Int. J. Anal. Appl. (2024), 22:26

International Journal of Analysis and Applications

Single-Valued Neutrosophic Ideal Approximation Spaces

Yaser Saber[1,2,*], Mohamed Abusalih[1], Esam Bader[1], Tawfik Elmasry[1,3], Abdelaziz Babiker[1], Florentin Smarandache[4]

[1]*Department of Business Administration, College of Science and Human Studies, Hotat Sudair, Majmaah University, Majmaah, 11952, Saudi Arabia*

[2]*Department of Mathematics, Faculty of Science, Al-Azhar University, Assiut, 71524, Egypt*

[3]*Department of Operations Research, Faculty of Graduate Studies Statistical Research, Cairo University, Egypt*

[4]*Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA*

[*]*Corresponding author:* y.sber@mu.edu.sa and m.abusalih@mu.edu.sa

Abstract. In this paper, we defined the basic idea of the single-valued neutrosophic upper $(\alpha_n)^\delta$, single-valued neutrosophic lower $(\alpha_n)_\delta$ and single-valued neutrosophic boundary sets $(\alpha_n)^B$ of a rough single-valued neutrosophic set α_n in a single-valued neutrosophic approximation space $(\tilde{\mathcal{F}}, \delta)$. Based on α_n and δ, we introduced the single-valued neutrosophic ideal approximation interior operator $\text{int}^\delta_{\alpha_n}$ and the single-valued neutrosophic ideal approximation closure operator $\text{Cl}^\delta_{\alpha_n}$. We joined the single-valued neutrosophic ideal notion with the single-valued neutrosophic approximation spaces and then introduced the single-valued neutrosophic ideal approximation closure and interior operators associated with a rough single-valued neutrosophic set α_n. single-valued neutrosophic ideal approximation connectedness and the single-valued neutrosophic ideal approximation continuity between single-valued neutrosophic ideal approximation spaces are introduced. The concepts of single-valued neutrosophic groups and their approximations have also been applied in the development of fuzzy systems, enhancing their ability to model and reason using uncertain and imprecise information.

1. Introduction

Sometimes, it is not convenient to apply practical problems to real-life applications. Data in medical sciences, economics, weather, climate changes, etc always involve various types of uncertainties. Moreover, fuzzy systems have been extensively studied and applied in various

Received: Dec. 1, 2023.

2020 *Mathematics Subject Classification.* 94D05.

Key words and phrases. Single-valued neutrosophic rough set; single-valued neutrosophic ideal; single-valued neutrosophic ideal approximation space; single-valued neutrosophic ideal continuity; single-valued neutrosophic ideal connectedness.

https://doi.org/10.28924/2291-8639-22-2024-26
ISSN: 2291-8639

domains due to their ability to handle uncertainty and imprecision. Fuzzy sets and fuzzy logic, provide a flexible framework for modeling and reasoning with vague and uncertain information. Fuzzy systems have found applications in areas such as control systems, decision support systems, and pattern recognition. The integration of neutrosophic set theory with fuzzy systems offers an intriguing avenue for further exploration. By incorporating the three membership functions of truth, indeterminacy, and falsity, neutrosophic sets can provide an enhanced representation of uncertainty. The combination of neutrosophic sets and fuzzy systems has the potential to improve the modeling and decision-making capabilities by capturing and managing a wider range of uncertainties that arise in real-world applications.

The notion of rough sets was given by Pawlak [1] referring to the uncertainty of intelligent systems characterized by insufficient and incomplete information. Rough sets are defined depending on some equivalence relation δ on a universal finite set $\tilde{\mathcal{F}}$. The pair $(\tilde{\mathcal{F}}, \delta)$ was called an approximation space based on an equivalence relation imposed on $\tilde{\mathcal{F}}$. In any approximation space, the notions of lower approximation, upper approximation, and boundary region operators of some subset could be induced. Many types of generalizations of Pawlak's rough set have been obtained by replacing equivalence relation with an arbitrary binary relation. On the other hand, the relationships between rough sets and topological spaces were studied in [2]. A lot of fuzzy generalizations of rough approximation have been proposed in the literature [3–7]. Irfan in [8] studied the connections between fuzzy set, rough set, and soft set ([9]) notions. Many papers studied the relationship between fuzzy rough set notions and fuzzy topologies [10, 11]. Recently, many researchers have used topological approaches in the study of rough sets and their applications. In [12], it was used the notion of ideal in soft rough ordinary topological space, and in [13], the authors introduced fuzzy soft connectedness in the sense of Chang [14].

To exceed the difficulties in using the traditional classical methods the word neutrosophy is understood to be a tool for handling problems involving incomplete, indeterminate and inconsistent information and the theory was initiated by Smarandache [15] as a generalization of fuzzy sets and intuitionistic fuzzy sets. He defined the neutrosophic sets to be characterized by three membership functions independently: truth, indeterminacy and falsity. For this reason, the neutrosophic set theory becomes an attractive field for scientists and researchers who like to develop their concerns and match up their works in this scope, such as Wang et al [16] who formed the single-valued neutrosophic sets, Yang et al., and Qiu at al., [17, 18] who proposed the single-valued neutrosophic relations and single-Valued neutrosophic rough sets, and also Saber et al. [19–30] who familiarized the concepts of single-valued neutrosophic ideal open local function and single-valued neutrosophic topological space.

The incentive of this paper is to present a new better single-valued neutrosophic lower and single-valued neutrosophic upper sets through which we get a more consistent single-valued neutrosophic boundary region set off a single-valued neutrosophic set α_n. From these single-valued neutrosophic lower and single-valued neutrosophic upper sets, we introduced concepts of new single-valued neutrosophic interior and single-valued neutrosophic closure operators related to a specific single-valued neutrosophic set $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$. and that single-valued neutrosophic relation δ on $\tilde{\mathcal{F}}$. In the single-valued neutrosophic approximation space $(\tilde{\mathcal{F}}, \delta)$, based on this single-valued neutrosophic interior and single-valued neutrosophic closure operators, we defined fuzzy approximation connectedness. Defining a single-valued neutrosophic ideal on $\tilde{\mathcal{F}}$ generates a single-valued neutrosophic ideal approximation space in which a single-valued neutrosophic local function was defined and many results are proved. Connectedness in single-valued neutrosophic ideal approximation spaces $(\tilde{\mathcal{F}}, \delta, \hbar)$ is defined and related with connectedness in single-valued neutrosophic approximation spaces. Also, single-valued neutrosophic ideal approximation continuity among two single-valued neutrosophic ideal approximation spaces were discussed.

In this study, $\tilde{\mathcal{F}}$ denotes to an initial universe, $\xi^{\tilde{\mathcal{F}}}$ is the collection of all single valued neutrosophic sets (simply, $\mathcal{SVNS}$) on $\tilde{\mathcal{F}}$ (where, $\xi = [0,1], \xi_0 = (0,1]$ and $\xi_1 = [0,1)$)

2. Preliminaries

Definition 2.1. *Let $\tilde{\mathcal{F}}$ be a finite set, with a generic element in $\tilde{\mathcal{F}}$ indicated by υ. A $\mathcal{SVNS}$ [4] is defined as:*

$$\alpha_n = \{\langle \upsilon, \tilde{\varrho}_{\alpha_n}(\upsilon), \tilde{\sigma}_{\sigma_n}(\upsilon), \tilde{\varsigma}_{\alpha_n}(\upsilon)\rangle : \upsilon \in \tilde{\mathcal{F}}\}$$

where $\tilde{\varrho}_{\alpha_n} : \tilde{\mathcal{F}} \to \xi$ ($\tilde{\varrho}_{\alpha_n}$ called membership function), $\tilde{\sigma}_{\alpha_n} : \tilde{\mathcal{F}} \to \xi$ ($\tilde{\sigma}_{\alpha_n}$ called nonmembership function) and $\tilde{\varsigma}_{\alpha_n} : \tilde{\mathcal{F}} \to \xi$ ($\tilde{\varsigma}_{\alpha_n}$ called nonmembership function) of υ to α_n with

$$0 \le \tilde{\varrho}_{\alpha_n}(\upsilon) + \tilde{\sigma}_{\alpha_n}(\upsilon) + \tilde{\varsigma}_{\alpha_n}(\upsilon) \le 3$$

All characterizations and concepts of $\mathcal{SVNS}$ originate in found [19] and [17].

Definition 2.2. *For any $\omega \in \tilde{\mathcal{F}}$, define a single valued neutrosophic coset (sample, $\mathcal{SVN}-coset$) $[\omega] : \tilde{\mathcal{F}} \to [0,1]$ as:*

$$\tilde{\sigma}_{[\omega]}(\nu) = \tilde{\sigma}_{\delta}(\omega, \nu), \quad \tilde{\varsigma}_{[\omega]}(\nu) = \tilde{\zeta}_{\delta}(\omega, \nu) \quad \tilde{\varrho}_{[\omega]}(\nu) = \tilde{\sigma}_{\delta}(\omega, \nu), \quad \forall \nu \in \tilde{\mathcal{F}}, \tag{2.1}$$

All elements $\nu \in \tilde{\mathcal{F}}$ with $\mathcal{SVNS}$ value $\tilde{\sigma}_{\delta}(\omega,\nu) \le 1, \tilde{s}_{\delta}(\omega,\nu) \le 1, \tilde{\varrho}_{\delta}(\omega,\nu) > 0$ are elements having a membership value in the $\mathcal{SVN}-coset$ $[\omega]$, and any element $\nu \in \tilde{\mathcal{F}}$ with $\tilde{\sigma}_{\delta}(\omega,\omega) = 0, \tilde{\varsigma}_{\delta}(\omega,\omega) = 0, \tilde{\varrho}_{\delta}(\omega,\omega) = 1$ is not included in the $\mathcal{SVN}-coset$ $[\omega]$. Any $\mathcal{SVN}-coset$ $[\omega]$ surely include the element $\omega \in \tilde{\mathcal{F}}$, and consequently

$$\tilde{\sigma}_{\bigwedge_{\mu\in\tilde{\mathcal{F}}}}([\omega](\mu)) = 0, \quad \tilde{\varsigma}_{\bigwedge_{\mu\in\tilde{\mathcal{F}}}}([\omega](\mu)) = 0, \quad \forall \omega \in \tilde{\mathcal{F}}, \quad \tilde{\varrho}_{\bigvee_{\mu\in\tilde{\mathcal{F}}}}([\omega](\mu)) = 1.$$

$$\tilde{\sigma}_{\bigwedge_{\mu\in\tilde{\mathcal{F}}}}([\omega](\nu)) = 0, \quad \tilde{\varsigma}_{\bigwedge_{\mu\in\tilde{\mathcal{F}}}}([\omega](\nu)) = 0, \quad \forall \nu \in \tilde{\mathcal{F}}, \quad \tilde{\varrho}_{\bigvee_{\mu\in\tilde{\mathcal{F}}}}([\omega](\nu)) = 1,$$

such that $\bigvee_{\nu\in\tilde{\mathcal{F}}}([\nu]) = \langle 0,1,1\rangle$.

