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A novel approach for solving neutrosophic fractional transportation problem with non- linear discounting cost

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Abstract: Fractional transportation problem that includes source and destination may have fractional objective functions in real- world applications to maximize the profitability ratio like profit/ cost or profit/ time. We refere to such transportation problems as fractional transportation problem.The paper considers the interval- valued neutrosophic numbers and its aritemematic operations. This paper deals with fractional transportation problem having discounting cost in neutrosophic environment, where the supply, demand and transportation costs are uncertain. The problem is considered by introducing all the parameters as neutrosophic numbers. Using the benefits of the score function definition, the problem is transformed into the corresponding deterministic form which can be illustrated by any method. and hence by applying of least cost method with the help of Kuhn- Tucker' optimality conditions, the optimal solution is resulted. Our strategy is to assess the issue and can rank different sort of neutrosophic numbers. To claify the proposed technique, a numerical example is given to show the adequacy of the new model.

Keywords: Optmization, Optimization problems; Fractional programming, Transportation problem, Non-linear programming, Discounting cost, Pentagonal fuzzy neutrosophic numbers, Score function, Vogel's approximation method, Kuhn- Tucker optimality conditions, Optimal neutrosophic solution,Decision making

1. Introduction

Transportation problem is one of the oldest applications of linear programming problems. The basic transportation problem was originally developed by Hitchcock [1]. In a transportation problem, products have to be transported from a number of sources to a number of destinations. Decisions have to be taken according to the amount of products transported between each two locations to minimize total transportation cost [2]. Typically, only a variable cost proportional to the number of products transported is afforded. However, in many real-world problems, a fixed/setup cost is also afforded when the transportation amount is positive [3]. The transportation problem can be modeled as a standard linear programming problem, that can be solved by the simplex method. We can get an initial basic feasible solution for the transportation problem by using the North-West corner rule, Row Minima, Column Minima, Matrix Minima or the Vogel's Approximation Method. To get an optimal solution for the transportation problem, we use the MODI method (Modified Distribution Method). Transportation problem (TP) is a special type of linear programming (LP) problem; where the objective is to minimize the cost of distributing product from m sources or origins to n distributions and their capacities are $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_m$, respectively. In other hand, there is a penalty c_{ij} connected with transportation a unit of product from source i to destination j. This penalty, perhaps cost or delivery time of safety of delivery, etc. A variable x_{ij} represents the unidentified quantity to be shipped from source i to destination j. Oheigeartaigh [4] developed an algorithm for fuzzy transportation problem (FTP) Chanas et al. [5] developed a parametric approach to solve single objective FTP. Thamaraiselvi and Santhi [6] studied FTP with hexagonal fuzzy numbers.

In Fractional problem (FP), decision problem arises to optimize the ratio subject to constraints. In real life decision conditions decision maker (DM) sometimes may face to evaluate ratio between inventory and sales, real cost and standard cost, output etc., with both denominator and numerator are linear. If only one ratio is considered as an objective function then under linear constraints, the problem is said to be linear fractional programming (LFP) problem. The Fractional programming problem, i. e., the maximization of a fraction of two functions subject to given conditions, arises in various decision making situations; for instance , fractional programming is used in the fields of traffic planning (Dantzig et al. [6]), network flows (Arisawa and Elmaghraby, [7]), and game theory (Isbell and Marlow, [8]). A review of various applications is given by Schaible, [9-11]. Tantawy [12-13] introduced two approaches to solve the LFP problem namely; a feasible direction approach and a duality approach. Odior [14] introduced an algebraic approach based on the duality concept and the partial fractions to solve the LFP problem. Gupta and Chakraborty [15] solved the LFP problem depending on the sign of the numerator under the assumption that the denominator is non -vanishing in the feasible region using the fuzzy programming approach. Stanojevic and Stancu- Minasian [16] proposed a method for solving fully fuzzified LFP problem. Buckley and Feuring (2000) studied fully fuzzified linear programming involving coefficients and decision variables as fuzzy quantities. Li and Chen [17] introduced a fuzzy LFP problem with fuzzy coefficients and present the concept of a fuzzy optimal solution. Pop and Stancu [18] studied LFP problem with all parameters and decision variables are triangular fuzzy numbers. Gomathi and Jayalakshmi [19] proposed an approach for solving linear fractional transportation problem. A nermous researchers studied fractional transportation (Veeramani et al. [20], Haque [21-24], Bas et al., [25], Akram et al., [26], El Sayed and Bakry [27], Khalifa et al., [28]).

In this paper, fractional transportation problem having discounting cost in neutrosophic environment is introduced. With the help of least cost method and the Kuhn- Tucker's optimality conditions, the optimal solution of the problem is resulted. The following are the study's main contributions and novelties:

1. Introducing suitable terminologies and measures that consider the properties of a possible optimal solution.
2. Presenting a parametric study by solving a parametric problem and determining the stability set of the first kind for collecting the most possible information about the possible optimal solutions in an uncertain situation
3. Interacting the analyst with the DM to assign a set of selected alternatives
4. Doing a multicriteria analysis by interacting with the DM for selecting one of the possible optimal as the satisfied optimal solution .

The rest of the paper is outlined as follows:

The following is how the paper is structured: Section 2 Presents some preliminaries and notation needed. Section 3, Formulates a neutrosophic fractional transportation problem with non- linear discounting cost. Section 4, proposes an algorithm combining with the least cost method and the Kuhn- Tucker's optimality conditions for solving the problem. Section 5, Introduces a numerical example for illustration. Section 6, Introduces discussion about the results. Section 7, introduces comparitive study with some existing relevant literature. Finally, some concluding remarks are reported.

2.Preliminaries

In This section, some of basic concepts and results related to neutrosophic set, single- valued trapezoidal neutrosophic numbers, and their arithmetic operations and its score function are recalled.

Definition1. (Atanason, [31]). A fuzzy set $\widetilde{B}$ is said to be an intuitionistic fuzzy set $\widetilde{B}^{IN}$of a non empty set X if $\widetilde{B}^{IN} = \{\langle x, \mu_{\widetilde{B}^{IN}}, \rho_{\widetilde{B}^{IN}} \rangle : x \in X\}$, where $\mu_{\widetilde{B}^{IN}}$, and $\rho_{\widetilde{B}^{IN}}$ are non-membership and membership functions such that $\mu_{\widetilde{B}^{IN}}$, $\rho_{\widetilde{B}^{IN}}: X \to [0,1]$ and $0 \le \mu_{\widetilde{B}^{IN}} + \rho_{\widetilde{B}^{IN}} \le 1$, for all $x \in X$.

