Algorithmic Structure of Smarandache-Lattice

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ABSTRACT: *In this paper, we introduced Smarandache-2-algebraic structure of Lattice. A Smarandache-2-algebraic structure on a set N means a weak algebraic structure S_1 on N such that there exist a proper subset M of N, which is embedded with a stronger algebraic structure S_2, stronger algebraic structure means satisfying more axioms, that is $S_1 << S_2$, by proper subset one can understand a subset different from the empty set, from the unit element if any, from the whole set. We define Smarandache-Lattice and construct its algorithms through orthomodular lattice ,residuated lattice,pseudocomplment lattice, arbitrary lattice and congruence and ideal lattice . For basic concept of near-ring we refer to Padilla Raul [21] and for smarandache algebraic structure we refer to Florentin Smarandache [8]*

1. Introduction

In order that, new notions are introduced in algebra to better study the congruence in number theory by Florentin smarandache [8]. By<proper subset> of a set A we consider a set P included in A, and different from A, different from empty set and from the unit element in A-if any they rank the algebraic structures using an order relationship:
They say that the algebraic structures $S_1<<S_2$ if : both are defined on the same set; all S_1 laws are also S_2 laws; all axioms of an S_1law are accomplished by the corresponding S_2 law; S_2 law accomplish strictly more axioms that S_1laws or S_2 has more laws than S_1.
For example: Semi group<< Monoid <<group<<ring<<field, or semi group<< commutative semi group, ring<<unitary ring etc. they define a general special structure to be a structure SM on a set A, different from a structure SN, such that a proper subset of A is a structure, where SM<<SN.In additionwe have published [13],[14],[15],[16].
Thecharacterization of Smarandache-lattice by the substructures of Lattice namely orthomodular lattice ,ideal lattice, pseudo complement lattice, Arbitrary lattice, Residuated lattice was studied. From that it is observed that orthomodular lattice of Boolean algebra are {0} and itself, ideals of Boolean algebra are {0} and itself, pseudo complement lattice of Boolean algebra are {0} and itself, arbitrary lattice of Boolean algebra are {0} and itself, residuated lattice of Boolean algebra are {0} and itself, The converse of the above are also true if non-zero substructures are considered .Then the Boolean algebra itself is orthomodular lattice ,ideals, pseudo complement lattice, Arbitrary lattice, Residuated lattice by this hypothesis, in this paper algorithms to construct the Smarandache lattice from its characterization obtained in paper[13],[14],[15],[16] are obtained.

2. Preliminaries

Definition: 2.1

A partially ordered set $(L,<)$ is said to form a **Lattice** if for every $a,b \in L$, Sup {a, b} and Inf {a, b} exist in L. In that case, we write Sup {a, b} = $a \vee b$, Inf{a, b} = $a \wedge b$ Other notations like a + b and a. b or $a \cup b$ and $a \cap b$ are also used for Sup {a, b} and Inf {a, b}.

Definition: 2.2 (Lattice as an Algebraic Structure)

A **lattice** as an algebraic structure is a set on which two binary operations are defined, called join and meet, denoted by ∨ and ∧, satisfying the following axioms (i) Commutative law (ii) Associative law (iii) Absorption law (iv) Idempotent law.

Definition: 2.3
A **Boolean algebra** consists of a set B, two binary operations $\wedge$ and $\vee$ (called meet and join respectively), a unary operation $'$ and two constants 0 and 1.These obey the following laws: (i) Commutative Laws (ii) Associative Laws (iii) Distributive Laws (iv) Identity Laws (v) Complement Laws (vi) Idempotent Laws (vii) Null Laws(viii) Absorption Laws (ix) DeMorgan's Laws(x) Involution Law.

Definition 2.4
The **Smarandache lattice** is defined to be a lattice S, such that a proper subset of S is a Boolean algebra with respect to with same induced operations. By proper subset we understand a set included in S, different from the empty set, from the unit element if any, and from S.

