambda,P_v]_U\right)^T\right)$$

P is an IV s-k-RS (By Theorem 3.1)

$\Rightarrow$ (ii) is true

(ii) & (iii) implies (i)

P is an IV s- κ- RS

$$\Rightarrow R\left(K[P_\mu,P_\lambda,P_v]_L K\right)=R\left(K[P_\mu,P_\lambda,P_v]_L{}^T K\right),R\left(K[P_\mu,P_\lambda,P_v]_U K\right)=R\left(K[P_\mu,P_\lambda,P_v]_U{}^T K\right)$$ Therefore,(ii) and (iii)

$$\Rightarrow R\left([P_\mu,P_\lambda,P_v]_L\right)=R\left(K[P_\mu,P_\lambda,P_v]_L{}^T K\right),R\left([P_\mu,P_\lambda,P_v]_U\right)=R\left(K[P_\mu,P_\lambda,P_v]_U{}^T K\right)$$

$P=<[P_{\mu L},P_{\lambda L},P_{vL}],[P_\mu,P_\lambda,P_v]_U>\in \mathrm{IVNFM}_{nn}$ is an IV κ- RS .

Therefore, (i) is true, Hence the theorem.

4. IV s – κ RS regular Neutrosophic fuzzy matrices

In this section, it was discovered that there are various generalized inverses of matrices in IVNFM. The comparable standards for different g-inverses of an IV s-k RS Neutrosophic fuzzy matrix to be IV s-k RS are also established. The generalized inverses of an IV s – κ RS P corresponding to the sets P{1, 2}, P{1, 2, 3} and P{1, 2, 4} are characterized.

Theorem 4.1: Let $P\in \mathrm{IVNFM}_{nn}$, Z belongs to P{1,2} and PW, ZW are an IV s- κ- RS. Then P is an IV s- κ - RS iff $W=<[W_\mu,W_\lambda,W_v]_L,[W_\mu,W_\lambda,W_v]_U>$ is an IV s- κ – RS.

Proof: Let $P=<[P_\mu,P_\lambda,P_v]_L,[P_\mu,P_\lambda,P_v]_U>\in IVNFM_{nn}$

$$R\left(KV[P_\mu,P_\lambda,P_v]_L\right)=R\left(KV[P_\mu,P_\lambda,P_v]_L W[P_{\mu L},P_{\lambda L},P_{vL}]\right)\subseteq R\left(W[P_\mu,P_\lambda,P_v]_L\right)$$

$$=R\left(WVV[P_\mu,P_\lambda,P_v]_L\right)\subseteq R\left(WVKKV[P_\mu,P_\lambda,P_v]_L\right)\subseteq R\left(KV[P_\mu,P_\lambda,P_v]_L\right)$$

Hence, $R\left(KV[P_\mu,P_\lambda,P_v]_L\right)=R\left(W[P_\mu,P_\lambda,P_v]_L\right)$

$$=R\left(KV\left(W[P_\mu,P_\lambda,P_v]_L\right)^T VK\right) \qquad [\text{WP is IV s-}\kappa\text{-RS}]$$

$$=R\left([P_\mu,P_\lambda,P_v]_L{}^T[W_\mu,W_\lambda,W_v]_L{}^T VK\right)$$

$$=R\left([W_\mu,W_\lambda,W_v]_L{}^T VK\right)=R\left(\left(KV[W_\mu,W_\lambda,W_v]_L\right)^T\right)$$

$$R\left(\left(KV[P_\mu,P_\lambda,P_v]_L\right)^T\right)=R\left([P_\mu,P_\lambda,P_v]_L{}^T VK\right)$$

$$=R\left(KV[P_\mu,P_\lambda,P_v]_L[W_\mu,W_\lambda,W_v]_L\right) \qquad [\text{VP is s-}\kappa-\text{IV RS}]$$

$$=R\left(KV[W_\mu,W_\lambda,W_v]_L\right)$$

Similarly,

Hence, $R\left(KV[W_\mu,W_\lambda,W_v]_U\right)=R\left(\left(KV[P_\mu,P_\lambda,P_v]_U\right)^T\right)$ (KVW is an IV RS)

