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The Rank Order of the Neutrosophic Triplets (T, I, F) May Change after Normalization

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Abstract

This paper examines the effects of normalization on the ranking order of single-valued neutrosophic triplets. Neutrosophic triplets are often normalized so that their sum equals one. However, we demonstrate that the ranking order of these triplets, when determined by score, accuracy, and certainty functions, can change upon normalization. Through formal proofs and illustrative counter-examples, we show that inequalities between unnormalized triplets do not always hold after normalization, affecting ranking stability. These findings highlight the importance of considering normalization's impact when using neutrosophic triplets for decision-making and uncertainty modeling.

Keywords: Neutrosophic set, Ranking functions, Score function, Accuracy function, Decision-making, Rank order consistency.

1 | Introduction

Neutrosophic theory [1], [2] extends the framework of fuzzy and intuitionistic fuzzy sets by incorporating an additional degree of indeterminacy, thereby representing truth, indeterminacy, and falsity as independent values within a triplet (T, I, F). Neutrosophic sets allow each component of this triplet to vary independently in the range [0, 1], providing a more nuanced representation of uncertainty and indeterminacy in data [3], which has proven valuable in fields such as decision-making [4–7], optimization [8–11], medical diagnosis [12], risk management [13], [14], graphs [15], Statistics [16], building contexts [17], supply chain [18], topology

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[19], [20], numerical analysis [21], performance evaluations [22–24], tourism strategies [25], and artificial intelligence [26–28]; see also [29–32].

One key aspect of neutrosophic triplets is their normalization. Normalization can simplify comparisons across triplets by standardizing values; however, the implications of this transformation for the ranking of neutrosophic triplets have not been fully examined. Ranking triplets is crucial in applications that require ordering based on degrees of truth, indeterminacy, or falsity, yet it is unclear whether normalization preserves ranking consistency.

This paper addresses this gap by exploring how normalization impacts the relative ordering of neutrosophic triplets according to three commonly used functions: the score function S(T, I, F), which reflects an aggregate measure of positiveness; the accuracy function A(T, I, F), which indicates the difference between truth and falsity components; and the certainty function C(T, I, F), which measures the degree of truth alone. We demonstrate that normalization can alter the ranking order of neutrosophic triplets as established by these functions, and we provide proofs and counter-examples illustrating conditions under which inconsistencies arise.

The findings of this study highlight the necessity of caution in applying normalized neutrosophic triplets in applications where ranking accuracy is essential. By showing that ranking reversals may occur post-normalization, we contribute to a more robust understanding of the neutrosophic framework, enhancing its applicability and reliability in uncertainty modeling. This work thus builds on prior studies of ranking functions for neutrosophic sets while offering new insights into the effects of normalization on rank stability.

2 | Ranking of Neutrosophic Triplets Preliminaries

In this section, we outline the essential definitions and functions related to neutrosophic triplets and discuss their properties in the context of ranking. These preliminaries are the foundation for analyzing how normalization influences the rank order of neutrosophic triplets; see [3].

2.1 | Neutrosophic Triplets and their Representation

A neutrosophic triplet (T, I, F) consists of three components: T, representing truth; I, representing indeterminacy; and F, representing falsity. Each component is defined on the interval [0,1], collectively capturing the uncertainty and indeterminacy associated with a given concept or decision. Formally, we define the set of un-normalized (non-normalized) neutrosophic triplets as

$$\mathrm{UN} = \{(\mathrm{T, I, F}), \mathrm{T, I, F} \in [0, 1], 0 \leq \mathrm{T+I+F} \leq 3\}.$$

For consistency, we exclude the trivial case where (T, I, F) = (0, 0, 0), as it cannot be normalized.

In many applications, it is desirable to normalize the triplet so that the sum T+I+F=1. We obtain the normalized triplet T', I', F') by dividing each component by the total sum

$$\mathrm{T}' = \frac{\mathrm{T}}{\mathrm{T+I+F}}, \mathrm{I}' = \frac{\mathrm{I}}{\mathrm{T+I+F}}, \mathrm{F}' = \frac{\mathrm{F}}{\mathrm{T+I+F}},$$

provided that T+I+F>0. This normalization ensures that all values are scaled relative to the total degree of truth, indeterminacy, and falsity, effectively mapping the triplet to a bounded simplex within [0,1].

