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Neutrosophic Almost Contra α-Continuous Functions

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Abstract: This study utilizes the notions of ℵ-α-open set to introduce and study new form of ℵ-ccontinuity termed as ℵ-almost contra α-continuous function. Besides, we also introduce ℵα-connected space, ℵ-weakly Hausdorff space, separation axioms, ℵα-normal and ℵ-strong normal spaces. Characterizations of ℵ-almost contra α-continuous functions is also discussed.

Keywords: ℵ-almost contra α-continuous function; ℵα-connected space; ℵ-weakly Hausdorff space; ℵ-locally αindiscrete space; ℵα-normal space; ℵ-strong normal space.

1. Introduction

Many real-world problems in Finance, Medical sciences, Engineering and Social sciences deals with uncertainties. There are difficulties in solving the uncertainties in these data by traditional mathematical models. There are approaches such as fuzzy sets [28], intuitionistic fuzzy sets [10], vague sets [13], and rough sets [18] which can be treated as mathematical tools to avert obstacles dealing with ambiguous data. But all these approaches have their implicit crisis in solving the problems involving indeterminate and inconsistent data due to inadequacy of parameterization tools. Smarandache [24] studied the idea of neutrosophic set as an approach for solving issues that cover unreliable, indeterminacy and persistent data. Neutrosophic topological space was introduced by Salama et.al. [19] in 2012. Further Neutrosophic topological spaces are studied in [20]. Applications of neutrosophic topology depend upon the properties of neutrosophic open sets, neutrosophic closed sets, neutrosophic interior operator and neutrosophic closure operator. Topologists studied the sets that are near to neutrosophic open sets and neutrosophic closed sets. In this order, Arokiarani et.al.[9] defined neutrosophic semi-open (resp. pre-open and α-open) functions and investigated their relations. In [9], the characterizations of characterizations of neutrosophic pre continuous (resp. α-continuous) functions is also discussed.

The idea of almost continuous functions is done in 1968 [21] in topology. Similarly, the notion of fuzzy almost contra continuous and fuzzy almost contra α-continuous functions were discussed in [16]}. Recently, Al-Omeri and Smarandache [26, 27] introduced and studied a number of the definitions of neutrosophic closed sets, neutrosophic mapping, and obtained several preservation properties and some characterizations about neutrosophic of connectedness and neutrosophic connectedness continuity. More recently, in [1, 8] authors have given how new trend of Neutrosophic theory is applicable in the field of Medicine and multimedia with a novel and powerful model.

In this paper, we define Almost contra-continuity in the context of neutrosophic topology such as Neutrosophic Almost α-contra-continuous function. We also discuss some characterizations of this concept. Moreover ℵα-connected space, ℵ- weakly Hausdorff space, separation axioms and ℵα -normal spaces are presented and investigated some properties.

2. Preliminaries

Definition: 2.1 [22, 23] Allow T,I,F as real standard or non standard members of $]0^{-},1^{+}[$, with

$sup_T = t_{sup}, inf_T = t_{inf}$,

$sup_I = i_{sup}, inf_I = i_{inf}$,

$sup_F = f_{sup}, inf_F = f_{inf}$

$n - sup = t_{sup} + i_{sup} + f_{sup}$

$n - inf = t_{inf} + i_{inf} + f_{inf}$. T,I,F are neutrosophic components.

Definition: 2.2 [22, 23] Let S_1 be a non-empty fixed set. A definition set (in short N-set) Λ is an object such that $\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), \gamma_{\Lambda}(x)\rangle : x \in S_1\}$ wherein $\mu_{\Lambda}(x), \sigma_{\Lambda}(x)$ and $\gamma_{\Lambda}(x)$ which represents the degree of membership function (viz $\mu_{\Lambda}(x)$), the degree of indeterminacy (viz $\sigma_{\Lambda}(x)$) as well as the degree of non-membership (viz $\gamma_{\Lambda}(x)$) respectively of each element $x \in S_1$ to the set Λ.

Remark: 2.3[22, 23]

I. *An N-set* $\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), \Gamma_{\Lambda}(x)\rangle : x \in S_1\}$ *can be identified to an ordered triple* $\langle \mu_{\Lambda}, \sigma_{\Lambda}, \Gamma_{\Lambda}\rangle$ *in* $]0^{-},1^{+}[$ *on* S_1.

