Issue 4 2017 Volume 13

The Journal on Advanced Studies in Theoretical and Experimental Physics, including Related Themes from Mathematics

PROGRESS IN PHYSICS

"All scientists shall have the right to present their scientific research results, in whole or in part, at relevant scientific conferences, and to publish the same in printed scientific journals, electronic archives, and any other media." — Declaration of Academic Freedom, Article 8

ISSN 1555-5534

The Journal on Advanced Studies in Theoretical and Experimental Physics, including Related Themes from Mathematics

PROGRESS IN PHYSICS

A quarterly issue scientific journal, registered with the Library of Congress (DC, USA). This journal is peer reviewed and included in the abstracting and indexing coverage of: Mathematical Reviews and MathSciNet (AMS, USA), DOAJ of Lund University (Sweden), Scientific Commons of the University of St. Gallen (Switzerland), Open-J-Gate (India), Referativnyi Zhurnal VINITI (Russia), etc.

Electronic version of this journal:
http://www.ptep-online.com

Postal Address

Department of Mathematics and Science,
University of New Mexico,
705 Gurley Ave., Gallup, NM 87301, USA

ISSN: 1555-5534 (print)
ISSN: 1555-5615 (online)

Standard Address Number: 297-5092
Printed in the United States of America

October 2017 **Vol. 13, Issue 4**

CONTENTS

Information for Authors

Progress in Physics has been created for rapid publications on advanced studies in theoretical and experimental physics, including related themes from mathematics and astronomy. All submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

All submissions should be designed in LaTeX format using *Progress in Physics* template. This template can be downloaded from *Progress in Physics* home page http://www.ptep-online.com

Preliminary, authors may submit papers in PDF format. If the paper is accepted, authors can manage LaTeXtyping. Do not send MS Word documents, please: we do not use this software, so unable to read this file format. Incorrectly formatted papers (i.e. not LaTeXwith the template) will not be accepted for publication. Those authors who are unable to prepare their submissions in LaTeXformat can apply to a third-party payable service for LaTeX typing. Our personnel work voluntarily. Authors must assist by conforming to this policy, to make the publication process as easy and fast as possible.

Abstract and the necessary information about author(s) should be included into the papers. To submit a paper, mail the file(s) to the Editor-in-Chief.

All submitted papers should be as brief as possible. Short articles are preferable. Large papers can also be considered. Letters related to the publications in the journal or to the events among the science community can be applied to the section *Letters to Progress in Physics*.

All that has been accepted for the online issue of *Progress in Physics* is printed in the paper version of the journal. To order printed issues, contact the Editors.

Authors retain their rights to use their papers published in *Progress in Physics* as a whole or any part of it in any other publications and in any way they see fit. This copyright agreement shall remain valid even if the authors transfer copyright of their published papers to another party.

Electronic copies of all papers published in *Progress in Physics* are available for free download, copying, and re-distribution, according to the copyright agreement printed on the titlepage of each issue of the journal. This copyright agreement follows the *Budapest Open Initiative* and the *Creative Commons Attribution-Noncommercial-No Derivative Works 2.5 License* declaring that electronic copies of such books and journals should always be accessed for reading, download, and copying for any person, and free of charge.

Consideration and review process does not require any payment from the side of the submitters. Nevertheless the authors of accepted papers are requested to pay the page charges. *Progress in Physics* is a non-profit/academic journal: money collected from the authors cover the cost of printing and distribution of the annual volumes of the journal along the major academic/university libraries of the world. (Look for the current author fee in the online version of *Progress in Physics*.)

Scale-Invariant Models of Natural Oscillations in Chain Systems and Their Cosmological Significance

Hartmut Müller

E-mail: hm@interscalar.com

In this paper we review scale-invariant models of natural oscillations in chain systems of harmonic quantum oscillators and derive measurable consequences. Basic model claims are verified in terms of fundamental particles, the cosmic microwave background and the solar system. The cosmological significance of some model statements is discussed.

Introduction

In the last 40 years many studies [1] were published which show that scale invariance (scaling) is a widely distributed phenomenon discovered in high energy physics [2–4], seismology [5, 6], biology [7–9] and stochastic processes of various nature [10].

As a property of power laws, scale invariance can be generated by very different mechanisms. The origin of power law relations and efforts to observe and validate them is a topic of research in many fields of science. However, the universality of scaling may have a mathematical origin that does not depend on the actual mechanism of manifestation.

In [11] we have shown that scale invariance is a fundamental property of natural oscillations in chain systems of similar harmonic oscillators. In [12] we applied this model on chain systems of harmonic quantum oscillators. In the case of a chain of protons as fundamental oscillators, particle rest masses coincide with the eigenstates of the system. This is valid not only for hadrons, but for mesons and leptons as well. Because of scale invariance, chains of electrons produce similar sets of natural frequencies.

In [13] Andreas Ries has shown that the complete description of elementary particle masses by the model of oscillations in chain systems is only possible if considering both, chains of protons and electrons. Furthermore, in [14] he was able to show that this model allows the prediction of the most abundant isotope for a given chemical element.

The core claims of scale-invariant models do not depend on the selection of the fundamental oscillator. Therefore, the rest mass of the fundamental oscillator can be even smaller than the electron mass. Consequently, all elementary particles can be interpreted as eigenstates in a chain system of harmonic quantum oscillators, in which the rest mass of each single oscillator goes to zero. This is how the transition of massless to massive states can be explained [15].

In [16] we have shown that scale-invariant models of natural oscillations in chain systems of protons also describe the mass distribution of large celestial bodies in the solar system.

The intention of this article is an adjustment of the basic claims of our model and an additional verification on fundamental particles, the cosmic microwave background and the solar system. Furthermore, we discuss the cosmological significance of some model claims.

1 Methods

Kyril Dombrowski [17] mentioned that oscillating systems – having the peculiarity to change their own parameters because of interactions inside the systems – have a tendency to reach a stable state where the individual oscillator frequencies are interrelated by specific numbers – namely minima of the rational number density on the number line.

Viktor and Maria Panchelyuga [18] showed that resonance phenomena appear more easily if they belong to maxima in the distribution of rational numbers, while maxima in the distribution of irrational numbers correspond with a high stability of the system, minimal interaction between parts of the system and minimal interaction with the surroundings.

In [11] we have shown that in the case of harmonic oscillations in chain systems, the set of natural frequencies is isomorphic to a discrete set of natural logarithms whose values are rational numbers.

Each real number (rational or irrational) has a biunique representation as a simple continued fraction. In addition, any rational number can be represented as a finite continued fraction and any finite continued fraction represents a rational number [19].

Consequently, the set of natural frequencies of a chain system of harmonic oscillators corresponds with a set of finite continued fractions $\mathcal{F}$, which are natural logarithms:

$$\ln(\omega_{jk}/\omega_{00}) = n_{j0} + \cfrac{z}{n_{j1} + \cfrac{z}{n_{j2} + \ddots + \cfrac{z}{n_{jk}}}} = [z, n_{j0}; n_{j1}, n_{j2}, \ldots, n_{jk}] = \mathcal{F}, \quad (1)$$

where ω_{jk} is the set of angular frequencies and ω_{00} is the fundamental frequency of the set. The denominators are integer numbers: $n_{j0}, n_{j1}, n_{j2}, \ldots, n_{jk} \in \mathbb{Z}$, the cardinality $j \in \mathbb{N}$ of the set and the number $k \in \mathbb{N}$ of layers are finite. In the canonical form, the numerator z is equal 1.

However, by means of the Euler equivalent transformation [20] every continued fraction with partial numerators

$z \neq 1$ can be changed into a continued fraction in the canonical form with $z = 1$.

Therefore, we will call the set $\mathcal{F}$ of finite continued fractions (1) with $z = 1$ the "Fundamental Fractal" of natural frequencies in chain systems of harmonic oscillators.

For rational exponents the natural exponential function is transcendental [21]. Therefore, $\mathcal{F}$ is a set of transcendental numbers that is isomorphic to the set of rational numbers represented by finite continued fractions. The function of isomorphism is the natural logarithm.

It seems that this transcendence and consequently the irrationality of $\mathcal{F}$ provides the high stability of the oscillating chain system because it avoids resonance interaction between the elements of the system.

2 Projections of the Fundamental Fractal

All elements of the continued fractions $\mathcal{F}$ are integers and can therefore be represented as unique products of prime factors. Consequently, we can distinguish classes of finite continued fractions (classes of rational numbers) in dependency on the divisibility of the numerators and denominators by prime numbers, as we have shown in [11]. Based on this, different projections of $\mathcal{F}$ can be studied.

Figure 1 demonstrates the formation of the canonical projection ($z = 1$). Each vertical line represents a rational number that is the logarithm of a natural frequency of a chain system of harmonic oscillators.

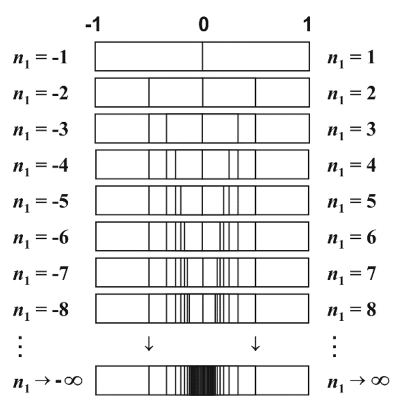

Fig. 1: The formation of the canonical projection ($z = 1$) of the $\mathcal{F}$ on the first layer $k = 1$ (natural logarithmic representation).

The distribution density increases hyperbolically with $|n_{j1}|$. In the range $1 < |n_{j1}| < 2$ the distribution density is minimum. Figure 2 shows that for finite continued fractions (1), ranges of high distribution density (nodes) arise near reciprocal integers $1, 1/2, 1/3, 1/4, \dots$ which are the attractor points of the distribution.

All the denominators of the continued fractions $\mathcal{F}$ are (positive and negative) integers. Therefore, the canonical projection is logarithmically symmetric, as figures 3 and 4 show.

Fig. 2: The canonical projection of the $\mathcal{F}$ in the range $0 \leqslant |n_{j0}| \leqslant 1$ for $k = 2$ (natural logarithmic representation).

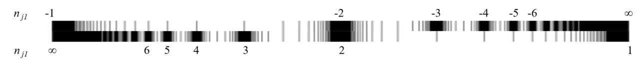

Fig. 3: The canonical projection of the $\mathcal{F}$ in the range $1 \leqslant |n_{j1}| < \infty$ for $k = 2$ (natural logarithmic representation).

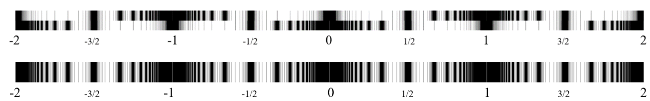

Fig. 4: The canonical projection of the $\mathcal{F}$ in the range $-2 \leqslant S \leqslant 2$ for $k = 2$ (natural logarithmic representation).

In the following we investigate continued fractions (1) which meet the Markov [22] convergence condition $|n| \geqslant |z| + 1$.

Figure 5 illustrates different projections generated by continued fractions (1) with denominators divisible by $2, 3, 4, \dots$ and the corresponding numerators $z = 1, 2, 3, \dots$

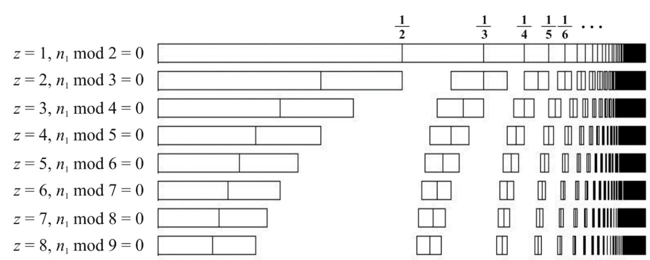

Fig. 5: Different projections generated by continued fractions (1) with denominators divisible by $2, 3, 4, \dots$ and corresponding numerators $z = 1, 2, 3, \dots$.

Figure 5 shows the nodes on the first layer $j = 1$ and also the borders of the node ranges, so the gaps are clearly visible. The borders of the gaps are determined by the alternating continued fractions $[z, 0; z+1, -z-1, z+1, -z-1, \dots] = 1$ and $[z, 0; z-1, -z+1, z-1, -z+1, \dots] = -1$, for $z \geqslant 1$.

Denominators that are divisible by 3 with $z = 2$ build the class of continued fractions (1) that generates the projection with the smallest gaps. These gaps remain empty even if the number of layers k increases infinitely.

In the 2/3-projection, free links n_{j0} of the continued fractions (1) that are divisible by 3 designate the main nodes, denominators divisible by 3 designate subnodes while all the other denominators designate the borders of gaps (see Figure 6 and 7).

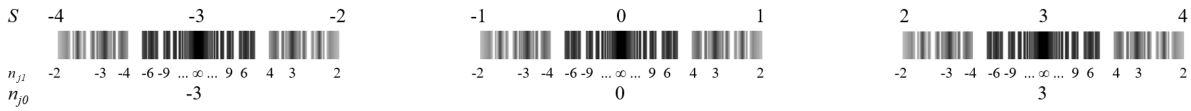

Fig. 6: The 2/3-projection of (1) with $z = 2$, divisible by 3 $|n_{j0}| = 3l$, $(l = 0, 1, 2, \dots)$ and denominators divisible by 3 $|n_{jk}| = 3d$, $(d = 1, 2, \dots)$ in the range of $-4 \leqslant \mathcal{F} \leqslant 4$).

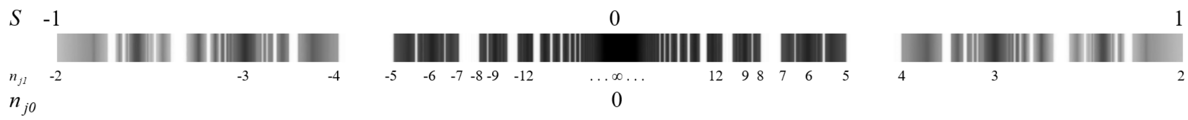

Fig. 7: The same 2/3-projection like in fig. 6, but in the range of $-1 \leqslant \mathcal{F} \leqslant 1$.

In [23] we have shown that in the 2/3-projection, ranges of gaps are connected with stochastic properties of natural oscillations in chain systems of protons. In the current paper we apply the canonical projection only.

3 Harmonic Scaling

Based on (1), we can now calculate the complete set ω_{jk} of natural angular frequencies of a chain system of similar harmonic oscillators, if the fundamental frequency ω_{00} or any other natural frequency of the set ω_{jk} is known:

$$\omega_{jk} = \omega_{00} \exp(\mathcal{F}) \, . \tag{2}$$

Here and in the following, $\mathcal{F}$ is considered in its canonical projection with $z = 1$. The natural angular oscillation period τ is defined as the reciprocal of the angular frequency:

$$\tau_{jk} = 1/\omega_{jk} \, . \tag{3}$$

The complete set of natural angular scale oscillation periods:

$$\tau_{jk} = \tau_{00} \exp(\mathcal{F}) \, . \tag{4}$$

In [12] we have shown that our model (1) can be applied also in the case of natural oscillations in chain systems of harmonic quantum oscillators where the oscillation energy E depends only on the frequency ($\hbar$ being the Planck constant):

$$E_{jk} = \hbar \omega_{jk} \, . \tag{5}$$

Consequently, the natural frequency set and the corresponding set of natural energies are isomorphic, so that chain systems of harmonic quantum oscillators generate discrete exponential energy series:

$$E_{jk} = E_{00} \exp(\mathcal{F}) \, , \tag{6}$$

where $E_{00} = \hbar\omega_{00}$ is the fundamental energy. Because of the mass-energy equivalence,

$$m_{jk} = E_{jk}/c^2 \tag{7}$$

the set of natural energies and the corresponding set of natural masses are isomorphic, so that chain systems of harmonic quantum oscillators generate discrete exponential series of masses:

$$m_{jk} = m_{00} \exp(\mathcal{F}) \, , \tag{8}$$

where $m_{00} = \omega_{00} \cdot \hbar/c^2$ is the fundamental mass.

Finally, the set of natural frequencies corresponds to an isomorphic set of natural wavelengths (c being the speed of light in vacuum),

$$\lambda_{jk} = c/\omega_{jk} \tag{9}$$

so that chain systems of harmonic quantum oscillators generate discrete exponential series of natural wavelengths:

$$\lambda_{jk} = \lambda_{00} \exp(\mathcal{F}) \, , \tag{10}$$

where $\lambda_{00} = c/\omega_{00}$ is the fundamental wavelength.

As a consequence of (3) and (9), the set of natural wavelengths and the set of natural oscillation periods in chain systems of harmonic quantum oscillators coincide with an isomorphic set of natural velocities:

$$\upsilon_{jk} = \lambda_{jk}/\tau_{jk} \, . \tag{11}$$

Therefore, chain systems of harmonic quantum oscillators generate discrete exponential series of natural velocities as well:

$$\upsilon_{jk} = \upsilon_{00} \exp(\mathcal{F}) \, , \tag{12}$$

where the fundamental velocity $\upsilon_{00} = c$ is the speed of light in a vacuum.

In relation to the anticipated harmonic exponential series of wavelengths, velocities, energies and masses as a consequence of harmonic oscillations in chain systems, we propose the term "harmonic scaling".

The natural exponential function of a real argument x is the unique nontrivial function that is its own derivative

$$\frac{d}{dx} e^x = e^x$$

and therefore its own anti-derivative as well. Because of the self-similarity of the natural exponential function regarding its derivatives, any real number, being the result of a measurement, can be thought of as a natural logarithm or as the logarithm of a logarithm. Therefore, harmonic scaling is not limited to exponentiation, but can be extended to tetration, pentation and other hyperoperations as well. In this case we will use the term “hyperscaling”.

4 Harmonic Scaling of Fundamental Particles

In [12] we have shown that physical properties of fundamental particles, for example the proton-to-electron mass ratio or the vector boson-to-electron mass ratio, can be derived from eigenstates in chain systems of harmonic quantum oscillators.

In fact, the natural logarithm of the proton/neutron to electron mass ratio is close to [7; 2] and the logarithm of the W/Z-boson to proton mass ratio is near [4; 2], so we can assume the equation:

$$\ln(m_{wz}/m_{pn}) = \ln(m_{pn}/m_e) - 3\,.$$

Consequently, the logarithm of the W/Z-boson to electron mass ratio is $4\frac{1}{2} + 7\frac{1}{2} = 12$:

$$\ln(m_{wz}/m_e) = 12\,,$$

where m_e, m_{pn}, m_{wz}, are the electron, proton/neutron and W/Z-boson rest masses. As table 1 shows, fundamental particle rest mass ratios correspond to attractor nodes of $\mathcal{F}$. Here and in the following we consider the continued fractions (1) in the canonical form, with the numerator $z = 1$ and write them in square brackets.

Table 1: Fundamental particle rest masses and the corresponding attractor nodes of $\mathcal{F}$, with the electron mass as fundamental. Data taken from Particle Data Group.

particle	particle rest mass m, MeV/c^2	$\mathcal{F}$	$\ln(m/m_e)$	$\ln(m/m_e) - \mathcal{F}$
H-boson	125090 ± 240	[12;2]	12.408	-0.092
Z-boson	91187.6 ± 2.1	[12;∞]	12.092	0.092
W-boson	80385 ± 15	[12;∞]	11.966	-0.034
neutron	939.565379 ± 0.000021	[7;2]	7.517	0.017
proton	938.272046 ± 0.000021	[7;2]	7.515	0.015
electron	$0.510998928 \pm 0.000000011$	[0;∞]	0.000	0.000

As table 1 shows, the logarithms of fundamental particle mass ratios are close to integer or half values that are rational numbers with the smallest possible numerators and denominators.

However, the natural logarithm of the W/Z-boson to proton mass ratio is not exactly 4.5, but between $11.966 - 7.515 = 4.451$ and $12.092 - 7.515 = 4.577$ that approximates $\exp(3/2) = 4.4817$. Thus, the properties of fundamental particle masses (table 1) also support our model of hyperscaling.

5 Fundamental Metrology and Planck Units

The electron and the proton are exceptionally stable and therefore accessible anywhere in the universe. Their lifespan tops everything that is measurable, exceeding 10^{29} years for protons and 10^{28} years for electrons [24]. In the framework of the standard theory of particle physics, the electron is stable because it is the least massive particle with non-zero electric charge. Its decay would violate charge conservation [25]. The proton is stable, because it is the lightest baryon and the baryon number is conserved as well. Therefore, the proton-to-electron mass ratio can be understood as a fundamental physical constant.

These unique properties of electrons and protons predestinate their physical characteristics as fundamental units. Table 2 shows the basic set of electron and proton units that can be considered as a fundamental metrology (c is the speed of light in a vacuum, $\hbar$ is the Planck constant, k_B is the Boltzmann constant).

Table 2: The basic set of physical properties of the electron and proton. Data taken from Particle Data Group. Frequencies, oscillation periods, temperatures and the proton wavelength are calculated.

property	electron	proton
mass m	$9.10938356(11) \cdot 10^{-31}$ kg	$1.672621898(21) \cdot 10^{-27}$ kg
energy $E = mc^2$	0.5109989461(31) MeV	938.2720813(58) MeV
angular frequency $\omega = E/\hbar$	$7.76344071 \cdot 10^{20}$ Hz	$1.42548624 \cdot 10^{24}$ Hz
oscillation period $\tau = 1/\omega$	$1.28808867 \cdot 10^{-21}$s	$7.01515 \cdot 10^{-25}$ s
wavelength $\lambda = c/\omega$	$3.8615926764(18) \cdot 10^{-13}$ m	$2.1030891 \cdot 10^{-16}$ m
temperature $T = mc^2/k_B$	$5.9298 \cdot 10^{9}$ K	$1.08881 \cdot 10^{13}$ m

In [15] we have shown that the Planck scale corresponds to a main attractor node of $\mathcal{F}$ and consequently, Planck units [26] are completely compatible with the fundamental metrology (tab. 2).

Originally proposed in 1899 by Max Planck, these units are also known as natural units, because the origin of their definition comes only from properties of nature and not from any human construct.

Max Planck wrote [27] that these units, “regardless of any particular bodies or substances, retain their importance for all times and for all cultures, including alien and non-human, and can therefore be called natural units of measurement”. Planck units are based only on the properties of space-time.

In fact, the logarithm of the Planck-to-proton mass ration is near the node [44; ∞] of the $\mathcal{F}$:

$$\ln\left(\frac{m_{\text{Planck}}}{m_{\text{proton}}}\right) = \ln\left(\frac{2.17647 \cdot 10^{-8}}{1.6726219 \cdot 10^{-27}}\right) = 44.012. \tag{13}$$

This fact does not only support our model (1), but allows us to derive the proton rest mass from the fundamental physical constants c, $\hbar$, G:

$$m_{\text{proton}} = \exp(-44)(\hbar c/G)^{1/2}\,. \tag{14}$$

In 1899, Max Planck noted that with his discovery of the quantum of action, sufficient fundamental constants were now

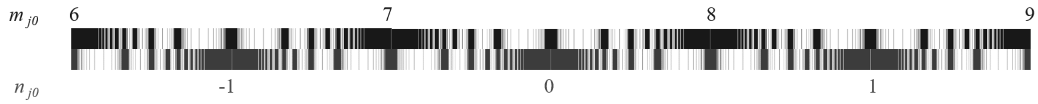

Fig. 8: The correspondence between electron-calibrated attractor nodes $[m_{j0}]$ and proton-calibrated attractor nodes $[n_{j0}]$ of $\mathcal{F}$ in its canonical projection.

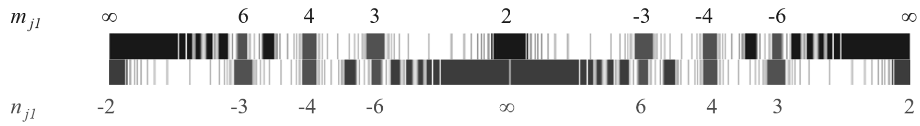

Fig. 9: The correspondence of electron-calibrated subnodes $[m_{j0}; m_{j1}]$ to proton-calibrated subnodes $[n_{j0}; n_{j1}]$ on the first layer of $\mathcal{F}$ in the canonical projection.

Fig. 10: The correspondence of the electron-calibrated $\mathcal{F}$ (above) to the proton-calibrated $\mathcal{F}$ (below) in the 2/3-projection.

known to define universal units for length, time, mass, and temperature.

This equation (14) may well be of cosmological significance, because it means that the values of proton and the electron rest masses are equally fundamental properties of space-time as are the speed of light, the Planck constant and the gravitational constant.

6 Cosmic Microwave Background

CMB data is critical to cosmology since any proposed model of the universe must explain this radiation. Within our model, the CMB can be understood as an eigenstate in a chain system of oscillating protons, because the black body temperature of the CMB corresponds to the main attractor node $[-29; \infty]$ of the $\mathcal{F}$ calibrated on the proton temperature (table 2):

$$\ln\left(\frac{T_{\text{CMB}}}{T_{\text{proton}}}\right) = \ln\left(\frac{2.726\,\text{K}}{1.08881 \cdot 10^{13}\,\text{K}}\right) = -29.016. \qquad (15)$$

7 Global Scaling

We hypothesise that harmonic scaling is a global phenomenon and continues in all scales, following the fundamental fractal (1) that is calibrated by this fundamental metrology (table 2). This hypothesis we have called 'Global Scaling' [23].

8 Calibration of the Fundamental Fractal

Table 1 shows that the natural logarithm of the proton-to-electron mass ratio is approximately 7.5 and consequently, the $\mathcal{F}$ calibrated on the proton will be shifted by 7.5 logarithmic units relative to the $\mathcal{F}$ calibrated on the electron. Figure 8 demonstrates this situation in the canonical projection.

As a consequence, all integer logarithms ($n_{j1} = \infty$) of the proton $\mathcal{F}$ correspond to half logarithms ($m_{j1} = \pm 2$) of the electron $\mathcal{F}$ and vice versa. In addition, the Diophantine equation (18) describes the correspondence of proton-calibrated subnodes $[n_{j0}; n_{j1}]$ with electron-calibrated subnodes $[m_{j0}; m_{j1}]$ on the first layer $k = 1$ of $\mathcal{F}$:

$$\frac{1}{n_{j1}} + \frac{1}{m_{j1}} = \frac{1}{2}\,. \qquad (16)$$

Only three pairs (n_{j1}, m_{j1}) of integers are solutions to this equation: (4, 4), (3, 6) and (6, 3). Figure 9 demonstrates this correspondence.

In fact, if a process property corresponds to a half logarithm ($m_{j1} = \pm 2$) of the electron calibrated $\mathcal{F}$ it also corresponds to an integer logarithm ($n_{j1} = \infty$) of the proton calibrated $\mathcal{F}$. Consequently, we must treat half logarithms and integer logarithms with equal (highest) priority. Furthermore, subnodes that satisfy the equation (16) are of high significance because the subnodes $m_{j1} = \pm 3$, $m_{j1} = \pm 4$ and $m_{j1} = \pm 6$ of the electron $\mathcal{F}$ coincide with the subnodes $n_{j1} = \pm 6$, $n_{j1} = \pm 4$ and $n_{j1} = \pm 3$ of the proton $\mathcal{F}$. It is likely that this correspondence amplifies the attractor effect of these subnodes.

As figure 10 shows, in the 2/3-projection, the electron-based $\mathcal{F}$ (above) fills the empty intervals $3l + 1 \leqslant S \leqslant 3l + 2$ ($l = 0, 1, 2, \ldots$) in the proton-based $\mathcal{F}$ (below). Furthermore, in the intervals $3l + 1/2 \leqslant S \leqslant 3l + 1$ ($l = 0, 1, 2, \ldots$) the proton $\mathcal{F}$ overlaps with the electron $\mathcal{F}$. In the 2/3-projection, the subnodes $[2, n_{j0}; 3, -6]$ and $[2, n_{j0}; -3, 6]$ in the logarithmic center of the overlapping area are the only nodes that are common to both the proton-based and electron-based $\mathcal{F}$.

In [23] we have applied the 2/3-projection on the Solar system. In the following, we will test our hypothesis of global scaling on the Solar system applying the canonical projection.

9 Applying Global Scaling on the Solar System

In 2010 we have shown [16] that the masses of large celestial bodies in the Solar system continue the scale-invariant sequence of fundamental particle rest masses (see table 1), corresponding with main attractor nodes of the fundamental fractal (1).

If we consider the Solar system as still evolving – at least in terms of small body collisions and matter exchanges with neighbouring systems – the expected attractor effect of nodes suggests applying $\mathcal{F}$ for the prediction of evolutionary trends.

Yet, the existence of stable orbits and large celestial bodies with stable rotation periods suggests testing our hypothesis of global scaling on the Solar system. Let us begin with the most noticeable examples.

The Sun

The current amount of the Solar mass supports our hypothesis of global scaling, because it corresponds to a main attractor node of the electron-calibrated $\mathcal{F}$ (8). In fact, the natural logarithm of the Sun-to-electron mass ratio is close to an integer number:

$$\ln\left(\frac{M_{\text{Sun}}}{m_{\text{electron}}}\right) = \ln\left(\frac{1.9884 \cdot 10^{30}\ \text{kg}}{9.10938356 \cdot 10^{-31}\ \text{kg}}\right) = 138.936.$$

Also, the Solar radius corresponds to a main attractor node of the electron $\mathcal{F}$ (10):

$$\ln\left(\frac{R_{\text{Sun}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{6.96407 \cdot 10^{8}\ \text{m}}{3.8615926764 \cdot 10^{-13}\ \text{m}}\right) = 48.945.$$

The Solar sidereal rotation period is in between $\tau_{\min} = 24.5$ days at the equator and $\tau_{\max} = 34.4$ days at the poles. The canonical projection of the electron $\mathcal{F}$ (4) shows that the Solar rotation period varies between the main attractor node $[63; \infty]$ and its nearest significant subnode $[63; -3]$:

$$\ln\left(\frac{\tau_{\max}}{\tau_{\text{electron}}}\right) = \ln\left(\frac{34.4 \cdot 86164\ \text{s}}{1.28808867 \cdot 10^{-21}\ \text{s}}\right) = 63.003\,,$$

$$\ln\left(\frac{\tau_{\min}}{\tau_{\text{electron}}}\right) = \ln\left(\frac{24.5 \cdot 86164\ \text{s}}{1.28808867 \cdot 10^{-21}\ \text{s}}\right) = 62.664\,.$$

Jupiter

Let's start with Jupiter's body mass:

$$\ln\left(\frac{M_{\text{Jupiter}}}{m_{\text{electron}}}\right) = \ln\left(\frac{1.8986 \cdot 10^{27}\ \text{kg}}{9.10938356 \cdot 10^{-31}\ \text{kg}}\right) = 131.981$$

we can see that the Jupiter body mass corresponds to the main attractor node $[132; \infty]$ of the electron $\mathcal{F}$ (8) and within our model, the body mass of Jupiter M_{Jupiter} can be calculated from the Solar Mass M_{Sun}, by simply dividing it seven times by the Euler number $e = 2.71828\ldots$:

$$M_{\text{Jupiter}} = \frac{M_{\text{Sun}}}{\exp(7)}\,. \tag{17}$$

Jupiter's body radius corresponds to the significant subnode $[47; -3]$ of the electron $\mathcal{F}$ (10):

$$\ln\left(\frac{R_{\text{Jupiter}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{7.1492 \cdot 10^{7}\ \text{m}}{3.8615926764 \cdot 10^{-13}\ \text{m}}\right) = 46.668\,.$$

The sidereal rotation period of Jupiter is 9.925 hours and corresponds with the main attractor node $[66; \infty]$ of the proton $\mathcal{F}$ (4):

$$\ln\left(\frac{\tau_{\text{Jupiter}}}{\tau_{\text{proton}}}\right) = \ln\left(\frac{9.925 \cdot 3600\ s}{7.01515 \cdot 10^{-25}\ s}\right) = 66.100\,.$$

In contrast to rotation as angular movement, the location of a celestial body in the Solar system in orbital movement changes permanently. Furthermore, in the case of non-zero eccentricity, the angular velocity of orbital movement is not constant. Therefore, we expect that the orbital periods coincide with attractor nodes of the $\mathcal{F}$ (4) with the electron oscillation period $2\pi\tau_e$ as the fundamental. For example, Jupiter's orbital period of 4332.59 days fulfils the conditions of global scaling very precisely:

$$\ln\left(\frac{T_{\text{Jupiter}}}{2\pi\tau_{\text{electron}}}\right) = \ln\left(\frac{4332.59 \cdot 86164\ \text{s}}{8.0932998 \cdot 10^{-21}\ \text{s}}\right) = 66.001.$$

When the logarithm of the sidereal rotation period of Jupiter slows down to $[66; \infty]$, the orbital-to-rotation period ratio of Jupiter can be described by the equation:

$$\frac{T_{\text{Jupiter}}}{\tau_{\text{Jupiter}}} = 2\pi\frac{\tau_{\text{electron}}}{\tau_{\text{proton}}}\,. \tag{18}$$

The orbital velocity of Jupiter is between $v_{\min} = 12.44$ and $v_{\max} = 13.72$ km/s. This velocity clearly approximates the main attractor node $[-10; \infty]$ of the $\mathcal{F}$ calibrated on the speed of light (12):

$$\ln\left(\frac{v_{\max}}{c}\right) = \ln\left(\frac{13720\ \text{m/s}}{299792458\ \text{m/s}}\right) = -9.992,$$

$$\ln\left(\frac{v_{\min}}{c}\right) = \ln\left(\frac{12440\ \text{m/s}}{299792458\ \text{m/s}}\right) = -10.090.$$

Consequently, the orbital distance of Jupiter between Perihelion = 4.95029 and Aphelion = 5.45492 astronomical units approximates the main attractor node $[56; \infty]$ of the electron-calibrated $\mathcal{F}$ (10):

$$\ln\left(\frac{A_{\text{Jupiter}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{5.45492 \cdot 149597870700\ \text{m}}{3.8615926764 \cdot 10^{-13}\ \text{m}}\right) = 56.011,$$

$$\ln\left(\frac{P_{\text{Jupiter}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{4.95029 \cdot 149597870700\ \text{m}}{3.8615926764 \cdot 10^{-13}\ \text{m}}\right) = 55.914.$$

By the way, the masses of Jupiter's largest moons fulfil the condition of global scaling as well. For example, the body mass of Ganymede fits perfectly with the main node [115; ∞] of the proton $\mathcal{F}$ (8):

$$\ln\left(\frac{M_{\text{Ganymede}}}{m_{\text{proton}}}\right) = \ln\left(\frac{1.4819 \cdot 10^{23}\ \text{kg}}{1.672621 \cdot 10^{-27}\ \text{kg}}\right) = 115.009.$$

On the other hand, the body mass of Io corresponds with the significant subnode [114; 2]:

$$\ln\left(\frac{M_{\text{Io}}}{m_{\text{proton}}}\right) = \ln\left(\frac{8.9319 \cdot 10^{22}\ \text{kg}}{1.672621 \cdot 10^{-27}\ \text{kg}}\right) = 114.502.$$

Venus

The morning star is another impressive example of global scaling. Like the Sun or Jupiter, the body mass of Venus corresponds to a main attractor node of the electron $\mathcal{F}$ (8):

$$\ln\left(\frac{M_{\text{Venus}}}{m_{\text{electron}}}\right) = \ln\left(\frac{4.8675 \cdot 10^{24}\ \text{kg}}{9.10938356 \cdot 10^{-31}\ \text{kg}}\right) = 126.015.$$

Although the rotation of Venus is reverse, its rotation period of 5816.66728 hours fits perfectly with the main attractor node [65; ∞] of the electron calibrated $\mathcal{F}$ (4):

$$\ln\left(\frac{\tau_{\text{Venus}}}{\tau_{\text{electron}}}\right) = \ln\left(\frac{5816.66728 \cdot 3600\ \text{s}}{1.28808867 \cdot 10^{-21}\ \text{s}}\right) = 64.958.$$

The sidereal orbital period of Venus of 224.701 days fulfils the condition of global scaling as well:

$$\ln\left(\frac{T_{\text{Venus}}}{2\pi\tau_{\text{electron}}}\right) = \ln\left(\frac{224.701 \cdot 86164\ \text{s}}{2\pi \cdot 1.28808867 \cdot 10^{-21}\ \text{s}}\right) = 63.042.$$

The orbital velocity of Venus ($v_{\text{min}} = 34.79$ and $v_{\text{max}} = 35.26$ km/s) corresponds well to the main attractor node [−9; ∞] of the speed of light calibrated $\mathcal{F}$ (12):

$$\ln\left(\frac{v_{\text{max}}}{c}\right) = \ln\left(\frac{35260\ \text{m/s}}{299792458\ \text{m/s}}\right) = -9.048,$$

$$\ln\left(\frac{v_{\text{min}}}{c}\right) = \ln\left(\frac{34790\ \text{m/s}}{299792458\ \text{m/s}}\right) = -9.062.$$

The orbital distance of Venus (Perihelion=0.71844 and Aphelion = 0.728213 astronomical units) corresponds precisely to the main attractor node [54; ∞] of the electron calibrated $\mathcal{F}$ (10):

$$\ln\left(\frac{A_{\text{Venus}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{0.728213 \cdot 149597870700\ \text{m}}{3.8615926764 \cdot 10^{-13}\ \text{m}}\right) = 53.997,$$

$$\ln\left(\frac{P_{\text{Venus}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{0.718440 \cdot 149597870700\ \text{m}}{3.8615926764 \cdot 10^{-13}\ \text{m}}\right) = 53.984.$$

The current body radius of Venus corresponds with the subnode [44; 5] of the electron $\mathcal{F}$ (10):

$$\ln\left(\frac{R_{\text{Venus}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{6.053 \cdot 10^{6}\ \text{m}}{3.8615926764 \cdot 10^{-13}\ \text{m}}\right) = 44.199.$$

However, its vicinity to the significant subnode [44; 4] gives reason to expect that Venus is still growing.

