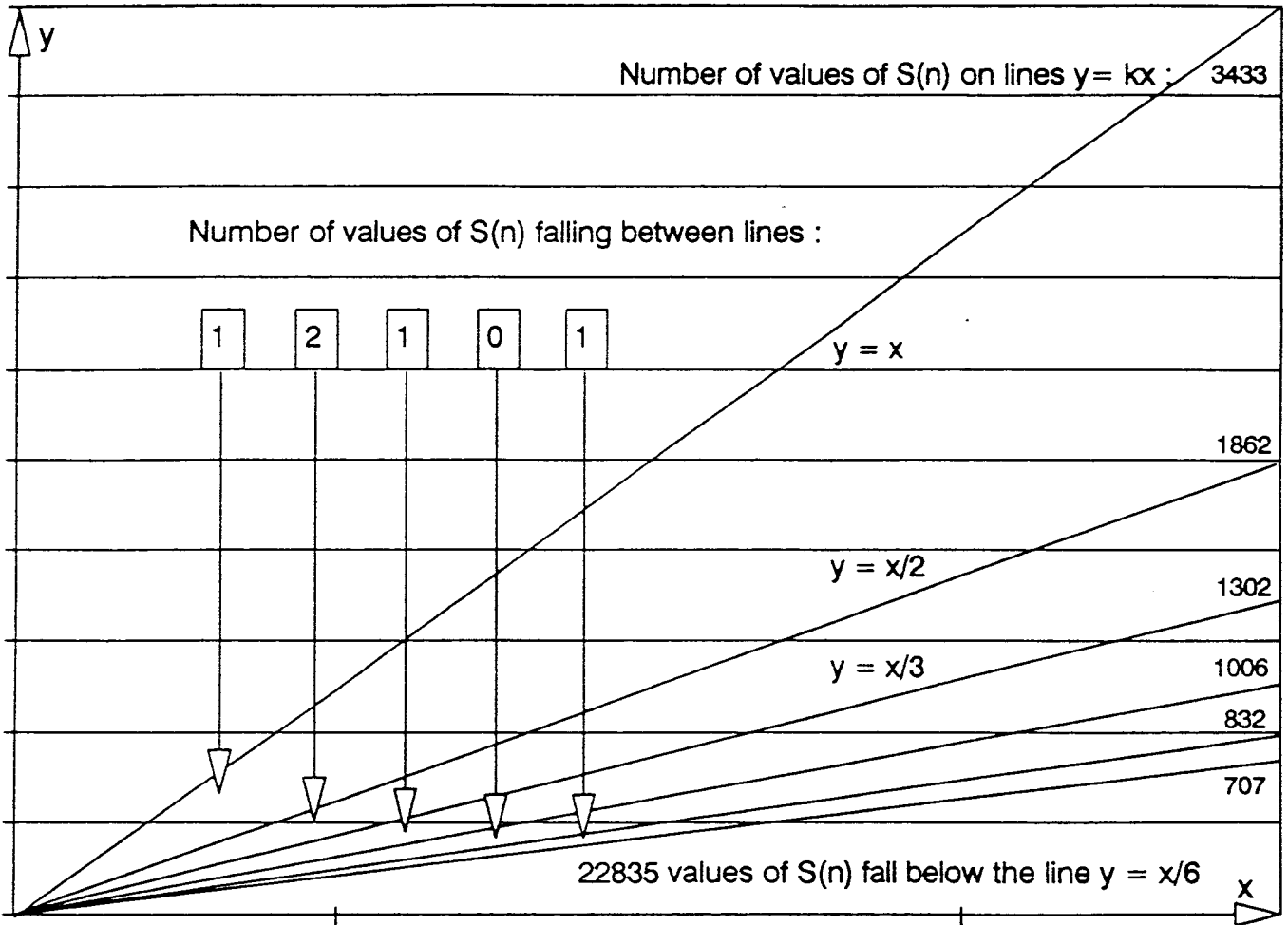


## An Illustration of the Distribution of the Smarandache Function

by Henry Ibstedt

The cover illustration is a representation of the values of the Smarandache function for  $n \leq 53$ . The group at the back of the diagram essentially corresponds to  $S(p) = p$ , the middle group to  $S(2p) = p$  ( $p \neq 2$ ) while the front group represents all the other values of  $S(n)$  for  $n \leq 53$ .