2.2 | Ranking Functions for Neutrosophic Triplets

Several ranking functions have been proposed in the literature to evaluate and compare neutrosophic triplets. These functions aim to assign a quantitative measure to each triplet based on its components. In this paper, we focus on three key functions commonly used in ranking neutrosophic sets:

I. Score function S(T, I, F): the score function aggregates the values of T, I, and F into a single measure of positiveness or overall degree. It is defined as

$$S: UN \rightarrow [0, 1],$$

$$S(T, I, F) = \frac{T+(1-I)+(1-F)}{3}.$$

Higher scores correspond to triplets with higher truth values and lower indeterminacy and falsity values, indicating greater positiveness.

II. Accuracy function A(T, I, F): the accuracy function represents the difference between the truth and falsity components, indicating certainty in the triplet. It is given by

$$A: UN \rightarrow [-1, 1],$$

$$A(T, I, F) = T - F.$$

This function takes values in $[-1,1]$, with higher values suggesting greater accuracy due to a larger truth component relative to falsity.

III. Certainty function C(T, I, F): the certainty function is defined as the truth component alone

$$C: UN \rightarrow [0, 1]$$

$$C(T, I, F) = T.$$

This function independently reflects the degree of truth without considering indeterminacy or falsity.

2.3 | Problem Statement

The primary goal of this paper is to investigate whether the rank order of neutrosophic triplets, as determined by the score, accuracy, and certainty functions, is preserved after normalization. Given two neutrosophic triplets $N_1=(T_1, I_1, F_1)$ and $N_2=(T_2, I_2, F_2)$, we examine how applying these ranking functions before and after normalization can lead to inconsistent ordering. Specifically, we seek to identify conditions under which $S(N_1)>S(N_2)$ in the unnormalized case may yield $S(N_1')\leq S(N_2')$ after normalization and similarly for the accuracy and certainty functions.

This theoretical analysis, supported by counter-examples, demonstrates that normalization can alter the relative order of neutrosophic triplets, which has implications for applications requiring reliable ranking of uncertain information.

3 | Ranking Algorithm and Theoretical Analysis

This section presents the algorithm used for ranking neutrosophic triplets based on the score, accuracy, and certainty functions. We then analyze the effects of normalization on these rankings, providing proofs and examples to illustrate cases where the rank order changes after normalization.

3.1 | Ranking Algorithm for Neutrosophic Triplets

Given two neutrosophic triplets $N_1 = (T_1, I_1, F_1)$ and $N_2 = (T_2, I_2, F_2)$, we propose a hierarchical ranking algorithm that sequentially applies the score, accuracy and certainty functions to determine the ordering of the triplets (no matter if they are un-normalized or normalized). The steps of the algorithm are as follows:

I. Apply the score function: if $S(N_1) > S(N_2)$, then $N_1 > N_2$; or if $S(N_1) < S(N_2)$, then $N_1 < N_2$.

II. Apply the accuracy function (if necessary): if $S(N_1) = S(N_2)$, then apply the accuracy function. If $A(N_1) > A(N_2)$, then $N_1 > N_2$; or if $A(N_1) < A(N_2)$, then $N_1 < N_2$.

III. Apply the certainty function (if necessary): if $A(N_1) = A(N_2)$, then apply the certainty function. If $C(N_1) > C(N_2)$, then $N_1 > N_2$; or if $C(N_1) < C(N_2)$, then $N_1 < N_2$. But, if $C(N_1) = C(N_2)$, then $N_1 = N_2$.

This algorithm ensures a total ordering for neutrosophic triplets based on their respective truth, indeterminacy, and falsity components. However, as we will show, normalization can affect these comparisons, leading to situations where the ranking order established by the algorithm changes post-normalization.

3.2 | Theoretical Analysis: Effects of Normalization on Ranking Order

We now analyze how normalization impacts the ranking of neutrosophic triplets by examining each ranking function. The following theorem formalizes conditions under which normalization can lead to rank reversals.

Theorem 1. Ranking order reversal upon normalization.