Mars

Again, the body mass of Mars corresponds to a main attractor node of the electron $\mathcal{F}$ (8):

$$\ln\left(\frac{M_{\text{Mars}}}{m_{\text{electron}}}\right) = \ln\left(\frac{6.4171 \cdot 10^{23}\ \text{kg}}{9.10938356 \cdot 10^{-31}\ \text{kg}}\right) = 123.989.$$

The sidereal rotation period of Mars is 24.62278 hours and coincides perfectly to the main node [67; ∞] of the proton $\mathcal{F}$ (4):

$$\ln\left(\frac{\tau_{\text{Mars}}}{\tau_{\text{proton}}}\right) = \ln\left(\frac{24.62278 \cdot 3600\ \text{s}}{7.01515 \cdot 10^{-25}\ \text{s}}\right) = 67.008.$$

The orbital velocity of Mars is between 21.97 and 26.50 km/s, approximating the subnode [−9; −2] of the speed of light calibrated $\mathcal{F}$ (12):

$$\ln\left(\frac{v_{\text{max}}}{c}\right) = \ln\left(\frac{26500\ \text{m/s}}{299792458\ \text{m/s}}\right) = -9.334,$$

$$\ln\left(\frac{v_{\text{min}}}{c}\right) = \ln\left(\frac{21970\ \text{m/s}}{299792458\ \text{m/s}}\right) = -9.521.$$

In addition, the orbital period of Mars 686.971 days meets precisely the condition of global scaling:

$$\ln\left(\frac{T_{\text{Mars}}}{2\pi\tau_{\text{electron}}}\right) = \ln\left(\frac{686.971 \cdot 86164\ \text{s}}{2\pi \cdot 1.28808867 \cdot 10^{-21}\ \text{s}}\right) = 65.997.$$

The orbital distance of Mars (Perihelion = 1.3814 and Aphelion = 1.6660 astronomical units) approximates the significant subnode [55; −4] of the electron $\mathcal{F}$ (10):

$$\ln\left(\frac{A_{\text{Mars}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{1.6660 \cdot 149597870700\ \text{m}}{3.8615926764 \cdot 10^{-13}\ \text{m}}\right) = 54.825,$$

$$\ln\left(\frac{P_{\text{Mars}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{1.3814 \cdot 149597870700\ \text{m}}{3.8615926764 \cdot 10^{-13}\ \text{m}}\right) = 54.637.$$

The current body radius of Mars is close to the significant subnode [44; −3] of the $\mathcal{F}$ (10):

$$\ln\left(\frac{R_{\text{Mars}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{3.396 \cdot 10^{6}\ \text{m}}{3.8615926764 \cdot 10^{-13}\ \text{m}}\right) = 43.621.$$

It is therefore likely that Mars, too, is still growing. From this point of view, the large Martian canyon (Valles Marineris) can be interpreted as a sign of crustal swelling [28].

Earth

The current mass of the Earth corresponds to the significant subnode [126; 4] of the electron $\mathcal{F}$ (8):

$$\ln\left(\frac{M_{\text{Earth}}}{m_{\text{electron}}}\right) = \ln\left(\frac{5.97237 \cdot 10^{24}\ \text{kg}}{9.10938356 \cdot 10^{-31}\ \text{kg}}\right) = 126.220.$$

Hence, we can expect that the Earth is slightly increasing its mass.

The body radius of the Earth approximates precisely the significant subnode [44; 4] of the electron $\mathcal{F}$ (10):

$$\ln\left(\frac{R_{\text{Earth equator}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{6.378 \cdot 10^3 \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 44.251,$$

$$\ln\left(\frac{R_{\text{Earth pole}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{6.357 \cdot 10^3 \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 44.248.$$

The sidereal rotation period of the Earth is 23.93444 hours and is located very close to the main node [67; ∞] in the proton $\mathcal{F}$ (4):

$$\ln\left(\frac{\tau_{\text{Earth}}}{\tau_{\text{proton}}}\right) = \ln\left(\frac{23.93444 \cdot 3600 \text{ s}}{7.01515 \cdot 10^{-25} \text{ s}}\right) = 66.980,$$

Therefore, we can expect that the rotation period of the Earth is also slightly increasing. Empirical studies [29] confirm the correlation between body mass and rotation period.

Earth's orbital period of 365.256363 days is close to the main attractor node [71] of the proton-based $\mathcal{F}$ (4):

$$\ln\left(\frac{T_{\text{Earth}}}{2\pi\tau_{\text{proton}}}\right) = \ln\left(\frac{365.256363 \cdot 86164 \text{ s}}{2\pi \cdot 7.01515 \cdot 10^{-25} \text{ s}}\right) = 71.043.$$

Earth's orbital velocity is between $v_{\min} = 29.29$ and $v_{\max} = 30.29$ km/s, approximating the significant subnode [−9; 4] of the speed of light-based $\mathcal{F}$ (12):

$$\ln\left(\frac{v_{\max}}{c}\right) = \ln\left(\frac{30290 \text{ m/s}}{299792458 \text{ m/s}}\right) = -9.200,$$

$$\ln\left(\frac{v_{\min}}{c}\right) = \ln\left(\frac{29290 \text{ m/s}}{299792458 \text{ m/s}}\right) = -9.234,$$

The orbital distance of the Earth (Perihelion = 0.9832687 and Aphelion = 1.01673 astronomical units) corresponds to the significant subnode [54; 3] of the electron-based $\mathcal{F}$ (10):

$$\ln\left(\frac{A_{\text{Earth}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{1.0167300 \cdot 149597870700 \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 54.331,$$

$$\ln\left(\frac{P_{\text{Earth}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{0.9832687 \cdot 149597870700 \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 54.297.$$

Mercury

Mercury's body mass is close to the significant subnode [123; 3] of the electron $\mathcal{F}$ (8):

$$\ln\left(\frac{M_{\text{Mercury}}}{m_{\text{electron}}}\right) = \ln\left(\frac{3.3011 \cdot 10^{23} \text{ kg}}{9.10938356 \cdot 10^{-31} \text{ kg}}\right) = 123.324.$$

Its body radius is close to the significant subnode [43; 3] of the electron $\mathcal{F}$ (10):

$$\ln\left(\frac{R_{\text{Mercury}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{2.44 \cdot 10^3 \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 43.290.$$

So we can expect that Mercury is slightly increasing its mass and size. The sidereal rotation period of Mercury is 1407.5 hours and corresponds to the main attractor node [71; ∞] of the proton $\mathcal{F}$ (4):

$$\ln\left(\frac{\tau_{\text{Mercury}}}{\tau_{\text{proton}}}\right) = \ln\left(\frac{1407.5 \cdot 3600 \text{ s}}{7.01515 \cdot 10^{-25} \text{ s}}\right) = 71.054.$$

The sidereal orbital period of Mercury of 87.9691 days is close to the main attractor node [62; ∞] of the electron $\mathcal{F}$ (4):

$$\ln\left(\frac{T_{\text{Mercury}}}{2\pi\tau_{\text{electron}}}\right) = \ln\left(\frac{87.9691 \cdot 86164 \text{ s}}{2\pi \cdot 1.28808867 \cdot 10^{-21} \text{ s}}\right) = 62.104.$$

The orbital velocity of Mercury oscillates between the main attractor node [−9; ∞] and the significant subnode [−9; 2] of the speed of light calibrated $\mathcal{F}$ (12):

$$\ln\left(\frac{v_{\max}}{c}\right) = \ln\left(\frac{58980 \text{ m/s}}{299792458 \text{ m/s}}\right) = -8.534,$$

$$\ln\left(\frac{v_{\min}}{c}\right) = \ln\left(\frac{38860 \text{ m/s}}{299792458 \text{ m/s}}\right) = -8.951.$$

Mercury's Aphelion corresponds to the main attractor node [61; ∞] of the proton calibrated $\mathcal{F}$ (10):

$$\ln\left(\frac{A_{\text{Mercury}}}{\lambda_{\text{proton}}}\right) = \ln\left(\frac{0.466697 \cdot 149597870700 \text{ m}}{2.1030891 \cdot 10^{-16} \text{ m}}\right) = 61.067.$$

Saturn

Saturn's body mass is close to the significant subnode [131; −4] of the electron calibrated $\mathcal{F}$ (8),

$$\ln\left(\frac{M_{\text{Saturn}}}{m_{\text{electron}}}\right) = \ln\left(\frac{5.6836 \cdot 10^{23} \text{ kg}}{9.10938356 \cdot 10^{-31} \text{ kg}}\right) = 130.776$$

so we suspect that Saturn is actually losing mass and that its ring system is part of the loss process.

The sidereal rotation period of Saturn is 10.55 hours and corresponds to the significant subnode [59; −3] of the electron $\mathcal{F}$ (4):

$$\ln\left(\frac{\tau_{\text{Saturn}}}{\tau_{\text{electron}}}\right) = \ln\left(\frac{10.55 \cdot 3600 \text{ s}}{1.28808867 \cdot 10^{-21} \text{ s}}\right) = 58.646.$$

Therefore, we may expect that Saturn is slightly slowing down its rotation. The orbital period of Saturn of 10759.22 days corresponds to the main attractor node [67; ∞] of the electron $\mathcal{F}$ (4):

$$\ln\left(\frac{T_{\text{Saturn}}}{2\pi\tau_{\text{electron}}}\right) = \ln\left(\frac{10759.22 \cdot 86164 \text{ s}}{2\pi \cdot 1.28808867 \cdot 10^{-21} \text{ s}}\right) = 66.911.$$

Therefore, we may predict that Saturn is slightly increasing its orbit.

The current orbital velocity of Saturn is between 9.09 and 10.18 km/s, approximating the significant subnode [−10; 3] of the speed of light calibrated $\mathcal{F}$ (12):

$$\ln\left(\frac{v_{max}}{c}\right) = \ln\left(\frac{10180 \text{ m/s}}{299792458 \text{ m/s}}\right) = -10.290,$$

$$\ln\left(\frac{v_{min}}{c}\right) = \ln\left(\frac{9090 \text{ m/s}}{299792458 \text{ m/s}}\right) = -10.404.$$

The orbital distance of Saturn is between Perihelion = 9.024 and Aphelion=10.086 astronomical units, oscillating between the significant subnodes [57; −2] and [57; −3] of the electron $\mathcal{F}$ (10):

$$\ln\left(\frac{A_{Saturn}}{\lambda_{electron}}\right) = \ln\left(\frac{10.086 \cdot 149597870700 \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 56.625,$$

$$\ln\left(\frac{P_{Saturn}}{\lambda_{electron}}\right) = \ln\left(\frac{9.024 \cdot 149597870700 \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 56.514.$$

Saturn's equatorial body radius is very close to the significant subnode [46; 2] of the electron $\mathcal{F}$ (10):

$$\ln\left(\frac{R_{Saturn}}{\lambda_{electron}}\right) = \ln\left(\frac{6.0268 \cdot 10^{7} \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 46.497$$

and consequently, to the main attractor node [54; ∞] of the proton $\mathcal{F}$ (10) as well:

$$\ln\left(\frac{R_{Saturn}}{\lambda_{proton}}\right) = \ln\left(\frac{6.0268 \cdot 10^{7} \text{ m}}{2.1030891 \cdot 10^{-16} \text{ m}}\right) = 54.012.$$

Furthermore, Titan's body mass is near the main node [115; ∞] of the proton $\mathcal{F}$ (8):

$$\ln\left(\frac{M_{Titan}}{m_{proton}}\right) = \ln\left(\frac{1.3452 \cdot 10^{23} \text{ kg}}{1.672621 \cdot 10^{-27} \text{ kg}}\right) = 114.912.$$

Uranus

To reach the nearby main attractor node [129; ∞] of the electron-based $\mathcal{F}$ (8), Uranus must increase its body mass by approx. 1/10 logarithmic units:

$$\ln\left(\frac{M_{Uranus}}{m_{electron}}\right) = \ln\left(\frac{8.681 \cdot 10^{25} \text{ kg}}{9.10938356 \cdot 10^{-31} \text{ kg}}\right) = 128.897.$$

The orbital period of Uranus of 30688.5 days corresponds to the main attractor node [68; ∞] of the electron-based $\mathcal{F}$ (4):

$$\ln\left(\frac{T_{Uranus}}{2\pi\tau_{electron}}\right) = \ln\left(\frac{30688.5 \cdot 86164 \text{ s}}{2\pi \cdot 1.28808867 \cdot 10^{-21} \text{ s}}\right) = 67.959,$$

Like Neptune, the body radius of Uranus is close to the significant subnode [46; −3] of the electron $\mathcal{F}$ (10):

$$\ln\left(\frac{R_{Uranus}}{\lambda_{electron}}\right) = \ln\left(\frac{2.5559 \cdot 10^{7} \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 45.639.$$

We may therefore expect that Uranus, like Neptune, is slightly swelling.

The orbital distance of Uranus (Perihelion = 18.33 and Aphelion = 20.11 astronomical units) approximates the significant subnode [57; 4] of the electron $\mathcal{F}$ (10):

$$\ln\left(\frac{A_{Uranus}}{\lambda_{electron}}\right) = \ln\left(\frac{20.11 \cdot 149597870700 \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 57.315,$$

$$\ln\left(\frac{P_{Uranus}}{\lambda_{electron}}\right) = \ln\left(\frac{18.33 \cdot 149597870700 \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 57.223.$$

The orbital velocity of Uranus is between 6.49 and 7.11 km/s, approximating the significant subnode [−11; 3] of the speed of light calibrated $\mathcal{F}$ (12):

$$\ln\left(\frac{v_{max}}{c}\right) = \ln\left(\frac{7110 \text{ m/s}}{299792458 \text{ m/s}}\right) = -10.741,$$

$$\ln\left(\frac{v_{min}}{c}\right) = \ln\left(\frac{6490 \text{ m/s}}{299792458 \text{ m/s}}\right) = -10.649.$$

The sidereal rotation period of Uranus is 17.24 hours and corresponds to the significant subnode [67; −3] of the proton $\mathcal{F}$ (4):

$$\ln\left(\frac{\tau_{Uranus}}{\tau_{proton}}\right) = \ln\left(\frac{17.24 \cdot 3600 \text{ s}}{7.01515 \cdot 10^{-25} \text{ s}}\right) = 66.652.$$

Therefore,we can expect that Uranus is slightly slowing down its rotation.

Neptune

Neptune's body mass corresponds to the main attractor node [129; ∞] of the electron calibrated $\mathcal{F}$ (8):

$$\ln\left(\frac{M_{Neptune}}{m_{electron}}\right) = \ln\left(\frac{1.0243 \cdot 10^{26} \text{ kg}}{9.10938356 \cdot 10^{-31} \text{ kg}}\right) = 129.062.$$

The sidereal rotation period of Neptune is 16.11 hours and coincides perfectly with the main attractor node [59; ∞] of the electron-calibrated $\mathcal{F}$ (4):

$$\ln\left(\frac{\tau_{Neptune}}{\tau_{electron}}\right) = \ln\left(\frac{16.11 \cdot 3600 \text{ s}}{1.28808867 \cdot 10^{-21} \text{ s}}\right) = 59.069.$$

The orbital velocity of Neptune is between 5.37 and 5.50 km/s, close to the main node [−11; ∞] of the speed of light calibrated $\mathcal{F}$ (12):

$$\ln\left(\frac{v_{max}}{c}\right) = \ln\left(\frac{5500 \text{ m/s}}{299792458 \text{ m/s}}\right) = -10.930,$$

$$\ln\left(\frac{v_{min}}{c}\right) = \ln\left(\frac{5370 \text{ m/s}}{299792458 \text{ m/s}}\right) = -10.906.$$

Neptune's current orbital distance (Perihelion = 29.81 and Aphelion = 30.33 astronomical units) corresponds to the significant subnode [58; −4] of the electron-calibrated $\mathcal{F}$ (10):

$$\ln\left(\frac{A_{\text{Neptune}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{30.33 \cdot 149597870700 \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 57.726,$$

$$\ln\left(\frac{P_{\text{Neptune}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{29.81 \cdot 149597870700 \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 57.709.$$

Because of the assumed attractor effect of the main node [−11; ∞] of the $\mathcal{F}$ (12), we can expect that the logarithm of Neptune's orbital velocity should decrease by nearly 1/10. At the same time, the logarithm of Neptune's orbital distance should increase by almost 1/20 due to the attractor effect of the significant subnode [58; −4] of the $\mathcal{F}$ (10). This trend forecast agrees with the Kepler laws: for circular Solar orbits, the orbital velocity of a planet changes with the square root of its orbital distance.

In addition, Neptune's orbital period of 60182 days is close to the significant subnode [69; −3] of the electron $\mathcal{F}$ (4):

$$\ln\left(\frac{T_{\text{Neptune}}}{2\pi\tau_{\text{electron}}}\right) = \ln\left(\frac{60182 \cdot 86164 \text{ s}}{2\pi \cdot 1.28808867 \cdot 10^{-21} \text{ s}}\right) = 68.632.$$

This value supports our trend estimation that Neptune's orbit is slightly growing.

The current body radius of Neptune is close to the significant subnode [46; −3] of $\mathcal{F}$ (10):

$$\ln\left(\frac{R_{\text{Neptune}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{2.4764 \cdot 10^{7} \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 45.607.$$

And so, we can expect that Neptune is still swelling.

Pluto

Although Pluto is no longer considered a planet, its body mass corresponds well with the main attractor node [120; ∞] of the electron $\mathcal{F}$ (8):

$$\ln\left(\frac{M_{\text{Pluto}}}{m_{\text{electron}}}\right) = \ln\left(\frac{1.305 \cdot 10^{22} \text{ kg}}{9.10938356 \cdot 10^{-31} \text{ kg}}\right) = 120.094.$$

The orbital period of Pluto of 90560 days corresponds to the main attractor node [69; ∞] of the electron $\mathcal{F}$ (4):

$$\ln\left(\frac{T_{\text{Pluto}}}{2\pi\tau_{\text{electron}}}\right) = \ln\left(\frac{90560 \cdot 86164 \text{ s}}{2\pi \cdot 1.28808867 \cdot 10^{-21} \text{ s}}\right) = 69.044.$$

The sidereal rotation period of Pluto is 152.87496 hours and corresponds to the significant subnode [61; 3] of the electron-calibrated $\mathcal{F}$ (4):

$$\ln\left(\frac{\tau_{\text{Pluto}}}{\tau_{\text{electron}}}\right) = \ln\left(\frac{152.87496 \cdot 3600 \text{ s}}{1.28808867 \cdot 10^{-21} \text{ s}}\right) = 61.319.$$

Therefore, we can expect that Pluto is slightly slowing down in its rotation.

The orbital velocity of Pluto oscillates between 3.71 and 6.10 km/s, approximating the main attractor node [−11; ∞] of the speed of light calibrated $\mathcal{F}$ (12):

$$\ln\left(\frac{v_{\max}}{c}\right) = \ln\left(\frac{6100 \text{ m/s}}{299792458 \text{ m/s}}\right) = -10.803,$$

$$\ln\left(\frac{v_{\min}}{c}\right) = \ln\left(\frac{3710 \text{ m/s}}{299792458 \text{ m/s}}\right) = -11.300.$$

The orbital distance of Pluto (Perihelion = 29.656 and Aphelion = 49.319 astronomical units) approximates the main attractor node [58; ∞] of the electron-calibrated $\mathcal{F}$ (10):

$$\ln\left(\frac{A_{\text{Pluto}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{49.319 \cdot 149597870700 \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 58.212,$$

$$\ln\left(\frac{P_{\text{Pluto}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{29.656 \cdot 149597870700 \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 57.704.$$

The body radius of Pluto 1187 ± 7 km is close to the significant subnode [42; 2] of the electron-calibrated $\mathcal{F}$ (10),

$$\ln\left(\frac{R_{\text{Pluto}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{1187 \cdot 10^{6} \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 42.570,$$

which is also close to the main attractor node [50; ∞] of the proton-calibrated $\mathcal{F}$ (10):

$$\ln\left(\frac{R_{\text{Pluto}}}{\lambda_{\text{proton}}}\right) = \ln\left(\frac{1187 \cdot 10^{6} \text{ m}}{2.1030891 \cdot 10^{-16} \text{ m}}\right) = 50.085.$$

Hence, we can expect that Pluto is slightly shrinking. This prognosis matches with new findings of surface-atmosphere interactions and mass wasting processes [30] on Pluto.

By the way, also Charon's body mass fits with the main node [118; ∞] of the electron $\mathcal{F}$ (8):

$$\ln\left(\frac{M_{\text{Charon}}}{m_{\text{electron}}}\right) = \ln\left(\frac{1.587 \cdot 10^{21} \text{ kg}}{9.10938356 \cdot 10^{-31} \text{ kg}}\right) = 117.944.$$

In conclusion, table 3 gives an overview of the current positions in the electron calibrated $\mathcal{F}$ (4), (8), (10), and (12) of the Sun and the planets (including Pluto) regarding their masses, sizes, rotation, orbital distances, periods and velocities.

Table 3 shows that our model (1) allows to see a connection between the stability of the Solar system and the stability of electron and proton. Jupiter, Neptune, Venus and Pluto occupy mostly main attractor nodes of the electron calibrated fundamental fractal $\mathcal{F}$ and therefore they can be understood as electron determined factors of stability in the Solar system. It is interesting that also the Sun occupies main nodes of the electron $\mathcal{F}$. Considering the coincidence of half logarithms in the electron $\mathcal{F}$ with integer logarithms (main attractor nodes) of the proton $\mathcal{F}$, the stability of Earth's rotation and orbit seems connected with the stability of the proton. Furthermore, Earth's mass and radius occupy the subnode $n_1 = 4$ that is maximum distant from any main attractor node of the $\mathcal{F}$. This position could be connected with some optimum of flexibility, if we consider the main nodes as islands of stability.

Table 3: The current positions in the electron calibrated $\mathcal{F}$ (4), (8), (10) and (12) of the largest bodies regarding their masses, sizes, rotation, orbital distances, periods and velocities. In the cases of large eccentricity*, the logarithmically average position is indicated.

celestial body	mass in $\mathcal{F}$ (8)	radius in $\mathcal{F}$ (10)	rotation period in $\mathcal{F}$ (4)	orbital period in in $\mathcal{F}$ (4)	orbital distance in $\mathcal{F}$ (10)	orbital velocity in $\mathcal{F}$ (12)
Sun	[139; ∞]	[49; ∞]	[63; ∞]			
Jupiter	[132; ∞]	[47; −3]	[58; 2]	[66; ∞]	[56; ∞]	[−10; ∞]
Saturn	[131;−4]	[46; 2]	[59; −3]	[67; ∞]	[56; 2]	[−10; −3]
Neptune	[129; ∞]	[46; -3]	[59; ∞]	[69; −3]	[58; −4]	[−11; ∞]
Uranus	[129; ∞]	[46; -3]	[59; 6]	[68; ∞]	[57; 4]	[−11; 3]
Earth	[126; 4]	[44; 4]	[59; 2]	[63; 2]	[54; 3]	[−9; −4]
Venus	[126; ∞]	[44; 4]	[65; ∞]	[63; ∞]	[54; ∞]	[-9; ∞]
Mars	[124; ∞]	[44; −3]	[59; 2]	[64; 6]	[55; −4]	[−9; −2]
Mercury	[123; 3]	[43; 3]	[63; 2]	[62; 6]	[53; 3]*	[−9; 3]*
Pluto	[120; ∞]	[42; 2]	[61; 3]	[69; ∞]	[58; ∞]*	[−11; ∞]*

Resume

Properties of fundamental particles, for example the proton-to-electron mass ratio or the vector boson-to-electron mass ratio (table 1), support our scale-invariant model (1) of eigenstates in chain systems of harmonic quantum oscillators and have allowed us to derive the proton rest mass from fundamental physical constants (14). In addition, the cosmic microwave background can be interpreted as an eigenstate of a chain system of oscillating protons (15).

In our scale-invariant model, physical properties of celestial bodies such as mass, size, rotation and orbital period can be understood as macroscopic quantized eigenstates of chain systems of oscillating protons and electrons. This understanding can be applied to evolutionary trend prognosis of the Solar system but may be of cosmological significance as well. Conceivably, the observable exponential expansion of the universe is a consequence of the scale-invariance of the fundamental fractal (1).

Acknowledgements

The author is grateful to Viktor Panchelyuga for valuable discussions and support.

Submitted on July 18, 2017

References

1. Barenblatt G. I. Scaling. Cambridge University Press, 2003.
2. Feynman R. P. Very High-Energy Collisions of Hadrons. *Phys. Rev. Lett.*, 1969, 23, 1415.
3. Kolombet V. Macroscopic fluctuations, masses of particles and discrete space-time. *Biofizika*, 1992, v. 36, 492–499 (in Russian).
4. Tatischeff B. Fractals and log-periodic corrections applied to masses and energy levels of several nuclei. arXiv:1107.1976v1 [physics.gen-ph] 11 Jul 2011.
5. Gutenberg B., Richter C. F., Seismicity of the Earth and Associated Phenomena, 2nd ed. Princeton, N.J., Princeton University Press, 1954.
6. Corral A. Universal local versus unified global scaling laws in the statistics of seismicity. arXiv:cond-mat/0402555 v1 23 Feb 2004.
7. Čislenko L. L. The Structure of the Fauna and Flora in connection with the sizes of the organisms. Moscow, 1981 (in Russian).
8. Schmidt-Nielsen K. Scaling. Why is the animal size so important? Cambridge University Press, 1984.
9. Zhirmunsky A. V., Kuzmin V. I. Critical levels in developmental processes of biological systems. Moscow, Nauka, 1982 (in Russian).
10. Shnoll S. E. Changes in the fine structure of stochastic distributions as a consequence of space-time fluctuations. arxiv.org/ftp/physics/papers/0602/0602017.
11. Müller H. Fractal Scaling Models of Resonant Oscillations in Chain Systems of Harmonic Oscillators. *Progress in Physics*, 2009, issue 2, 72–76.
12. Müller H. Fractal Scaling Models of Natural Oscillations in Chain Systems and the Mass Distribution of Particles. *Progress in Physics*, 2010, issue 3, 61–66.
13. Ries A. Bipolar Model of Oscillations in a Chain System for Elementary Particle Masses. *Progress in Physics*, 2012, issue 4, 20–28.
14. Ries A. Qualitative Prediction of Isotope Abundances with the Bipolar Model of Oscillations in a Chain System. *Progress in Physics*, 2015, issue 11, 183–186.
15. Müller H. Emergence of Particle Masses in Fractal Scaling Models of Matter. *Progress in Physics*, 2012, issue 4, 44–47.
16. Müller H. Fractal scaling models of natural oscillations in chain systems and the mass distribution of the celestial bodies in the Solar System. *Progress in Physics*, 2010, issue 3, 61–66.
Müller H. Scaling of body masses and orbital periods in the Solar system as consequence of gravity interaction elasticity. Abstracts of the XII. International Conference on Gravitation, Astrophysics and Cosmology, dedicated to the centenary of Einstein's General Relativity theory. Moscow, PFUR, 2015.
17. Dombrowski K. I. Rational numbers distribution and resonance. *Progress in Physics*, 2005, v. 1, 65–67.
18. Panchelyuga V. A., Panchelyuga M. S. Resonance and Fractals on the Real Numbers Set. *Progress in Physics*, 2012, issue 4, 48–53.
19. Khintchine A. Ya. Continued fractions. University of Chicago Press, Chicago, 1964.
20. Skorobogatko V. Ya. The Theory of Branched Continued Fractions and mathematical Applications. Moscow, Nauka, 1983.
21. Hilbert D. Über die Transcendenz der Zahlen e und π. *Mathematische Annalen*, 1893, 43, 216–219.
22. Markov A. A. Selected work on the continued fraction theory and theory of functions which are minimum divergent from zero. Moscow – Leningrad, 1948 (In Russian).
23. Müller H. Scaling as Fundamental Property of Natural Oscillations and the Fractal Structure of Space-Time. Foundations of Physics and Geometry. Peoples Friendship University of Russia, 2008 (in Russian).
24. Olive K. A. et al. (Particle Data Group), *Chin. Phys. C.*, 2016, v. 38, 090001.
25. Steinberg R. I. et al. Experimental test of charge conservation and the stability of the electron. *Physical Review D*, 1999, 61 (2), 2582–2586.
26. Astrophysical constants. Particle Data Group, pdg.lbl.gov
27. Max Planck. Über Irreversible Strahlungsvorgänge. In: *Sitzungsbericht der Königlich Preußischen Akademie der Wissenschaften*, 1899, vol. 1, 479–480.
28. Lin An. Structural analysis of the Valles Marineris fault zone: Possible evidence for large-scale strike-slip faulting on Mars. *Lithosphere*, 2012, 4 (4), 286–330.
29. Gizachew Tiruneh. Explaining Planetary-Rotation Periods Using an Inductive Method. arXiv: 0906.3531 [astro-ph.SR]
30. Stern S. A. et al. The Pluto system: Initial results from its exploration by New Horizons. *Science*, 2015, 350 (6258), 249–352; arXiv:1510.07704.

Notes on Extended Lorentz Transformations for Superluminal Reference Frames

Yaroslav I. Grushka

Institute of Mathematics NAS of Ukraine. 3, Tereschenkivska st., Kyiv, 01601, Ukraine. E-mail: grushka@imath.kiev.ua

The present paper is devoted to the analysis of different versions of extended Lorentz transformations, proposed for reference frames moving with the velocity, greater then the velocity of light. In particular we point out some errors of individual authors in this field.

This work is connected with the theory of tachyon movement. Research in this direction were initiated in the papers [1, 2] more than 50 years ago. Then, in the papers of E. Recami, V. Olkhovsky and R. Goldoni [3–5], the *extended Lorentz transformations* for reference frames, moving with the velocity, greater then the velocity of light c were proposed. Latter the above extended Lorentz transformations were rediscovered in [6, 7]. The ideas of E. Recami, V. Olkhovsky and R. Goldoni are still relevant in our time. In particular B. Cox and J. Hill published in [7] a new and elegant way to deduce the formulas of E. Recami, V. Olkhovsky and R. Goldoni's extended Lorentz transformations. Also in paper [8] the extended Lorentz transformations are obtained for the case, where the space of geometrical coordinates may be any real Hilbert space of any dimension, including infinity. Application of the E. Recami, V. Olkhovsky and R. Goldoni's extended Lorentz transformations to the problem of spinless tachyon localization can be found in [9].

In the paper [10] author tries to obtain several variants of new extended superluminal Lorentz transformations, different from transforms obtained by E. Recami, V. Olkhovsky and R. Goldoni. It should be emphasized, that the paper [10], together with incorrect statements, contains also valuable new results. For example, nonlinear extended Lorentz transformations, proposed in [10], may be applied in the theory of kinematic changeable sets [11] for construction some interesting examples or counterexamples. Now we focus on errors, committed by the author of [10].

At first view, the coordinate transformations (3)–(4) and (9)–(10) from [10] look like as new. But, actually, the formulas (3)–(4) and (9)–(10) from [10] are some, not quite correct, representations for well-known classical Lorentz transformations. Hence, these transformations can not be coordinate transformations for reference frames moving with the superluminal velocity.

For example, let us analyze in details the transformations (3)–(4) from [10] for the case of one space dimension:

$$x' = \gamma(v)\,(x - vf(v)t) \tag{a}$$

$$t' = \gamma(v)\left(t - \frac{vf(v)x}{c^2}\right), \tag{b}$$

where (x, t) are the space-time coordinates of any point in the fixed reference frame $\mathfrak{l}$ and (x', t') are the space-time coordinates of this point in the moving frame $\mathfrak{l}'$.

