Ovidiu-Ilie Șandru (editor)

Recent Advances in Extenics

Collected Papers of Romanian Scientists

Second edition

Politehnica
2025

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The volume is dedicated to Professor CAI WEN.

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ABSTRACT

This volume features contributions from Romanian scientists, covering various topics within the field of Extenics and its applications.

Key areas explored include multidimensional extension theory, which aims to expand the existing one-dimensional Extenics Theory to an n-dimensional case where n > 1.

The authors emphasize the importance of this extension for solving practical problems, ensuring technical consistency in its application.

Other significant topics addressed in the book are indicators of inclusion with applications in computer vision, and the development of a genetic algorithm for learning automata.

Additionally, the volume delves into Extenics models for the equilibrium control of bipedal robots and presents new progress in extension theory.

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An indicator of inclusion with applications to computer vision

- This article presents an algorithmic process for the automatic movement of a predefined object from a video image in a target region. This process is intended to facilitate the implementation of specialized software applications in solving such problems.

A Position Indicator with Applications In The Field of Designing Forms with Artificial Intelligence

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- This article focuses on an indicative application of Extenics Theory in modeling, monitoring, and controlling the equilibrium maintenance process of a humanoid bipedal robot within its environment.

New Progress in Extension Theory

- This paper presents a contribution from Romanian researchers to the development of Extension Theory. It highlights that the theory has received unanimous appreciation from specialists due to its multiple applications in various fields of human activity.

MULTIDIMENSIONAL EXTENICS THEORY[1]

Ovidiu Ilie ŞANDRU[2], Luige VLĂDAREANU[3], Paul ŞCHIOPU[4],
Victor VLĂDAREANU[5], Alexandra ŞANDRU[6]

This paper will "extend" Extenics Theory from the one-dimensional case known at present to the n-dimensional case, $n > 1$. Due to the importance of this theory in solving multiple practical problems, the extension presented in this paper brings a maximum of technical consistency to the applicative field.

Keywords: Extenics theory, Wen indicators, dynamic systems with status indicators.

1. Introduction

For solving certain paradoxical problems which are often met in daily practice, Prof. Cai Wen developed an efficient mathematical theory which he generically named "Extenics", namely the science of extending the means of investigation used in classical mathematics[7]. The results obtained up to the present by Prof. Cai deal exclusively with problems expressable by unidimensional mathematical models. Since this restricts the area of applicability of a means of investigation which even under these conditions proved to have widespread applications[8] it is only natural that one becomes concerned with its development in a way that allows a generalization of the existing theory which would comprise the practical problems that need to be mathematically expressed by multidimensional models[9]. This paper attempts to do just that.

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(7) For further details on this matter see papers [4, 9, 10, 11], where [9] represents the early work of prof. Cai Wen to which we shall refer expressly within our article and [4, 10, 11] represent developments of the points of view formulated in [9] followed by numerous applicative examples particularly for the engineering field.
(8) See in this respect papers [1-7], [10, 11, 12].
(9) Preoccupations in this direction do not represent a topic introduced by us exclusively. The first initiative of this kind belongs to prof. Forentin Smarandache (see paper [8]) who was able to develop prof Cai Wen's theory in two particular directions: one referring to the situation in which the sets of the considered mathematical model have central symmetry and the second in which the problems being studied admit the so-called "attraction point principle". However, the theory presented by us in this paper fundamentally differs from the approach of prof. Smarandache and the results that we obtained are in general, distinct. When compared to the theory of prof. Smarandache, our theory represents another variant of generalizing the theory of prof. Cai Wen, which leads to other implications upon the applicative field.

As can be seen from the content, the transition from the unidimensional case to the multidimensional is neither direct, as intuitive aspects do not function well within the generalization process, nor immediate, since reaching the general cadre of the theory necessitates a much more sophisticated mathematical apparatus.

2. Maximal extension of the notion of distance in Extenics Theory

By d we denote the euclidean distance on $\mathbb{R}^n$, i.e. $d(x,y)=\left(\sum_{k=1}^{n}(y_k-x_k)^2\right)^{1/2}$, where $x=(x_1,x_2,..,x_n)$, and $y=(y_1,y_2,..,y_n)$ represent two points from $\mathbb{R}^n$. This thing allows us to consider the relation

$$\delta(x,A)=\inf_{y\in A} d(x,y),$$

which, as is known, represents the distance from point $x\in\mathbb{R}^n$ to the set $A\subseteq\mathbb{R}^n$.

In classical mathematics the notion of distance from a given point to a certain set of points is sufficient to express whether that point belongs or not to the set considered. In Extenics theory, the relation that can exist between a point $x\in\mathbb{R}^n$ and a set $A\subseteq\mathbb{R}^n$ is to be extended, with the intention of expressing more than the simple idea that $x\in A$ or $x\notin A$. In order to obtain this result we propose replacing the indicator δ defined earlier with the indicator $\mathfrak{s}$ defined as follows:

$$\mathfrak{s}(x,A)=\begin{cases}\delta(x,A), & x\in \complement A\\ -\delta(x,\complement A), & x\in A\end{cases}, \tag{1}$$

where by $\complement A$ we denoted the absolute complement of A, i.e. $\complement A=\mathbb{R}^n\setminus A$.

We shall now show that this new indicator keeps all the properties of the indicator

$$\rho(x,[a,b])=\left|x-\frac{a+b}{2}\right|-\left|\frac{b-a}{2}\right|, \tag{2}$$

introduced by Cai Wen in [9] for the particular case where $x\in\mathbb{R}$ and set A is an interval of the real numbers' set of the form $[a,b]$, with $a<b$, which it also generalizes, in the sense that the restriction applied to our indicator, to the case studied by Prof. Cai, coincides with ρ. Indeed, the indicator $\mathfrak{s}$ verifies the properties detailed below:

Proposition 1. For any point $x\in\mathbb{R}^n$ and any set $A\subseteq\mathbb{R}^n$, if $x\in\overset{o}{A}$, where with $\overset{o}{A}$ is denoted the interior of the set A in topology induced by the metric d fixed earlier on the space $\mathbb{R}^n$, then $\mathfrak{s}(x,A)<0$, and reciprocally.

The proof of this sentence results directly from the definition of the indicator $\mathfrak{s}$.

Proposition 2. For any point $x \in \mathbb{R}^n$ and any set $A \subseteq \mathbb{R}^n$, we have $x \in \complement\overline{A} \Leftrightarrow \mathfrak{s}(x,A) > 0$, where with $\overline{A}$ is noted the closure of set A in topology induced by metric d on space $\mathbb{R}^n$.

As earlier the proof of this sentence results directly from the definition of indicator $\mathfrak{s}$.

Proposition 3. For any point $x \in \mathbb{R}^n$ and any sets A and B in $\mathbb{R}^n$, if $\overline{A} \subset \overset{o}{B}$ then $\mathfrak{s}(x,A) > \mathfrak{s}(x,B), \forall x \in \mathbb{R}^n$.

Proof. We need to analyze three distinct cases.

Case 1. $x \in A$. Let $x'' \in \partial B$ so that $\delta(x,\complement B) = d(x,x'')$ and $x' \in \partial A$ so that the points x, x', x'' be collinear, and x' be between x and x'', see Figure 1[10]. Under these conditions $\delta(x,\complement A) \le d(x,x') < d(x,x'') = \delta(x,\complement B)$. It follows $\mathfrak{s}(x,A) > \mathfrak{s}(x,B)$.

Note. The symbols ∂A and ∂B used above denote the boundaries of sets A and B respectively, meaning $\partial A = \overline{A} \setminus \overset{o}{A}$, respectively, $\partial B = \overline{B} \setminus \overset{o}{B}$.

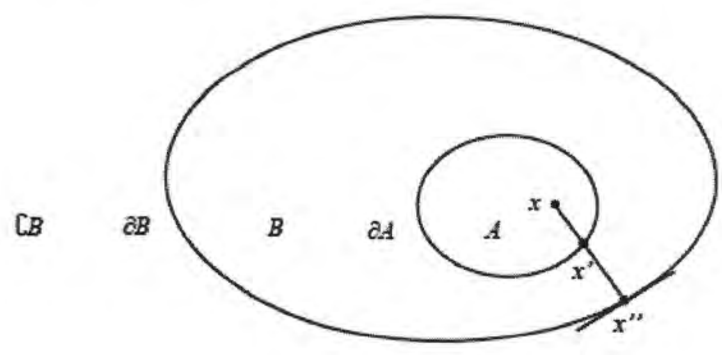

Figure 1.

Case 2. $x \in B \setminus A$. In this case we have $\delta(x,A) > -\delta(x,\complement B)$, or equivalently, $\mathfrak{s}(x,A) > \mathfrak{s}(x,B)$. See Figure 2.

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[10] Please note that both figure 1 and other figures, to which we shall refer to hereinafter, do not exhaustively cover the multitude of all situations envisaged by the demonstration (for example sets A and B must not necessarily be of domain type). The purpose of these figures is limited to only providing an intuitive visual framework meant to help fix the ideas within that demonstration.

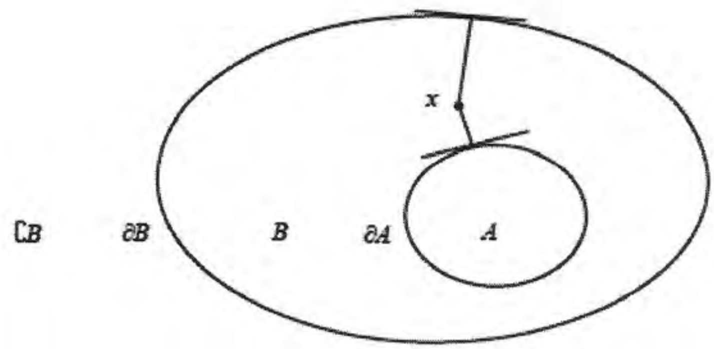

Figure 2.

Case 3. $x \in \complement B$. Let $x" \in \partial A$ so that $d(x,x") = \delta(x,A)$ and $x' \in [x,x"] \cap \partial B$, where $[x,x"]$ represents the set of all poinst of the straight line determined by points x and $x"$ situated between x and $x"$. See Figure 3. With this preamble we can write $\delta(x,A) = d(x,x") > d(x,x') \geq \delta(x,B)$, which means $\mathfrak{s}(x,A) > \mathfrak{s}(x,B)$.

