

Smarandache Curves, Surfaces and Geometries as Frameworks in Differential Geometry and Fundamental Mathematics

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Abstract: This paper presents the definition and distinctions between the Smarandache curves and all other types of curves, the definition and distinctions between Smarandache surfaces and all other types of surfaces, the definition and distinctions between Smarandache geometries and all other types of geometries, and includes nearly all research results so far.

Key Words: Smarandache curve, Smarandache surface, Smarandache geometry.

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§1. Introduction

The concepts of Smarandache curves, surfaces and geometries are frameworks in differential geometry and foundational mathematics, developed by Florentin Smarandache. Their primary distinction from classical counterparts lies in their hybrid structure and the partial negation of axioms.

1.1. Smarandache Curve. A Smarandache curve (or S-curve) is a regular curve in a geometric space (like Euclidean or Minkowski space) whose position vector is defined as a linear combination of the vectors of a moving frame (such as the Frenet-Serret frame, Bishop frame, or Darboux frame) of another base curve, first discussed in [28] and then, more and more research papers on Smarandache curves, surfaces [27]-[130] were published.

Definition Example: A regular curve in Minkowski space-time, whose position vector is composed by Frenet frame vectors on another regular curve, is called a Smarandache curve:

<https://fs.unm.edu/SG/NCSmarandacheCurvesOfMannheimCurve.pdf>

Distinctions from Other Curves:

- *Classical Curves:* Curves like helices or planar curves are typically defined by constant relationships between their intrinsic properties (e.g., constant curvature and torsion for a general helix).

- *Smarandache Curves:* They are derived curves whose geometric properties are intrinsi-

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cally linked to, and defined by, the moving frame of a different base curve. This gives them a more flexible and general structure, allowing for complex geometric analysis and applications in fields like robotics and physics. They are considered hybrid curves because they can satisfy properties of different types of curves simultaneously.

A Smarandache curve is a derived curve whose position vector is constructed by combining some of the Frenet frame vectors (the tangent T , normal N and binormal B) of another given regular space curve.

Formally, let $\alpha(s)$ be a regular space curve parameterized by arc length s with Frenet frame $\{T(s), N(s), B(s)\}$. Then, a Smarandache curve $\beta(s)$ is defined by

$$\beta(s) = aT(s) + bN(s) + cB(s),$$

where a, b, c are constants (or sometimes simple functions of s). Depending on which Frenet vectors are used, we have

Type	Definition
TN Smarandache curve	$\beta(s) = T(s) + N(s)$
TB Smarandache curve	$\beta(s) = T(s) + B(s)$
TNB Smarandache curve	$\beta(s) = T(s) + N(s) + B(s)$
etc.	Other combinations possible

Smarandache Curve

$$\beta(s) = aT(s) + bN(s) + cB(s)$$

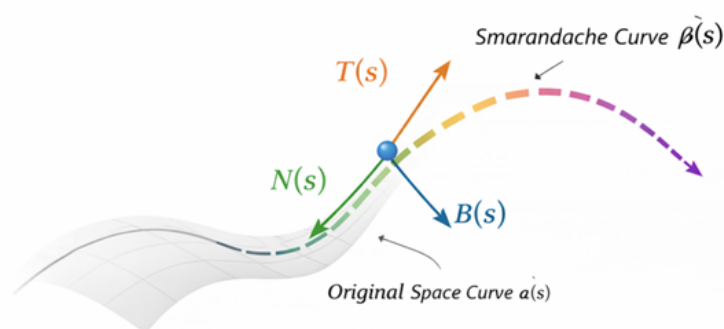


Figure 1.

1.2. Smarandache Surfaces. A Smarandache surface (or S-surface) is generally a surface, often a ruled surface, whose base curve and/or ruling direction vectors are themselves Smarandache curves.

Definition Example: A ruled surface is a curved surface which can be generated by the continuous motion of a straight line in space along a space curve called a directrix. This straight line is called a generator, or ruling, of the surface.

- A Smarandache ruled surface uses a Smarandache curve for the base curve (or the ruling vector). For instance, a type-2 Smarandache ruled surface uses a Smarandache curve as its base curve.

