Journal of Mahani Mathematical Research
Shahid Bahonar University of Kerman
Print ISSN: 2251-7952 Online ISSN: 2645-4505

APPLICATION OF SUPERHYPERGRAPHS-BASED DOMINATION NUMBER IN REAL WORLD

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Article type: Research Article
(Received: 07 March 2023, Received in revised form 27 May 2023)
(Accepted: 09 June 2023, Published Online: 11 June 2023)

Abstract. The concept of (quasi) superhypergraphs as a generalization of graphs makes a relation between some sets of elements in detail and in general (in the form of parts to parts, parts to whole, and whole to whole elements of sets) and is very useful in the real world. This paper considers the novel concept of (quasi) superhypergraphs and introduces the notation of dominating set and domination number of (quasi) superhypergraphs. Especially, we have analyzed the domination number of uniform (quasi) superhypergraphs and computed their domination number on different cases. The flows (from right to left, from left to right, and two-sided) as maps play a main role in (quasi) superhypergraphs and it is proved that domination numbers of (quasi) superhypergraphs are dependent on the flows. We define the valued-star (quasi) superhypergraphs for the design of hypernetworks and compute their domination numbers. We have shown that the domination numbers of valued-star (quasi) superhypergraphs are distinct in different flow states. In final, we introduce some applications of dominating sets of (quasi) superhypergraphs in hypernetwork as computer networks and treatment networks with the optimal application.

Keywords: (Quasi) superhypergraph, Dominating set,Domination number, r-star quasi superhypergraph, Flow.
2020 MSC: 05C69, 05C21,05C65.

1. Introduction

The concept of connected dominating sets was first introduced by Sampathkumar and Walikar [33]. They have applications in wireless sensor networks, wireless ad hoc networks, and in connection with some broadcast problems. Complex networks (networks have been used to model a vast array of phenomena) theory has been used to study complex systems. A complex network is a graph (a graph is a mathematical representation of a network) with non-trivial topological properties that do not occur in a simple graph such as lattices but often occur in graphs modeling with real systems. Since then,

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DOI: 10.22103/jmmr.2023.21203.1415
Publisher: Shahid Bahonar University of Kerman
How to cite: M. Hamidi, M. Taghinezhad, *Application of superhypergraphs-based domination number in real world*, J. Mahani Math. Res. 2024; 13(1): 211-228.

the studies of complex networks are undertaken in many disciplines including mathematics, physics, computer science, biology, social science, economics, and chemistry. Complex network models (such as brain interconnectivity map, ecosystem and food chains, gene, protein, molecule, and virus networks, political relations and lobbies, companies and their financial relations, bank transactions, power lines, phone lines, roads, highways, airlines, railways, paper citations, emails, web, Internet, mobile, SMS and phone networks, online friendship, and social networks) have many application in the real world such as Internet [34], world wide web [26], wireless communication networks [31], electric power systems [32], social [25], urban systems [11] and ecology networks [13]. However, many real-life systems involve multiple kinds of objects, and can't be described by simple graphs. To provide complete information on these systems were extended the concept of evolving models of complex networks to hypernetworks. Due to its powerful modeling ability, the hypernetwork framework has attracted attention in a variety of application domains, team sports performance [28], bioinformatics [14], finance [22] and picture fuzzy environment [24]. Hypergraph theory has been introduced by Berge as a generalization of graph theory around 1960 [8] as a model of hypernetworks. Since sometimes graph theory gives very limited information about networks, we say that the main motivation of hypergraph theory is for covering graph defects in applications. Also, the notion of hypergraphs has been considered a basic tool to present the systems by the clustering and participating methods. Today, hypergraphs have important applications and are used in hypernetworks extended in the branches of real problems such as computers, wireless sensors, treatment, food, and machine learning. There are also some researches about hypergraphs and their applications in hypernetworks such as creating and computing graphs from hypergraphs [16], single-valued neutrosophic directed (hyper)graphs, and applications in networks [17], achievable single-valued neutrosophic graphs in wireless sensor networks [18], on derivable tree [19] and accessible single-valued neutrosophic graphs [20]. Recently, Smarandache extended hypergraphs to a new concept as n-superhypergraph and pathogenic n-superhypergraph which have several properties and are connected with the real-world [30]. Indeed n-superhypergraphs are a generalization of hypergraphs, with this the advantage that they can communicate between the hyperedges. Recently, Hamidi et. al, introduced the concept of quasi superhypergraphs as a special concept of n-superhypergraphs and showed that in hypergraph theory, any hypergraph can relate a set of elements, while without any details that it makes some conflicts, defects, and shortcomings in hypergraph theory [15]. Thus by introducing superhypergraph, they try to eliminate defects of graphs (sometimes graph theory gives very limited information about complex networks) and hypergraphs (although the hypergraphs are for covering graph defects in the applications but in hypergraphs, the relation between vertices can't be described in full details). They introduced the incidence matrix of superhypergraph and computed the characteristic polynomial for the incidence matrix of superhypergraph, so they obtained the spectrum of superhypergraphs. They computed the number of superedges of any given superhypergraphs and based on superedges and partitions of an underlying set of superhypergraph, we obtained the number of all superhypergraphs on any non-empty set. Today, the concepts of dominating set, domination number and laplacian spectrum of hypergraphs are important in applications, decison making based on fuzzy graphs and some researchers work in this regard such as domination in hypergraphs [4], transversals and domination in uniform hypergraphs [6], locating-dominating sets in hypergraphs [12], an upper bound for the k-power domination number in r-uniform hypergraphs [5], infectious power domination of hypergraphs [7], the signless Laplacian matrix of hypergraphs [9], domination in intersecting hypergraphs [10], domination numbers and noncover complexes of hypergraphs [23], on the laplacian spectrum of k-uniform hypergraphs [29], a probabilistic linguistic three-way decision method with regret theory via fuzzy c-means clustering algorithm [35], three-way behavioral decision making with hesitant fuzzy information systems [36] and a three-way decision methodology with regret theory via triangular fuzzy numbers in incomplete multi-scale decision information systems [37]. The combination of fuzzy theory and graph topics has progressed rapidly in the past years and especially has many applications. Considering the importance of the subject of domination number, we refer to the relation of theory of fuzzy graphs, fuzzy hypergraphs, and domination numbers, such as fuzzy hypergraphs and related eextensions [1], certain types of domination in m-polar fuzzy graphs [2], novel decision making method based on domination in m-polar fuzzy graphs [3] and double domination on intuitionistic fuzzy graphs [27].

Motivation, contribution and advantage: Modeling based on (hyper)graphs is a clustering or grouping of elements based on certain properties, in which the properties of its elements are checked in each cluster, and this check has nothing to do with the properties of other clusters. We need more complete modeling to be able to analyze the effect of the elements in the entire modeling and with other cluster elements at the same time. Our motivation in introducing domination numbers of superhypergraphs is the coverage of this problem in (hyper)graphs and therefore we define it in such a way that in each cluster and group, the elements are related to the elements of other clusters and groups to analyze the effect of each element in the hypernetwork on the whole hypernetwork. In fact, in this modeling, for each complex hypernetwork, we examine the relationship between all the components of the systems that make up the complex hypernetwork in detail and analyze the impact of these components in the entire complex (super)hypernetwork. The highest advantage of modeling the world's issues based on the domination number of superhypergraph regarding the modeling of the domination number (hyper) graph is in the complex hypernetworks based on the superhypergraph, and the relationship between the components is checked only in the components of all clusters based on the map of clusters. This advantage leads to obtaining, optimal results in the complex (hypernetwork, based on our mathematical methods and computations.