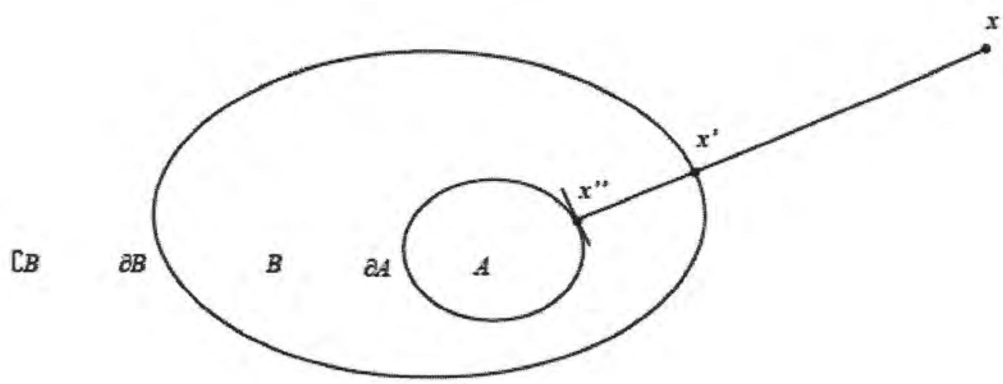

Figure 3.

Proposition 4. In the particular case $x \in \mathbb{R}$, $A = [a, b]$, $a, b \in \mathbb{R}$, $a < b$, the indicators $\rho(x,A)$ and $\mathfrak{s}(x,A)$ coincide.

The proof to this affirmation results directly from calculation.

3. Maximal extension of the Cai Wen indicator

With the help of the $\mathfrak{s}$ indicator we defined in the previous paragraph it is possible to build a new vital indicator for Extenics Theory, namely

$$\mathfrak{S}(x,A,B)=\frac{\mathfrak{s}(x,A)}{\mathfrak{s}(x,B)-\mathfrak{s}(x,A)}, \tag{3}$$

defined for any $x \in \mathbb{R}^n$ and for any sets A and B in $\mathbb{R}^n$ for which $\overline{A} \subset \overset{o}{B}$. This indicator generalizes the indicator

$$K(x,A,B)=\frac{\rho(x,A)}{\rho(x,B)-\rho(x,A)}, \tag{4}$$

introduced by Prof. Cai in [9] for the case $x \in \mathbb{R}$, $x \in \mathbb{R}$, $A=[a_0, b_0]$, $B=(a, b)$, $a_0, b_0, a, b \in \mathbb{R}$, $a < a_0 < b_0 < b$. The indicator $\mathfrak{S}$ has all the properties detailed below.

Proposition 5. For any two sets A and B in $\mathbb{R}^n$ for which $\overline{A} \subset \overset{o}{B}$ we have $\mathfrak{S}(x,A,B) < -1$, if $x \in \complement\overline{B}$; $-1 \le \mathfrak{S}(x,A,B) < 0$, if $x \in \overline{B} \setminus \overline{A}$; $\mathfrak{S}(x,A,B) \ge 0$, if $x \in \overline{A}$, and reciprocally.

Proof. Using the properties of the indicator $\mathfrak{s}$ detailed in the previous sentences it is easy to deduce that:

In the first case where $x \in \complement\overline{B}$, we have $\mathfrak{s}(x,A) > 0$ and $\mathfrak{s}(x,B) > 0$. It follows that $-\mathfrak{s}(x,B)+\mathfrak{s}(x,A) < \mathfrak{s}(x,A)$. Taking into account that $\mathfrak{s}(x,B)-\mathfrak{s}(x,A) < 0$, (see Proposition 3) we obtain the inequality $\mathfrak{S}(x,A,B) < -1$;

In the second case where $x \in \overline{B} \setminus \overline{A}$, we have $\mathfrak{s}(x,B) \le 0$ and $\mathfrak{s}(x,A) > 0$. Consequently, $-\mathfrak{s}(x,B)+\mathfrak{s}(x,A) \ge \mathfrak{s}(x,A)$. Since additionally $\mathfrak{s}(x,B)-\mathfrak{s}(x,A) < 0$, we can usually deduce that $-1 \le \mathfrak{S}(x,A,B) < 0$.

In the last case where $x \in \overline{A}$, we have $\mathfrak{s}(x,A) \le 0$. Since $\mathfrak{s}(x,B)-\mathfrak{s}(x,A) < 0$, it can be seen immediately that $\mathfrak{S}(x,A,B) \ge 0$.

Proposition 6. In the particular case $x \in \mathbb{R}$, $A=[a_0, b_0] \subset (a,b)$, $a_0, b_0, a, b \in \mathbb{R}$, $a < a_0 < b_0 < b$, the indicators $K(x,A,B)$ and $\mathfrak{S}(x,A,B)$ coincide.

Proof. In the hypotheses above the indicators ρ and $\mathfrak{s}$ coincide (see Proposition 4).

4. Aplication

Many of the state of the art technical installations such as those which emit high intensity radiation bring about areas which are dangerous for humans. We shall assume that we wish to secure such an installation through a centralized system using electronic sensors which oversees the danger areas and depending on the gravity of the situation can send warning signals for users or even stop the system. For this we shall note with X the area in the surrounding space (mathematically modeled through $\mathbb{R}^3$) inside which the radiation is over the admitted safety level and with X_0, ($\overline{X_0} \subset \overset{o}{X}$) that area inside X in which the level of radiation is unacceptable (fatal) for humans. The sensors mounted in the areas $\mathbb{R}^3 \setminus X$, $X \setminus X_0$ and X_0 send to the central monitoring and control unity the spatial coordinates of all persons implicated in the activity. The software application which interprets the data received from the sensors must accomplish the following functions: 1) When the spatial coordinates $x = (x_1, x_2\, x_3)$ of a user belong to the area $\mathbb{R}^3 \setminus X$ the installation is allowed to function unimpeded; 2) When the spatial coordinates $x = (x_1, x_2\, x_3)$ of a user belong to the area $X \setminus X_0$ the overseer system must send warning signals; 3) When the spatial coordinates $x = (x_1, x_2\, x_3)$ of a user belong to the area X_0 the overseer system must stop the installation.

Producing such a software application is much simplified by using the indicator $\mathfrak{S}(x, X_0, X)$ which was defined earlier in relation (3) in which $A = X_0$ and $B = X$. Indeed, according to Proposition 5, if $\mathfrak{S}(x_a(t), X_0, X) < -1$, for any $a \in \mathcal{A}$, where by $\mathcal{A}$ we have denoted the set of employees serving the installation which we refer to and by $x_a(t)$ the spatial coordinates of the employee a at the moment t, the monitoring and control system will not send out a warning signal – the installation is left to function at normal capacity; if $\mathfrak{S}(x_a(t), X_0, X) \in [-1, 0)$ for at least an $a \in \mathcal{A}$, then the monitoring and control system sends out warning signals for the installation users, but the installation is allowed to continue functioning; if $\mathfrak{S}(x_a(t), X_0, X) \geq 0$ for at least an $a \in \mathcal{A}$, then the centralized command system stops the installation unconditionally.

The problem analyzed previously referred to the simple case of a technological installation with only one risk factor. The solution presented can be extended to the general case of installations containing multiple risk factors. To exemplify this we shall analyze the case of a similar installation, but with $n > 1$ sources of radiation. The intended goal is the same as in the case previously studied: designing a monitoring system which will warn of approach into dangerous areas and if necessary interrupt the use of those radiation sources which could endanger humans. In order to fix the ideas we shall suppose $X_1, X_2, .., X_n$ to be the danger areas distributed for each of the n sources of the installation which emit radiation and $X_{01}, X_{02}, .., X_{0n}$ ($\overline{X_{01}} \subset \overset{o}{X}_1, \overline{X_{02}} \subset \overset{o}{X}_2, .., \overline{X_{0n}} \subset \overset{o}{X}_n$) to be the areas in their immediate vecinity of their sources, where the danger for personnel is maximum. Once these zones are established, we may then consider, for each of the alarms systems of the installation, an indicator

$$\mathfrak{S}(x, X_{0k}, X_k) = \frac{\mathfrak{s}(x, X_{0k})}{\mathfrak{s}(x, X_k) - \mathfrak{s}(x, X_{0k})},\ k = 1, 2, .., n, \qquad (5)$$

of the form considered earlier in the case of a one-source installation. The n alarm systems are designed to function according to the same principle as in the case of only one alarm system, but independently from one another, thus, if at a certain amount of time t all indicators $\mathfrak{S}\left(x_a(t), X_{0k}, X_k\right)$, $k=1, 2,.., n$, are strictly smaller than -1 for any $a \in \mathcal{A}$, where $\mathcal{A}$ is the set of personnel, and $x_a(t)$ are the spatial coordinates of employee a at time t, then the technological process is left unimpeded; if at a certain time t there is $a \in \mathcal{A}$ for which one or more of the indicators $\mathfrak{S}\left(x_a(t), X_{0k}, X_k\right)$, $k=1,2,..,n$, have values between -1, inclusively, and 0, exclusively, then the monitoring system or systems in that area will send out warning signals; and if at a certain time t there is $a \in \mathcal{A}$ for which one or more of the indicators $\mathfrak{S}\left(x_a(t), X_{0k}, X_k\right)$, $k=1,2,..,n$, have values greater or equal to 0 then the monitoring and control system or systems in those areas will automatically stop this or these sources of radiation emission.

In the case of the system with more degrees of freedom in regards to the possibility of producing an unwanted incident, situations may arise which are yet more complicated to monitor. For example, in an installation with n, $n>1$ risk factors, if stopping a certain subsystem of the installation is impossible (for technical reasons or those relating to disaster prevention) unless the entire installation is stopped, then the n monitoring and control systems of the installation, which supervise (each of them) the n sources of potential danger, will have to dispose of a relative independence of action only, a central unit for control and monitoring being required to synthesize the information from the local monitoring systems. Moreover, this central unit must have the power to override the local systems when the situation requires an overall interruption of the installation. In order to realize such a monitoring system we would propose that the n local monitoring and control systems are equipped with an indicator of the form (5), and the central unit with a temporal indicator of the form:

$$S(t) = \sup\left\{\mathfrak{S}\left(x_a(t),\, X_{0k},\, X_k\right) \mid 1 \le k \le n,\ a \in \mathcal{A}\right\}.$$

Parameter t appearing above designates the moment in time at which the monitoring is made.

The functioning principle of the protection system: As long as indicator $S(t)$ is strictly negative, the n local monitoring systems are allowed to act independently (meaning they will signal, whenever necessary, the presence of personnel in the moderate risk areas $X_k \setminus X_{0k}$, $k=1,2,..,n$). However, when a critical situation arises, where the indicator $S(t)$ would have a value greater or equal to 0, the centralized safety system would command the interruption of the entire installation.

5. Comments

The applicable examples of the indicators $\mathfrak{S}(x, X_0, X)$ and $S(t) = \sup\{\mathfrak{S}(x_a(t), X_{0k}, X_k) \mid 1 \leq k \leq n, a \in \mathcal{A}\}$ respectively, given in the previous paragraph suggest the introduction of a new concept in the theory of dynamic systems, that of "dynamic system with status indicator"(11). By this notion an important class of systems can be delimitated within dynamic systems, which, by the status indicators they are endowed with, can benefit from special methods to solve a great number of specific issues, such as those related to the real-time quality control process regarding their own functioning.