Definition 2. (Atanason, [32]). An intuitionistic fuzzy set $\widetilde{B}^{IN}$of a $\mathbb{R}$ is named an Intuitionistic fuzzy number if the following conditions hold:

1. There exists $c \in \mathbb{R}: \mu_{\widetilde{B}^{IN}}(c) = 1$, and $\rho_{\widetilde{B}^{IN}}(c) = 0$,
2. $\mu_{\widetilde{B}^{IN}}: \mathbb{R} \to [0,1]$ is continuous function such that

$$0 \le \mu_{\widetilde{B}^{IN}} + \rho_{\widetilde{B}^{IN}} \le 1, \text{ for all } x \in X,$$

3. The membership and nonmembership functions of $\widetilde{B}^{IN}$ are

$$\mu_{\widetilde{B}^{IN}}(x) = \begin{cases} 0, & -\infty < x < r, \\ h(x), & r \le x \le s, \\ 1, & x = s, \\ l(x), & s \le x \le t, \\ 0, & t \le x < \infty, \end{cases}$$

$$\rho_{\widetilde{B}^{IN}}(x) = \begin{cases} 0, & -\infty < x < a, \\ f(x), & a \le x \le s, \\ 1, & x = s, \\ g(x), & s \le x \le b, \\ 0, & b \le x < \infty. \end{cases}$$

Where $f, g, h, l: \mathbb{R} \to [0,1]$, h and g are completely increasing functions, l and f are completely decreasing functions with the constraints $0 \le f(x) + h(x) \le 1$, and $0 \le l(x) + g(x) \le 1$.

Definition 3. (Jianqiang and Zhong, [33]). A trapezoidal intuitionistic fuzzy number is denoted by $\widetilde{B}^{IN} = (r, s, t, u), (p, s, t, q)$, where $p \le r \le s \le t \le u \le q$ with non-membership and membership functions are defined as

A trapezoidal intuitionistic fuzzy number is denoted by $\widetilde{B}^{IN} = (r, s, t, u), (p, s, t, q)$, where $p \le r \le s \le t \le u \le q$ with membership and nonmembership functions are defined as:

$$\mu_{\bar{B}^{INT}}(x)=\begin{cases}\frac{x-r}{s-r}, & r\le x<s,\\ 1, & s\le x\le t,\\ \frac{u-x}{u-t}, & t\le x\le u,\\ 0, & \text{otherwise},\end{cases}\qquad \rho_{\bar{B}^{INT}}(x)=\begin{cases}\frac{s-x}{s-p}, & p\le x<s,\\ 0, & s\le x\le t,\\ \frac{x-t}{q-t}, & t\le x\le q,\\ 1, & \text{otherwise}\end{cases}$$

Definition 4. (Smarandache, [34]). A neutrosophic set $\bar{B}^N$ of non empty set X is defined as $\bar{B}^N=\{\langle x, I_{\bar{B}^N}(x), J_{\bar{B}^N}(x), V_{\bar{B}^N}(x)\rangle: x\in X, I_{\bar{B}^N}(x), J_{\bar{B}^N}(x), V_{\bar{B}^N}(x)\in]0_-,1^+[\}$, where $I_{\bar{B}^N}(x), J_{\bar{B}^N}(x)$, and $V_{\bar{B}^N}(x)$ are an indeterminacy- membership function, truth membership function, and a falsity- membership function and there is no limit on the sum of $I_{\bar{B}^N}(x), J_{\bar{B}^N}(x)$, and $V_{\bar{B}^N}(x)$, so $0^-\le I_{\bar{B}^N}(x)+J_{\bar{B}^N}(x)+V_{\bar{B}^N}(x)\le 3^+$, and $]0_-,1^+[$ is a nonstandard unit interval.

Definition 5. (Wang et al., [35]). A Single- valued neutrosophic set $\bar{B}^{SVN}$ of a non empty set X is defined as $\bar{B}^{SVN}=\{\langle x, I_{\bar{B}^N}(x), J_{\bar{B}^N}(x), V_{\bar{B}^N}(x)\rangle: x\in X\}$, where $I_{\bar{B}^N}(x), J_{\bar{B}^N}(x)$, and $V_{\bar{B}^N}(x)\in[0,1]$ for each $x\in X$ and $0\le I_{\bar{B}^N}(x)+J_{\bar{B}^N}(x)+V_{\bar{B}^N}(x)\le 3$.

Definition 6. (Thamariselvi and Santhi, [36]). Let $\tau_{\tilde{q}}, \varphi_{\tilde{q}}, \omega_{\tilde{q}}\in[0,1]$ and $r,s,t,u\in\mathbb{R}$ such that $r\le s\le t\le u$. Then a single valued trapezoidal neutrosophic number, $\tilde{b}^N=\langle (r,s,t,u): \tau_{\tilde{q}}, \varphi_{\tilde{q}}, \omega_{\tilde{q}}\rangle$ is a special neutrosophic set on $\mathbb{R}$, whose truth-membership, indeterminacy- membership, and falsity- membership functions are

$$\mu_{\tilde{q}}{}^N(x)=\begin{cases}\tau_{\tilde{q}^N}\left(\frac{x-r}{s-r}\right), & r\le x<s,\\ \tau_{\tilde{b}}, & s\le x\le t,\\ \tau_{\tilde{q}^N}\left(\frac{u-x}{u-t}\right), & t\le x\le u,\\ 0, & \text{otherwise},\end{cases}$$

$$\sigma_{\tilde{q}}{}^N(x)=\begin{cases}\frac{s-x+\omega_{\tilde{q}^N}(x-r)}{s-r}, & r\le x<s,\\ \omega_{\tilde{q}^N}, & s\le x\le t,\\ \frac{x-t+\omega_{\tilde{q}^N}(u-x)}{u-t}, & t\le x\le u,\\ 1, & \text{otherwise}.\end{cases}$$

Where $\tau_{\tilde{q}}, \varphi_{\tilde{q}}$, and $\omega_{\tilde{q}}$ indicate the maximum truth, minimum- indeterminacy, and minimum falsity membership degrees, respectively. A single- valued trapezoidal neutrosophic number $\tilde{q}^N=\langle (r,s,t,u): \tau_{\tilde{q}^N}, \varphi_{\tilde{q}^N}, \omega_{\tilde{q}^N}\rangle$ might express in ill- defined amount about q, which is roughly equal to $[s,t]$.

