

Review Article

Double Treatment in Product Mixture Problem Data Hyperparametric Function and SuperHyperparametric FunctionMaissam Ahmad Jdid^{a,b,*}, Florentin Smarandache^c^aFaculty of Science, Damascus University, Damascus, Syria^bDepartment of Requirements, International University for Science and Technology, Ghabageb, Syrian Arab Republic^cUniversity of New Mexico, Mathematics, Physics and Natural Sciences Division 705 Gurley Ave., Gallup, NM 87301, USA

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ABSTRACT

The fundamental focus of operations research is the existence of a problem requiring decision-making. The need for operations research methods increases as the complexity of the problem increases. One important method in operations research is linear programming, which relies on translating the actual situation under study into a linear mathematical model consisting of an objective function and constraints. This method uses data collected from the situation by experts. As we know, this data is suitable for operating conditions similar to those in which it was collected. In other words, this data is subject to change depending on the surrounding conditions. In light of this uncertainty, it was necessary to devise scientific methods suitable for all circumstances. In classical studies, researchers in the field of operations research introduced sensitivity analysis and parametric programming. This method is an expansion of sensitivity analysis because parametric programming studies the effect of simultaneous changes in the data when the coefficients change as a function of a single parameter. It also examines the effect of continuous changes in the coefficients of the objective function and the right-hand side of the constraints on the optimal solution. It provides us with a set of optimal acceptable solutions to the problem under study. In this research, we present a new approach to the product mix problem that aims to reformulate the mathematical model of this problem using the parametric programming method, which is an extension of sensitivity analysis, where the effect of simultaneous changes in the data is studied when the coefficients change as a function of a single parameter, Double treatment in product mixture problem data hyperparametric function and superhyperparametric function and the concept of a Hyperfunction, which associates each of the acceptable values provided by the model study using parametric programming to a subset of outputs. This generalizes classical parametric programming to represent multi-valued results. We will also reformulate it using parametric programming and SuperHyperFunction, through which sets (or groups of sets) are associated with values of higher-order power sets, which enables us to capture complex hierarchical or layered uncertainties. This enables us to obtain solutions that fit all the conditions that the operating environment of the system under study may experience.

1. Introduction

In all production institutions, for workflow to be optimal and achieve maximum profit, they must rely on a scientific study that determines the quantities of each product that must be produced using available resources (raw materials, labor, and working hours). The use of operations research methods fulfills the ambition of decision-makers in production institutions to obtain the greatest profit and lowest cost by transforming the issue under study into a mathematical model. In other words, a mathematical model is a simplified representation of a practical system from real life or a proposed idea for an executable system. The mathematical model is constructed based on data provided by experts. This data, collected under specific conditions, is subject to change with the changing circumstances surrounding the work environment. Therefore, the optimal solution for these models is appropriate for conditions similar to those in which the data was collected. To obtain an optimal solution suitable for all possible conditions in the work

environment, researchers in the field of operations research have developed methods that consider all changes and help decision-makers make optimal decisions appropriate for all circumstances. In older studies, sensitivity analysis and intermediate programming were used (Taha, 2006; Bakaja Ji et al., 1998), while in more recent studies, we have used Neutrosophic logic is a new vision of modeling designed to effectively address the uncertainties inherent in the real world. It replaces binary logic, which recognizes right and wrong, by introducing a third, neutral state that can be interpreted as indeterminate or uncertain. This can be used to reformulate linear mathematical models and some algorithms used to find their optimal solution (Jdid and Khalid, 2022; Jdid and Smarandache, 2023; Jdid et al., 2022; Smarandache and Jdid, 2023). To keep pace with the significant development of this logic, see Jdid et al., 2025, and Smarandache, 2016. In this research, we present a new study that doubles the uncertainty in the product mix problem data by reformulating the problem using parametric programming and the concept of a Hyperfunction, which associates each of the acceptable

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values provided by the model study using parametric programming to a subset of outputs. This generalizes classical parametric programming to represent multi-valued results. We will also reformulate it using parametric programming and SuperHyperFunction, through which sets (or groups of sets) are associated with values of higher-order power sets, which enables us to capture complex hierarchical or layered uncertainties. This enables us to obtain solutions that fit all the conditions that the operating environment of the system under study may experience

2. Discussion

The profit of any company that can produce a number of products is affected by the optimal selection of products that it will produce. To help these companies choose the products that achieve the maximum profit, operations research introduced the linear programming method, which uses data provided by experts to build a linear mathematical model whose optimal solution helps decision makers in the company to make the ideal decision and determine the optimal combination that achieves the maximum profit. The optimal solution depends on the data collected about the issue under study, and this data is subject to change according to the conditions in the work environment, meaning that it is uncertain data. To address the state of uncertainty, we present this study, which combines the old methods presented by researchers in the field of operations research and modern studies.