According to [10], the function $f(v)$ may be any real function, satisfying the following conditions:

1. $f(v) > 0,\ v \in \mathbb{R}$ and $f(0) = 1$;
2. $f(v)$ is even (that is $f(-v) = f(v),\ v \in \mathbb{R}$);

   The multiplier $\gamma(v)$ in (a)–(b) is connected with the function f by the formula,

$$\gamma(v) = \left(1 - \frac{v^2 f^2(v)}{c^2}\right)^{-1/2}.$$

   Thus, the following condition must be satisfied:

3. The transformations (a)–(b) are defined for such values $v \in \mathbb{R}$, for which the inequality $|v|\, f(v) < c$ is performed.

   In the paper [10], the parameter v is treated as the velocity of the moving reference frame $\mathfrak{l}'$. Thus, to include the subluminal diapason into the set of "allowed velocities", we may apply following condition:

4. $vf(v) < c$ for $0 < v < c$.

Note, that the condition 4 is not strictly necessary, and in the analysis of the transformations (a)–(b) we take into account only the conditions 1–3.

According to the paper [10], the parameter v in transformations (a)–(b) is the velocity of the moving reference frame $\mathfrak{l}'$. But now we are going to prove that the last statement is not true. For this purpose we calculate the inverse transform to (a)–(b), by means of solving the system (a)–(b) relatively the variables (x, t):

$$x = \gamma(v)\,(x' + vf(v)t') \tag{c}$$

$$t = \gamma(v)\left(t' + \frac{vf(v)x'}{c^2}\right). \tag{d}$$

The origin of the moving reference frame $\mathfrak{l}'$ at any fixed time point τ has the coordinates $(0, \tau)$ in the frame $\mathfrak{l}'$, and, according to the transformations (c)–(d), it has the coordinates $(\gamma(v)vf(v)\tau,\ \gamma(v)\tau)$ in the frame $\mathfrak{l}$. Consequently, the origin of the moving frame $\mathfrak{l}'$ will overpass the distance $\gamma(v)vf(v)\tau$ during the time interval $[\,0,\ \gamma(v)\tau\,]$ (where we select any $\tau \neq 0$). Hence, the velocity u of the moving reference frame $\mathfrak{l}'$ is equal to the following value:

$$u = \frac{\gamma(v)vf(v)\tau}{\gamma(v)\tau} = vf(v),$$

which is not v. Thus, the parameter v in (a)–(b) is expressed via the actual velocity u of the reference frame $\mathfrak{l}'$ by means of the formula, $v = \frac{u}{f(v)}$. And the substitution of the value $\frac{u}{f(v)}$ instead of v into transformations (a)–(b) leads to the classical Lorentz transformations.

Hence, we have seen, that *the formulas* (a)–(b) *(or the formulas* (3)–(4) *from* [10]*) are one of the representations for classical Lorentz transformations, and the actual velocity $u = vf(v)$ of the moving reference frame, according to the condition 3, can not exceed the velocity of light*.

Also, it should be noted, that the transformations (a)–(b) (or (3)–(4) from [10]) are preserving the Lorentz-Minkowski pseudo-metric:

$$\mathsf{M}_c(t, x) = x^2 - c^2 t^2$$

in the Minkowski space-time over real axis $x \in \mathbb{R}$. But *any bijective linear operator in the Minkowski space-time, preserving the Lorentz-Minkowski pseudo-metric, belongs to the general Lorentz group* [12]*, and it can not be coordinate transform for superluminal reference frame*.

The coordinate transformations (9)–(10) from [10], according to the author requirements, also are preserving the Lorentz-Minkowski pseudo-metric in the Minkowski space-time over $\mathbb{R}^3$. Therefore *they also can not be coordinate transformations for superluminal reference frames*. And they can be analyzed in details by a similar way as the transformations (3)–(4) from [10].

Submitted on July 25, 2017

References

1. O.-M. P. Bilaniuk, V.K. Deshpande, E.C.G. Sudarshan. "Meta" Relativity. *American Journal of Physics*, 1962, v. 30 (10), 718–723.
2. O.-M. P. Bilaniuk, E.C.G. Sudarshan. Particles beyond the Light Barrier. *Physics Today*, 1969, v. 22 (5), 43–51.
3. E. Recami, V.S. Olkhovsky. About Lorentz transformations and tachyons. *Lettere al Nuovo Cimento*, 1971, v. 1 (4), 165–168.
4. E. Recami, R. Mignani. More about Lorentz transformations and tachyons. *Lettere al Nuovo Cimento*, 1972, v. 4 (4), 144–152.
5. R. Goldoni. Faster-than-light inertial frames, interacting tachyons and tadpoles. *Lettere al Nuovo Cimento*, 1972, v. 5 (6), 495–502.
6. S.Yu. Medvedev On the Possibility of Broadening Special Relativity Theory Beyond Light Barrier. *Uzhhorod University Scientific Herald. Ser. Phys*, 2005, no. 18, 7–15 (in Ukrainian).
7. James M. Hill, Barry J. Cox. Einstein's special relativity beyond the speed of light. *Proc. of the Royal Society A*, 2012, v. 468 (2148), 4174–4192.
8. Ya.I. Grushka. Tachyon Generalization for Lorentz Transforms. *Methods of Functional Analysis and Topology*, 2013, v. 20 (2), 127–145.
9. Michele Mantegna. Revisiting Barry Cox and James Hill's theory of superluminal motion: a possible solution to the problem of spinless tachyon localization. *Proc. of the Royal Society A*, 2015, v. 471 (2175), 20140541.
10. Dara Faroughy. Extended Linear and Nonlinear Lorentz Transformations and Superluminality. *Advances in High Energy Physics*, 2013, article ID 760916, 20 pages.
11. Grushka Ya.I. Existence criteria for universal coordinate transforms in kinematic changeable sets. *Bukovinian Mathematical Journal*, 2014, v. 2 (2-3), 59-71, (In Ukrainian, English translation is available at https://www.researchgate.net/publication/270647695).
12. M. A. Naimark, Linear Representations of the Lorentz Group. *International Series of Monographs in Pure and Applied Mathematics*, v. 63 (XIV), 1964.

On Time Dilation, Space Contraction, and the Question of Relativistic Mass

Pierre A. Millette

E-mail: PierreAMillette@alumni.uottawa.ca, Ottawa, Canada

In this paper, we revisit the question of relativistic mass to clarify the meaning of this concept within special relativity, and consider time dilation and length contraction in more detail. We see that "length contraction" is a misnomer and that it should really be named "space contraction" to avoid confusion, and demonstrate the complementary nature of time dilation and space contraction. We see that relativistic mass is dependent on the difference in velocity v between an object's proper frame of reference that is at rest with the object and the frame of reference from which it is observed. We show that the inertial mass of a body is its proper mass while the relativistic mass m^* is in effect an effective mass. We find that relativistic mass results from dealing with dynamic equations in local time t in a frame of reference moving with respect to the object of interest, instead of the invariant proper time τ in the frame of reference at rest with the object. The results obtained are in agreement with the Elastodynamics of the Spacetime Continuum.

1 Introduction

The concept of relativistic mass has been a part of special relativistic physics since it was first introduced by Einstein [1, 2] and explored by the early relativists (see for example [3, 4]). Other terminology is also used for relativistic mass, representing the users' perspective on the concept. For example, Aharoni [5] refers to it as the "relative mass", while Dixon [6] refers to it as "apparent mass". Oas [7] and Okun [10] provide good overviews on the development of the historical use of the concept of relativistic mass. Oas [8] has prepared a bibliography of published works where the concept is used and where it is ignored.

There is no consensus in the physics community on the validity and use of the concept of relativistic mass. Some consider relativistic mass to represent an actual increase in the inertial mass of a body [12]. However, there have been objections raised against this interpretation (see Taylor and Wheeler [14], Okun [9–11], Oas [7]). The situation seems to arise from confusion on the meaning of the special relativistic dynamics equations. In this paper, we revisit the question of relativistic mass to clarify the meaning of this concept within special relativity, in light of the Elastodynamics of the Spacetime Continuum (*STCED*) [18, 19].

2 Relativistic mass depends on the frame of reference

The relativistic mass m^* is given by

$$m^* = \gamma m_0 \,, \tag{1}$$

where

$$\gamma = \frac{1}{(1-\beta^2)^{1/2}} \,, \tag{2}$$

$\beta = v/c$ and m_0 is the rest-mass or proper mass which is an invariant. Some authors [11] suggest that rest-mass should be denoted as m as this is the real measure of inertial mass. The relativistic mass of an object corresponds to the total energy of an object (invariant proper mass plus kinetic energy). The first point to note is that the relativistic mass is the same as the proper mass in the frame of reference at rest with the object, *i.e.* $m^* = m_0$ for $v = 0$. In any other frame of reference in motion with velocity v with respect to the object, the relativistic mass will depend on v according to (1).

For example, when the relativistic mass of a cosmic ray particle is measured† in an earth lab, it depends on the speed of the particle measured with respect to the earth lab. Similarly for a particle in a particle accelerator, where its speed is measured with respect to the earth lab. The relativistic mass of the cosmic ray particle measured from say a space station in orbit around the earth or a spaceship in transit in space would depend on the speed of the particle measured with respect to the space station or the spaceship respectively.

We thus see that relativistic mass is an effect similar to length contraction and time dilation in that it is dependent on the difference in velocity v between the object's frame of reference and the frame of reference from which it is measured. Observers in different moving frames will measure different relativistic masses of an object as there is no absolute frame of reference with respect to which an object's speed can be measured.

3 Time dilation and space contraction

To further understand this conclusion, we need to look into time dilation and length contraction in more detail. These special relativistic concepts are often misunderstood by physicists. Many consider these changes to be actual physical changes, taking the Lorentz-Fitzgerald contraction and the time dilation effect to be real.

† what is measured is the energy of the particle, not its mass.

For example, John Bell in [15] relates the problem of the thread tied between two spaceships and whether the thread will break at relativistic speeds due to length contraction. He insists that it will – he relates how "[a] distinguished experimental physicist refused to accept that the thread would break, and regarded my assertion, that indeed it would, as a personal misinterpretation of special relativity". Bell appealed to the CERN Theory Division for arbitration, and was dismayed that a clear consensus agreed that the thread would not break, as indeed is correct. As the number of special relativistic "paradoxes" attest, many physicists, scientists and engineers have similar misunderstandings, not clearly understanding the concepts.

This situation arises due to not realizing that v is the difference in velocity between an object's frame of reference and the frame of reference from which it is measured, not an absolute velocity, as discussed in the previous section 2. In a nutshell, time dilation and length contraction are apparent effects. In the frame of reference at rest with an object that is moving at relativistic speeds with respect to another frame of reference, there is no length contraction or time dilation.

The proper time in the frame of reference at rest with the object is the physical time, and the length of the object in the frame of reference at rest with the object is the physical length – there is no time dilation or length contraction. These are observed in other frames of reference moving with respect to that object and are only apparent dilations or contractions perceived in those frames only. Indeed, observers in frames of reference moving at different speeds with respect to the object of interest will see *different* time dilations and length contractions. These cannot all be correct – hence time dilation and length contraction are apparent, not real.

This can be demonstrated to be the case from physical considerations, and in so doing, we clarify further the nature of length contraction. Petkov [13] provides graphically a physical explanation of time dilation and length contraction, based on Minkowski's 1908 paper [16] where the latter first introduced the concept of a four-dimensional spacetime and the description of particles in that spacetime as worldlines. Worldlines of particles at rest are vertical straight lines in a *space−ct* diagram, while particles moving at a constant velocity v are oblique lines and accelerated particles are curved lines.

The basic physical reason for these effects can be seen from the special relativistic line element (using x to represent the direction of propagation and $c = 1$)

$$d\tau^2 = dt^2 - dx^2\,. \tag{3}$$

One sees that for a particle at rest, the vertical straight line in a *space−ct* diagram is equivalent to

$$d\tau^2 = dt^2\,, \tag{4}$$

which is the only case where the time t is equivalent to the proper time τ (in the object's frame of reference). In all other cases, in particular for the oblique line in the case of constant velocity v, (3) applies and there is a mixing of space x and time t, resulting in the perceived special relativistic effects observed in a frame of reference moving at speed v with respect to the object of interest.

Loedel diagrams [17], a variation on *space−ct* diagrams allowing to display the Lorentz transformation graphically, are used to demonstrate graphically length contraction, time dilation and other special relativistic effects in problems that involve two frames of reference. Figs. 1 and 2, adapted from Petkov's Figs. 4.18 [12, p. 86], and 4.20 [12, p. 91] respectively, and Sartori's Fig. 5.15 [17, p. 160], provide a graphical view of the physical explanation of time dilation and length contraction respectively.

From Fig. 1, we see that $\Delta t' > \Delta t$ as expected – the moving observer sees time interval $\Delta t'$ of the observed object to be dilated, while the observed object's time interval Δt is actually the physical proper time interval $\Delta\tau$. From Fig. 2, we see that space distance measurements, *i.e.* space intervals, $\Delta x' < \Delta x$ as expected – the moving observer sees space interval $\Delta x'$ of the observed object to be contracted, while the observed object's space interval Δx is actually the proper space interval.

This provides a physical explanation for length contraction as a manifestation of the reality of a particle's extended worldline, where the cross-section measured by an observer moving relative to it (*i.e.* at an oblique line in the *space−ct* diagram), creates the difference in perceived length between a body in its rest frame and a frame in movement, as seen in

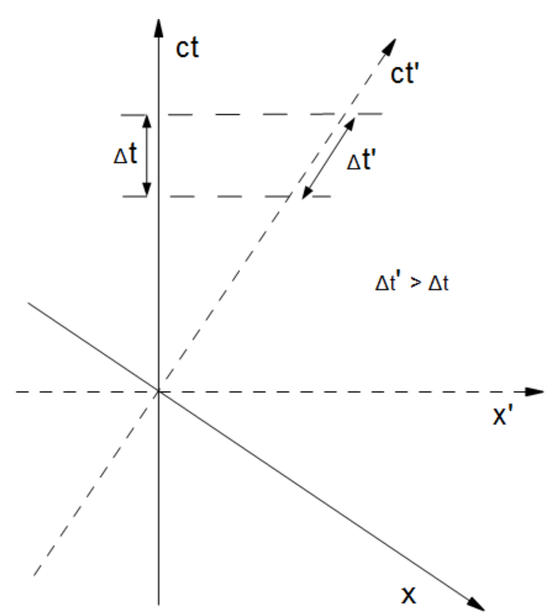

Fig. 1: Physical explanation of time dilation in a Loedel *space−ct* diagram

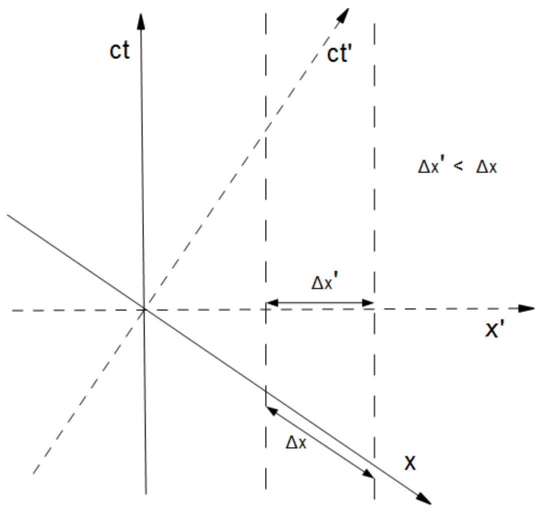

Fig. 2: Physical explanation of length contraction in a Loedel *space−ct* diagram

Fig. 2. It is important to understand that space itself is perceived to be contracted, not just objects in space. As seen in *STCED* [18], objects are not independent of spacetime, but are themselves deformations of spacetime, and are as such perceived to be contracted as space itself is. In actual practice, this phenomenon should be called *space contraction*, to avoid confusion, and demonstrate the complementary nature of time dilation and space contraction.

Thus we see that apparent time dilation and space contraction are perfectly valid physical results of Special Relativity, and there is nothing anomalous about them. Proper consideration of these phenomena eliminates the so-called paradoxes of Special Relativity as demonstrated by various authors, see for example [12, 14, 17]. We now explore the question of relativistic mass, which we first considered in section 2, in light of these considerations.

4 Relativistic mass as an effective mass

In this section, we show that the inertial mass of a body is its proper mass while the relativistic mass m^* is in effect an effective mass or, as Dixon [6] refers to it, an apparent mass. An effective mass is often introduced in dynamic equations in various fields of physics. An effective mass is not an actual mass – it represents a quantity of energy that behaves in dynamic equations similar to a mass. Using the effective mass, we can write the energy E as the sum of the proper mass and the kinetic energy K of the body, which is typically written as

$$E = m^* c^2 = m_0\, c^2 + K \tag{5}$$

to give

$$K = (\gamma - 1)\, m_0\, c^2 \,. \tag{6}$$

In reality, the energy relation in special relativity is quadratic, given by

$$E^2 = m_0^2\, c^4 + p^2 c^2 \,, \tag{7}$$

where p is the momentum. Making use of the effective mass (1) allows us to obtain a linear expression from (7), starting from

$$m^{*2} c^4 = \gamma^2 m_0^2\, c^4 = m_0^2\, c^4 + p^2 c^2 \,, \tag{8}$$

which becomes

$$pc = \sqrt{\gamma^2 - 1}\, m_0\, c^2 \tag{9}$$

or

$$pc = \beta\gamma m_0\, c^2 = \frac{v}{c}\,\gamma m_0\, c^2 = \frac{v}{c}\, E \,. \tag{10}$$

Then

$$p = m^* v \,. \tag{11}$$

As [12, p. 112] shows, the γ factor corresponds to the derivative of time with respect to proper time, *i.e.*

$$\frac{dt}{d\tau} = \frac{1}{(1-\beta^2)^{1/2}} = \gamma \,, \tag{12}$$

such that the velocity with respect to the proper time, u, is given by

$$u = \gamma v \,. \tag{13}$$

Hence using (13) in (11) yields the correct special relativistic relation

$$p = m_0\, u \,, \tag{14}$$

which again shows that m^* in (11) is an effective mass when dealing with dynamic equations in the local time t instead of the invariant proper time τ. It is easy to see that differentiating (14) with respect to proper time results in a force law that obeys Newton's law with the proper mass acting as the inertial mass.

Hence we find that relativistic mass results from dealing with mass in local time t in a frame of reference moving with respect to the object of interest, instead of the invariant proper time τ in the frame of reference at rest with the object, and, from that perspective, is an effect similar to space contraction and time dilation seen in section 3. We see that the rest-mass m_0 should really be referred to as the proper mass, to avoid any confusion about the invariant mass of a body.

Relativistic mass is not apparent as time dilation and space contraction are, but rather is a measure of energy that depends on the relative speed v between two frames of reference, and is not an intrinsic property of an object as there is no absolute frame of reference to measure an object's speed against. The relativistic mass energy $m^* c^2$ is actually the total energy of an object (proper mass plus kinetic energy) measured with respect to a given frame of reference and is not a mass *per se* as mass is a relativistic invariant, *i.e.* a four-dimensional scalar, while energy is the fourth component of a four-vector.

5 Relativistic mass and *STCED*

In *STCED*, the proper mass corresponds to the invariant longitudinal volume dilatation given by [19, p. 32]

$$\rho c^2 = 4\kappa_0\, \varepsilon \tag{15}$$

which is equivalent to the inertial mass. The constant κ_0 is the spacetime bulk modulus and ε is the spacetime volume dilatation. Clearly, the longitudinal volume dilatation does not increase with velocity as it is an invariant. The result (14) is as expected from *STCED*.

For a spacetime volume element, the apparent space contraction in the direction of motion will be cancelled out by the apparent time dilation, *i.e.* the γ factors will cancel out. Thus the volume dilatation ε and the proper mass density ρ of (15) remain unchanged from the perspective of all frames of reference.

The only quantity that is impacted by the observer's frame of reference is the kinetic energy K or alternatively the quantity pc. In the frame of reference at rest with the object (which we can call the proper frame of reference), the kinetic energy $K = 0$ as seen from (6), while $pc = 0$ as seen from (9). The relativistic mass of an object is an effective mass defined to correspond to the total energy of an object (invariant proper mass plus kinetic energy) as observed from the perspective of another frame of reference. It does not represent an increase in the proper mass of an object, which as we have seen in section 4, corresponds to the inertial mass of the object.

6 Discussion and conclusion

In this paper, we have revisited the question of relativistic mass to clarify the meaning of this concept within special relativity. We have also considered time dilation and length contraction in more detail to help clarify the concept of relativistic mass. We have seen that "length contraction" is a misnomer and that it should really be named "space contraction" to avoid confusion, and demonstrate the complementary nature of time dilation and space contraction.

We have seen that relativistic mass is dependent on the difference in velocity v between an object's proper frame of reference that is at rest with the object and the frame of reference from which it is observed. We showed that the inertial mass of a body is its proper mass while the relativistic mass m^* is in effect an effective mass. We showed that relativistic mass results from dealing with dynamic equations in local time t in a frame of reference moving with respect to the object of interest, instead of the invariant proper time τ in the frame of reference at rest with the object. The results obtained are in agreement with the Elastodynamics of the Spacetime Continuum.

Received on August 30, 2017

References

1. Einstein A. Zur Electrodynamik bewegter Körper. *Annalen der Physik*, 1905, vol. 17. English translation (On the Electrodynamics of Moving Bodies) reprinted in Lorentz H. A., Einstein A., Minkowski H, and Weyl H. The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity. Dover Publications, New York, 1952, pp. 37–65.
2. Einstein A. Ist die Trägheit eines Körpers von seinem Energiegehalt abhängig? *Annalen der Physik*, 1905, vol. 17. English translation (Does the Inertia of a Body Depend upon its Energy-Content?) reprinted in Lorentz H. A., Einstein A., Minkowski H, and Weyl H. The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity. Dover Publications, New York, 1952, pp. 69–71.
3. Tolman R. C. Relativity, Thermodynamics and Cosmology. Dover Publications, New York, (1934) 1987, pp. 42–58.
4. Torretti R. Relativity and Geometry. Dover Publications, New York, (1984) 1996, pp. 107–113.
5. Aharoni J. The Special Theory of Relativity, 2nd rev. ed. Dover Publications, New York, (1965) 1985, ch. 5.
6. Dixon W. G. Special Relativity: The Foundation of Macroscopic Physics. Cambridge University Press, Cambridge, 1982, ch. 3.
7. Oas G. On the Abuse and Use of Relativistic Mass. arXiv: physics.ed-ph/0504110v2.
8. Oas G. On the Use of Relativistic Mass in Various Published Works. arXiv: physics.ed-ph/0504111.
9. Okun L. B. The Concept of Mass. *Physics Today*, 1989, v. 42 (6), 31–36.
10. Okun L. B. The Einstein Formula: $E_0 = mc^2$. "Isn't the Lord Laughing?". *Physics–Uspekhi*, 2008, v. 51 (5), 513–527. arXiv: physics.hist-ph/0808.0437.
11. Okun L. B. Energy and Mass in Relativity Theory. World Scientific, New Jersey, 2009.
12. Petkov V. Relativity and the Nature of Spacetime, 2nd ed. Springer, New York, 2009, pp. 77–102, 111–114.
13. Petkov V. Inertia and Gravitation: From Aristotle's Natural Motion to Geodesic Worldlines in Curved Spacetime. Minkowski Institute Press, Montreal, 2012, pp. 78–82.
14. Taylor E. F., Wheeler J. A. Spacetime Physics: Introduction to Special Relativity, 2nd ed. Freeman, New York, 1992, pp. 250–251.
15. Bell J. S. How to Teach Special Relativity. *Progress in Scientific Culture*, 1976, v. 1 (2). Reprinted in Bell J. S. Speakable and Unspeakable in Quantum Mechanics. Cambridge University Press, Cambridge, 1987, pp. 67–80.
16. Minkowski H. Space and Time. 80*th* *Assembly of German Natural Scientists and Physicians*. Cologne, 21 September 1908. English translation reprinted in Lorentz H. A., Einstein A., Minkowski H, and Weyl H. The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity. Dover Publications, New York, 1952, pp. 73–91.
17. Sartori L. Understanding Relativity: A Simplified Approach to Einstein's Theories. University of California Press, Berkley, 1996, pp. 151–201.
18. Millette P. A. Elastodynamics of the Spacetime Continuum. *The Abraham Zelmanov Journal*, 2012, v. 5, 221–277.
19. Millette P. A. Elastodynamics of the Spacetime Continuum: A Spacetime Physics Theory of Gravitation, Electromagnetism and Quantum Physics. American Research Press, Rehoboth, NM, 2017.

Global Scaling as Heuristic Model for Search of Additional Planets in the Solar System

Hartmut Müller
E-mail: hm@interscalar.com

In this paper we apply scale-invariant models of natural oscillations in chain systems of harmonic quantum oscillators to search for additional planets in the Solar System and discuss the heuristic significance of those models in terms of our hypothesis of global scaling.

Introduction

In the last 8 years the heuristic significance of scale invariance (scaling) was demonstrated in various fields of physical research. In [1] we have shown that scale invariance is a fundamental property of natural oscillations in chain systems of similar harmonic oscillators. In [2] we applied this model on chain systems of harmonic quantum oscillators and could show that particle rest masses coincide with the eigenstates of the system. This is valid not only for hadrons, but for mesons and leptons as well. Andreas Ries [3] demonstrated that this model allows for the prediction of the most abundant isotope of a given chemical element. The interpretation of the Planck mass as eigenstate in a chain system of oscillating protons has allowed us to derive the proton rest mass from fundamental physical constants [4]. There we have proposed a new interpretation of the cosmic microwave background as a stable eigenstate of a chain system of oscillating protons.

Scale-invariant models of natural oscillations in chain systems of protons also give a good description of the mass distribution of large celestial bodies in the Solar System [5]. Physical properties of celestial bodies such as mass, size, rotation and orbital period can be understood as macroscopic quantized eigenstates of chain systems of oscillating protons and electrons [4]. This understanding can be applied to an evolutionary trend prognosis of the Solar System but may be of cosmological significance as well.

In this paper we apply our hypothesis of global scaling [4] to the search for additional planets in the Solar System.

Methods

In [1] we have shown that the set of natural frequencies of a chain system of harmonic oscillators coincides with a set of finite continued fractions $\mathcal{F}$, which are natural logarithms:

$$\ln(\omega_{jk}/\omega_{00}) = n_{j0} + \cfrac{z}{n_{j1} + \cfrac{z}{n_{j2} + \ddots + \cfrac{z}{n_{jk}}}} = \tag{1}$$

$$= [z, n_{j0}; n_{j1}, n_{j2}, \ldots, n_{jk}] = \mathcal{F},$$

where ω_{jk} is the set of angular frequencies and ω_{00} is the fundamental frequency of the set. The denominators are integer numbers: $n_{j0}, n_{j1}, n_{j2}, \ldots, n_{jk} \in \mathrm{Z}$, the cardinality $j \in \mathrm{N}$ of the set and the number $k \in \mathrm{N}$ of layers are finite. In the canonical form, the numerator z is equal 1.

Any finite continued fraction represents a rational number [6]. Therefore, all frequencies ω_{jk} in (1) are irrational, because for rational exponents the natural exponential function is transcendental [7]. This circumstance presumably provides for the high stability of the oscillating chain system because it avoids resonance interaction between the elements of the system [8].

In the case of harmonic quantum oscillators, the continued fraction (1) defines not only a fractal set of natural angular frequencies ω_{jk} and oscillation periods $\tau_{jk} = 1/\omega_{jk}$ of the chain system, but also fractal sets of natural energies $E_{jk} = \hbar \cdot \omega_{jk}$ and masses $m_{jk} = E_{jk}/c^2$ which correspond with the eigenstates of the system. For this reason, we have called the continued fraction (1) the "fundamental fractal" of eigenstates in chain systems of harmonic quantum oscillators [4].

The electron and the proton are exceptionally stable quantum oscillators and therefore the proton-to-electron rest mass ratio can be understood as a fundamental physical constant.

We hypothesize the cosmological significance of scale invariance based on the fundamental fractal $\mathcal{F}$ (1) that is calibrated by the physical characteristics of the electron and the proton. This hypothesis we have called 'global scaling' [9].

Results

In [4] we have shown that the masses of the largest bodies in the Solar System correlate with main attractor nodes of the $\mathcal{F}$ (1), supporting our hypothesis of global scaling as forming factor of the Solar System.

For example, the natural logarithm of the Sun-to-electron mass ratio is close to an integer number:

$$\ln(M_{\text{Sun}}/m_{\text{electron}}) =$$

$$= \ln(1.9884 \cdot 10^{30}\text{kg}/9.10938356 \cdot 10^{-31}\text{kg}) = 138.936$$

This is also valid for Jupiter's body mass:

$$\ln\left(M_{\text{Jupiter}}/m_{\text{electron}}\right) =$$

$$= \ln(1.8986 \cdot 10^{27}\text{kg}/9.10938356 \cdot 10^{-31}\text{kg}) = 131.981$$

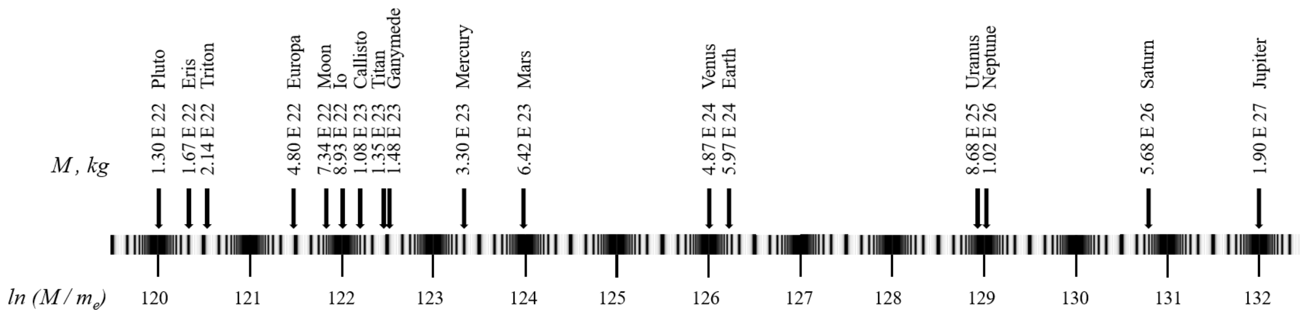

Fig. 1: The mass distribution of planets, heaviest planetoids and moons along the electron-calibrated fundamental fractal $\mathcal{F}$ (1). The nodes [130], [128], [127], [125], [123] and [121] are vacant.

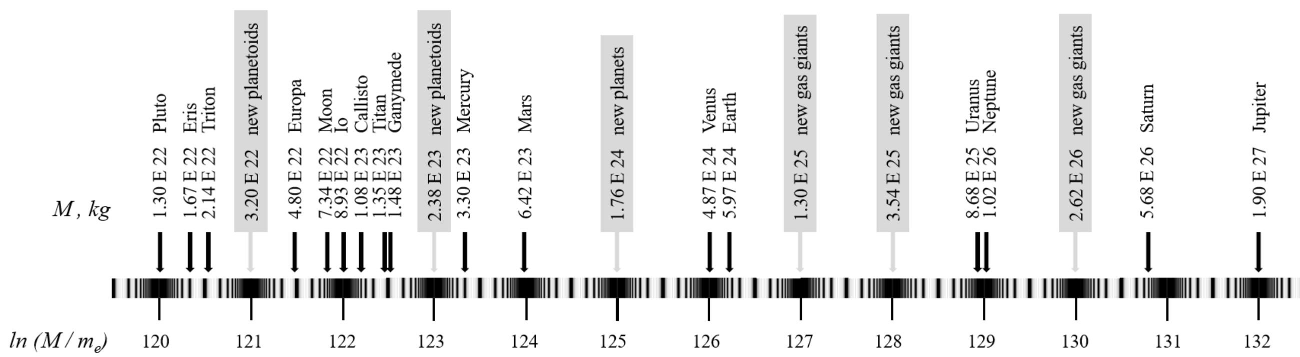

Fig. 2: This copy of fig. 1 shows the mass ranges of hypothetical planetoids, planets and gas giants which could occupy the vacant nodes [130], [128], [127], [125], [123] and [121] of the electron-calibrated fundamental fractal $\mathcal{F}$ (1).

And for Venus as well:

$$\ln\left(M_{\text{Venus}}/m_{\text{electron}}\right) =$$

$$= \ln(4.8675) \cdot 10^{24}\text{kg}/9.10938356 \cdot 10^{-31}\text{kg}) = 126.015$$

Table 1 gives an overview of the body masses of the planets and heaviest planetoids and their positions in the fundamental fractal $\mathcal{F}$ (1).
The electron rest mass $m_e = 9.10938356 \cdot 10^{-31}$ kg [10].

Table 1 shows that the body masses of Jupiter, Neptune, Uranus, Venus, Mars, Pluto, Charon and Haumea coincide with main attractor nodes (integer logarithms) of the electron-calibrated $\mathcal{F}$ (1). This also applies to the Sun. Figure 1 shows the mass distribution of planets, heaviest planetoids and moons along the electron-calibrated fundamental fractal $\mathcal{F}$ (1). The nodes [130], [128], [127], [125], [123], [121] are vacant.

The vacant nodes [121] and [123] indicate that in the mass ranges of 2 to $4 \cdot 10^{22}$ kg and in the range of 2 to $3 \cdot 10^{23}$ kg there should be planetoids still to be discovered. Furthermore, we may expect new planets in the range of 1 to $2 \cdot 10^{24}$ kg. The probability of new gas giants in the Solar System is also very high, because of the wide vacant mass ranges of 1 to $5 \cdot 10^{25}$ kg and of 2 to $3 \cdot 10^{26}$ kg. Figure 2 shows the distribution of these hypothetical bodies on the fundamental fractal $\mathcal{F}$ (1).

Conclusion

The discovery of new gas giants, planets and planetoids with the properties predicted above would be an important confirmation of our hypothesis of global scaling as a forming factor of the Solar System. Already in 2010 [5] we calculated the masses of some of these hypothetical bodies and in 2015 [11, 12] we estimated their orbital elements.

Our calculations correspond well with the hypothesis of Batygin and Brown [13] about a new gas giant called "planet 9" and with the hypothesis of Volk and Malhotra [14] about an unknown Mars-to-Earth mass "planet 10" beyond Pluto.