Definition 7. (Thamariselvi and Santhi, [36]). Let $\tilde{q}^N=\langle (r,s,t,u): \tau_{\tilde{q}^N}, \varphi_{\tilde{q}^N}, \omega_{\tilde{q}^N}\rangle$, and $\tilde{d}^N=\langle (r',s',t',u'): \tau_{\tilde{d}^N}, \varphi_{\tilde{d}^N}, \omega_{\tilde{d}^N}\rangle$ be two single- valued trapezoidal neutrosophic numbers and $v\neq 0$. The arithematic operations on $\tilde{q}^N$, and $\tilde{d}^N$ are

1. $\tilde{q}^N\oplus\tilde{d}^N=\langle (r+r', s+s', t+t', u+u'); \tau_{\tilde{q}^N}\wedge\tau_{\tilde{d}^N}, \varphi_{\tilde{q}^N}\vee\varphi_{\tilde{d}^N}, \omega_{\tilde{q}^N}\vee\omega_{\tilde{d}^N}\rangle$,
2. $\tilde{q}^N\ominus\tilde{d}^N=\langle (r-u', s-t', t-s', u'-r); \tau_{\tilde{q}^N}\wedge\tau_{\tilde{d}^N}, \varphi_{\tilde{q}^N}\vee\varphi_{\tilde{d}^N}, \omega_{\tilde{q}^N}\vee\omega_{\tilde{d}^N}\rangle$,
3. $$\tilde{q}^N\otimes\tilde{d}^N=\begin{cases}\langle (rr', ss', tt', uu'); \tau_{\tilde{q}^N}\wedge\tau_{\tilde{d}^N}, \varphi_{\tilde{q}^N}\vee\varphi_{\tilde{d}^N}, \omega_{\tilde{q}^N}\vee\omega_{\tilde{d}^N}\rangle, u,\ u'>0\\ \langle (ru', st', st', ru'); \tau_{\tilde{q}^N}\wedge\tau_{\tilde{d}^N}, \varphi_{\tilde{q}^N}\vee\varphi_{\tilde{d}^N}, \omega_{\tilde{q}^N}\vee\omega_{\tilde{d}^N}\rangle, u<0,\ u'>0\\ \langle (uu', ss', tt', rr'); \tau_{\tilde{q}^N}\wedge\tau_{\tilde{d}^N}, \varphi_{\tilde{q}^N}\vee\varphi_{\tilde{d}^N}, \omega_{\tilde{q}^N}\vee\omega_{\tilde{d}^N}\rangle, u<0,\ u'<0,\end{cases}$$
4. $$\tilde{q}^N\oslash\tilde{d}^N=\begin{cases}\langle (r/u', s/t', t/s', u/r'); \tau_{\tilde{q}^N}\wedge\tau_{\tilde{d}^N}, \varphi_{\tilde{q}^N}\vee\varphi_{\tilde{d}^N}, \omega_{\tilde{q}^N}\vee\omega_{\tilde{d}^N}\rangle, u,\ u'>0\\ \langle (u/u', t/t', s/s', r/r'); \tau_{\tilde{q}^N}\wedge\tau_{\tilde{d}^N}, \varphi_{\tilde{q}^N}\vee\varphi_{\tilde{d}^N}, \omega_{\tilde{q}^N}\vee\omega_{\tilde{d}^N}\rangle, u<0,\ u'>0\\ \langle (u/r', t/s', s/t', r/u'); \tau_{\tilde{q}^N}\wedge\tau_{\tilde{d}^N}, \varphi_{\tilde{q}^N}\vee\varphi_{\tilde{d}^N}, \omega_{\tilde{q}^N}\vee\omega_{\tilde{d}^N}\rangle, u<0,\ u'<0,\end{cases}$$

5. $k\tilde{d}^N = f(x) = \begin{cases} \langle (kr, ks, kt, k);\ \tau_{\tau_{\tilde{d}^N}},\ \varphi_{\tau_{\tilde{d}^N}}, \omega_{\tau_{\tilde{d}^N}}\ \rangle, k > 0, \\ \langle (ku, kt, ks, k\ r);\ \tau_{\tau_{\tilde{d}^N}},\ \varphi_{\tau_{\tilde{d}^N}}, \omega_{\tau_{\tilde{d}^N}}\ \rangle, k < 0, \end{cases}$

6. $\tilde{d}^{N^{-1}} = \langle (1/u', 1/t', 1/s', 1/r');\ \tau_{\tau_{\tilde{d}^N}},\ \varphi_{\tau_{\tilde{d}^N}}, \omega_{\tau_{\tilde{d}^N}}\ \rangle, \tilde{d}^N \neq 0.$

Definition 8. (Thamariselvi and Santhi, [37]). A two single- valued trapezoidal neutrosophic numbers$\tilde{b}$, and $\tilde{d}$ can be compared based on the score and accuracy functions as

1. Accuracy function $AC(\tilde{q}^N) = \left(\frac{1}{16}\right)[r + s + t + u] * [\mu_{\tilde{q}^N} + (1 - \rho_{\tilde{q}^N}(x) + \left(1 + \sigma_{\tilde{q}^N}(x)\right],$

2. Score function $SC(\tilde{q}^N) = \left(\frac{1}{16}\right)[r + s + t + u] * [\mu_{\tilde{q}^N} + (1 - \rho_{\tilde{q}^N}(x) + \left(1 - \sigma_{\tilde{q}^N}(x)\right].$

Definition 9. (Thamariselvi and Santhi, [37]). The order relations between $\tilde{b}^N$ and $\tilde{d}^N$ based on $SC(\tilde{q}^N)$ and $AC(\tilde{q}^N)$ are defined as

1. If $SC(\tilde{q}^N) < SC(\tilde{d}^N)$, then $\tilde{q}^N < \tilde{d}^N$
2. If $SC(\tilde{q}^N) = SC(\tilde{d}^N)$, then $\tilde{q}^N = \tilde{d}^N$,
3. If $AC(\tilde{q}^N) < AC(\tilde{d}^N)$, then $\tilde{q}^N < \tilde{d}^N$,
4. If $AC(\tilde{q}^N) > AC(\tilde{d}^N)$, then $\tilde{q}^N < \tilde{d}^N$,
5. If $AC(\tilde{q}^N) = AC(\tilde{d}^N)$, then $\tilde{q}^N = \tilde{d}^N$.

