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A NEW CHARACTERIZATION OF SMARANDACHE TNB CURVES OF HELICES IN THE SOL SPACE Sol^3

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Abstract

In this paper, we characterize Smarandache **TNB** curves of helices in the Sol space Sol^3. We characterize Smarandache **TNB** curves of helices in terms of their curvature and torsion. Finally, we find out their explicit parametric equations.

Keywords: General helix, Sol Space, Curvature, Torsion, Smarandache **TNB** curve.

1. INTRODUCTION

A fundamental advance in theory of curves was the advent of analytic geometry in the seventeenth century. This enabled a curve to be described using an equation rather than an elaborate geometrical construction. This not only allowed new curves to be defined and studied, but it enabled a formal distinction to be made between curves that can be defined using algebraic equations, algebraic curves. Some curves and surfaces have been also represented as motion by several authors [1-7].

The geometry of the Galilean Relativity works such as a bridge from Euclidean geometry to special Relativity. The geometry of curves in Euclidean space have been developed in the past [4] . In modern times, mathematicians have started to research curves and surfaces some different spaces [8-24] .

Helices are among easy and simple styles that are located in the filamentary and molecular improvements of mechanics. A nearby physical elements of such components have a inclination to be made by way of a standard elastic potential energy dependent on bending and opinion, which is accurately what we call a pole version.

In this paper, we study Smarandache **TNB** curves of helices in the Sol^3. We characterize Smarandache **TNB** curves of helices in terms of their curvature and torsion. Finally, we find out their explicit parametric equations.

2. MATERIAL AND METHODS

Sol space, one of Thurston's eight 3-dimensional geometries, can be viewed as $\mathbb{R}^3$ provided with Riemannian metric

$$g_{\mathrm{Sol}^3} = e^{2z}dx^2 + e^{-2z}dy^2 + dz^2,$$

where (x, y, z) are the standard coordinates in $\mathbb{R}^3$.

Note that the Sol metric can also be written as [25]:

$$g_{\mathrm{Sol}^3} = \sum_{i=1}^{3} \boldsymbol{\omega}^i \otimes \boldsymbol{\omega}^i,$$

where

$$\boldsymbol{\omega}^1 = e^z dx,\ \boldsymbol{\omega}^2 = e^{-z} dy,\ \boldsymbol{\omega}^3 = dz,$$

and the orthonormal basis dual to the 1-forms is

$$\mathbf{e}_1 = e^{-z}\frac{\partial}{\partial x},\ \mathbf{e}_2 = e^{z}\frac{\partial}{\partial y},\ \mathbf{e}_3 = \frac{\partial}{\partial z}.$$

Assume that $\{\mathbf{T},\mathbf{N},\mathbf{B}\}$ be the Frenet frame field along γ. Then, the Frenet frame satisfies the following Frenet--Serret equations [26,27]:

$$\begin{aligned} \nabla_{\mathbf{T}}\mathbf{T} &= \kappa\mathbf{N}, \\ \nabla_{\mathbf{T}}\mathbf{N} &= -\kappa\mathbf{T}+\tau\mathbf{B}, \\ \nabla_{\mathbf{T}}\mathbf{B} &= -\tau\mathbf{N}, \end{aligned}$$

where κ is the curvature of γ and τ its torsion and

$$\begin{aligned} g_{\mathrm{Sol}^3}(\mathbf{T},\mathbf{T}) &= 1, g_{\mathrm{Sol}^3}(\mathbf{N},\mathbf{N})=1, g_{\mathrm{Sol}^3}(\mathbf{B},\mathbf{B})=1, \\ g_{\mathrm{Sol}^3}(\mathbf{T},\mathbf{N}) &= g_{\mathrm{Sol}^3}(\mathbf{T},\mathbf{B})=g_{\mathrm{Sol}^3}(\mathbf{N},\mathbf{B})=0. \end{aligned}$$