2.1. Product mixture problem

2.1.1. General formula for the product mixture problem

Firm n produces a product type $A_1, A_2, \dots, A_n$ and requires $B_1, B_2, \dots, B_m$ of raw materials, of which the available quantities are $b_1, b_2, \dots, b_m$. The requirements for each of these products for each of the raw materials are shown in Table 1.

In addition to the raw materials, producing these products requires labor hours, of which K labor hours are available. Each unit of each product has a cost. Table 2 shows the labor hours required for each unit of each product, and the cost.

It is required to formulate the appropriate mathematical model through which the company can determine the optimal product mix at the lowest possible cost.

Mathematical model No. (1):

Let $x_1, x_2, \dots, x_n$. The quantities of products produced, respectively
Find:

$$\text{Minimize } Z = \sum_{j=1}^n c_j x_j$$

Table 1.
Raw materials data

Products \ Raw materials	A_1	A_2	-----	A_n
B_1	a_{11}	a_{12}	-----	a_{1n}
B_2	a_{21}	a_{22}	-----	a_{2n}
-----	-----	-----	-----	-----
B_m	a_{m1}	a_{m2}	-----	a_{mn}

Table 2.
Data on working hours and cost per unit

Products \ Working hours - Cost	A_1	A_2	-----	A_n
Working hours	k_1	k_2	-----	k_n
Cost	c_1	c_2	-----	c_n

Subject to:

$$\sum_{j=1}^n a_{ij} x_j \leq b_i \quad ; i = 1, 2, \dots, m \quad (\text{Raw materials})$$

$$\sum_{j=1}^n K_j x_j \leq K \quad (\text{Working hours})$$

$$x_j \geq 0 \quad ; j = 1, 2, \dots, n$$

2.1.2. Practical application no. (1)

A battery factory produces two different types of batteries: A_1, A_2 . It uses three types of raw materials: B_1, B_2, B_3 , of which the following quantities are available: 25, 50, 70, respectively. These products require labor hours. The total labor hours available in the factory are 90. Given that:

One unit of the first type of battery requires 2, 6, 3 of the three raw materials, respectively.

One unit of the second type of battery requires 1, 5, 4 of the three raw materials, respectively.

One unit of the first type of battery requires 3 labor hours, while one unit of the second type of battery requires 5 labor hours.

The cost of one unit of the first type of battery is 7\$, and the cost of one unit of the second type of battery is 5\$. We need to formulate an appropriate mathematical model that will enable the company to determine the optimal product mix at the lowest possible cost.

To construct the appropriate mathematical model, let x_1 be the quantity produced of the first type of battery and x_2 be the quantity produced of the second type of battery. We then obtain the following mathematical model:

Find:

$$\text{Min } Z = 5x_1 + 7x_2$$

Subject to:

$$2x_1 + x_2 \leq 25 \quad (\text{Raw material } B_1)$$

$$6x_1 + 5x_2 \leq 50 \quad (\text{Raw material } B_2)$$

$$3x_1 + 4x_2 \leq 70 \quad (\text{Raw material } B_3)$$

$$3x_1 + 5x_2 \leq 90 \quad (\text{Working hours})$$

$$x_1, x_2 \geq 0$$

We have obtained a classical linear mathematical model, and we can obtain its optimal solution using either the graphical method or the simplex method.

