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Chapter 13
Plithogenic Duplets and Plithogenic Triplets

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Abstract

A Neutrosophic Set is a mathematical framework that represents degrees of truth, indeterminacy, and falsehood to address uncertainty in membership values [41, 42]. In contrast, a Plithogenic Set extends this concept by incorporating attributes, their possible values, and the corresponding degrees of appurtenance and contradiction [50]. Among the related concepts of Neutrosophic Sets, Neutrosophic Duplets and Neutrosophic Triplets are well-known. This paper defines Plithogenic Duplets and Plithogenic Triplets as extensions of these concepts using the Plithogenic Set framework and briefly examines their relationship with existing concepts.

Keywords: Set Theory, Neutrosophic Set, Plithogenic Set, Neutrosophic Triplets

1 Preliminaries and Definitions

Some foundational concepts from set theory are applied in parts of this work.

1.1 Neutrosophic Set and Plithogenic Set

The Neutrosophic Set and Plithogenic Set are conceptual frameworks designed to handle uncertainty effectively. These frameworks are closely related to several other mathematical constructs, including Fuzzy Sets [67–71], Intuitionistic Fuzzy Sets [8–11], Neutrosophic Offsets [16, 18, 45, 46, 53, 59], Hyperneutrosophic Sets [17, 25–27], and Bipolar Neutrosophic Sets [2, 4, 5, 33]. Their definitions are provided below.

Definition 1.1 ((Single-valued) Neutrosophic Set)**.** [41–44, 56, 57] Let X be a given set. A (single-valued) Neutrosophic Set A on X is characterized by three membership functions:

$$T_A : X \to [0, 1], \quad I_A : X \to [0, 1], \quad F_A : X \to [0, 1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degree of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3.$$

Example 1.2 (Examples of Neutrosophic Sets)**.** Examples of several Neutrosophic Sets are provided below.

1. *Weather Prediction (cf. [12, 38, 61]):* Let $X = \{\text{Sunny}, \text{Rainy}, \text{Cloudy}\}$, representing weather conditions. A Neutrosophic Set A may be defined as:

$$T_A(\text{Sunny}) = 0.9, \quad I_A(\text{Sunny}) = 0.05, \quad F_A(\text{Sunny}) = 0.05,$$

$$T_A(\text{Rainy}) = 0.6, \quad I_A(\text{Rainy}) = 0.3, \quad F_A(\text{Rainy}) = 0.1,$$

$$T_A(\text{Cloudy}) = 0.4, \quad I_A(\text{Cloudy}) = 0.4, \quad F_A(\text{Cloudy}) = 0.2.$$

   - Sunny: High certainty (90
   - Rainy: Moderate likelihood of rain, with significant uncertainty.
   - Cloudy: Partial truth, indeterminacy, and falsity, reflecting ambiguity.

2. *Medical Diagnosis (cf. [7, 13, 14, 29, 64]):* Let $X = \{\text{Disease 1}, \text{Disease 2}, \text{Disease 3}\}$, representing possible diagnoses. Define a Neutrosophic Set A as:

$$T_A(\text{Disease 1}) = 0.8, \quad I_A(\text{Disease 1}) = 0.1, \quad F_A(\text{Disease 1}) = 0.1,$$

$$T_A(\text{Disease 2}) = 0.5, \quad I_A(\text{Disease 2}) = 0.3, \quad F_A(\text{Disease 2}) = 0.2,$$
$$T_A(\text{Disease 3}) = 0.2, \quad I_A(\text{Disease 3}) = 0.4, \quad F_A(\text{Disease 3}) = 0.4.$$

   - Disease 1: Highly likely, with minimal indeterminacy and falsity.
   - Disease 2: Moderate likelihood, higher indeterminacy.
   - Disease 3: Low likelihood, dominated by indeterminacy and falsity.

3. *Product Quality Assessment (cf. [30,36,66,73]):* Let $X = \{\text{High Quality}, \text{Medium Quality}, \text{Low Quality}\}$. A Neutrosophic Set A is defined as:

$$T_A(\text{High Quality}) = 0.7, \quad I_A(\text{High Quality}) = 0.2, \quad F_A(\text{High Quality}) = 0.1,$$

$$T_A(\text{Medium Quality}) = 0.5, \quad I_A(\text{Medium Quality}) = 0.3, \quad F_A(\text{Medium Quality}) = 0.2,$$
$$T_A(\text{Low Quality}) = 0.3, \quad I_A(\text{Low Quality}) = 0.4, \quad F_A(\text{Low Quality}) = 0.3.$$

   - High Quality: Considered mostly true with some uncertainty and minimal falsity.
   - Medium Quality: Equally distributed among truth, indeterminacy, and falsity.
   - Low Quality: More dominated by indeterminacy and falsity than truth.