Based on the vacancies in the fundamental fractal $\mathcal{F}$ (1), we hypothesize the existence of at least two unknown giant planets (see fig. 2). It is likely that they are gas giants. However, this conclusion cannot be made based on the estimation of their masses only, but requires an additional estimation of their radii, which should correspond with vacant positions in the fundamental fractal $\mathcal{F}$ (1) that is calibrated by the proton

Table 1: The logarithms of the body-to-electron mass ratio for the Sun, the planets, the heaviest planetoids (P) and the corresponding positions in the fundamental fractal $\mathcal{F}$ (1).

celestial body	**body mass m, kg**	**ln (m/m$_e$)**	$\mathcal{F}$
Sun	$1.9884 \cdot 10^{30}$	138.936	[139; ∞]
Jupiter	$1.8986 \cdot 10^{27}$	131.981	[132; ∞]
Saturn	$5.6836 \cdot 10^{26}$	130.776	[131; -4]
Neptune	$1.0243 \cdot 10^{26}$	129.062	[129; ∞]
Uranus	$8.681 \cdot 10^{25}$	128.897	[129; ∞]
Earth	$5.97237 \cdot 10^{24}$	126.220	[126; 4]
Venus	$4.8675 \cdot 10^{24}$	126.015	[126; ∞]
Mars	$6.4171 \cdot 10^{23}$	123.989	[124; ∞]
Mercury	$3.3011 \cdot 10^{23}$	123.324	[123; 3]
Eris (P)	$1.67 \cdot 10^{22}$	120.341	[120; 3]
Pluto (P)	$1.305 \cdot 10^{22}$	120.094	[120; ∞]
Haumea (P)	$4.006 \cdot 10^{21}$	118.913	[119; ∞]
Charon (P)	$1.587 \cdot 10^{21}$	117.944	[118; ∞]

and electron wavelengths.

Acknowledgements

The author is grateful to Viktor Panchelyuga for valuable discussions.

Received on September 9, 2017

References

1. Müller H. Fractal Scaling Models of Resonant Oscillations in Chain Systems of Harmonic Oscillators. *Progress in Physics*, 2009, vol. 2, 72–76.
2. Müller H. Fractal Scaling Models of Natural Oscillations in Chain Systems and the Mass Distribution of Particles. *Progress in Physics*, 2010, vol. 3, 61–66.
3. Ries A. Qualitative Prediction of Isotope Abundances with the Bipolar Model of Oscillations in a Chain System. *Progress in Physics*, 2015, vol. 11, 183–186.
4. Müller H. Scale-Invariant Models of Natural Oscillations in Chain Systems and their Cosmological Significance. *Progress in Physics*, 2017, vol. 4, 187–197.
5. Müller H. Fractal scaling models of natural oscillations in chain systems and the mass distribution of the celestial bodies in the Solar System. *Progress in Physics*, 2010, vol. 3, 61–66.
6. Khintchine A. Ya. Continued fractions. University of Chicago Press, Chicago 1964.
7. Hilbert D. Über die Transcendenz der Zahlen e und π. *Mathematische Annalen*, 1893, 43, 216–219.
8. Panchelyuga V. A., Panchelyuga M. S. Resonance and Fractals on the Real Numbers Set. *Progress in Physics*, October 2012, vol. 4, 48–53.
9. Müller H. Scaling as Fundamental Property of Natural Oscillations and the Fractal Structure of Space-Time. Foundations of Physics and Geometry. Peoples Friendship University of Russia, 2008 (in Russian).
10. Olive K. A. et al. (Particle Data Group), *Chin. Phys. C.* 38, 090001, 2016.
11. Müller H. Scaling of body masses and orbital periods in the Solar System. *Progress in Physics*, 2015, vol. 11, 133–135.
12. Müller H. Scaling of body masses and orbital periods in the Solar system as consequence of gravity interaction elasticity. Abstracts of the XII. International Conference on Gravitation, Astrophysics and Cosmology, dedicated to the centenary of Einstein's General Relativity theory. Moscow, PFUR, 2015.
13. Batygin K., Brown M.E. Evidence for a distant giant planet in the Solar System. *The Astronomical Journal*, 151/22, February 2016.
14. Volk K., Malhotra R. The curiously warped mean plane of the Kuiper belt. arXiv:1704.02444v2 [astro-ph.EP], 19 June 2017.

LETTERS TO PROGRESS IN PHYSICS

Discovered "Angel Particle", which is Both Matter and Antimatter, as a New Experimental Proof of Unmatter

Florentin Smarandache and Dmitri Rabounski

Department of Mathematics and Science, University of New Mexico, 705 Gurley Avenue, Gallup, NM 87301, USA
E-mails: smarand@unm.edu, rabounski@ptep-online.com

"Angel particle" bearing properties of both particles and anti-particles, which was recently discovered by the Stanford team of experimental physicists, is usually associated with Majorana fermions (predicted in 1937 by Ettore Majorana). In this message we point out that particles bearing properties of both matter and anti-matter were as well predicted without any connexion with particle physics, but on the basis of pure mathematics, namely — neutrosophic logic which is a generalization of fuzzy and intuitionistic fuzzy logics in mathematics.

Recently, a group of experimental physicists conducted by Prof. Shoucheng Zhang, in Stanford University, claimed about discovery of the particles that bear properties of both particles and anti-particles. The press-release [1] was issued on July 20, one day before the official publication [2].

Shoucheng Zhang told [1, 2] that the idea itself rose up from Ettore Majorana who in 1937 suggested that within the class of fermions a particle may exist which bear properties of particle and anti-particle in the same time. Such hypothetic particles are now know as "Majorana fermions".

In their experiment, the Stanford team used the following experimental setup. Two stacked films — the top film made of superconductor and the bottom film made of magnetic insulator — were stored together in a cooled down vacuum box. And an electrical current was sent through this "sandwich". Using a magnet mounted over the stackled films, the speed of the electrons in the film was able to be modifying. Varying the magnet's properties, the experimentalists registered Majorana particles which appeared in pairs in the electron flow but deviated from the electrons (so they were able to be registered separately). The experimentalists referred to the supposed new particle as "Angel particle" (meaning that, as well as angels are neither male nor female, the supposed particle is neither matter nor anti-matter).

Shoucheng Zhang also declared the importance of this discovery because, he thinks, the particles bearing properties of matter and anti-matter in the same time shows a fantastic perspective for computer industry and machinery.

In this background, we should note that particles bearing properties of matter and anti-matter were as well theoretically predicted being non-connected with particle physics, but only on the basis of pure mathematics. This is a series of works [3–8] based on neutrosophic logic (one of the multi-valued modern logics, a part of mathematics) authored by Florentin Smarandache.

So, following the neutrosophic logics, "between an entity <A> and its opposite <AntiA> there exist intermediate entities <NeutA> which are neither <A> nor <AntiA> [...] Thus, between "matter" and "antimatter" there must exist something which is neither matter nor antimatter, let's call it UNMATTER" [3]. Expanding this theory, a new type of matter — "unmatter" — was predicted.

Now, this theoretical study based on pure mathematics, elucidates that was discovered by the Stanford team conducted by Shoucheng Zhang. This fact shows that not only particle physics but also pure mathematics can make essential predictions that may change the wirld of science and techniques.

Submitted on September 18, 2017

References

1. Chui G. An experiment proposed by Stanford theorists finds evidence for the Majorana fermion, a particle that's its own antiparticle. *Stanford News Service*, 20 July 2017, http://news.stanford.edu/press/view/15528
2. He Q. L., Pan L., Stern A. L. et al. Chiral Majorana fermion modes in a quantum anomalous Hall insulator–superconductor structure. *Science*, 21 Jul 2017, v. 357, issue 6348, 294–299.
3. Smarandache F. A new form of matter — unmatter, composed of particles and anti-particles. *Progress in Physics*, 2005, v. 1, issue 1, 9–11.
4. Smarandache F. Verifying unmatter by experiments, more types of unmatter, and a Quantum Chromodynamics formula. *Progress in Physics*, 2005, v. 1, issue 2, 113–116.
5. Smarandache F. Unmatter entities inside nuclei, predicted by the Brightsen nucleon cluster model. *Progress in Physics*, 2006, v. 2, issue 1, 14–18.
6. Rabounski D., Smarandache F., Borissova L. Neutrosophic Methods in General Relativity. Hexis Publishers, Phoenix (Arizona), 2005.
7. Goldfain E. and Smarandache F. On emergent physics, "unparticles" and exotic "unmatter" states. *Progress in Physics*, 2008, v. 4, issue 4, 10–15.
8. Smarandache F. Unmatter plasma discovered. *Progress in Physics*, 2015, v. 11, issue 3, 246.

Vacuum Polarization by Scalar Field of Bose-Einstein Condensates and Experimental Design with Laser Interferences

B. J. Zhang[1], T. X. Zhang[2]

[1]Department of Electrical Engineering, University of Alabama in Huntsville, Huntsville, AL 35899. E-mail: bojunzhang333@gmail.com
[2]Department of Physics, Alabama A & M University, Normal, AL 35762. E-mail: tianxi.zhang@aamu.edu

In a five-dimensional gravitational theory (or 5D gravity), a scalar field is usually included to couple with the gravitational and electromagnetic fields, which are directly originated from or generated by the mass and electric charge of matter, respectively. Theoretical analyses have shown that the scalar field of 5D gravity can polarize the space (or vacuum) and shield gravity (or flatten spacetime), especially when the object that generates the fields is extremely compact, massive, and/or highly charged. Recently, the scalar field of 5D gravity has been directly connected to the Higgs field of 4D particle physics, so that it dramatically relates to the Ginzburg-Landau scalar field of Bose-Einstein condensates associated with superconductors and/or superfluids. Therefore, the scalar field effect on the properties of light and the weight of objects may be detectable in a laboratory of low temperature physics. In this study, we first analyze the index of refraction of the space or vacuum that is polarized by scalar field. We then explore approaches of detection and design experiments to test the space polarization or the effect of scalar field on light as well as the equivalence or connection between the scalar field of 5D gravity and that of 4D particle physics.

1 Introduction

In contrast to the vector field of electromagnetism and the tensor field of gravitation, a scalar field is a field that has no direction. Up to now, many physical phenomena are explained with the physics of scalar fields such as the cosmic inflation [1-2], dark matter [3-4], dark energy [5-6], particle mass generation [7-9], particle creation [10], gravitational field shielding [11-12], space or vacuum polarization [13-15], and so on. In the particle physics, the Higgs field, which generates masses of particles such as leptons and bosons, is a scalar field associated with particles of spin zero. In the 5D gravity, the gravitational and electromagnetic fields are coupled with a scalar field. Theoretical analyses have shown that the scalar field of 5D gravity can polarize the space or vacuum [13-15] and shield the gravity or flatten the spacetime [11-12,16-17], especially when the object of the fields is extremely compact, massive, and/or highly charged.

The scalar field of the 5D gravity has a direct relation or connection to the Higgs scalar field of the 4D particle physics [18]. The Higgs boson or Higgs particle is an elementary particle initially theorized in 1964 [7-9] and tentatively discovered to exist by the Large Hadron Collider at CERN [19]. This tentative discovery confirmed the existence of the Higgs scalar field, which led to the Nobel Prize of physics in 2013 to be awarded to Peter W. Higgs and Francois Englert. The Higgs mechanism is a process for particles to gain masses from the interaction with the Higgs scalar field. It describes the superconductivity of vacuum according to the Ginzburg-Landau model of the Bose-Einstein condensates.

Therefore, the scalar field of the 5D gravity can be considered as a type of Higgs scalar field of 4D particle physics. The latter can be considered as a type of Ginzburg-Landau scalar field of the Bose-Einstein condensates [20-21]. Then, that the scalar field of the 5D gravity can shield the gravitational field (or flatten the spacetime) and polarize the space or vacuum must imply that the Ginzburg-Landau scalar field of superconductors and superfluids in the state of Bose-Einstein condensates can also shield the gravitational field (or flatten the spacetime) and polarize the space or vacuum.

In fact, the experiment conducted about two decades ago had indeed shown that a rotating type-II ceramic superconductor disk at low temperature could have a moderate (∼2 – 3%) shielding effect against the Earth gravitational field [22]. The experiment conducted later for a static testing with the shielding effect of ∼0.4% [23]. Recently, we have explained these measurements as the gravitational field shielding [12] by the Ginzburg-Landau scalar field of Bose–Einstein condensates associated with the type II ceramic superconductor disk according to the 5D fully covariant gravity developed by Zhang [11,15,24].

In this paper, we will focus on the vacuum polarization by scalar field and its testing. We will explore some possible approaches and further design viable experiment setups to test the space or vacuum polarization by the scalar field (*i.e.* the effect of scalar field on light). We will, at first, apply the fully covariant 5D gravity with a scalar field that was developed by Zhang [11,15] and references therein to formulate the index of refraction in the vacuum that is polarized by the scalar field of this 5D gravity. Then, we will employ the Ginzburg-Landau scalar field generated by the Bose-Einstein condensates of superconductors and superfluids to replace or add the scalar field of the 5D gravity. Finally, we will design an experiment setup of laser light interferences that may detect the vacuum polarization by the Ginzburg-Landau scalar field and thus test Zhang's theory of vacuum polarization by scalar field as well as Wesson's equivalence and connection between the scalar field of 5D gravity and the scalar field of 4D particle physics.

2 Index of refraction of the vacuum polarized by the scalar field

According to the 5D fully covariant gravity with a scalar field [15] and references therein, we can determine the index of refraction of the vacuum that is polarized by the scalar field as

$$n \equiv \sqrt{\epsilon_r} = \Phi^{3/2} \exp\left(\frac{\lambda - \nu}{4}\right). \tag{1}$$

Here, Φ is the scalar field and the functions, e^{λ} and e^{ν}, are the $rr-$ and $tt-$components of the 4D spacetime metric. Both the scalar field and the metric components are completely determined according to the exact field solution obtained by Zhang [15] and references therein without any unknown parameter.

For objects in labs and the Earth itself, the fields of 5D gravity are weak, so that we can approximately represent $\Phi \sim 1 + \delta\Phi$, $e^{\lambda} \sim 1$, and $e^{\nu} \sim 1$. Then, the index of refraction in the vacuum that is polarized by scalar fields reduces to

$$n = 1 + \frac{3}{2} \sum \delta\Phi = 1 + \frac{3}{2} \left(\delta\Phi_{\rm 5D} + \delta\Phi_{\rm GL}\right). \tag{2}$$

Here, Σ refers to the summation of contributions from all kinds of scalar fields, including the scalar fields of the 5D gravity from the Earth and any other charged objects and the Ginzburg-Landau scalar fields of the 4D particle physics from the Bose-Einstein condensates associated with superconductors and superfluids.

According to Zhang's fully covariant 5D gravity [15] and references therein such as [11,24], the scalar field of a charged object with charge Q and mass M is given by,

$$\delta\Phi_{\rm 5D} = \frac{2GM(1 + 3\alpha^2)}{3\sqrt{1 + \alpha^2 c^2}} \frac{1}{r}, \tag{3}$$

where

$$\alpha = \frac{Q}{2\sqrt{G}M} \tag{4}$$

is a constant in cgs units, G is the gravitational constant, c is the light speed in free space, and r is the radial distance from the object. Considering an object with mass of 600 kg and charge of 0.01 C, we have $\alpha \sim 10^5$ and $\delta\Phi_{\rm 5D} \sim 10^{-19}$ at 1 m radial distance. For Earth, we have $\alpha \sim 0$ and $\delta\Phi_{\rm 5D} \sim 5 \times 10^{-10}$ on the surface. Therefore, via Earth or a charged object in labs, the scalar field of the 5D gravity is negligibly weak, *i.e.* $\delta\Phi_{\rm 5D} \sim 0$, and the effect on the vacuum polarization may be extremely difficult to detect. A new study by Zhang [25] has theoretically shown that the space or vacuum polarization by the scalar field of 5D gravity generated by a highly charged object may be directly detected by the extremely accurate Laser Interferometer Gravitational-Wave Observatory (LIGO), which has recently detected first ever the gravitational waves from a binary black hole merger as claimed in [26].

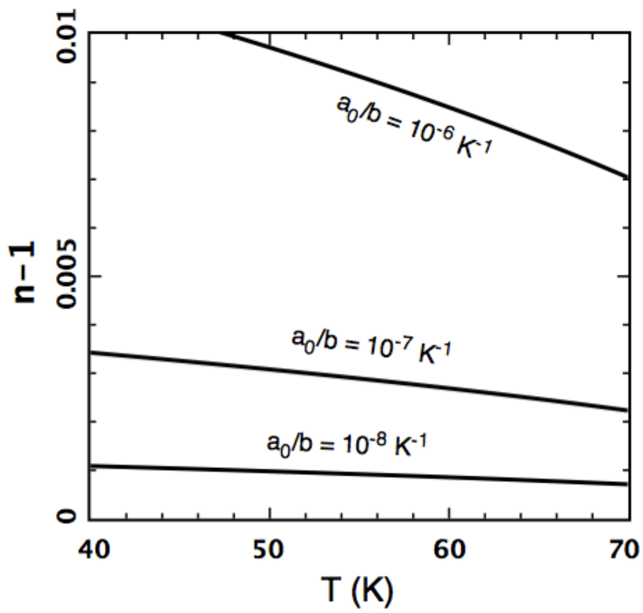

Fig. 1: The change for the index of refraction of the vacuum $(n - 1)$ versus the temperature of the superconductor (T). The vacuum is polarized by the Ginzburg-Landau scalar field of Bose-Einstein condensates associated with a type II superconductor whose transition temperature is $T_c = 92$ K. Three lines correspond to three cases for the ratio of the two phenomenological constants to be $a_0/b = 10^{-8}, 10^{-7}, 10^{-6}$ K, respectively.

The Ginzburg-Landau scalar field of Bose-Einstein condensates associated with superconductors and superfluids can be expressed as [20-21,27],

$$\delta\Phi_{\rm GL} = \sqrt{-\frac{a_0}{b}\left(T - T_c\right)}, \tag{5}$$

where a_0 and b are the phenomenological constants, T is the temperature, and T_c is the transition temperature. A type II superconductor, if its Ginzburg-Landau scalar field can produce a few percent (*e.g.* 2 – 3%) weight loss for a sample as experimentally shown by [22-23], can also polarize the vacuum by increasing the index of refraction about a detectable percentage. For a quantitative study, we plot in Fig. 1 the index of refraction in the vacuum that is polarized by the Ginzburg-Landau scalar field of Bose-Einstein condensates associated with a type II superconductor as a function of the temperature of the superconductor. In this plot, we have chosen the values $T_c = 92$ K and $a_0/b = 10^{-8}, 10^{-7}, 10^{-6}$ K^{-1} as done in [12].

It is seen that due to the polarization the index of refraction of the vacuum can be increased by $\sim 0.1 - 1\%$ for the ratio of the phenomenological constants to be in a range of $a_0/b = 10^{-8} - 10^{-6}$ K^{-1}, which could lead to ~2 – 3% weight loss for a sample as shown in [12]. This significant increase of the index of refraction should be detectable in an optical experiment. In the following section, we design an experiment to test this scalar field effect on light or space polarization here predicted according to Zhang's 5D fully covariant gravity and Wesson's scalar field equivalence or connection between the 5D gravity and the 4D particle physics. Superfluids, though the transition temperature is lower but if the ratio of phenomenological constants is higher, can also generate a significant scalar field to polarize the vacuum.

3 Experimental design and prediction

A laser light beam that has passed through a spatial filter can be separated into two beams by a beam separator. These two laser light beams once reflected by two mirrors into the same region will interfere. If the difference of their optical distances travelled by the two beams is a factor of a whole number of the light wavelength, the interference is constructive otherwise the interference is destructive. A bright and dark pattern of interference is formed in the interference region. Now, if one of the two laser light beams passes through the space or vacuum that is polarized by scalar fields, then the interference pattern will be changed. This is because the space polarization lengthens the optical length of the path of the light beam.

The interference pattern will change from bright to dark or dark to bright, if the extra optical distance traveled for the beam that has passed through the space or vacuum polarized by scalar fields is given by

$$(n-1)\,D = \left(m + \frac{1}{2}\right)\lambda\,, \tag{6}$$

where n is the index of refraction of the space or vacuum that is polarized by the scalar field and its relation to the scalar field is given by (1) or (2); D is the dimension of the object that produces the scalar field; $m + 1$ is the number of shifting the interference pattern from bright to dark (only one shift from bright to dark if $m = 0$); and λ is the wavelength of the laser light. The interference pattern does not change, if the extra optical distance is a whole number of the light wavelength, *i.e.* $(n-1)\,D = m\lambda$.

To polarize the space or vacuum that one of the two laser light beams travels through, we can place or put an electrically charged object, a type II ceramic superconductor disk, or a superfluid torus near the path of the beam (Fig. 2). Of course, we can put all of them together to enhance the total scalar field. Two superconductor disks can also double the effect. In these cases, the parameter D in (6) can be roughly estimated as the diameter of the charged object, superconductor disk, or superfluid torus.

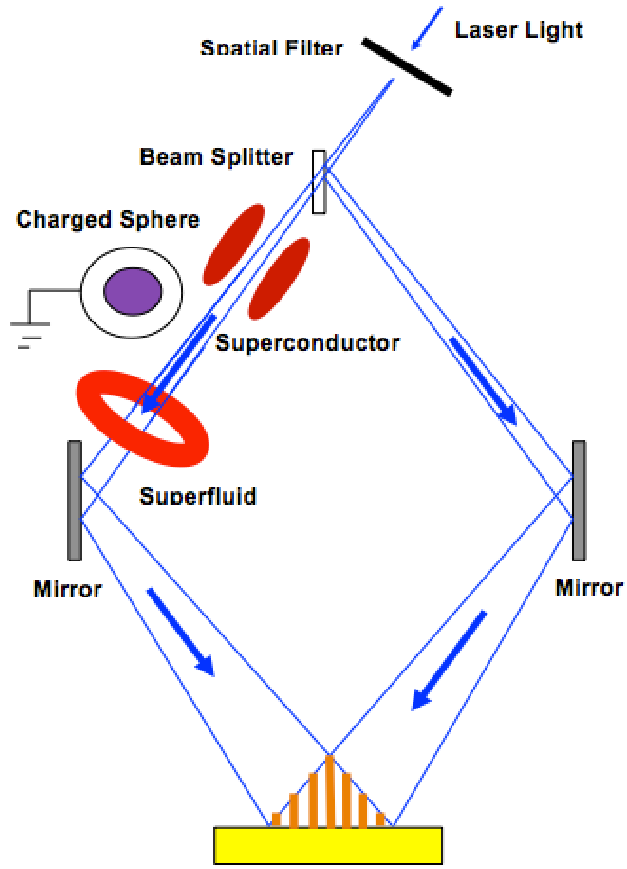

Fig. 2: A schematic diagram for the experimental setup to test the vacuum polarization by scalar field. A laser light that passes a spatial filter can be separated into two beams by a beam separator. The two beams once reflected by two mirrors into the same region will interfere and produce a bright-dark interference pattern. When the space or vacuum for the path of one beam is polarized by the scalar field generated by charged objects, superconductor disks, and/or superfluid toruses, the interference pattern will be varied or shifted. Therefore, the detection of any variation or shifting of the interference pattern will test the theory for the vacuum polarization by scalar field and the equivalence or connection for the scalar fields of 5D gravity and 4D particle physics.

As pointed out above, since it is not enough compact, massive, and/or highly charged, an object in labs cannot generate a significant scalar field to polarize the space or vacuum up to a detectable level, but except for LIGO [25-26]. The extra optical distance that a charged object can produce is $(n-1)\,D = 3/2\,\delta\Phi_{5D}D \sim 10^{-19}$, which is too small in comparison with the wavelength of light. Therefore, a charged object cannot lead to a measurable shifting of the interference pattern. The scalar field of 5D gravity due to the Earth can neither vary the interference pattern, because it evenly affects both the beams of laser light.

To see how significant for a type II ceramic supercon-

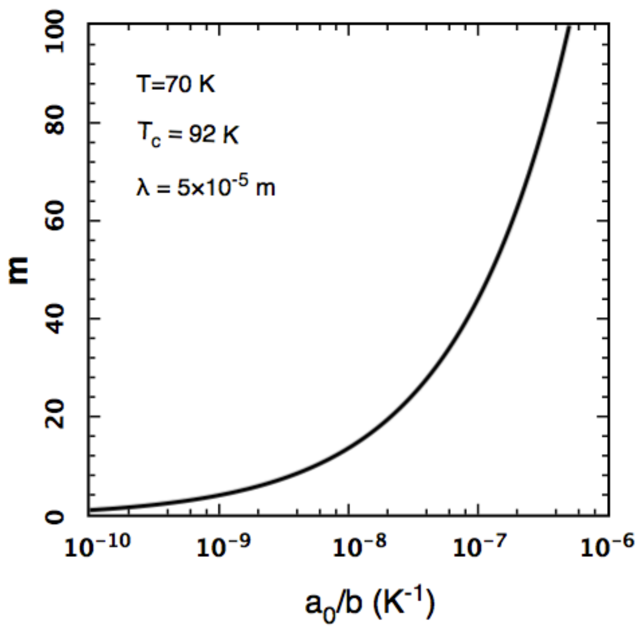

Fig. 3: The number of shifting the interference pattern from bright to dark, m, versus the ratio of the phenomenological constants, a_0/b. The temperature and transition temperature of the conductor are chosen as = 70 K and T_c = 92 K, respectvely.

ductor disk to vary the interference pattern, we plot in Fig. 3 the number of shift, m, as a function of the ratio of the phenomenological constants, a_0/b, according to (6) with (2) and (5). Here, we have chosen D = 0.11 m, T_c = 92 K, and T = 70 K according to the previous laboratory experiment [22] and analytical study [12]. The wavelength is chosen as a blue light with $\lambda \sim 5 \times 10^{-5}$ m. It is seen that the Ginzburg-Landau scalar field of Bose-Einstein condensates associated with a type II ceramic superconductor disk can lead to a significant shifting of the interference pattern. This varying of interference pattern is detectable only needing the ratio of the phenomenological constants to be greater than about 10^{-10} K^{-1}. Therefore, the effect of scalar field on light (or the space polarization) should be much more easily detected than the effect of scalar field on weight (or the gravitational field shielding).

4 Discussions and conclusions

We have investigated the vacuum polarization by the Ginzburg-Landau scalar field of Bose-Einstein condensates associated with superconductors and superfluids. First, we have formulated the index of refraction of the vacuum that is polarized by the scalar field according to Zhang's 5D fully covariant gravity and Wesson's equivalence or connection of scalar fields between 5D gravity and 4D particle physics. Then, we have designed an experimental setup with laser light interferences to detect the effect of scalar field on light and hence the vacuum polarization by the Ginzburg-Landau scalar field. Via this study, we have seen that the Ginzburg-Landau scalar field of Bose-Einstein condensates associated with a type II ceramic superconductor disk can cause a significant and thus detectable shifting of the laser light interference pattern. The ratio of the phenomenological constants can be much smaller than that for a detectable weight loss of a sample. Therefore, we have provided a possible approach and experimental setup for detecting the effect of scalar field on light in labs. In future, we will implement the design to conduct the experiment and perform the testing.

Acknowledgements

The author (B.J.Z.) is grateful for the support of educations in physics and engineering from Vanderbilt University, Georgia State University, and University of Alabama in Huntsville. He is especially appreciating very much for Dr. A. Kozhanov from Georgia State University and Dr. P. Reardon from University of Alabama in Huntsville for their great supervisions on his graduate study for Masters of Physics (2015) and Masters of Electrical Engineering in the concentration of optics (2017). He also thanks Dr. P. Guggilla, Dr. M. Edwards, and Dr. M. Aggarwal from Alabama A & M University for their support. This work was partially supported by the NSF/REU programs (Grant #: PHY-1263253, PHY-1559870) at Alabama A & M University.

Received on September 16, 2017

References

1. Guth A. Inflationary universe: A possible solution to the horizon and flatness problems. *Physical Review D*, 1981, v. 23, 347–356.
2. Watson G. S. An explosion of inflationary cosmology. arXiv: astro-ph/0005003.
3. Baldeschi M. R., Gelmini G. B., Ruffini R. On massive femions and bosons in galactic halos. *Physics Letters B*, 1983, v. 122, 221–224.
4. Matos T., Urena-Lepez L. A. Quintessence abd scalar dark matter in the universe. *Classical and Quantum Gravity*, 2000, v. 17, L75–L81.
5. Ratra B., Peebles P. J. E. Cosmological consequences of a rolling homogeneous scalar field. *Physical Review D*, 1988, v. 37, 3406–3427.
6. Sahni V. The cosmological constant problem and quintessence. *Classical and Quantum Gravity*, 2002, v. 19, 3435–3448.
7. Higgs P. W. Broken symmetries, massless particles and gauge fields. *Physics Letters*, 1964, v. 12, 132–133.
8. Higgs P. W. Broken symmetries and the masses of gauge bosons. *Physical Review Letters*, 1964, v. 13, 508–509.
9. Englert F., Brout R. Broken symmetry and the mass of gauge vector mesons. *Physical Review Letters*, 1964, v. 13, 321–323.
10. Sahni V., Habib S. Does inflationary particle production suggest $\Omega_m <$ 1. *Physical Review Letters*, 1998, v. 81, 1766–1769.
11. Zhang T. X. Gravitational field shielding and supernova explosions. *The Astrophysical Journal Letters*, 2010, v. 725, L117–L120.
12. Zhang B. J., Zhang T. X., Guggilla P., Dokhanian M. Gravitational field shielding by scalar field and type II superconductors. *Progress in Physics*, 2013, v. 1, 69–73.
13. Nodvik J. S. Suppression of singularities by the g^{55} field with mass and classical vacuum polarization in a classical Kaluza-Klein theory. *Physical Review Letters*, 1985, v. 55, L2519–L2522.

14. Dragilev V. M. Vacuum polarization of a scalar field in anisotropic multidimensional cosmology. *Theoretical Mathematics in Physics*, 1990, v. 84, 887–893.
15. Zhang T. X. The 5D fully-covariant theory of gravitation and its astrophysical application, *Galaxies*, 2014, v. 3, 18–51.
16. Zhang B. J., Zhang T. X., Guggilla P., Dokhanian M. Neutron star mass-radius relation with gravitational field shielding by a scalar field. *Research in Astronomy and Astrophysics*, 2013, v. 13, 571–578.
17. Zhang T. X. Gravitationless black holes. *Astrophysics and Space Science*, 2011, v. 334, 311–316.
18. Wesson P. S. The scalar field of 5D gravity and the Higgs field of 4D particle physics: A possible connection. arXiv:1003.2476.
19. Cho A. The discovery of the Higgs boson. *Science*, 2012, v. 338, 1524–1525.
20. Ginzburg V. I., Landau L. D. On the theory of superconductivity. *Zh. Eksp. Teor. Fiz.*, 1950, v. 20, 1064–1082.
21. Cyrot M. Ginzburg-Landau theory for superconductors. *Reports on Progress in Physics*, 1973, v. 36, 103–158.
22. Podkletnov E., Nieminen R. A possibility of gravitational force shielding by bulk $YBa_2Cu_3O_{7-x}$. *Physica C*, 1992, v. 203, 441–444.
23. Li N., Noever D., Robertson T., Koczer R., Brantley W. Static test for a gravitational force coupled to type II YBCO superconductor. *Physica C*, 1997, v. 281, 260–267.
24. Zhang T. X. Electric redshift and quasar. *The Astrophysical Journal Letters*, 2006, v. 636, L61–L63.
25. Zhang T. X. Testing 5D Gravity for Space Polarization by Scalar field with LIGO. *Progress in Physics*, 2017, v. 13, 180–186.
26. Abbott B. P. *et al.* Observation of gravitational waves from a binary black hole merger. *Physical Review Letters*, 2016, v. 116, id.061102.
27. Robertson G. A. Manipulating the vacuum scalar field with superconductor: A search for exotic material. *AIP Conference Proceedings*, 2005, v. 746, 1371–1378.

On the Question of Acceleration in Special Relativity

Pierre A. Millette

E-mail: PierreAMillette@alumni.uottawa.ca, Ottawa, Canada

In this paper, we consider the question of the impact of acceleration in special relativity. Some physicists claim that acceleration does not matter in special relativity based on the Clock Hypothesis. We find that the experimental support of the Clock Hypothesis usually provided by the Mössbauer spectroscopy experiment of Kündig [5] and the muon experiment of Bailey *et al* [2] is questionable at best. We consider the case for the impact of acceleration in special relativity and derive an expression for the time dilation in an accelerated frame of reference, based on the equivalence principle of general relativity. We also derive an expression for space contraction in an accelerated frame of reference. We note that the presence of acceleration in a frame of reference provides a means of determining the motion of that frame of reference as acceleration can be easily detected compared to constant velocity which cannot. We discuss the "twin paradox" of special relativity and note that this is not truly a special relativity problem for there is no way to avoid acceleration. We note that because of time dilation in accelerated frames of reference, the astronaut will age less than its earth-bound twin, but only during periods of acceleration.

1 Introduction

In a recent paper [1], we showed that time dilation and space contraction in inertial reference frames, that is unaccelerated reference frames moving at a constant velocity, are apparent effects perceived in a frame of reference moving with respect to an object of interest. The real physical time and length are in the frame of reference at rest with the object, and in that frame, there is no time dilation or space contraction as $v = 0$ (and acceleration $a = 0$). This is seen clearly in Fig. 1 where a time dilation is perceived in the frame of reference moving at speed v with respect to the object of interest ($\Delta t'$), while there is no dilation in the object's frame of reference (Δt).

This result would seem to be at odds with the often quoted experimental tests of special relativity confirming time dilation and length contraction. But if we consider, for example, Bailey *et al*'s muon experiment [2], we find that there is no contradiction with the experimental observations: a perceived time dilation is observed in the Earth's laboratory frame of reference while the muon, in its frame of reference has no time dilation – note that no measurements were carried out in the muon's frame of reference in the Bailey experiment.

Careful examination of experimental tests of special relativity also often reveals the presence of acceleration in the experiments, contrary to the conditions under which special relativity applies. The question of how to deal with acceleration in special relativity underlies many of the analytical and experimental conundrums encountered in the theory and is investigated in more details in this paper.