Definition 2.5 (Alternate definition for Smarandache lattice)
If there exists superset of a Boolean algebra is a Lattice with respect to the same induced operations, then that Boolean algebra is said to be Smarandache lattice.

Definition 2.6
A **Boolean algebra** is **a lattice** that contains a least element and a greatest element and that is both complemented and distributive

Definition 2.7
An ortho poset P is called **orthomodular** if for every pair $a, b \in P$ with $a < b$ there is $a, c \in P$ such that $c \perp a$ and $b = a \vee c$.We will write shortly P instead of $< P, \leq, \wedge, \vee, ' >$.For every $a, b \in P$ with $a \leq b$ let us denote $b - a = (b \wedge a') = (b' \vee a)' \in P$ According to the orthomodular law, $b = a \wedge (b - a) \in P$ $a, b \in P$ with $a \leq b$ and, moreover $a \perp (b - a)$.

Definition 2.8
The **orthomodular lattice** L is called **Boolean algebr**a if and if for every $a, b \in L$ the condition $a \wedge b = a \wedge b' = 0$ implies $a = 0$,

Definition 2.9
A **residuated lattice** is an algebra $< L \wedge, \vee \otimes, ' \rightarrow, 0, 1 >$ such that
(i) $< L \leq, \wedge, \vee, ' 0, 1 >$ is Lattice (the corresponding order will be denoted by $\leq$) with the least element 0 and the greatest element 1 (ii) $< L \wedge, \vee \otimes, ' \rightarrow, 0, 1 >$ is a commutative monoid (i.e. $\otimes$is commutative, associative, and $x \otimes x = 1$ holds) (iii) $x \otimes y \leq z$ if and only if $x \leq y \rightarrow z$ holds (adjointness condition).

Definition 2.10
A Boolean algebra is a **residuated lattice** which is both a Heyting algebra and an MV-algebra (relation to the usual axiomatization is $a \rightarrow b = \neg a \vee b$)

Definition 2.11
Let L be **a lattice** and $U \subseteq L$. U is said to be an **ideal** of L iff U is nonempty, $b \leq a \in U$ implies $B \in U$, and $a, b \in U$ implies $a \vee b \in U$.

Definition 2.12

The **Lattice ideal** L is called a **Boolean algebr**a if for all $0 \in I$, $a \in I \Rightarrow b \leq a$ then $b \in I$, $a \vee b \in I$

Definition 2.13

The arbitrary **Lattice ideal** L is called a **Boolean algebr**a if $L \cong \mathrm{e}\ (L)$

Definition 2.14

The **Pseudo complement Lattice** L is called a **Boolean algebr**a if $a \subseteq L$ and $b \subseteq L$ are such that $a\,\mathrm{I}\ a' = a$

3. Algorithms

In Gunder pliz[19] in section 1.60(d).The theorem by Gratzer and Fain is given the following conditions for a near ring $N \neq \{0\}$ are equivalent

1. $\mathrm{I}\ I \neq \{0\}, \{0\} \neq 1 \subseteq N$ 2. N contains a unique minimal ideal, contained in all other non-zero ideals.

Cosequently the following conditions for a lattice $N \neq \{0\}$ are equivalent

1. $\mathrm{I}\ I \neq \{0\}, \{0\} \neq 1 \subseteq N$ 2. N contains a unique minimal larttice ideal, contained in all other non-zero lattice ideals

Algorithms 3.1 (Orthomodular Lattice)

Step 1: Consider a non-empty set B
Step 2:
Verify that B is a Boolean algebra under meet and join
For, check the following conditions

1. (i) For all $a \in B$, $a \vee a = a \in B$
(ii) For all $a \in B$, $a \wedge a = a \in B$

2 (i) For all $a, b \in B$, $a \vee b = b \vee a \in B$,
(ii) For all $a, b \in B$, $a \wedge b = b \wedge a \in B$