Let the un-normalized triplets be (T_1, I_1, F_1) and $(T_2, I_2, F_2) \in UN$. Also, let their corresponding normalized forms (obtained by dividing each neutrosophic component T, I, F by the sum of all neutrosophic components $T + I + F \neq 0$) are $\left(\frac{T_1}{D_1}, \frac{I_1}{D_1}, \frac{F_1}{D_1}\right)$, where $T_1+ I_1+ F_1 = D_1$, and $\left(\frac{T_2}{D_2}, \frac{I_2}{D_2}, \frac{F_2}{D_2}\right)$, where $T_2+ I_2+ F_2 = D_2$.

I. If $S(T_1, I_1, F_1) > S(T_2, I_2, F_2)$, then $S\left(\frac{T_1}{D_1}, \frac{I_1}{D_1}, \frac{F_1}{D_1}\right)$ is not necessarily $> S\left(\frac{T_2}{D_2}, \frac{I_2}{D_2}, \frac{F_2}{D_2}\right)$.

II. If $A(T_1, I_1, F_1) > A(T_2, I_2, F_2)$, then $A\left(\frac{T_1}{D_1}, \frac{I_1}{D_1}, \frac{F_1}{D_1}\right)$ is not necessarily $> A\left(\frac{T_2}{D_2}, \frac{I_2}{D_2}, \frac{F_2}{D_2}\right)$.

III. If $C(T_1, I_1, F_1) > C(T_2, I_2, F_2)$, then $C\left(\frac{T_1}{D_1}, \frac{I_1}{D_1}, \frac{F_1}{D_1}\right)$ is not necessarily $> C\left(\frac{T_2}{D_2}, \frac{I_2}{D_2}, \frac{F_2}{D_2}\right)$.

Similarly, in the case of opposite inequalities (<).

Proof:

I. The score function:

Un-Normalized Triplets

Let $S(T_1, I_1, F_1) > S(T_2, I_2, F_2)$, then $\frac{T_1+(1-I_1)+(1-F_1)}{3} > \frac{T_2+(1-I_2)+(1-F_2)}{3}$

or $2 + T_1 - I_1 - F_1 > 2 + T_2 - I_2 - F_2$,

or $T_1 - I_1 - F_1 > T_2 - I_2 - F_2$.

Normalized Triplets

To normalize, we divide each component by the sum of all three components:

$(T, I, F) \rightarrow \left(\frac{T}{T+I+F}, \frac{I}{T+I+F}, \frac{F}{T+I+F}\right)$.

From $S\left(\frac{T_1}{D_1}, \frac{I_1}{D_1}, \frac{F_1}{D_1}\right) > S\left(\frac{T_2}{D_2}, \frac{I_2}{D_2}, \frac{F_2}{D_2}\right)$, one gets

$$\frac{\frac{T_1}{D_1} + \left(1 - \frac{I_1}{D_1}\right) + \left(1 - \frac{F_1}{D_1}\right)}{3} > \frac{\frac{T_2}{D_2} + \left(1 - \frac{I_2}{D_2}\right) + \left(1 - \frac{F_2}{D_2}\right)}{3},$$

or $\frac{T_1}{D_1} + 1 - \frac{I_1}{D_1} + 1 - \frac{F_1}{D_1} > \frac{T_2}{D_2} + 1 - \frac{I_2}{D_2} + 1 - \frac{F_2}{D_2}$,

or $\frac{T_1-I_1-F_1}{D_1} > \frac{T_2-I_2-F_2}{D_2}$,

or

$$T_1 - I_1 - F_1 > (T_2 - I_2 - F_2) \cdot \frac{D_1}{D_2}. \quad \textbf{(1)}$$

But, if for the un-normalized triplets (T_1, I_1, F_1) and (T_2, I_2, F_2) we have

$$T_1 - I_1 - F_1 > T_2 - I_2 - F_2.$$

The *Previous Inequality (1)* may not hold for some $D_2 < D_1$.

II. The accuracy function:

Un-Normalized Triplets

Let $A(T_1, I_1, F_1) > A(T_2, I_2, F_2)$, then $T_1 - F_1 > T_2 - F_2$.