The Plithogenic Set is known as a type of set that can generalize Neutrosophic Sets, Fuzzy Sets, and other similar sets [**?**, 1, 3, 15, 19–24, 28, 37, 49, 50, 58, 62, 63]. The definition of the Plithogenic Set is provided below.

Definition 1.3. [49, 50] Let S be a universal set, and $P \subseteq S$. A *Plithogenic Set* PS is defined as:

$$PS = (P, v, Pv, pdf, pCF)$$

where:

- v is an attribute.
- Pv is the range of possible values for the attribute v.
- $pdf : P \times Pv \to [0, 1]^s$ is the *Degree of Appurtenance Function (DAF)*.
- $pCF : Pv \times Pv \to [0, 1]^t$ is the *Degree of Contradiction Function (DCF)*.

These functions satisfy the following axioms for all $a, b \in Pv$:

1. *Reflexivity of Contradiction Function*:
$$pCF(a, a) = 0$$

2. *Symmetry of Contradiction Function*:

$$pCF(a, b) = pCF(b, a)$$

1.2 Neutrosophic Duplet

A Neutrosophic Duplet is defined as a pair $\langle a, \mathrm{neut}(a)\rangle$ within a set, where a represents an element of the set and $\mathrm{neut}(a)$ denotes the neutrosophic neutral element associated with a. The pair satisfies specific conditions related to neutrality and non-inversibility, as described in the literature [31, 32, 47, 60, 65, 72]. The formal definition is provided below.

Definition 1.4 (Neutrosophic Duplet)**.** [48] Let $\mathcal{U}$ be a universe of discourse, and $A \subseteq \mathcal{U}$ be a non-empty set endowed with a binary operation $*$. A pair $\langle a, \mathrm{neut}(a)\rangle$, where $a, \mathrm{neut}(a) \in A$, is called a *Neutrosophic Duplet* if the following conditions hold:

1. $\mathrm{neut}(a)$ is distinct from the unit element of A with respect to $*$ (if a unit element exists).
2. The operation satisfies:
$$a * \mathrm{neut}(a) = \mathrm{neut}(a) * a = a.$$
3. There does not exist $\mathrm{anti}(a) \in A$ such that:
$$a * \mathrm{anti}(a) = \mathrm{anti}(a) * a = \mathrm{neut}(a).$$

Example 1.5 (Example of Neutrosophic Duplets in $\mathbb{Z}_8$)**.** Consider $\mathbb{Z}_8 = \{0, 1, 2, \ldots, 7\}$ with the binary operation $*$ defined as regular multiplication modulo 8. The unit element with respect to $*$ is 1. The following are Neutrosophic Duplets in $\mathbb{Z}_8$:

$$\langle 2, 5\rangle, \quad \langle 4, 3\rangle, \quad \langle 4, 5\rangle, \quad \langle 4, 7\rangle, \quad \langle 6, 5\rangle.$$

For example:

- $2 * 5 = 5 * 2 = 10 \mod 8 = 2$, so $\mathrm{neut}(2) = 5 \neq 1$.
- There is no $\mathrm{anti}(2) \in \mathbb{Z}_8$ because $2 * x = 5 \mod 8$ is unsolvable as it implies $2x = 5 + 8k$, which contradicts even number = odd number.

1.3 Neutrosophic Triplet

A *NeutroStructure* generalizes classical structures by incorporating degrees of truth (T), indeterminacy (I), and falsehood (F). It is defined as follows [6, 32, 34, 35, 39, 40, 51, 52, 54, 55].

Definition 1.6 (Neutrosophic Triplet)**.** [52] A *Neutrosophic Triplet* represents a conceptual generalization of classical structures, incorporating degrees of truth (T), indeterminacy (I), and falsehood (F). Formally, for a given statement or mathematical object A in a space S:

$$\langle A, \mathrm{Neutro}A, \mathrm{Anti}A\rangle = \big\langle A(1, 0, 0),\ A(T, I, F),\ A(0, 0, 1)\big\rangle,$$

where:

- $A(1, 0, 0)$ (Classical Component): A is 100% true ($T = 1$), 0% indeterminate ($I = 0$), and 0% false ($F = 0$).
- $A(T, I, F)$ (Neutro Component): A is T% true, I% indeterminate, and F% false, such that $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$.
- $A(0, 0, 1)$ (Anti Component): A is 100% false ($F = 1$), 0% true ($T = 0$), and 0% indeterminate ($I = 0$).