$$\Leftrightarrow R\left(KV[P_\mu,P_\lambda,P_v]_L\right)=R\left(\left(KV[P_{\mu L},P_{\lambda L},P_{vL}]\right)^T\right),\ R\left(KV[P_\mu,P_\lambda,P_v]_U\right)=R\left(\left(KV[P_\mu,P_\lambda,P_v]_U\right)^T\right)$$

$$\Leftrightarrow R\left(KV[W_\mu,W_\lambda,W_v]_L\right)=R\left(\left(KV[W_\mu,W_\lambda,W_v]_L\right)^T\right),$$

$$R\left(KV[W_\mu,W_\lambda,W_v]_U\right)=R\left(\left(KV[W_\mu,W_\lambda,W_v]_U\right)^T\right)$$

$\Leftrightarrow KVX=\left[KV[W_\mu,W_\lambda,W_v]_L,KV[W_\mu,W_\lambda,W_v]_U\right]$ is an IV RS

$W=<[W_\mu,W_\lambda,W_v]_L,[W_\mu,W_\lambda,W_v]_U>$ is an IV RS.

Theorem 4.2:, Let $P\in IVNFM_{nn}$ $W=<[W_\mu,W_\lambda,W_v]_L,[W_\mu,W_\lambda,W_v]_U>\in$P{1,2,3}, $R(KV[P_\mu,P_\lambda,P_v]_L)=$ $R\left(KV[X_{\mu L},X_{\lambda L},X_{vL}]\right)^T$, $R(KV[P_\mu,P_\lambda,P_v]_U)=R\left(KV[Z_{\mu U},Z_{\lambda U},Z_{vU}]\right)^T$.Then

$P=<[P_\mu,P_\lambda,P_v]_L,[P_\mu,P_\lambda,P_v]_U>\in IVNFM_{nn}$ is IV s-κ- RS $\Leftrightarrow W=<[W_\mu,W_\lambda,W_v]_L,[W_\mu,W_\lambda,W_v]_U>$ is IV s- κ – RS.

Proof: Given P{1,2,3}, Hence ,

$[P_\mu, P_\lambda, P_v]_L [W_\mu, W_\lambda, W_v]_L [P_\mu, P_\lambda, P_v]_L = [P_\mu, P_\lambda, P_v]_L,$

$[W_\mu, W_\lambda, W_v]_L [P_{\mu L}, P_{\lambda L}, P_{vL}][W_\mu, W_\lambda, W_v]_L = [W_\mu, W_\lambda, W_v]_L,$

$\left([P_\mu, P_\lambda, P_v]_L [W_\mu, W_\lambda, W_v]_L\right)^T = [P_\mu, P_\lambda, P_v]_L [W_\mu, W_\lambda, W_v]_L$

Consider, $R\left(\left(KV[P_\mu, P_\lambda, P_v]_L\right)^T\right) = R\left([W_\mu, W_\lambda, W_v]_L{}^T [P_\mu, P_\lambda, P_v]_L{}^T VK\right)$[By using P = PW$P$]

$= R\left(KV\left([P_\mu, P_\lambda, P_v]_L [W_\mu, W_\lambda, W_v]_L\right)^T\right)$

$= R\left(\left([P_\mu, P_\lambda, P_v]_L [W_\mu, W_\lambda, W_v]_L\right)^T\right)$ [*By P*$_{\cdot 2.3}$]

$= R\left([P_\mu, P_\lambda, P_v]_L [W_\mu, W_\lambda, W_v]_L\right)$

$= R\left([W_\mu, W_\lambda, W_v]_L\right)$

[By using $[W_\mu, W_\lambda, W_v]_L = [W_\mu, W_\lambda, W_v]_L [P_\mu, P_\lambda, P_v]_L [W_\mu, W_\lambda, W_v]_L$]

$= R\left(KV[W_\mu, W_\lambda, W_v]_L\right)$ [*By P*$_{2.3}$]

Similarly, we can consider, $R\left(\left(KV[P_\mu, P_\lambda, P_v]_U\right)^T\right) = R\left([W_\mu, W_\lambda, W_v]_U{}^T [P_\mu, P_\lambda, P_v]_U{}^T VK\right)$

$= R\left(KV\left([P_\mu, P_\lambda, P_v]_U [W_\mu, W_\lambda, W_v]_U\right)^T\right)$