2.2. Parametric programming: (Taha, 2006; Bakaja Ji et al., 1998)

It is an extension of sensitivity analysis because it studies the effect of simultaneous changes in the data when the coefficients change as a function of a single parameter. To illustrate the idea of parametric programming and how it can be used for treatment in product mixture problem data, we study the following two cases:

2.2.1. The parametric cost problem

In intermediate programming, we replace c_j in the objective function with the intermediate function $c_j(\lambda)$. Then we write the mathematical model No. (1): as follows:

Mathematical model No. (2, c):

Find:

$$\text{Minimize } Z = \sum_{j=1}^n (c_j + \lambda c_j^*) x_j$$

Subject to:

$$\sum_{j=1}^n a_{ij} x_j \leq b_i \quad ; i = 1, 2, \dots, m \quad (\text{Raw materials})$$

$$\sum_{j=1}^n K_j x_j \leq K \quad (\text{Working hours})$$

$$x_j \geq 0 \quad ; j = 1, 2, \dots, n$$

where c_j is the cost vector, c_j^* is the cost variation vector, and λ is a positive or negative unknown parameter. Changing the value of λ changes the cost coefficients for all variables; what interests us is determining the set of optimal solutions for all values of λ in the range from $-\infty$ to $+\infty$.

Practical application No. (1) is written as follows:

2.2.2. Practical application No. (2, c)

If the vector of change of cost is $c^* = (1, -1)$.

Find:

$$\text{Min } Z = (5 + \lambda) x_1 + (7 - \lambda) x_2$$

Subject to:

$$2x_1 + x_2 \leq 25 \quad (\text{Raw material } B_1)$$

$$6x_1 + 5x_2 \leq 50 \quad (\text{Raw material } B_2)$$

$$3x_1 + 4x_2 \leq 70 \quad (\text{Raw material } B_3)$$

$$3x_1 + 5x_2 \leq 90 \quad (\text{Working hours})$$

$$x_1, x_2 \geq 0$$

$$c_j(\lambda) = (5 + \lambda, 7 - \lambda)$$

We obtained a linear neutrosophic mathematical model, and we can obtain its optimal solution using the graphical method or the simplex method used to solve the linear neutrosophic models, as explained in two research papers (Jdid and Smarandache, 2023; Jdid et al., 2022).

2.3. Parametric right-hand-side problem

The right-hand side constants in a linear programming problem represent the limits imposed on the available resources (labor, raw materials, working hours, etc.), which will necessarily affect the output. The resources are not necessarily independent of each other. In a practical problem, it is very likely that a shortage of one resource will be accompanied by a shortage of other resources, to varying degrees. This, in turn, will affect the output.

In all problems, we take into account simultaneous changes in the right-hand side constants when these changes depend on a single parameter, and study their effects on the optimal solution. Therefore, we take the right-hand side with the parameters $b_i(\lambda)$, and the critical values for λ are determined from the following condition. Then we write the mathematical model No. (1): as follows:

2.3.1. Mathematical model no. (2, b)

Find:

$$\text{Minimize } Z = \sum_{j=1}^n c_j x_j$$

Subject to:

$$\sum_{j=1}^n a_{ij} x_j \leq (b_i + \alpha b_i^*) ; i = 1, 2, \dots, m \quad (\text{Raw materials})$$

$$\sum_{j=1}^n K_j x_j \leq (K + \alpha K^*) \quad (\text{Working hours})$$

$$x_j \geq 0 \quad ; j = 1, 2, \dots, n$$

b_i, K right-hand vector known.

b_i^*, K^* vector variations.

α unknown parameter.

The values of the right-hand constants change whenever the value of the parameter α changes. Our interest is to determine the optimal set of solutions for all values of α between $-\infty$ and $+\infty$.

Practical application No. (1) is written as follows:

2.3.2. Practical application no. (2, b)

Find:

$$\text{Min } Z = 5x_1 + 7x_2$$

Subject to:

$$2x_1 + x_2 \leq (25 - 2\alpha) \quad (\text{Raw material } B_1)$$

$$6x_1 + 5x_2 \leq (50 + \alpha) \quad (\text{Raw material } B_2)$$

$$3x_1 + 4x_2 \leq (70 + 3\alpha) \quad (\text{Raw material } B_3)$$

$$3x_1 + 5x_2 \leq (90 - 5\alpha) \quad (\text{Working hours})$$

$$x_1, x_2 \geq 0$$

We obtained a linear neutrosophic mathematical model, and we can obtain its optimal solution using the graphical method or the simplex method used to solve the linear neutrosophic models, as explained in the two research papers (Jdid and Smarandache, 2023; Jdid et al., 2022).