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(11) In a very abstract manner the notion of dynamic system endowed with status indicator assumes, by definition, the existence of a set $(\Sigma, \mathfrak{I})$ made up of a dynamic system Σ and an indicator $\mathfrak{I}$ of the states which the system Σ passes through during its functioning.

An indicator of inclusion with applications to computer vision[12]

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ABSTRACT: In this paper we present an algorithmic process of necessary operations for the automatic movement of a predefined object from a video image in the target region of that image, intended to facilitate the implementation of specialized software applications in solving this kind of problems.

1. INTRODUCTION

The Algorithmic problem-solving procedures for automatic traveling objects within the video images were approached by us also in an earlier work, see [4]. The purpose of this paper is to point out a new method of solving these problems. As mentioned in the earlier work, the process which we shall indicate, will be based on the definition of an indicator of Extenics type specialized in signaling whether a particular set (of pixels, in our modeled case) is included in a target set on the monitor screen.

An aspect which has to be mentioned from the beginning is that both indicators, the one defined in [4], as well as the indicator that we shall define in this paper, fundamentally differ from the indicators currently used in the Extension theory because they make the leap from reporting the position of a single point in relation to one or two given sets to that of reporting the relationship between two sets - which is much more complex, thus establishing a factor that is meant to ensure the progress of this theory. The Extension theory, which we have referred to earlier, has been proposed by Professor Cai Wen in [5].

Because of the importance of this theory in both theoretical and practical field, it has been continuously extended, at the beginning by its founder himself, see [1, 6, 7], and then by other researchers from various fields of activity, see [2, 3, 4].

2. AN INDICATOR CAPABLE TO REPORT IF A SPECIFIC SET IS INCLUDED IN A GIVEN TARGET SET

This paragraph aims at presenting new results in order to complete and improve the existent Extension theory. The framework addressing these results is that of a metric space expressed through the doublet (X, d), where X is the set of points which make up the considered space, and d is the metric of this space.

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[12] This paper was published in "viXra.org, http://vixra.org/abs/1304.0133, 2013".

For any two nonempty sets A and B from X we introduce the indicator

$$\Delta(A, B) = \sup\{\delta(a, B) \mid a \in A\}, \tag{1}$$

where we denote by $\delta(a, B)$ the usual distance from point $a \in A$ to set B, that is

$$\delta(a, B) = \inf\{d(a, b) \mid b \in B\}.$$

Observations: 1) Relation $\Delta(A, B) = \Delta(B, A)$ is not always true, in other words, the value of indicator $\Delta(A, B)$ depends, in general, on the order in which sets A and B are considered.

2) Indicator $\Delta(A, B)$ can also take infinite values.

3) In the case of two bounded sets A and B [13] indicator $\Delta(A, B)$ is finite.

Proposition: Indicator Δ defined by relation (1) has the following properties:

1) $\Delta(A, B) = 0 \Rightarrow A \subseteq \overline{B}$, where $\overline{B}$ is the closure of set B in topology induced by metric d on space X. Reciprocally, $A \subseteq \overline{B} \Rightarrow \Delta(A, B) = 0$.

2) $\Delta(A, B) = \Delta(B, A) = 0 \Rightarrow \overline{A} = \overline{B}$. Reciprocally, $\overline{A} = \overline{B} \Rightarrow \Delta(A, B) = \Delta(B, A) = 0$.

Demonstration: 1) $\Delta(A, B) = 0 \Leftrightarrow \delta(a, B) = 0\ \forall\, a \in A \Leftrightarrow A \subseteq \overline{B}$. Reciprocally, if $A \subseteq \overline{B}$ then $\delta(a, B) = 0\ \forall\, a \in A \Leftrightarrow \Delta(A, B) = 0$.

2) $\Delta(A, B) = 0 \Rightarrow A \subseteq \overline{B}$, and $\Delta(B, A) = 0 \Rightarrow B \subseteq \overline{A}$. From $A \subseteq \overline{B}$, and $B \subseteq \overline{A}$ we deduce that $\overline{A} \subseteq \overline{B}$, respectively $\overline{B} \subseteq \overline{A}$. Consequently $\overline{A} = \overline{B}$. Reciprocally, if $\overline{A} = \overline{B}$ then $\delta(a, B) = 0\ \forall\, a \in A$, and $\delta(b, A) = 0\ \forall\, b \in B$, hence, $\Delta(A, B) = \Delta(B, A) = 0$.

Observations: 1) Due to property 1) from the above proposition, indicator Δ is named indicator of inclusion.

2) On the set $\mathcal{C}(X)$ of all non-empty compact subsets of X,

$$H(A, B) = \max\{\Delta(A, B), \Delta(B, A)\},$$

[13] A set Y from X is called bounded if its diameter $D(Y) = \sup\{d(y_1, y_2) \mid y_1, y_2 \in Y\}$ is bounded.

represents the Hausdorff distance between sets A and B.

3. APPLICATIONS

Pointer Δ specified by us in this paper can be used to solve the problems posed by the development of software applications for an automated movement of a specific object "$\mathcal{O}$" from a given video image "VIm" in a target region "$\mathcal{R}$" of that image. In order to achieve this goal, we use an algorithm similar to the one that was defined in [4]. In very general terms, the new algorithm has the following content: through a set of isometries $\mathcal{I}_i$, $i \in I$ of the plane, we move object $\mathcal{O}$ into different regions and positions of the image "VIm" by calculating, every time, the value of each indicator $\Delta\left(\mathcal{I}_i\left(\mathcal{O}\right), \mathcal{R}\right)$. The determination of the indices $i_0 \in I$ for which $\Delta\left(\mathcal{I}_{i_0}\left(\mathcal{O}\right), \mathcal{R}\right) = 0$ indicates the solution to the problem. Indeed, in the situation presented within the application, object $\mathcal{O}$ and region $\mathcal{R}$ can be abstractized through the means of compact sets, and given these hypotheses, point 1) of the proposition enunciated earlier admits the following restatement:

$$\Delta\left(\mathcal{I}_{i_0}\left(\mathcal{O}\right), \mathcal{R}\right) = 0 \Leftrightarrow \mathcal{I}_{i_0}\left(\mathcal{O}\right) \subseteq \mathcal{R}.$$

Observation: This algorithm can be easily adapted to solving some similar problems in the space of three dimensions, becoming even more useful in projecting artificial intelligence forms.

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A POSITION INDICATOR WITH APPLICATIONS IN THE FIELD OF DESIGNING FORMS WITH ARTIFICIAL INTELLIGENCE[14]

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The indicators used so far within the Theory of Extension, see papers [1], [2] and [4], can be synthetically expressed by the notion of "position indicators". More exactly, these indicators can be grouped in two main sub-categories: point-set position indicators and point-two sets position indicators. The secondary goal of this paper is to define these classifications, while the primary goal is that to extend the two notions to the most general notion of set-set position indicator.

Key words: Hausdorff measure, Extension theory, position indicators, computer vision, artificial intelligence.

1. Introduction

The first part of this paper aims at defining the notion of point-set position indicator and that of point-two sets position indicator, at discussing the main examples of such indicators and their relevance for the applicative field.

Point-set Position Indicators

For any point $x \in \mathbb{R}^n$ and any set $A \subset \mathbb{R}^n$, formula $\delta(x,A) = \inf\{d(x, a) | \, a \in A\}$, where d is the Euclidean distance on $\mathbb{R}^n$, defines (in classical mathematics) the distance from point x to set A. Based on the properties characteristic for the notion of distance, every time we have $\delta(x,A) > 0$ we can conclude that point x lies outside set $\overline{A}$ (closure of set A in relation to the usual topology of space $\mathbb{R}^n$) at distance $\delta(x,A)$ from the nearest point of set A. On this

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account expression $\delta(x,A)$ somehow takes the role of position indicator of x towards A, but not entirely, because in the case $\delta(x,A)=0$ the sole information provided is that $x \in A$, with no further indication of how far or how close to frontier ∂A of set A this point lies. In many concrete situations the knowledge of these details can be more useful. For example, when A symbolizes the 2-dimensional representation of a risk zone for a human (a very deep lake, a region contaminated with toxic substances etc.) and x symbolizes the position vector of a human situated in the interior of that zone, it would be useful for that human to know how close or far the exit ways are. It thus, becomes necessary the extension of the classical notion of distance by taking into account some indicators that deal with more requirements. This aspect has been pointed out by other researchers in their papers, as well, see, for example [4]. The most general point-set position indicator aiming at fulfilling the requirements formulated above is given by [2] and takes the form

$$\mathfrak{s}(x,A)=\begin{cases} \delta(x,A), & x\in \complement A \\ -\delta(x,\complement A), & x\in A \end{cases}, \tag{1}$$

where $\complement A$ represents the absolute complement of A, i.e. $\complement A=\mathbb{R}^n \setminus A$. Given the way it has been defined, indicator $\mathfrak{s}(x,A)$ has the following properties: $x\in \complement \overline{A} \Leftrightarrow \mathfrak{s}(x, A)>0$; $x\in \overset{o}{A}$[18] $\Leftrightarrow$ $\mathfrak{s}(x, A)<0$; $x\in \partial A$[19] $\Leftrightarrow$ $\mathfrak{s}(x, A)=0$, respectively.

Observations: 1) The indicator defined in expression (1) does not represent a distance; it expresses the distance from x to A only when point x is exterior to set A.

2) Other examples of point-set position indicators can be found in [1] and [4].

Point-two Sets Position Indicators

Paper [2], mentioned above, provides us with an example of a point-two sets indicator, as well. This takes the expression

[18] $\overset{o}{A}$ represents the interior of set A in the usual topology of space $\mathbb{R}^n$.

[19] ∂A represents the boundaries of set A, namely $\partial A=\overline{A}\setminus \overset{o}{A}$.

$$\mathfrak{S}(x,A,B)=\frac{\mathfrak{s}(x,A)}{\mathfrak{s}(x,B)-\mathfrak{s}(x,A)}, \tag{2}$$

where $x\in\mathbb{R}^n$, A and B are two sets from $\mathbb{R}^n$ with the property $\overline{A}\subset\overset{o}{B}$, and $\mathfrak{s}$ is indicator (1). This indicator has the following properties: $\mathfrak{S}(x,A,B)<-1 \Leftrightarrow x\in \complement\overline{B}$; $-1\le\mathfrak{S}(x,A,B)<0 \Leftrightarrow x\in\overline{B}\setminus\overline{A}$; $\mathfrak{S}(x,A,B)\ge 0 \Leftrightarrow x\in\overline{A}$.

Observations: 1) The properties presented earlier justify the designation of "point – two sets position indicator" given to indicator (2).
2) Other examples of point – two sets position indicators can be found in [1, 4].