II. *In this paper, we use the symbol* $\Lambda = \langle \mu_{\Lambda}, \sigma_{\Lambda}, \Gamma_{\Lambda}\rangle$ *for the N-set* $\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), \Gamma_{\Lambda}(x)\rangle : x \in S_1\}$.

Definition: 2.4[12] Let $S_1 \neq \emptyset$ and the N-sets Λ and Γ be defined as

$\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), \Gamma_{\Lambda}(x)\rangle : x \in S_1\}$, $\Gamma = \{\langle x, \mu_{\Gamma}(x), \sigma_{\Gamma}(x), \Gamma_{\Gamma}(x)\rangle : x \in S_1\}$. Then

I. $\Lambda \subseteq \Gamma$ *iff* $\mu_{\Lambda}(x) \leq \mu_{\Gamma}(x)$, $\sigma_{\Lambda}(x) \leq \sigma_{\Gamma}(x)$ *and* $\Gamma_{\Lambda}(x) \geq \Gamma_{\Gamma}(x)$ *for all* $x \in S_1$;

II. $\Lambda = \Gamma$ *iff* $\Lambda \subseteq \Gamma$ *and* $\Gamma \subseteq \Lambda$;

III. $\bar{\Lambda} = \{\langle x, \Gamma_{\Lambda}(x), \sigma_{\Lambda}(x), \mu_{\Lambda}(x)\rangle : x \in S_1\}$; *[Complement of* Λ*]*

IV. $\Lambda \cap \Gamma = \{\langle x, \mu_{\Lambda}(x) \wedge \mu_{\Gamma}(x), \sigma_{\Lambda}(x) \wedge \sigma_{\Gamma}(x), \Gamma_{\Lambda}(x) \vee \Gamma_{\Gamma}(x)\rangle : x \in S_1\}$;

V. $\Lambda \cup \Gamma = \{\langle x, \mu_{\Lambda}(x) \vee \mu_{\Gamma}(x), \sigma_{\Lambda}(x) \vee \sigma_{\Gamma}(x), \Gamma_{\Lambda}(x) \wedge \gamma_{\Gamma}(x)\rangle : x \in S_1\}$;

VI. $[\ ]\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), 1 - \mu_{\Lambda}(x)\rangle : x \in S_1\}$;

VII. $\langle\rangle\, \Lambda = \{\langle x, 1 - \Gamma_{\Lambda}(x), \sigma_{\Lambda}(x), \Gamma_{\Lambda}(x)\rangle : x \in S_1\}$.

Definition: 2.5[12] Let $\{\Lambda_i : i \in J\}$ be an arbitrary family of N-sets in S_1. Thereupon

I. $\cap \Lambda_i = \{\langle p, \wedge \mu_{\Lambda_i}(p), \wedge \sigma_{\Lambda_i}(p), \vee \Gamma_{\Lambda_i}(p)\rangle : p \in S_1\}$;

II. $\cup \Lambda_i = \{\langle p, \vee \mu_{\Lambda_i}(p), \vee \sigma_{\Lambda_i}(p), \wedge \Gamma_{\Lambda_i}(p)\rangle : p \in S_1\}$.

The main theme is to construct the tools for developing NTS, so we establish the neutrosophic sets $0_{\aleph}$ along with $1_{\aleph}$ in X as follows:

Definition: 2.6[12] $0_{\aleph} = \{\langle q, 0,0,1\rangle : q \in X\}$ and $1_{\aleph} = \{\langle q, 1,1,0\rangle : q \in X\}$.

Definition: 2.7[12] A definition topology (in short,ℵ-topology) on $S_1 \neq \emptyset$ is a family ξ_1 of N-sets in S_1 satisfying the laws given below:

I. $0_N, 1_N \in \xi_1$,

II. $W_1 \cap W_2 \in T$ *being* $W_1, W_2 \in \xi_1$,

III. $\cup W_i \in \xi_1$ *for arbitrary family* $\{W_i | i \in \Lambda\} \subseteq \xi_1$.

In this case the ordered pair (S_1, ξ_1) or simply S_1 is termed as NTS and each N-set in ξ_1 is named as neutrosophic open set (in short,ℵ-open set) . The complement $\overline{\Lambda}$ of an ℵ-open set Λ in S_1 is known as neutrosophic closed set (briefly,ℵ-closed set) in S_1.