Regarding these points, we generalize the concepts of the dominating set and the domination number of graphs to the novel concepts of dominating set and domination number of quasi superhypergraph. The concept of quasi superhypergraphs as an extension of graphs has some application in real life and solves problems and defects in graphs and hypergraphs. The main motivation for the computation of domination numbers of quasi superhypergraphs is the mathematical optimization of real-life problems, so we try to extend the concept of domination number of graphs to domination number of quasi superhypergraph. Indeed, the domination number of quasi superhypergraphs considers the mathematical optimization of hypernetworks. We define the star quasi superhypergraphs as a generalization of servers and compute their domination numbers. In this regard, we modify some hypernetworks similar to treatment networks with optimal applications and computer networks with optimal applications.

2. Preliminaries

In this section, we recall some definitions and results, which we need in what follows.

Definition 2.1. [8] Let X be a finite set. A *hypergraph* on X is a pair $H = (X, \{E_i\}_{i=1}^{m})$ such that for all $1 \leq i \leq m, \emptyset \neq E_i \subseteq X$ and $\bigcup\limits_{i=1}^{m} E_i = X$. The elements $x_1, x_2, \ldots, x_n$ of X are called (*hyper*) *vertices*, and the sets $E_1, E_2, \ldots, E_m$ are called the *hyperedges* of the hypergraph H. In any given hypergraph, hyperedges can contain an element (*loop*) two elements (*edge*) or more than three elements. If for all $1 \leq k \leq m$ $|E_k| = 2$, the hypergraph becomes an ordinary (undirected) graph. In a hypergraph its incidence matrix is a matrix $M_H = (m_{ij})_{n\times m}$, with m columns representing the hyperedges $E_1, E_2, \ldots, E_m$ and n rows representing the (hyper)vertices $x_1, x_2, \ldots, x_n$, where for all $1 \leq i \leq n$ and for all $1 \leq j \leq m$, we have $m_{ij} = 1$ if $x_i \in E_j$ and $m_{ij} = 0$ if $x_i \notin E_j$.

Definition 2.2. [16] A hypergraph $H = (X, \{E_i\}_{i=1}^{m})$ is called a complete hypergraph, if for any $x, y \in X$ there is $1 \leq i \leq m$ such that $\{x, y\} \subseteq E_i$. A hypergraph $H = (X, \{E_i\}_{i=1}^{n})$ is called as a joint complete hypergraph, if $|X| = n$ for all $1 \leq i \leq n, |E_i| = i$ and $E_i \subseteq E_{i+1}$.

Definition 2.3. [15] Let X be a non-empty set. Then

(*i*) $H = (X, S = \{S_i\}_{i=1}^{k}, \Phi = \{\varphi_{i,j} : S_i \to S_j\}_{i,j})$ is called a *quasi superhypergraph*, if $\Phi \neq \emptyset$ and $X = \bigcup\limits_{i=1}^{k} S_i$, where $k \geq 2$, and for all

$1 \leq i \leq k, S_i \in P^*(X)$, is called a *supervertex* and for any $i \neq j$, the map $\varphi_{i,j} : S_i \to S_j$(say S_i links to S_j) is called a *superedge*,

(*ii*) the quasi superhypergraph $H = (X, S = \{S_i\}_{i=1}^k, \Phi = \{\varphi_{i,j}\}_{i,j})$ is called a *superhypergraph*, if for any $S_i \in P^*(X)$, there exists at least one $S_j \in P^*(X)$, such that S_i links to S_j (it is not necessary all supervertices be linked),

(*iii*) the quasi superhypergraph $H = (X, S = \{S_i\}_{i=1}^k, \Phi = \{\varphi_{i,j}\}_{i,j})$ is called a *trivial* superhypergraph, if $k = 1$ (S_1 can't link to itself).

We will denote a quasi superhypergraph $H = (X, S = \{S_i\}_{i=1}^k, \Phi = \{\varphi_{i,j}\}_{i,j})$ by $H = (X, \{S_i\}_{i=1}^k, \{\varphi_{i,j}\}_{i,j})$ or $H = (X, S, \Phi)$, for simplify. Let X be a non-empty set, $\mathcal{SH}(X) = \{H \mid H \text{ is a superhypergraph on } X\}$ and $\mathcal{SH}(n_1, \ldots, n_k) = \{(X, \{S_i\}_{i=1}^k, \{\varphi_{i,j}\}) \in \mathcal{SH} \mid |S_i| = n_i\}$.

Theorem 2.4. [15] *Let X be a non-empty set and $|X| = n$. If $\alpha = \{(n_1, \ldots, n_r) \mid \sum_{i=1}^{r} n_i = n, n_i, r \in \mathbb{N}\}$ and $m = |\{i \mid n_i = n_j\}|$, then* $|\mathcal{SH}(X)| = \sum_{\alpha} (\frac{1}{m!} \prod_{i=1}^{r} \binom{n - \sum_{j=1}^{i-1} n_j}{n_i} (\sum_{1 \leq i \neq j \leq r} n_i^{n_j}))$.

3. Dominating set of superhypergraphs

In this section, we introduce the concept of the dominating set and the domination number of (quasi) superhypergraphs and investigate the main properties of the domination number of some types of (quasi) superhypergraphs.

Definition 3.1. Let $H = (X, S = \{S_i\}_{i=1}^k, \Phi = \{\varphi_{i,j}\}_{i,j})$ be a quasi superhypergraph and $D \subset X$. Then

(*i*) a set D is called an *dominating set* in H, if for all $x \in X \setminus D$, there exist $\varphi_{k,s} \in \Phi$ such that $x \in Dom(\varphi_{k,s})$ and $D \cap Dom(\varphi_{k,s}) \neq \emptyset$,

(*ii*) the *domination number* of H is defined by $\gamma_t(H) = |D^{min}|$, where D^{min} is a dominating set with minimum size.

Example 3.2. *Let $X = \{x_i\}_{i=1}^7$. Then $H = (X, \{S_i\}_{i=1}^3, \{\varphi_{1,2}, \varphi_{2,3}, \varphi_{3,2}\})$ is a quasi superhypergraph in Figure 1, where $\varphi_{1,2} = \{(x_1, x_4), (x_2, x_4), (x_3, x_5)\}$, $\varphi_{2,3} = \{(x_4, x_6), (x_5, x_7)\}$ and $\varphi_{3,2} = \{(x_6, x_4), (x_7, x_5)\}$. Hence, $D_1 = \{x_4, x_7\}$ is not a dominating set in H, $D_2 = \{x_1, x_4, x_7\}$ is a dominating set in H, because of $D \cap Dom(\varphi_{1,2}) \neq \emptyset, D \cap Dom(\varphi_{2,3}) \neq \emptyset$ and $D \cap Dom(\varphi_{3,2}) \neq \emptyset$. One can see that $\gamma_t(H) = 3$. Also $D_3 = \{x_1, x_2, x_4, x_7\}$ and $D_4 = \{x_1, x_2, x_4, x_5, x_7\}$ are dominating sets in H.*

Let $H = (X, \{S_i\}_{i=1}^m, \{\varphi_{i,j}\}_{i,j})$ be a quasi superhypergraph and $k \in \mathbb{N}$. We will denote it by uniform quasi superhypergraph, if for all $1 \leq i \neq j \leq m, S_i \cap S_j = \emptyset$ and a uniform quasi superhypergraph is called a k-uniform, if for all $1 \leq i \neq j \leq m, |S_i| = |S_j| = k$.