2. Set-set Position Indicators

This paragraph focuses on the presentation of new results aiming at further developing and improving the existent theory of Extension. The frame we shall refer to in our discussion is that of any metric space (X, d). The mathematical apparatus we would like to advance further on requires to take into consideration the notion of "Hausdorff measure". To ensure a better understanding of the concepts presented, we have synthetized the minimum of knowledge required in this regard in the appendix.

Let A and B be two non-empty sets from X. About set A we additionally assume that it admits a Hausdorff measure of dimension $r\ge 0$, $\mathcal{H}^r(A)$ is finite and nonzero. Under these conditions, by using indicator $\mathfrak{s}$ defined by the generalized relation (1) from the Euclidean metric space $\mathbb{R}^n$ for the actual metric space (X, d), we are able to consider the expression

$$\mathsf{S}(A, B)=\frac{\mathcal{H}^r\left(\{a\in A \mid \mathfrak{s}(a, B)\le 0\}\right)}{\mathcal{H}^r(A)}, \tag{3}$$

which accurately defines the indicator we wanted to introduce.

This indicator fulfills several mathematical properties important to the applicative field:

Proposition 1: $\mathsf{S}(A, B)=0 \Leftrightarrow A\cap B=\varnothing$ $\mathcal{H}^r$- almost everywhere (or differently expressed, $\mathcal{H}^r(A\cap B)=0$).

Demonstration: $\mathsf{S}(A, B)=0 \Leftrightarrow \mathcal{H}^r\left(\{a\in A \mid \mathfrak{s}(a, B)\le 0\}\right)=0$. Since $\mathfrak{s}(a, B)\le 0$ is equivalent to relation $a\in\overline{B}$, we deduce that $\mathcal{H}^r(A\cap B)=0$.

Proposition 2: $\mathsf{S}(A, B) > 0 \Rightarrow A \cap \overline{B} \neq \varnothing$.

Demonstration: $\mathsf{S}(A, B) > 0 \Leftrightarrow \mathcal{H}^r\left(\{a \in A \mid \mathfrak{s}(a, B) \leq 0\}\right) > 0$. From relation $\mathcal{H}^r\left(\{a \in A \mid \mathfrak{s}(a, B) \leq 0\}\right) > 0$ we deduce that there is $a \in A$ so that $\mathfrak{s}(a, B) \leq 0$. But $\mathfrak{s}(a, B) \leq 0$ implies that $a \in \overline{B}$, so $A \cap \overline{B} \neq \varnothing$.

Proposition 3: If besides the initial hypothesis made over set *A* and *B,* we assume additionally that set $\overline{B}$ is measurable[20] with respect to Hausdorff measure $\mathcal{H}^r$ (regarded as an outer measure on $\mathcal{P}(X)$, the family of all subsets of *X*), then relation $\mathsf{S}(A, B) = 1$ is equivalently with $A \subseteq \overline{B}$ $\mathcal{H}^r$ - almost everywhere.

Demonstration: $\mathsf{S}(A, B) = 1 \Leftrightarrow \mathcal{H}^r\left(\{a \in A \mid \mathfrak{s}(a, B) \leq 0\}\right) = \mathcal{H}^r(A)$. The way how indicator $\mathfrak{s}$ has been defined implies that $\{a \in A \mid \mathfrak{s}(a, B) \leq 0\} = A \cap \overline{B}$. Because set $\overline{B}$ is measurable we have $\mathcal{H}^r(A) = \mathcal{H}^r\left(A \cap \overline{B}\right) + \mathcal{H}^r\left(A \cap \complement\overline{B}\right)$. But $\mathcal{H}^r\left(A \cap \overline{B}\right) = \mathcal{H}^r\left(\{a \in A \mid \mathfrak{s}(a, B) \leq 0\}\right) = \mathcal{H}^r(A) \Rightarrow \mathcal{H}^r\left(A \cap \complement\overline{B}\right) = 0$, namely $A \subseteq \overline{B}$ $\mathcal{H}^r$ - almost everywhere.

Corollary: Let A and B be two closed nonempty sets from X for which there exists a Hausdorff measure of dimension $r \geq 0$ so that $\mathcal{H}^r(A)$ and $\mathcal{H}^r(B)$ are finite and nonzero. If sets A and B are $\mathcal{H}^r$ - measurable and if $\mathsf{S}(A,B) = 1$ and $\mathsf{S}(B,A) = 1$, then $A = B$ $\mathcal{H}^r$ - almost everywhere, and reciprocally.

Demonstration: From proposition 3 it results that $\mathsf{S}(A, B) = 1 \Leftrightarrow A \subseteq B$ $\mathcal{H}^r$ - almost everywhere and $\mathsf{S}(B, A) = 1 \Leftrightarrow B \subseteq A$ $\mathcal{H}^r$ - almost everywhere. Then $A = B$ $\mathcal{H}^r$ - almost everywhere.

---

[20] By definition we say that set $\overline{B}$ is $\mathcal{H}^r$ measurable if for any $T \subseteq X$ relation $\mathcal{H}^r(T) = \mathcal{H}^r\left(T \cap \overline{B}\right) + \mathcal{H}^r\left(T \cap \complement\overline{B}\right)$ takes place.

Observations: 1) From the definition of indicator S, (relation (3)) it can be easily deduced that $0 \le \mathsf{S}(A,B) \le 1$, for any pair of non-empty subsets A and B of space X for which set A admits a Hausdorff measure $\mathcal{H}^r$ of dimension $r \ge 0$, so that $\mathcal{H}^r(A) \ne 0$ and $\mathcal{H}^r(A) < \infty$.
2) The properties presented earlier within propositions 1 - 3 aim at justifying the designation of "set-set position indicator" that indicator (3) receives.
3) Another example of set-set position indicator can be found in [3].

3. Applications

Indicator S defined by us in this paper can be used as an example for computer vision while developing software applications regarding the automatic inclusion of a certain object $\mathcal{O}$ into a target region $\mathcal{R}$ of a given video image (*VIm*). To realize this, we propose an algorithm which in broad terms has the following content: by means of a set of isometries $\mathcal{I}_i$, $i \in I$ of the plan, we move object $\mathcal{O}$ to different regions and positions of image *VIm* by calculating the value of indicator $\mathsf{S}(\mathcal{I}_i(\mathcal{O}), \mathcal{R})$, each time. Finding that index $i_0 \in I$ for which $\mathsf{S}(\mathcal{I}_{i_0}(\mathcal{O}), \mathcal{R}) = 1$, is equivalent to finding the solution to the problem.

Observations: 1) In some cases solution $\mathcal{I}_{i_0}$ found by using the method presented above can not be fully satisfactory because relation $\mathcal{I}_{i_0}(\mathcal{O}) \subseteq \mathcal{R}$, concerned in $\mathsf{S}(\mathcal{I}_{i_0}(\mathcal{O}), \mathcal{R}) = 1$, (see proposition 3, applied in the case when the sets by which objects $\mathcal{O}$ and $\mathcal{R}$ are being abstractized, are supposed to be compact[21]) is only guaranteed by $\mathcal{H}^r$ - almost everywhere.
2) Just like the algorithm presented in [3], this algorithm can be easily adapted to solving any similar problem in a space with three dimensions, becoming, thus, more useful to the field of designing forms with artificial intelligence.

4. Appendix

Hausdorff measure

Let (X, d) be a metric space, Y a subset from X, and δ a strictly positive real number. A finite or countable collection of sets $\{U_1, U_2, ...\}$ from X with diameter $D(U_1) \le \delta$, $D(U_2) \le \delta$,.., for which $Y \subset U_1 \cup U_2 \cup \cdots$, is called δ-cover of set Y. By virtue of this notion, for any subset Y from X and for any two real numbers $r \ge 0$, and $\delta > 0$, we can define indicator

[21] Given the application we analyze, these hypotheses are as natural as possible

$$\mathcal{H}_\delta^r(Y) = \inf_{\{U_1, U_2, \ldots\} \in \mathcal{C}_\delta(Y)} \{D^r(U_1) + D^r(U_2) + \cdots\},$$

where $\mathcal{C}_\delta(Y) = \{\{U_1, U_2, \ldots\} \mid \{U_1, U_2, \ldots\} \text{ is a } \delta-\text{cover of } Y\}$. This indicator defines a decreasing function $\delta \to \mathcal{H}_\delta^r(Y)$. This property guaranties the existence of the limit

$$\mathcal{H}^r(Y) = \lim_{\delta \to 0} \mathcal{H}_\delta^r(Y),$$

which, by definition, is called the *r*-dimensional Hausdorff measure of Y.

Among the properties of the Hausdorff measure, $\mathcal{H}^r(\cdot)$, we mention:

1) $\mathcal{H}^r(\varnothing) = 0$;

2) $\mathcal{H}^r(Y_1) \leq \mathcal{H}^r(Y_2)$, if $Y_1 \subseteq Y_2,\ Y_1, Y_2 \in \mathcal{P}(X)$;

3) $\mathcal{H}^r\left(\bigcup_{n=1}^{\infty} Y_n\right) \leq \sum_{n=1}^{\infty} \mathcal{H}^r(Y_n)$, if $\{Y_n \mid n \in \mathbb{N}^*\} \subset \mathcal{P}(X)$, is any countable collection of sets.

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GENETIC ALGORITHM FOR LEARNING AUTOMATA(22)

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Abstract: This paper presents a new type of genetic algorithm of environment exploration artificial intelligence automata based on various elements of Extenics Theory.

Key Words: Extenics Theory, Cell Decomposition, Genetic Algorithm, Machine Learning, AI, robot motion planning, collision-free path.

1. INTRODUCTION

In the field of software application for the coordination of walking robots equipped with artificial vision it is well understood that methods oriented to the development of that function specific to human intelligence, namely the ability of formulation an own strategy based on the data received from the environment and on experience accumulated over time, are preferable to the more wide-spread methods which feature a pre-determined strategy (also depending on the concrete conditions in the environment), selected by the human operator in the actual software application. The motivation for this is very simple: this solves not only problems relating to the services offered by a certain type of robot but also the problems relating to the continual improvement of artificial intelligent forms.

This paper presents a new method based on the implication of mathematical instruments specific to Extenics Theory, meant to contribute to the development of artificial self-learning automata by exploring the environment and making decisions based on experience acquired autonomously.

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2. NECESSARY ELEMENTS OF EXTENICS THEORY

The so-called Extenics Theory, founded by Prof. Cai Wen, is a collection of unconventional methods and mathematical instruments conceived to serve in solving problems of a practical and experimental nature where a consecrated methodology is not available. For a short list of paper promoting this theory see [1, 6 – 8]. This section will define the mathematical notions we need to attain the goal proposed at the beginning of this paper.