2.4. Formulation of parametric programming using the concept of a hyperfunction

Hyperoperation [7,10]:

A hyperoperation is a generalization of a binary operation where the result of combining two elements is a set, not a single element. Formally, for a set X , a hyperoperation $\circ$ is defined as:

$$\circ : X \times X \rightarrow P(X)$$

Hyperfunction (Fujita and Smarandache, 2025):

A Hyperfunction is a function where the domain remains a classical set X , but the codomain is extended to the powerset of X , denoted $P(X)$. Formally, a Hyperfunction f is defined as:

$$f : X \rightarrow P(X) \text{ for any } x \in X,$$

$$f(x) \subseteq X$$

Is a subset of X .

2.4.1. Hyperparametric programming - linear objective function

The Hyperparametric cost problem:

Let $X = (x_1, x_2, \dots, x_n)$ be the quantities produced of products $(A_1, A_2, \dots, A_n)$.

We take the vector of variations $C_j^* = ([\mu_j, \beta_j])$. Then the coefficient of the variables in the objective function is equal to $c_j + \lambda c_j^* = c_j + \lambda [\mu_j, \beta_j]$, where λ is an unknown parameter, positive or negative.

Where:

$$C(\lambda) = \prod_{j=1}^n [c_j + \lambda \mu_j, c_j + \lambda \beta_j] \subseteq \mathbb{R}^n$$

The hyperparametric linear objective function is defined as:

$$H_j : S \rightarrow P(\mathbb{R}) ; H_j(x, \lambda) = \left\{ (c_j + \lambda c_j^*)^T X \mid (c_j + \lambda c_j^*) \in C(\lambda) \right\}$$

Since the optimal solution is associated with the critical value λ_j , for each acceptable value of the parameter λ_r , a hyperparametric linear objective function is defined as follows:

$$H_r : S \rightarrow P(\mathbb{R}) , H_r(x, \lambda) = \left\{ (c_r + \lambda c_r^*)^T X \mid (c_r + \lambda c_r^*) \in C(\lambda) \right\}$$

This function assigns to each decision vector X a set of acceptable optimal values $X = (x_1, x_2, \dots, x_n)$

This means we will obtain a set of acceptable optimal solution sets, eliminating uncertainty and adapting to all possible conditions in the work environment.

3. Result

For each acceptable value of the parameter λ_r , the objective function is a hyperfunction, from which we obtain a set of acceptable optimal values for the decision vector X . This means we obtain a set of acceptable optimal solution sets, eliminating uncertainty and adapting to all possible conditions in the working environment.

Therefore, mathematical model No. (1) is written as follows:

Mathematical model (3, c):

Find:

$$\min_{x_j, \lambda} (Z_{\min}(x, \lambda), Z_{\max}(x, \lambda))$$

Subject to:

$$\sum_{j=1}^n a_{ij} x_j \leq b_i \quad ; i = 1, 2, \dots, m \quad (\text{Raw materials})$$

$$\sum_{j=1}^n K_j x_j \leq K \quad (\text{Working hours})$$

$$x_j \geq 0 \quad ; j = 1, 2, \dots, n$$

Where:

$$Z_{\min}(x, \lambda) = \sum_{j=1}^n (c_j + \lambda \mu_j) x_j, Z_{\max}(x, \lambda) = \sum_{j=1}^n (c_j + \lambda \beta_j) x_j$$

Practical application No. (1) is written as follows:

Practical application No. (3, c):

$$\text{Let } C_j^* = ([1, 2], [-1, 1]) \rightarrow C(\lambda) = [5 + \lambda, 5 + 2\lambda] \times [7 - \lambda, 7 + \lambda] \subseteq \mathbb{R}^2$$

$$\begin{aligned} Z_{\min}(x, \lambda) &= (5 + \lambda)x_1 + (7 - \lambda)x_2, Z_{\max}(x, \lambda) \\ &= (5 + 2\lambda)x_1 + (7 + \lambda)x_2 \end{aligned}$$