3. Problem statement and solution concepts

Consider the following general neutrosophic fractional transportation problem

(NFTP) $\max \tilde{F}^N(x) = \frac{\tilde{P}^N(x)}{\tilde{Q}^N(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{p}_{ij}^N x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{q}_{ij}^N x_{ij}}$

Subject to

$\sum_{j=1}^{n} x_{ij} = \tilde{a}_i^N, i = \overline{1, m},$
$\sum_{i=1}^{m} x_{ij} = \tilde{b}_j^N, j = \overline{1, n},$
$x_{ij} \geq 0;\ \forall i, j.$

Where, $\tilde{p}_{ij}^N, \tilde{q}_{ij}^N, \tilde{a}_i^N$, and $\tilde{b}_j^N$, are neutrosophic numbers. Based on the score function introduced in Definition 8, the NFTP is converted into the following FTP as

(FTP) $\max F(x) = \frac{P(x)}{Q(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} q_{ij} x_{ij}}$

Subject to

$\sum_{j=1}^{n} x_{ij} \leq a_i, i = \overline{1, m},$
$\sum_{i=1}^{m} x_{ij} \geq b_j, j = \overline{1, n},$
$x_{ij} \geq 0;\ \forall i, j.$

It is supposed that $Q(x) > 0;\ \forall x = (x_{ij}) \in G$, where G is the feasible domain and $a_i > 0, b_j > 0$. Also, it is assumed that $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$.

Definition 10. (Bajalinov. [38]). A point $\bar{x} = \{\bar{x}_{ij}: i = \overline{1, m}; \overline{1, n}\}$ is said to be feasible solution to FTP if $\bar{x}$ satisfies the constraints in it.

Definition 11. A feasible point $\bar{x} = \{\bar{x}_{ij}: i = \overline{1, m}; \overline{1, n}\}$ is called an optimal solution to FTP if $F(\bar{x}) \geq F(x);\ \forall x$.

The Lagrange function for the FTP can be formulated as

$$L(x, \zeta) = \frac{P(x)}{Q(x)} - \zeta_i\left(\sum_{j=1}^{n} x_{ij} - a_i\right) - \zeta_j\left(b_j - \sum_{i=1}^{m} x_{ij}\right) - \xi_{ij} x_{ij} = 0.$$

Where, ζ_i and ζ_j are Lagrange multipliers.

The optimal point $\overline{x}$ satisfies the Kuhn- Tucker's optimality conditions:

$$\frac{\partial F}{\partial x_{ij}} = \frac{\partial F(\overline{x})}{\partial x_{ij}} - (\zeta_i + \zeta_j) - \xi_{ij} = 0,$$

$$\zeta_i\left(\sum_{j=1}^{n} x_{ij} - a_i\right) = 0,$$

$$\zeta_j\left(b_j - \sum_{i=1}^{m} x_{ij}\right) = 0,$$

$$\sum_{j=1}^{n} x_{ij} \le a_i, i = \overline{1, m},$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j, j = \overline{1, n},$$

$$\xi \hat{x}_{ij} = 0,$$

$$\zeta_i \ge 0 \text{ and } \zeta_j \ge 0$$

.

4. Solution Algorithm

In This section, a solution approach for solving NFTP is illustrated in the following steps:

Step1: Convert the NFTP into the corresponding crisp FTP based on the score function.

Step2: Consider the FTP $\left(\frac{p_{ij}}{q_{ij}}\right)$.

Step3: Search for the initial basic feasible solution of FTP using the least cost method.

Step4: Estimate the objective function value at $\overline{x}$ (i.e., $\frac{P(\overline{x})}{Q(\overline{x})}$). Add s_i and t_j to the R.H.S and the bottom of the TP Table 1

Step5: Add s_i and t_j to the R.H.S and the bottom of the TP tableau, respectively as

Table 1. Fractional transportation representation

$\frac{\partial F(\overline{x})}{\partial x_{ij}}$	…	…	$\frac{\partial F(\overline{x})}{\partial x_{1m}}$	a_1	s_1
…	…	…	…	…	…
…	$\frac{\partial F(\overline{x})}{\partial x_{ij}}$	…	…	a_i	s_i
$\frac{\partial F(\overline{x})}{\partial x_{n1}}$	…	…	$\frac{\partial F(\overline{x})}{\partial x_{nm}}$	a_n	s_n
b_1	…	…	b_m		
t_1	…	…	t_m		

Step 6: Calculate the values of s_i and t_j from the relation $\frac{\partial F}{\partial x_{B_{ij}}} = s_i + t_j$

$$\frac{\partial F}{\partial x_{B_{ij}}} = s_i + t_j \quad (1)$$

Step 7: If $M_{ij} = \frac{\partial F}{\partial x_{NB_{ij}}} - s_i - t_j \ge 0;\ \forall x_{ij}$ (2)

(non- basic variables), then $\overline{x}$ is Kuhn- Tucker point. Otherwise, go to step8 as x_{ij} (non- basic variables).

Step 8: Termination conditions:

(i). If all $\frac{\partial F}{\partial x_{NB_{ij}}} > 0 \Rightarrow$ the optimality and the uniqueness of the solution.

(ii). If all $\frac{\partial F}{\partial x_{NB_{ij}}} \ge 0$ with at least one $\frac{\partial F}{\partial x_{NB_{ij}}} = 0 \Rightarrow$ the optimality of the solution and the alternative solution exists.