Let (X, d) be a metric space. For any and all points $x \in X$ and any set $A \subset X$, the relation $\delta(x,A) = \inf\{d(x,a) \mid a \in A\}$ defines, in classical mathematics, the distance from point x to the set A. The indicator

$$\mathfrak{s}(x,A) = \begin{cases} \delta(x,A), & x \in \complement A \\ -\delta(x,\complement A), & x \in A \end{cases}, \tag{1}$$

where $\complement A$ is the complementary set to set A, which we introduced in [4] to model the monitoring and control process for access into restricted areas overseen by optical sensors, better "positions" the point x in relation to the set A than the indicator $\delta(x,A)$. Indeed, in case $x \in \complement A$, due to the relation $\mathfrak{s}(x,A) = \delta(x,A)$, the indicator $\mathfrak{s}(x,A)$ expresses "the distance" from point x to the set A, and when $x \in A$, the indicator $\mathfrak{s}(x,A)$ expresses the "opposite sign distance" from point x to the frontier ∂A of the set A, while the indicator $\delta(x,A)$ only lets us know the distance from point x to set A equals zero, meaning that the point x belongs to set A.

In essence, Extenics Theory is founded on the idea of extending classical notions and mathematic instruments with the aim of obtaining new notions and instruments which would be better adapted to the requirements of applicative queries. With a view to this and due to the special characteristics of the indicator $\mathfrak{s}(x,A)$ mentioned above, we can safely conclude that it is a perfect fit to the specifics of Extenics Theory.

With the help of indicator $\mathfrak{s}(x,A)$ defined by the relation (1), this paper will introduce and use a new Extenics indicator, able to model the capacity of learning automata to take own decisions based on the conclusions drawn from repeated attempts. This new indicator is defined by the relation

$$\mathcal{E}(A,B) = \inf\{\mathfrak{s}(a, B) \mid a \in A\}. \tag{2}$$

3. PROBLEM FORMULATION

The problem to be solved is representative both due to its inter-disciplinary nature, as well as to its degree of generality for the current technologic and scientific level. It consists in developing those components of artificial intelligence which would allow a mobile automaton, fitted with the possibility of gathering data from the environment, to investigate on its own the topography of the surrounding space, recognize obstacle around it, execute similar knowledge and learning actions to a human's, and based on the accumulated experience to self-perfect its space orientation abilities so that it will be capable to travel between two given locations, surrounded by obstacles, while avoiding collisions. The following figure shows a simple version of this problem. More specifically, for establishing the concepts, this figure is an abstract version of a monitor screen, on which are configured a robot, marked with R, an obstacle and a destination. The robot will need to find, through different experiments, its own access paths to the arrival location and finally choose the most economical with regard to the stages to be fulfilled to accomplish the movement to the desired place.

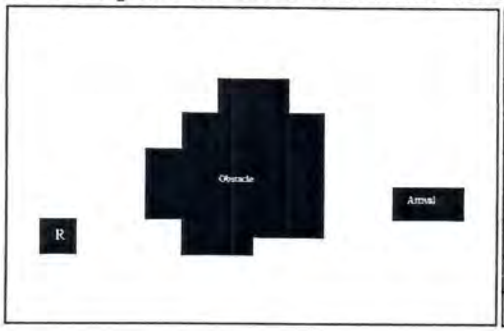

Figure 1.

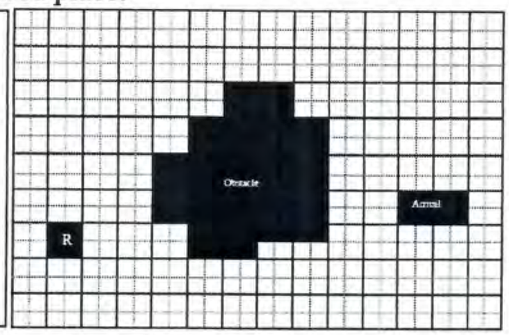

Figure 2.

The first step in solving the problem formulated earlier is a standard operation for such hypotheses, namely the monitor screen is partitioned into squares, whose dimension is chosen so that all object appearing on the screen can be covered exactly by an integer number of such squares. This is illustrated in Figure 2. For the algorithm that is to be created it is important to note that the number of squares (cells) in such a partition is finite (the smallest sized squares on the monitor screen are the pixels which make up its surface).

In many acclaimed scientific papers dealing with the subject, the robot is attributed eight degrees of freedom (in other words the robot can move in the network of squares of the partition established previously in each of the eight directions shown in Figure 3). Through chaotic movement (remember the robot does not have an initial movement strategy, this must be undertaken by the robot experimentally) the robot has the opportunity to know the surrounding environment and to later decide which the favourable access paths would be to the desired objective.

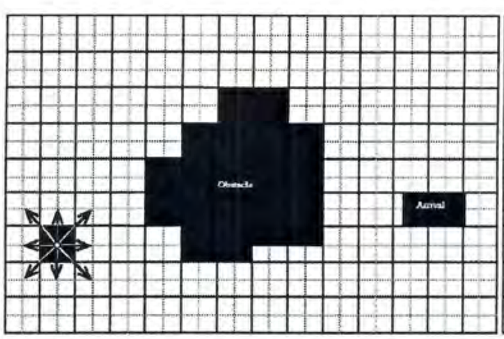

Figure 3.

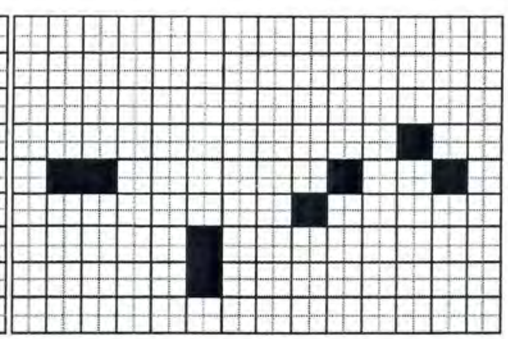

Figure 4.

Under the conditions of eight degree of freedom movement, we call road segment any reunion of partition squares – established as a working base, which have at least one point and at most one side of contact in common. The minimal segments of the situation considered (eight degree of freedom movement) are shown in Figure 4. Any reunion of such segments which are subject to the previously established rule: the segments that reunite must have in common at least one point but no more than the points situated on a side of the squares (cells) which come into contact, will be called road, trajectory, or access path.

Figure 5 illustrates an example of road linking the initial position R of the robot to its destination location.

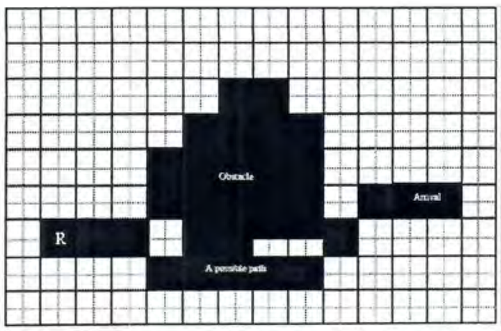

Figure 5.

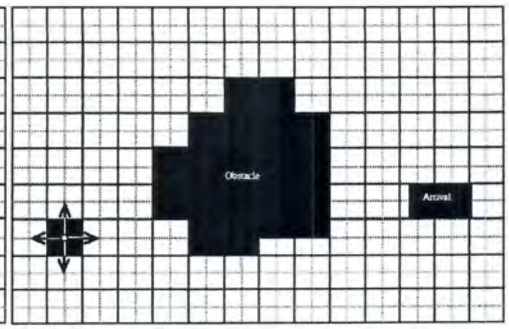

Figure 6.

One of the impediments of the method for investigating the surrounding environment presented earlier is the large number of variants that must be verified with the aim of finding the solution, which not only increases the robot's reaction time, but which can also lead to memory blockages. Indeed, with the exception of squares (positions) occupied by the objects populating the environment (in our case the monitor screen), in any other screen position, the robot can move in three, five or eight different directions, most of them (namely $(n-1)^2$ minus the number of squares which compose the objects on the screen, if the total number of squares (cells) in the used partition is n^2) allowing movement in eight different directions. In order to decrease this number we will reduce the number of degrees of freedom of the robot to four, as is shown in Figure 6. This is not however the only reason for this. The second, perhaps more important even than the first, is that this excludes the absolutely absurd situations when the robot can pass between two obstacles even if there is insufficient space for this to happen in real situations. For clarification, this is explained in Figure 7.

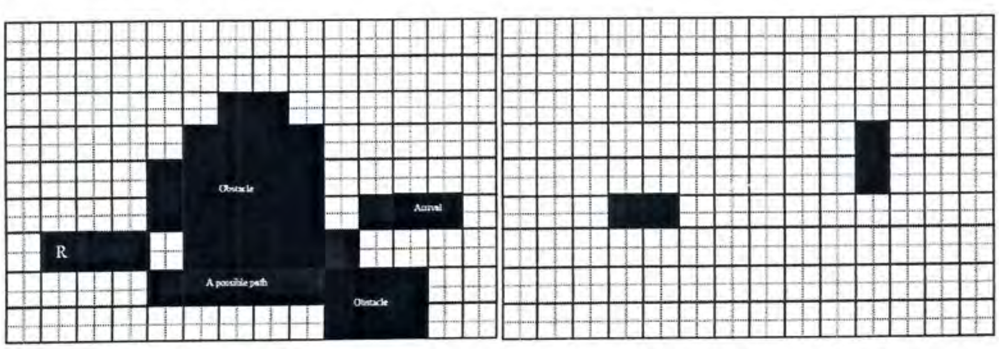

Figure 7. Figure 8.

The minimal road segments in the case of four degree of freedom movement are of the type shown in Figure 8. These are reunions of squares of the partition applied to the environment (in our case the monitor screen) which have in common exactly (meaning at least and at most) one side of the squares (cells) participating in the reunion. Admissible roads in this case will be by definition reunions of such segments that satisfy the relation mentioned earlier referring to the need to have exactly one side of contact.

For clarification, Figure 9 shows a possible path for a robot with four degrees of freedom.

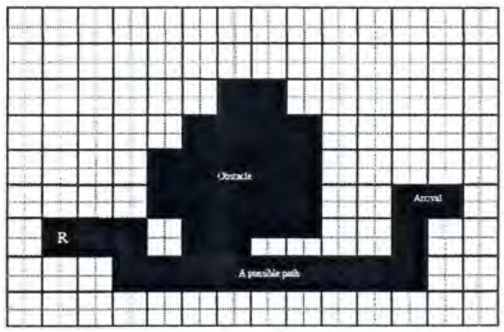

Figure 9.

4. PROPOSED SOLUTION

The proposed algorithm for coordinating the exploration process of the surrounding environment maintains the known working philosophy: according to the degrees of freedom of the robot, this moves a square in all possible directions, thus detecting the objects in the environment it is exploring. With the data thus obtained, which it stores beforehand, the robot develops a map of the environment and later, based on this map, establishes the road (access paths) to the target location, as well as the movement strategy.