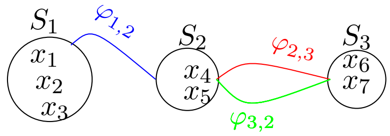

FIGURE 1. Quasi superhypergraph $H = (X, \{S_i\}_{i=1}^{3}, \{\varphi_{1,2}, \varphi_{2,3}, \varphi_{3,2}\})$

Theorem 3.3. *Let $H = (X, \{S_i\}_{i=1}^{k}, \{\varphi_{i,j}\}_{i,j})$ be a uniform superhypergraph. Then $\gamma_t(H) = k$.*

Proof. Since $H = (X, \{S_i\}_{i=1}^{k}, \{\varphi_{i,j}\}_{i,j})$ is a superhypergraph, for any $S_i \in P^*(X)$ there exists at least one $S_j \in P^*(X)$ such that S_i links to S_j. Suppose $D = \{x_1, x_2, \ldots, x_k\}$, where for all $1 \leq i \leq k, x_i \in S_i$. Hence for any given $x_i \in X \setminus D$ there exists a map $\varphi_{i,j} : S_i \to S_j$ such that $x_i \in Dom(\varphi_{i,j})$ and so $D \cap Dom(\varphi_{i,j}) \neq \emptyset$. Suppose that $D' \subset D$ is a dominating set. Thus there exists $x_i \in D \setminus D'$ and so $x_i \in ((X \setminus D') \cap S_i)$. Hence there exists $\varphi_{i,j}$ such that $x_i \in Dom(\varphi_{i,j}) \subseteq S_i$ and $x_i \notin D'$. It follows that $Dom(\varphi_{i,j}) \cap D' = \emptyset$, which is a contradiction. Thus D is a dominating set of H, therefore $\gamma_t(H) = k$. □

Theorem 3.4. *Let $H = (X, \{S_i\}_{i=1}^{2}, \{\varphi_{i,j}\}_{i,j}\})$ be a uniform quasi superhypergraph and $|S_1| \neq |S_2|$. Then $\gamma_t(H) = 1 + \max\{|S_1|, |S_2|\}$ or $\gamma_t(H) = 1 + \min\{|S_1|, |S_2|\}$.*

Proof. Let $|S_1| < |S_2|$. Since $H = (X, \{S_i\}_{i=1}^{2}, \{\varphi_{i,j}\}_{i,j})$ is a quasi superhypergraph, there exists $\varphi_{1,2}$ or $\varphi_{2,1}$ that $\Phi \cap \{\varphi_{1,2}, \varphi_{2,1}\} \neq \emptyset$. If $\varphi_{1,2} \in \Phi$, then $D = S_2 \cup \{x\}$, where $x \in S_1$. It condudes that $X \setminus D = S_1 \setminus \{x\}$ and so $\gamma_t(H) = 1 + max\{|S_1|, |S_2|\}$. If $\varphi_{2,1} \in \Phi$, then $D = S_1 \cup \{x\}$, where $x \in S_2$. It condudes that $X \setminus D = S_2 \setminus \{x\}$ and so $\gamma_t(H) = 1 + min\{|S_1|, |S_2|\}$. □

Corollary 3.5. *Let $H = (X, \{S_i\}_{i=1}^{2}, \{\varphi_{i,j}\}_{i,j}\})$ be a uniform quasi superhypergraph and $|S_1| = |S_2|$. Then $\gamma_t(H) = 1 + |S_1|$.*

Let $H = (X, S, \Phi)$ be a uniform quasi superhypergraph. Then denote $\mathcal{B} = \{x \in X \mid \exists\ \varphi \in \Phi$ such that $x \in Dom(\varphi)\}$ and $\mathcal{C} = \{S_i \in S \mid \exists\ \varphi \in \Phi$ such that $Dom(\varphi) \subseteq S_i\}$.

Theorem 3.6. *Let $H = (X, \{S_i\}_{i=1}^{m}, \{\varphi_{i,j}\}_{i,j}\})$ be a uniform quasi superhypergraph. If $X = \mathcal{B}$, then $\gamma_t(H) = |X| - |\mathcal{C}|$.*

Proof. Let $H = (X, \{S_i\}_{i=1}^{m}, \{\varphi_{i,j}\}_{i,j}\})$ be a uniform quasi superhypergraph and $|\mathcal{B}| = l$. We claim that $D^{min} = \mathcal{A} \cup \{x \mid x \notin \mathcal{B}, x \notin \mathcal{A}\}$, where $\mathcal{A} = \{s_1, s_2, \ldots, s_m \mid s_i \in S_i\}$. Let $x \in X \setminus D^{min}$. Then $x \in \mathcal{B}$, so there exists a map $\varphi \in \Phi$ such that $x \in Dom(\varphi)$. In addition, $Dom(\varphi) \cap D^{min} \neq \emptyset$, since $Dom(\varphi) \cap \mathcal{A} \neq \emptyset$ and so D^{min} is a dominating set. If there exists $D' \subseteq X$ such that it is a dominating set and $D' \subset D^{\min}$, then $X \setminus D^{\min} \subset X \setminus D'$ and so there exists $x_0 \in X \setminus D'$ such that $x_0 \notin X \setminus D^{\min}$, which is a contradication.. Thus there exsists $1 \leq i \leq m$ such that $x_0 \in S_i$, where either $S_i \in \mathcal{C}$ or $S_i \notin \mathcal{C}$. If $x_0 \in S_i$, where $S_i \in \mathcal{C}$, then $x_0 \in D^{min}$, since there exists $\varphi \in \Phi$ such that $x_0 \in Dom(\varphi) \cap D^{\min} \neq \emptyset$, because of $Dom(\varphi) \cap \mathcal{B}$. If $x_0 \in S_i$, where $S_i \notin \mathcal{C}$, then for all $\varphi \in \Phi, x_0 \notin Dom(\Phi)$, which is a contradication. Hence $D' = D^{\min}$ and so $\gamma_t(H) = |X| - |\mathcal{C}|$. □

Example 3.7. *(i) Let* $X = \{x_i\}_{i=1}^{8}$. *Then* $H = (X, \{S_i\}_{i=1}^{4}, \Phi)$ *be a quasi superhypergraph as shown in Figure 2, where* $\Phi = \{\varphi_{1,2}, \varphi_{1,4}, \varphi_{3,1}, \varphi_{3,2}, \varphi_{4,3}\}$. *Then* $\gamma_t(H) = 4$.

(ii) Let $X = \{x_i\}_{i=1}^{6}$. *Then* $H = (X, \{S_i\}_{i=1}^{3}, \Phi')$ *be a quasi superhypergraph as shown in Figure 3, where* $\Phi' = \{\varphi_{1,3}\}$. *It is easy to see that* $|\mathcal{C}| = 1$ *and* $\gamma_t(H) = |X| - |\mathcal{C}|$, *while* $|X| \neq |\mathcal{B}|$. *It shows that the converse of Theorem 3.6, is not necessarily true.*

Corollary 3.8. *Let* $H = (X, \{S_i\}_{i=1}^{m}, \{\varphi_{i,j}\}_{i,j}\})$ *be a uniform superhypergraph. Then* $\gamma_t(H) = m$.

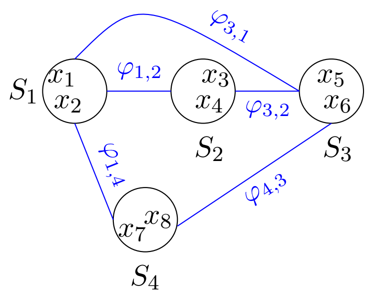

FIGURE 2. Quasi superhypergraph $H = (X, \{S_i\}_{i=1}^{4}, \Phi)$

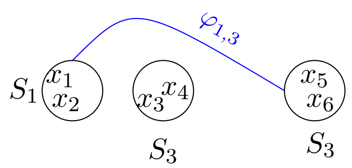

FIGURE 3. Quasi superhypergraph $H = (X, \{S_i\}_{i=1}^{3}, \Phi)$

Dominating set of star quasi superhypergraphs

In this subsection, we introduce the concept of r-star quasi superhypergraphs and compute the domination number of r-star quasi superhypergraphs.