Examples:

1. *Theorem Triplet:* $\langle$Theorem, NeutroTheorem, AntiTheorem$\rangle$:

- A classical theorem holds universally true ($T = 1, I = 0, F = 0$).
- A NeutroTheorem is partially true, indeterminate, or false ($T, I, F \neq 1, 0, 0$).
- An AntiTheorem is universally false ($T = 0, I = 0, F = 1$).

2. *Definition Triplet:* ⟨Definition, NeutroDefinition, AntiDefinition⟩:

- A classical definition is universally true.
- A NeutroDefinition applies with partial uncertainty.
- An AntiDefinition is universally invalid or false.

Example 1.7 (Examples of Neutrosophic Triplets)**.** Several specific examples of Neutrosophic Triplets are provided below.

1. *Weather Prediction:* Let A be the statement "It will rain tomorrow."

- Classical Component: $A(1, 0, 0)$ means the prediction is absolutely certain to be true (e.g., $T = 1, I = 0, F = 0$).
- Neutro Component: $A(T, I, F) = (0.6, 0.3, 0.1)$ means there is 60% certainty it will rain, 30% uncertainty, and 10% certainty it will not rain.
- Anti Component: $A(0, 0, 1)$ means the prediction is absolutely false (e.g., $F = 1, T = 0, I = 0$).

2. *Quality Control:* Consider A as "This product meets quality standards."

- Classical Component: $A(1, 0, 0)$ means the product unquestionably meets quality standards.
- Neutro Component: $A(T, I, F) = (0.8, 0.1, 0.1)$ means there is 80% certainty the product meets the standards, with 10% uncertainty and 10% certainty it does not meet them.
- Anti Component: $A(0, 0, 1)$ means the product categorically does not meet quality standards.

3. *Medical Diagnosis:* Let A be "The patient has a specific disease."

- Classical Component: $A(1, 0, 0)$ means the diagnosis is definitively correct.
- Neutro Component: $A(T, I, F) = (0.7, 0.2, 0.1)$ indicates a 70% likelihood of the disease, 20% uncertainty, and 10% likelihood of not having the disease.
- Anti Component: $A(0, 0, 1)$ means the diagnosis is definitively wrong.

2 Results of This Paper

This section highlights the main contributions of this paper.

2.1 Plithogenic Duplet

The Plithogenic Duplet extends the Neutrosophic Duplet by utilizing the Plithogenic Set framework. The definitions and related concepts are detailed below.

Definition 2.1 (Plithogenic Duplet)**.** Let $\mathcal{U}$ be a universe of discourse, and $A \subseteq \mathcal{U}$ be a non-empty set endowed with a binary operation $*$. A pair $\langle a, \text{plitho}(a) \rangle$, where $a, \text{plitho}(a) \in A$, is called a *Plithogenic Duplet* if the following conditions hold:

1. *Plithogenic Degree of Appurtenance Function (DAF):* $\text{plitho}(a)$ represents a value determined by the DAF:
$$pdf(a, v_a) = (T_a, I_a, F_a),$$
where $v_a \in Pv$ (attribute value) and $T_a, I_a, F_a \in [0, 1]$ represent the degrees of truth, indeterminacy, and falsehood, respectively.

2. *Neutrality Condition:* The operation $*$ satisfies:
$$a * \text{plitho}(a) = \text{plitho}(a) * a = a,$$
ensuring $\text{plitho}(a)$ acts as a plithogenic neutral element with respect to a.

3. *Non-Inversibility Condition:* There does not exist $\text{anti}(a) \in A$ such that:
$$a * \text{anti}(a) = \text{anti}(a) * a = \text{plitho}(a).$$

4. *Degree of Contradiction Function (DCF):* A DCF pCF applies to attribute values $v_a, v_b \in Pv$, satisfying:
$$pCF(v_a, v_a) = 0, \quad pCF(v_a, v_b) = pCF(v_b, v_a).$$

Example 2.2 (Example of a Plithogenic Duplet)**.** Let $\mathcal{U} = \{x, y, z\}$ and $A = \{x, y\}$ with the operation $*$ defined as follows:
$$x * y = x, \quad y * x = y, \quad x * x = x, \quad y * y = y.$$

Define a Plithogenic Set:
$$PS = (A, v, Pv, pdf, pCF),$$
where:

- v is the attribute "weight" with possible values $Pv = \{v_1, v_2\}$,
- $pdf(x, v_1) = (0.8, 0.1, 0.1)$, $pdf(y, v_2) = (0.7, 0.2, 0.1)$,
- $pCF(v_1, v_1) = 0$, $pCF(v_1, v_2) = 0.3$.