The main difference in our implementation to the standard procedure is the way in which the robot exercises its exploration function of the environment (this will be done with mathematical instruments belonging to Extenics Theory), as well as the strategy it applies with the aim of reducing the number of experimental verifications it has to undertake in the recognition phase of the environment, which, even under the restraint of four degrees of freedom for the robot, is still overwhelming, especially in the situations where the robot passes from "virtual demonstrations" to "real applicative" implementations.

The exploration function for the environment which we are proposing in this paper is done after the following procedure: the central coordination unit of the robot (the on-board processor) calculates the values of the Extenics indicator $\mathcal{E}(A, B)$ defined by the formula (2) in which A will represent, one at a time, the neighbouring squares (cells)to the current position of the robot, and B the reunion of all objects that populate the space surrounding the robot (in our case the reunion of objects presented on the monitor screen), thus establishing: 1.) the fact that both A and B signify, from a mathematical standpoint, the set of points comprising these entities and 2.) the fact that the neighbour relationship between a square (cell) of the network partitioning the monitor screen and the robot is defined with relation to the degrees of freedom available to the robot, such as in Figure 10, where neighbouring positions to the robot are only the squares 1 through 4. After these operations, at the next step, from the movement possibilities for the robot are excluded the squares (cells) for which the value of indicator (2) is negative, considering as admissible movement (possible for the robot) only those that lead to the robot occupying a square (cell) for which the value of indicator (2) is greater than or equal to zero.

More precisely, using the example in Figure 10, if we note A_1, A_2, $A_3$and A_4as the surfaces of the squares (cells) 1, 2, 3 and 4 and with B the reunion of sets by which the obstacle and robot destination are synthesized, the first stage is to calculate the values of the Extenics indicators$\varepsilon(A_1, B)$, $\varepsilon(A_2, B)$,$\varepsilon(A_3, B)$and respectively$\varepsilon(A_4, B)$and the second stage is to decide which of the four possible movement directions are valid (admissible) and which are not (in the case of Figure 10, all of them are).

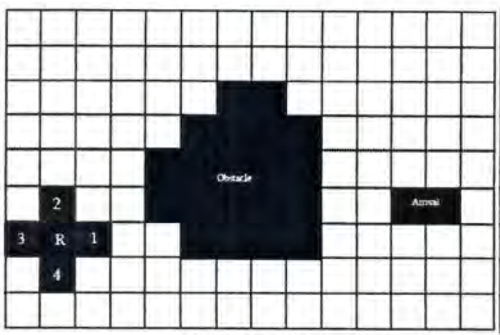

Figure 10.

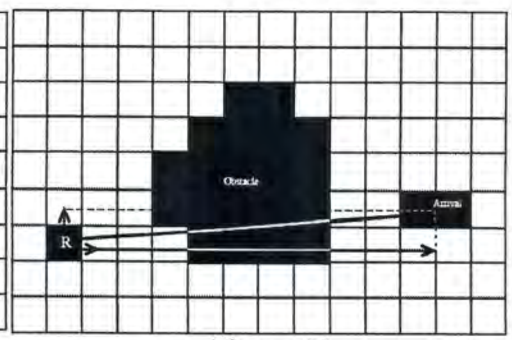

Figure 11.

As is well known (see example in [2]), in the vast majority of cases, the succession of experiments undertaken by the learning algorithms during the processes is not done after a particular design, but rather the exploratory activity is at random. With the aim of optimizing the performance of our algorithm (reducing the time to solution, minimizing the necessary memory resource for the learning processes, increasing execution precision), we endow the algorithm with a decisional function which will allow it to coordinate its actions after a certain strategy.

The function is meant to provide a hierarchy of possible directions for the movement of the robot after a certain rule which monitors the permanent orientation of the robot movement towards the destination. This is achieved in the following way: in the list of possible directions for the robot at a given time, the priority directions coincide with the vertical and horizontal projections (or, more accurately, the axis of the reference system to which the robot is considered) of the vector determined by the centre of the robot, considered as origin and the centre of the destination, considered as extremity. For clarification see Figures 11, 12 and 13. Figure 11 shows the priority directions for the movement of robot R corresponding to the given situation, while Figures 12 and 13 present the alternative robot positions in case it had followed one of these directions.

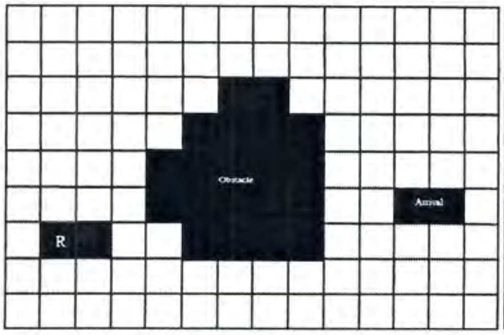

Figure 12.

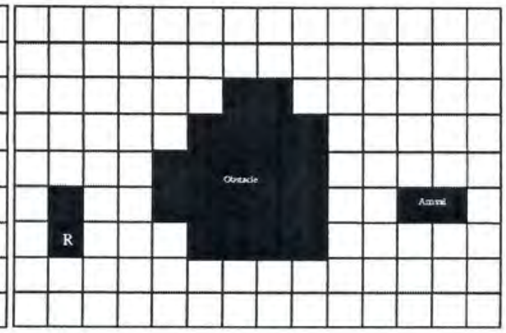

Figure 13.

When the robot is able to move in one of these directions determined by the rule established earlier, it is obvious that by choosing such a direction it maximizes its chances to reach the desired target. It is equally obvious that, in practice, there may arise situations where the above rule cannot be applied. Such an example is presented in Figure 14. Indeed, this figure shows one of the variants for robot movement, satisfying the mentioned rule, which, as can be seen, is blocked in position 3. In such a situation, the algorithm will allow the robot to execute movements in directions that do not take into account the general movement rule, including executing a number of steps backward along its path. This is however considered a last resort movement for removing the robot from this predicament.

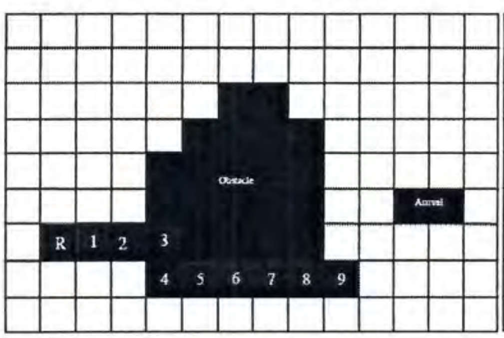

Figure 14.

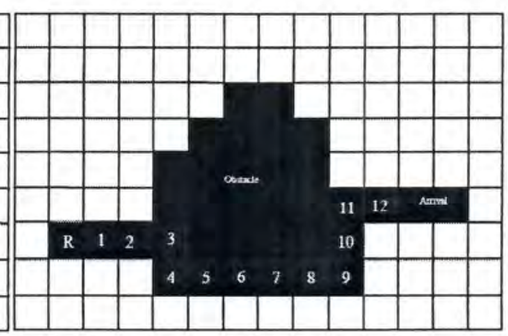

Figure 15.

Figure 14 shows the way the proposed algorithm takes the robot from position 3 (a blockage for the adopted rule) in which it can no longer move according to the rule for orientation to the target. From position 4, which the robot reaches by completely ignoring the general rule for movement, until position 9, the robot moves in accordance with the rule set forth.

Once it is in position 9, the robot has a number of options for accomplishing its activity. In Figures 15, 16 and 17 three alternative endings are presented. These are all optimal with respect to the number of necessary steps. Indeed, in their case, the number of steps needed for successfully finalizing the entire operation is minimal.

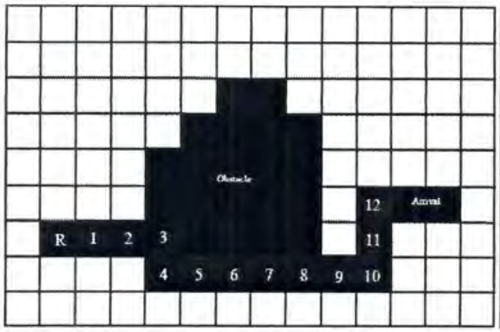

Figure 16.

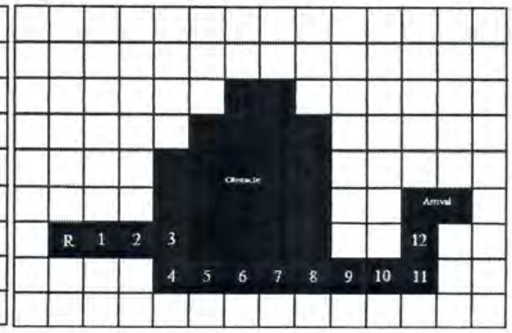

Figure 17.

5. CONCLUSIONS

The learning through exploration method proposed in this paper accrues certain defining qualities: one is the simplicity of conception which allows for a smooth transition to practice, while another is the ability for application to a large number of varied situations; also there should be noted the attributes referring to economical use of the computer's memory resources and the additional increase in reaction time.

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EXTENICS MODEL FOR EQUILIBRIUM CONTROL OF BIPEDAL ROBOTS[23]

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INTRODUCTION

As is becoming well known (see [1-6]), what is designated as Extenics Theory is a collection of mathematical methods and instruments, some of which unconventional, especially conceived to serve in tackling apparently contradictory problems, which practicing researchers are often confronted with. The father of this theory is Prof. Cai Wen, whose basic ideas were published in many papers. Of these we would point out [4], which is the very beginning of this field, as well as [1], [5] and [6] in which the theory has been detailed and applied to different scientific pursuits, including engineering. After the publishing of these papers, Prof. Cai Wen's Extenics Theory has also been developed by other researchers, see for example [2]. Among those interested in developing the mathematical apparatus of Extenics are some of the authors of this paper. The results thus obtained have been published in [3] and describe the generalization from the 1-dimensional case to the *n*-dimensional case of fundamental indicators for the whole of Extenics Theory.

In this paper we will highlight the utility of Extenics Theory's applicative toolbox in modeling the monitoring and control process for maintaining equilibrium with a (humanoid) bipedal robot during its movement in the environment. This objective will be accomplished by using a special indicator of a mathematical nature, whose definition will be handled in the following section.

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AN INDICATOR TO CHARACTERISE THE POSITION OF A POINT TO A SET

Let (X, d) be a metric space. For every $x \in X$ and any $A \subset X$, the formula $\delta(x,A) = \inf\{d(x,a) \mid a \in A\}$ defines (in classical mathematics) the distance from point x to set A. According to the properties of the notion of distance, any time there is $\delta(x,A) > 0$ we can conclude that the point x is outside the set $\overline{A}$ (the closure of set A with respect to the metric space topology (X, d)) being at distance $\delta(x,A)$ to the closest point of set A. This is why the expression $\delta(x,A)$ is to some extent an indicator of the position of x to A, but not completely, since if $\delta(x,A) = 0$ it offers us only the information $x \in A$, with no indication of how far or close the point is to the frontier ∂A of set A. In many situations this can prove vital. From the examples given [3] arises the need to extend the classical notion of distance and to introduce some specialized indicator which would satisfy these demands. This was in fact elaborated by Prof. Cai Wen in [4].