Diagram 1. Distribution of  $S(n)$  up to  $n = 32000$  (not to scale)



It may be interesting to take this graphical presentation a bit further. All the values of  $S(n)$  for  $n \leq 32000$  (conveniently chosen in order to use short integers only) have been sorted as shown in table 1. Of the 19114 points  $(n, S(n))$  situated above the line  $y = x/50$  only 61 points fall between lines. All of these of course correspond to cases where  $n$  is not square free. Diagram 1 illustrates this for the lines  $y = x, y = x/2, y = x/3, y = x/4, y = x/5$  and  $y = x/6$ . The top line contains 3433 points  $(n, S(n))$  although there are only 3432 primes less than 32000. This is because  $(4, S(4))$  belongs to this line.

TABLE 1. On the distribution of the Smarandache Function  $S(n)$  for  $n \leq 32000$ .

$N$  = number of values of  $S(n)$  on the line  $y=x/k$ , i.e.  $S(n)=n/k$ . The points  $(n,S(n))$  are the only ones between lines  $y=x/k$  and  $y=x/(k+1)$  for  $k < 50$ .

| k  | N    | Points $(n,S(n))$ between lines: |
|----|------|----------------------------------|
| 1  | 3433 | ( 9, 6)                          |
| 2  | 1862 | ( 16, 6) ( 25, 10)               |
| 3  | 1302 | ( 49, 14)                        |
| 4  | 1006 |                                  |
| 5  | 832  | ( 121, 22)                       |
| 6  | 707  | ( 169, 26)                       |
| 7  | 616  | ( 45, 6) ( 75, 10)               |
| 8  | 550  | ( 125, 15) ( 289, 34)            |
| 9  | 495  | ( 361, 38)                       |
| 10 | 450  | ( 147, 14)                       |
| 11 | 417  | ( 529, 46)                       |
| 12 | 387  |                                  |
| 13 | 359  | ( 80, 6)                         |
| 14 | 336  | ( 841, 58)                       |
| 15 | 321  | ( 961, 62)                       |
| 16 | 301  | ( 250, 15) ( 343, 21) ( 363, 22) |
| 17 | 283  | ( 175, 10) ( 245, 14)            |
| 18 | 273  | (1369, 74)                       |
| 19 | 256  | ( 507, 26)                       |
| 20 | 250  | ( 243, 12) (1681, 82)            |
| 21 | 239  | (1849, 86)                       |
| 22 | 227  | ( 225, 10)                       |
| 23 | 213  | (2209, 94)                       |
| 24 | 218  |                                  |
| 25 | 204  | ( 256, 10) ( 867, 34)            |
| 26 | 196  | (2809,106)                       |
| 27 | 190  | ( 605, 22)                       |
| 28 | 187  | (1083, 38)                       |
| 29 | 176  | (3481,118)                       |
| 30 | 179  | (3721,122)                       |
| 31 | 163  | ( 441, 14) ( 625, 20)            |
| 32 | 164  | ( 686, 21) ( 845, 26)            |
| 33 | 159  | ( 500, 15) (4489,134)            |
| 34 | 154  | (1587, 46)                       |
| 35 | 154  | (5041,142)                       |
| 36 | 153  | (5329,146)                       |
| 37 | 139  |                                  |
| 38 | 139  | ( 539, 14) ( 847, 22)            |
| 39 | 136  | (6241,158)                       |
| 40 | 139  | ( 486, 12) (1331, 33)            |
| 41 | 125  | (6889,166)                       |
| 42 | 133  | ( 512, 12) (1445, 34)            |
| 43 | 119  | (2523, 58)                       |
| 44 | 125  | (7921,178)                       |
| 45 | 126  | ( 637, 14) (1183, 26)            |
| 46 | 117  | (2883, 62)                       |
| 47 | 109  | (1805, 38)                       |
| 48 | 120  | ( 729, 15) (9409,194)            |
| 49 | 114  | (1089, 22)                       |
| 50 | 112  |                                  |

Number of elements below  $y = x/50$ : 12774 .