With respect to the orthonormal basis $\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$, we can write

$$\begin{aligned} \mathbf{T} &= T_1\mathbf{e}_1+T_2\mathbf{e}_2+T_3\mathbf{e}_3, \\ \mathbf{N} &= N_1\mathbf{e}_1+N_2\mathbf{e}_2+N_3\mathbf{e}_3, \\ \mathbf{B} &= \mathbf{T}\times\mathbf{N}=B_1\mathbf{e}_1+B_2\mathbf{e}_2+B_3\mathbf{e}_3. \end{aligned}$$

Theorem 2.1. ([28]) *Let* $\gamma : I \to \mathrm{Sol}^3$ *be a unit speed non-geodesic general helix. Then, the parametric equations of* γ *are*

$$x(s) = \frac{\sin \mathrm{P} e^{-\cos \mathrm{P} s - \mathrm{C}_3}}{\mathrm{C}_1^2+\cos^2\mathrm{P}}[-\cos\mathrm{P}\cos[\mathrm{C}_1 s+\mathrm{C}_2]+\mathrm{C}_1\sin[\mathrm{C}_1 s+\mathrm{C}_2]]+\mathrm{C}_4,$$

$$y(s) = \frac{\sin \mathrm{P} e^{\cos \mathrm{P} s + \mathrm{C}_3}}{\mathrm{C}_1^2+\cos^2\mathrm{P}}[-\mathrm{C}_1\cos[\mathrm{C}_1 s+\mathrm{C}_2]+\cos\mathrm{P}\sin[\mathrm{C}_1 s+\mathrm{C}_2]]+\mathrm{C}_5,$$

$$z(s) = \cos\mathrm{P}s+\mathrm{C}_3,$$

where $\mathrm{C}_1,\mathrm{C}_2,\mathrm{C}_3,\mathrm{C}_4,\mathrm{C}_5$ *are constants of integration.*

3. RESULTS AND DISCUSSION

Definition 3.1. *Let* $\gamma : I \to \mathrm{Sol}^3$ *be a unit speed helix in the Sol Space* Sol^3 *and* $\{\mathbf{T},\mathbf{N},\mathbf{B}\}$ *be its moving Frenet frame. Smarandache* **TNB** *curves are defined by*

$$\gamma_{\mathbf{TNB}} = \frac{1}{\sqrt{2\kappa^2 - 2\kappa\tau + 2\tau^2}}(\mathbf{T}+\mathbf{N}+\mathbf{B}).$$

Theorem 3.2. *Let* $\gamma : I \to \mathrm{Sol}^3$ *be a unit speed non-geodesic helix in the Sol Space* Sol^3. *Then, the equation of Smarandache* **TNB** *curve of a unit speed non-geodesic helix is given by*