Definition 2.3. *Let us define the single-valued neutrosophic difference between two* `SVNSs` *as next:*

$$\tilde{\varrho}_{\alpha_n} \overline{\wedge} \tilde{\varrho}_{\varepsilon_n}(\omega) = \begin{cases} 0, & \text{if} \quad \tilde{\varrho}_{\alpha_n}(\omega) \le \tilde{\varrho}_{\varepsilon_n}(\omega), \\ \tilde{\varrho}_{\alpha_n} \wedge \tilde{\varrho}_{(\varepsilon_n)^c}(\omega), & \text{otherwise.} \end{cases}$$

$$\tilde{\sigma}_{\alpha_n}\overline{\vee}\tilde{\sigma}_{\varepsilon_n}(\omega) = \begin{cases} 1, & if \quad \tilde{\sigma}_{\alpha_n}(\omega) \geq \tilde{\sigma}_{\varepsilon_n}(\omega), \\ \tilde{\sigma}_{\alpha_n} \vee \tilde{\sigma}_{(\varepsilon_n)^c}(\omega), & otherwise. \end{cases} \tag{2.2}$$

$$\tilde{\zeta}_{\alpha_n}\overline{\vee}\tilde{\zeta}_{\varepsilon_n}(\omega) = \begin{cases} 1, & if \quad \tilde{\zeta}_{\alpha_n}(\omega) \geq \tilde{\zeta}_{\varepsilon_n}(\omega), \\ \tilde{\zeta}_{\alpha_n} \vee \tilde{\zeta}_{(\varepsilon_n)^c}(\omega), & otherwise. \end{cases}$$

3. Single-Valued Neutrosophic Ideal Approximation Spaces

Definition 3.1. *A nonempty collection of* `SVNS`*s* $\hbar$ *of a set* $\tilde{\mathcal{F}}$ *is called single valued neutrosophic ideal [8] (briefly,* `SVNI`*) on* $\tilde{\mathcal{F}}$ *if it satisfies the following:*

1. $\langle 0,1,1\rangle \in \hbar$,
2. *If* $\tilde{\varrho}_{\alpha_n}(\omega) \leq \tilde{\varrho}_{\varepsilon_n}(v), \tilde{\sigma}_{\alpha_n}(\omega) \geq \tilde{\sigma}_{\varepsilon_n}(\omega), \tilde{\zeta}_{\alpha_n}(\omega) \geq \tilde{\zeta}_{\varepsilon_n}(\omega)$ *and* $\alpha_n \in \hbar$, *then,* $\varepsilon_n \in \hbar, \forall \omega \in \tilde{\mathcal{F}}$ *and* $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$
3. *If* $\alpha_n, \varepsilon_n \in \hbar$, *then* $\langle (\tilde{\varrho}_{\alpha_n} \vee \tilde{\varrho}_{\varepsilon_n})(\omega), (\tilde{\sigma}_{\alpha_n} \wedge \tilde{\sigma}_{\varepsilon_n})(\omega)(\tilde{\zeta}_{\alpha_n} \wedge \tilde{\zeta}_{\varepsilon_n})(\omega)\rangle \in \hbar, \forall\ \omega \in \tilde{\mathcal{F}}$ *and* $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$.

If $\hbar_1$ *and* $\hbar_2$ *are* `SVNI`*s on* $\tilde{\mathcal{F}}$, *we obtain* $\hbar_1$ *is finer than* $\hbar_1$ $[\hbar_2$ *is coarser than* $\hbar_1]$ *if* $\hbar_1 \supseteq \hbar_2$. *The triple* $(\tilde{\mathcal{F}}, \delta, \hbar)$ *is said to be a single-valued neutrosophic ideal approximation space (briefly,* `SVNIAS`*). Denote the trivial* `SVNI` $\hbar^\circ$ *as a* `SVNI` *including only* $\langle 0,1,1\rangle$.

Definition 3.2. *Let* $(\tilde{\mathcal{F}}, \delta, \hbar)$ *be a* `SVNIAS` *and* $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$. *Then, the single valued neutrosophic local set* $[\varepsilon_n]^\star_{\alpha_n}(\delta,\hbar)$ *of a set* $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ *is defined by:*

$$\begin{aligned}[\varepsilon_n]^\star_{\alpha_n}(\delta,\hbar) = & \bigwedge\{\pi_n \in \zeta^{\tilde{\mathcal{F}}} : \varepsilon_n \overline{\wedge} \pi_n = \langle \tilde{\varrho}_{\varepsilon_n} \overline{\wedge} \tilde{\varrho}_{\pi_n}(\omega), \tilde{\sigma}_{\sigma_n} \overline{\vee} \tilde{\sigma}_{\pi_n}(\omega), \\ & \tilde{\zeta}_{\varepsilon_n} \overline{\vee} \tilde{\zeta}_{\pi_n}(\omega)\rangle \in \hbar, \mathrm{Cl}^{\alpha_n}_{\delta}(\pi_n) = \pi_n\}\end{aligned}$$

Briefly, we will write $[\varepsilon_n]^\star_{\alpha_n}$ *or* $[\varepsilon_n]^\star_{\alpha_n}(\hbar)$ *instead of* $[\varepsilon_n]^\star_{\alpha_n}(\delta,\hbar)$.

Corollary 3.1. *Let* $(\tilde{\mathcal{F}}, \delta, \hbar)$ *be a* `SVNIAS` *and* $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$, *where* $\hbar^\diamond$ *is the trivial single valued neutrosophic ideal on* $\tilde{\mathcal{F}}$. *Then, for each* $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$, *we have* $[\varepsilon_n]^\star_{\alpha_n} = \mathrm{Cl}^{\alpha_n}_{\delta}(\varepsilon_n)$.

Proof. Since, $\hbar^\diamond = \langle 0,1,1\rangle$, we obtain

$$[\varepsilon_n]^\star_{\alpha_n}(\delta,\hbar) = \bigwedge\{\pi_n \in \zeta^{\tilde{\mathcal{F}}} : \varepsilon_n \overline{\wedge} \pi_n = \langle \tilde{\varrho}_{\varepsilon_n} \overline{\wedge} \tilde{\varrho}_{\pi_n}(\omega), \tilde{\sigma}_{\sigma_n} \overline{\vee} \tilde{\sigma}_{\pi_n}(\omega), \tilde{\zeta}_{\varepsilon_n} \overline{\vee} \tilde{\zeta}_{\pi_n}(\omega)\rangle = \Big\langle 0,1,1\rangle, \mathrm{Cl}^{\alpha_n}_{\delta}(\pi_n) = \pi_n\Big\}$$

this implies

$$[\varepsilon_n]^\star_{\alpha_n}(\delta,\hbar) = \bigwedge\{\pi_n \in \zeta^{\tilde{\mathcal{F}}} : \tilde{\varrho}_{\varepsilon_n}(\omega) \leq \tilde{\varrho}_{\pi_n}(\omega), \tilde{\sigma}_{\varepsilon_n}(\omega) \geq \tilde{\sigma}_{\pi_n}(\omega)\tilde{\zeta}_{\varepsilon_n}(\omega) \geq \tilde{\zeta}_{\pi_n}(\omega), \mathrm{Cl}^{\alpha_n}_{\delta}(\pi_n) = \pi_n\}.$$

Since $\varepsilon_n \leq \mathrm{Cl}^{\alpha_n}_{\delta}(\varepsilon_n), \mathrm{Cl}^{\alpha_n}_{\delta}\left(\mathrm{Cl}^{\alpha_n}_{\delta}(\varepsilon_n)\right) = \mathrm{Cl}^{\alpha_n}_{\delta}(\varepsilon_n)$, then $[\varepsilon_n]^\star_{\alpha_n} \leq \mathrm{Cl}^{\alpha_n}_{\delta}(\varepsilon_n)$. Let $\mathrm{Cl}^{\alpha_n}_{\delta}(\varepsilon_n) \nleq [\varepsilon_n]^\star_{\alpha_n}$, then there exists $\pi_n \in \zeta^{\tilde{\mathcal{F}}}, \tilde{\varrho}_{\varepsilon_n}(\omega) \leq \tilde{\varrho}_{\pi_n}(\omega), \tilde{\sigma}_{\varepsilon_n}(\omega) \geq \tilde{\sigma}_{\pi_n}(\omega), \tilde{\zeta}_{n_n}(\omega) \geq \tilde{\zeta}_{\pi_n}(\omega), \mathrm{Cl}^{\alpha_n}_{\delta}(\pi_n) = \pi_n$, so that $\mathrm{Cl}^{\alpha_n}_{\delta}(\varepsilon_n) > \pi_n$ But $\tilde{\varrho}_{\varepsilon_n}(\omega) \leq \tilde{\varrho}_{\pi_n}(\omega), \tilde{\sigma}_{\varepsilon_n}(\omega) \geq \tilde{\sigma}_{\pi_n}(\omega), \tilde{\zeta}_{\varepsilon_n}(\omega) \geq \tilde{\zeta}_{\pi_n}(\omega)$ implies that $(\varepsilon_n)^\delta \leq (\pi_n)^\delta$ and then

$$\mathrm{Cl}^{\alpha_n}_{\delta}(\varepsilon_n) = [(\alpha_n)_\delta]^c \vee [\varepsilon_n]^\delta \leq [(\alpha_n)_\delta]^c \vee [\pi_n]^\delta = \mathrm{Cl}^{\alpha_n}_{\delta}(\pi_n) = \pi_n.$$

Contradiction, and then $[\varepsilon_n]^\star_{\alpha_n} = \mathrm{Cl}^{\alpha_n}_{\delta}(\varepsilon_n)$. □

δ	ω_1	ω_2	ω_3	ω_4	ω_5
ω_1	(1, 0, 0)	(0, 1, 1)	(0, 1, 1)	(0, 1, 1)	(0, 1, 1)
ω_2	(0, 1, 1)	(1, 0, 0)	(0, 1, 1)	(0, 1, 1)	(0, 1, 1)
ω_3	(0, 1, 1)	(0, 1, 1)	(1, 0, 0)	(0, 1, 1)	(0, 1, 1)
ω_4	(0, 1, 1)	(0, 1, 1)	(0, 1, 1)	(1, 0, 0)	(0.2, 0.2, 0.2)
ω_5	(0, 1, 1)	(0, 1, 1)	(0, 1, 1)	(0.2, 0.2, 0.2)	(1, 0, 0)

Table 1. SVNRof δ

Example 3.1. *This is an example that shows that* $[\varepsilon_n]^{\star}_{\alpha_n} \leq \mathrm{Cl}^{\alpha_n}_{\delta}(\varepsilon_n)$.

Let δ be SVNR on a set $\tilde{\mathcal{F}} = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$ as shown down.