Definition: 2.8[12] Let Λ be an $\aleph$-set in an $NTSS_1$. Thereupon

$\aleph int(\Lambda) = \cup \{G|G$ is an $\aleph$-open set in S_1 and $G \subseteq \Lambda\}$ is termed as neutrosophic interior (in brief $\aleph$-interior) of Λ;

$\aleph cl(\Lambda) = \cap \{G|G$ is an $\aleph$-closed set in S_1 and $G \supseteq \Lambda\}$ is termed as neutrosophic closure (shortly $\aleph cl$) of Λ.

Definition: 2.9[12] Let X be a nonempty set. Whenever r, t, s be real standard or non standard subsets of $]0^{-}, 1^{+}[$ then the neutrosophic set $x_{r,t,s}$ is termed as neutrosophic point(in short NP)in X given by $x_{r,t,s}(x_p) = \begin{cases} (r,t,s), & if \quad x = x_p \\ (0,0,1), & if \quad x \neq x_p \end{cases}$ for $x_p \in X$ is termed as the support of $x_{r,t,s}$, wherein r indicates the degree of membership value, t indicates the degree of indeterminacy along with s as the degree of non-membership value of $x_{r,t,s}$.

Definition: 2.10[12] Allow (S_1, ξ_1) be a NTS. A neutrosophic set Λ in (S_1, ξ_1) is termed as $\mathfrak{g}\aleph$ closed set if $Ncl(\Lambda) \subseteq \Gamma$ whenever $\Lambda \subseteq \Gamma$ and Γ is a $\aleph$-open set. The complement of a $\mathfrak{g}\aleph$-closed set is named as $\mathfrak{g}\aleph$-open set.

Definition: 2.11[12] Let (X, T) be a NTS and Λ be a neutrosophic set in X. Subsequently, the neutrosophic generalized closure and neutrosophic generalized interior of Λ are defined by,

$$(i) NGcl(\Lambda) = \cap \{G: G \text{ is a generalized neutrosophic closed set in } S_1 \text{ and } \Lambda \subseteq G\}.$$
$$(ii) NGint(\Lambda) = \cup \{G: G \text{ is a generalized neutrosophic open set in } S_1 \text{ and } \Lambda \supseteq G\}.$$

3. Neutrosophic Almost Contra α-Continuous Functions.

A new form of $\aleph\alpha$-continuity termed as $\aleph$-almost contra α-continuity is discussed along with some of their properties.

Definition 3.1 *Let (S_1, ξ_1) and (S_2, ξ_2) be any two NTS . A function $g: (S_1, \xi_1) \to (S_2, \xi_2)$ is named as $\aleph$-almost contra α-continuous if inverse image of each $\aleph$-regular open set in S_2 is $\aleph\alpha$-closed set in S_1.*

Recall that, for a function $f: S_1 \to S_2$, the subset $G_f = \{x, f(x): x \in S_1\} \subset S_1 \times S_2$ is said to be graph of f.

Theorem 3.2 *Let $f: S_1 \to S_2$ be a function along with $g: S_1 \to S_1 \times S_2$ be the graph function defined by $g(x) = (x, f(x))$ being each $x \in S_1$. Whenever g is $\aleph$-almost contra α-continuous function, thereupon f is $\aleph$-almost contra α-continuous function.*

Proof. Let M be a $\aleph$-regular closed set in S_2 accordingly $S_1 \times M$ is a $\aleph$-regular closed set in $S_1 \times S_2$. In view of g is $\aleph$-almost contra α-continuous, so that $f^{-1}(M) = g^{-1}(S_1 \times M)$ is a $\aleph\alpha$-open in S_1. Thus f is $\aleph$-almost contra α-continuous.

Definition: 3.3

1. *A nonempty family $\mathbb{F}$ of $\aleph$ −open sets on (S_1, ξ_1) is known as $\aleph$ −filter if*
   - *I.* $0_\aleph \notin \mathbb{F}$
   - *II.* *If $A, B \in \mathbb{F}$ then $A \cap B \in \mathbb{F}$*
   - *III.* *If $A \in \mathbb{F}$ and $A \subset B$ then $B \in \mathbb{F}$*
2. *A nonempty family $\mathbb{B}$ of $\aleph$ − open sets on $\mathbb{F}$ is named as $\aleph$ − filter base if*
   - *I.* $0_\aleph \notin \mathbb{B}$