$$\begin{aligned}
\gamma_{\mathbf{TNB}} \;=\;& \mathrm{W}[\sin \mathrm{P} \cos[\mathrm{C}_1 s + \mathrm{C}_2] + \frac{1}{\kappa}[-\frac{1}{\mathrm{C}_1}\sin \mathrm{P} \sin[\mathrm{C}_1 s + \mathrm{C}_2] + \cos \mathrm{P} \sin \mathrm{P} \cos[\mathrm{C}_1 s + \mathrm{C}_2]] \\
&+ [\frac{1}{\kappa}\sin \mathrm{P} \sin[\mathrm{C}_1 s + \mathrm{C}_2]][\sin^2 \mathrm{P} \sin^2[\mathrm{C}_1 s + \mathrm{C}_2] - \sin^2 \mathrm{P} \cos^2[\mathrm{C}_1 s + \mathrm{C}_2] \\
&- \frac{1}{\kappa}\cos \mathrm{P}[\frac{1}{\mathrm{C}_1}\sin \mathrm{P} \cos[\mathrm{C}_1 s + \mathrm{C}_2] - \cos \mathrm{P} \sin \mathrm{P} \sin[\mathrm{C}_1 s + \mathrm{C}_2]]]\mathbf{e}_1 \\
&+ \mathrm{W}[\sin \mathrm{P} \sin[\mathrm{C}_1 s + \mathrm{C}_2] + \frac{1}{\kappa}[\frac{1}{\mathrm{C}_1}\sin \mathrm{P} \cos[\mathrm{C}_1 s + \mathrm{C}_2] - \cos \mathrm{P} \sin \mathrm{P} \sin[\mathrm{C}_1 s + \mathrm{C}_2]] \\
&- [\frac{1}{\kappa}\sin \mathrm{P} \cos[\mathrm{C}_1 s + \mathrm{C}_2]][\sin^2 \mathrm{P} \sin^2[\mathrm{C}_1 s + \mathrm{C}_2] - \sin^2 \mathrm{P} \cos^2[\mathrm{C}_1 s + \mathrm{C}_2] \\
&- \frac{1}{\kappa}\cos \mathrm{P}[-\frac{1}{\mathrm{C}_1}\sin \mathrm{P} \sin[\mathrm{C}_1 s + \mathrm{C}_2] + \cos \mathrm{P} \sin \mathrm{P} \cos[\mathrm{C}_1 s + \mathrm{C}_2]]]]\mathbf{e}_2 \\
&+ \mathrm{W}[\cos \mathrm{P} + \frac{1}{\kappa}[\sin^2 \mathrm{P} \sin^2[\mathrm{C}_1 s + \mathrm{C}_2] - \sin^2 \mathrm{P} \cos^2[\mathrm{C}_1 s + \mathrm{C}_2]] \\
&+ \frac{1}{\kappa}\sin \mathrm{P} \cos[\mathrm{C}_1 s + \mathrm{C}_2][\frac{1}{\mathrm{C}_1}\sin \mathrm{P} \cos[\mathrm{C}_1 s + \mathrm{C}_2] - \cos \mathrm{P} \sin \mathrm{P} \sin[\mathrm{C}_1 s + \mathrm{C}_2]] \\
&- \frac{1}{\kappa}\sin \mathrm{P} \sin[\mathrm{C}_1 s + \mathrm{C}_2][-\frac{1}{\mathrm{C}_1}\sin \mathrm{P} \sin[\mathrm{C}_1 s + \mathrm{C}_2] + \cos \mathrm{P} \sin \mathrm{P} \cos[\mathrm{C}_1 s + \mathrm{C}_2]]]\mathbf{e}_3,
\end{aligned}$$

where $\mathrm{C}_1, \mathrm{C}_2$ *are constants of integration and*

$$\mathrm{W} = \frac{1}{\sqrt{2\kappa^2 - 2\kappa\tau + 2\tau^2}}.$$

Corollary 3.3. *Let* $\gamma : I \to \mathrm{Sol}^3$ *be a unit speed non-geodesic helix in the Sol Space* Sol^3*. Then, the parametric equations of Smarandache* **TNB** *curves of a unit speed non-geodesic helix are given by*

$$
\begin{aligned}
x_{\mathbf{TNB}}(s) \;=\; & \exp[-\mathrm{W}[\cos\mathrm{P} + \frac{1}{\kappa}[\sin^2\mathrm{P}\sin^2[\mathrm{C}_1 s + \mathrm{C}_2] - \sin^2\mathrm{P}\cos^2[\mathrm{C}_1 s + \mathrm{C}_2] \\
& + \frac{1}{\kappa}\sin\mathrm{P}\cos[\mathrm{C}_1 s + \mathrm{C}_2][\frac{1}{\mathrm{C}_1}\sin\mathrm{P}\cos[\mathrm{C}_1 s + \mathrm{C}_2] - \cos\mathrm{P}\sin\mathrm{P}\sin[\mathrm{C}_1 s + \mathrm{C}_2] \\
& - \frac{1}{\kappa}\sin\mathrm{P}\sin[\mathrm{C}_1 s + \mathrm{C}_2][-\frac{1}{\mathrm{C}_1}\sin\mathrm{P}\sin[\mathrm{C}_1 s + \mathrm{C}_2] + \cos\mathrm{P}\sin\mathrm{P}\cos[\mathrm{C}_1 s + \mathrm{C}_2]]]] \\
& \mathrm{W}[\sin\mathrm{P}\cos[\mathrm{C}_1 s + \mathrm{C}_2] + \sin\mathrm{P}\cos[\mathrm{C}_1 s + \mathrm{C}_2] + [\frac{1}{\kappa}\sin\mathrm{P}\sin[\mathrm{C}_1 s + \mathrm{C}_2][\sin^2\mathrm{P}(1 - 2\cos^2[\mathrm{C}_1 s + \mathrm{C}_2]) \\
& - \frac{1}{\kappa}\cos\mathrm{P}[\frac{1}{\mathrm{C}_1}\sin\mathrm{P}\cos[\mathrm{C}_1 s + \mathrm{C}_2] - \cos\mathrm{P}\sin\mathrm{P}\sin[\mathrm{C}_1 s + \mathrm{C}_2]]],
\end{aligned}
$$