Distinctions from Other Surfaces:

- *Classical Surfaces:* Surfaces like cylinders, cones, or minimal surfaces are defined by standard properties (e.g., zero Gaussian curvature for developable surfaces, zero mean curvature for minimal surfaces).
- *Smarandache Surfaces:* Their construction is based on the hybrid nature of S-curves, meaning S-surfaces themselves are often surfaces of hybrid geometrical structures. This makes them a more complex and general category, incorporating the unique geometric characteristics inherited from the S-Curves used in their definition.

Smarandache Surface

$$\Phi(s, v) = \alpha(s) + a(v)T(s) + b(v)N(s) + c(v).B(s),$$

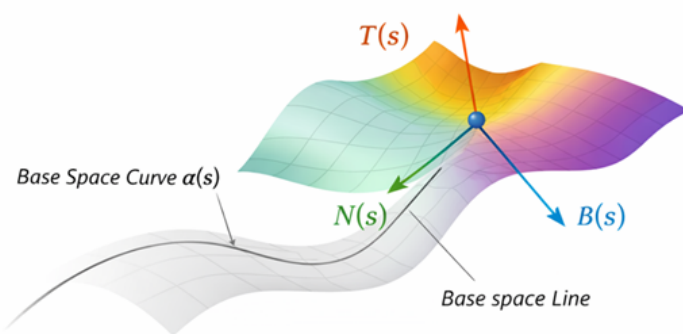


Figure 2.

1.3. Smarandache Geometries. A Smarandache geometry (or S-geometry) is a geometry in which at least one axiom is Smarandachely denied (<https://fs.unm.edu/SG/>). A Smarandache geometry (or S-geometry) is a geometry first discussed in [10] on in which at least one axiom is Smarandachely denied (<https://fs.unm.edu/SG/>) and then, more books and articles [1]-[26] were published.

Definition: An axiom is Smarandachely denied if it behaves in at least two different ways within the same space. This can mean the axiom is:

- (1) Validated and Invalidated (partially true and partially false);
- (2) Only Invalidated but in at least two distinct ways.

Distinctions from Other Geometries:

- *Classical geometries* (Euclidean, Riemannian, Hyperbolic, Elliptic): These are homogeneous spaces where a given axiom (like the parallel postulate) holds uniformly throughout the space (e.g., in Euclidean geometry, there is exactly one parallel line through an external point).

- *Smarandache Geometries*: They are hybrid or heterogeneous multispaces that combine structures from different classical geometries into a single space. For example, a single S-Geometry can be partially Euclidean and partially Non-Euclidean. This concept introduces the degree of negation of an axiom, which is analogous to the degrees of truth and falsehood in fuzzy logic or neutrosophic logic.

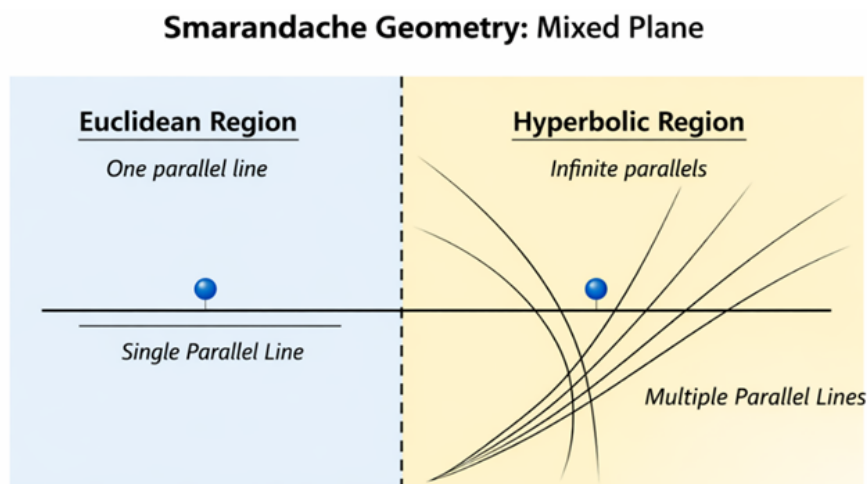


Figure 3.