3 (i) For all $a, b, c \in B$, $a \vee (b \vee c) = (a \vee b) \vee c \in B$
(ii) For all $a, b, c \in B$, $a \wedge (b \wedge c) = (a \wedge b) \wedge c \in B$

4 (i) For all $a, b \in B$, $a \vee (a \wedge b) = a \in B$
(ii) For all $a, b \in B$, $a \wedge (a \vee b) = a \in B$

5(i). For all $a, b, c \in B$, $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \in B$
(ii) For all $a, b, c \in B$, $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \in B$

6 (i) For all $a'' = a$

7 (i) For all $a \in B$, $a \vee a' = 1 \in B$,
(ii) For all $a \in B$, $a \wedge a' = 0 \in B$

8(i) For all $a \in B$, $a \vee 0 = a \in B$,

(ii) For all $a \in B$, $a \wedge 1 = a \in B$

9(i) For all $a \in B$, $a \vee 1 = 1 \in B$,

(ii) For all $a \in B$, $a \wedge 0 = 0 \in B$

10(i) $a,b \in B$, $\overline{(a \vee b)} = \bar{a} \wedge \bar{b} \in B$

(ii) $a,b \in B$, $\overline{(a \wedge b)} = \bar{a} \vee \bar{b} \in B$

If the above conditions are satisfied,
$(B, \wedge, \vee, ', 0, 1)$ is a Boolean algebra.

Step 3: Let $B = B_0$

Step 4: Let B_i , $i = 0, 1, 2 \ldots n$ be supersets of B_0

Step 5: Let $S = \bigcup_{i=1}^{n} B_i$

Step 6: Choose sets B_j 's from B_i 's subject to for all $a, b \in B$ such that
$a \wedge b = a \wedge b' \Rightarrow a = 0$,

Step 7: $\bigcap B_j = B_0 \neq \{0\} \subset S$

Step 8: If step (7) is a true, then we write S is a Smarandache-Lattice.

Algorithms 3.2 (Pseudo complemented lattice)

Step 1: Consider a non-empty set M
Step 2: Verify that B is a Boolean algebra under meet and join
For, check the following conditions

1. (i) For all $a \in B$, $a \vee a = a \in B$

(ii) For all $a \in B$, $a \wedge a = a \in B$

2 (i) For all $a,b \in B$, $a \vee b = b \vee a \in B$,

(ii) For all $a,b \in B$, $a \wedge b = b \wedge a \in B$

3. (i) For all $a,b,c \in B$, $a \vee (b \vee c) = (a \vee b) \vee c \in B$

(ii) For all $a,b,c \in B$, $a \wedge (b \wedge c) = (a \wedge b) \wedge c \in B$

4.(i) For all $a,b \in B$, $a \vee (a \wedge b) = a \in B$

(ii) For all $a,b \in B$, $a \wedge (a \vee b) = a \in B$

5(i). For all $a,b,c \in B$, $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \in B$

(ii) For all $a,b,c \in B$, $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \in B$

6(i) For all $a'' = a$

7(i) For all $a \in B$, $a \vee a' = 1 \in B$,

(ii) For all $a \in B$, $a \wedge a' = 0 \in B$

8(i) For all $a \in B$, $a \vee 0 = a \in B$,

(ii) For all $a \in B$, $a \wedge 1 = a \in B$

9(i) For al $a \in B$, $a \vee 1 = 1 \in B$,

(ii) For all $a \in B$, $a \wedge 0 = 0 \in B$

10(i) $a,b \in B$, $\overline{(a \vee b)} = \bar{a} \wedge \bar{b} \in B$

(ii) $a,b \in B$, $\overline{(a \wedge b)} = \bar{a} \vee \bar{b} \in B$

If the above conditions are satisfied,
$(B, \wedge, \vee, ', 0, 1)$ is a Boolean algebra.