Let $H = (X, \{S_i\}_{i=1}^2, \{\varphi_{i,j}\}_{i,j})$ be a quasi superhypergraph. If $\Phi(S_i, S_j) = \{\varphi_{i,j} \mid \varphi_{i,j} : S_i \to S_j, i, j \geq 1\}$, will say S_i *flows* to S_j and will denote by $S_i \rightsquigarrow S_j$.

Let $r \in \mathbb{N}$. Define an r-star superhypergraph $H = (X, S, \Phi)$, where $S = \{S_i^j \mid 1 \leq j \leq r, 1 \leq i \leq m_j\}$ and $\Phi = \{\varphi_{i,j} \mid 1 \leq j \leq r, 1 \leq i \leq m_j\}\}$(See Figure 4). If $r = 1$, $H = (X, S, \Phi)$ is denoted by a star superhypergraph.

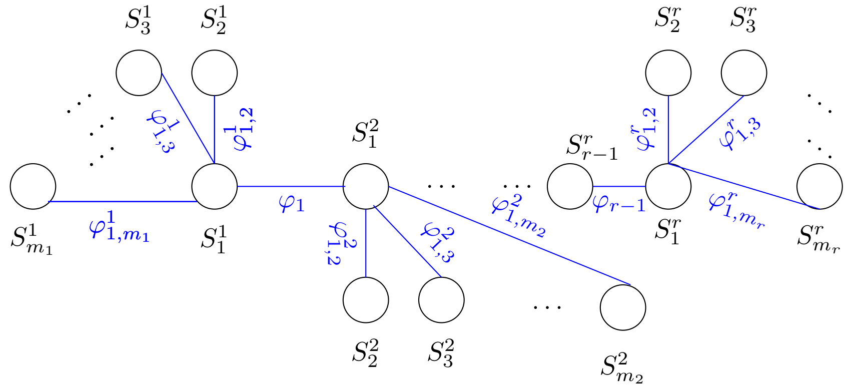

FIGURE 4. r-star superhypergraph $H = (X, \{S_i^j\}_{i,j}, \{\varphi_{i,j}\}_{i,j})$

Proposition 3.9. *Let $H = (X, \{S_\alpha, S_\beta\}, \Phi\})$ be a quasi superhypergraph, where S_α flows to S_β and $|S_\alpha| \geq 2$. Then $\gamma_t(H) = 1 + |S_\beta|$.*

Proof. Consider the quasi superhypergraph $H = (X, \{S_\alpha, S_\beta\}, \Phi\})$ that is shown in Figure 5. It is clear that $D := D(\alpha, \beta) = \{x_\alpha\} \cup S_\beta$, where $x_\alpha \in S_\alpha$. Thus $\gamma_t(H) = 1 + |S_\beta|$. □

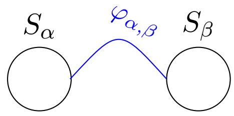

FIGURE 5. Quasi superhypergraph $H = (X, \{S_\alpha, S_\beta\}, \Phi\})$

Theorem 3.10. *Let $H = (X, \{S_i\}_{i=1}^m, \{\varphi_{i,j}\}_{i,j}\})$ be a star quasi superhypergraph, where for any $2 \leq i \leq m$, S_1 flows to S_i and $|S_1| \geq 2$. Then $\gamma_t(H) = 1 + \sum_{i=2}^{m} |S_i|$.*

Proof. Let $H = (X, \{S_i\}_{i=1}^m, \{\varphi_{i,j}\}_{i,j}\})$ be a star quasi superhypergraph. Claim that $D^{\min} = \{s\} \cup \bigcup_{i=2}^{m} S_i$, where $s \in S_1$. Since $|S_1| \geq 2$, we have $X \setminus D^{\min} =$

$S_1 \setminus \{s\} \neq \emptyset$ and for all $x \in X \setminus D^{\min}$, we get that there exists $2 \leq j \leq m$ such that $x \in D_{\varphi_{1,j}} \cap D^{\min} \neq \emptyset$, since for any $2 \leq i \leq m$, S_1 flows to S_i. Hence $D^{\min}$ is a dominating set of X. If there exists $D' \subseteq X$ such that it is a dominating set and $D' \subset D^{\min}$, then $X \setminus D^{\min} \subset X \setminus D'$ and so there exists $x_0 \in X \setminus D'$ such that $x_0 \notin X \setminus D^{\min}$. If $x_0 \in S_1$, it follows that there exists $2 \leq k \leq m$ such that $x_0 \in Dom(\varphi_{1,k}) \cap D' \neq \emptyset$ and $x_0 \in Dom(\varphi_{1,k}) \cap D = \emptyset$, which it is a contradication, because of $D' \subset D^{\min}$. If $x_0 \notin S_1$, it follows that there exists $2 \leq k \leq m$ such that $x_0 \in S_k$, so for any $2 \leq k \leq m$, $x_0 \notin Dom(\varphi_{1,k}) \cap D' \neq \emptyset$, which is a contradiction, because for any $2 \leq i \leq m$, S_1 flows to S_i. Thus in any cases, get a contradication. Hence $D^{\min} = \{s\} \cup \bigcup_{i=2}^{m} S_i$ and so $\gamma_t(H) = 1 + \sum_{i=2}^{m} |S_i|$. □

Theorem 3.11. *Let $H = (X, S, \Phi)$ be an r-star quasi superhypergraph, where for any $1 \leq j \leq r$ and $2 \leq i \leq m_j$, $S_1^j \rightsquigarrow S_i^j$ and $S_1^1 \rightsquigarrow S_1^2 \rightsquigarrow S_1^3 \rightsquigarrow \ldots \rightsquigarrow S_1^r$. If for any $1 \leq j \leq r, |S_1^j| \geq 2$, then*

(*i*) $\gamma_t(H) = r + \sum_{\substack{1 \leq j \leq r \\ 2 \leq i \leq m_j}} |S_i^j|$.

(*ii*) *If $H = (X, S, \Phi)$ is k-uniform ($k \in \mathbb{N}$), then* $\gamma_t(H) = r(1-k) + k(\sum_{i=1}^{r} m_i)$.

Proof. (*i*) Since for any $1 \leq j \leq r$ and $2 \leq i \leq m_j$, $S_1^j \rightsquigarrow S_i^j$ and $S_1^1 \rightsquigarrow S_1^2 \rightsquigarrow S_1^3 \rightsquigarrow \ldots \rightsquigarrow S_1^r$, by Theorem 3.10, for every $1 \leq j \leq r, |S_1^j| \geq 2$, imply that $|D_j| = 1 + \sum_{i=2}^{m_j} |S_i^j|$, where $D_j = \{s_j\} \bigcup_{i=2}^{m_j} S_i^j$ and $s_j \in S_1^j$. Hence $D = \bigcup_{j=1}^{r} D_j$ and so

$$\gamma_t(H) = (1 + \sum_{i=2}^{m_1} |S_i^1|) + (1 + \sum_{i=2}^{m_2} |S_i^2|) + \ldots + (1 + \sum_{i=2}^{m_r} |S_i^r|) = r + \sum_{j=1}^{r} \sum_{i=2}^{m_j} |S_i^j|.$$

In addition, $S_1^1 \rightsquigarrow S_1^2 \rightsquigarrow S_1^3 \rightsquigarrow \ldots \rightsquigarrow S_1^r$. If consider $D(i, i+1)$ the dominating set for $S_1^i \rightsquigarrow S_1^{i+1}$, by Proposition 3.9, we get that $D(i, i+1) = \{s_i\} \cup S_1^{i+1}$, where $s_i \in S_1^i$, and $1 \leq i \leq r$. It is clear that for any $1 \leq i \leq r$, $D(i, i+1) \subseteq D$ and so $D = \bigcup_{j=1}^{r} D_j$. Thus $\gamma_t(H) = r + \sum_{\substack{1 \leq j \leq r \\ 2 \leq i \leq m_j}} |S_i^j|$.