(iii). If at least one $\frac{\partial F}{\partial x_{NB_{ij}}} < 0 \Rightarrow$ the solution is not optimal

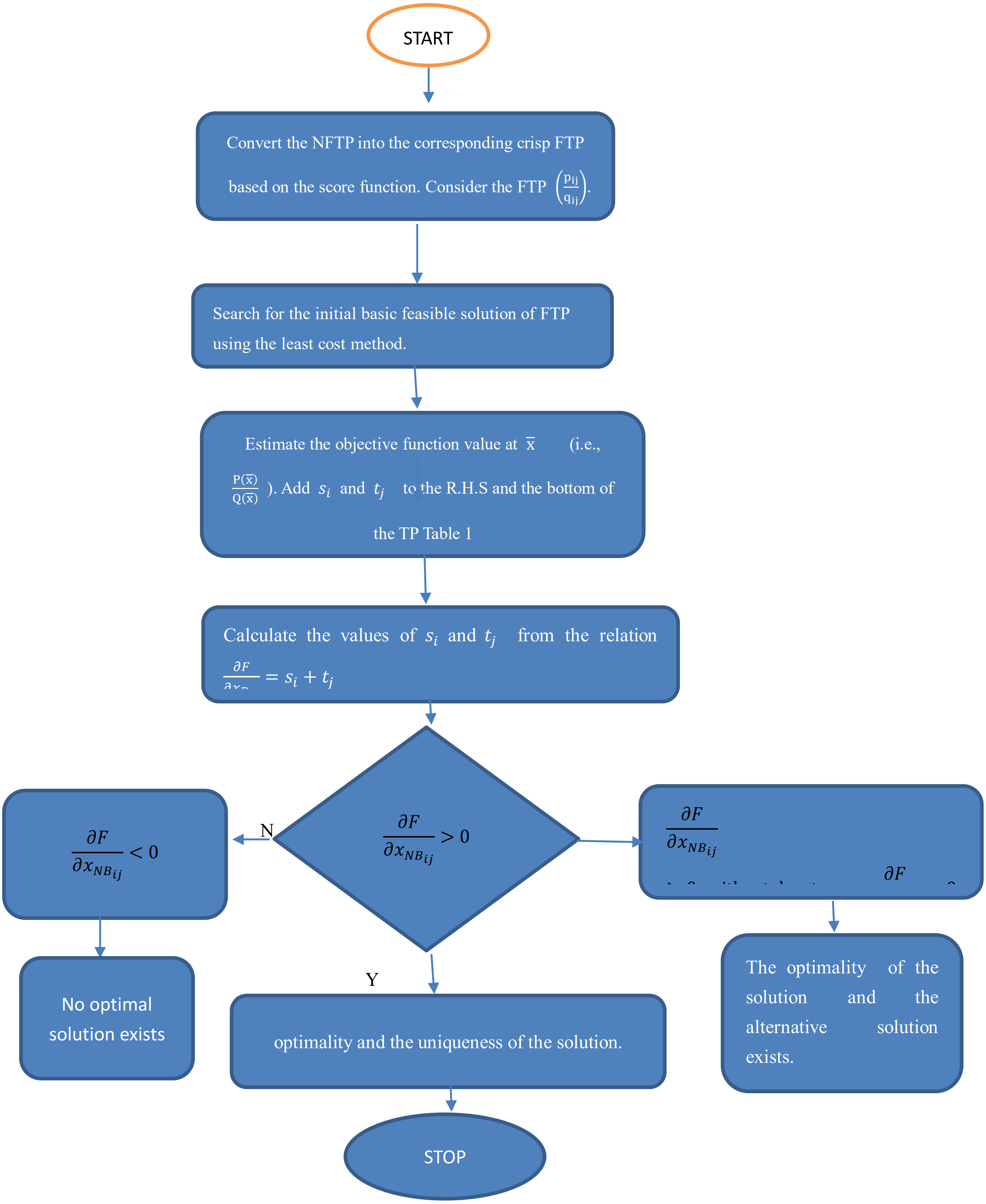

Fig. 1 The flow chart of the proposed solution procedure

5.Numerical example

Consider the following NFTP in which the objective function is the maximization of ratio of total profit given the total cost.

The following table illustrated the transportation company profit gained

Table 2. Input data of neutrosophic profit associated with shipment (in $)

	D_1	D_2	D_3	D_4
O_1	$\langle (14,17,21,28); 0.7,0.1$	$\langle (28,30,35,44); 0.9,0.2$	$\langle (10,16,18,20); 0.8,0.1$	$\langle (17,25,30,35); 0.7,0.2$
O_2	$\langle (18,20,22,25); 0.8,0.2$	$\langle (14,17,21,28); 0.7,0.1$	$\langle (31,35,40,45); 0.6,0.4$	$\langle (16,18,20,26); 0.8,0.2$
O_3	$\langle (18,21,23,26); 0.8,0.2$	$\langle (14,18,20,24); 0.6,0.4$	$\langle (25,30,35,40); 0.6,0.2$	$\langle (18,21,23,26); 0.8,0.2$

The cost of the shipping unit of the commodity from the supply to the demand is shown in the following table

Table 3. Input data of neutrosophic cost associated of shipment (in $)

	D_1	D_2	D_3	D_4
O_1	$\langle (25,30,35,40); 0.6,0.2$	$\langle (17,25,30,35); 0.7,0.2$	$\langle (28,30,35,44); 0.9,0.2$	$\langle (18,20,22,25); 0.8,0.2$
O_2	$\langle (18,20,22,25); 0.8,0.2$	$\langle (14,18,20,24); 0.6,0.4$	$\langle (23,27,30,35); 0.7,0.2$	$\langle (17,25,30,35); 0.7,0.2$
O_3	$\langle (23,28,30,34); 0.7,0.2$	$\langle (25,30,35,40); 0.6,0.2$	$\langle (14,17,21,28); 0.7,0.1$	$\langle (14,17,21,28); 0.7,0.1$

Supplies: $\tilde{a}_1^N = \langle (190,250,260,300); 0.7,0.2,0.1\rangle$, , $\tilde{a}_2^N = \langle (340,380,447,500); 0.7,0.2,0.1\rangle, \tilde{a}_3^N = \langle (283,300,350,400); 0.7,0.2,0.1\rangle$

Demands: $\tilde{b}_1^N = \langle (157,163,169,178); 0.7,0.1,0.2\rangle$, $\tilde{b}_2^N = \langle (340,380,446,500); 0.7,0.1,0.2\rangle$, $\tilde{b}_3^N = \langle (157,163,169,178); 0.7,0.1,0.2\rangle$, $\tilde{b}_4^N = \langle (190,250,260,300); 0.7,0.2,0.1\rangle$.