Find:

$$\min_{x_j, \lambda} (Z_{\min}(x, \lambda), Z_{\max}(x, \lambda))$$

Subject to:

$$2x_1 + x_2 \leq 25 \quad (\text{Raw material } B_1)$$

$$6x_1 + 5x_2 \leq 50 \quad (\text{Raw material } B_2)$$

$$3x_1 + 4x_2 \leq 70 \quad (\text{Raw material } B_3)$$

$$3x_1 + 5x_2 \leq 90 \quad (\text{Working hours})$$

$$x_1, x_2 \geq 0$$

Here we are required to use the information given in (Jdid and Smarandache, 2023; Jdid et al., 2022) to solve the following two neutrosophic linear models:

First Model:

$$Z_{\min}(x, \lambda) = (5 + \lambda)x_1 + (7 - \lambda)x_2$$

Conditions:

$$2x_1 + x_2 \leq 25 \quad (\text{Raw material } B_1)$$

$$6x_1 + 5x_2 \leq 50 \quad (\text{Raw material } B_2)$$

$$3x_1 + 4x_2 \leq 70 \quad (\text{Raw material } B_3)$$

$$3x_1 + 5x_2 \leq 90 \quad (\text{Working hours})$$

$$x_1, x_2 \geq 0$$

The second model:

$$Z_{\max}(x, \lambda) = (5 + 2\lambda)x_1 + (7 + \lambda)x_2$$

Conditions:

$$2x_1 + x_2 \leq 25 \quad (\text{Raw material } B_1)$$

$$6x_1 + 5x_2 \leq 50 \quad (\text{Raw material } B_2)$$

$$3x_1 + 4x_2 \leq 70 \quad (\text{Raw material } B_3)$$

$$3x_1 + 5x_2 \leq 90 \quad (\text{Working hours})$$

$$x_1, x_2 \geq 0$$

The optimal solution is:

$$\min_{x_j, \lambda} (Z_{\min}(x, \lambda), Z_{\max}(x, \lambda))$$

This example illustrates how H_{obj} models cost uncertainty in parametric programming.

3.1. Forming parametric programming using the concept of a SuperHyperFunction:

Using the following definitions:

3.1.1. SuperHyperOperations (Smarandache, 2024)

Let X be a non-empty set, and let $P_k(X)$ be the k -th powerset of X . Define:

$$P^0(H) = H, P^{k+1}(H) = P(P^k(H)) ; k \geq 0$$

A SuperHyperOperation of order (m, n) is an m -ary operation:

$$\circ^{(m,n)} : H^m \rightarrow P_n^*(X)$$

If the codomain excludes the empty set, it is classical-type; if it includes it, it is Neutrosophic-type.

A SuperHyperFunction extends this idea further by mapping sets (or sets of sets) to higher-order powerset values, thereby capturing complex hierarchical or layered uncertainties. We explore the use of SuperHyperFunction in Parametric Programming specifically the SuperHyperparametric cost problem:

3.1.2. Linear objective n -SuperHyperfunction (Jdid et al., 2025)

Let $S = \mathbb{R}^n$ be the decision space and $C \subseteq \mathbb{R}^n$ a nonempty compact set of coefficient vectors. Define for each $x \in S$

$$H_{obj}^1 = \{c^T x \mid c \in C\} \subseteq \mathbb{R}$$

and recursively for $k = 2, 3, \dots, n$:

$$H_{obj}^k = P\{H_{obj}^{k-1}(X)\} \setminus \{\emptyset\}$$

the family of all nonempty subsets of $H_{obj}^{k-1}(X)$. Then,

$$H_{obj}^n : S \rightarrow P^n(\mathbb{R}), x \mapsto H_{obj}^n(x)$$

is called the Linear Objective n -SuperHyperfunction.

Let $X = (x_1, x_2, \dots, x_n)$ be the quantities produced of products $(A_1, A_2, \dots, A_n)$.

We take the vector of variations $C_j^* = [\mu_j, \beta_j]$. Then the coefficient of the variables in the objective function is equal to $c_j + \lambda c_j^* = c_j + \lambda [\mu_j, \beta_j]$, where λ is an unknown parameter, positive or negative.

$$H_{obj}^1 = \left\{ \left\{ (c_j + \lambda c_j^*)^T X \mid (c_j + \lambda c_j^*) \in C(\lambda) \right\} \right\} \subseteq \mathbb{R}$$

and recursively for $k = 2, 3, \dots, n$:

$$H_{obj}^k = P\{H_{obj}^{k-1}(X, \lambda)\} \setminus \{\emptyset\}$$

the family of all nonempty subsets of $H_{obj}^{k-1}(X, \lambda)$. Then

$$H_{obj}^n(X, \lambda) : S \rightarrow P^n(\mathbb{R}), x \mapsto H_{obj}^n(x, \lambda)$$

$H_{obj}^n(X, \lambda)$, Is called the parametric objective n -SuperHyperfunction.