2 Measuring the impact of acceleration in special relativity

The theory of special relativity applies to unaccelerated (constant velocity) frames of reference, known as inertial frames of reference, in a four-dimensional Minkowski spacetime [3], of which the three-dimensional Euclidean space is a subspace. When the Lorentz-Fitzgerald contraction was first introduced, it was considered to be a real physical effect in Euclidean space to account for the null results of the Michelson-Morley experiment. Einstein derived length contraction and time dilation as effects originating in special relativity. These depend on the velocity of the frame of reference with respect to which an object is being observed, not the object's velocity

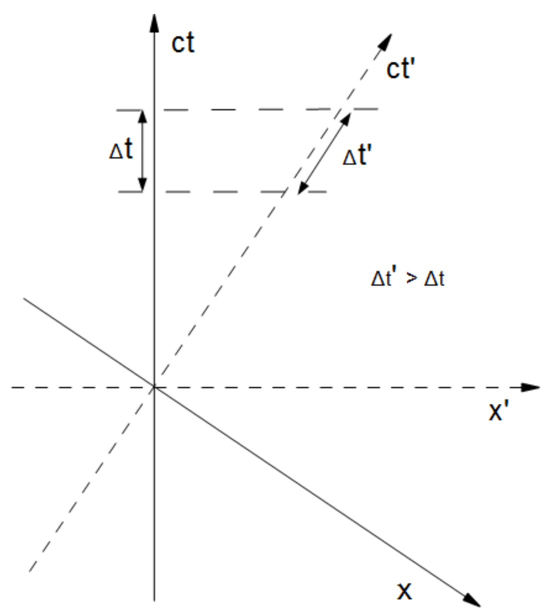

Fig. 1: Physical explanation of time dilation in a Loedel *space−ct* diagram

which can only be relative to another frame of reference, as there is no absolute frame of reference against which to measure the object's velocity. Indeed, if time dilation and length contraction were real effects in special relativity, this would be equivalent to saying that there is an absolute frame of reference against which it is possible to measure an object's velocity, contrary to the theory.

Increasingly, special relativity has been applied to accelerated frames of reference for which the theory does not apply. Some physicists claim that acceleration does not matter in special relativity and that it has no impact on its results, but there are many indications that this is not the case. The Clock Hypothesis (or Postulate) is used to justify the use of accelerated frames in special relativity: "when a clock is accelerated, the effect of motion on the rate of the clock is no more than that associated with its instantaneous velocity – the acceleration adds nothing" [4, p. 9], and further postulates that if the Clock Hypothesis applies to a clock, " then the clock's proper time will be proportional to the Minkowski distance along its worldline" [4, p. 95] as required.

Two experimental confirmations of the Clock Hypothesis are usually given. The postulate is claimed to have been shown to be true for accelerations of $\sim 10^{16} g$ in a Mössbauer spectroscopy experiment by Kündig [5] and of $\sim 10^{18} g$ in Bailey *et al*'s muon experiment [2], which uses rotational motion of particles to generate the acceleration – one obtains the quoted acceleration for a particle velocity close to the speed of light. However, a close examination of these experiments shows that they don't quite provide the experimental confirmation they are purported to give.

Kholmetskii *et al* [6] reviewed and corrected the processing of Kündig's experimental data and obtained an appreciable difference of the relative energy shift $\Delta E/E$ between emission and absorption resonant lines from the predicted relativistic time dilation $\Delta E/E = -v^2/2c^2$ (to order c^{-2}), where v is the tangential velocity of the resonant radiation absorber. Writing the relative energy shift as $\Delta E/E = -k\,v^2/c^2$, they found that $k = 0.596 \pm 0.006$ instead of $k = 0.5$ as predicted by special relativity and Kündig's original reported result of $k = 0.5003 \pm 0.006$. They then performed a similar Mössbauer spectroscopy experiment [7] with two absorbers with a substantially different isomer shift to be able to correct the Mössbauer data for vibrations in the rotor system at various rotational frequencies. They obtained a value of $k = 0.68 \pm 0.03$, a value similar to $2/3$. Since then Kholmetskii and others [8–12] have performed additional experimental and theoretical work to try to explain the difference, but the issue remains unresolved at this time, and is a clear indication that acceleration is not compatible with special relativity.

In their experiment of the measurement of the lifetime of positive and negative muons in a circular orbit, Bailey *et al* [2] obtained lifetimes of high-speed muons which they then reduced to a mean proper lifetime at rest, assuming that special relativity holds in their accelerated muon experimental setup. This experiment was carried out at CERN's second Muon Storage Ring (MSR) [13, 14] which stores relativistic muons in a ring in a uniform magnetic field. The MSR was specifically designed to carry out muon $(g-2)$ precession experiments (g is the Landé g-factor) with muons of momentum 3.094 GeV/c corresponding to a γ-factor of 29.3 (effective relativistic mass [1]), so that the electrons emitted from muon decay in the lab frame were very nearly parallel to the muon momentum. The decay times of the emitted electrons were measured in shower counters inside the ring to a high precision, and the muon lifetimes in the laboratory frame were calculated by fitting the experimental decay electron time spectrum to a six-parameter exponential decay modulated by the muon spin precession frequency, using the maximum likelihood method – one of the six parameters is the muon relativistic lifetime.

It is important to note that the decay electrons would be ejected at the instantaneous velocity of the muon ($0.9994c$ from the $\gamma = 29.3$ factor) tangential to the muon's orbit. Thus the ejected electron moves at the constant velocity of ejection to the shower counter and acceleration does not play a role. Even though the muons are accelerated, the detected electrons are not, and the experiment is not a test of the Clock Hypothesis under acceleration as claimed. There is thus no way of knowing the impact of acceleration from the experimental results as acceleration is non-existent in the detection and measurement process.

It should also be noted that Hafele *et al* [17] in their time dilation "twin paradox" experiment applied a correction for centripetal acceleration to their experimental results. in addition to a gravitational time dilation correction, to obtain results in agreement with Lorentz time dilation. The effect of acceleration cannot be disregarded in that experiment. This will be considered in more details in section 4. We thus find that the experimental support of the Clock Hypothesis is questionable at best.

3 The case for the impact of acceleration in special relativity

Having determined that there is little experimental support for the validity of the Clock Hypothesis in accelerated frames of reference in special relativity, we consider the case for the impact of acceleration in special relativity. Einstein developed general relativity to deal with accelerated frames of reference – if acceleration can be used in special relativity, why bother to develop a more general theory of relativity? Inspection of an accelerated worldline in a Minkowski *space-ct* diagram shows that indeed there is no basis for the Clock Hypothesis, as seen in Fig. 2. The accelerated worldline suffers an increasing rate of time dilation, somewhat like gravitational time dilation where increasing height in the gravitational potential results in increasing time dilation.

This brings to mind Einstein's equivalence principle introduced in the analysis of accelerated frames of reference in general relativity. The simplest formulation of this principle states that on a local scale, the physical effects of a gravitational field are indistinguishable from the physical effects of an accelerated frame of reference [15] (*i.e.* an accelerated frame of reference is locally equivalent to a gravitational field). Hence, as displayed graphically for the accelerated worldline in the Minkowski *space-ct* diagram of Fig. 2, an accelerated frame of reference undergoes time dilation similar to gravitational time dilation [15]. Indeed, assuming that acceleration has no impact in special relativity cannot be correct as it violates the equivalence principle of general relativity.

We explore the connection between gravitational time dilation and the time dilation in an accelerated frame of reference in greater details. Gravitational time dilation can be derived starting from the Schwarzschild metric with signature (+ - - -) [16, p. 40]

$$c^2 d\tau^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - \\ - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right) , \tag{1}$$

where τ is the proper time, (r, θ, φ, t) are the spherical polar coordinates including time, G is the gravitational constant, M is the mass of the earth and c is the speed of light in vacuo. The gravitational time dilation is obtained from the dt^2 term to give

$$\Delta t = \left(1 - \frac{2GM}{rc^2}\right)^{-\frac{1}{2}} \Delta t_0 , \tag{2}$$

where Δt_0 is the undilated (proper) time interval and Δt is the dilated time interval in the earth's gravitational field. This can be rewritten as

$$\Delta t = \left(1 - \frac{2GMr}{r^2 c^2}\right)^{-\frac{1}{2}} \Delta t_0 , \tag{3}$$

where the term GM/r^2 is an acceleration a equal to g for $r = R$, the earth's radius, and finally

$$\Delta t = \left(1 - \frac{2ar}{c^2}\right)^{-\frac{1}{2}} \Delta t_0 . \tag{4}$$

By the equivalence principle, this is also the time dilation in an accelerated frame of reference. For small accelerations, using the first few terms of the Taylor expansion, this time dilation expression can be written as

$$\Delta t \simeq \left(1 + \frac{ar}{c^2}\right) \Delta t_0 . \tag{5}$$

The impact of acceleration on time dilation for small acceleration will usually be small due to the c^{-2} dependency.

We note in particular the expressions for centripetal acceleration $a = v^2/r$ in the case of circular motion

$$\Delta t = \left(1 - \frac{2v^2}{c^2}\right)^{-\frac{1}{2}} \Delta t_0 , \tag{6}$$

which becomes for small accelerations, again using the first few terms of the Taylor expansion,

$$\Delta t \simeq \left(1 + \frac{v^2}{c^2}\right) \Delta t_0 . \tag{7}$$

In this case, the impact can be significant, of the same order as the relativistic Lorentz time dilation. Hence there is no doubt that accelerated frames of reference also undergo time dilation compared to unaccelerated (inertial) frames of reference.

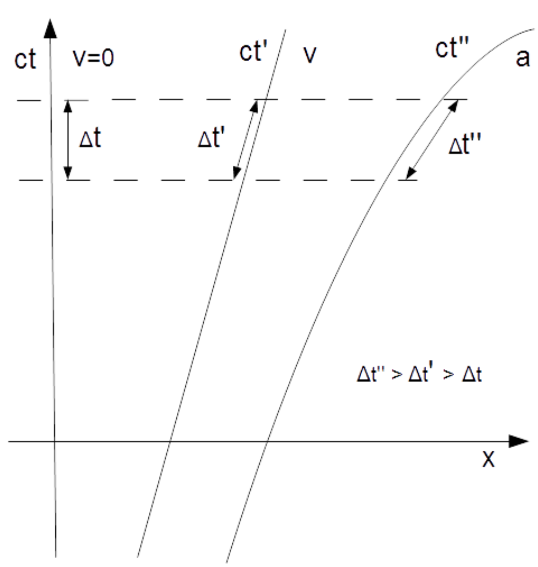

Fig. 2: Physical explanation of an accelerated worldline in a Minkowski *space−ct* diagram

4 The consequences of acceleration in special relativity

The presence of acceleration in a frame of reference provides a means of determining the motion of that frame of reference as acceleration can be easily detected compared to constant velocity which cannot. Whereas in an inertial frame of reference there is no way of determining one's velocity, this limitation disappears in accelerated frames of reference.

Physical time dilation due to acceleration is a reality, as is physical space contraction, which, from (1), is seen to have the inverse of the functional form of (4), to give the acceleration space contraction relation

$$\Delta x = \left(1 - \frac{2ar}{c^2}\right)^{\frac{1}{2}} \Delta x_0 \tag{8}$$

which for small accelerations, using the first few terms of the Taylor expansion, becomes

$$\Delta x \simeq \left(1 - \frac{ar}{c^2}\right) \Delta x_0 . \tag{9}$$

Till now, we have not discussed the so-called "twin paradox" of special relativity. This is not truly a paradox for there is no way to avoid acceleration in the problem and it is thus not a special relativity problem. Assume that by some miracle we have twins moving at constant velocity with respect to one another from departure to return with no acceleration and that they are able to compare their age. It is important to notice that in their inertial frames of reference, both proper times $d\tau$, the one in the frame of reference at rest with the earth, and the one in the frame of reference at rest with the spaceship, are equal to the physical time in both the frame of reference at rest with the earth and the frame of reference at rest with the spaceship. From the earth, it looks like the spaceship's time is dilated, and from the spaceship, it looks like the earth's time is dilated. It doesn't matter as the time dilation in one location as seen from the other location is apparent as seen in [1]. When the spaceship comes back to earth, the twins would see that indeed they have the same age.

The problem can be recast in a simpler fashion. Suppose instead of the earth and a spaceship, we have two spaceships moving at constant relativistic speed with respect to one another from start to finish with no acceleration, and that the twins are able to compare their age at the start and the finish. One spaceship moves slowly because of engine problems, while the other moves at relativistic speeds. The resolution would be as described in the previous paragraph: the twins would see that indeed they have the same age at the finish.

The complication in this problem is that forces have to be applied to accelerate the spaceship, then decelerate it to turn around, accelerate it again and finally decelerate it when it comes back to the earth. The problem then needs to be treated using accelerated frames of reference for those periods on the spaceship. As we have seen in section 3, because of time dilation in accelerated frames of reference, the astronaut will age less than its earth-bound twin, but only during periods of acceleration. During periods of unaccelerated constant velocity travel, there will be no differential aging between the twins. However, the earth-bound twin is itself in an accelerated frame of reference the whole time, so its time will also be dilated. The details of who is older and younger will depend on the details of the acceleration periods, with the earth-bound twin's time dilation depending on (2) and (6), and the spaceship-bound twin's time dilation depending on (4).

Comparing how these findings line up with the results of Hafele's circumglobal experiment [17, 18], it is important to note that Hafele's experiment was done the whole time in a non-inertial accelerated frame of reference. Its results were corrected for gravitational time dilation and centripetal acceleration time dilation, the latter correction clearly showing that acceleration has an impact on special relativity. The centripetal acceleration time dilation correction used by Hafele *et al* [17] is similar to (6). One side effect of the experiment being conducted in gravitational and accelerated frames of reference is that it was possible to determine their motion, contrary to special relativity. The Lorentz time dilation would then become a real effect in this purported test of the "twin paradox". There was no symmetry in the relative motions that would have seen the plane stationary and the earth moving given that gravitational and centripetal accelerations clearly showed who was moving and at what velocity.

5 Discussion and conclusion

In this paper, we have considered the question of the impact of acceleration in special relativity. Some physicists claim that acceleration does not matter in special relativity – this view is part of the Clock Hypothesis which is used to justify the use of accelerated frames in special relativity. We have found that the experimental support of the Clock Hypothesis usually provided by the Mössbauer spectroscopy experiment of Kündig [5] and the muon experiment of Bailey *et al* [2] is questionable at best.

We have considered the case for the impact of acceleration in special relativity and have derived an expression for the time dilation in an accelerated frame of reference, based on the equivalence principle of general relativity. We have also derived an expression for space contraction in an accelerated frame of reference.

As a consequence, we have noted that the presence of acceleration in a frame of reference provides a means of determining the motion of that frame of reference as acceleration can be easily detected compared to constant velocity which cannot – whereas in an inertial frame of reference there is no way of determining one's velocity, this limitation disappears in accelerated frames of reference.

We have discussed the "twin paradox" of special relativity and have noted that this is not truly a paradox for there is no way to avoid acceleration in the problem and it is thus not a special relativity problem. We have noted that because of time dilation in accelerated frames of reference, the astronaut will age less than its earth-bound twin, but only during periods of acceleration, while during periods of unaccelerated constant velocity travel, there will be no differential aging between the twins. However, as the earth-bound twin is itself in an accelerated frame of reference the whole time, the details of who is older and who is younger will depend on the details of the acceleration periods of both twins. Finally we have reviewed how these findings line up with the results of Hafele's circumglobal experiment [17, 18] and find no contradiction.

Received on September 28, 2017

References

1. Millette P. A. On Time Dilation, Space Contraction and the Question of Relativistic Mass. *Progress in Physics*, 2017, v. 13, 200–203.
2. Bailey J., Borer K., Combley F., Drumm H., Krienen F., Lange F., Picasso E., von Ruden W., Farley F. J. M., Field, J. H., Flegel, W., Hattersley P. M. Measurements of Relativistic Time Dilatation for Positive and Negative Muons in a Circular Orbit. *Nature*, 1977, v. 268, 301–305.

3. Minkowski H. Space and Time. 80^{th} *Assembly of German Natural Scientists and Physicians*. Cologne, 21 September 1908. English translation reprinted in Lorentz H. A., Einstein A., Minkowski H, and Weyl H. The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity. Dover Publications, New York, 1952, pp. 73–91.
4. Brown H. Physical Relativity: Spacetime Structure from a Dynamical Perspective. Oxford University Press, Oxford, 2005.
5. Kündig W. *Phys. Rev.*, 1963, v. 129, 2371.
6. Kholmetskii A. L., Yarman T. and Missevitch O. V. Kündig's Experiment on the Transverse Doppler Shift Re-analyzed. *Phys. Scr.*, 2008, v. 77, 035302–035306.
7. Kholmetskii A. L., Yarman T., Missevitch O. V. and Rogozev B. I. A Mössbauer Experiment in a Rotating System on the Second-order Doppler Shift: Confirmation of the Corrected Result by Kündig. *Phys. Scr.*, 2009, v. 79, 065007.
8. Kholmetskii A. L., Yarman T., Missevitch O. V. and Rogozev B. I. Mössbauer Experiments in a Rotating System on the Time Dilation Effect. *Int. J. Phys. Sci.*, 2011, v. 6, 84–92.
9. Yarman T., Kholmetskii A. L., Arik M., Akku, B., Öktem Y., Susam A., Missevitch O. V. Novel Mössbauer Experiment in a Rotating System and the Extra Energy Shift Between Emission and Absorption Lines. arXiv: physics.gen-ph/1503.05853.
10. Yarman T., Kholmetskii A. L., Arik M. Mössbauer Experiments in a Rotating System: Recent Errors and Novel Interpretation. *The European Physical Journal Plus*, 2015, v. 130, 191.
11. Corda C. The Mössbauer Rotor Experiment and the General Theory of Relativity. arXiv: gr-qc/1602.04212.
12. Kholmetskii A. L., Yarman T., Yarman O., Arik M. Unabridged Response to "The Mössbauer Rotor Experiment and the General Theory of Relativity" by C. Corda: General Relativity Cannot Supply an Answer to the Extra Time Dilation in Rotor Mössbauer Experiments. arXiv: physics.gen-ph/1610.04219.
13. Combley F., Farley F. J. M., and Picasso E. The CERN Muon $(g-2)$ Experiments. *Physics Reports (Review Section of Physics Letters)*, 1981, v. 68 (2), 93–119.
14. Miller J. P., de Rafael E., and Roberts B. L. Muon $(g-2)$: Experiment and Theory. *Rep. Prog. Phys.*, 2007, v. 70, 7953–881.
15. Nolan P.J. The Fundamentals of the Theory of Relativity. Farmingdale State College, New York, pp. 7-2, 7-13.
16. Ciufolini I. and Wheeler J. A. Gravitation and Inertia. Princeton University Press, Princeton, NJ, 1995.
17. Hafele J. C. and Keating R. E. Around-the-World Atomic Clocks: Predicted Relativistic Time Gains. *Science*, 1972, v. 177, 166–168.
18. Hafele J. C. and Keating R. E. Around-the-World Atomic Clocks: Observed Relativistic Time Gains. *Science*, 1972, v. 177, 168–170.

Theorem of Non-Returning and Time Irreversibility of Tachyon Kinematics

Yaroslav I. Grushka
Institute of Mathematics NAS of Ukraine, 3, Tereschenkivska st., 01601, Kyiv, Ukraine.
E-mail: grushka@imath.kiev.ua

Using the recently developed mathematical apparatus of the theory of universal kinematic sets, we prove that the hypothesis of the existence of material objects and inertial reference frames moving with superluminal velocities in the general case does not lead to the violation of the principle of causality, that is, to a possibility of the returning to the own past. This result is obtained as the corollary of the abstract theorem on irreversibility, which gives the sufficient condition of time irreversibility for universal kinematic sets.

1 Introduction

Subject of constructing the theory of super-light movement, had been posed in the papers [1, 2] more than 50 years ago. Despite the fact that on today tachyons (ie objects moving at a velocity greater than the velocity of light) are not experimentally detected, this subject remains being actual.

It is well known that among physicists it is popular the belief that the hypothesis of the existence of tachyons leads to temporal paradoxes, connected with the possibility of changing the own past. Conditions of appearing these time paradoxes were carefully analyzed in [3]. It should be noted, that in [3] superluminal motion is allowed only for particles or signals whereas superluminal motion for reference frames is forbidden. This fact does not give the possibility to bind the own time with tachyon particle, and, therefore to determine real direction of motion of the particle. In the paper [4] for tachyon particles the own reference frames are axiomatically introduced only for the case of one space dimension. Such approach allows to determine real direction of motion of the tachyon particle by more correct way, and so to obtain more precise results.

In particular, in the paper [4] it was shown, that the hypothesis of existence of material objects, moving with the velocity, greater than the velocity of light, does not lead to formal possibility of returning to the own past in general. Meanwhile in the papers of E. Recami, V. Olkhovsky and R. Goldoni [5–7], and and later in the papers of S. Medvedev [8] as well as J. Hill and B. Cox [9] the generalized Lorentz transforms for superluminal reference frames are deduced in the case of three-dimension space of geometric variables. In the paper [10] it was proven, that the above generalized Lorentz transforms may be easy introduced for the more general case of arbitrary (in particular infinity) dimension of the space of geometric variables.

Further, in [11], using theory of kinematic changeable sets, on the basis of the transformations [10], the mathematically strict models of kinematics, allowing the superluminal motion for particles as well as for inertial reference frames, had been constructed. Thus, the tachyon kinematics in the sense of E.Recami, V. Olkhovsky and R. Goldoni are surely mathematically strict objects. But, these kinematics are impossible to analyze on the subject of time irreversibility (that is on existence the formal possibility of returning to the own past), using the results of the paper [4], because in [4] complete, multidimensional superluminal reference frames are missing.

Moreover, it can be proved, that the axiom "AxSameFuture" from [4, subsection 2.1] for these tachyon kinematics is not satisfied. The paper [12] [1] is based on more general mathematical apparatus in comparison with the paper [4], namely on mathematical apparatus of the theory of kinematic changeable sets. In [12] the strict definitions of time reversibility and time irreversibility for universal kinematics were given, moreover in this paper it was proven, that all tachyon kinematics, constructed in the paper [11], are time reversible in principle. In connection with the last fact the following question arises:

Is it possible to build the certainly time-irreversible universal kinematics, which allows for reference frames moving with any speed other than the speed of light, using the generalized Lorentz-Poincare transformations in terms of E. Recami, V. Olkhovsky and R. Goldoni?

In the present paper we prove the abstract theorem on non-returning for universal kinematics and, using this theorem, we give the positive answer on the last question.

For further understanding of this paper the main concepts and denotation system of the theories of changeable sets, kinematic sets and universal kinematics, are needed. These theories were developed in [11, 13–17]. Some of these papers were published in Ukrainian. That is why, for the convenience of readers, main results of these papers were "converted" into English and collected in the preprint [18], where one can find the most complete and detailed explanation of these theories. Hence, we refer to [18] all readers who are not familiar with the essential concepts. So, during citation of needed main results we sometimes will give the dual reference of these results (in one of the papers [11, 13–17] as well as in [18]).

[1] Note, that main results of the paper [12] were announced in [19].

2 Elementary-time states and changeable systems of universal kinematics

Definition 1. *Let $\mathcal{F}$ be any universal kinematics* [1]*, $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ be any reference frame of $\mathcal{F}$ and $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ be any elementary-time state in the reference frame $\mathfrak{l}$. The set*

$$\omega^{\{\mathfrak{l},\mathcal{F}\}} = \{(\mathfrak{m}, \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega) \mid \mathfrak{m} \in \mathcal{L}k(\mathcal{F})\}$$

(where (x, y) is the ordered pair, composed of x and y) is called by ***elementary-time state of the universal kinematics*** *$\mathcal{F}$, generated by ω in the reference frame $\mathfrak{l}$.*

Remark 1. In the case, where the universal kinematics $\mathcal{F}$ is known in advance, we use the abbreviated denotation $\omega^{\{\mathfrak{l}\}}$ instead of the denotation $\omega^{\{\mathfrak{l},\mathcal{F}\}}$.

Assertion 1. *Let $\mathcal{F}$ be any universal kinematics and $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$. Then for arbitrary elementary-time states $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ and $\omega_1 \in \mathbb{B}\mathfrak{s}(\mathfrak{m})$ the following assertions are equivalent:*
1) $\omega^{\{\mathfrak{l}\}} = \omega_1^{\{\mathfrak{m}\}}$; 2) $\omega_1 = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega$.

Proof. **1.** First, we prove, that statement 2) leads to the statement 1). Consider any $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ and $\omega_1 \in \mathbb{B}\mathfrak{s}(\mathfrak{m})$ such that $\omega_1 = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega$. Applying Definition 1 and [18, Property 1.12.1(3)] [2], we deduce

$$\begin{aligned}\omega_1^{\{\mathfrak{m}\}} &= \{(\mathfrak{p}, \langle !\, \mathfrak{p} \leftarrow \mathfrak{m}\rangle\, \omega_1) \mid \mathfrak{p} \in \mathcal{L}k(\mathcal{F})\} = \\ &= \{(\mathfrak{p}, \langle !\, \mathfrak{p} \leftarrow \mathfrak{m}\rangle \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega) \mid \mathfrak{p} \in \mathcal{L}k(\mathcal{F})\} = \\ &= \{(\mathfrak{p}, \langle !\, \mathfrak{p} \leftarrow \mathfrak{l}\rangle\, \omega) \mid \mathfrak{p} \in \mathcal{L}k(\mathcal{F})\} = \omega^{\{\mathfrak{l}\}}.\end{aligned}$$

2. Inversely, suppose, that $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$, $\omega_1 \in \mathbb{B}\mathfrak{s}(\mathfrak{m})$ and $\omega^{\{\mathfrak{l}\}} = \omega_1^{\{\mathfrak{m}\}}$. Then, by Definition 1, we have

$$\begin{aligned}&\{(\mathfrak{p}, \langle !\, \mathfrak{p} \leftarrow \mathfrak{l}\rangle\, \omega) \mid \mathfrak{p} \in \mathcal{L}k(\mathcal{F})\} = \\ &\qquad = \{(\mathfrak{p}, \langle !\, \mathfrak{p} \leftarrow \mathfrak{m}\rangle\, \omega_1) \mid \mathfrak{p} \in \mathcal{L}k(\mathcal{F})\}. \qquad (1)\end{aligned}$$

According to [18, Property 1.12.1(1)], we have, $\langle !\, \mathfrak{l} \leftarrow \mathfrak{l}\rangle\, \omega = \omega$. Hence, in accordance with (1), for element $(\mathfrak{l}, \omega) = (\mathfrak{l}, \langle !\, \mathfrak{l} \leftarrow \mathfrak{l}\rangle\, \omega) \in \{(\mathfrak{p}, \langle !\, \mathfrak{p} \leftarrow \mathfrak{l}\rangle\, \omega) \mid \mathfrak{p} \in \mathcal{L}k(\mathcal{F})\}$ we obtain the correlation, $(\mathfrak{l}, \omega) \in \{(\mathfrak{p}, \langle !\, \mathfrak{p} \leftarrow \mathfrak{m}\rangle\, \omega_1) \mid \mathfrak{p} \in \mathcal{L}k(\mathcal{F})\}$. Therefore, there exists the reference frame $\mathfrak{p}_0 \in \mathcal{L}k(\mathcal{F})$ such that $(\mathfrak{l}, \omega) = (\mathfrak{p}_0, \langle !\, \mathfrak{p}_0 \leftarrow \mathfrak{m}\rangle\, \omega_1)$. Hence we deduce $\mathfrak{l} = \mathfrak{p}_0$, as well $\omega = \langle !\, \mathfrak{p}_0 \leftarrow \mathfrak{m}\rangle\, \omega_1 = \langle !\, \mathfrak{l} \leftarrow \mathfrak{m}\rangle\, \omega_1$. So, based on [18, Properties 1.12.1(1,3)], we conclude, $\omega_1 = \langle !\, \mathfrak{m} \leftarrow \mathfrak{m}\rangle\, \omega_1 = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle \langle !\, \mathfrak{l} \leftarrow \mathfrak{m}\rangle\, \omega_1 = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega$. □

The next corollary follows from Assertion 1.

Corollary 1. *Let $\mathcal{F}$ be any universal kinematics. Then for every $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ and $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ the following equality holds:*

$$\omega^{\{\mathfrak{l}\}} = (\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega)^{\{\mathfrak{m}\}}.$$

[1] Definition of universal kinematics can be found in [11, page 89] or [18, page 156].

[2] Reference to Property 1.12.1(3) means reference to the item 3 from the group of properties "Properties 1.12.1".

Assertion 2. *Let $\mathcal{F}$ be any universal kinematics. Then the set*

$$\mathbb{B}\mathfrak{s}[\mathfrak{l}, \mathcal{F}] = \left\{\omega^{\{\mathfrak{l},\mathcal{F}\}} \mid \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})\right\} \qquad (2)$$

does not depend of the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ (ie $\forall \mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ $\mathbb{B}\mathfrak{s}[\mathfrak{l}, \mathcal{F}] = \mathbb{B}\mathfrak{s}[\mathfrak{m}, \mathcal{F}]$).

Proof. Consider arbitrary $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$. Using Corollary 1, we have

$$\begin{aligned}\mathbb{B}\mathfrak{s}[\mathfrak{l}, \mathcal{F}] &= \left\{\omega^{\{\mathfrak{l}\}} \mid \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})\right\} = \\ &= \left\{(\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega)^{\{\mathfrak{m}\}} \mid \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})\right\}.\end{aligned}$$

Hence, according to [18, Corollary 1.12.6], we obtain

$$\begin{aligned}\mathbb{B}\mathfrak{s}[\mathfrak{l}, \mathcal{F}] &= \left\{(\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega)^{\{\mathfrak{m}\}} \mid \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})\right\} = \\ &= \left\{\omega_1^{\{\mathfrak{m}\}} \mid \omega_1 \in \mathbb{B}\mathfrak{s}(\mathfrak{m})\right\} = \mathbb{B}\mathfrak{s}[\mathfrak{m}, \mathcal{F}]. \qquad \square\end{aligned}$$

Definition 2. *Let $\mathcal{F}$ be any universal kinematics.*

1. The set $\mathbb{B}\mathfrak{s}(\mathcal{F}) = \mathbb{B}\mathfrak{s}[\mathfrak{l}, \mathcal{F}]$ $(\forall\, \mathfrak{l} \in \mathcal{L}k(\mathcal{F}))$ is called by the set of all elementary-time states of $\mathcal{F}$.

2. Any subset $\widehat{\mathbf{A}} \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$ is called by the (common) ***changeable system of the universal kinematics*** *$\mathcal{F}$.*

Assertion 3. *Let $\mathcal{F}$ be any universal kinematics and $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ be any reference frame of $\mathcal{F}$. Then for every element $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ only one element $\omega_0 \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ exists such, that $\hat{\omega} = \omega_0^{\{\mathfrak{l}\}}$.*

Proof. Consider any $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ and $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$. By Definition 2 and Assertion 2 (formula (2)), we have

$$\mathbb{B}\mathfrak{s}(\mathcal{F}) = \mathbb{B}\mathfrak{s}[\mathfrak{l}, \mathcal{F}] = \left\{\omega^{\{\mathfrak{l}\}} \mid \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})\right\}.$$

So, since $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$, the element $\omega_0 \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ must exist such that the following equality is performed:

$$\hat{\omega} = \omega_0^{\{\mathfrak{l}\}}. \qquad (3)$$

Let us prove that such element ω_0 is unique. Assume that $\hat{\omega} = \omega_1^{\{\mathfrak{l}\}}$, where $\omega_1 \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$. Then, from the equality (3) we deduce, $\omega_0^{\{\mathfrak{l}\}} = \omega_1^{\{\mathfrak{l}\}}$. Hence, according to Assertion 1 and [18, Property 1.12.1(1)], we obtain, $\omega_1 = \langle !\, \mathfrak{l} \leftarrow \mathfrak{l}\rangle\, \omega_0 = \omega_0$. □

Definition 3. *Let $\mathcal{F}$ be any universal kinematics, $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ be any elementary-time state of $\mathcal{F}$ and $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ be any reference frame of $\mathcal{F}$. Elementary-time state $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ is named by* ***image*** *of elementary-time state $\hat{\omega}$* ***in the reference frame*** *$\mathfrak{l}$ if and only if $\hat{\omega} = \omega^{\{\mathfrak{l}\}}$.*

In accordance with Assertion 3, every elementary-time state $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ always has only one image in any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$. Image of elementary-time state $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ in the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$ will be denoted via $\hat{\omega}_{\{\mathfrak{l},\mathcal{F}\}}$ (in the cases, where the universal kinematics $\mathcal{F}$ is known in advance, we use the abbreviated denotation $\hat{\omega}_{\{\mathfrak{l}\}}$).

Thus, according to Definition 3, for arbitrary $\hat{\omega} \in \mathbb{Bs}(\mathcal{F})$ the following equality holds:

$$\left(\hat{\omega}_{\{\mathfrak{l}\}}\right)^{\{\mathfrak{l}\}} = \hat{\omega}. \tag{4}$$

From the other hand, if for any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ and any fixed elementary-time state $\omega \in \mathbb{Bs}(\mathfrak{l})$, we denote $\hat{\omega} := \omega^{\{\mathfrak{l}\}}$, then by Definition 3, we will receive, $\omega = \hat{\omega}_{\{\mathfrak{l}\}}$. Therefore we have:

$$\left(\omega^{\{\mathfrak{l}\}}\right)_{\{\mathfrak{l}\}} = \omega \quad (\forall \mathfrak{l} \in \mathcal{L}k(\mathcal{F})\ \forall \omega \in \mathbb{Bs}(\mathfrak{l})). \tag{5}$$

From equalities (4) and (5) we deduce the following corollary:

Corollary 2. *Let $\mathcal{F}$ be any universal kinematics and $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ be any reference frame of $\mathcal{F}$. Then:*

1. The mapping $(\cdot)^{\{\mathfrak{l}\}}$ is bijection from $\mathbb{Bs}(\mathfrak{l})$ onto $\mathbb{Bs}(\mathcal{F})$.

2. The mapping $(\cdot)_{\{\mathfrak{l}\}}$ is bijection from $\mathbb{Bs}(\mathcal{F})$ onto $\mathbb{Bs}(\mathfrak{l})$.

3. The mapping $(\cdot)_{\{\mathfrak{l}\}}$ is inverse to the mapping $(\cdot)^{\{\mathfrak{l}\}}$.

Assertion 4. *Let $\mathcal{F}$ be any universal kinematics and $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ be any reference frames $\mathcal{F}$. Then the following statements are performed:*

1. For every $\hat{\omega} \in \mathbb{Bs}(\mathcal{F})$ the equality $\hat{\omega}_{\{\mathfrak{m}\}} = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \hat{\omega}_{\{\mathfrak{l}\}}$ holds.

2. For each $\omega \in \mathbb{Bs}(\mathfrak{l})$ the equality $\left(\omega^{\{\mathfrak{l}\}}\right)_{\{\mathfrak{m}\}} = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega$ is true.