$$
\begin{aligned}
y_{\mathbf{TNB}}(s) \;=\; & \exp[\mathrm{W}[\cos\mathrm{P} + \frac{1}{\kappa}[\sin^2\mathrm{P}\sin^2[\mathrm{C}_1 s + \mathrm{C}_2] - \sin^2\mathrm{P}\cos^2[\mathrm{C}_1 s + \mathrm{C}_2] \\
& + \frac{1}{\kappa}\sin\mathrm{P}\cos[\mathrm{C}_1 s + \mathrm{C}_2][\frac{1}{\mathrm{C}_1}\sin\mathrm{P}\cos[\mathrm{C}_1 s + \mathrm{C}_2] - \cos\mathrm{P}\sin\mathrm{P}\sin[\mathrm{C}_1 s + \mathrm{C}_2] \\
& - \frac{1}{\kappa}\sin\mathrm{P}\sin[\mathrm{C}_1 s + \mathrm{C}_2][-\frac{1}{\mathrm{C}_1}\sin\mathrm{P}\sin[\mathrm{C}_1 s + \mathrm{C}_2] + \cos\mathrm{P}\sin\mathrm{P}\cos[\mathrm{C}_1 s + \mathrm{C}_2]]]] \\
& \mathrm{W}[\sin\mathrm{P}\sin[\mathrm{C}_1 s + \mathrm{C}_2] + \sin\mathrm{P}\sin[\mathrm{C}_1 s + \mathrm{C}_2] \\
& -[\frac{1}{\kappa}\sin\mathrm{P}\cos[\mathrm{C}_1 s + \mathrm{C}_2][\sin^2\mathrm{P}(1 - 2\cos^2[\mathrm{C}_1 s + \mathrm{C}_2]) \\
& - \frac{1}{\kappa}\cos\mathrm{P}[-\frac{1}{\mathrm{C}_1}\sin\mathrm{P}\sin[\mathrm{C}_1 s + \mathrm{C}_2] + \cos\mathrm{P}\sin\mathrm{P}\cos[\mathrm{C}_1 s + \mathrm{C}_2]]]],
\end{aligned}
$$

$$
\begin{aligned}
z_{\mathbf{TNB}}(s) \;=\; & \mathrm{W}[\cos\mathrm{P} + \frac{1}{\kappa}[\sin^2\mathrm{P}\sin^2[\mathrm{C}_1 s + \mathrm{C}_2] - \sin^2\mathrm{P}\cos^2[\mathrm{C}_1 s + \mathrm{C}_2] \\
& + \frac{1}{\kappa}\sin\mathrm{P}\cos[\mathrm{C}_1 s + \mathrm{C}_2][\frac{1}{\mathrm{C}_1}\sin\mathrm{P}\cos[\mathrm{C}_1 s + \mathrm{C}_2] - \cos\mathrm{P}\sin\mathrm{P}\sin[\mathrm{C}_1 s + \mathrm{C}_2] \\
& - \frac{1}{\kappa}\sin\mathrm{P}\sin[\mathrm{C}_1 s + \mathrm{C}_2][-\frac{1}{\mathrm{C}_1}\sin\mathrm{P}\sin[\mathrm{C}_1 s + \mathrm{C}_2] + \cos\mathrm{P}\sin\mathrm{P}\cos[\mathrm{C}_1 s + \mathrm{C}_2]]],
\end{aligned}
$$

where $\mathrm{C}_1, \mathrm{C}_2$ *are constants of integration and*

$$\mathrm{W} = \frac{1}{\sqrt{2\kappa^2 - 2\kappa\tau + 2\tau^2}}.$$

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