Normalized Triplets

$$A(\frac{T_1}{D_1},\frac{I_1}{D_1},\frac{F_1}{D_1}) > A(\frac{T_2}{D_2},\frac{I_2}{D_2},\frac{F_2}{D_2}),$$

or $\frac{T_1}{D_1}-\frac{F_1}{D_1} > \frac{T_2}{D_2}-\frac{F_2}{D_2}$,

or $\frac{T_1-F_1}{D_1} > \frac{T_2-F_2}{D_2}$,

or $T_1-F_1 > (T_2-F_2)\cdot\frac{D_1}{D_2}$.

Similarly, this inequality may differ for some $D_2 < D_1$.

III. The certainty function:

Un-Normalized Triplets

Let $C(T_1, I_1, F_1) > C(T_2, I_2, F_2)$,

or $T_1 > T_2$.

Normalized Triplets

$$C(\frac{T_1}{D_1},\frac{I_1}{D_1},\frac{F_1}{D_1}) > C(\frac{T_2}{D_2},\frac{I_2}{D_2},\frac{F_2}{D_2}),$$

or $\frac{T_1}{D_1} > \frac{T_2}{D_2}$,

or $T_1 > T_2\cdot\frac{D_1}{D_2}$, which may not hold for some $D_2 < D_1$.

This analysis shows that the normalized ranking order of neutrosophic triplets can differ from their unnormalized ranking order under certain conditions. In the next section, we illustrate these scenarios with specific counter-examples.

4 | Counter-Examples Demonstrating Rank order Reversal

To illustrate the theoretical results, we present counter-examples showing cases where the ranking order of neutrosophic triplets changes after normalization. These examples demonstrate the impact of normalization on the score, accuracy, and certainty functions, highlighting instances where inequalities between triplets are reversed.

4.1 | Counter-Example for the Score Function

Un-Normalized Neutrosophic Triplets

Let $N_1 = (0.2, 0, 0)$ and $N_2 = (0.1, 0. 0)$. Using the score function, we calculate

$S(N_1) = 0.2 + (1 - 0) + (1 – 0) = 2.2.$

$S(N_2) = 0.1 + (1 - 0) + (1 – 0) = 2.1.$

Hence, $S(N_1) > S(N_2)$ indicates that N_1 ranks higher than N_2 in the unnormalized form.

Normalized Neutrosophic Triplets:

I. Normalized $N_1 = (\frac{0.2}{0.2+0+0}, \frac{0}{0.2+0+0}, \frac{0}{0.2+0+0}) = (1, 0, 0).$

II. Normalized $N_2 = (\frac{0.1}{0.1+0+0}, \frac{0}{0.1+0+0}, \frac{0}{0.1+0+0}) = (1, 0, 0).$

We see that normalizedN_1 = normalizedN_2, and of course:

$S(normalizedN_1) = S(normalizedN_2) = 1 + (1 – 0) + (1 – 0) = 3.$

Therefore, $S(N_1) > S(N_2)$ but, after normalization,

$S(normalizedN_1) = S(normalizedN_2).$

The inequality was transformed into equality.

Second Counter-Example: Consider the triplets $N_1 = (0, 0.8, 0.8)$ and $N_2 = (0.1, 0.9. 0.9)$. So we have:

$S(N_1) = 0 + (1 – 0.8) + (1 – 0.8) = 0.4,$

$S(N_2) = 0.1 + (1 – 0.9) + (1 – 0.9) = 0.3.$

Whence $S(N_1) > S(N_2)$.

Normalized Triplets:

I. Normalized $N_1 = (\frac{0}{0+0.8+0.8}, \frac{0.8}{0+0.8+0.8}, \frac{0.8}{0+0.8+0.8}) = (0, 0.5, 0.5).$

II. Normalized $N_2 = (\frac{0.1}{0.1+0.9+0.9}, \frac{0.9}{0.1+0.9+0.9}, \frac{0.9}{0.1+0.9+0.9}) = (\frac{1}{19}, \frac{9}{19}, \frac{9}{19}).$

$S(normalizedN_1) = 0 + (1 – 0.5) + (1 – 0.5) = 1$

$S(normalizedN_2) = \frac{1}{19} + (1 - \frac{9}{19}) + (1 - \frac{9}{19}) = \frac{1}{19} + \frac{18}{19} + \frac{18}{19} = \frac{37}{19} = 1.94\ldots$

whence $S(normalizedN_1) < S(normalizedN_2)$.