(*ii*) We consider the stes

$$C_1 = \{S_1^1, S_2^1, \ldots, S_{m_1}^1\}, C_2 = \{S_1^2, S_2^2, \ldots, S_{m_2}^2\}, \ldots, C_r = \{S_1^r, S_2^r, \ldots, S_{m_r}^r\},$$

because of for any $1 \leq j \leq r$ and $2 \leq i \leq m_j$
, $S_1^j \rightsquigarrow S_i^j$. In addition, for $r \in \mathbb{N}$, we have $S_1^1 \rightsquigarrow S_1^2 \rightsquigarrow S_1^3 \rightsquigarrow \ldots \rightsquigarrow S_1^r$.
Applying (i), we get that

$$\begin{aligned}\gamma_t(H) &= r + \sum_{\substack{1\leq j\leq r\\2\leq i\leq m_j}} |S_i^j| = r + km_1 + km_2 + \ldots + km_r \\ &- (|S_1^1| + |S_1^2| + |S_1^3| + \ldots + |S_1^r|) \\ &= r + \sum_{\substack{1\leq j\leq r\\2\leq i\leq m_j}} |S_i^j| = r + km_1 + km_2 + \ldots + km_r - kr \\ &= r(1-k) + k(\sum_{i=1}^{r} m_i),\end{aligned}$$

because of, for any $1 \leq j \leq r$ and $2 \leq i \leq m_j, |S_i^j| = k$. □

Theorem 3.12. *Let $H = (X, \{S_i\}_{i=1}^m, \{\varphi_{i,j}\}_{i,j})$ be a star quasi superhypergraph, where for any $2 \leq i \leq m$, S_i flows to S_1 and $|S_i| \geq 2$. Then $\gamma_t(H) = m + (|S_1| - 1)$.*

Proof. Let $H = (X, \{S_i\}_{i=1}^m, \{\varphi_{i,j}\}_{i,j})$ be a star quasi superhypergraph. Claim that $D^{\min} = \{s_2, s_3, \ldots,$
$s_m \mid s_i \in S_i\} \cup S_1$. Since for any $2 \leq i \leq m$, S_i, $|S_i| \geq 2$, we have $X \setminus D^{\min} = (\bigcup_{i=2}^{m} S_i) \setminus \{s_2, s_3, \ldots, s_m \mid s_i \in S_i\} \neq \emptyset$ and for all $x \in X \setminus D^{\min}$, we get that there exists $2 \leq j \leq m$ such that $x \in D_{\varphi_{j,1}} \cap D^{\min} \neq \emptyset$, since for any $2 \leq i \leq m$, S_i flows to S_1. Hence $D^{\min}$ is a dominating set of X. If there exists $D' \subseteq X$ such that it is a dominating set and $D' \subset D^{\min}$, then $X \setminus D^{\min} \subset X \setminus D'$ and so there exists $x_0 \in X \setminus D'$ such that $x_0 \notin X \setminus D^{\min}$. If $x_0 \in S_1$, it follows that there exists $2 \leq k \leq m$ such that $x_0 \in Dom(\varphi_{1,k}) \cap D' \neq \emptyset$, which it is a contradication, because for any $2 \leq i \leq m$, S_i flows to S_1. If $x_0 \notin S_1$, it follows that there exists $2 \leq k \leq m$ such that $x_0 \in S_k$, and so there exists $2 \leq k \leq m$, such that $x_0 \in Dom(\varphi_{k,1}) \cap D' \neq \emptyset$, which is a contradiction, because of $x_0 \notin D'$. Thus in any cases, get a contradication. Hence $\gamma_t(H) = m + (|S_1| - 1)$. □

Theorem 3.13. *Let $H = (X, S, \Phi)$ be an r-star quasi superhypergraph, where for any $1 \leq j \leq r$ and $2 \leq i \leq m_j$, $S_i^j \rightsquigarrow S_1^j$ and $S_1^r \rightsquigarrow S_1^{r-1} \rightsquigarrow \ldots \rightsquigarrow S_1^2 \rightsquigarrow S_1^1$. If for any $1 \leq j \leq r, |S_1^j| \geq 2$, then*

(i) $\gamma_t(H) = (|S_1| - 1) + \sum_{i=1}^{r} m_i$.

(ii) *If $H = (X, S, \Phi)$ is k-uniform ($k \in \mathbb{N}$), then* $\gamma_t(H) = (k-1) + \sum_{i=1}^{r} m_i$.

Proof. (i) Since for any $1 \leq j \leq r$ and $2 \leq i \leq m_j$, $S_i^j \rightsquigarrow S_1^j$ and $S_1^r \rightsquigarrow S_1^{r-1} \rightsquigarrow \ldots \rightsquigarrow S_1^2 \rightsquigarrow S_1^1$, by Theorem 3.12, for every $1 \leq j \leq r, |S_1^j| \geq 2$, imply that $|D_j| = m_j + (|S_j| - 1)$, where $D_j = (S_i^j) \cup \{s_2^j, s_3^j, \ldots, s_{m_j}^j \mid s_i^j \in S_i^j\}$. Hence $D = \bigcup_{j=1}^{r} D_j = S_1^1 \cup \{s_i^j \mid s_i^j \in S_i^j, i \neq 1\}$. Thus

$$\gamma_t(H) = |S_1^1|) + (m_1 - 1 + m_2 + \ldots + m_1) = (|S_1| - 1) + \sum_{i=1}^{r} m_i.$$

In addition, $S_1^r \rightsquigarrow S_1^{r-1} \rightsquigarrow \ldots \rightsquigarrow S_1^2 \rightsquigarrow S_1^1$. If consider $D(i+1, i)$ the dominating set for $S_1^{i+1} \rightsquigarrow S_1^i$, by Proposition 3.9, we get that $D(i+1, i) = \{s_{i+1}\} \cup S_1^i$, where $s_{i+1} \in S_{i+1}$ and $1 \leq i \leq r$. It is clear that for any $1 \leq i \leq r$, $D(i+1, i) \subseteq D$ and so $D = \bigcup_{j=1}^{r} D_j = S_1^1 \cup \{s_i^j \mid s_i^j \in S_i^j, i \neq 1\}$. Thus $\gamma_t(H) = r + \sum_{\substack{1 \leq j \leq r \\ 2 \leq i \leq m_j}} |S_i^j|$.