The discounting cost related to the commodity unit of purchased, transported and the discounted (%) resulted from some shipping policy is given in the following table

Table 4. Discount cost associated of shipment (%)

	D_1	D_2	D_3	D_4
O_1	0.02	0.03	0.05	0.02
O_2	0.03	0.01	0.005	0.02
O_3	0.014	0.04	0.013	0.04

In Table 2, 3: the profit and cost Based on the score function of the neutrosophic number are converted into:

Table 5. Input data of profit associated with shipment (in $)

	D_1	D_2	D_3	D_4	**Supply**
O_1	12	18	10	14	150

O_2	10	14	16	10	250
O_3	11	8	17	11	200
Demand	100	250	100	150	

Table 6. Input data of cost associated of shipment (in $)

	D_1	D_2	D_3	D_4	**Supply**
O_1	17	14	18	10	150
O_2	12	8	15	14	250
O_3	15	17	14	12	200
Demand	100	250	100	150	

From Table 5 and Table 6, the fractional transportation problem can be formulated as follows

$$\max F(x) = \frac{\begin{pmatrix} 12x_{11}+18x_{12}+10x_{13}+14x_{14} \\ +10x_{21}+14x_{22}+16x_{23}+10x_{24} \\ +11x_{31}+8x_{32}+17x_{33}+11x_{34} \end{pmatrix}}{\begin{pmatrix} 17x_{11}+14x_{12}+18x_{13}+10x_{14} \\ +12x_{21}+8x_{22}+15x_{23}+14x_{24} \\ +15x_{31}+17x_{32}+14x_{33}+12x_{34} \end{pmatrix}}$$

Subject to

$$x_{11} + x_{12} + x_{13} + x_{14} = 150,$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 250,$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 200,$$
$$x_{11} + x_{21} + x_{31} = 100,$$
$$x_{12} + x_{22} + x_{32} = 250,$$
$$x_{13} + x_{23} + x_{33} = 100,$$
$$x_{14} + x_{24} + x_{34} = 150,$$
$$x_{ij} \geq 0, i = 1, 2, 3; j = 1, 2, 3, 4.$$

Then, the cost function terms are:

$$\frac{p_{11}}{q_{11}}x_{11} = \frac{12}{17}x_{11} - d_{11}x_{11}^2 \Rightarrow \frac{p_{11}}{q_{11}}x_{11} = 0.706x_{11} - 0.02x_{11}^2,$$

$$\frac{p_{12}}{q_{12}}x_{12} = \frac{18}{14}x_{12} - d_{12}x_{12}^2 \Rightarrow \frac{p_{12}}{q_{12}}x_{12} = 1.286x_{12} - 0.03x_{12}^2,$$

$$\frac{p_{13}}{q_{13}}x_{13} = \frac{10}{18}x_{13} - d_{13}x_{13}^2 \Rightarrow \frac{p_{13}}{q_{13}}x_{13} = 0.556x_{13} - 0.05x_{13}^2,$$

$$\frac{p_{14}}{q_{14}}x_{14} = \frac{18}{14}x_{14} - d_{14}x_{14}^2 \Rightarrow \frac{p_{14}}{q_{14}}x_{14} = 1.4x_{14} - 0.02x_{14}^2,$$

$$\frac{p_{21}}{q_{21}}x_{21} = \frac{10}{12}x_{21} - d_{21}x_{21}^2 \Rightarrow \frac{p_{21}}{q_{21}}x_{21} = 0.833x_{21} - 0.03x_{21}^2,$$

$$\frac{p_{22}}{q_{22}}x_{22} = \frac{14}{8}x_{22} - d_{22}x_{22}^2 \Rightarrow \frac{p_{22}}{q_{22}}x_{22} = 1.75x_{22} - 0.01x_{22}^2,$$

$$\frac{p_{23}}{q_{23}}x_{23} = \frac{16}{15}x_{23} - d_{23}x_{23}^2 \Rightarrow \frac{p_{23}}{q_{23}}x_{23} = 1.067x_{23} - 0.005x_{23}^2,$$

$$\frac{p_{24}}{q_{24}}x_{24} = \frac{10}{14}x_{24} - d_{24}x_{24}^2 \Rightarrow \frac{p_{24}}{q_{24}}x_{24} = 0.714x_{24} - 0.02x_{24}^2,$$

$$\frac{p_{31}}{q_{31}}x_{31} = \frac{11}{15}x_{31} - d_{31}x_{31}^2 \Rightarrow \frac{p_{31}}{q_{31}}x_{31} = 0.733x_{31} - 0.014x_{31}^2,$$

$$\frac{p_{32}}{q_{32}}x_{32} = \frac{8}{17}x_{32} - d_{32}x_{32}^2 \Rightarrow \frac{p_{32}}{q_{32}}x_{32} = 0.4706x_{32} - 0.04x_{32}^2,$$

$$\frac{p_{33}}{q_{33}}x_{33} = \frac{17}{14}x_{33} - d_{33}x_{33}^2 \Rightarrow \frac{p_{33}}{q_{33}}x_{33} = 1.2143x_{33} - 0.013x_{33}^2,$$

$$\frac{p_{34}}{q_{34}}x_{34} = \frac{11}{12}x_{34} - d_{34}x_{34}^2 \Rightarrow \frac{p_{34}}{q_{34}}x_{34} = 0.917x_{34} - 0.04x_{34}^2,$$

Now, let us apply the Vogel's approximation method to determine the initial basic feasible solution for the transportation as

Table 7. Initial basic feasible solution tableau

	D_1		D_1		D_1		D_1		Supply	Raw penalty
O_1	12	**50**	18		10	**100**	14		150	4
	17		14		18		10			
O_2	10	**50**	14	**50**	16		10	**150**	250	4
	12		8		15		14			
O_3	11		8	**200**	17		11		200	2
	15		17		14		12			
Demand	100		250		100		150			
Column penalty	3		6		1		2			

Then, the initial basic feasible solution is

$\overline{x} = (\overline{x}_{11}, \overline{x}_{13}, \overline{x}_{21}, \overline{x}_{22}, \overline{x}_{24}, \overline{x}_{32}) = (50, 100, 50, 50, 150, 200)$, and $\overline{F} = \frac{P(\overline{x})}{Q(\overline{x})} = 0.6448$

To improve the solution, let us apply the Kuhn- Tucker's optimality conditions as

$F_{x_{11}} = -1.294$, $F_{x_{12}} = 1.286$, $F_{x_{13}} = -9.444$, $F_{x_{14}} = 1.4$,
$F_{x_{21}} = -2.167$, $F_{x_{22}} = 0.75$, $F_{x_{23}} = 1.067$, $F_{x_{24}} = -5.286$,
$F_{x_{31}} = 0.733$, $F_{x_{32}} = -15.5294$, $F_{x_{33}} = 1.2143$, $F_{x_{34}} = 0.917$.