Step 3: Let $B = B_0$

Step 4: Let B_i , $i = 0,1,2\ldots n$ be supersets of B_0

Step 5: Let $S = \bigcup_{i=1}^{n} B_i$

Step 6: Choose sets B_j's from B_i's subject to for all $a \subseteq L$ and $b \subseteq L$ are such that $a \cap a' = a$

Step 7: $\bigcap B_j = B_0 \neq \{0\} \subset S$

Step 8: If step (7) is a true, then we write S is a Smarandache-Lattice

Algorithmsn 3.3 (Residuated lattice**)**

Step 1: Consider a non-empty set M
Step 2: Verify that B is a Boolean algebra under meet and join
For, check the following conditions

1. (i) For all $a \in B$, $a \vee a = a \in B$

(ii) For all $a \in B$, $a \wedge a = a \in B$

2.(i) For all $a,b \in B$, $a \vee b = b \vee a \in B$,

(ii) For all $a,b \in B$, $a \wedge b = b \wedge a \in B$

3. (i) For all $a,b,c \in B$, $a \vee (b \vee c) = (a \vee b) \vee c \in B$

(ii) For all $a,b,c \in B$, $a \wedge (b \wedge c) = (a \wedge b) \wedge c \in B$

4.(i) For all $a,b \in B$, $a \vee (a \wedge b) = a \in B$

(ii) For all $a,b \in B$, $a \wedge (a \vee b) = a \in B$

5 (i). For all $a,b,c \in B$, $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \in B$

(ii) For all $a,b,c \in B$, $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \in B$

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6 (i) For all $a'' = a$

7 (i) For all $a \in B$, $a \vee a' = 1 \in B$,

(ii) For all $a \in B$, $a \wedge a' = 0 \in B$

8 (i) For all $a \in B$, $a \vee 0 = a \in B$,

(ii) For all $a \in B$, $a \wedge 1 = a \in B$

9 (i) For al $a \in B$, $a \vee 1 = 1 \in B$,

(ii) For all $a \in B$, $a \wedge 0 = 0 \in B$

10 (i) $a,b \in B$, $\overline{(a \vee b)} = \bar{a} \wedge \bar{b} \in B$

(ii) $a,b \in B$, $\overline{(a \wedge b)} = \bar{a} \vee \bar{b} \in B$

If the above conditions are satisfied, $(B, \wedge, \vee, ', 0, 1)$ is a Boolean algebra.

Step 3: Let $B = B_0$

Step 4: Let B_i , $i = 0,1,2 \ldots n$ be supersets of B_0

Step 5: Let $S = \bigcup_{i=1}^{n} B_i$

Step 6: Choose sets B_j 's from B_i 's subject to for all $a,b \in B$ such that $a \to b = \neg a \vee b$,

Step 7: I $B_j = B_0 \neq \{0\} \subset S$

Step 8: If step (7) is a true, then we write S is a Smarandache-Lattice

Algorithms 3.4 (Arbitrary lattice and Congruence's)

Step 1: Consider a non-empty set M

Step 2: Verify that B is a Boolean algebra under meet and join

For, check the following conditions

1. (i) For all $a \in B$, $a \vee a = a \in B$

(ii) For all $a \in B$, $a \wedge a = a \in B$

2.(i) For all $a,b \in B$, $a \vee b = b \vee a \in B$,

(ii) For all $a,b \in B$, $a \wedge b = b \wedge a \in B$

3. (i) For all $a,b,c \in B$, $a \vee (b \vee c) = (a \vee b) \vee c \in B$

(ii) For all $a,b,c \in B$, $a \wedge (b \wedge c) = (a \wedge b) \wedge c \in B$

4.(i) For all $a,b \in B$, $a \vee (a \wedge b) = a \in B$

(ii) For all $a,b \in B$, $a \wedge (a \vee b) = a \in B$

5(i). For all $a,b,c \in B$, $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \in B$