(ii) Applying (i), we get that

because of, for any $1 \leq j \leq r$ and $2 \leq i \leq m_j, |S_i^j| = k$. □

Example 3.14. *Let* $X = \{x_i\}_{i=1}^{14}$. *Then* $H = (X, \{S_i\}_{i=1}^{7}, \{\varphi_{2,1}, \varphi_{3,1}, \varphi_{4,1}, \varphi_{5,4}, \varphi_{6,4}, \varphi_{7,4}\})$ *is a 2-quasi superhypergraph in Figure 6, where*

$$\varphi_{2,1} = \{(x_3, x_1), (x_4, x_2)\}, \varphi_{3,1} = \{(x_5, x_1), (x_6, x_2)\},$$
$$\varphi_{4,1} = \{(x_7, x_1), (x_8, x_2)\}, \varphi_{5,4} = \{(x_9, x_7), (x_{10}, x_8)\},$$
$$\varphi_{6,4} = \{(x_{11}, x_7), (x_{12}, x_8)\}, \varphi_{7,4} = \{(x_{13}, x_7), (x_{14}, x_8).$$

Hence, $D = \{x_1, x_2, x_3, x_5, x_7, x_9, x_{11}, x_{13}\}$ *is a dominating set in* H *and so* $\gamma_t(H) = 8$.

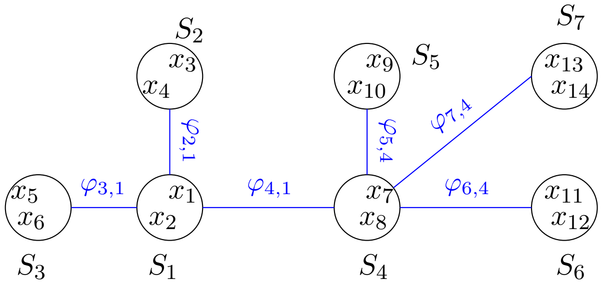

FIGURE 6. 2-star quasi superhypergraph $H = (X, \{S_i\}_{i=1}^{7}, \{\varphi_{2,1}, \varphi_{3,1}, \varphi_{4,1}, \varphi_{5,4}, \varphi_{6,4}, \varphi_{7,4}\})$

Theorem 3.15. *Let $H = (X, \{S_i\}_{i=1}^m, \{\varphi_{i,j}\}_{i,j})$ be a star quasi superhypergraph, where for any $2 \leq i \leq m$, S_i flows to S_1 and S_1 flows to S_i. Then $\gamma_t(H) = m$.*

Proof. Let $H = (X, \{S_i\}_{i=1}^m, \{\varphi_{i,j}\}_{i,j})$ be a star quasi superhypergraph. We claim that $D^{\min} = \{s_1, s_2, \ldots, s_m \mid s_i \in S_i\}$, is a dominating set. Since for all $2 \leq i \leq m, S_1$ flows to S_i and S_i flows to S_1, we get that for all $x \in X \setminus D^{\min}$, there exist $\varphi_{1,i} : S_1 \to S_i$ and $\varphi_{i,1} : S_i \to S_1$ such that $x \in Dom(\varphi_{1,i}) \cap Dom(\varphi_{i,1}) \cap D^{\min} \neq \emptyset$. It follows that $D^{\min}$ is a dominating set. If there exists $D' \subseteq X$ such that it is a dominating set and $D' \subset D^{\min}$, then $X \setminus D^{\min} \subset X \setminus D'$ and so there exists $x_0 \in X \setminus D'$ such that $x_0 \notin X \setminus D^{\min}$. Because of D' is a dominating set in X and $x_0 \in X \setminus D'$ we get that $x \in Dom(\varphi_{i,1}) \cap Dom(\varphi_{1,j})$ but $(\varphi_{i,1}) \cap D' = (\varphi_{1,j}) \cap D' = \emptyset$, while it is a contradiction. Thus D^{min} is the smallest dominating set in X and so $\gamma_t(H) = m$. □

Theorem 3.16. *Let $r \in \mathbb{N}$ and $H = (X, S, \Phi)$ be an r-star quasi superhypergraph, where for any $1 \leq j \leq r$ and $2 \leq i \leq m_j$, $S_i^j \leftrightsquigarrow S_1^j$ and $S_1^r \leftrightsquigarrow S_1^{r-1} \leftrightsquigarrow \ldots \leftrightsquigarrow S_1^2 \leftrightsquigarrow S_1^1$. If for any $1 \leq j \leq r, |S_1^j| \geq 2$. Then $\gamma_t(H) = r$.*

Proof. Let $r \in \mathbb{N}$ and $H = (X, S, \Phi)$ be an r-star quasi superhypergraph. We claim that $D = \{s_{i_1}^1, s_{i_2}^2, \ldots, s_{i_r}^r \mid 1 \leq j \leq r, 1 \leq i_j \leq m_j, s_{i_j}^j \in S_i^j\}$. Is a dominating set. The proof similar to Theorem 3.13, and is obtained from Theorem 3.15. □

4. Application of Domination Number Based on Superhypergraph

In this section, we consider the concepts of domination number of quasi superhypergraphs and introduce some of their applications in real-life related to these concepts, especially in hypernetwork.

Computer Network in Optimal Conditions:

Let $\mathcal{C} = \{pc_1, pc_2, pc_3, \ldots pc_{10}\}$ be a set of computers set in some different department of a university such that $S_1 = \{pc_1, pc_2, pc_3\}, S_2 = \{pc_4, pc_5\}, S_3 = \{pc_6, pc_7\}$ and $S_3 = \{pc_8 pc_9, pc_{10}\}$ are computers site of different department, respectively. We know that the pc_2 in S_1, pc_4 in S_2, pc_6 in S_3, and pc_9 in S_4 have the server role and are the best computers. We want to monitor the functions of each of the computers by one or a small number of computers in such a way that every one of these computers can control the weak computers. Now, we design the modify a superhypergraph $H = (X, \{S_i\}_{i=1}^4, \{\varphi_{1,3}, \varphi_{4,2}, \varphi_{2,3}, \varphi_{3,2}\})$ in Figure 7 as a computer network, where $\varphi_{13} = \{(2, 7), (1, 7), (3, 6)\}, \varphi_{13} = \{(9, 5), (8, 5), (10, 4)\}, \varphi_{23} = \{(4, 7), (5, 6)\}$ and $\varphi_{32} = \{(7, 4), (6, 5)\}$. Clearly $D = \{2, 9, 4, 6\}$ is a dominating set and so $\gamma_t(H) = 4$. It follows that a minimum number of computers is required to completely monitor the communication network is 4 and must we protect the computer sets pc_2, pc_4, pc_6 and pc_9 in more details.

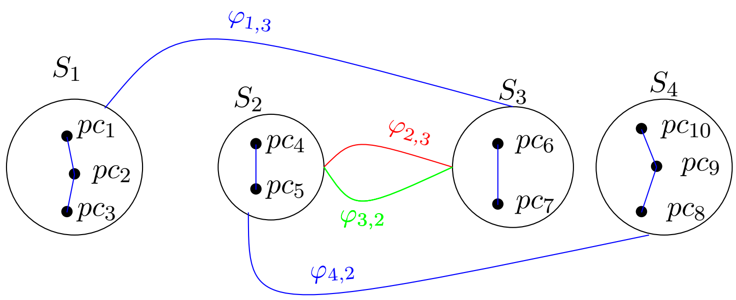

FIGURE 7. Computer network in opitomal conditions

Treatment Network With Optimal Application:

Let $T = \{T_1, T_2, T_3, T_4\}$ be a set of clinics in a town as named by T, where $T_1 = \{d_1 := Doctor - 1, d_2 := Doctor - 2, d_3 := Doctor - 3, n_1 := Nurse - 1, e := Equipmene\}, T_2 = \{d'_1 := Doctor - 1, n'_1 := Nurse - 1, e' := Equipmene\}, T_3 = \{n''_1 := Nurse - 1, e'' := Equipmene\}$ and $T_4 = \{d'''_1 := Doctor - 1, n'''_1 := Nurse - 1, e'''_1 := Equipmene, e'''_2 := Equipmene\}$. Researches show that the above clinics are different in services and facilities such that in $T_1, e :=$ *equipmene* is very strong, $T_2, d'_1 := Doctor - 1$ has good skills, $T_3, n''_1 := Nurse - 1$, is with commitment and $T_4, d'''_1 := Doctor - 1, e'''_2 := Equipmene$ are in satisfactory. We want to make an ideal network in this town such that people should benefit from good medical facilities and prevent much death. We modify this treatment network such that reaches an optimal state in terms of the time of the patient's arrival at the clinic and the use of excellent medical services. In this regard we design a network as a quasi hypergraph $H = (T, \{T_i\}_{i=1}^{4}, \{\varphi_{41}, \varphi_{43}, \varphi_{34}, \varphi_{24}, \varphi_{21}, \varphi_{23}\})$ in Figure 8, where

$$\varphi_{41} = \{(d'''_1, d_1), (e'''_1, e), (e'''_2, d_2), (n''_1, n_1)\},$$
$$\varphi_{43} = \{(n'''_1, n_1), (d'''_1, n''_1), (e'''_2, e''), (e'''_1, e'')\},$$
$$\varphi_{34} = \{(e'', e'''_2), (e'''_1, e), (n''_1, n'''_1)\}, \varphi_{24} = \{(d'_1, n'''_1), (n'_1, n'''_1), (e', e'''_2)\},$$
$$\varphi_{21} = \{(d'_1, d_1), (e', e), (n'_1, n_1) \text{ and } \varphi_{23} = \{(d'_1, n''_1), (n'_1, n''_1), , (e', e'')\}\}.$$

By above computations $D = \{d'''_1, e, n''_1, d'_1\}$ is a dominating set and so $\gamma_t(H) = 4$. It means that in this town, for the optimal and balanced use of medical services, it is necessary to apply the dominating set D for this model.

5. Discussion

In graph theory, the study of domination and related subset problems, such as matching, independence, or covering has had significant growth. In particular, the problem of finding dominating sets in graphs started in 1960 [21], but it was in 1962 when the concept of domination number of a graph was defined.

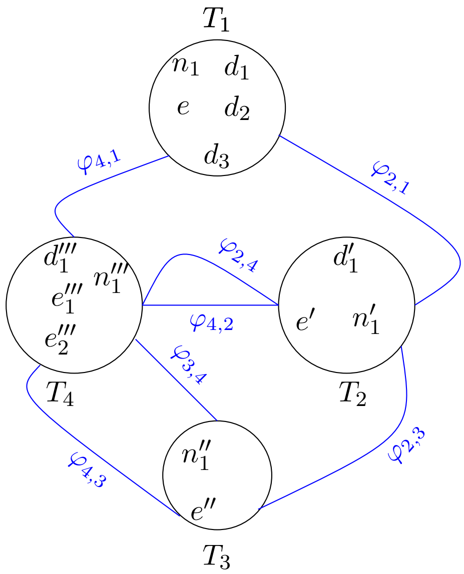

FIGURE 8. Treatment network with optimal application

The application of domination in graphs lies in various fields for solving real-life problems, therefore, it is of particular importance. The outcomes of this research provide insight into the optimization of existing hypernetwork in the real world. We introduce the concept of the dominating set in superhypergraphs as a generalization of the dominating set of hypergraphs and graphs. Considert the graph $G = (X = \{a, b, c, d\}, E = \{\{a, b\}, \{a, d\}, \{c, b\}, \{c, d\}, \{e, d\}\})$ in Figure 9. We try to describe the graph $G = (\{a, b, c, d\}, E)$ based on the maps $\Phi = \{\varphi_{\{a,b\}}, \varphi_{\{a,d\}}, \varphi_{\{c,b\}}, \varphi_{\{c,d\}}, \varphi_{\{d,e\}}\}$, which for any $\alpha, \beta \in X, \Phi_{\{\alpha,\beta\}} : \{\alpha, \beta\} \to \{\alpha, \beta\}$ is defined by $\Phi_{\{\alpha,\beta\}}(\alpha) = \beta$ and $\Phi_{\{\alpha,\beta\}}(\beta) = \alpha$. Consider $D = \{b, d\}$, based on the Definition 3.1, one can see that D is a dominating set in G. Thus it is easy to see that a superhypergraph is a generalization of a graph and the concept of dominating set in superhypergraphs is a generalization of the concept of the dominating set in graphs. But in graph theory, a graph can relate two vertices, therefore, it can't deal with group relation of more than two elements. In addition, in the design of the connection of several groups of elements, apart from the discussion of the connection of all the elements, the discussion of the analysis of the relation between part and part, whole and part, and part and whole is also important, and this analysis can't be done in graph theory. As an outcome, it is a difficult task to calculate the domination number due to the limitations mentioned in graph theory. All the limitations mentioned in graph theory motivated us to generalize graphs to superhypergraphs and lead the concept of domination numbers in superhypergraph. Therefore, solving the above-mentioned defects is one of the advantages of this study, which is mainly developed based on a superhypergraph. Of course, our study has some shortcomings and limitations that can be mentioned as follows:

(i) The optimal design of real problems under the hypernetworks based on the superhypergraphs is a very time-consuming task because there are no special theorems for choosing the suitable flows in this study and it takes a lot of work.

(ii) This study can't analyze the optimal problems in complex hypernetworks, so it must introduce the complex superhypergraphs in the future.

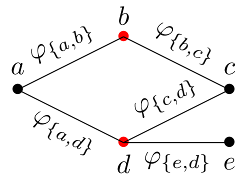

FIGURE 9. Graph $G = (X, E)$

6. **Conclusion**

The current paper has defined and considered the notion of domination numbers of quasi superhypergraphs. This study tries to prove that the maps in the concept of quasi-superhypergraphs are fundamental and play a main role in the domination numbers of quasi superhypergraphs. The main motivation of this work is to apply domination numbers based on quasi superhypergraphs in the real world and it is a generalization of the application of domination numbers of graphs in the real world. Indeed, all results of domination numbers of graphs can be extended to domination numbers of quasi superhypergraphs both theoretically and practically. The merits of the proposed method are to fix the defects of domination numbers in graph theory. Indeed, domination number in graph theory investigate the optimal case for limited elements, while domination number in quasi superhypergraphs considers the optimal case for the set of elements or object(object can be set).

Also, we:

(i) introduced the notation of uniform superhypergraph and computed its domination number,

(ii) proved that the domination number of uniform superhypergraphs is equal to their supervertices,

(iii) introduced the notation of r-star superhypergraphs and investigated their properties,

(iv) computed the domination number of r-star superhypergraphs, especialy for uniform r-star superhypergraphs,

(v) showed that the domination number of r-star superhypergraphs is dependend to flows,
(vi) presented some of the applications of superhypergraphs in real-life, especially in hypernetwork.

We hope that these results are helpful for further studies in zero divisor quasi superhypergraphs graphs via algebra structure and hyperstructures and fuzzy quasi superhypergraphs. In our future studies, we hope to obtain more results regarding the comparison of the method with some existing methods, prove the effectiveness of the method, fundamental relation on quasi superhypergraphs, fuzzy zero divisor quasi superhypergraphs, and their applications in other research.

7. Data Availability Statement

The data used to support the findings of this study are included within this article and can be obtained from the corresponding author upon request for more details on the data.

8. Aknowledgement

We would like to thank the reviewers for their thoughtful comments and efforts towards improving our manuscript.