II. *If* $B_1, B_2 \in \mathbb{B}$ *then* $B_3 \subset B_1 \cap B_2$ *for some* $B_3 \in \mathbb{B}$

3. *A* $\aleph$ *–filter* $\mathbb{F}$ *is known as* $\aleph$ *–convergent to a* $\aleph$ *–point* $x_{r,s,t}$ *of a NTS*(S_1, ξ_1) *if for each* $\aleph$ *–open set A of* (S_1, ξ_1)*containing*$x_{r,s,t}$, *there exists a* $\aleph$ *–set* $B \in \mathbb{F}$ *so as* $B \subseteq A$.
4. *A* $\aleph$ *–filter* $\mathbb{F}$ *is said to be* $\aleph\alpha$*-convergent to a* $\aleph$ *–point* $x_{r,s,t}$ *of a NTS*(S_1, ξ_1) *if for each* $\aleph\alpha$ *–open set A of* (S_1, ξ_1)*containing*$x_{r,s,t}$*,there exists a* $\aleph - setB \in \mathbb{F}$ *thereby* $B \subseteq A$.
5. *A* $\aleph$ *–filter* $\mathbb{F}$ *is said to be* $\aleph$ *rc-convergent to a* $\aleph$ *–point* $x_{r,s,t}$ *of a NTS*(S_1, ξ_1) *if for each* $\aleph$ *regular closed set A of* (S_1, ξ_1) *containing* $x_{r,s,t}$*,there exists a* $\aleph$ *–set* $B \in \mathbb{F}$ *so as* $B \subseteq A$.

Proposition 3.4 *If a function* $\mu: S_1 \to S_2$ *is* $\aleph$*-almost contra* α*-continuous function and each* $\aleph$ *–filter base* $\mathbb{F}$ *in* S_1 *is* $\aleph\alpha$*-converging to* $x_{r,s,t}$*, the* $\aleph$ *–filter base* $\mu(\mathbb{F})$ *is* $\aleph$*rc-convergent to* $\mu(x_{r,s,t})$.

Proof. Let $x_{r,s,t} \in S_1$ and $\mathbb{F}$ be any $\aleph$-filter base in S_1 is $\aleph\alpha$-converging to $x_{r,s,t}$. As μ is $\aleph$-almost contra α-continuous, subsequently for any $\aleph$ regular closed R in S_2 including $\mu(x_{r,s,t})$, there exists $\aleph\alpha$-open W in S_1 involving $x_{r,s,t}$ so as $\mu(W) \subset R$. As $\mathbb{F}$ is $\aleph\alpha$-convergent to $x_{r,s,t}$, there occurs $A \in \mathbb{F}$ thereby $A \subset W$. This means that $\mu(A) \subset R$ and consequently the $\aleph$-filter base $\mu(\mathbb{F})$ is $\aleph$rc-convergent to $\mu(x_{r,s,t})$.

Definition: 3.5

1. *A space* S_1 *is termed as* $\aleph\alpha$*-connected if* S_1 *can't be expressed as union of two disjoint non-empty* $\aleph\alpha$*-open sets.*
2. *A space* S_1 *is named as* $\aleph$*-connected if* S_1 *cannot be written as union of two disjoint non-empty* $\aleph$*-open sets.*

Theorem 3.6 *If* $f: S_1 \to S_2$ *is a* $\aleph$*-almost contra* α*-continuous surjection along with* S_1 *is* $\aleph\alpha$*-connected space, then*S_2 *is* $\aleph$*-connected.*

Proof. Let $f: S_1 \to S_2$ be a $\aleph$-almost contra α-continuous surjection with S_1 is $\aleph\alpha$-connected space. Assuming S_2 is a not $\aleph$-connected space. Accordingly, there existdisjoint$\aleph$-open sets W and Rsuch that$S_2 = W \cup R$. Then, $W$and$R$ are $\aleph$-clopen in S_2. As f is $\aleph$-almost contra α-continuous, $f^{-1}(W)$and$f^{-1}(R)$ are $\aleph\alpha$-open sets in S_1. In addition $f^{-1}(W)$and$f^{-1}(R)$ are disjoint non-empty and$S_1 = f^{-1}(W) \cup f^{-1}(R)$. It is contradiction to the fact thatS_1 is $\aleph\alpha$-connected space. Hence, S_2 is $\aleph$-connected.

Definition 3.6 *A space* S_1 *is named as* $\aleph$*-locally* α*-indiscrete if every* $\aleph\alpha$*-open set is* $\aleph$*-closed in* S_1.