Assume that $\alpha_n = \langle(1,0,0),(1,0,0),(1,0,0),(0.1,0.1,0.1),(0.2,0.2,0.2)\rangle$. *Then,*

$$\tilde{\varrho}_{(\alpha_n)^\delta}(\omega_1) = \tilde{\varrho}_{\alpha_n}(\omega_1) \wedge \bigvee_{\alpha_n(\mu)>0, \mu\neq\omega_1} \tilde{\varrho}_{[\omega]}(\mu) = 0,$$

$$\tilde{\sigma}_{(\alpha_n)^\delta}(\omega_1) = \tilde{\sigma}_{\alpha_n}(\omega_1) \vee \bigwedge_{\alpha_n(\mu)>0, \mu\neq\omega_1} (\tilde{\sigma}_{[\omega]})(\mu) = 1,$$

$$\tilde{\varsigma}_{(\alpha_n)^\delta}(\omega_1) = \tilde{\varrho}_{\alpha_n}(\omega_1) \vee \bigwedge_{\alpha_n(\mu)>0, \mu\neq\omega_1} \tilde{\varsigma}_{[\omega]}(\mu) = 1,$$

Hence, $(\alpha_n)^\delta(\omega_1) = (0,1,1)$. *Similarly, we can obtain* $(\alpha_n)^\delta(\omega_2) = (0,1,1)$ *and* $(\alpha_n)^\delta(\omega_3) = (0,1,1)$. *Also,*

$$\tilde{\varrho}_{(\alpha_n)^\delta}(\omega_4) = \tilde{\varrho}_{\alpha_n}(\omega_4) \wedge \bigvee_{\alpha_n(\mu)>0, \mu\neq\omega_4} \tilde{\varrho}_{[\omega]}(\mu) = 0.1,$$

$$\tilde{\sigma}_{(\alpha_n)^\delta}(\omega_4) = \tilde{\sigma}_{\alpha_n}(\omega_4) \vee \bigwedge_{\alpha_n(\mu)>0, \mu\neq\omega_4} (\tilde{\sigma}_{[\omega]})(\mu) = 0.2,$$

$$\tilde{\varsigma}_{(\alpha_n)^\delta}(\omega_4) = \tilde{\varrho}_{\alpha_n}(\omega_4) \vee \bigwedge_{\alpha_n(\mu)>0, \mu\neq\omega_4} \tilde{\varsigma}_{[\omega]}(\mu) = 0.2,$$

Hence, $(\alpha_n)^\delta(\omega_4) = (0.1,0.2,0.2)$, *and*

$$\tilde{\varrho}_{(\alpha_n)^\delta}(\omega_5) = \tilde{\varrho}_{\alpha_n}(\omega_5) \wedge \bigvee_{\alpha_n(\mu)>0, \mu\neq\omega_4} \tilde{\varrho}_{[\omega]}(\mu) = 0.2,$$

$$\tilde{\sigma}_{(\alpha_n)^\delta}(\omega_5) = \tilde{\sigma}_{\alpha_n}(\omega_5) \vee \bigwedge_{\alpha_n(\mu)>0, \mu\neq\omega_4} (\tilde{\sigma}_{[\omega]})(\mu) = 0.2,$$

$$\tilde{\varsigma}_{(\alpha_n)^\delta}(\omega_5) = \tilde{\varrho}_{\alpha_n}(\omega_5) \vee \bigwedge_{\alpha_n(\mu)>0, \mu\neq\omega_4} \tilde{\varsigma}_{[\omega]}(\mu) = 0.2,$$

Thus $(\alpha_n)^\delta(\omega_5) = (0.2,0.2,0.2)$ *and than,*

$$(\alpha_n)^\delta = \langle(0,1,1),(0,1,1),(0,1,1),(0.1,0.2,0.2),(0.2,0.2,0.2)\rangle,$$

$$(\alpha_n)_\delta = \langle (1,0,0),(1,0,0),(1,0,0),(0.2,0.1,0.1),(0.2,0.2,0.2)\rangle,$$

$$[(\alpha_n)_\delta]^c = \langle (0,1,1),(0,1,1),(0,1,1),(0.1,0.9,0.2),(0.2,0.8,0.2)\rangle.$$

Let, $\varepsilon_n = \langle (0.3,0.3,0.3),(0,1,1),(0,1,1),(0,0.8,0.2),(0,0.8,0.2)\rangle$*, then* $(\varepsilon_n)^\delta = \langle (0,1,1),\ (0,1,1),\ (0,1,1),(0,0.8,0.2),(0,0.8,0.2)\rangle$ *and thus*

$$\mathrm{Cl}_\delta^{\alpha_n}(\varepsilon_n) = [(\alpha_n)_\delta]^c \vee (\varepsilon_n)^\delta = \langle (0,1,1),(0,1,1),(0,1,1),(0.1,0.8,0.2),(0.2,0.8,0.2)\rangle.$$

Assume that a `SVNI` *is defined on* $\tilde{\mathcal{F}}$ *as follows*

$$\hbar = \{\pi_n \in \zeta^{\tilde{\mathcal{F}}} : \pi_n \leq \langle (0.5,0.3,0.3),(0.5,0.5,0.5),(0.5,0.5,0.5),(0.5,0.5,0.5),(0.5,0.5,0.5)\rangle\}$$

Note that every $\pi_n \in \zeta^{\tilde{\mathcal{F}}}$*, we have* $\mathrm{Cl}_\delta^{\alpha_n}(\pi_n) \geq \langle (0,1,1),(0,1,1),(0,1,1),(0.1,0.9,0.2),(0.2,0.8,0.2)\rangle$*, and recall that if* $\varepsilon_n \in \hbar$*, then* $[\varepsilon_n]^\star_{\alpha_n} = \langle 0,1,1\rangle$ *[By equations 2.1.,* $\varepsilon_n \overline{\wedge} \langle 0,1,1\rangle = \varepsilon_n \in \hbar$*,* $\mathrm{Cl}_\delta^{\alpha_n}(\langle 0,1,1\rangle) = \langle 0,1,1\rangle$*] and if* $\mathrm{Cl}_\delta^{\alpha_n}(\varepsilon_n) = \varepsilon_n$ *and* $\varepsilon_n \notin \hbar$*, then* $[\varepsilon_n]^\star_{\alpha_n} = \varepsilon_n$*.*

Now, we get that $\forall\ \pi_n = \langle \rho,(0,1,1),(0,1,1),(0.1,0.9,0.2),(0.2,0.8,0.2)\rangle$ *for all* $\rho \in I$*, we get*

$$\mathrm{Cl}_\delta^{\alpha_n}(\pi_n) = \pi_n,$$

Form equations 2.1, we have $\varepsilon_n \overline{\wedge} \pi_n \in \hbar$*, and thus*

$$[\varepsilon_n]^\star_{\alpha_n} = \langle (0,1,1),(0,1,1),(0,1,1),(0.1,0.9,0.2),(0.2,0.8,0.2)\rangle \ngeq \mathrm{Cl}_\delta^{\alpha_n}(\varepsilon_n).$$

Hence, $[\varepsilon_n]^\star_{\alpha_n} \leq \mathrm{Cl}_\delta^{\alpha_n}(\varepsilon_n)$*.*

Proposition 3.1. *Let* $(\tilde{\mathcal{F}},\delta,\hbar)$ *be a* `SVNIAS` *and* $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$*. Then,*

1. $\varepsilon_n \leq \upsilon_n$ *implies* $[\varepsilon_n]^\star_{\alpha_n} \leq [\upsilon_n]^\star_{\alpha_n}$*.*
2. *If* $\hbar_1,\hbar_2$ *are* `SVNIs` *ideals on* $\tilde{\mathcal{F}}$ *and* $\hbar_1 \subseteq \hbar_2$*, then* $[\varepsilon_n]^\star_{\alpha_n}(\hbar_1) \geq [\varepsilon_n]^\star_{\alpha_n}(\hbar_2)$*.*
3. $\mathrm{int}_\delta^{\alpha_n}\left([\varepsilon_n]^\star_{\alpha_n}\right) \leq [\varepsilon_n]^\star_{\alpha_n} = \mathrm{Cl}_\delta^{\alpha_n}\left([\varepsilon_n]^\star_{\alpha_n}\right) \leq \mathrm{Cl}_\delta^{\alpha_n}(\varepsilon_n)$*.*
4. $([\varepsilon_n]^\star_{\alpha_n})^\star_{\alpha_n} \leq \mathrm{Cl}_\delta^{\alpha_n}\left([\varepsilon_n]^\star_{\alpha_n}\right)$*.*
5. $[\varepsilon_n]^\star_{\alpha_n} \vee [\upsilon_n]^\star_{\alpha_n} \leq [\varepsilon_n \vee \upsilon_n]^\star_{\alpha_n}$ *and* $[\varepsilon_n]^\star_{\alpha_n} \wedge [\upsilon_n]^\star_{\alpha_n} \geq [\varepsilon_n \wedge \upsilon_n]^\star_{\alpha_n}$*.*

Proof. Suppose that $[\varepsilon_n]^\star_{\alpha_n} \nleq [\upsilon_n]^\star_{\alpha_n}$, then there exists $\eta_n \in \zeta^{\tilde{\mathcal{F}}}$ with $\mathrm{Cl}_\delta^{\alpha_n}(\eta_n) = \eta_n$ and $\upsilon_n \overline{\wedge} \eta_n = \left\langle \tilde{\varrho}_{\upsilon_n} \overline{\wedge} \tilde{\varrho}_{\eta_n}(\omega), \tilde{\sigma}_{\upsilon_n} \overline{\vee} \tilde{\sigma}_{\eta_n}(\omega), \tilde{\varsigma}_{\upsilon_n} \overline{\vee} \tilde{\varsigma}_{\eta_n}(\omega)\right\rangle \in \hbar$ such that

$$\begin{aligned}
&\tilde{\varrho}_{[\varepsilon_n]^\star_{\alpha_n}}(\omega) > \tilde{\varrho}_{\eta_n}(\omega) \geq \tilde{\varrho}_{[\upsilon_n]^\star_{\alpha_n}}(\omega),\\
&\tilde{\sigma}_{[\varepsilon_n]^\star_{\alpha_n}}(\omega) < \tilde{\sigma}_{\eta_n}(\omega) \leq \tilde{\sigma}_{[\upsilon_n]^\star_{\alpha_n}}(\omega),\\
&\tilde{\varsigma}_{[\varepsilon_n]^\star_{\alpha_n}}(\omega) < \tilde{\varsigma}_{\eta_n}(\omega) \leq \tilde{\varsigma}_{[\upsilon_n]^\star_{\alpha_n}}(\omega).
\end{aligned} \tag{3.1}$$

Since $\varepsilon_n \leq \upsilon_n$, then $\tilde{\varrho}_{\varepsilon_n} \overline{\wedge} \tilde{\varrho}_{\eta_n}(\omega) \leq \tilde{\varrho}_{\upsilon_n} \overline{\wedge} \tilde{\varrho}_{\eta_n}(\omega)$, $\tilde{\sigma}_{\varepsilon_n} \overline{\vee} \tilde{\sigma}_{\eta_n}(\omega) \geq \tilde{\sigma}_{\upsilon_n} \overline{\vee} \tilde{\sigma}_{\eta_n}(\omega)$, $\tilde{\varsigma}_{\varepsilon_n} \overline{\vee} \tilde{\varsigma}_{\eta_n}(\omega) \geq \tilde{\varsigma}_{\upsilon_n} \overline{\vee} \tilde{\varsigma}_{\eta_n}(\omega)$ and $\left\langle \tilde{\varrho}_{\varepsilon_n} \overline{\wedge} \tilde{\varrho}_{\eta_n}(\omega), \tilde{\sigma}_{\varepsilon_n} \overline{\vee} \tilde{\sigma}_{\eta_n}(\omega), \tilde{\varsigma}_{\varepsilon_n} \overline{\vee} \tilde{\varsigma}_{\eta_n}(\omega)\right\rangle \in \hbar$ with $\mathrm{Cl}_\delta^{\alpha_n}(\eta_n) = \eta_n$. Thus,

$$\tilde{\varrho}_{[\varepsilon_n]^\star_{\alpha_n}}(\omega) \leq \tilde{\varrho}_{[\eta_n]}(\omega), \quad \tilde{\sigma}_{[\varepsilon_n]^\star_{\alpha_n}}(\omega) \geq \tilde{\sigma}_{[\eta_n]}(\omega), \quad \tilde{\varsigma}_{[\varepsilon_n]^\star_{\alpha_n}}(\omega) \geq \tilde{\varsigma}_{[\eta_n]}(\omega),$$

which is a contradiction for equation (3) and hence $[\varepsilon_n]^\star_{\alpha_n} \leq [\upsilon_n]^\star_{\alpha_n}$.
(2): Suppose that $[\varepsilon_n]^\star_{\alpha_n}(\hbar_1) \ngeq [\varepsilon_n]^\star_{\alpha_n}(\hbar_2)$, then there exists $\eta_n \in \zeta^{\tilde{\mathcal{F}}}$ with $\mathrm{Cl}^{\alpha_n}_\delta(\eta_n) = \eta_n$ and $\upsilon_n \overline{\wedge} \eta_n = \left\langle \tilde{\varrho}_{\upsilon_n} \overline{\wedge} \tilde{\varrho}_{\eta_n}(\omega), \tilde{\sigma}_{\upsilon_n} \overline{\nabla} \tilde{\sigma}_{\eta_n}(\omega), \tilde{\varsigma}_{\upsilon_n} \overline{\nabla} \tilde{\varsigma}_{\eta_n}(\omega) \right\rangle \in \hbar_1$ such that

$$\begin{aligned} &\tilde{\varrho}_{[\varepsilon_n]^\star_{\alpha_n}(\hbar_1)}(\omega) \leq \tilde{\varrho}_{\eta_n}(\omega) < \tilde{\varrho}_{[\varepsilon_n]^\star_{\alpha_n}(\hbar_2)}(\omega), \\ &\tilde{\sigma}_{[\varepsilon_n]^\star_{\alpha_n}(\hbar_1)}(\omega) \geq \tilde{\sigma}_{\eta_n}(\omega) > \tilde{\sigma}_{[\varepsilon_n]^\star_{\alpha_n}(\hbar_2)}(\omega) \\ &\tilde{\varsigma}_{[\varepsilon_n]^\star_{\alpha_n}(\hbar_1)}(\omega) \geq \tilde{\varsigma}_{\eta_n}(\omega) > \tilde{\varsigma}_{[\varepsilon_n]^\star_{\alpha_n}(\hbar_2)}(\omega) \end{aligned} \tag{3.2}$$