Proof. 1) Chose any $\hat{\omega} \in \mathbb{Bs}(\mathcal{F})$. Applying Corollary 1 to the elementary-time state $\hat{\omega}_{\{\mathfrak{l}\}} \in \mathbb{Bs}(\mathfrak{l})$ and using equality (4), we obtain

$$\left(\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \hat{\omega}_{\{\mathfrak{l}\}}\right)^{\{\mathfrak{m}\}} = \left(\hat{\omega}_{\{\mathfrak{l}\}}\right)^{\{\mathfrak{l}\}} = \hat{\omega}.$$

Thence, using equality (5), we have

$$\hat{\omega}_{\{\mathfrak{m}\}} = \left(\left(\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \hat{\omega}_{\{\mathfrak{l}\}}\right)^{\{\mathfrak{m}\}}\right)_{\{\mathfrak{m}\}} = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \hat{\omega}_{\{\mathfrak{l}\}}.$$

2) Consider any $\omega \in \mathbb{Bs}(\mathfrak{l})$. Applying Corollary 1 as well as equality (5), we deliver

$$\left(\omega^{\{\mathfrak{l}\}}\right)_{\{\mathfrak{m}\}} = \left(\left(\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega\right)^{\{\mathfrak{m}\}}\right)_{\{\mathfrak{m}\}} = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega. \qquad \square$$

Let $\mathcal{F}$ be any universal kinematics. The set $\widehat{\mathbf{A}}_{\{\mathfrak{l},\mathcal{F}\}} = \left\{\hat{\omega}_{\{\mathfrak{l},\mathcal{F}\}} \mid \hat{\omega} \in \widehat{\mathbf{A}}\right\}$ is called ***image of changeable system*** $\widehat{\mathbf{A}} \subseteq \mathbb{Bs}(\mathcal{F})$ in the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$.

Any changeable system $A \subseteq \mathbb{Bs}(\mathfrak{l})$ in the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ always generates the (common) changeable system $A^{\{\mathfrak{l},\mathcal{F}\}} := \left\{\omega^{\{\mathfrak{l},\mathcal{F}\}} \mid \omega \in A\right\} \subseteq \mathbb{Bs}(\mathcal{F})$.

Remark 2. In the cases, where universal kinematics $\mathcal{F}$ is known in advance, we use the abbreviated denotations $\widehat{\mathbf{A}}_{\{\mathfrak{l}\}}$ and $A^{\{\mathfrak{l}\}}$ instead of $\widehat{\mathbf{A}}_{\{\mathfrak{l},\mathcal{F}\}}$ and $A^{\{\mathfrak{l},\mathcal{F}\}}$ (correspondingly).

Applying equalities (4) and (5), we obtain the equalities:

$$\left(\widehat{\mathbf{A}}_{\{\mathfrak{l}\}}\right)^{\{\mathfrak{l}\}} = \widehat{\mathbf{A}} \quad \text{and} \quad \left(A^{\{\mathfrak{l}\}}\right)_{\{\mathfrak{l}\}} = A$$

(for arbitrary universal kinematics $\mathcal{F}$, reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ and changeable systems $\widehat{\mathbf{A}} \subseteq \mathbb{Bs}(\mathcal{F})$ as well $A \subseteq \mathbb{Bs}(\mathfrak{l})$).

3 Chain paths of universal kinematics and definition of time irreversibility

Definition 4. *Let $\mathcal{F}$ be any universal kinematics. Changeable system $\widehat{\mathbf{A}} \subseteq \mathbb{Bs}(\mathcal{F})$ is called **piecewise chain** changeable system if and only if there exist the sequences of changeable systems $\widehat{\mathbf{A}}_1, \cdots, \widehat{\mathbf{A}}_n \subseteq \mathbb{Bs}(\mathcal{F})$ and reference frames $\mathfrak{l}_1, \cdots, \mathfrak{l}_n \in \mathcal{L}k(\mathcal{F})$ $(n \in \mathbb{N})$ satisfying the following conditions:*

(a) $\left(\widehat{\mathbf{A}}_k\right)_{\{\mathfrak{l}_k\}} \in \mathbb{L}l(\mathfrak{l}_k) \quad \left(\forall k \in \overline{1,n}\right)$,[1] *where definition of set $\mathbb{L}l(\mathfrak{l}_k) = \mathbb{L}l((\mathfrak{l}_k)^\wedge)$ can be found in* [18, pages 63, 88, 156]*;*

(b) $\bigcup_{k=1}^{n} \widehat{\mathbf{A}}_k = \widehat{\mathbf{A}}$,

and, moreover, in the case $n \geq 2$ the following additional conditions are satisfied:

(c) $\widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k+1} \neq \emptyset \ \left(\forall k \in \overline{1,n-1}\right)$*;*

(d) *For each $k \in \overline{1,n-1}$ and arbitrary $\omega_1 \in \left(\widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k+1}\right)_{\{\mathfrak{l}_k\}}$, $\omega_2 \in \left(\widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k+1}\right)_{\{\mathfrak{l}_k\}}$ the inequality $\mathsf{tm}(\omega_1) <_{\mathfrak{l}_k} \mathsf{tm}(\omega_2)$ holds.*

(e) *For every $k \in \overline{2,n}$ and arbitrary $\omega_1 \in \left(\widehat{\mathbf{A}}_{k-1} \cap \widehat{\mathbf{A}}_k\right)_{\{\mathfrak{l}_k\}}$, $\omega_2 \in \left(\widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k-1}\right)_{\{\mathfrak{l}_k\}}$ the inequality $\mathsf{tm}(\omega_1) <_{\mathfrak{l}_k} \mathsf{tm}(\omega_2)$ is performed.*

*In this case the ordered composition $\mathscr{A} = \left(\widehat{\mathbf{A}}, \left(\widehat{\mathbf{A}}_1, \mathfrak{l}_1\right), \cdots, \left(\widehat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ will be named by the **chain path** of universal kinematics $\mathcal{F}$.*

Definition 5. *Let $\mathcal{F}$ be any universal kinematics.*

(a) *Changeable system $A \subseteq \mathbb{Bs}(\mathfrak{l})$ is refereed to as **geometrically-stationary** in the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ if and only if $A \in \mathbb{L}l(\mathfrak{l})$ and for arbitrary $\omega_1, \omega_2 \in A$ the equality $\mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}(\omega_1)\right) = \mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}(\omega_2)\right)$ holds.*

(b) *The set of all geometrically-stationary changeable systems in the reference frame $\mathfrak{l}$ is denoted via $\mathbb{L}g(\mathfrak{l}, \mathcal{F})$. In the cases, where the universal kinematics $\mathcal{F}$ is known in advance, we use the abbreviated denotation $\mathbb{L}g(\mathfrak{l})$.*

(c) *The chain path $\mathscr{A} = \left(\widehat{\mathbf{A}}, \left(\widehat{\mathbf{A}}_1, \mathfrak{l}_1\right), \cdots, \left(\widehat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ in $\mathcal{F}$ $(n \in \mathbb{N})$ is called by **piecewise geometrically-stationary** if and only if $\forall k \in \overline{1,n}$ $\left(\widehat{\mathbf{A}}_k\right)_{\{\mathfrak{l}_k\}} \in \mathbb{L}g(\mathfrak{l}_k)$.*

From the physical point of view piecewise geometrically-stationary chain path may be interpreted as process of "vagancy" of observer (or some material particle or signal), which moves by means of "jumping" from previous reference frame to the next frame with a finite number of times.

Definition 6. *Let $\mathcal{F}$ be any universal kinematics and let $\mathscr{A} = \left(\widehat{\mathbf{A}}, \left(\widehat{\mathbf{A}}_1, \mathfrak{l}_1\right), \cdots, \left(\widehat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ be arbitrary chain path in $\mathcal{F}$.*

[1] Further we denote via $\overline{m,n}$ $(m, n \in \mathbb{N},\ m \leq n)$ the set $\overline{m,n} = \{m, \cdots, n\}$.

1. *Element $\hat{\omega}_s \in \mathbb{Bs}(\mathcal{F})$ is called by **start** element of the path $\mathscr{A}$, if and only if $\hat{\omega}_s \in \widehat{\mathbf{A}}_1$ and for every $\hat{\omega} \in \widehat{\mathbf{A}}_1$ the inequality $\mathsf{tm}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right) \leq_{\mathfrak{l}_1} \mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}_1\}}\right)$ is performed.*
2. *Element $\hat{\omega}_f \in \mathbb{Bs}(\mathcal{F})$ is called by **final** element of the path $\mathscr{A}$, if and only if $\hat{\omega}_f \in \widehat{\mathbf{A}}_n$ and for every $\hat{\omega} \in \widehat{\mathbf{A}}_n$ the inequality $\mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}_n\}}\right) \leq_{\mathfrak{l}_n} \mathsf{tm}\left(\left(\hat{\omega}_f\right)_{\{\mathfrak{l}_n\}}\right)$ holds.*
3. *The chain path $\mathscr{A}$, which owns (at least one) start element and (at least one) final element, is called by **closed**.*

Assertion 5. *Any chain path $\mathscr{A}$ of arbitrary universal kinematics $\mathcal{F}$ can not have more, than one start element and more, than one final element.*

Proof. **(a)** Let $\hat{\omega}_s$, $\hat{\omega}_x$ be two start elements of the chain path $\mathscr{A} = \left(\widehat{\mathbf{A}}, \left(\widehat{\mathbf{A}}_1, \mathfrak{l}_1\right), \cdots, \left(\widehat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$. Then, by Definition 6, we have $\hat{\omega}_s, \hat{\omega}_x \in \widehat{\mathbf{A}}_1$, $\mathsf{tm}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right) \leq_{\mathfrak{l}_1} \mathsf{tm}\left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right)$ and $\mathsf{tm}\left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right) \leq_{\mathfrak{l}_1} \mathsf{tm}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right)$. Therefore we get

$$\mathsf{tm}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right) = \mathsf{tm}\left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right). \tag{6}$$

Since $\hat{\omega}_s, \hat{\omega}_x \in \widehat{\mathbf{A}}_1$, then $(\hat{\omega}_s)_{\{\mathfrak{l}_1\}}, (\hat{\omega}_x)_{\{\mathfrak{l}_1\}} \in \left(\widehat{\mathbf{A}}_1\right)_{\{\mathfrak{l}_1\}}$, where, in accordance with Definition 4 (subitem (a)), we have, $\left(\widehat{\mathbf{A}}_1\right)_{\{\mathfrak{l}_1\}} \in \mathbb{L}l(\mathfrak{l}_1)$. That is, according to [18, Assertion 1.7.5 (item 1)], $\left(\widehat{\mathbf{A}}_1\right)_{\{\mathfrak{l}_1\}}$ is a function from $\mathbf{Tm}(\mathfrak{l}_1)$ into $\mathfrak{Bs}(\mathfrak{l}_1)$. So, using equality $\omega = (\mathsf{tm}(\omega), \mathsf{bs}(\omega))$ $(\omega \in \mathbb{Bs}(\mathfrak{l}_1))$ as well as formula (6), we obtain

$$\mathsf{bs}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right) = \left(\widehat{\mathbf{A}}_1\right)_{\{\mathfrak{l}_1\}}\left(\mathsf{tm}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right)\right) = \\ = \left(\widehat{\mathbf{A}}_1\right)_{\{\mathfrak{l}_1\}}\left(\mathsf{tm}\left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right)\right) = \mathsf{bs}\left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right).$$

Using the last equality and equality (6), we deduce, $(\hat{\omega}_s)_{\{\mathfrak{l}_1\}} = \left(\mathsf{tm}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right), \mathsf{bs}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right)\right) = \left(\mathsf{tm}\left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right), \mathsf{bs}\left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right)\right) = (\hat{\omega}_x)_{\{\mathfrak{l}_1\}}$. Hence, according to formula (4), we deliver $\hat{\omega}_s = \left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right)^{\{\mathfrak{l}_1\}} = \left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right)^{\{\mathfrak{l}_1\}} = \hat{\omega}_x$.

(c) Similarly it can be proven that the chain path $\mathscr{A}$ can not have more, than one final element. □

Further the start element of the chain path $\mathscr{A}$ of the universal kinematics $\mathcal{F}$ will be denoted via $\mathsf{po}(\mathscr{A}, \mathcal{F})$, or via $\mathsf{po}(\mathscr{A})$. The final element of the chain path $\mathscr{A}$ will be denoted via $\mathsf{ki}(\mathscr{A}, \mathcal{F})$, or via $\mathsf{ki}(\mathscr{A})$. Where the denotations $\mathsf{po}(\mathscr{A})$ and $\mathsf{ki}(\mathscr{A})$ are used in the cases when they do not cause misunderstanding. Thus, for every closed chain path $\mathscr{A}$ both start and final elements ($\mathsf{po}(\mathscr{A})$ and $\mathsf{ki}(\mathscr{A})$) always exist.

Definition 7. *Closed chain path $\mathscr{A}$ of universal kinematics $\mathcal{F}$ is refereed to as **geometrically-cyclic** in the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ if and only if $\mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{po}(\mathscr{A})_{\{\mathfrak{l}\}}\right)\right) = \mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{ki}(\mathscr{A})_{\{\mathfrak{l}\}}\right)\right)$.*

Definition 8. *Universal kinematics $\mathcal{F}$ is called **time irreversible** if and only if for every reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ and for each chain path $\mathscr{A}$, geometrically-cyclic in the frame $\mathfrak{l}$ and piecewise geometrically-stationary in $\mathcal{F}$, it is performed the inequality $\mathsf{tm}\left(\mathsf{po}(\mathscr{A})_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{ki}(\mathscr{A})_{\{\mathfrak{l}\}}\right)$.*

*Universal kinematics $\mathcal{F}$ is called **time reversible** if and only if it is not time irreversible.*

The physical sense of time irreversibility notion is that in time irreversible kinematics there is not any process or object which returns to the begin of the own path at the past, moving by means of "jumping" from previous reference frame to the next frame. So, there are not temporal paradoxes in these kinematics.

4 Direction of time between reference frames of universal kinematics

For formulation main theorem we need some notions, connected with direction of time between reference frames.

Definition 9. *Let $\mathcal{F}$ be any universal kinematics.*

1. *We say that reference frame $\mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ is **time-nonnegative** relatively the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ (in the universal kinematics $\mathcal{F}$) (denotation is $\mathfrak{m} \Uparrow_{\mathcal{F}} \mathfrak{l}$) if and only if for arbitrary $\mathrm{w}_1, \mathrm{w}_2 \in \mathbb{M}k(\mathfrak{l})$ such that $\mathsf{bs}(\mathrm{w}_1) = \mathsf{bs}(\mathrm{w}_2)$ and $\mathsf{tm}(\mathrm{w}_1) \leq_{\mathfrak{l}} \mathsf{tm}(\mathrm{w}_2)$ it is performed the inequality, $\mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) \leq_{\mathfrak{m}} \mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$.*
2. *We say that reference frame $\mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ is **time-positive** in $\mathcal{F}$ relatively the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ (denotation is $\mathfrak{m} \Uparrow^{+}_{\mathcal{F}} \mathfrak{l}$) if and only if for arbitrary $\mathrm{w}_1, \mathrm{w}_2 \in \mathbb{M}k(\mathfrak{l})$ such that $\mathsf{bs}(\mathrm{w}_1) = \mathsf{bs}(\mathrm{w}_2)$ and $\mathsf{tm}(\mathrm{w}_1) <_{\mathfrak{l}} \mathsf{tm}(\mathrm{w}_2)$ it is performed the inequality, $\mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) <_{\mathfrak{m}} \mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$.*
3. *We say that reference frame $\mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ is **time-nonpositive** in $\mathcal{F}$ relatively the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ (denotation is $\mathfrak{m} \Downarrow_{\mathcal{F}} \mathfrak{l}$) if and only if for arbitrary $\mathrm{w}_1, \mathrm{w}_2 \in \mathbb{M}k(\mathfrak{l})$ such that $\mathsf{bs}(\mathrm{w}_1) = \mathsf{bs}(\mathrm{w}_2)$ and $\mathsf{tm}(\mathrm{w}_1) \leq_{\mathfrak{l}} \mathsf{tm}(\mathrm{w}_2)$ it is performed the inequality, $\mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) \geq_{\mathfrak{m}} \mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$.*
4. *We say that reference frame $\mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ is **time-negative** in $\mathcal{F}$ relatively the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ (denotation is $\mathfrak{m} \Downarrow^{-}_{\mathcal{F}} \mathfrak{l}$) if and only if for arbitrary $\mathrm{w}_1, \mathrm{w}_2 \in \mathbb{M}k(\mathfrak{l})$ such that $\mathsf{bs}(\mathrm{w}_1) = \mathsf{bs}(\mathrm{w}_2)$ and $\mathsf{tm}(\mathrm{w}_1) <_{\mathfrak{l}} \mathsf{tm}(\mathrm{w}_2)$ it is performed the inequality, $\mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) >_{\mathfrak{m}} \mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$.*
5. *The universal kinematics $\mathcal{F}$ is named by **weakly time-positive** if and only if there exist at least one reference frame $\mathfrak{l}_0 \in \mathcal{L}k(\mathcal{F})$ such that the correlation $\mathfrak{l}_0 \Uparrow^{+}_{\mathcal{F}} \mathfrak{l}$ holds for every reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$.*

Remark 3. Apart from weak time-positivity we can introduce other, more strong, form of time-positivity. We say that universal kinematics $\mathcal{F}$ is **time-positive** if and only if for arbitrary reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ the correlation $\mathfrak{l} \Uparrow^{+}_{\mathcal{F}} \mathfrak{m}$

holds. It is not hard to prove that every kinematics of kind $\mathcal{F} = \mathfrak{UP}(\mathfrak{H}, \mathcal{B}, c)$ (connected with classical special relativity and introduced in [11] and [18, Section 24]) is time-positive.

Assertion 6. *For arbitrary reference frames* $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ *of any universal kinematics* $\mathcal{F}$ *the following statements are performed.*

1) If $\mathfrak{m} \Uparrow^+_{\mathcal{F}} \mathfrak{l}$, *then* $\mathfrak{m} \Uparrow_{\mathcal{F}} \mathfrak{l}$.

2) If $\mathfrak{m} \Downarrow^-_{\mathcal{F}} \mathfrak{l}$, *then* $\mathfrak{m} \Downarrow_{\mathcal{F}} \mathfrak{l}$.

Proof. **1)** Indeed, let $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ and $\mathfrak{m} \Uparrow^+_{\mathcal{F}} \mathfrak{l}$. Then for every $\mathrm{w}_1, \mathrm{w}_2 \in \mathbb{M}k(\mathfrak{l})$ such, that $\mathsf{bs}(\mathrm{w}_1) = \mathsf{bs}(\mathrm{w}_2)$ and $\mathsf{tm}(\mathrm{w}_1) \leq_{\mathfrak{l}} \mathsf{tm}(\mathrm{w}_2)$, we deduce the following:

(a) In the case $\mathsf{tm}(\mathrm{w}_1) <_{\mathfrak{l}} \mathsf{tm}(\mathrm{w}_2)$, by Definition 9, item 2, we get, $\mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) <_{\mathfrak{m}} \mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$.

(b) In the case $\mathsf{tm}(\mathrm{w}_1) = \mathsf{tm}(\mathrm{w}_2)$, we have $\mathrm{w}_1 = (\mathsf{tm}(\mathrm{w}_1), \mathsf{bs}(\mathrm{w}_1)) = (\mathsf{tm}(\mathrm{w}_2), \mathsf{bs}(\mathrm{w}_2)) = \mathrm{w}_2$, and so $\mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) = \mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$.

2) Second item of this Assertion can be proven similarly. □

5 Theorem of Non-Returning

Theorem 1. *Any weakly time-positive universal kinematics* $\mathcal{F}$ *is time irreversible.*

To prove Theorem 1 we need a few auxiliary assertions.

Assertion 7. *Let* $\widehat{\mathbf{A}} \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$ *be changeable system of universal kinematics* $\mathcal{F}$ *such, that* $\widehat{\mathbf{A}}_{\{\mathfrak{l}_0\}} \in \mathbb{L}g(\mathfrak{l}_0)$ *for some reference frame* $\mathfrak{l}_0 \in \mathcal{L}k(\mathcal{F})$. *Let* $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ *be reference frame, satisfying condition* $\mathfrak{l} \Uparrow_{\mathcal{F}} \mathfrak{l}_0$.

Then for arbitrary $\hat{\omega}_1, \hat{\omega}_2 \in \widehat{\mathbf{A}}$ *the inequality* $\mathsf{tm}\left((\hat{\omega}_1)_{\{\mathfrak{l}_0\}}\right) \leq_{\mathfrak{l}_0} \mathsf{tm}\left((\hat{\omega}_2)_{\{\mathfrak{l}_0\}}\right)$ *assures the the inequality* $\mathsf{tm}\left((\hat{\omega}_1)_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left((\hat{\omega}_2)_{\{\mathfrak{l}\}}\right)$.

Proof. Suppose that, under conditions of the assertion, we have $\hat{\omega}_1, \hat{\omega}_2 \in \widehat{\mathbf{A}}$ and $\mathsf{tm}\left((\hat{\omega}_1)_{\{\mathfrak{l}_0\}}\right) \leq_{\mathfrak{l}_0} \mathsf{tm}\left((\hat{\omega}_2)_{\{\mathfrak{l}_0\}}\right)$. According to Definition of Minkowski coordinates (see [11, formula (2)] or [18, formula (2.3)]), we have $\mathsf{tm}(\omega) = \mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}(\omega)\right)$ $(\forall\, \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l}_0))$. So, we get

$$\mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left((\hat{\omega}_1)_{\{\mathfrak{l}_0\}}\right)\right) \leq_{\mathfrak{l}_0} \mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left((\hat{\omega}_2)_{\{\mathfrak{l}_0\}}\right)\right). \tag{7}$$

Since $(\hat{\omega}_1)_{\{\mathfrak{l}_0\}}, (\hat{\omega}_2)_{\{\mathfrak{l}_0\}} \in \widehat{\mathbf{A}}_{\{\mathfrak{l}_0\}}$ (where $\widehat{\mathbf{A}}_{\{\mathfrak{l}_0\}} \in \mathbb{L}g(\mathfrak{l}_0)$) then, by Definition 5 (items (a),(b)), we have

$$\mathsf{bs}\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left((\hat{\omega}_1)_{\{\mathfrak{l}_0\}}\right)\right) = \mathsf{bs}\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left((\hat{\omega}_2)_{\{\mathfrak{l}_0\}}\right)\right). \tag{8}$$

Taking into account that $\mathfrak{l} \Uparrow_{\mathcal{F}} \mathfrak{l}_0$ and using Definition 9 (item 1) as well as formulas (7), (8), we get the inequality:

$$\mathsf{tm}\left([\mathfrak{l} \leftarrow \mathfrak{l}_0]\,\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left((\hat{\omega}_1)_{\{\mathfrak{l}_0\}}\right)\right) \leq_{\mathfrak{l}} \mathsf{tm}\left([\mathfrak{l} \leftarrow \mathfrak{l}_0]\,\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left((\hat{\omega}_2)_{\{\mathfrak{l}_0\}}\right)\right).$$

Thence, using [18, formula (3.2)], we obtain

$$\mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}\left(\langle !\, \mathfrak{l} \leftarrow \mathfrak{l}_0 \rangle\,(\hat{\omega}_1)_{\{\mathfrak{l}_0\}}\right)\right) \leq_{\mathfrak{l}} \leq_{\mathfrak{l}} \mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}\left(\langle !\, \mathfrak{l} \leftarrow \mathfrak{l}_0 \rangle\,(\hat{\omega}_2)_{\{\mathfrak{l}_0\}}\right)\right).$$

Applying the last inequality as well as Assertion 4, we deduce the inequality:

$$\mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}\left((\hat{\omega}_1)_{\{\mathfrak{l}\}}\right)\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}\left((\hat{\omega}_2)_{\{\mathfrak{l}\}}\right)\right). \tag{9}$$

According to Definition of Minkowski coordinates (see [11, formula (2)] or [18, formula (2.3)]), for every $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ we have the equality $\mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega)\right) = \mathsf{tm}(\omega)$. That is why from the inequality (9) it follows the desired inequality $\mathsf{tm}\left((\hat{\omega}_1)_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left((\hat{\omega}_2)_{\{\mathfrak{l}\}}\right)$. □

Assertion 8. *Let,* $\mathscr{A} = \left(\widehat{\mathbf{A}}, \left(\widehat{\mathbf{A}}_1, \mathfrak{l}_1\right), \cdots, \left(\widehat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ $(n \in \mathbb{N})$ *be closed, piecewise geometrically-stationary chain path of universal kinematics* $\mathcal{F}$ *and* $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ *be reference frame such that* $\mathfrak{l} \Uparrow_{\mathcal{F}} \mathfrak{l}_i$ *for every* $i \in \overline{1,n}$. *Then for arbitrary* $\hat{\omega} \in \widehat{\mathbf{A}}$ *the following inequality holds:*

$$\mathsf{tm}\left(\mathsf{po}(\mathscr{A})_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}(\hat{\omega}_{\{\mathfrak{l}\}}) \leq_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{ki}(\mathscr{A})_{\{\mathfrak{l}\}}\right). \tag{10}$$

Proof. Let $\mathcal{F}$ be universal kinematics and $\mathscr{A} = \left(\widehat{\mathbf{A}}, \left(\widehat{\mathbf{A}}_1, \mathfrak{l}_1\right), \cdots, \left(\widehat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ $(n \in \mathbb{N})$ be closed, piecewise geometrically-stationary chain path of $\mathcal{F}$. Let, $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ be reference frame such that $\mathfrak{l} \Uparrow_{\mathcal{F}} \mathfrak{l}_i$ $(\forall\, i \in \overline{1,n})$.

1) First we prove that for any $\hat{\omega} \in \widehat{\mathbf{A}}$ it holds the inequality:

$$\mathsf{tm}\left(\mathsf{po}(\mathscr{A})_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}(\hat{\omega}_{\{\mathfrak{l}\}}). \tag{11}$$

By Definition 4 (item (b)), $\widehat{\mathbf{A}} = \bigcup_{k=1}^{n} \widehat{\mathbf{A}}_k$. So, it is sufficient to prove the inequality (11) for the cases $\hat{\omega} \in \widehat{\mathbf{A}}_k$ $(k \in \overline{1,n})$.

1.a) First we prove the inequality (11) for $\hat{\omega} \in \widehat{\mathbf{A}}_1$. According to Definition 6 (item 1), for $\hat{\omega} \in \widehat{\mathbf{A}}_1$ we obtain that $\mathsf{po}(\mathscr{A}) \in \widehat{\mathbf{A}}_1$ and

$$\mathsf{tm}\left(\mathsf{po}(\mathscr{A})_{\{\mathfrak{l}_1\}}\right) \leq_{\mathfrak{l}_1} \mathsf{tm}(\hat{\omega}_{\{\mathfrak{l}_1\}}). \tag{12}$$

According to the above, we have $\hat{\omega} \in \widehat{\mathbf{A}}_1$ and $\mathsf{po}(\mathscr{A}) \in \widehat{\mathbf{A}}_1$. Moreover, by Definition 5 (item (c)), we get, $\left(\widehat{\mathbf{A}}_1\right)_{\{\mathfrak{l}_1\}} \in \mathbb{L}g(\mathfrak{l}_1)$. By conditions of Assertion, we have, $\mathfrak{l} \Uparrow_{\mathcal{F}} \mathfrak{l}_1$. So, in accordance with Assertion 7, the correlation (12) stipulates the inequality $\mathsf{tm}\left(\mathsf{po}(\mathscr{A})_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}(\hat{\omega}_{\{\mathfrak{l}\}})$. Hence, in the case $\hat{\omega} \in \widehat{\mathbf{A}}_1$, the inequality (11) has been proven. Moreover, the last inequality has been proven for all $\hat{\omega} \in \widehat{\mathbf{A}}$ in the case $n = 1$. So, further we consider, that $n > 1$.

1.b) Assume, that inequality (11) is performed for all $\hat{\omega} \in \widehat{\mathbf{A}}_{k-1}$, where $k \in \overline{2,n}$. And, let us prove, that then this inequality is true for each $\hat{\omega} \in \widehat{\mathbf{A}}_k$.

In the case $\hat{\omega} \in \widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k-1}$ the inequality (11) is true in accordance with inductive hypothesis. Hence, it remains to prove the last inequality for every $\hat{\omega} \in \widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k-1}$. According to item (c) of Definition 4, we have $\widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k-1} \neq \emptyset$. Hence, at least one element $\hat{\eta} \in \widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k-1}$ exists. Since,

$$\hat{\eta} \in \widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k-1} \quad \text{and} \quad \hat{\omega} \in \widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k-1}, \tag{13}$$

then we get $\hat{\eta}_{\{\mathfrak{l}_k\}} \in \left(\widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k-1}\right)_{\{\mathfrak{l}_k\}}$, $\hat{\omega}_{\{\mathfrak{l}_k\}} \in \left(\widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k-1}\right)_{\{\mathfrak{l}_k\}}$. Therefore, according to item (e) of Definition 4, we deliver

$$\mathsf{tm}\left(\hat{\eta}_{\{\mathfrak{l}_k\}}\right) \leq_{\mathfrak{l}_k} \mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}_k\}}\right). \tag{14}$$

According to (13), we have $\hat{\eta}, \hat{\omega} \in \widehat{\mathbf{A}}_k$, where, by item (c) of Definition 5, $\left(\widehat{\mathbf{A}}_k\right)_{\{\mathfrak{l}_k\}} \in \mathbb{L}g\left(\mathfrak{l}_k\right)$. Since $\mathfrak{l} \Uparrow_{\mathcal{F}} \mathfrak{l}_k$, then taking into account inequality (14) and Assertion 7 we deduce

$$\mathsf{tm}\left(\hat{\eta}_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}\}}\right). \tag{15}$$

According to (13), we have $\hat{\eta} \in \widehat{\mathbf{A}}_{k-1}$. So, by inductive hypothesis, we deliver

$$\mathsf{tm}\left(\mathsf{po}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\hat{\eta}_{\{\mathfrak{l}\}}\right). \tag{16}$$

Inequalities (15) and (16) assure inequality (11).

Thus, by Principle of mathematical induction, inequality (11) is true for arbitrary $\hat{\omega} \in \bigcup_{k=1}^{n} \widehat{\mathbf{A}}_k = \widehat{\mathbf{A}}$.

2) Now we are aiming to prove, that for any $\hat{\omega} \in \widehat{\mathbf{A}}$ it holds the inequality:

$$\mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right). \tag{17}$$

2.a) First we prove the inequality (17) for $\omega \in \widehat{\mathbf{A}}_n$. According to Definition 6 (item 2), for $\hat{\omega} \in \widehat{\mathbf{A}}_n$ we obtain that $\mathsf{ki}\left(\mathscr{A}\right) \in \widehat{\mathbf{A}}_n$ and

$$\mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}_n\}}\right) \leq_{\mathfrak{l}_n} \mathsf{tm}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}_n\}}\right). \tag{18}$$

According to the above, we have $\hat{\omega} \in \widehat{\mathbf{A}}_n$ and $\mathsf{ki}\left(\mathscr{A}\right) \in \widehat{\mathbf{A}}_n$. Moreover, by Definition 5 (item (c)), we get $\left(\widehat{\mathbf{A}}_n\right)_{\{\mathfrak{l}_n\}} \in \mathbb{L}g\left(\mathfrak{l}_n\right)$. By conditions of Assertion, we have $\mathfrak{l} \Uparrow_{\mathcal{F}} \mathfrak{l}_n$. So, in accordance with Assertion 7, the correlation (18) stipulates the inequality (17). Hence, in the case $\hat{\omega} \in \widehat{\mathbf{A}}_n$, the inequality (17) is proven. Moreover, the last inequality is proven for all $\hat{\omega} \in \widehat{\mathbf{A}}$ in the case $n = 1$. So, further we consider, that $n > 1$.

2.b) Assume, that inequality (17) is performed for all $\hat{\omega} \in \widehat{\mathbf{A}}_{k+1}$, where $k \in \overline{1, n-1}$. And, let us prove, that then this inequality is true for each $\hat{\omega} \in \widehat{\mathbf{A}}_k$.

In the case $\omega \in \widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k+1}$ the inequality (17) is true in accordance with inductive hypothesis. Hence, it remains to prove the last inequality for every $\hat{\omega} \in \widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k+1}$. According to item (c) of Definition 4, we have $\widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k+1} \neq \emptyset$. Hence, at least one element $\hat{\eta} \in \widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k+1}$ exists. Taking into account that

$$\hat{\eta} \in \widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k+1} \quad \text{and} \quad \hat{\omega} \in \widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k+1}, \tag{19}$$

we get $\hat{\eta}_{\{\mathfrak{l}_k\}} \in \left(\widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k+1}\right)_{\{\mathfrak{l}_k\}}$, $\hat{\omega}_{\{\mathfrak{l}_k\}} \in \left(\widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k+1}\right)_{\{\mathfrak{l}_k\}}$. Therefore, according to item (d) of Definition 4, we deliver

$$\mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}_k\}}\right) \leq_{\mathfrak{l}_k} \mathsf{tm}\left(\hat{\eta}_{\{\mathfrak{l}_k\}}\right). \tag{20}$$

According to (19), we have $\hat{\eta}, \hat{\omega} \in \widehat{\mathbf{A}}_k$, where $\left(\widehat{\mathbf{A}}_k\right)_{\{\mathfrak{l}_k\}} \in \mathbb{L}g\left(\mathfrak{l}_k\right)$ by item (c) of Definition 5. Since $\mathfrak{l} \Uparrow_{\mathcal{F}} \mathfrak{l}_k$ then, taking into account inequality (20) and Assertion 7, we deduce

$$\mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\hat{\eta}_{\{\mathfrak{l}\}}\right). \tag{21}$$

According to (19), we have $\hat{\eta} \in \widehat{\mathbf{A}}_{k+1}$. So, by inductive hypothesis, we deliver

$$\mathsf{tm}\left(\hat{\eta}_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right). \tag{22}$$

Inequalities (21) and (22) assure inequality (17). Thus, by Principle of mathematical induction, inequality (17) is true for arbitrary $\hat{\omega} \in \bigcup_{k=1}^{n} \widehat{\mathbf{A}}_k = \widehat{\mathbf{A}}$.