Definition 3.7 *A function* $g: S_1 \to S_2$ *is termed as* $\aleph$*-almost continuous if* $g^{-1}(V)$ *is* $\aleph$*-open in* S_1 *for each* $\aleph$*-regular open set* V *in* S_2.

Definition 3.8 *A function* $f: S_1 \to S_2$ *is known as* $\aleph$*-almost* α*-continuous if* $f^{-1}(V)$ *is* $\aleph\alpha$*-open in* S_1 *for each* $\aleph$*-regular open set* V *in* S_2.

Theorem 3.9 *If a function*$\eta: S_1 \to S_2$ *is* $\aleph$*-almost contra* α*-continuous function and* S_1 *is* $\aleph$*-locally* α*-indiscrete space, then* f *is* $\aleph$*-almost continuous function.*

Proof. Let$W$ be a $\aleph$-regular closed set in S_2. Since η is $\aleph$-almost contra α-continuous function, $\eta^{-1}(W)$ is $\aleph\alpha$-open set in S_1 and S_1 is $\aleph$-locally α-indiscrete space, which implies $\eta^{-1}(W)$ is a $\aleph$-closed set in S_1. Hence,η is $\aleph$-almost continuous function.

Definition 3.10 *A space S_1 named as ℵ-weakly Hausdorff if each element of S_1 is an intersection of ℵ-regular closed sets.*

Definition 3.11 *A space S_1 is named as*

1. *ℵα-T_0 if for each pair of distinct ℵ-points $x_{r,s,t}$ and $y_{r,s,t}$ in S_1, there exists ℵα-open set U such that $x_{r,s,t} \in U$, $y_{r,s,t} \notin U$ or $x_{r,s,t} \notin U$, $y_{r,s,t} \in U$.*

2. *ℵα-T_1 if for each pair of distinct ℵ-points $x_{r,s,t}$ and $y_{r,s,t}$ in S_1, there exist ℵα-open sets U and V containing $x_{r,s,t}$ and $y_{r,s,t}$ respectively, so as $y_{r,s,t} \notin U$ and $x_{r,s,t} \notin V$.*

3. *ℵα-T_2 if for each pair of distinct ℵ-points $x_{r,s,t}$ and $y_{r,s,t}$ in S_1, there exists ℵα-open set U containing $x_{r,s,t}$ and ℵα-open set V containing $y_{r,s,t}$ so as $U \cap V = 0_N$.*

4. *A space S_1 is termed as ℵα-normal if each pair of non-empty disjoint ℵ-closed sets can be separated by disjoint ℵα-open sets.*

5. *A space S_1 is termed as ℵ-strongly-normal ifr each pair of disjoint non-empty ℵ-closed sets U and V there exists disjoint ℵ-open sets W and R such that $U \subset W$, $V \subset R$ and ℵ$cl(W) \cup$ ℵ$cl(R) = 0_{ℵ}$.*

6. *A space S_1 is called a ℵ-ultra normal if each pair of non-empty disjoint ℵ-closed sets can be separated by disjoint ℵ-clopen sets.*

Theorem 3.12 *If $f: S_1 \rightarrow S_2$ is an ℵ-almost contra α-continuous injection and S_2 is ℵ-weakly Hausdorff space, then S_1 is ℵα-T_1.*

Proof. Let S_2 be a ℵ-weakly Hausdorff space. For any distinct ℵ points $x_{r,s,t}$ and $y_{r,s,t}$ in S_1, there exist V and W, ℵ-regular closed sets in S_2 such that $f(x_{r,s,t}) \in V$, $f(y_{r,s,t}) \notin V$, $f(y_{r,s,t}) \in W$ and $f(x_{r,s,t}) \notin W$. As f is ℵ-almost contra α-continuous, $f^{-1}(V)$and$f^{-1}(W)$ are ℵα-open subsets of S_1 such that $x_{r,s,t} \in f^{-1}(V)$, $y_{r,s,t} \notin f^{-1}(V)$, $y_{r,s,t} \in f^{-1}(W)$ and $x_{r,s,t} \notin f^{-1}(W)$. Hence, S_1 is ℵα-T_1.