Since $\hbar_1 \subseteq \hbar_2$, then $\left\langle \tilde{\varrho}_{\varepsilon_n} \overline{\wedge} \tilde{\varrho}_{\eta_n}(\omega), \tilde{\sigma}_{\varepsilon_n} \overline{\nabla} \tilde{\sigma}_{\eta_n}(\omega), \tilde{\varsigma}_{\varepsilon_n} \overline{\nabla} \tilde{\varsigma}_{\eta_n}(\omega) \right\rangle \in \hbar_2$ and $\mathrm{Cl}^{\alpha_n}_\delta(\eta_n) = \eta_n$. Thus,

$$\tilde{\varrho}_{[\varepsilon_n]^\star_{\alpha_n}(\hbar_2)}(\omega) \leq \tilde{\varrho}_{[\eta_n]}(\omega), \quad \tilde{\sigma}_{[\varepsilon_n]^\star_{\alpha_n}(\hbar_2)}(\omega) \geq \tilde{\sigma}_{[\eta_n]}(\omega), \quad \tilde{\varsigma}_{[\varepsilon_n]^\star_{\alpha_n}(\hbar_2)}(\omega) \geq \tilde{\varsigma}_{[\eta_n]}(\omega),$$

it is a contradiction for equation (4). Hence $[\varepsilon_n]^\star_{\alpha_n}(\hbar_1) \geq [\varepsilon_n]^\star_{\alpha_n}(\hbar_2)$.
(3): $\mathrm{int}^{\alpha_n}_\delta\left([\varepsilon_n]^\star_{\alpha_n}\right) \leq [\varepsilon_n]^\star_{\alpha_n} \leq \mathrm{Cl}^{\alpha_n}_\delta\left([\varepsilon_n]^\star_{\alpha_n}\right)$ direct. Since,

$$[\varepsilon_n]^\star_{\alpha_n} \leq \mathrm{Cl}^{\alpha_n}_\delta(\varepsilon_n),$$

then $\mathrm{Cl}^{\alpha_n}_\delta\left([\varepsilon_n]^\star_{\alpha_n}\right) \leq \mathrm{Cl}^{\alpha_n}_\delta(\varepsilon_n)$.
(4): Since $[\varepsilon_n]^\star_{\alpha_n} \leq \mathrm{Cl}^{\alpha_n}_\delta(\varepsilon_n)$, then $\left[[\varepsilon_n]^\star_{\alpha_n}\right]^\star_{\alpha_n} \leq \mathrm{Cl}^{\alpha_n}_\delta([\varepsilon_n]^\star_{\alpha_n})$.
(5): From (1), we have $\varepsilon_n \leq \upsilon_n \Longrightarrow [\varepsilon_n]^\star_{\alpha_n} \leq [\upsilon_n]^\star_{\alpha_n}$, and so (5) is satisfied. □

Definition 3.3. *Let $(\tilde{\mathcal{F}}, \delta, \hbar)$ be a* SVNIAS *and $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$. Then,*

$$(\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n) = \mathrm{Cl}^{\alpha_n}_\delta(\varepsilon_n) \vee ((\alpha_n)^\delta)^\star_{\alpha_n} \quad \forall \varepsilon \in \zeta^{\tilde{\mathcal{F}}}. \tag{3.3}$$

$$(\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n) = \mathrm{int}^{\alpha_n}_\delta(\varepsilon_n) \wedge (((\alpha_n)^\delta)^\star_{\alpha_n})^c \quad \forall \varepsilon \in \zeta^{\tilde{\mathcal{F}}}. \tag{3.4}$$

$(\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}$ and $(\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}$ are single valued neutrosophic operators from $\zeta^{\tilde{\mathcal{F}}}$ into $\zeta^{\tilde{\mathcal{F}}}$ based on a specific SVNS *α_n and* SVNI *$\hbar$ in the* SVNAS *$(\tilde{\mathcal{F}}, \delta)$.*

Now, if $\hbar = \hbar^\diamond$, then

$$(\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n) = \mathrm{Cl}^{\alpha_n}_\delta(\varepsilon_n \vee \alpha_n) \geq \mathrm{Cl}^{\alpha_n}_\delta(\varepsilon_n) = \mathrm{Cl}^{\alpha_n}_\delta((\varepsilon_n)^\star_{\alpha_n}) = (\varepsilon_n)^\star_{\alpha_n} \text{ and}$$

$$(\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n) = \mathrm{int}^{\alpha_n}_\delta(\varepsilon_n \vee (\alpha_n)^c) \geq \mathrm{int}^{\alpha_n}_\delta(\varepsilon_n) = \mathrm{int}^{\alpha_n}_\delta((((\varepsilon_n)^c)^\star_{\alpha_n})^c) = (((\varepsilon_n)^c)^\star_{\alpha_n})^c.$$

Proposition 3.2. *Let $(\tilde{\mathcal{F}}, \delta, \hbar)$ be a* SVNIAS *with $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ fixed. Then, for any $\varepsilon_n, \upsilon \in \zeta^{\tilde{\mathcal{F}}}$, we have:*

1. $(\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n) \leq \mathrm{int}^{\alpha_n}_\delta(\varepsilon_n) \leq \varepsilon_n \leq \mathrm{Cl}^{\alpha_n}_\delta(\varepsilon_n) \leq (\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n)$.
2. $(\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}\left((\varepsilon_n)^c\right) = \left((\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n)\right)^c$ *and* $(\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}\left((\varepsilon_n)^c\right) = \left((\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n)\right)^c$.
3. $(\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n \vee \pi_n) = (\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n) \vee (\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}(\pi_n)$ *and* $(\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n \wedge \pi_n) = (\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n) \wedge (\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}(\pi_n)$.
4. $(\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n \wedge \pi_n) = (\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n) \wedge (\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}(\pi_n)$ *and* $(\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n \vee \pi_n) = (\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n) \vee (\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}(\pi_n)$.
5. $(\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}((\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n)) \geq (\mathrm{Cl}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n)$ *and* $(\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}((\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n)) \leq (\mathrm{int}^{\alpha_n}_\delta)^\star_{\alpha_n}(\varepsilon_n)$.

6. *If* $\varepsilon_n \leq \pi_n$, *then* $(\mathrm{Cl}_\delta^{\alpha_n})^\star_{\alpha_n}(\varepsilon_n) \leq (\mathrm{Cl}_\delta^{\alpha_n})^\star_{\alpha_n}(\pi_n)$ *and* $(\mathrm{int}_\delta^{\alpha_n})^\star_{\alpha_n}(\varepsilon_n) \leq (\mathrm{int}_\delta^{\alpha_n})^\star_{\alpha_n}(\pi_n)$.

Proof. (1): From equations 3.3 and 3.4, we get the proof.

(2): From the Definition 3.3, we get that

$$\begin{aligned}\left[\left(\mathrm{int}_\delta^{\alpha_n}\right)^\star_{\alpha_n}(\varepsilon_n)\right]^c &= \left[\left(\mathrm{int}_\delta^{\alpha_n}\right)(\varepsilon_n) \wedge \left(\left((\alpha_n)^\delta\right)^\star_{\alpha_n}\right)^c\right]^c \\ &= \left[\mathrm{int}_\delta^{\alpha_n}(\varepsilon_n)\right]^c \vee \left((\alpha_n)^\delta\right)^\star_{\alpha_n} \\ &= \mathrm{Cl}_\delta^{\alpha_n}\left((\varepsilon_n)^c\right) \vee \left((\alpha_n)^\delta\right)^\star_{\alpha_n} \\ &= \left(\mathrm{Cl}_\delta^{\alpha_n}\right)^\star_{\alpha_n}((\varepsilon_n)^c).\end{aligned}$$

By the same way, it can be shown that $(\mathrm{int}_\delta^{\alpha_n})^\star_{\alpha_n}\left((\varepsilon_n)^c\right) = \left((\mathrm{Cl}_\delta^{\alpha_n})^\star_{\alpha_n}(\varepsilon_n)\right)^c$.

(3)-(5): from the Definition $(\mathrm{int}_\delta^{\alpha_n})^\star_{\alpha_n}$ and $(\mathrm{Cl}_\delta^{\alpha_n})^\star_{\alpha_n}$, we get the proof.

(6): From the Definition $(\mathrm{int}_\delta^{\alpha_n})^\star_{\alpha_n}$ and $(\mathrm{Cl}_\delta^{\alpha_n})^\star_{\alpha_n}$, we get $\varepsilon_n \leq \pi_n \Rightarrow \mathrm{Cl}_\delta^{\alpha_n}(\varepsilon_n) \leq \mathrm{Cl}_\delta^{\alpha_n}(\pi_n), \forall \varepsilon_n, \pi_n \in \zeta^{\tilde{\mathcal{F}}}$, and then $\left(\mathrm{Cl}_\delta^{\alpha_n}\right)^\star_{\alpha_n}(\varepsilon_n) \leq \left(\mathrm{Cl}_\delta^{\alpha_n}\right)^\star_{\alpha_n}(\pi_n)$. Similarly, $\left(\mathrm{int}_\delta^{\alpha_n}\right)^\star_{\alpha_n}(\varepsilon_n) \leq \left(\mathrm{int}_\delta^{\alpha_n}\right)^\star_{\alpha_n}(\pi_n)$. □

4. Connectedness in Single Valued Neutrosophic Ideal Approximation Spaces

We begin this section by defining the notion of connectedness in single valued neutrosophic ideal approximation spaces. Some of its characteristic properties are considered.