9. Funding

This research received no external funding.

10. Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] M. Akram, and A. Luqman, *Fuzzy Hypergraphs and Related Extensions*, Studies in Fuzziness and Soft Computing DOI: 10.1007/978-981-15-2403-5 vol. 390 (2020).

[2] M. Akram, N. Waseem, and B. Davvaz, *Certain Types of Domination in m-polar Fuzzy Graphs*, Journal of Multiple-Valued Logic and Soft Computing vol. 29, no. 5 (2017) 619–646.

[3] M. Akram, N. Waseem, *Novel decision making method based on domination in m-polar fuzzy graphs*, Communications of the Korean Mathematical Society vol. 32, no. 4 (2017) 1077-1097.

[4] B. D. Acharya, *Domination in Hypergraphs*, AKCE International Journal of Graphs and Combinatorics vol. 4, no. 2 (2007) 117–126.

[5] J. S. Alameda, F. Kenter, K. Meagher, M. Young, *An upper bound for the k-power domination number in r-uniform hypergraphs*, Discrete Mathematics vol. 345, no. 11 (2022)113038 1-9.

[6] C. Bujtas, M. A. Henning, Z. Tuz, *Transversals and domination in uniform hypergraphs*, European Journal of Combinatorics vol. 33, no. 1 (2012) 62-71.

[7] B. Bjorkman, *Infectious power domination of hypergraphs*, Discrete Mathematics vol. 343, no.3 (2020) 111724 1-12.

[8] C. Berge, *Graphs and Hypergraphs*, North Holland 1979.
[9] K. Cardoso and V. Trevisan, *The signless Laplacian matrix of hypergraphs*, Special Matrices vol. 10 (2022) 327-342.
[10] Y. Dong, E. Shana, L. Kanga, S. Li, *Domination in intersecting hypergraphs*, Discrete Applied Mathematics vol. 251, no. 31 (2018) 155–159.
[11] M. W. Y. Dong, *A Critical Analysis on Complex Urban Systems and Complex Systems Theory*, Journal of Computing and Information Science in Engineering vol. 3, no. 1 (2023) 24–34.
[12] M. Fazil, I. Javaid, M. Salman & U. Ali, *Locating-dominating sets in hypergraphs*, Periodica Mathematica Hungarica vol. 72 (2016) 224–234.
[13] M. Goberna and M. Verdu, *Cautionary notes on the use of co-occurrence networks in soil ecology*, Soil Biology & Biochemistry vol. 166 (2022) 108534 1-10.
[14] T. Gaudelet, N. M. Dognin and N. Przulj, *Higher-order molecular organization as a source of biological function*, Journal of Bioinformatics and Computational Biology vol. 34 (2018) i944–i953.
[15] M. Hamidi, F. Smarandache, and E. Davneshvar, *Spectrum of Superhypergraphs via Flows*, Journal of Mathematics vol. 2022 (2022) 1-12.
[16] M. Hamidi and A. Borumand Saeid, *Creating and Computing Graphs from Hypergraphs*, Kragujevac Journal of Mathematics vol. 43, no. 1 (2019) 139–164.
[17] M. Hamidi and F. Smarandache, *Single-Valued Neutrosophic Directed (Hyper)Graphs And Applications in Networks*, Journal of Intelligent & Fuzzy Systems vol. 37, no. 2 (2019) 2869–2885.
[18] M. Hamidi and A. Borumand Saeid, *Achievable Single-Valued Neutrosophic Graphs in Wireless Sensor Networks*, New Mathematics and Natural Computation vol. 14, no. 2 (2018) 157–185.
[19] M. Hamidi, A. Borumand Saeid, *On Derivable Tree*, Transactions on Combinatorics, vol. 8, no. 2 (2019) 21–43.
[20] M. Hamidi, A. Borumand saied, *Accessible single-valued neutrosophic graphs*, Journal of Applied Mathematics and Computing vol. 57 (2018) 121–146.
[21] C. F. De. Jaenisch, *Applications de Lanaluse Mathematique an Jen des Echecs*, Petrograd, 1862.
[22] A. Jing, *The technology and digital financial risk management model using intelligent data processing*, Journal of Optics vol. 273 (2023)170410 1-13.
[23] J. Kim, M. Kim, *Domination numbers and noncover complexes of hypergraphs*, Journal of Combinatorial Theory, Series A vol. 180 (2021)105408 1-27.
[24] A. Luqman, M. Akram, and A. N. A. Koam, *Granulation of Hypernetwork Models under the q-Rung Picture Fuzzy Environment*, Journal of Mathematics vol. 7, no. 6 (2019)496 1–25.
[25] C. Y. Lee, H. Y. Chong, P. C. Liao X. Wang, *Critical Review of Social Network Analysis Applications in Complex Project Management*, Journal of Management in Engineering vol. 34, no. 2 04017061-2– 04017061-15.
[26] A. S. d. Mata, *Complex Networks: a Mini-review*, Brazilian Journal of Physics vol. 50 (2020) 658–672.
[27] A. Nagoorgani, Mu. Akram and S. Anupriya, *Double domination on intuitionistic fuzzy graphs* , Journal of Computational and Applied Mathematics vol. 52, no. 1-2 (2016) 515-528.
[28] J. Ribeiro, K. Davids, D. Araujo, P. Silva, J. Ramos, R.Lopes and J. Garganta, *The Role of Hypernetworks as a Multilevel Methodology for Modelling and Understanding Dynamics of Team Sports Performance*, The American Journal of Sports Medicine vol. 49 (2019) 1337–1344.
[29] S . S. Sahak, S. K. Panda, *On the Laplacian spectrum of k-uniform hypergraphs*, Linear Algebra and its Applications vol. 655, no. 15 (2022) 1-27.

[30] F. Smarandache, *Extension of Hypergraph to n-Super hypergraph and to Plithogenic n-Super hypergraph, and Extension of Hyperalgebra to n-ary (Classical-/Neutro-/Anti-)Hyperalgebra*, Neutrosophic Sets and Systems vol. 33 (2020) 290-296.
[31] C. Sergiou, M. Lestas, P. Antonious, C. Liaskos and A. Pitsillides, *Complex Systems: A Communication Networks Perspective Towards 6G*, IEEE Access vol. 8 (2020) 89007-89030.
[32] M. Saleh, Y. Esa and A. Mohamed, *Applications of Complex Network Analysis in Electric Power Systems*, Energy vol. 11, no. 6 1381 (2018) 1–16.
[33] E. Sampathkumar and Hanumappa B Walikar, *The connected domination number of a graph*, Journal of Mathematical Physics vol. 13, no. 6 (1979) 607-613.
[34] L. Trajkovic, *Analysis of Internet topologies*, IEEE Circuits and Systems Magazine vol. 10, no. 3 (2010) 48–54.
[35] J. Zhu, X. Ma, L. Martınez and J. Zhan, *A probabilistic linguistic three-way decision method with regret theory via fuzzy c-means clustering algorithm*, IEEE Transactions on Fuzzy Systems DOI: 10.1109/Tfuzz.2023.3236386 (2023).
[36] J. Zhan, J. Wang, W.Ding, and Y. Yao, *Three-way behavioral decision making with hesitant fuzzy information systems: survey and challenges*, IEEE/CAA Journal of Automatica Sinica vol. 2 (2023) 330-350.
[37] J. Zhan, J. Deng, Z. Xu, L. Martinez, *A three-way decision methodology with regret theory via triangular fuzzy numbers in incomplete multi-scale decision information systems*, IEEE Transactions on Fuzzy Systems DOI: 10.1109/Tfuzz.2023.3237646 (2023).

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