To write mathematical model No. (1) using the SuperHyperparametric function:

We take the cost vector $C_j = (c_1, c_2, \dots, c_3)$ and the variation vector

$$C_j^* = \left[\begin{matrix} \mu^t, \beta^t \end{matrix} \right] ; t = 1, 2, \dots, n,$$

Let

$$S = \{x = (x_1, x_2) \in \mathbb{R}^2 \mid x_1, x_2 \geq 0\}$$

be the decision space for producing n products. Suppose the cost per unit is uncertain under three scenarios:

$$C(\lambda) = \left(c^{(1)}, c^{(2)}, c^{(3)} \right) = \left(\begin{matrix} c_1 + \lambda \mu^1, c_1 + \lambda \beta^1 \\ c_2 + \lambda \mu^2, c_2 + \lambda \beta^2 \\ c_3 + \lambda \mu^3, c_3 + \lambda \beta^3 \end{matrix} \right)$$

First-level objective values:

$$H_{obj}^1(x, \lambda) = \{c^T(\lambda) X \mid c(\lambda) \in C(\lambda)\} \subseteq \mathbb{R}$$

$$H_{obj}^1(X, \lambda) = \left\{ \begin{matrix} (c_1 + \lambda \mu^1)x_1 + (c_1 + \lambda \beta^1)x_2, (c_2 + \lambda \mu^2)x_1 \\ + (c_2 + \lambda \beta^2)x_2, (c_3 + \lambda \mu^3)x_1 + (c_3 + \lambda \beta^3)x_2 \end{matrix} \right\} = \{r_1, r_2, r_3\}$$

Second and third levels:

$$H_{obj}^k(x, \lambda) = P\{H_{obj}^{k-1}(X, \lambda)\} \setminus \{\emptyset\} ; k = 2, 3, \dots, n$$

$$H_{obj}^2(x, \lambda) = P\{H_{obj}^1(X, \lambda)\} \setminus \{\emptyset\}$$

Concrete combinations:

$$H_{obj}^2(X, \lambda) = \left\{ \{r_i\}, \{r_i, r_j\}, \{r_1, r_2, r_3\} ; 1 \leq i < j \leq 3 \right\}$$

$$r_1 = (c_1 + \lambda \mu^1)x_1 + (c_1 + \lambda \beta^1)x_2,$$

$$r_2 = (c_2 + \lambda \mu^2)x_1 + (c_2 + \lambda \beta^2)x_2,$$

$$r_3 = (c_3 + \lambda \mu^3)x_1 + (c_3 + \lambda \beta^3)x_2$$

Consider the linear parametric model:
minimize:

$$H_{obj}^2(x, \lambda)$$

subject to:

$$\sum_{j=1}^2 a_{ij}x_j \leq b_i ; i = 1, 2, \dots, m \text{ (Raw materials)}$$

$$\sum_{j=1}^2 K_j x_j \leq K \text{ (Working hours)}$$

$$x_1, x_2 \geq 0$$

Here, minimizing the third-level SuperHyperfunction $H_{obj}^3(x, \lambda)$ captures hierarchical profit uncertainty across all single-scenario, two-scenario, and three-scenario combinations.

This summarizes cost uncertainty via interval $[Z_{Min}, Z_{Max}]$.

3.1.3. Linear cost hyperparametric

Let $S = R^n$ be the allocation space, and let $C(\lambda) \subseteq R^n$ be a nonempty compact set of possible marginal-cost vectors.

Linear Cost Hyperparametric is the mapping:

$$H_{Co} : S \rightarrow P(R) = \{c^T x \mid c \in C(\lambda)\}$$

which assigns to each X allocation the set of all possible cost values $c^T x$ as c varies over $C(\lambda)$.