In order to accomplish the main objective of this paper, to model the monitoring and control process of a humanoid robot's equilibrium in different stages of action, we need to defined an indicator capable of characterising the position of a point with respect to a set. The best option to attain this goal is considering the relation

$$\mathsf{s}(x,A) = \begin{cases} -\delta(x,A), & x \in \complement A \\ \delta(x,\complement A), & x \in A \end{cases}, \tag{1}$$

where $\complement A$ is the complementary set of set A. Indeed, from the way it was defined, the indicator $\mathsf{s}(x,A)$ has the properties $x \in \complement\overline{A} \Leftrightarrow \mathsf{s}(x,A) < 0$, $x \in \overset{o}{A} \Leftrightarrow \mathsf{s}(x,A) > 0$, respectively, $x \in \partial A \Leftrightarrow \mathsf{s}(x,A) = 0$.

Observation: The indicator $\mathfrak{s}(x,A), x \in \mathbb{R}^n, A \subset \mathbb{R}^n$ introduced in [3] with the aim of generalizing the indicator $\rho(x,[a,b]),\ a,b \in \mathbb{R}, a < b$, considered by Cai Wen in [4], differs in the special case where $X = \mathbb{R}^n$ from the indicator defined in this paper only in its sign, that is to say, when $X = \mathbb{R}^n$, we have satisfied the relation $\mathfrak{s}(x,A) = -\mathsf{s}(x,A),\ \forall x \in \mathbb{R}^n,\ \forall A \subset \mathbb{R}^n$.

2) When the indicator $s(G',c_1)$ is strictly positive, its value is directly proportional with the robot's degree of stability.

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New Progress in Extension Theory(30)

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ABSTRACT: This paper presents the contribution of Romanian researchers to the development of Extension Theory as founded by Cai Wen, a theory which has enjoyed and enjoys unanimous appreciation from specialists due to its multiple applications in very varied fields of human activity.

INTRODUCTION

Professor Cai Wen from Guangdong University of Technology in China has established in his studies [1], [7], [8], [9] a set of mathematical methods and instruments meant to serve in solving those problems that appear in day-to-day activities, practical applications or experimental research, which cannot be broached with conventional mathematics, which he banded together in the general notion of "Extension Theory". For example, by considering the indicator

$$\rho(x, [a,b]) = \left|x - \frac{a+b}{2}\right| - \left|\frac{b-a}{2}\right|, \qquad (1)$$

defined by the Cartesian product of the set of real numbers $\mathbb{R}$ and the family $\mathcal{C}$ of the subsets of $\mathbb{R}$ of the form $[a, b]$, $a, b \in \mathbb{R}, a < b$, Prof. Wen has extended the capacity of classical notion of "distance from a point $x \in \mathbb{R}$ to a set $A \subset \mathbb{R}$, of the form $A = [a, b]$",
$\delta(x, [a, b]) = \inf\{|y-x| \mid y \in [a, b]\}$, for expressing the way point x relates to set A. Indeed, when $x \in \mathbb{R} \setminus [a, b]$, $\rho(x, [a,b]) = \delta(x, [a,b])$. Thus the Wen indicator $\rho(x, [a,b])$ expresses the distance from point x to set $[a, b]$. However, when x is within the interval (a, b), the Wen indicator $\rho(x, [a,b])$ is strictly negative and its absolute value $|\rho(x, [a,b])|$ coincides with the distance from point x to the frontier $\{a, b\}$ of the interval $[a, b]$, unlike the indicator $\delta(x, [a,b])$ which is equal to 0 and only expresses the membership of point x in the set (a, b).

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This indicator verifies a number of important properties:

Proposition 1. For any point $x \in \mathbb{R}$ and any interval $[a, b] \subset \mathbb{R}$, if $x \in (a, b)$ then $\rho(x, [a,b]) < 0$, and reciprocally.

Proposition 2. For any point $x \in \mathbb{R}$ and any interval $[a, b] \subset \mathbb{R}$ takes place $x \in \mathbb{R} \setminus [a, b] \Leftrightarrow \rho(x,[a, b]) > 0$.

Proposition 3. For any point $x \in \mathbb{R}$ and any intervals $[a_0, b_0] \subset \mathbb{R}$ and $[a, b] \subset \mathbb{R}$ in $\mathbb{R}$, if $[a_0, b_0] \subset (a, b)$ then $\rho(x,[a_0, b_0]) > \rho(x,[a, b]), \forall x \in \mathbb{R}^n$.

Based on the indicator $\rho(x, [a,b])$ and its properties, Prof. Cai Wen defined a second indicator

$$K(x,[a_0, b_0],(a, b)) = \frac{\rho(x,[a_0, b_0])}{\rho(x,(a, b)) - \rho(x,[a_0, b_0])}, \tag{2}$$

where $[a_0, b_0] \subset \mathbb{R}$ and $[a, b] \subset \mathbb{R}$ are two intervals in $\mathbb{R}$ vith $a_0, b_0, a, b \in \mathbb{R}$, $a < a_0 < b_0 < b$. The indicator $K(x,[a_0, b_0],(a, b))$ has all the properties detailed below:

Proposition 4. For any two intervals $[a_0, b_0]$ and $[a, b]$ in $\mathbb{R}$ for which $[a_0, b_0] \subset (a, b)$ we have $K(x,[a_0, b_0],(a, b)) < -1$, if $x \in \mathbb{R} \setminus [a, b]$; $-1 \le K(x,[a_0, b_0],(a, b)) < 0$, if $x \in [a, b] \setminus [a_0, b_0]$; $K(x,[a_0, b_0],(a, b)) \ge 0$, if $x \in [a_0, b_0]$, and reciprocally.

This second indicator has great practical value as it allows the mathematical modeling of problems relating to the control of dynamic systems. For example, when there is to be designed a dynamic system with three states which can be expresses mathematically by the following membership relations: 1) $x \in \mathbb{R} \setminus (a, b)$; 2) $x \in (a, b) \setminus [a_0, b_0]$; 3) $x \in [a_0, b_0]$, (where between the intervals $[a_0, b_0]$ and (a, b) exists the relation $[a_0, b_0] \subset (a, b)$) which, when passing from one state to another must react differently, a software application meant to control the system activity is made easier by using the indicator $K(x,[a_0, b_0],(a, b))$. This is due to the ability of indicator $K(x,[a_0, b_0],(a, b))$ to characterize through numerical values the three states of the system.

Starting from the ever-varying requirements in the application domain, which require finding indicators capable of characterizing more general and complex mathematical relations, a generalization of the existing set theory was necessary. In this activity the nucleus of Romanian researchers constituted around the Chinese researchers coordinated by Cai Wen have managed remarkable results meant to decisively impact the development of Extension Theory from a fundamenta, as well as applicative, viewpoint. These results will be synthesized, according to their nature, in the following two chapters.

2 FUNDAMENTAL EXTENSION THEORY

2.1 *Founding n-dimensional Extension Theory*

In this subsection we will present in short the results obtained and published in [3 – 6] which represent the optimal option for generalizing the Cai Wen indicators from the one – dimensional case, presented earlier, to the n - dimensional case. It should be noted that there is another variant for generalizing Cai Wen indicators, presented by Prof. Florentin Smarandache in [2]. In this paper we have chosen to present the main ideas of the theory in [3] as they represent the mathematical method founding n - dimensional Extension Theory that is most abstract and free of additional hypotheses.

Let x be a point and A a set in $\mathbb{R}^n$. By

$$\delta(x,A)=\inf_{y\in A} d(x,y),$$

where d is the Euclidean distance on $\mathbb{R}^n$, we note the distance from point $x\in\mathbb{R}^n$ to set $A\subseteq\mathbb{R}^n$. With the aid of distance δ we will define the indicator

$$\mathfrak{s}(x,A)=\begin{cases}\delta(x,A), & x\in \complement A\\ -\delta(x,\complement A), & x\in A\end{cases}, \tag{3}$$

where by $\complement A$ we noted the absolute complement of A, i.e. $\complement A=\mathbb{R}^n\setminus A$. In the particular case $n=1$ and $A=[a,b]$ this indicator coincides with the Cai Wen indicator (1). Moreover it enjoys the following properties:

Proposition 5. For any point $x\in\mathbb{R}^n$ and any set $A\subseteq\mathbb{R}^n$, if $x\in\overset{o}{A}$, where with $\overset{o}{A}$ is denoted the interior of the set A in topology induced by the metric d fixed earlier on the space $\mathbb{R}^n$, then $\mathfrak{s}(x,A)<0$, and reciprocally.

Proposition 6. For any point $x\in\mathbb{R}^n$ and any set $A\subseteq\mathbb{R}^n$, we have $x\in\complement\overline{A}\Leftrightarrow\mathfrak{s}(x,A)>0$, where with $\overline{A}$ is noted the closure of set A in topology induced by metric d on space $\mathbb{R}^n$.

Proposition 7. For any point $x\in\mathbb{R}^n$ and any sets A and B in $\mathbb{R}^n$, if $\overline{A}\subset\overset{o}{B}$ then $\mathfrak{s}(x,A)>\mathfrak{s}(x,B),\forall x\in\mathbb{R}^n$.

Proof of these can be found in [3].

Due to the properties named earlier of the indicator $\mathfrak{s}$ we can now define a new most important indicator for Extension Theory, namely

$$\mathfrak{S}(x,A,B)=\frac{\mathfrak{s}(x,A)}{\mathfrak{s}(x,B)-\mathfrak{s}(x,A)}, \tag{4}$$

defined for any $x\in\mathbb{R}^n$ and for any sets A and B in $\mathbb{R}^n$ which fulfil the property $\overline{A}\subset\overset{o}{B}$. This indicator generalizes the indicator

$$K(x,A,B)=\frac{\rho(x,A)}{\rho(x,B)-\rho(x,A)}, \tag{5}$$

introduced by Prof. Wen in [7] for the case $x\in\mathbb{R}$, $A=[a_0,b_0]$, $B=(a,b)$, $a_0, b_0, a, b\in\mathbb{R}$, $a<a_0<b_0<b$. Moreover, the indicator $\mathfrak{S}$ has all of the following properties:

Proposition 8. For any two sets A and B in $\mathbb{R}^n$ for which $\overline{A}\subset\overset{o}{B}$ we have $\mathfrak{S}(x,A,B)<-1$, if $x\in\complement\overline{B}$; $-1\le\mathfrak{S}(x,A,B)<0$, if $x\in\overline{B}\setminus\overline{A}$; $\mathfrak{S}(x,A,B)\ge 0$, if $x\in\overline{A}$, and reciprocally.