Definition 4.1. *Let* $(\tilde{\mathcal{F}}, \delta)$ *be a* `SVNAS` *and* $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$. *Then,*

1. *The* `SVNSs` $\varepsilon_n, \upsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ *are called single valued neutrosophic approximation separated (briefly, SVNA-separated) if*

$$\mathrm{Cl}_\delta^{\alpha_n}(\varepsilon_n) \wedge \upsilon_n = \varepsilon_n \wedge \mathrm{Cl}_\delta^{\alpha_n}(\upsilon_n) = \langle 0,1,1\rangle .$$

2. *A* `SVNS` $\pi_n \in \zeta^{\tilde{\mathcal{F}}}$ *is called single valued neutrosophic approximation disconnected set (briefly, SVNA-disconnected) if there exist SVNA-separated sets* $\varepsilon_n, \upsilon_n \in \zeta^{\tilde{\mathcal{F}}}$, *such that* $\varepsilon_n \vee \upsilon_n = \pi_n$. *A* `SVNS` π_n *is called single valued neutrosophic approximation connected (briefly,* `SVNA-connected`*) if it is not* `SVNA-disconnected`*.*
3. $(\tilde{\mathcal{F}}, \delta)$ *is called single valued neutrosophic approximation disconnected space (briefly, SVNA-disconnected space) if there exist SVNA-separated sets* $\varepsilon_n, \upsilon_n \in \zeta^{\tilde{\mathcal{F}}}$, *such that* $\varepsilon_n \vee \upsilon_n = \langle 1,0,0\rangle$. *A* `SVNAS` $(\tilde{\mathcal{F}}, \delta)$ *is called* `SVNA-connected` *if it is not* `SVNA-disconnected`

Definition 4.2. *Let* $(\tilde{\mathcal{F}}, \delta, \hbar)$ *be a* `SVNIAS` *and* $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$. *Then,*

1. *The* `SVNSs` $\varepsilon_n, \upsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ *are called single valued neutrosophic ideal approximation separated (briefly,* `SVNIA-separated`*) if*

$$\left(\mathrm{Cl}_\delta^{\alpha_n}\right)^\star_{\alpha_n}(\varepsilon_n) \wedge \upsilon_n = \varepsilon_n \wedge \left(\mathrm{Cl}_\delta^{\alpha_n}\right)^\star_{\alpha_n}(\upsilon_n) = \langle 0,1,1\rangle .$$

2. *A* `SVNS` $\pi_n \in \zeta^{\tilde{\mathcal{F}}}$ *is called single valued neutrosophic ideal approximation disconnected set (briefly,* `SVNIA-disconnected`*) if there exist* `SVNIA-separated` *sets* $\varepsilon_n, \upsilon_n \in \zeta^{\tilde{\mathcal{F}}}$, *such that*

$\varepsilon_n \vee \upsilon_n = \pi_n$. A `SVNS` π_n is called single valued neutrosophic ideal approximation connected (briefly, `SVNIA-connected`) if it is not `SVNIA-disconnected`.

3. *$(\tilde{\mathcal{F}}, \delta, \hbar)$ is called single valued neutrosophic ideal approximation disconnected space (briefly, SVNIA-disconnected space) if there exist `SVNIA-separated` sets $\varepsilon_n, \upsilon_n \in \zeta^{\tilde{\mathcal{F}}}$, such that $\varepsilon_n \vee \upsilon_n = \langle 1, 0, 0 \rangle$. A `SVNIAS` $(\tilde{\mathcal{F}}, \delta, \hbar)$ is called `SVNIA-connected` if it is not `SVNIA-disconnected`.*

Remark 4.1. *Any two `SVNIA-separated` sets $\varepsilon_n, \upsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ are `SVNA-separated` as well (from that $\mathrm{Cl}_\delta^{\alpha_n}(\pi_n) \leq \left(\mathrm{Cl}_\delta^{\alpha_n}\right)^\star_{\alpha_n}(\varepsilon_n)\ \forall\ \pi_n \in \zeta^{\tilde{\mathcal{F}}}$). That is, neutrosophic ideal approximation disconnectedness implies single-valued neutrosophic approximation disconnectedness and thus, single-valued neutrosophic approximation connectedness implies single-valued neutrosophic ideal approximation connectedness.*

Example 4.1. *This is an example that shows that $[\varepsilon_n]^\star_{\alpha_n} \leq \mathrm{Cl}_\delta^{\alpha_n}(\varepsilon_n)$.*

Let δ be `SVNR` on a set $\tilde{\mathcal{F}} = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$ as shown down.

δ	ω_1	ω_2	ω_3	ω_4	ω_5
ω_1	(1, 0, 0)	(1, 0, 0)	(0, 1, 1)	(0, 1, 1)	(0, 1, 1)
ω_2	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(0, 1, 1)	(0, 1, 1)
ω_3	(0, 1, 1)	(1, 0, 0)	(1, 0, 0)	(0, 1, 1)	(0, 1, 1)
ω_4	(0, 1, 1)	(0, 1, 1)	(0, 1, 1)	(1, 0, 0)	(1, 0, 0)
ω_5	(0, 1, 1)	(0, 1, 1)	(0, 1, 1)	(1, 0, 0)	(1, 0, 0)

Table 2. SVNRof δ

Assume that $\alpha_n = \langle (0,1,1), (0,1,1), (0.2, 0.2, 0.2), (1,0,0), (1,0,0) \rangle$. Then,

$$\tilde{\varrho}_{(\alpha_n)^\delta}(\omega_1) = \tilde{\varrho}_{\alpha_n}(\omega_1) \wedge \bigvee_{\alpha_n(\mu)>0, \mu \neq \omega_1} \tilde{\varrho}_{[\omega]}(\mu) = 0,$$

$$\tilde{\sigma}_{(\alpha_n)^\delta}(\omega_1) = \tilde{\sigma}_{\alpha_n}(\omega_1) \vee \bigwedge_{\alpha_n(\mu)>0, \mu \neq \omega_1} (\tilde{\sigma}_{[\omega]})(\mu) = 1,$$

$$\tilde{\varsigma}_{(\alpha_n)^\delta}(\omega_1) = \tilde{\varrho}_{\alpha_n}(\omega_1) \vee \bigwedge_{\alpha_n(\mu)>0, \mu \neq \omega_1} \tilde{\varsigma}_{[\omega]}(\mu) = 1,$$

Hence, $(\alpha_n)^\delta(\omega_1) = (0, 1, 1)$. Similarly, we can obtain $(\alpha_n)^\delta(\omega_2) = (0, 1, 1)$ and

$$\tilde{\varrho}_{(\alpha_n)^\delta}(\omega_3) = \tilde{\varrho}_{\alpha_n}(\omega_3) \wedge \bigvee_{\alpha_n(\mu)>0, \mu \neq \omega_3} \tilde{\varrho}_{[\omega]}(\mu) = 0.2,$$

$$\tilde{\sigma}_{(\alpha_n)^\delta}(\omega_3) = \tilde{\sigma}_{\alpha_n}(\omega_3) \vee \bigwedge_{\alpha_n(\mu)>0, \mu \neq \omega_3} (\tilde{\sigma}_{[\omega]})(\mu) = 0.2,$$

$$\tilde{\varsigma}_{(\alpha_n)^\delta}(\omega_3) = \tilde{\varrho}_{\alpha_n}(\omega_3) \vee \bigwedge_{\alpha_n(\mu)>0, \mu \neq \omega_3} \tilde{\varsigma}_{[\omega]}(\mu) = 0.2,$$

Hence, $(\alpha_n)^\delta(\omega_3) = (0.2, 0.2, 0.2)$ *and*

$$\tilde{\varrho}_{(\alpha_n)^\delta}(\omega_4) = \tilde{\varrho}_{\alpha_n}(\omega_4) \wedge \bigvee_{\alpha_n(\mu)>0, \mu\neq\omega_4} \tilde{\varrho}_{[\omega]}(\mu) = 1,$$

$$\tilde{\sigma}_{(\alpha_n)^\delta}(\omega_4) = \tilde{\sigma}_{\alpha_n}(\omega_4) \vee \bigwedge_{\alpha_n(\mu)>0, \mu\neq\omega_4} (\tilde{\sigma}_{[\omega]})(\mu) = 0,$$

$$\tilde{\varsigma}_{(\alpha_n)^\delta}(\omega_4) = \tilde{\varrho}_{\alpha_n}(\omega_4) \vee \bigwedge_{\alpha_n(\mu)>0, \mu\neq\omega_4} \tilde{\varsigma}_{[\omega]}(\mu) = 0,$$

Hence, $(\alpha_n)^\delta(\omega_4) = (1, 0, 0)$*. Similarly,* $(\alpha_n)^\delta(\omega_5) = (1, 0, 0)$*. Thus, by equations (3) and (4) we get than*

$$(\alpha_n)^\delta = \langle (0, 1, 1), (0, 1, 1), (0.2, 0.2, 0.2), (1, 0, 0), (1, 0, 0) \rangle = (\alpha_n)_\delta,$$

$$[(\alpha_n)_\delta]^c = \langle (1, 0, 0), (1, 0, 0), (0.2, 0.8, 0.2), (0, 1, 1), (0, 1, 1) \rangle.$$

Suppose that

$$\varepsilon_n = \langle (0, 1, 1), (0, 1, 1), (0, 1, 1), (0.6, 0.6, 0.6), (0, 1, 1) \rangle,$$
$$\upsilon_n = \langle (0, 1, 1), (0, 1, 1), (0, 1, 1), (0, 1, 1), (0.6, 06, 0.6) \rangle,$$

then

$$(\varepsilon_n)^\delta = \langle (0, 1, 1), (0, 1, 1), (0, 1, 1), (0.6, 0.6, 0.6), (0, 1, 1) \rangle,$$
$$(\upsilon_n)^\delta = \langle (0, 1, 1), (0, 1, 1), (0, 1, 1), (0, 1, 1), (0.6, 06, 0.6) \rangle,$$

and thus from Definitions $\mathrm{Cl}_\delta^{\alpha_n}, \mathrm{int}_\delta^{\alpha_n}$ *in [14], we get than*

$$\mathrm{Cl}_\delta^{\alpha_n}(\varepsilon_n) = [(\alpha_n)_\delta]^c \vee [\varepsilon_n]^\delta = \langle (1, 0, 0), (1, 0, 0), (0.2, 08, 0.2), (0.6, 0.6, 0.6), (0, 1, 1) \rangle$$

, and also, $\mathrm{Cl}_\delta^{\alpha_n}(\upsilon_n) = [(\alpha_n)_\delta]^c \vee [\upsilon_n]^\delta = \langle (1, 0, 0), (1, 0, 0), (0.2, 08, 0.2), (0, 1, 1), (0.6, 0.6, 0.6) \rangle$*, which means that*

$$\mathrm{Cl}_\delta^{\alpha_n}(\varepsilon_n) \wedge \upsilon_n = \varepsilon_n \wedge \mathrm{Cl}_\delta^{\alpha_n}(\upsilon_n) = \langle 0, 1, 1 \rangle.$$

Thus, $\varepsilon_n, \upsilon_n$ *are* `SVNA-separated` *sets, and moreover the* `SVNS`

$$(\varepsilon_n \vee \upsilon_n) = \langle (1, 0, 0), (1, 0, 0), (0, 1, 1), (0.6, 0.6, 0.6), (0.6, 0.6, 0.6) \rangle$$

is `SVNA-disconnected` *set.*

Now, suppose

$$\hbar = \{\eta_n \in \zeta^{\tilde{\mathcal{F}}} : \eta_n \leq \langle (0.7, 0.7, 0.7), (0.7, 0.7, 0.7), (0.7, 0.7, 0.7), (0.7, 0.7, 0.7), (0.7, 0.7, 0.7) \rangle \}.$$

Then

$$((\alpha_n)^\delta)^\star_\delta = \langle (1, 0, 0), (1, 0, 0), (0.2, 0.2, 0.2), (0.3, 0.3, 0.3), (0.3, 0.3, 0.3) \rangle.$$

Hence,

$$\left(\mathrm{Cl}_\delta^{\alpha_n}\right)^\star_{\alpha_n}(\varepsilon_n) = \langle (1, 0, 0), (1, 0, 0), (0.2, 0.2, 0.2), (0.6, 0.3, 0.3), (0.3, 0.3, 0.3) \rangle,$$

which means that

$$(\mathrm{Cl}_{\delta}^{\alpha_n})_{\alpha_n}^{\star}(\varepsilon_n) \wedge \upsilon_n = \langle (0,1,1),(0,1,1),(0,1,1),(0,1,1),(0.3,06,0.6)\rangle \neq \langle 0,1,1\rangle.$$