Here, the Plithogenic Duplets are:
$$\langle x, \text{plitho}(x)\rangle = \langle x, v_1\rangle, \quad \langle y, \text{plitho}(y)\rangle = \langle y, v_2\rangle,$$
with the following properties:

1. Neutrality: $x * \text{plitho}(x) = x$, $y * \text{plitho}(y) = y$.
2. Non-inversibility: There is no $\text{anti}(x)$ or $\text{anti}(y)$ in A.

Theorem 2.3. *The Plithogenic Duplet generalizes the Neutrosophic Duplet by incorporating the Plithogenic Set framework, allowing for attribute-based degrees of truth, indeterminacy, and falsehood through a Degree of Appurtenance Function (DAF) and a Degree of Contradiction Function (DCF).*

Proof. Let $\mathcal{U}$ be a universe of discourse, $A \subseteq \mathcal{U}$ a non-empty set, and $*$ a binary operation defined on A. Consider the definitions of Neutrosophic Duplet and Plithogenic Duplet:

From Definition 1.4, a Neutrosophic Duplet $\langle a, \text{neut}(a)\rangle$ satisfies:

1. $\text{neut}(a)$ is distinct from the unit element (if it exists).
2. The operation satisfies:
$$a * \text{neut}(a) = \text{neut}(a) * a = a.$$

3. No anti(a) exists such that:
$$a * \text{anti}(a) = \text{anti}(a) * a = \text{neut}(a).$$

From Definition 2.1, a Plithogenic Duplet $\langle a, \text{plitho}(a)\rangle$ satisfies the following conditions:

1. The value plitho(a) is determined by the Plithogenic Degree of Appurtenance Function (DAF):
$$pdf(a, v_a) = (T_a, I_a, F_a),$$
where $v_a \in Pv$ and $T_a, I_a, F_a \in [0, 1]$.
2. The neutrality condition:
$$a * \text{plitho}(a) = \text{plitho}(a) * a = a.$$
3. The non-inversibility condition:
$$\nexists \text{anti}(a) \in A \quad \text{such that} \quad a * \text{anti}(a) = \text{anti}(a) * a = \text{plitho}(a).$$
4. The Degree of Contradiction Function (DCF):
$$pCF(v_a, v_a) = 0, \quad pCF(v_a, v_b) = pCF(v_b, v_a).$$

The Neutrosophic Duplet is a specific case of the Plithogenic Duplet where:

1. The attribute value v_a and $pdf(a, v_a) = (T_a, I_a, F_a)$ reduce to the fixed values:
$$(T_a, I_a, F_a) = (1, 0, 0) \quad \text{(Classical Component)}.$$
2. No additional attributes or contradiction functions (pCF) are defined.
3. The operation $*$ remains identical in both cases, preserving neutrality and non-inversibility conditions.

By introducing the Plithogenic Set framework, the Plithogenic Duplet allows for attribute-based customization and a richer representation of truth, indeterminacy, and falsehood via pdf and pCF. This subsumes the fixed membership structure of the Neutrosophic Duplet as a special case. Therefore, the Plithogenic Duplet is a generalization of the Neutrosophic Duplet. □

2.2 Plithogenic Triplet

The Plithogenic Triplet extends the Neutrosophic Triplet by incorporating the concepts of attributes, their values, and the Degree of Appurtenance and Contradiction Functions, fundamental to Plithogenic Sets. The formal definition is as follows:

Definition 2.4 (Plithogenic Triplet)**.** Let S be a universal set, and $P \subseteq S$ a Plithogenic Set defined by $PS = (P, v, Pv, pdf, pCF)$, where:

- v is an attribute.
- Pv is the set of possible values of v.
- $pdf : P \times Pv \to [0, 1]^s$ is the Degree of Appurtenance Function (DAF).
- $pCF : Pv \times Pv \to [0, 1]^t$ is the Degree of Contradiction Function (DCF).