2.2 *New Perspectives in Extension Theory: A New Type of Extension Theory*

This subsection will show two options for the development of existing Extension Theory. These consist of considering some new types of indicator, capable of characterising the different relations between two subsets A and B of an abstract set X chosen as a base space.

The first type of indicator is an inclusion indicator, introduced in [5]. We shall consider a metric measurable space $(X, d, \mathcal{B}_X, \mu)$, in which X designated the set of points of the chosen space, d is the metric of said space , $\mathcal{B}_X$ denotes the family of Borelian parts of X (31), and μ is the considered measure on $\mathcal{B}_X$. After these preparations, for any two nonempty sets A and B in X we can define the indicator

$$\Delta(A, B)=\sup\{\delta(a, B)\mid a\in A\}, \tag{6}$$

where $\delta(a, B)$ is the usual distance from point $a\in A$ to set B, defined by us early in the subsection 2.1.

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(31) We define $\mathcal{B}_X$ as the smallest collection of subsets of X with the following properties: 1) $\mathcal{B}_X$ contains every open set and every closed set of the metric space (X, d); 2) $\mathcal{B}_X$ contains the union of every finite or countable collection of sets from $\mathcal{B}_X$; 3) $\mathcal{B}_X$ contains the intersection of every finite or countable collection of sets from $\mathcal{B}_X$.

Observations: 1) The relation $\Delta(A, B)=\Delta(B, A)$ is not always true, in other words the value of indicator $\Delta(A, B)$ depends in general on the order in which sets A and B are considered.

2) The indicator $\Delta(A, B)$ can also have infinite values.

3) For the case of two bounded sets A and B (32) the indicator $\Delta(A, B)$ is finite.

This indicator has the following properties:

1) $\Delta(A, B)=0 \Rightarrow A \subseteq B$ μ - almost everywhere. Reciprocally, $A \subseteq B \Rightarrow \Delta(A, B)=0$.
2) $\Delta(A, B)=\Delta(B, A)=0 \Rightarrow A=B$ μ - almost everywhere. Reciprocally, $A=B \Rightarrow \Delta(A, B)=\Delta(B, A)=0$.
3) $H(A, B)=\max\{\Delta(A,B), \Delta(B, A)\}$ represents the Hausdorff distance between the sets A and B.

The proof to these can be found in [5].

The second type of indicator, which is called a positioning indicator was introduced in [6]. Let (X, d) be a metric space and A, B two nonempty sets in X. The set A can be assumed additionally to allow for a Hausdorff measure of dimension $r \geq 0$, $\mathcal{H}^r(A)$ (33) finite and nonzero. Under these conditions, using the indicator $\mathfrak{s}$ defined by relation (3) we can now consider the relation

$$\mathsf{S}(A, B)=\frac{\mathcal{H}^r\left(\{a \in A \mid \mathfrak{s}(a, B) \leq 0\}\right)}{\mathcal{H}^r(A)}, \tag{7}$$

which defines the indicator we would like to introduce.

This indicator fulfils a number of mathematical properties important for the application field:

Proposition 9: $\mathsf{S}(A, B)=0 \Leftrightarrow A \cap B=\varnothing$ $\mathcal{H}^r$ - almost everywhere (or in other words, sets A and B are exterior to one another, without taking into account an eventual set of null $\mathcal{H}^r$ measure).

Proposition 10: $\mathsf{S}(A, B)>0 \Rightarrow A \cap B \neq \varnothing$ (sets A and B have common elements).

Proposition 11: $\mathsf{S}(A, B)=1 \Leftrightarrow A \subseteq B$, $\mathcal{H}^r$ - almost everywhere.

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(32) A subset Y of X is called bounded if its diameter $D(Y)=\sup\{d(y_1, y_2) \mid y_1, y_2 \in Y\}$ is bounded.
(33) For clarifications relating to the notion of r-dimensional Hausdorff measure see the Appendix included at the end of the paper.

Corollary: If $\mathsf{S}(A,B)=1$ and $\mathsf{S}(B,A)=1$, then $A=B$ $\mathcal{H}^r$ - almost everywhere, and reciprocally.

Observations: 1) Proof of the above can be found in [6].

2) From the definition of indicator S, (relation (7)) it can be easily deduced that $0 \le \mathsf{S}(A,B) \le 1$, for any pair of subsets A and B of space X with the properties: A is measurable in relation to a certain Hausdorff measure $\mathcal{H}^r$ of dimension $r \ge 0$, for which $\mathcal{H}^r(A) \ne 0$ and $\mathcal{H}^r(A) < \infty$, while B is a nonempty set.

3) Under the conditions in point 2), $\mathcal{H}^r(\{a \in A | \mathfrak{s}(a, B) \le 0\}) = \mathcal{H}^r(A \cap B)$.

3 APPLICATIVE EXTENSION THEORY

3.1 *Applications of Extension Theory in the Field of Designing Autonomous Warning Ssystems*

Many of the current technological installation such as those that emit high intensity radiation create danger zones for human activity. Assume we would like to secure such an installation through a centralized system of electronic sensor which would monitor danger areas and, depending on the gravity of the situation, send out warning signals for users or even interrupt the plant. For this we note with X the area in the surrounding space (mathematically modelled by $\mathbb{R}^3$) inside which the radiation exceeds the admissible safety level and X_0 as the zone inside X in which the radiation is unacceptable (fatal) for humans. Sensors mounted in the zones $\mathbb{R}^3 \setminus X$, $X \setminus X_0$ and X_0 send to the central monitoring and control unit the spatial coordinates of all persons involved with the activity. The software application which receives the sensor data must fulfil the following functions: 1) when the position vector $x=(x_1,x_2,x_3)$ of a user belongs to the zone $\mathbb{R}^3 \setminus X$ the plant is left to function uninterrupted; 2) when the position vector of a user $x=(x_1,x_2,x_3)$ belongs to the zone $X \setminus X_0$ the monitoring system must send out a warning signal; 3) when the position vector $x=(x_1,x_2,x_3)$ of a user belongs to the zone X_0 the monitoring system must shut down the functioning process.

Designing such a software application is much simplified by using the indicator $\mathfrak{S}(x,X_0,X)$ defined earlier by the relation (4) in which $A=X_0$ and $B=X$. Indeed, according to Proposition 8, if $\mathfrak{S}(x_a(t),X_0,X)<-1$, for any $a \in \mathcal{A}$, where $\mathcal{A}$ designates the set of employees servicing the installation and $x_a(t)$ is the position vector of person a at time t, the monitoring and control system does not send out a warning - the plant is left to function at full capacity; if $\mathfrak{S}(x_a(t),X_0,X) \in [-1,0)$ for at least one $a \in \mathcal{A}$, the monitoring and control system sends out warning messages for the users, but the process is left uninterrupted; if $\mathfrak{S}(x_a(t), X_0, X) \geq 0$ for at least one $a \in \mathcal{A}$, the centralized command system unequivocal shuts down the plant process.

Observations: 1) The example of practical utility of indicator $\mathfrak{S}(x, X_0, X)$ presented earlier leads to the introduction of a new concept in the theory of dynamic systems, namely "state indicator dynamic system" [34]. With the aid of this notion, within the field of dynamic systems, can be outlined an important class of systems, which due to their embedded state indicators could benefit from special solutions to a number of specific problems with respect to real time control of the quality of their functioning system.

2) The application presented in this subsection was taken from [3].

3.2 *Applications of Extension Theory in the Field of Humanoid Robot Design*

Let us considering a given walking robot for which we would like to design a monitoring and control algorithm for equilibrium. This robot we note as R. In order for this robot to be ensured equilibrium in a static position as well as during the movement process, it is well understood that its centre of gravity must project permanently inside a region situated within its legs. For a better understanding of the following, see Figure 1 below. The point G is the robot's centre of gravity, the set G is the region where the projection G' of point G so that the robot's equilibrium is ensured.

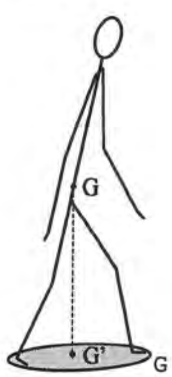

Figure 1. Graphic representation of the region G in which the projection *G'* of the centre of gravity *G* of robot R must fall so that its equilibrium is ensured.

[34] In a very abstract manner the notion of dynamic system endowed with status indicator assumes, by definition, the existence of a set $(\Sigma, \mathfrak{I})$ made up of a dynamic system Σ and an indicator $\mathfrak{I}$ of the states which the system Σ passes through during its functioning.

As can be seen from earlier, the main function that any algorithm which ensures the equilibrium of a biped robot must accomplish is in essence monitoring at every moment whether the projection G' of the robot's centre of gravity R falls within the safety region, noted by G, and to evaluate how stabe is the equilibrium status of robot R.

The method which we propose is based on using the indicator $s(G', G)$ obtained from the relation

$$s(G', G) = -s(G', G),$$

where s is the indicator (3) for the particular case when $X = \mathbb{R}^2$ and

$$d((x_1,y_1),(x_2,y_2)) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \ \forall (x_1,y_1),(x_2,y_2) \in \mathbb{R}^2.$$

Indeed, while the indicator $s(G', G) \geq 0$, the robot R is in equilibrium.

The indicator $s(G', G)$ can also provide us a measure of the security of the robot: a high positive value indicates good stability, a low positive value indicates a growing danger of falling, and a negative value indicates losing equilibrium.

Observation: The application presented in this subsection was taken from [4].

3.3 *Applications of Extension Theory in the field of Artificial Intelligence Forms Design*

Indicators Δ and S presented earlier (in relations (6), respectively (7)) can be used for example in computer vision in designing software applications for the automatic inclusion of a certain object $\mathcal{O}$ in a target region $\mathcal{R}$ of a given video image ImV. In order to attain this goal the following would be required of our algorithm: through a set of isometrics $\mathcal{I}_i, i \in I$ of the plane, we move the object $\mathcal{O}$ into different regions and positions of the image ImV, computing each time the value of one of the two indicator $\Delta(\mathcal{I}_i(\mathcal{O}), \mathcal{R})$ or $\mathsf{S}(\mathcal{I}_i(\mathcal{O}), \mathcal{R})$. Finding the index $i_0 \in I$ for which $\Delta(\mathcal{I}_{i_0}(\mathcal{O}), \mathcal{R}) = 0$ or $\mathsf{S}(\mathcal{I}_{i_0}(\mathcal{O}), \mathcal{R}) = 1$ constitutes solving the problem.

Observations: 1) This algorithm can be easily adapted to solving similar problems in 3-dimensional space, thus becoming even more useful in the field of projecting AI forms.

2) The application presented in this subsection was taken from [5] and [6].

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