To determine the cost equation, let us use the equation (1)

$$\frac{\partial F}{\partial x_{B_{ij}}} = s_i + t_j$$

Since,

$F_{x_{11}} = s_1 + t_1 \Rightarrow s_1 + t_1 = -1.294$, $F_{x_{12}} = s_1 + t_2 \Rightarrow s_1 + t_2 = 1.286$,
$F_{x_{13}} = s_1 + t_3 \Rightarrow s_1 + t_3 = -9.444$, $F_{x_{14}} = s_1 + t_4 \Rightarrow s_1 + t_4 = 1.4$,
$F_{x_{21}} = s_2 + t_1 \Rightarrow s_2 + t_1 = -2.167$, $F_{x_{22}} = s_2 + t_2 \Rightarrow s_2 + t_2 = 0.75$,
$F_{x_{23}} = s_2 + t_3 \Rightarrow s_2 + t_3 = 1.067$, $F_{x_{24}} = s_2 + t_4 \Rightarrow s_2 + t_4 = -5.286$,
$F_{x_{31}} = s_3 + t_1 \Rightarrow s_3 + t_1 = 0.733$, $F_{x_{32}} = s_3 + t_2 \Rightarrow s_3 + t_2 = -15.5294$,
$F_{x_{33}} = s_3 + t_3 \Rightarrow s_3 + t_3 = 1.2143$, $F_{x_{34}} = s_3 + t_4 \Rightarrow s_3 + t_4 = 0.917$.
Put $s_1 = 0$, we have
$t_1 = -1.294, t_2 = 1.286, t_3 = -9.444, t_4 = 1.4, s_2 = -0.873, t_3 = 1.94, t_4 = -4.413$,
$s_3 = 2.027$,
Let us estimate the reduced cost from the equation (2) as

$$M_{12} = \frac{\partial F}{\partial x_{NB_{12}}} - s_1 - t_2 = -35,$$

$$M_{14} = \frac{\partial F}{\partial x_{NB_{14}}} - s_1 - t_4 = 5.81,$$

$$M_{23} = \frac{\partial F}{\partial x_{NB_{23}}} - s_2 - t_3 = 11.43,$$

$$M_{31} = \frac{\partial F}{\partial x_{NB_{14}}} - s_1 - t_4 = 19.18,$$

$$M_{33} = \frac{\partial F}{\partial x_{NB_{14}}} - s_1 - t_4 = 28.33,$$

$$M_{34} = \frac{\partial F}{\partial x_{NB_{14}}} - s_1 - t_4 = 22.99.$$

It is clear that $M_{12} = -35$, so x_{12} should be entered as basic variables. Then the next iteration resulted in the initial basic feasible solution

$\overline{x} = (\overline{x}_{12}, \overline{x}_{13}, \overline{x}_{21}, \overline{x}_{24}, \overline{x}_{32}) = (50, 100, 100, 100, 200)$, and $\overline{F} = \frac{P(\overline{x})}{Q(\overline{x})} = 0.5294$

By repeating the previous procedure, we obtain
$F_{x_{11}} = 0.706$, $F_{x_{12}} = -1.714$, $F_{x_{13}} = -9.444$, $F_{x_{14}} = 1.4$,
$F_{x_{21}} = -5.176$, $F_{x_{22}} = 1.75$, $F_{x_{23}} = 1.067$, $F_{x_{24}} = -3.286$,
$F_{x_{31}} = 0.733$, $F_{x_{32}} = -15.5294$, $F_{x_{33}} = 1.2143$, $F_{x_{34}} = 0.917$.
Let us use the equation (1), to determine the cost equation as
$F_{x_{11}} = s_1 + t_1 \Rightarrow s_1 + t_1 = 0.706$, $F_{x_{12}} = s_1 + t_2 \Rightarrow s_1 + t_2 = -1.714$,
$F_{x_{13}} = s_1 + t_3 \Rightarrow s_1 + t_3 = -9.444$, $F_{x_{14}} = s_1 + t_4 \Rightarrow s_1 + t_4 = 1.4$,
$F_{x_{21}} = s_2 + t_1 \Rightarrow s_2 + t_1 = -5.176$, $F_{x_{22}} = s_2 + t_2 \Rightarrow s_2 + t_2 = 1.75$,
$F_{x_{23}} = s_2 + t_3 \Rightarrow s_2 + t_3 = 1.067$, $F_{x_{24}} = s_2 + t_4 \Rightarrow s_2 + t_4 = -3.286$,
$F_{x_{31}} = s_3 + t_1 \Rightarrow s_3 + t_1 = 0.733$, $F_{x_{32}} = s_3 + t_2 \Rightarrow s_3 + t_2 = --15.5294$,
$F_{x_{33}} = s_3 + t_3 \Rightarrow s_3 + t_3 = 1.2143$, $F_{x_{34}} = s_3 + t_4 \Rightarrow s_3 + t_4 = 0.917$.
Set $s_1 = 0$, we get
$t_1 = 0.706, t_2 = -1.714, t_3 = -9.444, t_4 = 1.4, s_2 = -5.882, s_3 = 0.027$.
By applying the equation (2), let us estimate the reduced cost from as