Theorem 3.13 *If $h: S_1 \rightarrow S_2$ is a ℵ-almost contra α-continuous injective mapping from space S_1 into a ℵ-Ultra Hausdroff space S_2, thernS_1 is ℵα-T_2.*

Proof. Let$x_{r,s,t}$and$y_{r,s,t}$ be any two distinct ℵ elements in S_1. As f is an injective $h(x_{r,s,t}) \neq h(y_{r,s,t})$ and S_2 is ℵ-Ultra Hausdroff space, there exist disjoint ℵ-clopen sets U and V of S_2 containing $h(x_{r,s,t})$ and $h(y_{r,s,t})$ respectively. Subsequently, $x_{r,s,t} \in h^{-1}(U)$ and $y_{r,s,t} \in h^{-1}(V)$, wherein $h^{-1}(U)$ and $h^{-1}(V)$ are disjoint ℵα-open sets in S_1. Then,S_1 is ℵα-T_2.

Proposition 3.14 *If S_2 is ℵ strongly-normal and$\mu: S_1 \rightarrow S_2$ is a ℵ almost contra-α-continuous closed injection, thenS_1 is ℵα-normal.*

Proof. Suppose$J$ and $K$are disjoint ℵ-closed members of S_1. Let μ is ℵ-closed and injective $f(J)$ and $f(K)$ are disjoint ℵ-closed sets in S_2. As S_2 is ℵ strongly-normal, there exist ℵ-open sets W and R in Y so that $\mu(J) \subset W$ and $\mu(K) \subset R$ and ℵ$cl(W) \cap$ ℵ$cl(R) = 0_{ℵ}$. Then, since ℵ$cl(W)$ and ℵ$cl(V)$ are ℵ regular closed, and μ is an ℵ almost contra α-continuous, μ^{-1}(ℵ$cl(W)$) and μ^{-1}(ℵ$cl(R)$) are ℵα-open sets in S_1. This implies $J \subseteq \mu^{-1}$(ℵ$cl(W)$), $K \subseteq \mu^{-1}$(ℵ$cl(R)$) and μ^{-1}(ℵ$cl(W)$) and μ^{-1}(ℵ$cl(R)$) are disjoint,so S_1 is ℵα-normal.

Theorem 3.15 *If $f: S_1 \to S_2$ is a ℵ-almost contra α-continuous, ℵ-closed injection along with S_2 is ℵ-ultra normal, then S_1 is ℵα-normal.*

Proof. Let P and Q be disjoint ℵ-closed sets of S_1. As f is ℵ-closed as well as injective, $f(P)$ along with $f(Q)$ are disjoint ℵ-closed sets in S_2. Since S_2 is ℵ-ultra normal, there exist disjoint ℵ-clopen sets U and V in S_2such that$f(P) \subset U$ and $f(Q) \subset V$. This implies $P \subset f^{-1}(U)$ with $Q \subset f^{-1}(V)$. As f is a ℵ-almost contra α-continuous injection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint ℵα-open sets in S_1. Therefore, S_1 is ℵα-normal.

Definition 3.16 *A function $f: S_1 \to S_2$ is called ℵ-weakly almost contra α continuous if for each ℵ-point $x_{r,s,t}$ in S_1 and each ℵ regular closed set V of S_2containing$f(x_{r,s,t})$, there exists a ℵα-open set U in S_1, such that ℵ$cl(f(U)) \subseteq V$.*

Definition 3.17 *A function $f: S_1 \to S_2$ is termed as ℵ(α, S)-open if the image of each ℵ-open set is ℵ-semi open.*

Theorem 3.18 *If $f: S_1 \to S_2$ is a ℵ-weakly almost contra α-continuous and ℵ(α, S)-open then, f is ℵ-almost contra α continuous.*

Proof. Let $x_{r,s,t}$ be a ℵ point in S_1 and V be a ℵ-regular closed set containing $f(x_{r,s,t})$. Since f is ℵ-weakly almost contra α continuous, there exist a ℵα-open set U in S_1 containing $x_{r,s,t}$ so as ℵ$cl(f(U)) \subseteq V$. Since f is a ℵ(α, S)-open, $f(U)$ is a ℵ-semi open set in S_2 and $f(U) \subseteq$ ℵ$cl($ℵ$int(f(U))) \subseteq V$.This shows f is ℵ almost contra α continuous.

4. Conclusions

In this paper, we have introduced and studied the concepts like, Neutrosophic Almost α-contra-continuous function, ℵα-connected space, ℵ-weakly Hausdorff space, separation axioms and ℵα-normal spaces and investigated some properties. Some preservation theorems are also discussed. It will be necessary to carry out more theoretical research to establish a general framework for decision-making and to define patterns for complex network conceiving and practical application.

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