Inequality (10) follows from (11) and (17). □

Proof of Theorem 1. Let $\mathcal{F}$ be weakly time-positive universal kinematics. Then, by Definition 9, there exists the reference frame $\mathfrak{l}_0 \in \mathcal{L}k\left(\mathcal{F}\right)$ such that

$$\forall\, \mathfrak{m} \in \mathcal{L}k\left(\mathcal{F}\right) \quad \mathfrak{l}_0 \Uparrow_{\mathcal{F}}^{+} \mathfrak{m}. \tag{23}$$

Let $\mathscr{A} = \left(\widehat{\mathbf{A}}, \left(\widehat{\mathbf{A}}_1, \mathfrak{l}_1\right), \cdots, \left(\widehat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ $(n \in \mathbb{N})$ be piecewise geometrically-stationary chain path in $\mathcal{F}$ and, moreover, $\mathscr{A}$ is geometrically-cyclic relatively some reference frame $\mathfrak{l} \in \mathcal{L}k\left(\mathcal{F}\right)$. By Definition 7, $\mathscr{A}$ is closed chain path. According to Assertion 6, correlation (23) leads to the correlation $\mathfrak{l}_0 \Uparrow_{\mathcal{F}} \mathfrak{l}_k$ $(\forall k \in \overline{1, n})$. Hence, applying Assertion 8, we ensure

$$\mathsf{tm}\left(\mathsf{po}\left(\mathscr{A}\right)_{\{\mathfrak{l}_0\}}\right) \leq_{\mathfrak{l}_0} \mathsf{tm}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}_0\}}\right). \tag{24}$$

Assume, that $\mathsf{tm}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right) <_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{po}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right)$. Then, by Definition of Minkowski coordinates (see [11, formula (2)] or [18, formula (2.3)]), we obtain

$$\mathsf{tm}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right)\right) <_{\mathfrak{l}} \mathsf{tm}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{po}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right)\right). \tag{25}$$

Since the path $\mathscr{A}$ is geometrically-cyclic relatively the reference frame $\mathfrak{l}$, then, by Definition 7, we have

$$\mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{po}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right)\right) = \mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right)\right). \tag{26}$$

Since (in accordance with (23)) $\mathfrak{l}_0 \Uparrow_{\mathcal{F}}^{+} \mathfrak{l}$, then, by Definition 9 (item 2), from the correlations (25), and (26), we get the inequality:

$$\mathsf{tm}\left(\left[\mathfrak{l}_0 \leftarrow \mathfrak{l}\right] \mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right)\right) <_{\mathfrak{l}_0}$$
$$<_{\mathfrak{l}_0} \mathsf{tm}\left(\left[\mathfrak{l}_0 \leftarrow \mathfrak{l}\right] \mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{po}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right)\right).$$

Thence, using [18, formula (3.2)] , we deduce the inequality:

$$\mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left(\langle ! \ \mathfrak{l}_0 \leftarrow \mathfrak{l}\rangle \, \mathsf{ki}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right)\right) <_{\mathfrak{l}_0}$$
$$<_{\mathfrak{l}_0} \mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left(\langle ! \ \mathfrak{l}_0 \leftarrow \mathfrak{l}\rangle \, \mathsf{po}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right)\right).$$

Taking into account Assertion 4, the last inequality can be reduced to the form, $\mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left(\mathsf{ki}\,(\mathscr{A})_{\{\mathfrak{l}_0\}}\right)\right) <_{\mathfrak{l}_0} \mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left(\mathsf{po}\,(\mathscr{A})_{\{\mathfrak{l}_0\}}\right)\right)$, and, by Definition of Minkowski coordinates (see [11, formula (2)] or [18, formula (2.3)])), we assure

$$\mathsf{tm}\left(\mathsf{ki}\,(\mathscr{A})_{\{\mathfrak{l}_0\}}\right) <_{\mathfrak{l}_0} \mathsf{tm}\left(\mathsf{po}\,(\mathscr{A})_{\{\mathfrak{l}_0\}}\right).$$

But, the last inequality contradicts to the correlation (24). Therefore, hypothesis affirming, that $\mathsf{tm}\left(\mathsf{ki}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right) <_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{po}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right)$ is false. Consequently we have

$$\mathsf{tm}\left(\mathsf{po}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{ki}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right). \tag{27}$$

Thus, for each reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ and for each chain path $\mathscr{A}$, geometrically-cyclic in the frame $\mathfrak{l}$ and piecewise geometrically-stationary in $\mathcal{F}$, it holds the inequality (27). So, by Definition 8, kinematics $\mathcal{F}$ is time irreversible, which must be proved. □

6 Certainly time irreversibility. Strengthened version of theorem of non-returning

Recall, that in the papers [17, Definition 6], [18, Definition 3.25.2] the notion of equivalence of universal kinematics relatively coordinate transform had been introduced. According to these papers, we denote equivalent relatively coordinate transform kinematics $\mathcal{F}_1$ and $\mathcal{F}_2$ via $\mathcal{F}_1\,[\equiv]\,\mathcal{F}_2$.

Definition 10. *We say that universal kinematics $\mathcal{F}$ is **certainly time irreversible** if and only if arbitrary universal kinematics $\mathcal{F}_1$ such, that $\mathcal{F}\,[\equiv]\,\mathcal{F}_1$ is time irreversible. In the opposite case we will say that universal kinematics $\mathcal{F}$ is **conditionally time reversible**.*

Since, according to [17, Assertion 3] (see also [18, Assertion 3.25.1]), for each universal kinematics $\mathcal{F}$ it is fulfilled the correlation $\mathcal{F}\,[\equiv]\,\mathcal{F}$, then we receive the following Corollary from Definition 10:

Corollary 3. *Any certainly time irreversible universal kinematics $\mathcal{F}$ is time irreversible.*

The physical sense of certain time irreversibility notion is that in certainly time irreversible kinematics temporal paradoxes are impossible basically, that is there is not potential possibility to affect the own past by means of "traveling" and "jumping" between reference frames. Whereas, in time irreversible, but conditionally time reversible kinematics such potential possibility exists, but it is not realized in the scenario of evolution, acting in this kinematics.

Assertion 9. *Let universal kinematics $\mathcal{F}$ be weakly time-positive. Then every universal kinematics $\mathcal{F}_1$ such that $\mathcal{F}_1\,[\equiv]\,\mathcal{F}$ is weakly time-positive also.*

Proof. Let $\mathcal{F}$ be weakly time-positive universal kinematics and $\mathcal{F}_1\,[\equiv]\,\mathcal{F}$. Recall, that in [18, Definition 3.27.3] for every reference frame $\mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ it was introduced the reference frame $\mathfrak{m} \downharpoonleft_{\mathcal{F}_1}$, related with $\mathfrak{m}$ in the universal kinematics $\mathcal{F}_1$:

$$\mathfrak{m} \downharpoonleft_{\mathcal{F}_1} := \mathbf{lk}_{\mathtt{ind}(\mathfrak{m})}(\mathcal{F}_1). \tag{28}$$

Since kinematics $\mathcal{F}$ is weakly time-positive then, by Definition 9, the reference frame $\mathfrak{l}_0 \in \mathcal{L}k(\mathcal{F})$ exists such that for each reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ the correlation $\mathfrak{l}_0 \Uparrow^{+}_{\mathcal{F}} \mathfrak{l}$ holds. Denote:

$$\mathfrak{l}_0^{(1)} := \mathfrak{l}_0 \downharpoonleft_{\mathcal{F}_1}.$$

Let us consider any reference frame $\mathfrak{l}^{(1)} \in \mathcal{L}k(\mathcal{F}_1)$. Denote: $\mathfrak{l} := \mathfrak{l}^{(1)} \downharpoonleft_{\mathcal{F}} \in \mathcal{L}k(\mathcal{F})$. Then, according to [18, Properties 3.27.1] and formula (28), we have

$$\mathfrak{l}^{(1)} = \mathfrak{l} \downharpoonleft_{\mathcal{F}_1} = \mathbf{lk}_{\mathtt{ind}(\mathfrak{l})}(\mathcal{F}_1).$$

Hence, taking into account [18, Definition 3.25.2 (item 2)], formula (28) and [18, Property 3.25.1(1)], we get

$$\begin{aligned}
\mathbb{M}k\left(\mathfrak{l}_0^{(1)};\mathcal{F}_1\right) &= \mathbb{M}k\left(\mathbf{lk}_{\mathtt{ind}(\mathfrak{l}_0)}(\mathcal{F}_1);\mathcal{F}_1\right) = \\
&= \mathbb{M}k\left(\mathbf{lk}_{\mathtt{ind}(\mathfrak{l}_0)}(\mathcal{F});\mathcal{F}\right) = \mathbb{M}k\left(\mathfrak{l}_0;\mathcal{F}\right); \\
\mathbb{M}k\left(\mathfrak{l}^{(1)};\mathcal{F}_1\right) &= \mathbb{M}k\left(\mathfrak{l};\mathcal{F}\right).
\end{aligned} \tag{29}$$

Similarly applying [18, Definition 3.25.2 (item 2)] we ensure the equalities:

$$\mathbb{T}\mathbf{m}\left(\mathfrak{l}_0^{(1)}\right) = \mathbb{T}\mathbf{m}\left(\mathfrak{l}_0\right); \quad \mathbb{T}\mathbf{m}\left(\mathfrak{l}^{(1)}\right) = \mathbb{T}\mathbf{m}\left(\mathfrak{l}\right) \tag{30}$$

(where (in accordance with [18, Subsection 6.3]) $\mathbb{T}\mathbf{m}(\mathfrak{m}) = (\mathbf{Tm}(\mathfrak{m}), \leq_{\mathfrak{m}})$ $(\forall\, \mathfrak{m} \in \mathcal{L}k(\mathcal{F}) \cup \mathcal{L}k(\mathcal{F}_1))$). Moreover, according to [18, Property 3.25.1(1) and Definition 3.25.2 (item 3)], we obtain

$$\begin{aligned}
\left[\mathfrak{l}_0 \leftarrow \mathfrak{l},\, \mathcal{F}\right] &= \left[\mathbf{lk}_{\mathtt{ind}(\mathfrak{l}_0)}(\mathcal{F}) \leftarrow \mathbf{lk}_{\mathtt{ind}(\mathfrak{l})}(\mathcal{F}),\, \mathcal{F}\right] = \\
&= \left[\mathbf{lk}_{\mathtt{ind}(\mathfrak{l}_0)}(\mathcal{F}_1) \leftarrow \mathbf{lk}_{\mathtt{ind}(\mathfrak{l})}(\mathcal{F}_1),\, \mathcal{F}_1\right] = \\
&= \left[\mathfrak{l}_0 \downharpoonleft_{\mathcal{F}_1} \leftarrow \mathfrak{l} \downharpoonleft_{\mathcal{F}_1},\, \mathcal{F}_1\right] = \left[\mathfrak{l}_0^{(1)} \leftarrow \mathfrak{l}^{(1)},\, \mathcal{F}_1\right].
\end{aligned} \tag{31}$$

Taking into account (29), let us consider any elements $w_1, w_2 \in \mathbb{M}k\left(\mathfrak{l}^{(1)};\mathcal{F}_1\right) = \mathbb{M}k(\mathfrak{l};\mathcal{F})$ such that $\mathsf{bs}(w_1) = \mathsf{bs}(w_2)$ and $\mathsf{tm}(w_1) <_{\mathfrak{l}^{(1)}} \mathsf{tm}(w_2)$. Then, in accordance with (30), we obtain the inequality $\mathsf{tm}(w_1) <_{\mathfrak{l}} \mathsf{tm}(w_2)$. Since (as it was mentioned before) $\mathfrak{l}_0 \Uparrow^{+}_{\mathcal{F}} \mathfrak{l}$, then, by Definition 9 (item 2), we obtain the inequality $\mathsf{tm}\left([\mathfrak{l}_0 \leftarrow \mathfrak{l},\, \mathcal{F}]\, w_1\right) <_{\mathfrak{l}_0} \mathsf{tm}\left([\mathfrak{l}_0 \leftarrow \mathfrak{l},\, \mathcal{F}]\, w_2\right)$. Thence, using (31) and (30), we ensure the inequality, $\mathsf{tm}\left(\left[\mathfrak{l}_0^{(1)} \leftarrow \mathfrak{l}^{(1)},\, \mathcal{F}_1\right] w_1\right) <_{\mathfrak{l}_0^{(1)}} \mathsf{tm}\left(\left[\mathfrak{l}_0^{(1)} \leftarrow \mathfrak{l}^{(1)},\, \mathcal{F}_1\right] w_2\right)$. By Definition 9 (item 2), taking into account the arbitrariness of choice elements $w_1, w_2 \in \mathbb{M}k\left(\mathfrak{l}^{(1)}; \mathcal{F}_1\right)$ such, that $\mathsf{bs}(w_1) = \mathsf{bs}(w_2)$ and $\mathsf{tm}(w_1) <_{\mathfrak{l}^{(1)}} \mathsf{tm}(w_2)$, we obtain the correlation $\mathfrak{l}_0^{(1)} \Uparrow_{\mathcal{F}_1}^{+} \mathfrak{l}^{(1)}$ (for every reference frame $\mathfrak{l}^{(1)} \in \mathcal{L}k(\mathcal{F}_1)$). Hence, by Definition 9, kinematics $\mathcal{F}_1$ is weakly time-positive. □

Applying Assertion 9 as well as Theorem 1, we obtain the following (strengthened) variant of Theorem of Non-Returning:

Theorem 2. *Any weakly time-positive universal kinematics $\mathcal{F}$ is **certainly** time irreversible.*

7 Example of certainly time irreversible tachyon kinematics

In this section we build the certainly time-irreversible universal kinematics, which allows for reference frames moving with any speed other than the speed of light, using the generalized Lorentz-Poincare transformations in terms of E. Recami, V. Olkhovsky and R. Goldoni.

Let $(\mathfrak{H}, \|\cdot\|, \langle\cdot,\cdot\rangle)$ be a Hilbert space over the real field such, that $\dim(\mathfrak{H}) \geq 1$, where $\dim(\mathfrak{H})$ is dimension of the space $\mathfrak{H}$. Emphasize, that the condition $\dim(\mathfrak{H}) \geq 1$ should be interpreted in a way that the space $\mathfrak{H}$ may be infinite-dimensional. Let $\mathcal{L}(\mathfrak{H})$ be the space of (homogeneous) linear continuous operators over the space $\mathfrak{H}$. Denote by $\mathcal{L}^{\times}(\mathfrak{H})$ the space of all operators of affine transformations over the space $\mathfrak{H}$, that is $\mathcal{L}^{\times}(\mathfrak{H}) = \{\mathbf{A}_{[\mathbf{a}]} \mid \mathbf{A} \in \mathcal{L}(\mathfrak{H}),\ \mathbf{a} \in \mathfrak{H}\}$, where $\mathbf{A}_{[\mathbf{a}]}x = \mathbf{A}x + \mathbf{a}$, $x \in \mathfrak{H}$. The ***Minkowski space*** over the Hilbert space $\mathfrak{H}$ is defined as the Hilbert space $\mathcal{M}(\mathfrak{H}) = \mathbb{R} \times \mathfrak{H} = \{(t, x) \mid t \in \mathbb{R},\ x \in \mathfrak{H}\}$, equipped by the inner product and norm: $\langle w_1, w_2\rangle = \langle w_1, w_2\rangle_{\mathcal{M}(\mathfrak{H})} = t_1 t_2 + \langle x_1, x_2\rangle$, $\|w_1\| = \|w_1\|_{\mathcal{M}(\mathfrak{H})} = \left(t_1^2 + \|x_1\|^2\right)^{1/2}$ (where $w_i = (t_i, x_i) \in \mathcal{M}(\mathfrak{H})$, $i \in \{1, 2\}$) ([10, 18]). In the space $\mathcal{M}(\mathfrak{H})$ we select the next subspaces: $\mathfrak{H}_0 := \{(t, \mathbf{0}) \mid t \in \mathbb{R}\}$, $\mathfrak{H}_1 := \{(0, x) \mid x \in \mathfrak{H}\}$ with $\mathbf{0}$ being zero vector. Then, $\mathcal{M}(\mathfrak{H}) = \mathfrak{H}_0 \oplus \mathfrak{H}_1$, where $\oplus$ means the orthogonal sum of subspaces. Denote: $\mathbf{e}_0 := (1, \mathbf{0}) \in \mathcal{M}(\mathfrak{H})$. Introduce the orthogonal projectors on the subspaces $\mathfrak{H}_1$ and $\mathfrak{H}_0$:

$$\mathbf{X}w = (0, x) \in \mathfrak{H}_1;\ \widehat{\mathbf{T}}w = (t, \mathbf{0}) = \mathcal{T}(w)\,\mathbf{e}_0 \in \mathfrak{H}_0,$$
$$\text{where } \mathcal{T}(w) = t \quad (w = (t, x) \in \mathcal{M}(\mathfrak{H})).$$

Let $\mathbf{B}_1(\mathfrak{H}_1)$ be the unit sphere in the space $\mathfrak{H}_1$ ($\mathbf{B}_1(\mathfrak{H}_1) = \{x \in \mathfrak{H}_1 \mid \|x\| = 1\}$). Any vector $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ generates the following orthogonal projectors, acting in $\mathcal{M}(\mathfrak{H})$:

$$\mathbf{X}_1[\mathbf{n}]\,w = \langle \mathbf{n}, w\rangle\,\mathbf{n} \quad (w \in \mathcal{M}(\mathfrak{H}));$$
$$\mathbf{X}_1^{\perp}[\mathbf{n}] = \mathbf{X} - \mathbf{X}_1[\mathbf{n}].$$

Recall, that an operator $U \in \mathcal{L}(\mathfrak{H})$ is referred to as ***unitary*** on $\mathfrak{H}$, if and only if $\exists\, U^{-1} \in \mathcal{L}(\mathfrak{H})$ and $\forall\, x \in \mathfrak{H}$ $\|Ux\| = \|x\|$. Let $\mathfrak{U}(\mathfrak{H}_1)$ be the set of all ***unitary*** operators over the space $\mathfrak{H}_1$.

Fix some real number c such, that $0 < c < \infty$. Denote:

$$\mathfrak{P}\mathfrak{T}_{\mathsf{fin}}^{\mp}(\mathfrak{H}, c) := \left\{ \mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}] \;\middle|\; \begin{array}{l} \lambda \in [0, \infty) \setminus \{c\}, \\ s = \operatorname{sign}(c - \lambda), \\ J \in \mathfrak{U}(\mathfrak{H}_1),\ \mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1), \\ \mathbf{a} \in \mathcal{M}(\mathfrak{H}) \end{array} \right\}, \quad (32)$$

where $\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}] \in \mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$ ($\lambda \in [0, \infty) \setminus \{c\}$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$, $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$) are operators of generalized Lorentz-Poincare Transformations in the sense of E. Recami, V. Olkhovsky and R. Goldoni, introduced in [10, 11, 18]:

$$\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}]w = \mathbf{W}_{\lambda,c}[s, \mathbf{n}, J](w + \mathbf{a}), \quad \text{where}$$
$$\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J]w = \frac{\left(s\mathcal{T}(w) - \frac{\lambda}{c^2}\langle \mathbf{n}, w\rangle\right)}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}}\mathbf{e}_0 + $$
$$+ J\left(\frac{\lambda\mathcal{T}(w) - s\langle \mathbf{n}, w\rangle}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}}\mathbf{n} + \mathbf{X}_1^{\perp}[\mathbf{n}]\,w\right). \quad (33)$$

According to [18, 20], every operator of kind $\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}]$ belongs to $\mathbf{Pk}(\mathfrak{H})$, where $\mathbf{Pk}(\mathfrak{H})$ is the set of all operators $\mathbf{S} \in \mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$, which have the continuous inverse operator $\mathbf{S}^{-1} \in \mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$. Using results of the papers [18, 20], we can calculate the operators, inverse to the operators of kind $\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J]$ and $\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}]$.

Lemma 1. *For arbitrary $c \in (0, \infty)$, $\lambda \in [0, \infty) \setminus \{c\}$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$ and $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ the following equality holds:*

$$(\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J])^{-1} =$$
$$= \mathbf{W}_{\lambda,c}\left[s\operatorname{sign}(c - \lambda), \operatorname{sign}(c - \lambda)J\mathbf{n}, J^{-1}\right]. \quad (34)$$

Proof. Consider arbitrary $0 < c < \infty$, $\lambda \in [0, \infty) \setminus \{c\}$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$ and $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$. According to [10, page 143] or [18, formula (2.86)], operator $\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J]$ may be represented in the form:

$$\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J] = \mathbf{U}_{\theta,c}[s, \mathbf{n}, J], \quad (35)$$

where

$$\theta = \frac{1 - \frac{\lambda}{c}}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}} \quad \left(\lambda = c\frac{1 - \theta|\theta|}{1 + \theta|\theta|}\right),\ -1 \leq \theta \leq 1.$$

Hence, according to [20, Corollary 5.1] or [18, Corollary 2.18.3], we obtain, that $(\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J])^{-1} \in \mathcal{L}(\mathcal{M}(\mathfrak{H}))$, and moreover:

$$(\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J])^{-1} = (\mathbf{U}_{\theta,c}[s, \mathbf{n}, J])^{-1} =$$
$$= \mathbf{U}_{\theta^s,c}\left[s_\theta, s_\theta J\mathbf{n}, J^{-1}\right], \quad (36)$$

$$\text{where } s_\theta = \mathfrak{S}(s, \theta) = \begin{cases} 1, & s, \theta > 0 \\ -1, & s < 0 \text{ or } \theta < 0. \end{cases}$$

In the case $s = 1$ we have, $s_\theta = \operatorname{sign}\theta = \operatorname{sign}\left(\frac{1-\frac{\lambda}{c}}{\sqrt{\left|1-\frac{\lambda^2}{c^2}\right|}}\right) = \operatorname{sign}(c-\lambda)$. Hence, in this case, using (36) and (35), we obtain

$$\begin{aligned}(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J])^{-1} &= \mathbf{U}_{\theta,c}\left[s_\theta, s_\theta J\mathbf{n}, J^{-1}\right] = \\ &= \mathbf{W}_{\lambda,c}\left[s_\theta, s_\theta J\mathbf{n}, J^{-1}\right] = \\ &= \mathbf{W}_{\lambda,c}\left[\operatorname{sign}(c-\lambda), \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right] \quad (s=1). \end{aligned} \tag{37}$$

Now we consider the case $s = -1$ $(\theta^s = \theta^{-1})$. Applying (36) and [18, formula (2.90)], in this case we deduce

$$\begin{aligned}(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J])^{-1} &= \mathbf{U}_{\theta^{-1},c}\left[s_\theta, s_\theta J\mathbf{n}, J^{-1}\right] = \\ &= \mathbf{U}_{\theta,c}\left[s_\theta \operatorname{sign}\theta, -s_\theta(\operatorname{sign}\theta)J\mathbf{n}, J^{-1}\right] = \\ &= \mathbf{U}_{\theta,c}\left[-\operatorname{sign}\theta, (\operatorname{sign}\theta)J\mathbf{n}, J^{-1}\right] = \\ &= \mathbf{U}_{\theta,c}\left[-\operatorname{sign}(c-\lambda), \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right] = \\ &= \mathbf{W}_{\lambda,c}\left[-\operatorname{sign}(c-\lambda), \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right] \\ &\qquad (s=-1). \end{aligned} \tag{38}$$

Taking into account (37) and (38) in the both cases we obtain (34). □

Using Lemma 1, we obtain the following corollary.

Corollary 4. *For arbitrary $c \in (0,\infty)$, $\lambda \in [0,\infty)\setminus\{c\}$, $s \in \{-1,1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ and $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$ the following equality is fulfilled:*

$$\begin{aligned}(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}])^{-1}\,\mathrm{w} &= \\ = \mathbf{W}_{\lambda,c}&\left[s\operatorname{sign}(c-\lambda), \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right]\mathrm{w} - \mathbf{a} \\ &\qquad (\mathrm{w} \in \mathcal{M}(\mathfrak{H})).\end{aligned}$$

Let $\mathcal{B}$ be any base changeable set such, that $\mathfrak{Bs}(\mathcal{B}) \subseteq \mathfrak{H}$ and $\mathbb{T}\mathbf{m}(\mathcal{B}) = (\mathbb{R},\leq)$, where $\leq$ is the standard order in the field of real numbers $\mathbb{R}$. Denote:

$$\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c) := \mathfrak{Ku}\left(\mathfrak{PT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},c), \mathcal{B}; \mathfrak{H}\right), \tag{39}$$

where the denotation $\mathfrak{Ku}(\cdot,\cdot;\cdot)$ is introduced in [11], [18, page 166]. From [18, Assertion 2.17.5] it follows, that in the case $\dim(\mathfrak{H}) = 3$ universal kinematics $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)$ may be considered as tachyon extension of kinematics of classical special relativity, which allows for reference frames moving with arbitrary speed other than the speed of light.

According to [18, Property 3.23.1(1)], the set $\mathcal{L}k\left(\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)\right)$ of all reference frames of universal kinematics $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)$, defined by (39), can be represented in the form:

$$\begin{aligned}\mathcal{L}k\left(\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)\right) &= \\ &= \left\{(\mathbf{U},\mathbf{U}[\mathcal{B},\mathbb{T}\mathbf{m}(\mathcal{B})]) \mid \mathbf{U} \in \mathfrak{PT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},c)\right\} = \\ &= \left\{(\mathbf{U},\mathbf{U}[\mathcal{B}]) \mid \mathbf{U} \in \mathfrak{PT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},c)\right\}.\end{aligned} \tag{40}$$

In accordance with [18, Corollary 2.19.5], subclass of operators

$$\begin{aligned}\mathfrak{P}_+(\mathfrak{H},c) &= \\ &= \left\{\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}] \;\middle|\; \begin{array}{l}\lambda \in [0,c),\ s=1,\\ J \in \mathfrak{U}(\mathfrak{H}_1),\\ \mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1),\ \mathbf{a} \in \mathcal{M}(\mathfrak{H})\end{array}\right\} \subseteq \\ &\qquad \subseteq \mathfrak{PT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},c)\end{aligned}$$

is group of operators over the space $\mathcal{M}(\mathfrak{H})$. So, the identity operator $\mathbb{I}_{\mathcal{M}(\mathfrak{H})}\mathrm{w} = \mathrm{w}$ $(\forall \mathrm{w} \in \mathcal{M}(\mathfrak{H}))$ belongs to the class $\mathfrak{PT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},c)$. Hence, in accordance with (40), we may define the following reference frame:

$$\begin{aligned}\mathfrak{l}_{0,\mathcal{B}} &:= (\mathbb{I}_{\mathcal{M}(\mathfrak{H})}, \mathbb{I}_{\mathcal{M}(\mathfrak{H})}[\mathcal{B}]) = \\ &= (\mathbb{I}_{\mathcal{M}(\mathfrak{H})}, \mathcal{B}) \in \mathcal{L}k\left(\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)\right)\end{aligned} \tag{41}$$

(recall, that, according to [18, Remark 1.11.3], $\mathbb{I}_{\mathcal{M}(\mathfrak{H})}[\mathcal{B}] = \mathcal{B}$).

Lemma 2. *For each reference frame $\mathfrak{l} \in \mathcal{L}k\left(\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)\right)$ the following correlation holds:*

$$\mathfrak{l}_{0,\mathcal{B}} \Uparrow^{+}_{\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)} \mathfrak{l}.$$

Proof. Consider any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{UPT}^{\mp}(\mathfrak{H},\mathcal{B},c))$. According to (40) and (32), frame $\mathfrak{l}$ can be represented in the form:

$$\mathfrak{l} = (\mathbf{U},\mathbf{U}[\mathcal{B}]), \quad \text{where} \tag{42}$$

$$\begin{aligned}\mathbf{U} = \mathbf{W}_{\lambda,c}&[\operatorname{sign}(c-\lambda),\mathbf{n},J;\mathbf{a}], \\ &0 \leq \lambda < +\infty,\ \lambda \neq c, \\ \mathbf{n} \in \mathbf{B}_1&(\mathfrak{H}_1),\ J \in \mathfrak{U}(\mathfrak{H}_1),\ \mathbf{a} \in \mathcal{M}(\mathfrak{H}).\end{aligned} \tag{43}$$

Applying [18, Properties 3.23.1(3,4,7)] as well (42), (43), (41) and Corollary 4 we obtain

$$\mathbb{T}\mathbf{m}(\mathfrak{l}) = \mathbb{T}\mathbf{m}(\mathfrak{l}_{0,\mathcal{B}}) = \mathbb{T}\mathbf{m}(\mathcal{B}) = (\mathbb{R},\leq); \tag{44}$$

$$\begin{aligned}\mathbb{M}k(\mathfrak{l}) &= \mathbb{M}k(\mathfrak{l}_{0,\mathcal{B}}) = \mathbf{Tm}(\mathcal{B}) \times \mathfrak{H} = \\ &= \mathbb{R} \times \mathfrak{H} = \mathcal{M}(\mathfrak{H}); \\ [\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w} &= \mathbb{I}_{\mathcal{M}(\mathfrak{H})}\mathbf{U}^{-1}\mathrm{w} = \\ &= (\mathbf{W}_{\lambda,c}[\operatorname{sign}(c-\lambda),\mathbf{n},J;\mathbf{a}])^{-1}\,\mathrm{w} = \\ &= \mathbf{W}_{\lambda,c}\left[(\operatorname{sign}(c-\lambda))^2, \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right]\mathrm{w} - \mathbf{a} = \\ &= \mathbf{W}_{\lambda,c}\left[1, \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right]\mathrm{w} - \mathbf{a} \\ &\qquad (\mathrm{w} \in \mathbb{M}k(\mathfrak{l}) = \mathcal{M}(\mathfrak{H})).\end{aligned} \tag{45}$$

Now we consider any $\mathrm{w}_1, \mathrm{w}_2 \in \mathbb{M}k(\mathfrak{l}) = \mathcal{M}(\mathfrak{H})$ such that $\mathsf{bs}(\mathrm{w}_1) = \mathsf{bs}(\mathrm{w}_2)$ and $\mathsf{tm}(\mathrm{w}_1) <_{\mathfrak{l}} \mathsf{tm}(\mathrm{w}_2)$. According to (44), inequality $\mathsf{tm}(\mathrm{w}_1) <_{\mathfrak{l}} \mathsf{tm}(\mathrm{w}_2)$ is equivalent to the inequality $\mathsf{tm}(\mathrm{w}_1) < \mathsf{tm}(\mathrm{w}_2)$. From the equality $\mathsf{bs}(\mathrm{w}_1) = \mathsf{bs}(\mathrm{w}_2)$ it follows that

$$\mathbf{X}(\mathrm{w}_2 - \mathrm{w}_1) = \\ = \mathbf{X}(\mathsf{tm}(\mathrm{w}_2) - \mathsf{tm}(\mathrm{w}_1), \mathsf{bs}(\mathrm{w}_2) - \mathsf{bs}(\mathrm{w}_1)) = \\ = (0, \mathsf{bs}(\mathrm{w}_2) - \mathsf{bs}(\mathrm{w}_1)) = \mathbf{0}.$$

Thence, using (45) and (33) we deduce

$$\begin{aligned} &\mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_2) - \mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) = \\ &= \mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_2 - [\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) = \\ &= \mathsf{tm}\Big(\mathbf{W}_{\lambda,c}\left[1, \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right]\mathrm{w}_2 - \\ &- \mathbf{W}_{\lambda,c}\left[1, \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right]\mathrm{w}_1\Big) = \\ &= \mathsf{tm}\Big(\mathbf{W}_{\lambda,c}\left[1, \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right](\mathrm{w}_2 - \mathrm{w}_1)\Big) = \\ &= \mathcal{T}\Big(\mathbf{W}_{\lambda,c}\left[1, \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right](\mathrm{w}_2 - \mathrm{w}_1)\Big) = \\ &= \frac{\mathcal{T}(\mathrm{w}_2 - \mathrm{w}_1) - \frac{\lambda}{c^2}\langle \operatorname{sign}(c-\lambda)J\mathbf{n}, \mathrm{w}_2 - \mathrm{w}_1\rangle}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}} = \\ &= \frac{\mathcal{T}(\mathrm{w}_2 - \mathrm{w}_1) - \frac{\lambda}{c^2}\langle \operatorname{sign}(c-\lambda)\mathbf{X}J\mathbf{n}, \mathrm{w}_2 - \mathrm{w}_1\rangle}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}} = \\ &= \frac{\mathcal{T}(\mathrm{w}_2 - \mathrm{w}_1) - \frac{\lambda}{c^2}\langle \operatorname{sign}(c-\lambda)J\mathbf{n}, \mathbf{X}(\mathrm{w}_2 - \mathrm{w}_1)\rangle}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}} = \\ &= \frac{\mathcal{T}(\mathrm{w}_2 - \mathrm{w}_1)}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}} = \frac{\mathcal{T}(\mathrm{w}_2) - \mathcal{T}(\mathrm{w}_1)}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}} > 0 \end{aligned}$$

Therefore, $\mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) < \mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$, ie, according to (44), we have, $\mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) <_{\mathfrak{l}_{0,\mathcal{B}}} \mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$. Thus, for arbitrary $\mathrm{w}_1, \mathrm{w}_2 \in \mathbb{M}k(\mathfrak{l}) = \mathcal{M}(\mathfrak{H})$ such, that $\mathsf{bs}(\mathrm{w}_1) = \mathsf{bs}(\mathrm{w}_2)$ and $\mathsf{tm}(\mathrm{w}_1) <_{\mathfrak{l}} \mathsf{tm}(\mathrm{w}_2)$ it is true the inequality $\mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) <_{\mathfrak{l}_{0,\mathcal{B}}} \mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$. And, taking into account Definition 9 (item 2), we have seen, that $\mathfrak{l}_{0,\mathcal{B}} \Uparrow^{+}_{\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)} \mathfrak{l}$. □

Corollary 5. *Every universal kinematics of kind* $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)$ $(0 < c < \infty)$ *is certainly time irreversible.*

Proof. According to Lemma 2 and Definition 9 (item 5), kinematics of kind $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)$ $(0 < c < \infty)$ is weakly time-positive. Hence, by Theorem 2, kinematics $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)$ is certainly time irreversible. □

Remark 4. Kinematics of kind $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)$ $(0 < c < \infty)$ is weakly time-positive, but it is not time-positive. Similarly to Lemma 2 it can be proved, that for any (superluminal) reference frame of kind:

$$\begin{aligned} &\mathfrak{l} = (\mathbf{U}, \mathbf{U}[\mathcal{B}]) \in \mathcal{L}k\left(\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)\right), \quad \text{where} \\ &\mathbf{U} = \mathbf{W}_{\lambda,c}[\operatorname{sign}(c-\lambda), \mathbf{n}, J; \mathbf{a}] = \mathbf{W}_{\lambda,c}[-1, \mathbf{n}, J; \mathbf{a}], \\ &c < \lambda < +\infty,\ \mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1),\ J \in \mathfrak{U}(\mathfrak{H}_1),\ \mathbf{a} \in \mathcal{M}(\mathfrak{H}) \end{aligned}$$

the correlation $\mathfrak{l} \Downarrow^{-}_{\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)} \mathfrak{l}_{0,\mathcal{B}}$ is true despite the fact that $\mathfrak{l}_{0,\mathcal{B}} \Uparrow^{+}_{\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)} \mathfrak{l}$ (according to Lemma 2).

