AN IMPROVEMENT ON THE SMARANDACHE DIVISIBILITY THEOREM

Maohua Le

Department of Mathematics, Zhanjiang Normal College
Zhanjiang, Guangdong, P.R.China.

Abstract. Let a, n be positive integers. In this paper we prove that $n \mid (a^n - a)[n/2]!$

For any positive integer a and n, Smarandache [3] proved that

(1) $$n \mid (a^n - a)(n-1)!.$$

The above division relation is the Smarandache divisibility theorem (see [1, Notions 126]). In this paper we give an improvement on (1) as follows:

Theorem. For any positive integers a and n, we have

(2) $$n \mid (a^n - a)[n/2]!,$$

where $[n/2]$ is the largest integer which does not exceed $n/2$.

Proof. The division relation (2) holds for $n \le 9$, we may assume that $n > 9$. By Fermat's theorem (see [2, Theorem 71]), if n is a prime, then we have

(3) $$n \mid (a^n - a),$$

for any a. We see from (3) that (2) is true.

If n is a composite number, then we have $n = pd$, where p,d are integers satisfying $p \ge q \ge 2$. Further, if $p \ne q$, then we have $n|p!$ It implies that $n|(n/q)!$ Since $q \ge 2$, we get

(4) $n \mid [n/2]!$

If $p = q$, Then $n = p^2$ and

(5) $n \mid (2p)!$

Since $n > 9$, we have $n \ge 4^2$, $p \ge 4$ and $2p \le n/2$. Hence, we see from (5) that (4) is also true in this case. The combination of (3) and (4), the theorem is proved.

References

1. Dumitrescu and Seleacu, Some Notions and Questions In Number Theory, Erhus Univ. Press, Glendale, 1994.
2. G.H.Hardy and E. M.Wright, An Introduction to the Theory of Numbers, Oxford Univ. Press, Oxforf, 1936.
3. F.Smarandache, Problemes avec et sans ... problemes!, Somipress, Fes, Morocco, 1983.