Practical application No. (1) is written as follows:

Practical application No. (4, c):

To build the appropriate mathematical model, let x_1 be the quantity produced from the first type of batteries and x_2 be the quantity produced from the second type of batteries. Then we get the following mathematical model:

$$X = (x_1, x_2); x_1, x_2 \geq 0$$

Due to stakeholder disagreement, the marginal-cost vector lies in:

$$\text{Let } C_j^* = ([1, 2], [-1, 1]) \rightarrow C(\lambda) = [5 + \lambda, 5 + 2\lambda] \times [7 - \lambda, 7 + \lambda] \subseteq R^2$$

$$C(\lambda) = \prod_{j=1}^n [c_j + \lambda \mu_j, c_j + \lambda \beta_j] \subseteq R^n$$

$$[c_j + \lambda \mu_j, c_j + \lambda \beta_j]$$

$$C(\lambda) = (c_1(\lambda), c_2(\lambda))^T \mid c_1(\lambda) \in [5 + \lambda, 5 + 2\lambda], c_2(\lambda) \in [7 - \lambda, 7 + \lambda]$$

Then:

$$H_{co}(x, \lambda) = \left\{ \left(c(\lambda)^T x \mid c(\lambda) \in C(\lambda) \right) = c_1(\lambda)x_1 + c_2(\lambda)x_2 \mid \left[\begin{array}{l} c_1(\lambda) \in [5 + \lambda, 5 + 2\lambda], c_2(\lambda) \in [7 - \lambda, 7 + \lambda] \\ (5 + \lambda)x_1 + (7 - \lambda)x_2, (5 + 2\lambda)x_1 + (7 + \lambda)x_2 \end{array} \right] \right\}$$

Resource constraints are:

$$2x_1 + x_2 \leq 25 \text{ (Raw material } B_1)$$

$$6x_1 + 5x_2 \leq 50 \text{ (Raw material } B_2)$$

$$3x_1 + 4x_2 \leq 70 \text{ (Raw material } B_3)$$

$$3x_1 + 5x_2 \leq 90 \text{ (Working hours)}$$

$$x_1, x_2 \geq 0$$

A robust-optimistic linear program uses the extremal values of the Hyperparametric:

$$C_{\min}(x, \lambda) = (5 + \lambda)x_1 + (7 - \lambda)x_2, \\ C_{\max}(x, \lambda) = (5 + 2\lambda)x_1 + (7 + \lambda)x_2$$

We solve:

minimize:

$$(C_{\min}(x, \lambda), C_{\max}(x, \lambda))$$

Subject to:

$$2x_1 + x_2 \leq 25 \text{ (Raw material } B_1)$$

$$6x_1 + 5x_2 \leq 50 \text{ (Raw material } B_2)$$

$$3x_1 + 4x_2 \leq 70 \text{ (Raw material } B_3)$$

$$3x_1 + 5x_2 \leq 90 \text{ (Working hours)}$$

$$x_1, x_2 \geq 0$$

Here also we solve two linear neutrosophy models as in the previous study.

4. Conclusions

In this research, we present a study of the product mix problem, aiming to reformulate the mathematical model that represents it using the concept of parametric programming. The solution to the model we obtain provides an acceptable optimal solution for every acceptable value of the parameter when the cost vector changes according to the surrounding conditions. To obtain highly accurate optimal solutions, we used the concept of hyperfunctions and hyper hyperfunctions to formulate the problem with other mathematical models. Since hyperfunctions and hyper hyperfunctions link each input to a subset of outputs that is, they link sets (or groups of sets) to values of higher-order power sets the optimal solution to these models enables them to capture complex, hierarchical, or stratified uncertainty situations suitable for all possible operating environment conditions. Since the primary goal of this research is to present various mathematical models for the product mix problem, we note the following: any researcher interested in obtaining solutions to the models we have arrived at in this research can use the methods provided by classical operations research and the modern studies we presented in (Jdid and Smarandache, 2023; Jdid et al., 2022).

CRedit authorship contribution statement

Maissam Ahmad Jdid: Conceptualization, formal analysis, investigation, writing—original draft, writing—review and editing. **Florentin Smarandache:** Conceptualization, writing—review and editing. All authors reviewed and gave their approval for the final manuscript.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Declaration of generative AI and AI-assisted technologies in the writing process

The authors confirm that there was no use of artificial intelligence (AI)-assisted technology for assisting in the writing or editing of the manuscript.

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