(ii) For all $a,b,c \in B$, $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \in B$

6(i) For all $a'' = a$

7(i) For all $a \in B$, $a \vee a' = 1 \in B$,

(ii) For all $a \in B$, $a \wedge a' = 0 \in B$

8(i) For all $a \in B$, $a \vee 0 = a \in B$,

(ii) For all $a \in B$, $a \wedge 1 = a \in B$

9(i) For al $a \in B$, $a \vee 1 = 1 \in B$,

(ii) For all $a \in B$, $a \wedge 0 = 0 \in B$

10(i) $a,b \in B$, $\overline{(a \vee b)} = \bar{a} \wedge \bar{b} \in B$

(ii) $a,b \in B$, $\overline{(a \wedge b)} = \bar{a} \vee \bar{b} \in B$

If the above conditions are satisfied,
$(B, \wedge, \vee, ', 0, 1)$ is a Boolean algebra.

Step 3: Let $B = B_0$

Step 4: Let B_i , $i = 0,1,2 \ldots n$ be supersets of B_0

Step 5: Let $S = \bigcup_{i=1}^{n} B_i$

Step 6: Choose sets B_j 's from B_i 's subject to for all $L \cong \text{e}\ (L)$ (Isomorphic to congruence of L)

Step 7: $\mathrm{I}\ B_j = B_0 \neq \{0\} \subset S$

Step 8: If step (7) is a true, then we write S is a Smarandache-Lattice

Algorithms 3.5 (Lattice Ideal)

Step 1: Consider a non-empty set M

Step 2: Verify that B is a Boolean algebra under meet and join

For, check the following conditions

1. (i) For all $a \in B$, $a \vee a = a \in B$

(ii) For all $a \in B$, $a \wedge a = a \in B$

2.(i) For all $a,b \in B$, $a \vee b = b \vee a \in B$,

(ii) For all $a,b \in B$, $a \wedge b = b \wedge a \in B$

3. (i) For all $a,b,c \in B$, $a \vee (b \vee c) = (a \vee b) \vee c \in B$

(ii) For all $a,b,c \in B$, $a \wedge (b \wedge c) = (a \wedge b) \wedge c \in B$

4.(i) For all $a,b \in B$, $a \vee (a \wedge b) = a \in B$

(ii) For all $a,b \in B,\ a \wedge (a \vee b) = a \in B$

5(i). For all $a,b,c \in B,\ a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \in B$

(ii) For all $a,b,c \in B,\ a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \in B$

6 (i) For all $a'' = a$

7 (i) For all $a \in B,\ a \vee a' = 1 \in B$,

(ii) For all $a \in B,\ a \wedge a' = 0 \in B$

8 (i) For all $a \in B,\ a \vee 0 = a \in B$,

(ii) For all $a \in B,\ a \wedge 1 = a \in B$

9 (i) For all $a \in B,\ a \vee 1 = 1 \in B$,

(ii) For all $a \in B,\ a \wedge 0 = 0 \in B$

10 (i) For all $a,b \in B,\ \overline{(a \vee b)} = \bar{a} \wedge \bar{b} \in B$

(ii) For all $a,b \in B,\ \overline{(a \wedge b)} = \bar{a} \vee \bar{b} \in B$

If the above conditions are satisfied,
$(B, \wedge, \vee, ', 0, 1)$ is a Boolean algebra.

Step 3: Let $B = B_0$

Step 4: Let $B_i\ ,\ i = 0,1,2....n$ be supersets of B_0

Step 5: Let $S = \bigcup_{i=1}^{n} B_i$

Step 6: Choose sets B_j's from B_i's subject to for all $0 \in I$ if $a \in I \Rightarrow b \le a$ then $b \in I$
$a \vee b \in I$

Step 7: $\bigcap B_j = B_0 \neq \{0\} \subset S$

Step 8: If step (7) is a true, then we write S is a Smarandache-Lattice

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