The overall significance of these Smarandache notions is the creation of hybrid structures that allow for the simultaneous existence of multiple, sometimes contradictory, properties within a single defined entity, providing a more general and flexible framework for theoretical and applied mathematics. The Smarandache curve example, the most elementary type is an involute-evolute curve pair or a curve defined by a simple linear combination of the base curve's moving frame. The Smarandache surfaces are often constructed as ruled surfaces where the base curve or ruling vector is a Smarandache curve. The Smarandache geometries involve the partial denial of an axiom within a single space.

§2. Smarandache Euclidean-Hyperbolic Geometry

Axiom Smarandachely Denied: The Euclid's fifth postulate (the parallel postulate).

- *Space (S)*: Consider a single geometric plane.
- *Subspace (Euclidean region)*: This is the standard Euclidean plane, where for a given line ($L1$) and an external point ($P1$), there is exactly one line through $P1$ parallel to ($L1$).
- *Subspace (Hyperbolic Region)*: This is a region (e.g., a disk or portion of the plane) where for a given line ($L2$) and an external point ($P2$), there are infinitely many lines through $P2$ parallel to $L2$.
- *Smarandache Geometry*: The space (S) is the union of the two distinct regions ($R1$ and $R2$). The parallel postulate is *Smarandachely denied* because it holds true (validated) in the $R1$ part, and is false (invalidated) in the $R2$ part, all within the same space (S).

Distinction: Classical geometries are homogeneous (e.g., Euclidean or hyperbolic, but not both). Smarandache geometry is a heterogeneous multispace, where two (or more) types of geometry coexist and influence properties based on location (<https://fs.unm.edu/SG/>).

Connecting Smarandache geometries with neutrosophic logic helps explain the foundational philosophy behind these hybrid structures.

§3. Smarandache Geometries and Neutrosophic Logic

Neutrosophy is a branch of philosophy and logic, developed by Florentin Smarandache, that studies the origin, nature, and scope of neutralities. It is characterized by the representation of any idea, concept, or proposition as a triplet.

Neutrosophic logic (NL) is the corresponding mathematical tool where a proposition is defined by three independent components in the real unit interval:

- (*Truth*): The degree to which it is true;
- (*Indeterminacy*): The degree to which it is indeterminate or neutral;
- (*Falsehood*): The degree to which it is false.

Conceptual Relationship. The fundamental idea of Smarandache geometry (S-geometry) is a direct application of Neutrosophic logic to the structure of axioms:

(1) *Classical Geometry:* An axiom is either true ($T = 1, I = 0, F = 0$), as in Euclidean geometry where the parallel postulate is true, or false ($T = 0, I = 0, F = 1$), as in Hyperbolic geometry where it is false.

(2) *Smarandache Geometry:* An S-geometry exists when a single axiom is simultaneously true in one part of the space and false in another part of the space. This hybrid structure is the geometric manifestation of an Indeterminate or Neutral state.

Concept	Geometric Example (Parallel Postulate)	Neutrosophic Logic Equivalent
Axiom is Validated (True)	Exactly one parallel line exists (Euclidean region)	–
Exactly one parallel line exists (Euclidean region)	Infinitely many parallel lines exist (Hyperbolic region) Infinitely many parallel lines exist (Hyperbolic region)	–
Axiom is Smarandachely Denied	Both the Euclidean and hyperbolic cases exist within the same space	and simultaneously, which implies indeterminacy

In essence, S-geometries are the geometric spaces that model the philosophical concepts of Neutrosophy, allowing for the rigorous study of hybrid systems and contradictions in mathe-

ematics. This is further explored in fields like:

(a) NeutroGeometry which is a geometry that has at least one axiom that is partially false ($0 < F < 1$) and no axiom that is totally false;

(b) AntiGeometry, which is a geometry that has at least one axiom that is totally false ($F = 1$), for example the Non-Euclidean geometries are particular cases of the AntiGeometry, as a newer development (<https://fs.unm.edu/NG/>);

(c) unlike the classical geometry, where all axioms are totally true, $T = 1$.

Update books, research papers on Smarandache geometries, Smarandache curves and Smarandache surfaces with resources on the internet can be found in references [1]-[130] following.

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