Therefore, $S(N_1) > S(N_2)$, but $S(normalizedN_1) < S(normalizedN_2)$.

4.2 | Counter-Example for the Accuracy Function

Unnormalized Neutrosophic Triplets: Let $N_1 = (0.9, 0.8, 0.6)$ and $N_2 = (0.8, 0.1, 0.6)$. Using the accuracy function, we calculate $A(N_1) = 0.9 – 0.6 = 0.3$, $A(N_2) = 0.8 – 0.6 = 0.2$.

Thus $A(N_1) > A(N_2)$, so N_1 ranks higher than N_2 in terms of accuracy in the unnormalized form.

Normalized Neutrosophic Triplets:

I. Normalized $N_1 = (\frac{9}{23}, \frac{8}{23}, \frac{6}{23}).$

II. normalized $N_2 = (\frac{8}{15}, \frac{1}{15}, \frac{6}{15})$.

$A(normalizedN_1) = \frac{9}{23} - \frac{6}{23} = \frac{3}{23} = 0.130434...$

$A(normalizedN_2) = \frac{8}{15} - \frac{6}{15} = \frac{2}{15} = 0.333333...$

Thus, $A(normalizedN_1) < A(normalizedN_2)$.

Hence, from $A(N_1) > A(N_2)$ one arrives by normalization to the opposite inequality: $A(normalizedN_1) < A(normalizedN_2)$. This reversal illustrates how normalization can impact accuracy-based ranking.

4.3 | Counter-Example for the Certainty Function

Un-Normalized Neutrosophic Triplets:

Let $N_1 = (0.9, 0.8, 0.6)$ and $N_2 = (0.8, 0.3, 0.4)$. Using the certainty function $C(T,I,F)=T$, we compute:

$C(N_1) = 0.9$, $C(N_2) = 0.8$.

Thus, $C(N_1) > C(N_2)$, so N_1 ranks higher than N_2 in terms of certainty in the unnormalized form.

Normalized Neutrosophic Triplets:

I. Normalized $N_1 = (\frac{9}{23}, \frac{8}{23}, \frac{6}{23})$.

II. Normalized $N_2 = (\frac{8}{15}, \frac{3}{15}, \frac{4}{15})$.

III. $C(normalizedN_1) = \frac{9}{23} = 0.130434...$

IV. And $C(normalizedN_2) = \frac{8}{15} = 0.533333...$

As such, from $C(N_1) > C(N_2)$, one gets, after normalization, the opposite inequality: $C(normalizedN_1) < C(normalizedN_2)$.

This result shows that even certainty-based ranking can be affected by normalization.

These counter-examples confirm that normalization can lead to reversals in the ranking order of neutrosophic triplets across all three ranking functions. This inconsistency suggests that normalization may introduce significant biases in applications requiring stable rankings, which could impact decision-making processes.

Remark 1. There are numerous cases when the inequalities of score, accuracy, and certainty functions, respectively, for un-normalized and normalized neutrosophic triplets, are the same, but also numerous other cases when the inequalities are respectively opposite to each other, or they become equalities as shown in the previous counter-examples.

Remark 2. This proves, once more, that the ranking of the neutrosophic triplets (T, I, F) is, in general, different from the ranking of the intuitionistic fuzzy set duplets (T, F) or triplets (T, H, F), where H is hesitancy and $T + H + F = 1$.

5 | Conclusion

In this study, we explored how normalization affects the ranking order of neutrosophic triplets (T, I, F), showing that it can lead to inconsistencies across score, accuracy, and certainty functions. Through theoretical analysis and counter-examples, we demonstrated that normalization, while often used to facilitate comparison, can unintentionally alter ranking orders, potentially impacting decision-making and uncertainty modeling applications. Our findings underscore the need for caution when normalizing neutrosophic data, suggesting practitioners consider using normalized and unnormalized rankings or alternative methods to preserve consistency. This work lays the foundation for further research into ranking methods that balance standardization with reliability, fostering more robust applications of neutrosophic theory in fields that rely on accurately handling indeterminate information.

Author Contributions

S Smarandache was involved in conceptualization, methodology development, software implementation, validation, and paper writing.

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Data Availability

Data is not available due to privacy.

Conflicts of Interest

The authors declare no conflict of interest.

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