$= R\left(\left([P_\mu, P_\lambda, P_v]_U [W_\mu, W_\lambda, W_v]_U\right)^T\right)$ [*By P*$_{\cdot 2.3}$]

$= R\left([P_\mu, P_\lambda, P_v]_U [W_\mu, W_\lambda, W_v]_U\right)$ $\left[(PW)^T = \mathrm{PW}\right]$

$= R\left([W_\mu, W_\lambda, W_v]_U\right)$ [By using W = WPW]

$= R\left(KV[W_\mu, W_\lambda, W_v]_U\right)$ [By $P_{2.3}$]

If KVA is an IV RS

$\Leftrightarrow R\left(KV[P_\mu, P_\lambda, P_v]_L\right) = R\left(\left(KV[P_\mu, P_\lambda, P_v]_L\right)^T\right),$

$R\left(KV[P_\mu, P_\lambda, P_v]_U\right) = R\left(\left(KV[P_\mu, P_\lambda, P_v]_U\right)^T\right)$

$\Leftrightarrow R\left(KV[W_\mu, W_\lambda, W_v]_L\right) = R\left(\left(KV[W_\mu, W_\lambda, W_v]_L\right)^T\right),$

$KVX = [KV[W_\mu, W_\lambda, W_v]_L, KV[W_\mu, W_\lambda, W_v]_U]$ is an IV RS.

$W = <[W_\mu, W_\lambda, W_\nu]_L, [W_\mu, W_\lambda, W_\nu]_U >$ is an IV s-k RS.

Theorem 4.3: Let $P \in \text{IVNFM}_{nn}$, Z ∈ A {1, 2, 4}, $\text{R}(\text{KV}[\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_L)^T = R\left(\text{KV}[W_\mu, W_\lambda, W_\nu]_L\right)$,

$\text{R}(\text{KV}[\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_U)^T = R\left(\text{KV}[W_\mu, W_\lambda, W_\nu]_U\right)$. Then KVP is an IV s- κ-Ks iff

$W = <[W_\mu, W_\lambda, W_\nu]_L, [W_\mu, W_\lambda, W_\nu]_U >$ is an IV s- κ- RS.

Proof: Given, P {1, 2, 4}, Hence $[\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_L [W_\mu, W_\lambda, W_\nu]_L [\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_L = [\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_L$,

$[W_\mu, W_\lambda, W_\nu]_L [\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_L [W_\mu, W_\lambda, W_\nu]_L = [W_\mu, W_\lambda, W_\nu]_L$,

$\left([W_\mu, W_\lambda, W_\nu]_L [\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_L\right)^T = [W_\mu, W_\lambda, W_\nu]_L [\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_L$

Consider, $\text{R}\left(\left(KV[\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_L\right)^T\right) = \text{R}\left([W_\mu, W_\lambda, W_\nu]_L{}^T [\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_L{}^T VK\right)$ $\left[\text{By using P} = \text{PW}P\right]$

$= R\left(KV\left([\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_L [W_\mu, W_\lambda, W_\nu]_L\right)^T\right)$

$= R\left(\left([\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_L [W_\mu, W_\lambda, W_\nu]_L\right)^T\right)$ $\left[By\ P_{2.3}\right]$

$= R\left([\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_L [W_\mu, W_\lambda, W_\nu]_L\right) = R\left([W_\mu, W_\lambda, W_\nu]_L\right)$

$= R\left(\text{KV}[W_\mu, W_\lambda, W_\nu]_L\right)$ $\left[By\ P_{2.3}\right]$

$R\left(\left(KV[\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_U\right)^T\right) = \text{R}\left([W_\mu, W_\lambda, W_\nu]_U{}^T [\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_U{}^T VK\right)$ $\left[\text{By using P} = \text{PW}P\right]$

$= R\left(KV\left([\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_U [W_\mu, W_\lambda, W_\nu]_U\right)^T\right)$

$= R\left(\left([\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_U [W_\mu, W_\lambda, W_\nu]_U\right)^T\right)$ $\left[By\ P_{2.3}\right]$

$= R\left([\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_U [W_\mu, W_\lambda, W_\nu]_U\right)$ $\left[\ (\text{P}W)^T = PW\right]$