Hence, not every `SVNA-separated` *sets are* `SVNIA-separated` *sets, and moreover, the* `SVNS` *set* $(\varepsilon_n \vee \upsilon_n)$ *will be* `SVNIA-connected` *set whenever* $\hbar \neq \zeta^{\tilde{\mathcal{F}}}$ *and* $\hbar \neq \langle 0,1,1\rangle$*, that is, whenever* $\hbar$ *is a proper* `SVNI` *on* $\tilde{\mathcal{F}}$*.*

Theorem 4.1. *Let* $(\tilde{\mathcal{F}},\delta,\hbar)$ *be a* `SVNIAS` *and* $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$*. Then, the following are equivalent.*

1. $(\tilde{\mathcal{F}},\delta,\hbar)$ *is* `SVNIA-connected`*.*
2. $\varepsilon_n \wedge \upsilon_n = \langle 0,1,1\rangle, \left(\mathrm{int}_{\delta}^{\alpha_n}\right)_{\alpha_n}^{\star}(\varepsilon_n) = \varepsilon_n, \left(\mathrm{int}_{\delta}^{\alpha_n}\right)_{\alpha_n}^{\star}(\upsilon_n) = \upsilon_n$ *and* $\varepsilon_n \vee \upsilon_n = \langle 1,0,0\rangle$ *imply* $\varepsilon_n = \langle 0,1,1\rangle$ *or* $\upsilon_n = \langle 0,1,1\rangle$*.*
3. $\varepsilon_n \wedge \upsilon_n = \langle 0,1,1\rangle, \left(\mathrm{Cl}_{\delta}^{\alpha_n}\right)_{\alpha_n}^{\star}(\varepsilon_n) = \varepsilon_n, \left(\mathrm{Cl}_{\delta}^{\alpha_n}\right)_{\alpha_n}^{\star}(\upsilon_n) = \upsilon_n$ *and* $\varepsilon_n \vee \upsilon_n = \langle 1,0,0\rangle$ *imply* $\varepsilon_n = \langle 0,1,1\rangle$ *or* $\upsilon_n = \langle 0,1,1\rangle$*.*

Proof. $(1) \Rightarrow (2)$: Let $\varepsilon_n, \upsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ with $\left(\mathrm{int}_{\delta}^{\alpha_n}\right)_{\alpha_n}^{\star}(\varepsilon_n) = \varepsilon_n, \left(\mathrm{int}_{\delta}^{\alpha_n}\right)_{\alpha_n}^{\star}(\upsilon_n) = \upsilon_n$ such that $\varepsilon_n \wedge \upsilon_n = \langle 0,1,1\rangle$ and $\varepsilon_n \vee \upsilon_n = \langle 1,0,0\rangle$. Then, from (2) in Proposition 3.2., we get that

$$(\mathrm{Cl}_{\delta}^{\alpha_n})_{\alpha_n}^{\star}(\varepsilon_n) = (\mathrm{Cl}_{\delta}^{\alpha_n})_{\alpha_n}^{\star}((\upsilon_n)^c) = ((\mathrm{int}_{\delta}^{\alpha_n})_{\alpha_n}^{\star}(\upsilon_n))^c = (\upsilon_n)^c = \varepsilon_n,$$

$$(\mathrm{Cl}_{\delta}^{\alpha_n})_{\alpha_n}^{\star}(\upsilon_n) = (\mathrm{Cl}_{\delta}^{\alpha_n})_{\alpha_n}^{\star}((\varepsilon_n)^c) = ((\mathrm{int}_{\delta}^{\alpha_n})_{\alpha_n}^{\star}(\varepsilon_n))^c = (\varepsilon_n)^c = \upsilon_n,$$

Hence, $\left(\mathrm{Cl}_{\delta}^{\alpha_n}\right)_{\alpha_n}^{\star}(\varepsilon_n) \wedge \upsilon_n = \varepsilon_n \wedge \left(\mathrm{Cl}_{\delta}^{\alpha_n}\right)_{\alpha_n}^{\star}(\upsilon_n) = \varepsilon_n \wedge \upsilon_n = \langle 0,1,1\rangle$. That is, $\varepsilon_n, \upsilon_n$ are `SVNIA-separated` sets so that $\varepsilon_n \vee \upsilon_n = \langle 1,0,0\rangle$. But $(\tilde{\mathcal{F}},\delta,\hbar)$ is `SVNIA-connected` implies that $\varepsilon_n = \langle 0,1,1\rangle$ or $\upsilon_n = \langle 0,1,1\rangle$.

$(2) \Rightarrow (3)$:, $(3) \Rightarrow (1)$: Clear. □

Theorem 4.2. *Let* $(\tilde{\mathcal{F}},\delta,\hbar)$ *be a* `SVNIAS` *and* $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$*. Then, the following are equivalent.*

1. ε_n *is* `SVNIA-connected` *set.*
2. *If* υ_n, π_n *are* `SVNIA-separated` *sets with* $\varepsilon_n \leq (\upsilon_n \vee \pi_n)$*, then* $\varepsilon_n \wedge \upsilon_n = \langle 0,1,1\rangle$ *or* $\varepsilon_n \wedge \pi_n = \langle 0,1,1\rangle$*.*
3. *If* υ_n, π_n *are* `SVNIA-separated` *sets with* $\varepsilon_n \leq (\upsilon_n \vee \pi_n)$*, then* $\varepsilon_n \leq \upsilon_n$ *or* $\varepsilon_n \leq \pi_n$*.*

Proof. $(1) \Rightarrow (2)$: Let υ_n, π_n are `SVNIA-separated` sets with $\varepsilon_n \leq (\upsilon_n \vee \pi_n)$. That is,

$$\left(\mathrm{Cl}_{\delta}^{\alpha_n}\right)_{\alpha_n}^{\star}(\upsilon_n) \wedge \pi_n = \upsilon_n \wedge \left(\mathrm{Cl}_{\delta}^{\alpha_n}\right)_{\alpha_n}^{\star}(\pi_n) = \langle 0,1,1\rangle.$$

So that $\varepsilon_n \leq (\upsilon_n \vee \pi_n)$. Since

$$\begin{aligned}(\mathrm{Cl}_{\delta}^{\alpha_n})_{\alpha_n}^{\star}(\varepsilon_n \wedge \upsilon_n) \wedge (\varepsilon_n \wedge \pi_n) &= (\mathrm{Cl}_{\delta}^{\alpha_n})_{\alpha_n}^{\star}(\varepsilon_n) \wedge (\mathrm{Cl}_{\delta}^{\alpha_n})_{\alpha_n}^{\star}(\upsilon_n) \wedge (\varepsilon_n \wedge \pi_n)\\ &= (\mathrm{Cl}_{\delta}^{\alpha_n})_{\alpha_n}^{\star}(\varepsilon_n) \wedge \varepsilon_n \wedge (\mathrm{Cl}_{\delta}^{\alpha_n})_{\alpha_n}^{\star}(\upsilon_n) \wedge \pi_n\\ &= \langle 0,1,1\rangle.\end{aligned}$$

$$\begin{aligned}(\mathrm{Cl}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}(\varepsilon_n \wedge \pi_n) \wedge (\varepsilon_n \wedge \upsilon_n) &= (\mathrm{Cl}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}(\varepsilon_n) \wedge (\mathrm{Cl}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}(\pi_n) \wedge (\varepsilon_n \wedge \upsilon_n)\\ &= (\mathrm{Cl}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}(\varepsilon_n) \wedge \varepsilon_n \wedge (\mathrm{Cl}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}(\pi_n) \wedge \upsilon_n\\ &= \langle 0,1,1\rangle.\end{aligned}$$

Then, $(\mu \wedge v)$ and $(\mu \wedge \rho)$ are SVNIA-separated sets with $\varepsilon_n = (\varepsilon_n \wedge \upsilon_n) \vee (\varepsilon_n \wedge \pi_n)$. But ε_n is SVNIA-connected means that $\varepsilon_n \wedge \upsilon_n = \langle 0,1,1\rangle$ or $\varepsilon_n \wedge \pi_n = \langle 0,1,1\rangle$.

(2) $\Rightarrow$ (3) : If $\varepsilon_n \wedge \upsilon_n = \langle 0,1,1\rangle$, $\varepsilon_n \leq (\upsilon_n \vee \pi_n)$ means that $\varepsilon_n = \varepsilon_n \wedge (\upsilon_n \vee \pi_n) = (\varepsilon_n \wedge \upsilon_n) \vee (\varepsilon_n \wedge \pi_n) = (\varepsilon_n \wedge \pi_n)$ and thus $(\varepsilon_n \leq \pi_n)$. Also, if $\varepsilon_n \wedge \pi_n = \langle 0,1,1\rangle$, then $\varepsilon_n \leq \upsilon_n$.

(3) $\Rightarrow$ (1): Let υ_n, π_n be SVNIA-separated sets so that $\varepsilon_n = \upsilon_n \vee \pi_n$. Then, from (3), $\varepsilon_n \leq \upsilon_n$ or $\varepsilon_n \leq \pi_n$. If $\varepsilon_n \leq \upsilon_n$, then

$$\pi_n = (\upsilon_n \vee \pi_n) \wedge \pi_n = \varepsilon_n \wedge \pi_n \leq \upsilon_n \wedge \pi_n \leq (\mathrm{Cl}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}(\upsilon_n) \wedge \pi_n = \langle 0,1,1\rangle.$$

Also, if $\mu \leq \rho$, then

$$\upsilon_n = (\upsilon_n \vee \pi_n) \wedge \upsilon_n = \varepsilon_n \wedge \upsilon_n \leq \pi_n \wedge \upsilon_n \leq (\mathrm{Cl}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}(\pi_n) \wedge \upsilon_n = \langle 0,1,1\rangle.$$

Hence, ε_n is SVNIA-connected set. □

Definition 4.3. *Let $(\tilde{\mathcal{F}},\delta)$, $(\tilde{\mathcal{G}},\delta^{\star})$ be two SVNAS and $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$, $\varepsilon_n \in \zeta^{\tilde{\mathcal{G}}}$ are SVNSs. Then, the mapping $f : (\tilde{\mathcal{F}},\delta) \to (\tilde{\mathcal{G}},\delta^{\star})$ is called single valued neutrosophic approximation continuous (briefly, SVNA-continuous) if $f^{-1}(\mathrm{int}_{\delta^{\star}}^{\varepsilon_n}(\upsilon_n)) \leq \mathrm{int}_{\delta}^{\alpha_n}(f^{-1}(\upsilon_n))\ \ \forall\ \ \upsilon_n \in \zeta^{\tilde{\mathcal{G}}}$.*
Equivalently. f is called SVNA-continuous if $f^{-1}(\mathrm{Cl}_{\delta^{\star}}^{\varepsilon_n}(\upsilon_n)) \geq \mathrm{Cl}_{\delta}^{\alpha_n}(f^{-1}(\upsilon_n))\ \ \forall\ \ \upsilon_n \in \zeta^{\tilde{\mathcal{G}}}$.

Definition 4.4. *A mapping $f : (\tilde{\mathcal{F}},\delta,\hbar) \to (\tilde{\mathcal{G}},\delta^{\star})$ is called single valued neutrosophic ideal approximation continuous (briefly, SVNIA-continuous) if $f^{-1}(\mathrm{int}_{\delta^{\star}}^{\varepsilon_n}(\upsilon_n)) \leq (\mathrm{int}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}\left(f^{-1}(\upsilon_n)\right) \forall \upsilon_n \in \zeta^{\tilde{\mathcal{G}}}$.*
Equivalently. f is called SVNIA-continuous if $f^{-1}(\mathrm{Cl}_{\delta^{\star}}^{\varepsilon_n})(\upsilon_n) \geq (\mathrm{Cl}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}(f^{-1}(\upsilon_n)) \forall \upsilon_n \in \zeta^{\tilde{\mathcal{G}}}$.