Remark 5. It is easy to see that the binary relation $\Uparrow^{+}_{\mathcal{F}}$ is reflexive on the set $\mathcal{L}k(\mathcal{F})$ of all reference frames of arbitrary universal kinematics $\mathcal{F}$. From Remark 4 it follows that in the general case this relation is not symmetric. Using the results of [10, Section 7, paragraph 4] it can be proven that this relation is not transitive in the general case.

8 On the physical interpretation of main result

The aim of this section is to explain main Theorem 2 in the physical language. We can imagine, that any universal kinematics $\mathcal{F}$ is some abstract "world", which not necessarily coincides with the our. In every such "world" $\mathcal{F}$ there exists the fixed for this "world" set of reference frames $\mathcal{L}k(\mathcal{F})$. We reach the agreement that for any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ the arrows of the clock, fixed in the frame $\mathfrak{l}$ are rotating clockwise relatively the frame $\mathfrak{l}$. We say, that the reference frame $\mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ is time-positive relatively the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ (ie $\mathfrak{m} \Uparrow^{+}_{\mathcal{F}} \mathfrak{l}$) if and only if the observer in the reference frame $\mathfrak{m}$ (fixed relatively $\mathfrak{m}$) observes that the arrows of the clock, fixed in the frame $\mathfrak{l}$ are rotating clockwise in the frame $\mathfrak{m}$ as well (cf. Definition 9, item 2). We abandon the physical question, how can the observer in $\mathfrak{m}$ "see" the clock, fixed in the other frame $\mathfrak{l}$. From the mathematical point of view, the possibility of observation the clock, attached to another reference frame, is guaranteed by existence of universal coordinate transform between every two reference frames (see definition of universal kinematics in [11, 18]). According to Remark 5, the binary relation $\Uparrow^{+}_{\mathcal{F}}$ always is reflexive, but, in the general case, it is not symmetric and is not transitive on the set $\mathcal{L}k(\mathcal{F})$ of all reference frames of the "world" $\mathcal{F}$.

We also suppose, that in the "world" $\mathcal{F}$ the interframe voyagers can exist. Such voyagers may move from one reference frame to the another frame, passing near them (similarly as, standing near the tram track, we can jump into the tram, passing near us).

From the physical point of view Theorem 2 asserts, that *if in the "world"* $\mathcal{F}$ *there exists at least one reference frame* $\mathfrak{l}_0 \in \mathcal{L}k(\mathcal{F})$*, which is time-positive relatively the every frame* $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$*, then in this "world" the temporal paradoxes, connected with the possibility of the returning to the own past are impossible*. This means, that any interframe voyager, starting in some reference frame $\mathfrak{l}$ in some fixed point x can not finish its travel in the frame $\mathfrak{l}$ and in the point x at the past time.

9 Conclusions

1. According to Corollary 5, kinematics of kind $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)$ (in the case $\dim(\mathfrak{H}) = 3$) gives the example of certainly time-irreversible tachyon extension of kinematics of classical special relativity, which

allows for reference frames moving with arbitrary velocity other than the velocity of light. Thus, the main conclusion of Theorem 2 is the following:

In the general case the hypothesis of existence of material objects and inertial reference frames, moving with the velocity, greater than the velocity of light, does not lead to temporal paradoxes, connected with existence of formal possibility of returning to the own past.

2. In [9] authors have deduced two variants of generalized superluminal Lorentz transforms for the case, when two inertial frames are moving along the common x-axis:

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{\left(\frac{v}{c}\right)^2 - 1}}, \quad x' = \frac{x - vt}{\sqrt{\left(\frac{v}{c}\right)^2 - 1}}, \quad \begin{array}{l} y' = y, \\ z' = z, \end{array} \qquad (46)$$

where $v \in \mathbb{R}$, $|v| > c$ (see [9, formula (3.16)]) and:

$$t' = \frac{-t + \frac{vx}{c^2}}{\sqrt{\left(\frac{v}{c}\right)^2 - 1}}, \quad x' = \frac{-x + vt}{\sqrt{\left(\frac{v}{c}\right)^2 - 1}}, \quad \begin{array}{l} y' = y, \\ z' = z \end{array} \qquad (47)$$

(see [9, formula (3.18)]). Transforms (46) are particular cases of the transforms of kind (33) for the case, where $\dim(\mathfrak{H}) = 3$, $\lambda > c$ and $s = 1$, whereas transforms (47) belong to the transforms of kind (33) for the case, where $\dim(\mathfrak{H}) = 3$, $\lambda > c$ and $s = -1$. If we chose in (33) the value $s = 1$ for subluminal as well as superluminal diapason, we obtain the class of operators $\mathfrak{PT}_+(\mathfrak{H}, c)$, defined in [13, 18] and based on this class of operators universal kinematics of kind $\mathfrak{UPT}(\mathfrak{H}, \mathcal{B}, c)$. According to results, announced in [19] and published in [12], this kinematics is conditionally time reversible [1]. But, if we chose in (33) the value $s = 1$ for subluminal diapason and value $s = -1$ for superluminal diapason, we reach the class of operators $\mathfrak{PT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, c)$, defined in (32) and based on this class of operators universal kinematics of kind $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)$. According to Corollary 5, kinematics $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)$ is certainly time irreversible. Thus we can formulate the following conclusion, concerning two variants of superluminal Lorentz transforms, deduced in [9]:

From the standpoint of time-irreversibility, transforms (47) or [9, formula (3.18)] are more suitable for representation of the tachyon continuation of Einstein's special theory of relativity than (46) or [9, formula (3.16)].

Main results of this paper had been announced in [19].

Received on September 19, 2017

[1] In fact, class of operators $\mathfrak{PT}_+(\mathfrak{H}, c)$ contains apart from operators of kind (33) (with $s = 1$) also operators, corresponding tachyon inertial reference frames with infinite velocities. However, using results of the paper [12], it is not hard to deduce that the "subkinematics" of kinematics $\mathfrak{UPT}(\mathfrak{H}, \mathcal{B}, c)$, which includes only all reference frames from $\mathfrak{UPT}(\mathfrak{H}, \mathcal{B}, c)$ with finite velocities, also is conditionally time reversible.

References

1. Bilaniuk O.-M. P., Deshpande V.K., Sudarshan E.C.G. "Meta" Relativity. *American Journal of Physics*, 1962. v. 30, no. 10, 718–723.
2. Bilaniuk O.-M. P. , Sudarshan E.C.G. Particles beyond the light barrier. *Physics Today*, 1969, v. 22, no. 5, 43–51.
3. V.I. Ilyashevytch, S.Y. Medvedev. Tachyon acausal loops. *Uzhhorod University Scientific Herald. Series Physics*, 2010, no. 27, 103-111 (in Ukrainian).
4. Andréka H., Madarász J.X., Németi I., Stannett M., Székely G. Faster than light motion does not imply time travel. *Classical and Quantum Gravity*, 2014, v. 31, no. 9, 095005.
5. Recami E., Olkhovsky V.S. About Lorentz transformations and tachyons. *Lettere al Nuovo Cimento*, 1971, v. 1, no. 4, 165–168.
6. Goldoni R. Faster-than-light inertial frames, interacting tachyons and tadpoles. *Lettere al Nuovo Cimento*, 1972, v. 5, no. 6, 495–502.
7. Recami E. Classical Tachyons and Possible Applications. *Riv. Nuovo Cimento*, 1986, v. 9, s. 3, no. 6, 1–178.
8. Medvedev S.Yu. On the possibility of broadening special relativity theory beyond light barrier. *Uzhhorod University Scientific Herald. Ser. Phys.*, 2005, no. 18, 7–15 (in Ukrainian).
9. Hill J.M., Cox B.J. Einstein's special relativity beyond the speed of light. *Proc. of the Royal Society A*, 2012, v. 468, no. 2148, 4174–4192.
10. Grushka Ya.I. Tachyon generalization for lorentz transforms. *Methods of Functional Analysis and Topology*, 2013, v. 20, no. 2, 127–145.
11. Grushka Ya.I. Kinematic changeable sets with given universal coordinate transform. *Proceedings of Institute of Mathematics NAS of Ukraine (Zb. Pr. Inst. Mat. NAN Ukr.)*, 2015, v. 12, no. 1, 74–118 (in Ukrainian).
12. Grushka Ya.I. On time reversibility of tachyon kinematics. *Proceedings of Institute of Mathematics NAS of Ukraine (Zb. Pr. Inst. Mat. NAN Ukr.)*, 2016, v. 13, no. 2, 125–174 (in Ukrainian).
13. Grushka Ya.I. Changeable sets and their application for the construction of tachyon kinematics. *Proceedings of Institute of Mathematics NAS of Ukraine (Zb. Pr. Inst. Mat. NAN Ukr.)*, 2014, v. 11, no. 1, 192–227 (in Ukrainian).
14. Grushka Ya.I. Existence criteria for universal coordinate transforms in kinematic changeable sets. *Bukovinian Mathematical Journal*, 2014, v. 2, no. 2-3, 59–71 (in Ukrainian).
15. Grushka Ya.I. Abstract concept of changeable set. Preprint: arXiv:1207.3751v1, (2012), 54 p.
16. Grushka Ya.I. Coordinate transforms in kinematic changeable sets. *Reports of the National Academy of Sciences of Ukraine*, 2015, no. 3, 24–31, (in Ukrainian).
17. Grushka Ya.I. Evolutionary extensions of kinematic sets and universal kinematics. *Proceedings of Institute of Mathematics NAS of Ukraine (Zb. Pr. Inst. Mat. NAN Ukr.)*, 2015, v. 12, no. 2, 139–204 (in Ukrainian).
18. Grushka Ya.I. Draft introduction to abstract kinematics. (Version 2.0). Preprint: viXra: 1701.0523v2, (2017), 208 p.
19. Grushka Ya.I. On time irreversibility of universal kinematics. *Reports of the National Academy of Sciences of Ukraine*, 2016, no. 7, 14–21, (in Ukrainian).
20. Grushka Ya.I. Algebraic properties of tachyon lorentz transforms. *Proceedings of Institute of Mathematics NAS of Ukraine (Zb. Pr. Inst. Mat. NAN Ukr.)*, 2013, v. 10, no. 2, 138–169 (in Ukrainian).

Chain Systems of Harmonic Quantum Oscillators as a Fractal Model of Matter and Global Scaling in Biophysics

Hartmut Müller

E-mail: hm@interscalar.com

In this paper we introduce chain systems of harmonic quantum oscillators as a fractal model of matter and apply it to the analysis of frequency ranges of cyclical biological processes. The heuristic significance of global scaling in biophysics and medicine is discussed.

Introduction

Normal matter is formed by nucleons and electrons because they are exceptionally stable. Their lifespan tops everything that is measurable, exceeding 10^{29} years for the proton and 10^{28} years for the electron [1]. The proton-to-electron mass ratio is approximately 1836, so that the mass contribution of the proton to normal matter is very high, for example in the hydrogen atom (protium) it is $1 - 1/1836 \approx 99.95$ percent. Consequently, the mass contribution of the electron is only 0.05 percent. In heavier atoms which contain neutrons, the electron contribution to atomic mass is even smaller.

In addition, protons and neutrons have similar rest masses (the difference being only 0.14 percent) which allows us to interpret the proton and the neutron as similar quantum oscillators with regard to their rest masses. the framework of the standard particle model [2], protons and neutrons are baryons, with the proton connecting to a lower quantum energy level and a much more stable state than the neutron.

Therefore, in [3] we have introduced a fractal model of matter as a chain system of oscillating protons. In [4] we have shown that scale invariance is a fundamental property of this model. As a consequence of this scale invariance, chain systems of oscillating electrons produce similar series of eigenstates so that the proton model mass can be derived from the electron rest mass and vice versa. Furthermore, the interpretation of the Planck mass as an eigenstate in a chain system of oscillating protons has allowed us to derive the proton rest mass from fundamental physical constants [5].

Scale-invariant models of natural oscillations in chain systems of protons also provide a good description of the mass distribution of large celestial bodies in the Solar System [6]. Physical properties of celestial bodies such as mass, size, rotation and orbital period can be understood as macroscopic quantized eigenstates in chain systems of oscillating protons and electrons [7]. This understanding can be applied to an evolutionary trend prognosis of the Solar System but may be of cosmological significance as well. In [8] we have calculated the model masses of unknown planets in the Solar System.

In this paper we apply our fractal model of matter as a chain system of oscillating protons and our hypothesis of global scaling [7] to the domain of biophysics, especially to the analysis of frequency ranges of cyclical biological processes.

Methods

In [4] we have shown that the set of natural frequencies of a chain system of harmonic oscillators coincides with a set of finite continued fractions $\mathcal{F}$, which are natural logarithms:

$$\ln(\omega_{jk}/\omega_{00}) = n_{j0} + \cfrac{z}{n_{j1} + \cfrac{z}{n_{j2} + \ddots + \cfrac{z}{n_{jk}}}} = [z, n_{j0}; n_{j1}, n_{j2}, \ldots, n_{jk}] = \mathcal{F}, \quad (1)$$

where ω_{jk} is the set of angular frequencies and ω_{00} is the fundamental frequency of the set. The denominators are integer: $n_{j0}, n_{j1}, n_{j2}, \ldots, n_{jk} \in \mathrm{Z}$, the cardinality $j \in \mathrm{N}$ of the set and the number $k \in \mathrm{N}$ of layers are finite. In the canonical form, the numerator z is equal 1.

Any finite continued fraction represents a rational number [9]. Therefore, all frequencies ω_{jk} in (1) are irrational, because for rational exponents the natural exponential function is transcendental [10]. This circumstance presumably provides for the high stability of the oscillating chain system because it avoids resonance interaction between the elements of the system [11].

In the case of harmonic quantum oscillators, the continued fraction (1) defines not only a fractal set of natural angular frequencies ω_{jk} and oscillation periods $\tau_{jk} = 1/\omega_{jk}$ of the chain system, but also fractal sets of natural energies $E_{jk} = \hbar \cdot \omega_{jk}$ and masses $m_{jk} = E_{jk}/c^2$ which correspond with the eigenstates of the system. For this reason, we have called the continued fraction (1) the "fundamental fractal" of eigenstates in chain systems of harmonic quantum oscillators [4].

We hypothesize the scale invariance based on the fundamental fractal $\mathcal{F}$ (1) , calibrated by the properties of the proton and electron, is a universal characteristic of matter. This hypothesis we have called 'global scaling' [7].

In order to test global scaling on frequencies of cyclical biological processes we must calculate the natural logarithm of the process-to-proton frequency ratio. The proton angular frequency is $\omega_p = m_p c^2/\hbar = 1.425486 \cdot 10^{24}$ Hz,

Table 1: Frequency ranges of some cyclical biological processes and the corresponding attractor nodes of the fundamental fractal $\mathcal{F}$ (1), with the proton frequency $\omega_p = 1.425486 \cdot 10^{24}$ Hz as fundamental.

cyclic process of human physiology	**frequency range ω, Hz**	**ln (ω / ω_p)**	$\mathcal{F}$
adult relaxed breathing [13]	0.22..0.27	−57.13.. − 56.94	[−57; ∞]
adult relaxed heart rate [14]	0.83..1.5	−55.80.. − 55.21	[−55; −2]
brain activity delta	0.15..3	−57.52.. − 54.52	[−57; −2]..[−54; −2]
brain activity theta [12]	3..8	−54.52.. − 53.53	[−54; −2]..[−54; 2]
brain activity alpha	8..13	−53.53.. − 53.06	[−53; −2]..[−53; ∞]
brain activity beta	14..34	−52.97.. − 52.06	[−53; ∞]..[−52; ∞]
brain activity gamma	35..250	−52.05.. − 50.10	[−52; ∞]..[−50; ∞]
muscle vibration [15]	22..24	−52.53.. − 52.44	[−52; −2]
flicker fusion threshold [16]	60..120	−51.52.. − 50.83	[−51; −2]..[−51; ∞]
newborn baby cry [17]	400..500	−49.62.. − 49.41	[−49; −2]
threshold of hearing [18, 19]	1900..2100	−40.55.. − 40.45	[−40; −2]

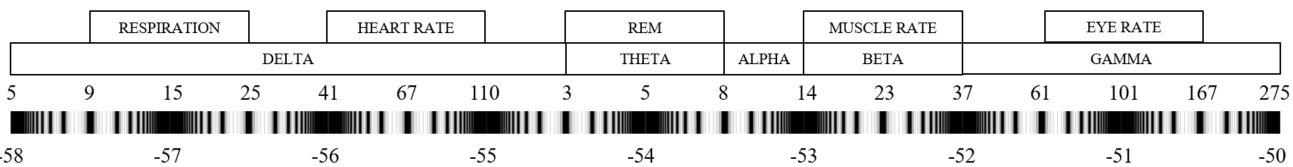

Fig. 1: Distribution (logarithmic representation) of frequency ranges (positive numbers) of human brain wave activity and other cyclical biological processes in the canonical projection of the fundamental fractal $\mathcal{F}$ (1) with the proton angular frequency $\omega_p = 1.42548624 \cdot 10^{24}$ Hz as fundamental. Negative numbers are logarithms and denote attractor nodes. Data taken from table 1.

where $m_p = 1.672621 \cdot 10^{-27}$ kg [1] is the proton rest mass, $\hbar$ is the Planck constant, c is the speed of light in vacuum. In the canonical form ($z = 1$), nodes of the fundamental fractal $\mathcal{F}$ (1) concur with integer and half logarithms.

For example, the frequency range of the theta electrical brain activity (theta waves, oscillatory pattern in electroencephalographic signals) is between 3 and 8 Hz [12] and the natural logarithm of the theta-to-proton frequency ratio is between $[-54; -2]$ and $[-54; 2]$ approximating the main node $[54; \infty]$ of the proton calibrated fundamental fractal $\mathcal{F}$ (1):

$$\ln\left(\frac{\omega_{\max\, theta}}{\omega_{\text{proton}}}\right) = \ln\left(\frac{8\,\text{Hz}}{1.425486 \cdot 10^{24}\,\text{Hz}}\right) = -53.53,$$

$$\ln\left(\frac{\omega_{\min\, theta}}{\omega_{\text{proton}}}\right) = \ln\left(\frac{3\,\text{Hz}}{1.425486 \cdot 10^{24}\,\text{Hz}}\right) = -54.52.$$

Results

Table 1 shows the logarithms of frequency ranges of some cyclical biological processes and the corresponding attractor nodes (integer and half logarithms) of the fundamental fractal $\mathcal{F}$ (1).

Figure 1 shows the distribution (in logarithmic representation) of frequency ranges of brain wave activity and other cyclical processes of human physiology in the fundamental fractal $\mathcal{F}$ (1) with the proton angular frequency $\omega_p = 1.42548624 \cdot 10^{24}$ Hz as fundamental. Negative numbers are logarithms and denote attractor nodes. Positive numbers are frequencies, given in cycles per minute within the delta-range, and given in Hz within the theta, alpha, beta and gamma ranges.

Although the analyzed processes are of very high complexity, figure 1 shows that the frequency ranges of electrical brain activity (oscillatory patterns in electroencephalographic signals) and of other cyclical biological processes correspond with attractor nodes of the fundamental fractal $\mathcal{F}$ (1). This fact supports our hypothesis of global scaling.

Conclusion

Frequency ranges of electrical brain activity and of some other cyclical biological processes coincide well with the proton calibrated fundamental fractal $\mathcal{F}$ (1) which would indicate that these cycles may have a subatomic origin. It should also be considered that the frequency ranges of electrical brain activity are common to most mammalian species [20, 21].

The accordance of the brain wave frequency ranges with the proton calibrated fundamental fractal $\mathcal{F}$ (1) not only supports our hypothesis of global scaling, but also suggests an understanding of the biological organism as an oscillating chain system. This view could be of medical significance as well.

Scale invariance as a property of biological processes is well studied [22, 23] and it is not an exclusive characteristic of adult physiology. For example, the heart rate and the respiratory cycle of the fetus are related in the same way as in the adult [24]. Perhaps even the Weber-Fechner law – "intensity of sensation is proportional to the logarithm of stimulation" [25] – can be understood as a consequence of scale invariance in chain systems of cyclical biological processes.

Furthermore, global scaling suggests that the electrical brain activity continues beyond the known gamma range, because higher frequency processes like voice and hearing have to be brain-controlled as well. It is likely that traditional methods of electroencephalographic signal analysis are unable to separate high frequency patterns because of their very low amplitude. However, global scaling allows us to calculate the frequency ranges of such ultra-gamma activity (for which we propose the name "epsilon"). The frequency ranges of this very dynamic "epsilon" activity should be between $\omega_p \exp(-50) = 275$ Hz and $\omega_p \exp(-49) = 747$ Hz.

Acknowledgements

The author is grateful to Viktor Panchelyuga and Leili Khosravi for valuable discussions.

Received on September 19, 2017

References

1. Olive K.A. et al. (Particle Data Group), *Chin. Phys. C*, 2016, v. 38, 090001.
2. Patrignani C. et al. (Particle Data Group), *Chin. Phys. C*, 2016, v. 40, 100001.
3. Müller H. Fractal Scaling Models of Natural Oscillations in Chain Systems and the Mass Distribution of Particles. *Progress in Physics*, 2010, v. 3, 61–66.
4. Müller H. Fractal Scaling Models of Resonant Oscillations in Chain Systems of Harmonic Oscillators. *Progress in Physics*, 2009, v. 2, 72–76.
5. Müller H. Emergence of Particle Masses in Fractal Scaling Models of Matter. *Progress in Physics*, 2012, v. 4, 44–47.
6. Müller H. Fractal scaling models of natural oscillations in chain systems and the mass distribution of the celestial bodies in the Solar System. *Progress in Physics*, 2010, v. 3, 61–66.
7. Müller H. Scale-Invariant Models of Natural Oscillations in Chain Systems and their Cosmological Significance. *Progress in Physics*, 2017, v. 4, 187–197.
8. Müller H. Global Scaling as Heuristic Model for Search of Additional Planets in the Solar System.
9. Khintchine A. Ya. Continued fractions. University of Chicago Press, Chicago 1964.
10. Hilbert D. Über die Transcendenz der Zahlen e und π. *Mathematische Annalen*, 1893, v. 43, 216–219.
11. Panchelyuga V.A., Panchelyuga M. S. Resonance and Fractals on the Real Numbers Set. *Progress in Physics*, October 2012, v. 4, 48–53.
12. Tesche C.D., Karhu J. Theta oscillations index human hippocampal activation during a working memory task. *PNAS*, 2000, v. 97(2).
13. Ganong's Review of Medical Physiology (23 ed.) p. 600.
14. Spodick D.H. Survey of selected cardiologists for an operational definition of normal sinus heart rate. *The American Journal of Cardiology*. 1993, v. 72(5), 487–88.
15. Schaefer L.V. et al. Synchronization of muscular oscillations between two subjects during isometric interaction. *European Journal Trans Myology. Basic Appl. Myol.* 2014, v. 24(3), 195–202.
16. Davis J. Humans perceive flicker artefacts at 500 Hz, Sci Rep, Wiley, 5, 7861, PMC 4314649, 1986.
17. Lei G. et al. Pitch Analysis of Infant Crying. *National Journal of Digital Content Technology and its Applications*. 2013, v. 7(6).
18. Carhart, R. Clinical application of bone conduction audiometry. *Archives of Otolaryngology*, 1950, v. 51, 798–808.
19. Homma K., Shizmu Y., Puria, S. Ossicular resonance modes of the human middle ear for bone and air conduction. *Journal of the Acoustical Society of America*, 2009, v. 125, 968–979.
20. Sainsbury R.S., Heynen A., Montoya C.P. Behavioral correlates of hippocampal type 2 theta in the rat. *Physiol Behav.* 1987, v. 39(4), 513–519. PMID 3575499
21. Stewart M., Fox S.E. Hippocampal theta activity in monkeys. *Brain Res.* 1991, v. 538(1), 59–63. PMID 2018932
22. Barenblatt G.I. Scaling. Cambridge University Press, 2003.
23. Schmidt-Nielsen K., Scaling. Why is the animal size so important? Cambridge University Press, 1984.
24. Breathing and control of heart rate. *British Medical Journal*, 16.12.1978.
25. Fechner G.T. Elemente der Psychophysik, Band 2, Leipzig: Breitkopf und Härtel, 1860.

LETTERS TO PROGRESS IN PHYSICS

A Comment on "Can the One-way Speed of Light be Used for Detection of Violations of the Relativity Principle?"

Joseph Catania
E-mail: jcatania1@verizon.net

I show in this Letter that Spavieri et. al.'s clock transport delay calculations are incorrectly determined because of a sign error. Thus, the results of Roland De Witte (1991) should be considered significant.

1 Details

Assume for simplicity that what Spavieri et. al. [1] mean by $\mathbf{u}(t)$ is, a velocity of constant magnitude u, with a varying direction, yielding a total effective absolute velocity $\mathbf{V} \simeq \mathbf{v} + \mathbf{u}(t)$. Spavieri et. al.'s Equation (5) is reproduced here for convenience,

$$\delta\tau \simeq \frac{dt}{\gamma_V} - \frac{dt}{\gamma_v} \simeq \frac{(v^2 - V^2)\,dt}{2c^2} = -\frac{\mathbf{v}\cdot\mathbf{u}(t)\,dt}{c^2}\,. \qquad \text{(S5)}$$

Notice that Equation (6),

$$\Delta\tau = -\frac{1}{c^2}\int_A^B \mathbf{v}\cdot\mathbf{u}(t)\,dt = -\frac{L}{c^2}\,v\,(\cos\theta_A - \cos\theta_B) \qquad \text{(S6)}$$

is supposedly the integral of (5). Referring to Fig. 1 in [1] the projection of $\mathbf{u}(t)$ on $\mathbf{v}$ is $-u\cdot\cos(\pi/2-\theta) = -u\cdot\sin\theta$ and $|\,\mathbf{u}(t)\,| = L\omega = L\cdot\frac{d\theta}{dt}$ giving,

$$\Delta\tau = -\frac{1}{c^2}\int_A^B \mathbf{v}\cdot\mathbf{u}(t)\,dt = -\frac{L}{c^2}\,v\int_A^B -\sin\theta\cdot\frac{\mathrm{d}\theta}{\mathrm{dt}}\,dt =$$

$$= -\frac{L}{c^2}\,v\,(\cos\theta_B - \cos\theta_A). \qquad \text{(C1)}$$

Thus, Spavieri et. al. does not correctly calculate $\Delta\tau$, a quantity which they call clock transport delay (CTD). A simple sign check on $\delta\tau$ in (S5) and $\Delta\tau$ in (S6) shows they aren't the same. $|\,\mathbf{V}\,| < |\,\mathbf{v}\,|$ thus (S5) is positive, whereas since $-[\cos 0 - \cos(0 - d\theta)]$ is negtive, (S6) is negative. Replacing (C1) with (S6), the signs now agree.

2 Comments

The De Witte effect is given by,

$$t_{OB} - t_{OA} = \frac{L}{c^2}\,v\,(\cos\theta_B - \cos\theta_A) \qquad \text{(C2)}$$

and shows a decreasing effect as θ increases or decreases from its alignment with v (which we take as $\theta = 0$). Eqs. (C1) and (S5) show an increasing effect, whereas (S6), which is evidentally a harmonized version of (S5), shows a decreasing effect. So (S6), which supports Spavieri et. al.'s thesis, that the De Witte Effect is merely due to slow clock transport, is incorrect due to a sign error. The result is that if Spavieri et. al. is to be taken seriously the effect measured by De Witte will be due to twice what is derived in [1, 2, 4], which derivations do not ignore Fresnel drag. For instance Spavieri et. al's Equation (4) would be modified to,

$$\bar{t}_{OA} - \bar{t}_{OB} = \Delta\tau + \frac{L}{c^2}\,v\,(\cos\theta_A - \cos\theta_B) =$$

$$= \frac{2L}{c^2}\,v\,(\cos\theta_A - \cos\theta_B). \qquad \text{(C3)}$$

It must be noted at this point that Spavieri et. al. cites [5] (ref. 16 at the end of §3 in [1]) in which they claim that CTD is equivalent to Einstein Synchronization (ES). Unfortunately, the derivation in [5] §2 is riddled with error. For example Eq. (2) should be $t = \frac{h}{w}$ instead of $t = \frac{h}{\Delta w}$ and Eq. (6) should be $t_1 = \frac{\gamma h}{c-v}$ instead of $t_1 = \frac{h}{c-v}$. Thus, CTD and ES agree in [5] up to second order only after a harmonization.

3 Comments on synchronization

The discussion in [1] on clock transport time delay would seem to be completely spurious. An Einstein clock synchronization (ES) performed from O to A will guarantee synchronization throughout rotation about O. Such a vacuum synchronization will give the same result no matter whether the clock is at A, B or any other point as long as the laboratory frame path length is the same. This is guaranteed by the constant propagation velocity of light in the ether and the Lorentz transformation (LT), as shown by Maxwell's luminiferous ether theory and confirmed by two-way speed of light measurements in vacuo. Thus, Einstein's 'On the Electrodynamics of Moving Bodies' is based on ether theoretical dogma, as any treatment needs to be in order to be predictive.

Consider the case where the lab frame is moving at velocity v wrt the ether and the dielectric rod in this frame is rotating at constant velocity u. By ES any clock at rest wrt O can be synchronized to O and all such clocks at distance L wrt O have the same synchronization. Any clock at velocity u and distance L wrt O has the same synchronization wrt O. Therefore, if A is synchronized with O it will remain synchronized.

According to [1] the CTD, due to time dilation as clock A moves slowly due to Earth's rotation, can be calculated from [1] using,

$$\frac{1}{\gamma_0} - \frac{1}{\gamma_0} = 0. \tag{C6}$$

since in the frame of the rotating clocks they have no relative velocity wrt each other. They do have relative velocity wrt each other in the ether frame but that leads to (C1) and (C3) instead of (S6). Since no measurements are made from the ether frame but are made from the frame of the atomic clocks we must refer synchronization to this frame, as LT teaches that the two synchronizations aren't the same. LT also guarantees that the time dilation effects of CTD are the same for the signal propagation time on De Witte's cable as they are for the measuring clocks, negating relative effect between the two.

Alternatively, since the CTD of A wrt to O equals, by symmetry the CTD of O wrt A, they must cancel. This is an example of The Clock Paradox and ensures that no dissynchronization will occur between O and A, as opposed to what is taught in [1].

One might also ask, How do we ascribe CTD as the cause of De Witte's effect in the vacuum case when there is no De Witte Effect in the vacuum? Too, in De Witte's Experiment [3] when the North-South signal and the South-North signal are subtracted any biases or dissynchronizations would cancel. Additionally, if De Witte's results could be ascribed to clock transport delay it would still obtain that a measurement of velocity wrt the ether had been made in contradiction to SR canon.

4 Closing comments

Using the sidereal rotation period of Earth,

$$\omega \simeq \frac{2\pi}{86164.1}\ \mathrm{s}^{-1} \simeq 7.3 \cdot 10^{-5}\ \mathrm{s}^{-1} \tag{C4}$$

and,

$$dt = \frac{L}{c} = 5 \cdot 10^{-6}\ \mathrm{s}; \quad u(t) = L\omega \simeq 0.11\ \mathrm{m/s} \tag{C5}$$

from (C2) and [3] the absolute motion velocity is,

$$v = \frac{\left(14 \cdot 10^{-9}\right)\left(9 \cdot 10^{16}\right)\left(\cos 0 - \cos \frac{\pi}{2}\right)}{1500} = 8.4 \cdot 10^{5}\ \mathrm{m/s}.$$

[As an aside, this absolute motion velocity of 840 km/s is larger than those stated in [3] for the De Witte Experiment, larger than Earth's velocity wrt the Cosmic Microwave Background and larger than most author's estimates. Also, since the declinations of De Witte's cable and the absolute motion vector of Earth wrt vacuum are estimated to be as much as about 25° apart we should expect a velocity from 840-930 km/s. Note that this result is stated with some reservation (see below).]

Some have expressed the belief [1, 4] that Fresnel drag may not be acting in certain cases where a refractive material is known to be present. Fresnel drag is a dogmatic phenomenon equivalent to the LT with excellent experimental confirmation. It shouldn't be possible to turn physics on or off like a light switch, it is always present with refractive materials but the effect is not always correctly anticipated formally. In fact according to detailed calculations by the author, De Witte cannot be explained by a predictive ether-based formalism (Michelson-Lorentz formalism) with a final transformation to the lab frame. Such calculations, be they for one-way, two-way, with or without refractive media, always return results which speak of no unusual effects. Thus the Roland De Witte Effect remains a mystery.

Submitted on October 4, 2017

References

1. Spavieri G., et. al. Can the one-way speed of light be used for detection of violations of the relativity principle? *Phys. Lett. A*, 2012, v.376, 795–797.
2. Cahill R. T., Brotherton D. Experimental investigation of the Fresnel drag effect in RF coaxial cables. *Prog. Phys.*, 2011, v.7, issue 1, 43.
3. Cahill R. T. The Roland De Witte 1991 Experiment. *Prog. Phys.*, 2006, v.2, issue 3, 60–65.
4. Cahill R. T. One-way speed of light measurements without clock synchronisation. *Prog. Phys.*, 2012, v.8, issue 3, 43–45.
5. Cavalleri G., Spinelli G. Problems of synchronization in Special Relativity and possible links with stochastic electrodynamics. *Found. Phys.*, 1983, v.13, 1221.

Progress in Physics is an American scientific journal on advanced studies in physics, registered with the Library of Congress (DC, USA): ISSN 1555-5534 (print version) and ISSN 1555-5615 (online version). The journal is peer reviewed and listed in the abstracting and indexing coverage of: Mathematical Reviews of the AMS (USA), DOAJ of Lund University (Sweden), Scientific Commons of the University of St.Gallen (Switzerland), Open-J-Gate (India), Referential Journal of VINITI (Russia), etc. Progress in Physics is an open-access journal published and distributed in accordance with the Budapest Open Initiative: this means that the electronic copies of both full-size version of the journal and the individual papers published therein will always be acessed for reading, download, and copying for any user free of charge. The journal is issued quarterly (four volumes per year).

Electronic version of this journal: http://www.ptep-online.com

Postal address:
Department of Mathematics and Science, University of New Mexico,
705 Gurley Avenue, Gallup, NM 87301, USA