$= N\left([\text{X}_{\mu U}, \text{X}_{\lambda U}, \text{X}_{\nu U}]\right) = R\left(\text{KV}[W_\mu, W_\lambda, W_\nu]_U\right)$ $\left[By\ P_{2.3}\right]$

If KVP is an IV RS

$\Leftrightarrow R\left(KV[\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_L\right) = \text{R}\left(\left(KV[\text{P}_\mu, \text{P}_\lambda, \text{P}_\nu]_L\right)^T\right)$,

$R\left(KV[\text{P}_{\mu U}, \text{P}_{\lambda U}, \text{P}_{\nu U}]\right) = \text{R}\left(\left(KV[\text{P}_{\mu U}, \text{P}_{\lambda U}, \text{P}_{\nu U}]\right)^T\right)$

$\Leftrightarrow R\left(KV[W_\mu, W_\lambda, W_\nu]_L\right) = \text{R}\left(\left(KV[W_\mu, W_\lambda, W_\nu]_L\right)^T\right)$,

$KVX = [KV[W_{\mu}, W_{\lambda}, W_{v}]_L, KV[W_{\mu}, W_{\lambda}, W_{v}]_U]$ is an IV RS.

$W = < [W_{\mu}, W_{\lambda}, W_{v}]_L, [W_{\mu}, W_{\lambda}, W_{v}]_U >$ is an IV s-k RS.

The aforementioned Theorems reduce to comparable criteria, in particular for K = I, for different g-inverses of interval valued s- RS to be IV secondary RS.

Corollary 4.1: For $P \in$ IVNFM$_{nn}$, Z$\in$ P {1, 2} and $PW = < [P_{\mu}, P_{\lambda}, P_{v}]_L [W_{\mu}, W_{\lambda}, W_{v}]_L$ $, [P_{\mu}, P_{\lambda}, P_{v}]_U [W_{\mu}, W_{\lambda}, W_{v}]_U >$, $WP = < [W_{\mu}, W_{\lambda}, W_{v}]_L [P_{\mu}, P_{\lambda}, P_{v}]_L, [W_{\mu}, W_{\lambda}, W_{v}]_U [P_{\mu}, P_{\lambda}, P_{v}]_U >$, are is an IV s- RS. Then P is an IV s- RS iff $W = < [W_{\mu}, W_{\lambda}, W_{v}]_L, [W_{\mu}, W_{\lambda}, W_{v}]_U >$ is an IV s- RS.

Corollary 4.2: For $P \in$ IVNFM$_{nn}$, W$\in$ P {1, 2, 3}, $R(KV[P_{\mu}, P_{\lambda}, P_{v}]_L) = R\left(V[W_{\mu}, W_{\lambda}, W_{v}]_L\right)^T$ $, R(KV[P_{\mu U}, P_{\lambda U}, P_{v U}]) = R\left(V[W_{\mu}, W_{\lambda}, W_{v}]_U\right)^T$. Then P is an IV s- RS iff $W = < [W_{\mu}, W_{\lambda}, W_{v}]_L$ $, [W_{\mu}, W_{\lambda}, W_{v}]_U >$ is an IV s- RS.

Corollary 4.3: For $P \in$ IVNFM$_{nn}$, W$\in$P, $R(V[P_{\mu}, P_{\lambda}, P_{v}]_L)^T = R\left(V[W_{\mu}, W_{\lambda}, W_{v}]_L\right)^T$, $R(V[P_{\mu}, P_{\lambda}, P_{v}]_U)^T$, $, = R\left(V[W_{\mu}, W_{\lambda}, W_{v}]_U\right)$. Then P is an IV s- RS iff W is an IV s-RS.

5. Conclusion:

We present equivalent characterizations of an IV k- RS, IV RS, IV s- RS, IV s-k RS NFM. Also, we give the example of s-k-symmetric fuzzy matrix is s-k- RS Neutrosophic fuzzy matrix the opposite isn't always true. We discuss various g-inverse associated with a regular matrices and obtain characterization of set of all inverses. Equivalent conditions for various g-inverses of an Interval Valued s-k-range Symmetric and s-range Symmetric NFMs are determined. In future, we shall prove some related properties of Interval Valued Secondary k-range Symmetric Neutrosophic Fuzzy Matrices.

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