Every SVNIA-continuous mapping will be SVNA-continuous as well (from (1) in Proposition 3.2.) but not converse.

Remark 4.2. *Since $\hbar$ and $\hbar^{\star}$ are independent SVNIs on $\tilde{\mathcal{F}}$ and $\tilde{G}$ respectively, then the mapping $f : (\tilde{\mathcal{F}},\delta,\hbar) \to (\tilde{\mathcal{G}},\delta^{\star},\hbar^{\star})$ still not SVNIA-continuous in general even if we have taken f is a bijective map with respect to $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ and $f(\alpha_n) \in \zeta^{\tilde{\mathcal{G}}}$ and the SVNR δ an $\tilde{\mathcal{F}}$ and $\delta^{\star}$ on $\tilde{\mathcal{G}}$ wrere $\delta^{\star} = \delta \circ \left(f^{-1} \times f^{-1}\right) = (f \times f)(\delta)$. This special case itself could be as an example of a SVNA-continuous mapping but not SVNIA-continuous in general*

Theorem 4.3. *Let $(\tilde{\mathcal{F}},\delta,\hbar),(\tilde{\mathcal{G}},\delta^{\star},\hbar^{\star})$, associated with $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$, $\varepsilon_n \in \zeta^{\tilde{\mathcal{G}}}$ respectively, be SVNIAS and $f : (\tilde{\mathcal{F}},\delta,\hbar) \to (\tilde{\mathcal{G}},\delta^{\star},\hbar^{\star})$ is a SVNIA-continuous mapping. Then, $f(\eta_n) \in \zeta^{\tilde{\mathcal{G}}}$ is a SVNIA-connected set if η_n is a SVNIA-connected set in $\tilde{\mathcal{F}}$.*

Proof. Let $\upsilon_n, \pi_n \in \zeta^{\tilde{\mathcal{G}}}$ be SVNIA-separated sets with $f(\eta_n) = \upsilon_n \vee \pi_n$. That is, $(\mathrm{Cl}_{\delta^{\star}}^{\varepsilon_n})^{\star}_{\varepsilon_n}(\upsilon_n) \wedge \pi_n = \mathrm{Cl}_{\delta^{\star}}^{\varepsilon_n})^{\star}_{\varepsilon_n}(\pi_n) \wedge \upsilon_n = \langle 0,1,1\rangle$. Then, $\eta_n \leq \left(f^{-1}(\upsilon_n) \vee f^{-1}(\pi_n)\right)$, and from f is SVNIA-continuous, we get that

$$\begin{aligned}(\mathrm{Cl}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}(f^{-1}(\upsilon_n)) \wedge f^{-1}(\pi_n) &\le f^{-1}(\mathrm{Cl}_{\delta^{\star}}^{\varepsilon_n}(\upsilon_n)) \wedge f^{-1}(\pi_n)\\ &= f^{-1}(\mathrm{Cl}_{\delta^{\star}}^{\varepsilon_n}(\upsilon_n) \wedge \pi_n) = f^{-1}(\langle 0,1,1\rangle)\\ &= \langle 0,1,1\rangle.\end{aligned}$$

and similarly, we have

$$\begin{aligned}(\mathrm{Cl}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}(f^{-1}(\pi_n)) \wedge f^{-1}(\upsilon_n) &\le f^{-1}(\mathrm{Cl}_{\delta^{\star}}^{\varepsilon_n}(\pi_n)) \wedge f^{-1}(\upsilon_n)\\ &= f^{-1}(\mathrm{Cl}_{\delta^{\star}}^{\varepsilon_n}(\pi_n) \wedge \upsilon_n) = f^{-1}(\langle 0,1,1\rangle)\\ &= \langle 0,1,1\rangle.\end{aligned}$$

Hence, $f^{-1}(\upsilon_n)$ and $f^{-1}(\pi_n)$ are `SVNIA-separated` sets in $\tilde{\mathcal{F}}$ so that $\eta_n \le (f^{-1}(\upsilon_n) \vee f^{-1}(\pi_n))$. But by (3) in Theorem (4.2), we get that $\eta_n \le f^{-1}(\upsilon_n)$ or $\eta_n \le f^{-1}(\pi_n)$ which means that $f(\eta_n) \le \upsilon_n$ or $f(\eta_n) \le \pi_n$. Thus, from that η_n is `SVNIA-connected` set in $\tilde{\mathcal{F}}$, and again from (3) in Theorem (4.2), we get that $f(\eta_n)$ is `SVNIA-connected` in $\tilde{\mathcal{G}}$. □

The implications in the following diagram are satisfied whenever f is `SVNIA-continuous`.

$$\begin{array}{ccc}\eta_n \textit{ is } \texttt{SVNA-connected} & \longrightarrow & \eta_n \textit{ is } \texttt{SVNIA-connected}\\ \downarrow & & \downarrow\\ f(\eta_n) \textit{ is } \texttt{SVNA-connected} & \longrightarrow & f(\eta_n) \textit{ is } \texttt{SVNIA-connected}\\ \uparrow & & \uparrow\\ \eta_n \textit{ is } \texttt{SVNIA-connected} & & \eta_n \textit{ is } \texttt{SVNA-connected}\end{array}$$

Only the implications in the following diagram are satisfied whenever f is `SVNA-continuous`.

$$\begin{array}{ccc}\eta_n \textit{ is } \texttt{SVNA-connected} & \longrightarrow & \eta_n \textit{ is } \texttt{SVNIA-connected}\\ \downarrow & & \downarrow\\ f(\eta_n) \textit{ is } \texttt{SVNA-connected} & \longrightarrow & f(\eta_n) \textit{ is } \texttt{SVNIA-connected}\\ \uparrow & & \\ \eta_n \textit{ is } \texttt{SVNIA-connected} & & \end{array}$$

Example 4.2. *Let $\tilde{\mathcal{F}} = \tilde{\mathcal{G}} = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ and $f : \tilde{\mathcal{F}} \to \tilde{\mathcal{G}}$ such that*

$$f(\omega_1) = f(\omega_2) = \omega_1, \quad f(\omega_3) = \omega_2, \quad f(\omega_4) = \omega_4,$$

δ and δ^* are SVNR on $\tilde{\mathcal{F}}$, $\tilde{\mathcal{G}}$ respectively as follows:

Assume that

$$\alpha_n = \langle (1,0,0), (1,0,0), (1,0,0), (0.2,0.2,0.2)\rangle \in \zeta^{\tilde{\mathcal{F}}},$$

and

$$\eta_n = \langle (0,1,1), (0.3,0.3,0.3), (0.5,0.5,0.5), (0.2,0.2,0.2)\rangle \in \zeta^{\tilde{\mathcal{G}}}$$

δ	ω_1	ω_2	ω_3	ω_4
ω_1	(1, 0, 0)	(0, 1, 1)	(0, 1, 1)	(0, 1, 1)
ω_2	(0, 1, 1)	(1, 0, 0)	(0, 1, 1)	(0, 1, 1)
ω_3	(0, 1, 1)	(0, 1, 1)	(1, 0, 0)	(0.3, 0.3, 0.3)
ω_4	(0, 1, 1)	(0, 1, 1)	(0.3, 0.3, 0.3)	(1, 0, 0)

TABLE 3. SVNRof δ

δ^*	ω_1	ω_2	ω_3	ω_4
ω_1	(1, 0, 0)	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(0, 1, 1)
ω_2	(0.5, 0.5, 0.5)	(1, 0, 0)	(0.5, 0.5, 0.5)	(0, 1, 1)
ω_3	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)	(1, 0, 0)	(0, 1, 1)
ω_4	(0, 1, 1)	(0, 1, 1)	(0, 1, 1)	(1, 0, 0)

TABLE 4. SVNRof δ

Then,

$$\tilde{\varrho}_{(\alpha_n)^\delta}(\omega_1) = \tilde{\varrho}_{\alpha_n}(\omega_1) \wedge \bigvee_{\alpha_n(\mu)>0, \mu\neq\omega_1} \tilde{\varrho}_{[\omega]}(\mu) = 0,$$

$$\tilde{\sigma}_{(\alpha_n)^\delta}(\omega_1) = \tilde{\sigma}_{\alpha_n}(\omega_1) \vee \bigwedge_{\alpha_n(\mu)>0, \mu\neq\omega_1} (\tilde{\sigma}_{[\omega]})(\mu) = 1,$$

$$\tilde{\varsigma}_{(\alpha_n)^\delta}(\omega_1) = \tilde{\varrho}_{\alpha_n}(\omega_1) \vee \bigwedge_{\alpha_n(\mu)>0, \mu\neq\omega_1} \tilde{\varsigma}_{[\omega]}(\mu) = 1,$$

Hence, $(\alpha_n)^\delta(\omega_1) = (\alpha_n)^\delta(\omega_2) = (\alpha_n)^\delta(\omega_3) = (0, 1, 1)$. *Also,*

$$\tilde{\varrho}_{(\alpha_n)^\delta}(\omega_4) = \tilde{\varrho}_{\alpha_n}(\omega_4) \wedge \bigvee_{\alpha_n(\mu)>0, \mu\neq\omega_4} \tilde{\varrho}_{[\omega]}(\mu) = 0.2,$$

$$\tilde{\sigma}_{(\alpha_n)^\delta}(\omega_4) = \tilde{\sigma}_{\alpha_n}(\omega_4) \vee \bigwedge_{\alpha_n(\mu)>0, \mu\neq\omega_4} (\tilde{\sigma}_{[\omega]})(\mu) = 0.3,$$

$$\tilde{\varsigma}_{(\alpha_n)^\delta}(\omega_4) = \tilde{\varrho}_{\alpha_n}(\omega_4) \vee \bigwedge_{\alpha_n(\mu)>0, \mu\neq\omega_4} \tilde{\varsigma}_{[\omega]}(\mu) = 0.3,$$

Thus, $(\alpha_n)^\delta(\omega_4) = (0.2, 0.3, 0.3)$ *and than,*

$$(\alpha_n)^\delta = \langle (0,1,1), (0,1,1), (0,1,1), (0.2, 0.3, 03) \rangle,$$

$$(\alpha_n)_\delta = \langle (1,0,0), (1,0,0), (1,0,0), (0.3, 0.2, 0.2) \rangle,$$

$$[(\alpha_n)_\delta]^c = \langle (0,1,1), (0,1,1), (0,1,1), (0.2, 0.8, 0.3) \rangle.$$

For $\beta_n = \langle(0,1,1),(0,1,1),,(0.2,0.8,0.2),(0,1,1)\rangle \in \zeta^{\tilde{\mathcal{G}}}$, *we get that*

$$f^{-1}(\beta_n) = \langle 0,1,1\rangle$$

and then $\mathrm{Cl}_{\delta}^{\alpha_n}(f^{-1}(\beta_n)) = \langle 0,1,1\rangle$. *Since*

$$(\eta_n)_{\delta^*} = \langle(0.5,0.5,0.5),(0.5,0.3,0.3),(0.5,0.5,0.5),(1,0,0\rangle,$$

$$(\beta_n)^{\delta^*} = \langle(0,1,1),(0,1,1),(0.2,0.8,0.5),(0,1,1)\rangle,$$

$$[(\eta_n)_{\delta^*}]^c = \langle(0.5,0.5,0.5),(0.3,0.7,0.5),(0.5,0.5,0.5),(0,1,1)\rangle.$$