A *Plithogenic Triplet* for an element $x \in P$ with respect to an attribute v is defined as:
$$\langle x, \text{Plitho}x, \text{Anti}x\rangle = \big\langle x(1, 0, 0),\ x(pdf, pCF),\ x(0, 0, 1)\big\rangle,$$
where:

- $x(1,0,0)$: Represents the classical membership of x, being fully true ($pdf = 1, pCF = 0$).
- $x(pdf, pCF)$: Represents the Plithogenic membership of x, where the Degree of Appurtenance Function and Degree of Contradiction Function vary between 0 and 1.
- $x(0,0,1)$: Represents the anti-membership of x, being fully false ($pdf = 0, pCF = 1$).

Example 2.5 (Plithogenic Triplet Example)**.** Consider a universal set $S = \{A, B, C\}$ representing three different projects. Define an attribute v = Difficulty Level with possible values $Pv = \{\text{Low}, \text{Medium}, \text{High}\}$. Let the Degree of Appurtenance Function pdf and the Degree of Contradiction Function pCF for each project $x \in S$ be given as follows:

$$pdf(A, \text{Low}) = 0.8,\ \ pdf(A, \text{Medium}) = 0.15,\ \ pdf(A, \text{High}) = 0.05,$$

$$pCF(\text{Low}, \text{High}) = 0.7,\ \ pCF(\text{Low}, \text{Medium}) = 0.3.$$

Then, the Plithogenic Triplet for project A is:

$$\langle A, \text{Plitho}A, \text{Anti}A\rangle = \left\langle A(1,0,0),\ A(pdf, pCF),\ A(0,0,1)\right\rangle,$$

where $A(pdf, pCF)$ reflects the varying degrees of appurtenance and contradiction for A with respect to the attribute Difficulty Level.

Theorem 2.6. *The Plithogenic Triplet generalizes the Neutrosophic Triplet by utilizing the Plithogenic Set framework, allowing for attribute-based customization through the Degree of Appurtenance Function (DAF) and Degree of Contradiction Function (DCF).*

Proof. Let S be a universal set, $P \subseteq S$ a Plithogenic Set, and $PS = (P, v, Pv, pdf, pCF)$ as defined in Definition 2.4. Consider the definitions of Neutrosophic Triplet and Plithogenic Triplet:

From Definition 1.6, a Neutrosophic Triplet $\langle A, \text{Neutro}A, \text{Anti}A\rangle$ satisfies:

1. $A(1,0,0)$: Represents the classical component, being fully true ($T = 1, I = 0, F = 0$).
2. $A(T, I, F)$: Represents the neutrosophic component, where T, I, F can take values in $[0,1]$ such that $T + I + F \leq 3$.
3. $A(0,0,1)$: Represents the anti-component, being fully false ($T = 0, I = 0, F = 1$).

From Definition 2.4, a Plithogenic Triplet $\langle x, \text{Plitho}x, \text{Anti}x\rangle$ satisfies:

1. $x(1,0,0)$: Represents the classical membership of x, being fully true ($pdf = 1, pCF = 0$).
2. $x(pdf, pCF)$: Represents the plithogenic membership of x, where:

   $$pdf(x, v_x) = (T_x, I_x, F_x), \quad pCF(v_x, v_y),$$

   and $T_x, I_x, F_x \in [0,1]$, $pCF(v_x, v_x) = 0$, $pCF(v_x, v_y) = pCF(v_y, v_x)$.
3. $x(0,0,1)$: Represents the anti-membership of x, being fully false ($pdf = 0, pCF = 1$).

The Neutrosophic Triplet is a specific case of the Plithogenic Triplet where:

1. The attribute v and its possible values Pv are fixed and not explicitly considered.
2. The Degree of Appurtenance Function (DAF) simplifies to:

   $$pdf(x, v_x) = (T_x, I_x, F_x),$$

   where v_x is implicit, and the values T_x, I_x, F_x satisfy the same conditions as in the Neutrosophic Triplet.
3. The Degree of Contradiction Function (DCF) is not used, effectively setting $pCF(v_x, v_y) = 0$ for all v_x, v_y.

The Plithogenic Triplet incorporates attributes, their possible values, and the Degree of Contradiction Function (DCF), thereby extending the flexibility and expressiveness of the Neutrosophic Triplet. Consequently, the Neutrosophic Triplet is a special case of the Plithogenic Triplet. □

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Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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