$$M_{11} = \frac{\partial F}{\partial x_{NB_{12}}} - s_1 - t_1 = 23.2,$$

$$M_{14} = \frac{\partial F}{\partial x_{NB_{14}}} - s_1 - t_4 = 10.15,$$

$$M_{22} = \frac{\partial F}{\partial x_{NB_{23}}} - s_2 - t_2 = 4.0522,$$

$$M_{23} = \frac{\partial F}{\partial x_{NB_{14}}} - s_2 - t_3 = 7.1,$$

$$M_{31} = \frac{\partial F}{\partial x_{NB_{14}}} - s_3 - t_1 = 23.2,$$

$$M_{33} = \frac{\partial F}{\partial x_{NB_{14}}} - s_3 - t_3 = 24.5,$$

$$M_{34} = \frac{\partial F}{\partial x_{NB_{14}}} - s_3 - t_4 = 23.5.$$

Since al of $\frac{\partial F}{\partial x_{NB_{ij}}} > 0$ at $\bar{x} = (\bar{x}_{12}, \bar{x}_{13}, \bar{x}_{21}, \bar{x}_{24}, \bar{x}_{32}) = (50, 100, 100, 100, 200)$ this leads to the optimal solution with the optimum value equal to $\bar{F} = 0.5294$, and in neutrosophic is

$$\tilde{F}^N = \frac{\langle(8600, 10500, 11550, 14100); 0.6, 0.4, 0.7\rangle}{\langle(9850, 14750, 17200, 20150); 0.6, 0.4, 0.7\rangle} = \langle(0.4268, 0.61045, 0.78305, 1.4315); 0.6, 0.4, 0.7\rangle$$

6. Results and Discussions

It is clear that the neutrosophic optimum value is:
$\tilde{F}^N = \langle(0.4268, 0.61045, 0.78305, 1.4315); 0.6, 0.4, 0.7\rangle$ is better than the primary basic feasible solution, where it lies between 0.4268 *and* 1.4315. Also, as the optimum value lies between 0.61045 *and* 0.78305, the overall acceptance level is60%. Also, the degrees of truthfulness and indeterminacy, respectively are given by:

$$\mu(x) = \begin{cases} 0.6\left(\frac{x-0.4268}{0.61045-0.4268}\right), & 0.4268 \le x < 0.61045, \\ 0.6, & 0.61045 \le x \le 0.78305, \\ 0.6\left(\frac{1.4315-x}{1.4315-0.78305}\right), & 0.78305 \le x \le 1.4315, \\ 0, & \text{otherwise,} \end{cases}$$

$$\rho(x) = \begin{cases} \frac{0.61045-x+0.4(x-0.4268)}{0.61045-0.4268}, & 0.4268 \le x < 0.61045, \\ 0.4, & 0.61045 \le x \le 0.78305, \\ \frac{x-0.78305+0.4(1.4315-x)}{1.4315-0.78305}, & 0.78305 \le x \le 1.4315, \\ 1, & \text{otherwise,} \end{cases}$$

$$\sigma(x) = \begin{cases} \frac{0.61045-x+0.7(x-0.4268)}{0.61045-0.4268}, & 0.4268 \le x < 0.61045, \\ 0.7, & 0.61045 \le x \le 0.78305, \\ \frac{x-0.78305+0.7(1.4315-x)}{1.4315-0.78305}, & 0.78305 \le x \le 1.4315, \\ 1, & \text{otherwise,} \end{cases}$$

Hence, the decision maker concludes that the optimum value range in between 0.4268 and 1.4315. On the other hand, the unit profit maximum is achieved with the supply 50 units from O_1 to D_2 with discount 3%, 100 units from O_1 to D_3 with discount 5%, 100 units from O_2 to D_1 with discount 3%, 100 units from O_2 to D_4 with discount 2%, and 200 units from O_3 to D_2 with discount 4%.

6.1 **Advantages/Limitations of the proposed algorithm**

The proposed algorithm's principal advantage is a novel combination of a parametric study, a multicriteria analysis, and the DM's vision. This combination uses the benefit of a parametric study that is used to scan the searching space smartly, the benefit of the multicriteria analysis that is used to rank the alternative solutions by employing the vision of the DM, and the benefit of involving the vision of the DM. Applying the proposed algorithm to real-life problems may encounter some limitations such as:

1- It does not take into account the complete parametric space, which has an endless number of possible scenarios. But, no other techniques can handle such situations where there are infinite scenarios.
2- It is impossible to assign a unified technique for assigning the interesting scenarios for the DM i.e. the approach does not involve a unified method; where the DM's vision and weights differ from one to another.
3- Many factors must be considered such as; (i) the possibility of formulating the problem as a FTP problem, (ii) the possibility of formulating the KKT conditions and solving it, and (iii) the capability of solving the PFTP problem's selected scenarios and finding their exact optimal solutions.

7. Comparitive Study

In this section, the proposed study is compared with some existing relevant literature to carve out the advantageous aspect of the proposed study. The Table 8 presents this comparison under certain parameters. It's obvious that the result obtained by the proposed approach is less than the result by Gomathi and Jayalakshmi [19]

Table 8. Comparisons of different researcher's contributions

Author's name	Vogel's approximation method	Kuhn- Tucker optimality conditions	Optimal neutrosophic solution	Environment
Gomathi and Jayalakshmi [19]	√	√	×	crisp
Bas et al., [25],	×	×	√	Crisp
Akram et al., [26]	×	×	√	Fuzzy
Our proposed approach	√	√	√	Neutrosophic

8. Conclusions and future works

In this paper, the maximization fractional transportation problem has solved efficiently in neutrosophic environment. The method which has applied is can be used in all of road tax, discount cost and others. In addition, the analysis process in the proposed approach depends upon some proposed characteristics that consider the uncertainty in determining the optimal solution. Fundamental definitions for NINP problems, such as optimistic-optimal solutions, pessimistic-optimal solutions, satisfactory-optimal solutions, and feasibility-risk factors, were also introduced. Furthermore, the proposed approach involves the vision of the DM in the process of finding the optimal solution, and a utility function is used to rank the different alternatives so that the satisfactory optimal solution can be easily identified. Finally, an example is introduced to clarify the efficiency of the proposed approach. Finally, an example is introduced to clarify the efficiency of the proposed approach and compare the results obtained by one of the most prominent evolutionary algorithms, the genetic algorithm (GA), to validate the accuracy and reliability of simulation results. Future work may include the further extension of this study to other fuzzy- like structure (i. e., interval- valued fuzzy set, Neutrosophic set, Pythagorean fuzzy set, Spherical fuzzy set etc. with more discussion and suggestive comments.

Data Availibity

No data were used to support this study.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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