Then, $\mathrm{Cl}_{\delta^*}^{\eta_n}(\beta_n) = [(\eta_n)_{\delta^*}]^c \vee (\beta_n)^{\delta^*} = \langle(0.5,0.5,0.5),(0.3,0.7,0.5),(0.5,0.5,0.5),(0,1,1)\rangle$. *Thus,*

$$f^{-1}(\mathrm{Cl}_{\delta^*}^{\eta_n}(\beta_n)) = \langle(0.5,0.5,0.5),(0.5,0.5,0.5),(0.3,0.7,0.5),(0,1,1)\rangle \geq \langle 0,1,1\rangle = \mathrm{Cl}_{\delta}^{\alpha_n}(f^{-1}(\beta_n))$$

. Hence, there is a fuzzy set $\beta_n \in \zeta^{\tilde{\mathcal{G}}}$ *satisfying the condition of single valued neutrosophic approximation continuity. Next, we will show that* β_n *itself will not satisfy the condition of fuzzy ideal approximation continuity.*

Since, $\mathrm{Cl}_{\delta}^{\eta_n}(f^{-1}(\beta_n)) = \langle 0,1,1\rangle$, *then*

$$(\mathrm{Cl}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}\left(f^{-1}(\beta_n)\right) = \mathrm{Cl}_{\delta}^{\alpha_n}\left(f^{-1}(\beta_n)\right) \vee ((\alpha_n)^{\delta})^{\star}_{\alpha_n} = ((\alpha_n)^{\delta})^{\star}_{\alpha_n},$$

that is,

$$(\mathrm{Cl}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}\left(f^{-1}(\beta_n)\right) = \{\langle(0,1,1),(0,1,1),(0,1,1),(0.2,0.3,03)\rangle\}^{\star}_{\alpha_n}.$$

Now, define a single-valued neutrosophic ideal $\hbar$ *over* $\tilde{\mathcal{F}}$ *as next*

$$\varphi \in \hbar \Leftrightarrow \varphi \leq \{\langle(1,0,0),(1,0,0),(0.2,0.2,0.2),(0.2,0.2,0.2)\rangle\}$$

. Then, from being $\mathrm{Cl}_{\delta}^{\eta_n}(\mu_n) = (\alpha_n)^c_{\delta} \vee (\mu_n)^{\delta} \geq \langle(0,1,1),(0,1,1),(0,1,1),(0.2,0.8,0.3)\rangle$, *we get that,*

$$(\mathrm{Cl}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}\left(f^{-1}(\beta_n)\right) = \{\langle(0,1,1),(0,1,1),(0.2,0.8,0.2),(0.2,0.8,0.2)\rangle\}.$$

according to the definition of $\hbar$ *and the definition of*

$$[\mu_n]^{\star}_{\alpha_n}(\delta,\hbar) = \bigwedge\{\nu_n \in \zeta^{\tilde{\mathcal{F}}} : \mu_n \overline{\wedge} \nu_n = \langle \tilde{\varrho}_{\mu_n}\overline{\wedge}\tilde{\varrho}_{\nu_n}(\omega), \tilde{\sigma}_{\mu_n}\overline{\vee}\tilde{\sigma}_{\nu_n}(\omega), \tilde{\varsigma}_{\mu_n}\overline{\vee}\tilde{\varsigma}_{\nu_n}(\omega)\rangle \in \hbar, \mathrm{Cl}_{\delta}^{\alpha_n}(\nu_n) = \nu_n\}$$

. Hence, we obtain

$$f^{-1}(\mathrm{Cl}_{\delta^*}^{\eta_n}(\beta_n)) = \langle(0.5,0.5,0.5),(0.5,0.5,0.5),(0.3,0.7,0.5),(0,1,1)\rangle \ngeq (\mathrm{Cl}_{\delta}^{\alpha_n})^{\star}_{\alpha_n}\left(f^{-1}(\beta_n)\right)$$

$$= \{\langle(0,1,1),(0,1,1),(0.2,0.8,0.2),(0.2,0.8,0.2)\rangle\}$$

Therefore, not any single-valued neutrosophic approximation continuous map must be a single-valued neutrosophic ideal approximation continuous but the converse is a must.

5. Conclusion

In this paper, we introduced the notion of single-valued neutrosophic approximation space. Joining a single-valued neutrosophic ideal to the single-valued neutrosophic approximation space, we got a single-valued neutrosophic ideal approximation space with other properties different from those of single-valued neutrosophic approximation spaces. In future work, we will define single-valued neutrosophic approximation rough groups and single-valued neutrosophic approximation rough rings as applications of this paper.

Discussion for further works: The theories that were used in this article could be extended to study some similar notions in the neutrosophic metric topological spaces.

Acknowledgments: The authors would like to thank Deanship of Scientific Research at Majmaah University for supporting this work under Project Number No: R-2024-937.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] Z. Pawlak, Rough Sets, Int. J. Comput. Inform. Sci. 11 (1982), 341–356. https://doi.org/10.1007/bf01001956.

[2] Z. Pei, D. Pei, L. Zheng, Topology vs Generalized Rough Sets, Int. J. Approx. Reason. 52 (2011), 231–239. https://doi.org/10.1016/j.ijar.2010.07.010.

[3] X. Chen, Q. Li, Construction of Rough Approximations in Fuzzy Setting, Fuzzy Sets Syst. 158 (2007), 2641–2653. https://doi.org/10.1016/j.fss.2007.05.016.

[4] D. Dubois, H. Prade, Rough Fuzzy Sets and Fuzzy Rough Sets, Int. J. Gen. Syst. 17 (1990), 191–209. https://doi.org/10.1080/03081079008935107.

[5] Z. Li, T. Xie, Roughness of Fuzzy Soft Sets and Related Results, Int. J. Comput. Intell. Syst. 8 (2015), 278–296. https://doi.org/10.1080/18756891.2015.1001951.

[6] G. Liu, Generalized Rough Sets Over Fuzzy Lattices, Inform. Sci. 178 (2008), 1651–1662. https://doi.org/10.1016/j.ins.2007.11.010.

[7] W. Wu, Generalized Fuzzy Rough Sets, Inform. Sci. 151 (2003), 263–282. https://doi.org/10.1016/s0020-0255(02)00379-1.

[8] M.I. Ali, A Note on Soft Sets, Rough Soft Sets and Fuzzy Soft Sets, Appl. Soft Comput. 11 (2011), 3329–3332. https://doi.org/10.1016/j.asoc.2011.01.003.

[9] D. Molodtsov, Soft Set Theory–First Results, Comput. Math. Appl. 37 (1999), 19–31. https://doi.org/10.1016/s0898-1221(99)00056-5.

[10] D. Boixader, J. Jacas, J. Recasens, Upper and Lower Approximations of Fuzzy Sets, Int. J. Gen. Syst. 29 (2000), 555–568. https://doi.org/10.1080/03081070008960961.

[11] W.Z. Wu, A Study on Relationship Between Fuzzy Rough Approximation Operators and Fuzzy Topological Spaces, Springer-Verlag, Berlin, Heidelberg, (2005).

[12] A.M. Abd El-Latif, Generalized Soft Rough Sets and Generated Soft Ideal Rough Topological Spaces, J. Intell. Fuzzy Syst. 34 (2018), 517–524. https://doi.org/10.3233/jifs-17610.

[13] J. Mahanta, P.K. Das, Fuzzy Soft Topological Spaces, J. Intell. Fuzzy Syst. 32 (2017), 443–450. https://doi.org/10.3233/jifs-152165.

[14] C.L. Chang, Fuzzy Topological Spaces, J. Math. Anal. Appl. 24 (1968), 182–190. https://doi.org/10.1016/0022-247x(68)90057-7.

[15] F. Smarandache, A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics, 6th ed. InfoLearnQuest, Ann Arbor, MI, USA. (2007).

[16] H. Wang, F. Smarandache, Y.Q. Zhang, R. Sunderraman, Single Valued Neutrosophic Sets, Multispace Multistruct. 4 (2010), 410–413.

[17] H.L. Yang, Z.L. Guo, Y. She, X. Liao, On Single Valued Neutrosophic Relations, J. Intell. Fuzzy Syst. 30 (2016), 1045–1056. https://doi.org/10.3233/ifs-151827.

[18] Q. Jin, K. Hu, C. Bo, L. Li, A New Single-Valued Neutrosophic Rough Sets and Related Topology, J. Math. 2021 (2021), 5522021. https://doi.org/10.1155/2021/5522021.

[19] F. Alsharari, Y.M. Saber, $\mathcal{G}\Theta_{\tau_i}^{\star\tau_j}$-Fuzzy Closure Operator, New Math. Nat. Comput. 16 (2020), 123–141. https://doi.org/10.1142/s1793005720500088.

[20] F. Alsharari, Y.M. Saber, F. Smarandache, Compactness on Single-Valued Neutrosophic Ideal Topological Spaces, Neutrosophic Sets Syst. 41 (2021), 127–145.

[21] Y.M. Saber, M.A. Abdel-Sattar, Ideals on Fuzzy Topological Spaces, Appl. Math. Sci. 8 (2014), 1667–1691. https://doi.org/10.12988/ams.2014.33194.

[22] Y. Saber, F. Alsharari, F. Smarandache, An Introduction to Single-Valued Neutrosophic Soft Topological Structure, Soft Comput. 26 (2022), 7107–7122. https://doi.org/10.1007/s00500-022-07150-4.

[23] Y.M. Saber, F. Alsharari, Generalized Fuzzy Ideal Closed Sets on Fuzzy Topological Spaces in Sostak Sense, Int. J. Fuzzy Log. Intell. Syst. 18 (2018), 161–166. https://doi.org/10.5391/ijfis.2018.18.3.161.

[24] Y. Saber, F. Alsharari, F. Smarandache, On Single-Valued Neutrosophic Ideals in Sostak Sense, Symmetry. 12 (2020), 193. https://doi.org/10.3390/sym12020193.

[25] Y. Saber, F. Alsharari, F. Smarandache, M. Abdel-Sattar, Connectedness and Stratification of Single-Valued Neutrosophic Topological Spaces, Symmetry. 12 (2020), 1464. https://doi.org/10.3390/sym12091464.

[26] Y. Saber, F. Alsharari, F. Smarandache, M. Abdel-Sattar, On Single Valued Neutrosophic Regularity Spaces, Comput. Model. Eng. Sci. 130 (2022), 1625–1648. https://doi.org/10.32604/cmes.2022.017782.

[27] Y. Saber, Connectedness in Single-Valued Neutrosophic Soft Grill Topological Spaces, Int. J. Anal. Appl. 21 (2023), 137. https://doi.org/10.28924/2291-8639-21-2023-137.

[28] Y. Saber, H. Alohali, T. Elmasry, F. Smarandache, On Single-Valued Neutrosophic Soft Uniform Spaces, AIMS Math. 9 (2024), 412–439. https://doi.org/10.3934/math.2024023.

[29] A.M. Zahran, S.A.A. El-Baki, Y.M. Saber, Decomposition of Fuzzy Ideal Continuity via Fuzzy Idealization, Int. J. Fuzzy Log. Intell. Syst. 9 (2009), 83–93. https://doi.org/10.5391/ijfis.2009.9.2.083.

[30] A.M. Zahran, S.E. Abbas, S.A. Abd El-baki, Y.M. Saber, Decomposition of Fuzzy Continuity and Fuzzy Ideal Continuity via Fuzzy Idealization, Chaos Solitons Fractals. 42 (2009), 3064–3077. https://doi.org/10.1016/j.chaos.2009.04.010.